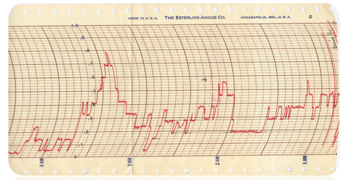
e+e- → **photon** : first charging in a Lab
(Frascati 21st February 1961)



A democratic resummation procedure for infrared gluons with an application to survival probabilities

Giulia Pancheri

INFN Frascati

+ D. A. Fagundes, A. Grau, O. Shekhovtsova and Y.N.Srivastava

ECT*-LFC17

The problem

- How to resum QCD effects to all orders ?
- Region of interest in resumming soft gluons is
  ~ zero momentum
- As $k_t \to 0$ single gluons disappear → $k^2 \sim 0$ → virtual and real gluons mix →
  finite order gives way to integration over k_t
- What matters now is the **integral** which needs to **be finite**

$$\int d^3\bar{n}_{soft-gluon}(k_t)$$

Our approach

see G.P.&Y.N.Srivastava, **Eur.Phys.J. C77 (2017) no.3, 150**

- We study the **infrared limit** through
  - A theoretical limit on the large distance behaviour

    of
  - an experimentally well defined observable

- The Froissart bound on the scattering amplitude

$$L_{max} = kb_{max}$$

$$a_\ell \lesssim s^{\in}$$

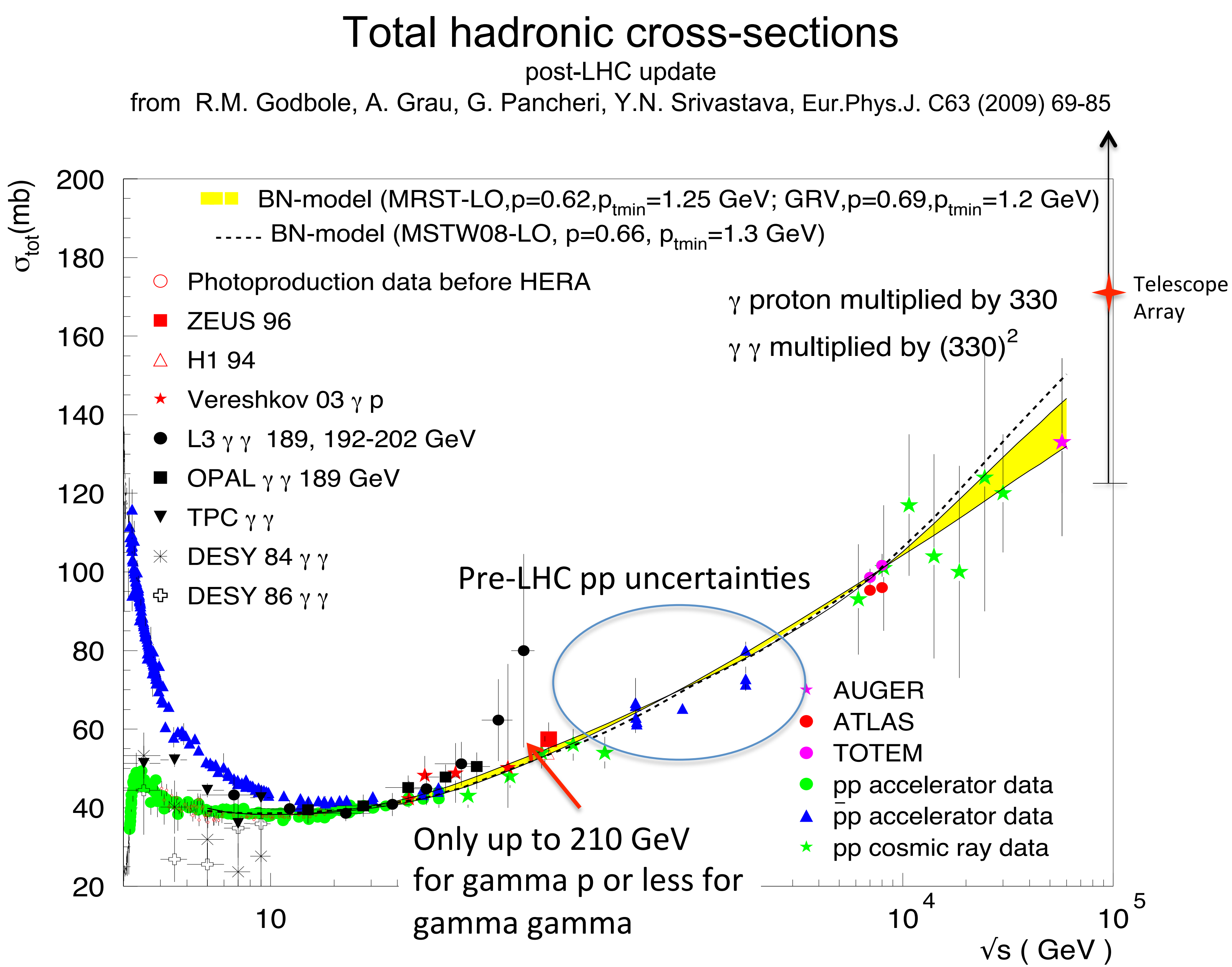
→ The total cross-section at very high energy

$$\sigma_{total} \lesssim [\log s]^2$$

OUTLINE

- The energy dependence of the total hadronic cross-section
- The Bloch Nordsieck + B. Touschek (BNT) soft quanta resummation procedure
- The model for the total and inelastic x-section
- Survival probability outside the diffractive region with all order resummation model

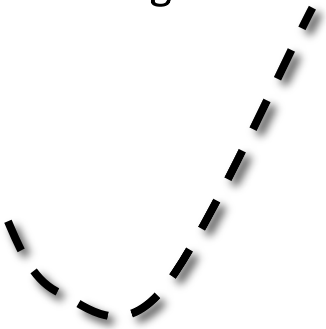
Total hadronic cross-sections



All total cross-sections rise… but not too much (Froissart dixit)

