CHERENKOV AND PARAMETRIC (QUASI-CHERENKOV) RADIATIONS PRODUCED BY A RELATIVISTIC CHARGED PARTICLE MOVING THROUGH CRYSTALS BUILT FROM METALLIC WIRES

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Charged & Neutral Particles Channeling Phenomena

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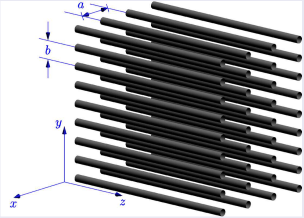
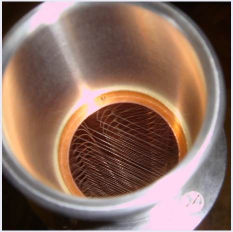
Crystals Built From Metallic Wires (Wire Media)





Photonic crystals built from parallel metallic wires have drawn much attention for their potential to serve diverse purposes: subwavelength imaging, surface field enhancement used in so-called field-enhanced nanosensing, subwavelength image magnification. These structures are also proposed for use as resonators in electromagnetic radiation sources. Depending on crystal period and wire thickness, considered crystals can operate in a wide range of frequencies – from microwave and terahertz to infrared and optical.

[1] C.R. Simovski, P.A. Belov [et. al.] Advanced Materials, 24(31), 4229–4248, 2012.

Parametric Radiation in Wire Media

According to results obtained in [2] the intensity of parametric radiation grows essentially when the wavelength $\lambda = 2\pi/k$ becomes comparable with the wire radius R: $kR \sim 1$. As a result, with electron bunches in which the number of electrons $n_e \sim 10^9$–10^{11} (such as produced by laser acceleration) it is possible to obtain THz radiation pulses of GW power.

Until recently, interaction of electromagnetic waves with wire media was analyzed for $kR \ll 1$ case, when scattering of the electromagnetic wave by a single wire is isotropic.
The analysis lead in [2] was based on the extrapolation of theoretical results true at $kR \ll 1$, to frequency range $kR \sim 1$, where electromagnetic wave scattering on a single wire is anisotropic (i.e., scattering amplitude depends on the angle of scattering).

[2] V.G. Baryshevsky, A.A. Gurinovich, Nucl. Instr. Meth. B, 355, 69–75, 2015.

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Study objectives

- Derive the equations that describe diffraction of electromagnetic waves in crystals built from metallic wires taking into account an angular dependence of scattering amplitude.
- Consider spontaneous Cherenkov and parametric (quasi-Cherenkov) radiation in such crystals under these conditions.

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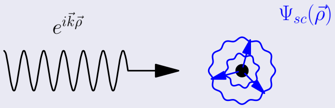
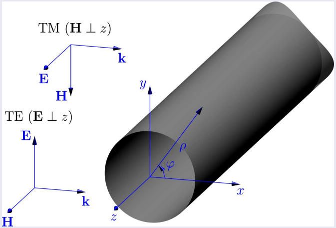
Electromagnetic Wave Scattering by a Single Wire



$$\Psi_{sc} = A(\varphi) \int_{-\infty}^{\infty} \frac{e^{ik\sqrt{\rho^2+z^2}}}{\sqrt{\rho^2+z^2}} dz.$$



Isotropic scattering (TM-wave):

$k_\rho R \ll 1 \Rightarrow A(\varphi) \approx A_0,$
$\Psi_{sc} \approx i\pi A_0 H_0(k_\rho \rho),$

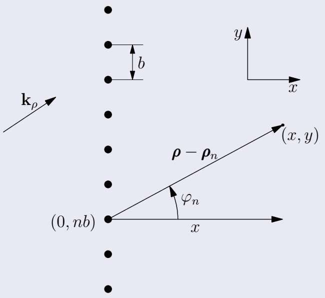
Anisotropic scattering (TM-wave):

$k_\rho R \lesssim 1 \Rightarrow A(\varphi) \approx A_0 + A_1 \cos\varphi,$
$\Psi_{sc} \approx i\pi A_0 H_0(k_\rho \rho) - \pi A_1 H_1(k_\rho \rho) \cos\varphi,$

where $A(\varphi)$ is the scattering amplitude, φ is the azimuthal angle of polar coordinate system centered at wire location, and H_n are the Hankel cylindrical functions of the first kind of the n-th order, $k_\rho = \sqrt{k^2 - k_z^2}$.