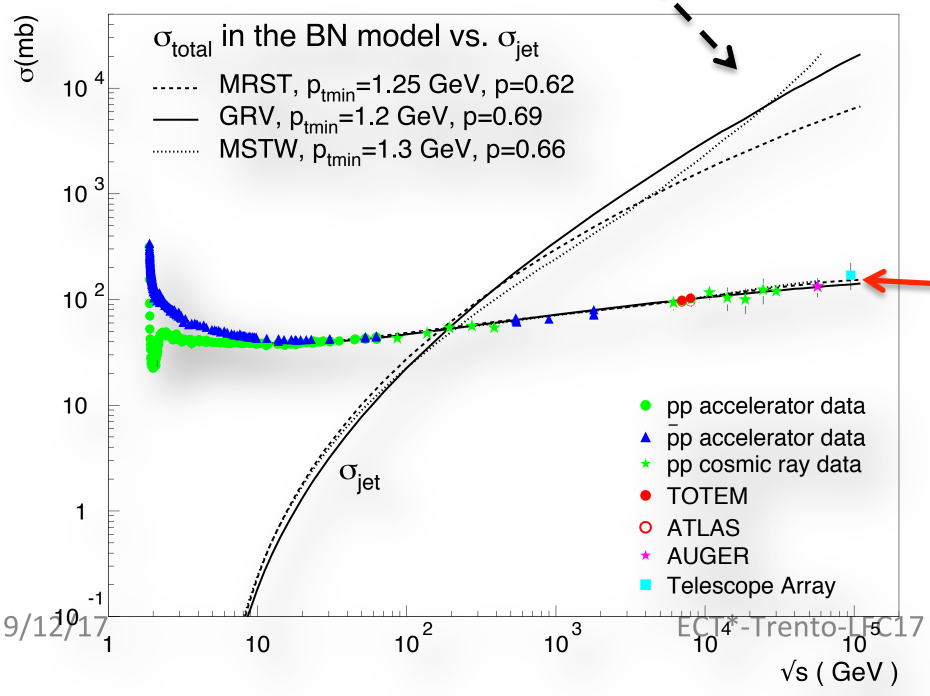
Cline,Halzen &Luthe 1973
Gaisser, Halzen,Stanev 1985
G.P., Y.N. Srivastava 1986
Durand,Pi 1987
Sjostrand, van Zijl 1987
…

What generates the rise? Low-x parton collisions

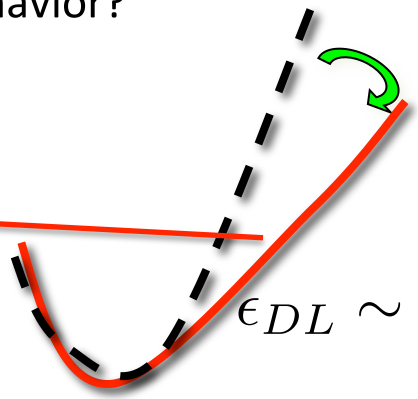


s^{ϵ} $\epsilon_{mini-jets} \sim 0.3$

pp



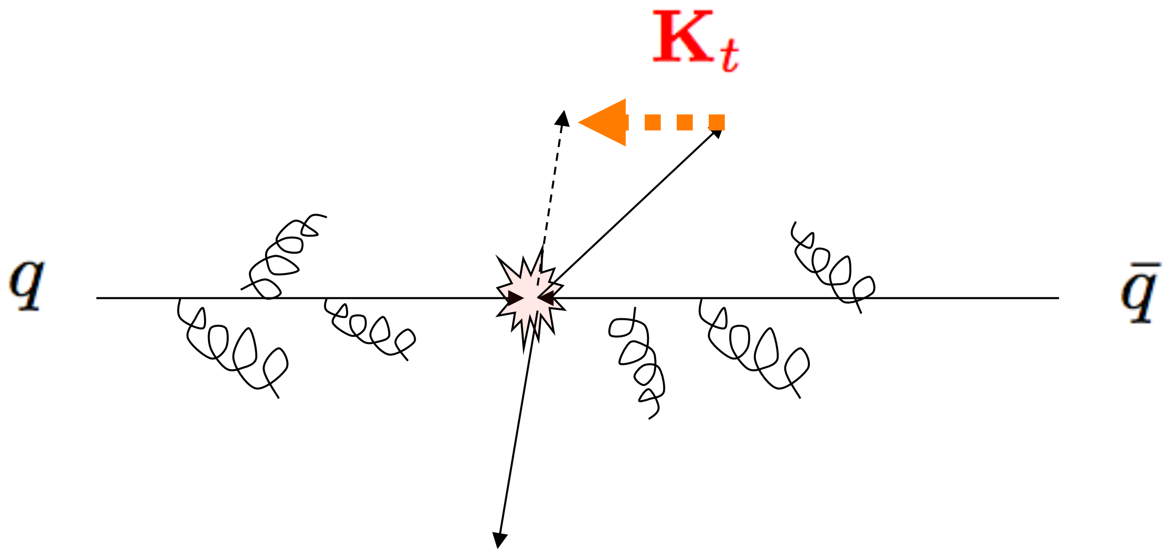
But, what tames the rise into a Froissart-like behavior?



$\epsilon_{DL} \sim 0.08$

How to go from hard to soft ?

Soft gluon emission tames the hard rise as it introduces parton acollinearity

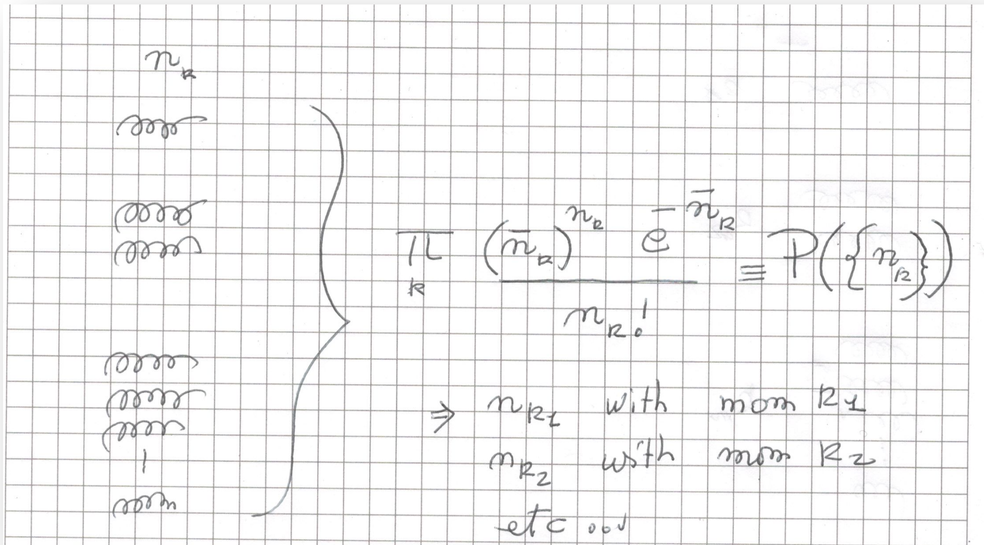


Acollinearity reduces the collision cross-section as
partons do not scatter head-on any more, also explained as the gluon cloud becoming too thick for partons to see each other : gluon saturation

A.Corsetti, A. Grau. G.P., Y.N. Srivastava, PLB 1996
A. Grau, G.P., Y.N. Srivastava. PRD60 (1999) 114020; R.M. Godbole, A. Grau, G.P., Y.N. Srivastava, PRD72 (2005) 076001

Taming the rise with All Order Soft Gluon Resummation

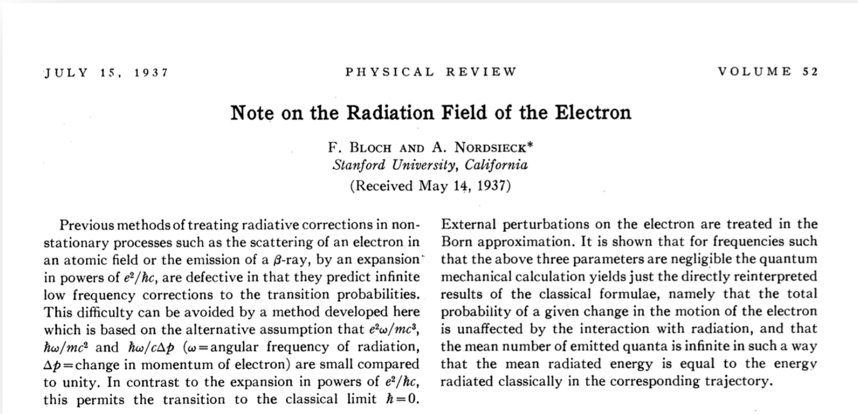
- Hadronic interactions show a large distance cut-off
- → large distance cut-off ←→ zero momentum gluons
- Need for a formalism for infrared gluons → semiclassical “democratic” resummation



The infrared region in hadronic collisions → large distance QCD

To access **the infra-red** region and tame the rise of total x-sections from minijet contributions, we use a resummation procedure different from the usual LLA or Sudakov:

- **Energy Momentum Conservation**
- The **Bloch and Nordsieck** theorem on infinite "photon" emission Phys Rev 1937



JULY 15, 1937 PHYSICAL REVIEW VOLUME 52

Note on the Radiation Field of the Electron

F. Bloch and A. Nordsieck*
Stanford University, California
(Received May 14, 1937)

Previous methods of treating radiative corrections in non-stationary processes such as the scattering of an electron in an atomic field or the emission of a β-ray, by an expansion in powers of $e^2/\hbar c$, are defective in that they predict infinite low frequency corrections to the transition probabilities. This difficulty can be avoided by a method developed here which is based on the alternative assumption that $e^2\omega/mc^3$, $\hbar\omega/mc^2$ and $\hbar\omega/c\Delta p$ (ω=angular frequency of radiation, Δp=change in momentum of electron) are small compared to unity. In contrast to the expansion in powers of $e^2/\hbar c$, this permits the transition to the classical limit $\hbar=0$. External perturbations on the electron are treated in the Born approximation. It is shown that for frequencies such that the above three parameters are negligible the quantum mechanical calculation yields just the directly reinterpreted results of the classical formulae, namely that the total probability of a given change in the motion of the electron is unaffected by the interaction with radiation, and that the mean number of emitted quanta is infinite in such a way that the mean radiated energy is equal to the energy radiated classically in the corresponding trajectory.

- **Maximally allowed divergence** of the coupling of soft gluons to quarks

From Bloch Norsdieck (1937) (B-N) theory of emission from a classical source to our proposal for maximally allowed infrared singularity in QCD (1984)

- F. Bloch and A. Nordsieck
  - Neglecting recoil → Poisson distribution of soft photons (gluons) emitted

$$P(\{n_{\mathbf{k}}\}) = \Pi_{\mathbf{k}} \frac{[\bar{n}_{\mathbf{k}}]^{n_{\mathbf{k}}}}{n_{\mathbf{k}}!} e^{-\bar{n}_{\mathbf{k}}}$$

  - Only emission of infinite number of soft photons (gluons) is finite

- B. Touschek
  - 1952 with W. Thirring : covariant formulation
  - 1968 (with E.Etim+GP) : summation cum energy-momentum conservation

$$d^4P(K) = \sum_{n_{\mathbf{k}}} P(\{n_{\mathbf{k}}\})\delta^4(\sum_k k n_{\mathbf{k}} - K) d^4K \qquad (1)$$

$$\delta^4(\sum_k k n_{\mathbf{k}} - K) = \int \frac{d^4x}{(2\pi)^4} e^{iK\cdot x} e^{-n_{\mathbf{k}} k\cdot x}$$

In (1) Exchange Sum in $n_{\mathbf{k}}$ with Product on **k**

Semi-classical and democratic (no branching nor ordering) summation

→ $$d^4P(K) = d^4K \int \frac{d^4x}{(2\pi)^4} e^{-h(x)+iK\cdot x}$$

Soft "photon" spectrum is exponentiated and regularized

$$h(x) = \sum_k (1 - e^{-ik\cdot x})\bar{n}_{\mathbf{k}} \to \int d^3\bar{n}_{\mathbf{k}}(1 - e^{-ik\cdot x})$$

→ Integrate over K_0 *and* K_3 → $$d^2P(K_t, s) = d^2K_t \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{-i\mathbf{K}_t\cdot\mathbf{b} - h(b,s)}$$

$h(b,s)$?