[3] V.G. Baryshevsky, E.A. Gurnevich, Journal of Nanophotonics, 6, 061713 (2012).

Scattering by a Single Crystal Plane



The general solution of this problem looks like

$$\Psi = e^{i\mathbf{k}_\rho \boldsymbol{\rho}} + i\pi F_0 \sum_{n=-\infty}^{\infty} e^{ik_y bn} H_0(k_\rho|\boldsymbol{\rho}-\boldsymbol{\rho}_n|)$$

$$-\pi F_1 \sum_{n=-\infty}^{\infty} e^{ik_y bn} H_1(k_\rho|\boldsymbol{\rho}-\boldsymbol{\rho}_n|)\cos\varphi_n$$

$$-\pi F_1' \sum_{n=-\infty}^{\infty} e^{ik_y bn} H_1(k_\rho|\boldsymbol{\rho}-\boldsymbol{\rho}_n|)\sin\varphi_n \Leftrightarrow$$

$$\Leftrightarrow \Psi = e^{i\mathbf{k}_\rho \boldsymbol{\rho}} + \sum_{n=-\infty}^{\infty}\left(F_0 + F_1\frac{k_{xn}}{k_\rho}\operatorname{sgn} x + F_1'\frac{k_{yn}}{k_\rho}\right)\frac{2i\pi}{k_{xn}b}e^{ik_{yn}y}e^{ik_{xn}|x|},$$

where $k_{xn}^2 = k_\rho^2 - (k_y - 2\pi n/b)^2$, coefficients F_0, F_1, F_1' can be expressed in terms of amplitudes A_0, A_1.

Wave Propagation in Crystal

The analysis of wave propagation in crystal built from a large number of such gratings (crystal planes) regularly spaced at a distance a from one another, was performed in [3,4]. The obtained dispersion equation for finding the wave vector in crystal $\mathbf{q}$ is quite cumbersome and we do not present it here.

The effective permittivity of crystal expands in a Fourier series by reciprocal lattice vectors: $\varepsilon(\mathbf{r},\omega) - 1 = \sum_{\boldsymbol{\tau}} g_{\boldsymbol{\tau}}(\omega) e^{-i\boldsymbol{\tau}\mathbf{r}}$. Coefficients $g_{\boldsymbol{\tau}}$ are necessary to know for examination of radiation processes in crystal. If $kR \ll 1$, then from dispersion equation we have

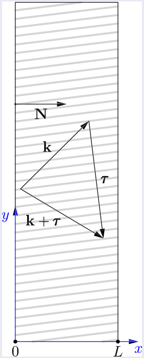
$$g(\boldsymbol{\tau}) \approx \frac{4\pi}{k_\rho^2 ab}\left(\frac{A_0}{1+i\pi A_0} + \frac{A_1}{1+i\frac{\pi}{2}A_1}\cdot\cos(\mathbf{k},\mathbf{k}+\boldsymbol{\tau})\right). \qquad (1)$$

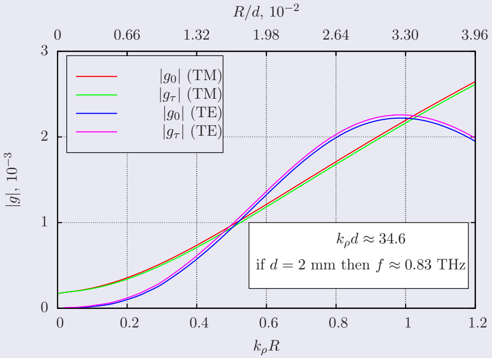
In case $kR \sim 1$, coefficients $g_{\boldsymbol{\tau}}$ should be calculated by numerical solution of the obtained dispersion equation.