→ QED or QCD : integrand can be finite or singular but in order to be finite → **integrable** is the minimal condition

Application to K-t resummation in QCD

G.Parisi, R.Petronzio 1979 (PP) and Yu.L. Dokshitzer, D.I.D'Yakonov, S.I.Troyan 1978 (DDT)

With Asymptotic Freedom

$$h^{(PP)}(b,s) = \frac{4}{3\pi^2}\int_{M^2}^{Q^2} d^2k_\perp [1 - e^{i\mathbf{k}_\perp\cdot\mathbf{b}}]\alpha_s(k_\perp^2)\frac{\ln(Q^2/k_\perp^2)}{k_\perp^2}$$

Dropped in LLA

Our Proposal (ZPC 1984) of maximally allowed singularity

1. Keep the **full** $[1 - e^{i\mathbf{k}_\perp\cdot\mathbf{b}}]$

2. $M^2 \to 0$

3. $\alpha_{IR}(k_t) \propto [\frac{\Lambda}{k_t}]^{2p}$ $p < 1$

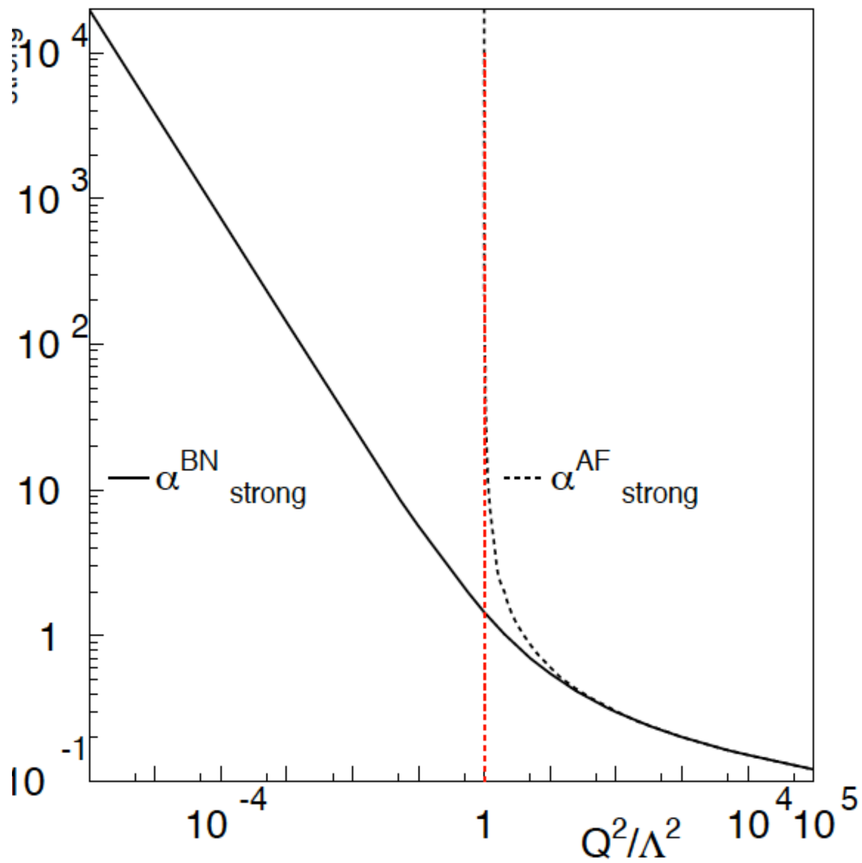
An ansatz (not yet implemented) for a maximally allowed singular expression for coupling of zero momentum gluons

$$\alpha_{strong}(k_t^2) = \frac{1}{\ln[1 + (\frac{k_t^2}{\Lambda^2})^{b_0}]}$$

Photon 2013

arXiv:1403.8050

D. Fagundes, A. Grau, GP,
O.Shekhovtsova and Y. Srivastava



Eikonal (unitarity respectful) model for the total cross-section

$$\sigma_{total} = 2\int d^2\mathbf{b}[1 - e^{-\chi_I(b,s)}]$$

- The **eikonal** function ~ real
- The rise is from pQCD → **minijets** with actual PDFs
- The taming (Froissart bound) of minijet rise is from all order resummation of **soft gluons**

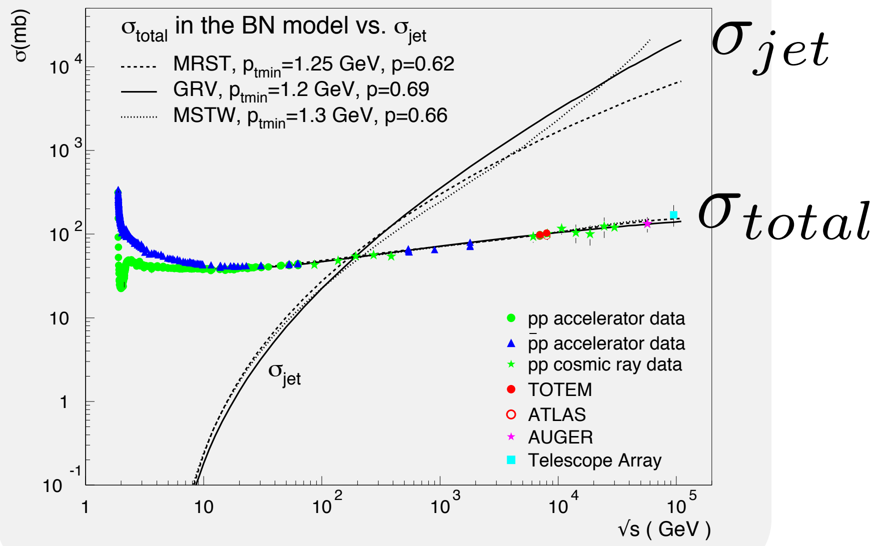
PDF driven eikonal minijet model: Minijets vs total cross-section

$$\sigma_{\text{jet}}^{AB}(s;p_{t_{min}}) = \int_{p_{t_{min}}}^{\sqrt{s}/2} dp_t$$
$$\int_{4p_t^2/s}^{1} dx_1 \int_{4p_t^2/(x_1 s)}^{1} dx_2$$
$$\sum_{i,j,k,l} f_{i|A}(x_1,p_t^2) f_{j|B}(x_2,p_t^2) \frac{d\hat{\sigma}_{ij}^{kl}(\hat{s})}{dp_t}.$$

Negligible at low energies

But around

$$\sqrt{s} \simeq 20 \div 30\ GeV$$



x ≤ 0.1 and with $p_t \geq 1$ GeV → pQCD can be applied

$$\sigma_{\text{jet}}^{AB}(s;p_{t_{min}} \simeq 1\ GeV) \sim 10\%\ \sigma_{total}^{AB}$$

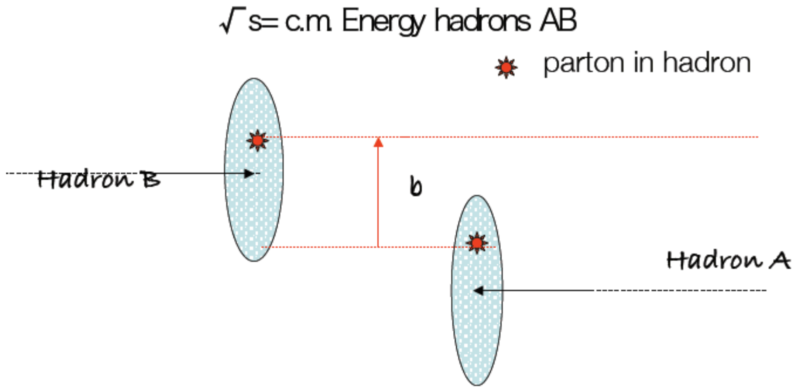
We model the impact parameter distribution in eikonal formulation as the Fourier-transform of ISR soft k_t distribution and thus obtain a cut-off at large distances : Froissart bound?

$$A_{BN}(b,s) = N \int d^2\mathbf{K}_\perp \; e^{-i\mathbf{K}_\perp \cdot \mathbf{b}} \frac{d^2P(\mathbf{K}_\perp)}{d^2\mathbf{K}_\perp} = \frac{e^{-h(b,q_{max})}}{\int d^2\mathbf{b}\; e^{-h(b,q_{max})}}$$

$$h(b,E) = \frac{16}{3\pi}\int_0^{qmax} \frac{dk_t}{k_t}\alpha_{eff}(k_t)\ln(\frac{2q_{max}}{k_t})[1 - J_0(bk_t)]$$

$$\boxed{\alpha_{eff}(k_t \to 0) \sim k_t^{-2p}}$$

$$\boxed{A_{BN}(b,s) \sim e^{-(b\bar{\Lambda})^{2p}}}$$



qmax ?

Fixed by single gluon emission kinematics

b-distributions of partons

$$2\chi_I(b,s) = \bar{n}_{soft}(b,s) + \bar{n}_{hard}(b,s) = A_{FF}(b)\sigma_{soft}(s) + A_{BN}(b,s)\sigma_{mini-jet}(s,p_{tmin})$$

- For partons with no contribution rising with energy → Form factors

$$A_{FF}(b) = \frac{\mu^2}{96\pi}(\mu b)^3 K_3(\mu b)$$

$$\sigma_{soft} \simeq constant\ +\ decreasing$$

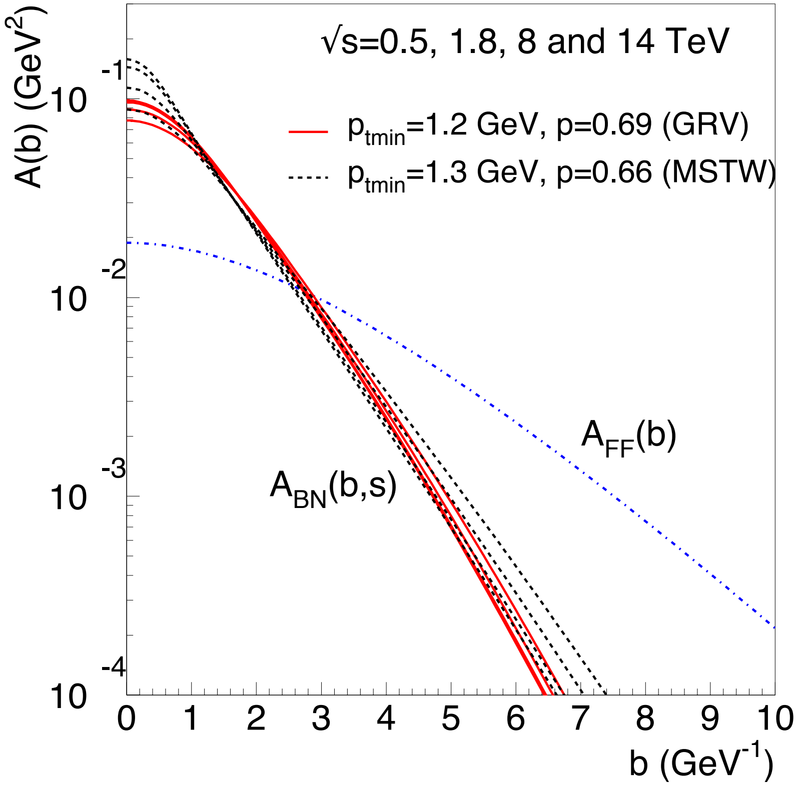
- For partons -> minijets, PDFs and p_{tmin}
  calculate upper integration limit in k_t from single gluon kinematics

  Chiappetta,Greco Phys.Lett 1981

$$q_{max}(s; PDF, p_{tmin})$$

  Choose p value (for integration in the infrared)

$$A_{BN}(b,s)$$

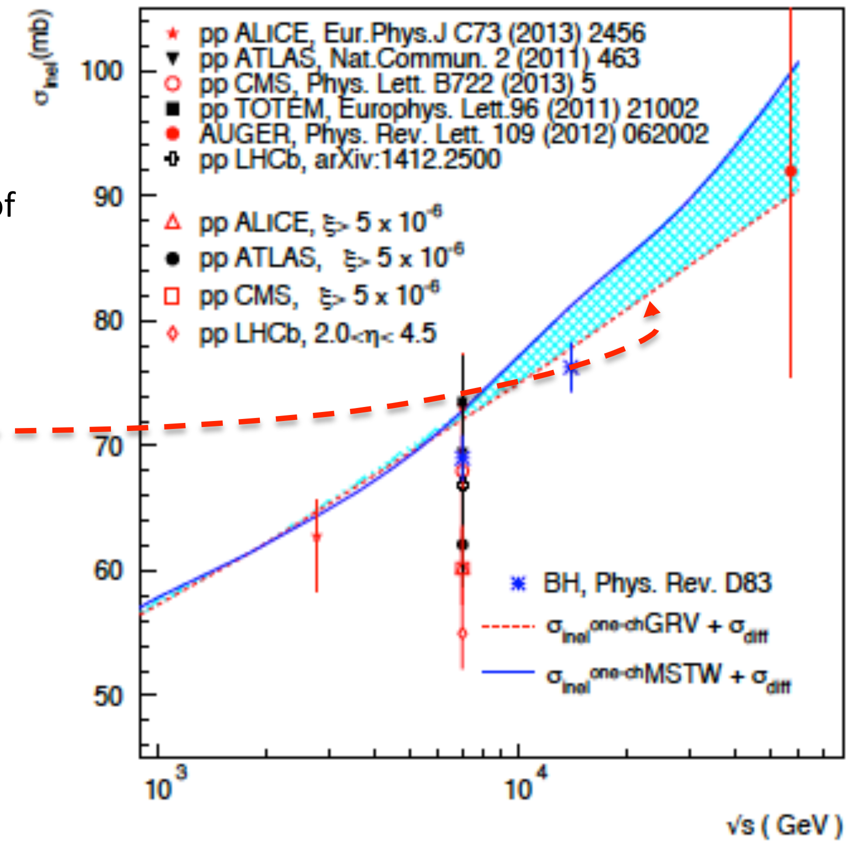


Caveat: Single Diffraction rises with energy but is not described by minijets nor by n_{soft}

How do we compare this approach with data and other models?

- A single parton-parton collision dressed with resummed soft gluon emission is analogue of a Pomeron in Regge-Pomeron (R-P) approaches
- Eikonalization → includes multiple Pomerons
- Our b-distribution is the corresponding realization of the dipole in Regge-Pomeron models
- Not satisfactory for Single Diffraction
  -> inelastic x-section is too low
- This is (so far) a single channel model with two components, soft and minijets → no Single Diffraction … work is in progress
- Including SD by ``hand" (fit to SD) gets close to extrapolations for inelastic
- R-P is two or three channels (GLM, KMR) → allows for SD and DD and for elastic dsigma/dt

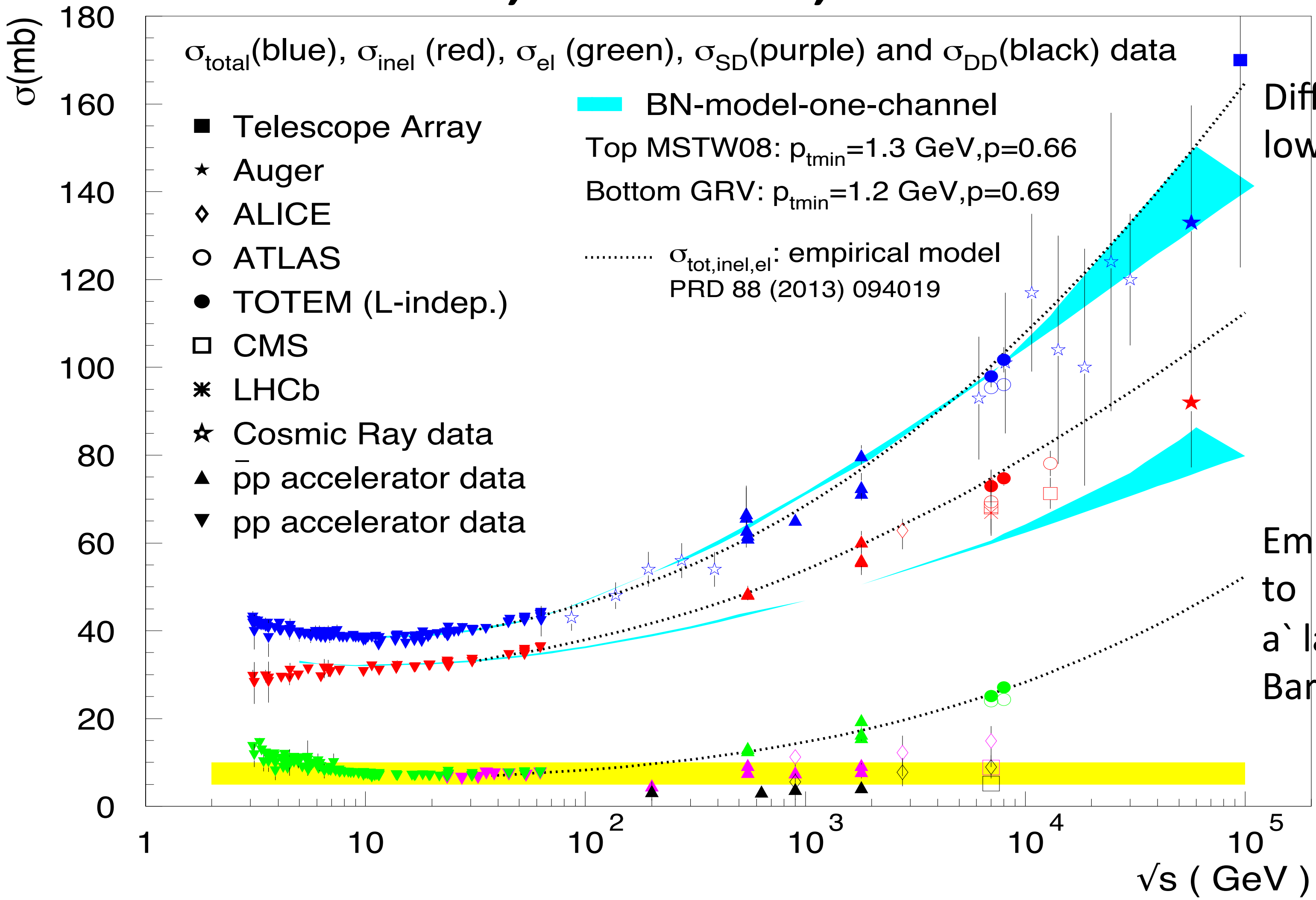
σ_{inel}



Fagundes, Grau, GP, Shekhovtsova,Srivastava, PRD 2015

$$2\chi_I(b,s) = \bar{n}_{soft}(b,s) + \bar{n}_{hard}(b,s) = A_{FF}(b)\sigma_{soft}(s) + A_{BN}(b,s)\sigma_{mini-jet}(s,p_{tmin})$$

Total, elastic, inelastic



Different low-x behaviour

Empirical fit to $d\sigma/dt$ a` la Phillips & Barger (1974)

D.Fagundes, A.Grau, GP, O.Shekhovtsova, Y.N. Srivastava arXiv:1706.00093 PRD 96 (2017)

Why bother with the inelastic x-section?

- Check of models
- Used in extraction of σ^{pp}_{total} from σ^{p-air}_{prod}
- Used for estimates of survival probabilities for hard processes

Our recent (2017) proposal for SPG in central rapidity region (non-diffractive)

$$< |S(b)|^2 >_{hard} = \int d^2\mathbf{b}\ A_{BN}(b,s) e^{-\bar{n}_{hard}(b,s)}$$

All order in impact space so as **to exclude minijet** contribution in central region

$$< |S(b)|^2 >_{soft} = \int d^2\mathbf{b}\ A_{FF}(b,s) e^{-\bar{n}_{soft}(b,s)}$$

To **exclude soft** Contributions with Form factor distribution

$$\bar{\mathcal{S}}^2_{total}(s) = \bar{\mathcal{S}}^2_{soft}(s) + \bar{\mathcal{S}}^2_{hard}(s)$$
$$\equiv w_{soft}(s) < |S(b)|^2 >_{soft} + w_{hard}(s) < |S(b)|^2 >_{hard}$$

$$w_{soft/hard}(s) \equiv \frac{\sigma_{soft/jet}(s)}{\sigma_{soft}(s) + \sigma_{jet}(s)} \equiv \frac{\sigma_{soft/jet}(s)}{\sigma_B(s)}$$

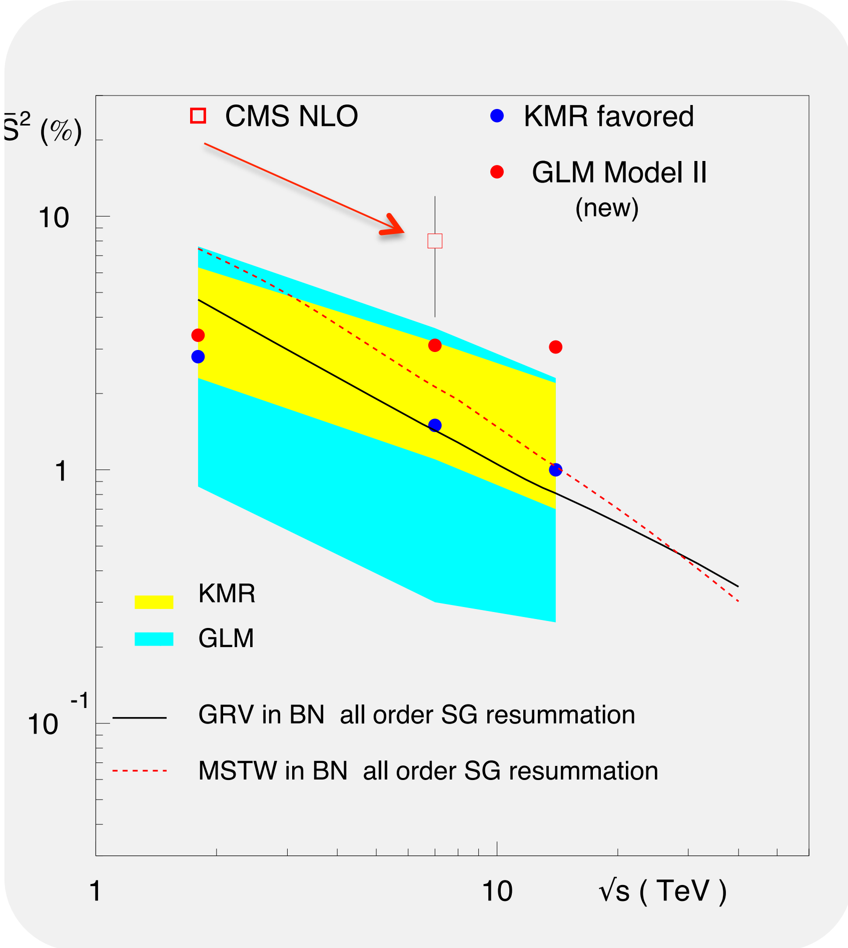
Comparison with Regge-Pomeron

- NLO CMS (also ATLAS) for di-jet diffractive production is still higher than shown models

- Wide band of predictions from
  - Khoze, A. D. Martin, and M. G. Ryskin, (KMR), Eur. Phys. J. C73, 2503 (2013)
  - E. Gotsman, E. Levin, and U. Maor (GLM), Eur. Phys. J. C76, 177 (2016)

- KMR in ~ good ~ agreement
  with our proposal

- We disagree with Block, Durand,Ha and Halzen 2015 estimate (see backup slides)



Conclusions

- We have developed a mini-jet LO PDF based model to describe
  - the energy dependence of the total cross-section at very high energies
  - non –diffractive soft and semi-hardc ollisions in hadronic processes

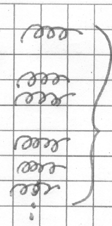
- Our proposed “democratic” soft gluon resummation approach with an ansatz for a maximally allowed singularity leads to

$$\sigma^{total} \lesssim [logs]^{1/p} \qquad 1/2 < p < 1$$

→ consistent with the Froissart bound **R.M.Godbole, A. Grau, G.P., Y.N.Srivastava PLB B682 (2009) 55-60**

And possibly providing a glimpse into the confinement region.

Derivation of Bloch Nordsieck Touschek Formula in one page

①  $\Big\}\, k, n_k$

n_k photons of momentum $\vec{k}$ all independently Poisson emitted

$$P_{n_k}(\bar{n}_k) = \frac{1}{n_k!}\, e^{-\bar{n}_k}\, (\bar{n}_k)^{n_k}$$

$\Rightarrow$ only ∞ # is finite: Bloch Nordsieck 1937

② E. Etim, G.P., Bruno Touschek 1968

$$d^4P(K) = \sum_{n_k} \prod_k P_{n_k}(\bar{n}_k)\, d^4K\, \delta^4\Big(\sum k\, n_k - K\Big)$$

$$\downarrow$$

$$\int \frac{d^4x}{(2\pi)^4}\, e^{ix\cdot(\;)}$$

$\Rightarrow$ any # of soft quanta $\to$ not ordered

$\hookrightarrow$ not sequential

<u>but</u> ENERGY-MOMENTUM CONSERVATION is IMPOSED

③ Exchange $\sum_{n_k}$ with $\prod_k$ $\Rightarrow$ exponentiation

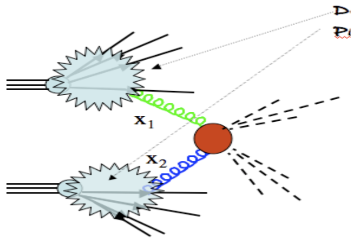
④ $$d^4P(K) = d^4K \int \frac{d^4x}{(2\pi)^4}\, e^{iK\cdot x} \left\{ e^{-h(x)} \right\}$$

⑤ continuum limit $h(x) = \int d^3\bar{n}_k \left[1 - e^{-ik\cdot x}\right]$

Keeps trace of EN-MOM conservation

Bloch-Nordsieck (BN) model for pp → pion proton, gamma p, gamma gamma etc.

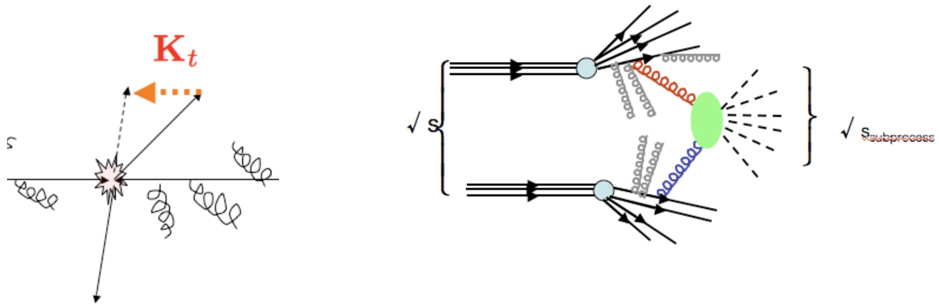
1.



PDFs DGLAP

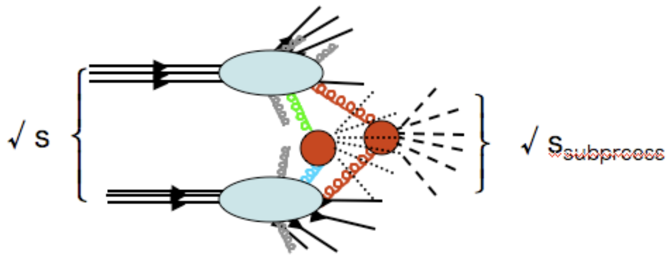
Mini-jet cross-sections
Pt ~ 1 GeV

2.



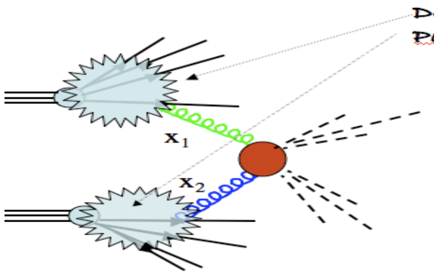
Dress with Initial State
Infrared Radiation

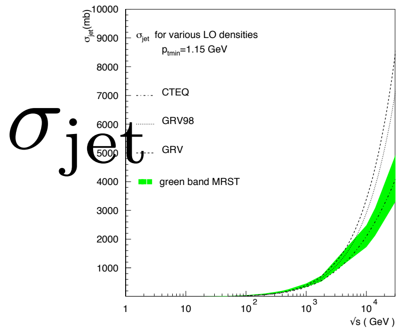
3.



Eikonalize the collision probability

$$\sigma_{total} = 2\int d^2\mathbf{b}[1 - e^{-\chi(b,s)}]$$

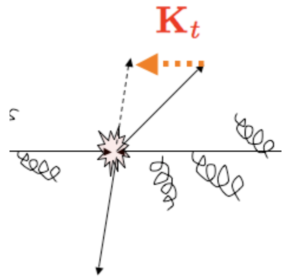


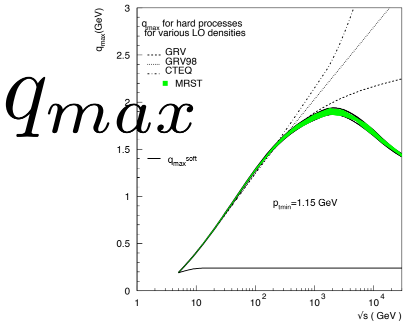


1. Calculate mini-jet cross-section choosing a band of PDFs and p_{tmin}

$$\sigma_{mini-jet} \simeq s^{\epsilon}$$

$$\epsilon \simeq 0.3 - 0.4$$

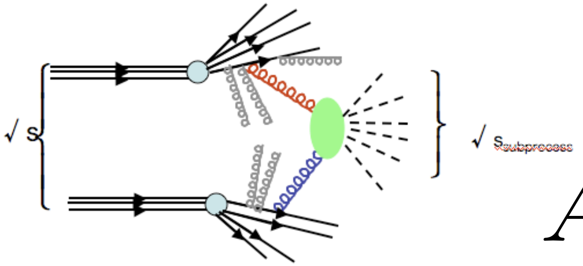


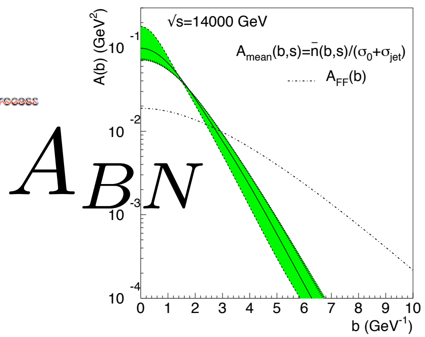


2. Calculate single soft gluon upper scale, for given PDF, ptmin

$$q_{max} \simeq p_{tmin}$$

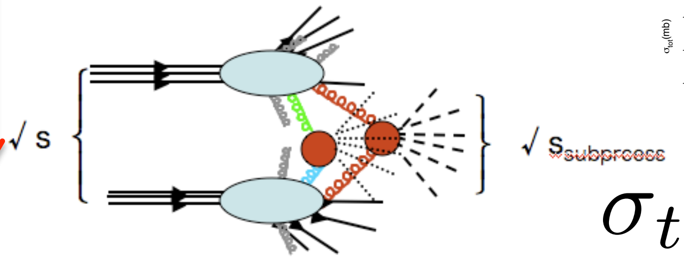
$$\lesssim 2 - 3\ GeV$$

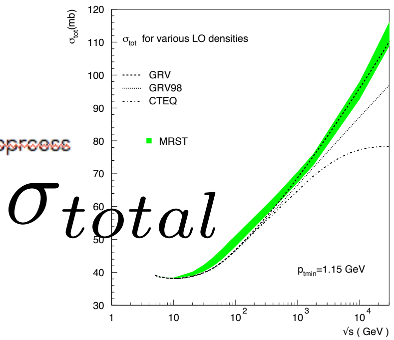




3. Calculate impact parameter distribution for calculated q_{max} and chosen infrared parameter p

$$2\chi_I(b,s) = A_{FF}(b)\sigma_{soft}(s) +$$
$$+ A_{BN}(b,s)\sigma_{mini-jet}(s, p_{tmin})$$





4. Eikonalize

$$\sigma_{total} = 2\int d^2\mathbf{b}[1 - e^{-\chi(b,s)}]$$

Survival probabilities

$$\mathcal{S}^2(s) \equiv < |S(b,s)|^2 >= \int d^2\mathbf{b} A(b,s) e^{-\bar{n}(b,s)}$$

???

- Probability of no collisions $e^{-\bar{n}(b,s)}$
- Choosing $\bar{n}(b,s)$ from fit to total x-section

  and

- Impact parameter distribution from soft component
  - LO : Form factors type in BDHH or in our 2008 model
  - LO for CMS data and diffractive di-jet production