[3] V.G. Baryshevsky, E.A. Gurnevich, Journal of Nanophotonics, 6, 061713 (2012).
[4] V.G. Baryshevsky, E.A. Gurnevich, LANL e-print arXiv:1609.05689, 2016.

Example: Two-Wave Laue Diffraction

For example lets calculate g_0, g_τ for case of two-wave diffraction in Laue geometry. For simplicity, we shall assume that the crystal has a square lattice ($a = b \equiv d$).





Spectral-Angular Distribution of Radiation

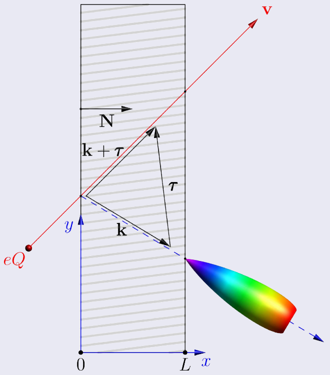
For analysis of radiation process common expressions for spectral-angular distribution of Cherenkov, transition and parametric radiation were used. For example, for the parametric radiation emitted in diffraction direction in Laue geometry

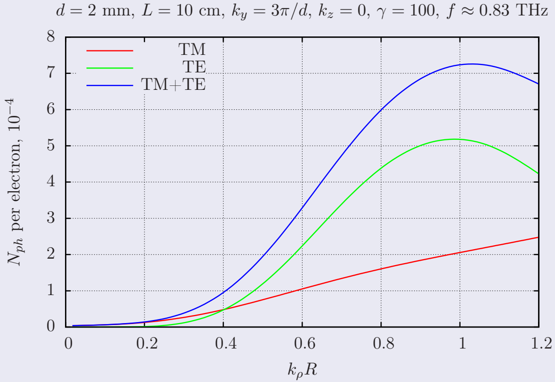
$$\frac{d^2 N_\tau^s}{d\omega d\Omega} = \frac{e^2 Q^2 \omega}{4\pi^2 \hbar c^3} (\mathbf{e}_s^\tau \mathbf{v})^2 \left| \sum_{\mu=1,2} \beta_1 \xi_{\mu s}^\tau e^{i \frac{\omega}{c\gamma_0} \varepsilon_{\mu s} L} \left[\frac{1}{\omega - \mathbf{k}_\tau \mathbf{v}} - \frac{1}{\omega - \mathbf{q}_{\tau\mu s} \mathbf{v}} \right] \times \right.$$
$$\left. \times \left(e^{i(\omega - \mathbf{q}_{\tau\mu s}\mathbf{v}) \frac{L}{c\gamma_0}} - 1 \right) \right|^2 ,$$

Similar expressions can be obtained for parametric radiation in Bragg geometry as well as for Cherenkov and transition radiation. To use these expressions it is enough to know the diffraction geometry and quantities g_τ.

[2] V.G Baryshevsky, A.A. Gurinovich, Nucl. Inst. and Meth. B, 355, 69–75

Intensity of Parametric Radiation (Laue Case)

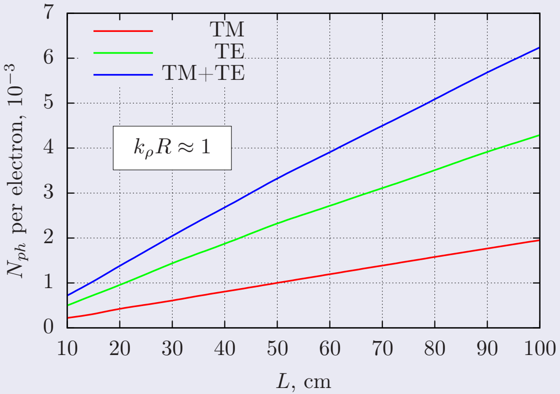




Intensity of radiation in diffraction direction reaches its maximum at $k_\rho R \approx 1$, which corresponds to wires radius of $R \approx 65\ \mu$m if crystal period $d = 2$ mm.

For an electron bunch with $n_e \sim 1.25 \cdot 10^9$ and $t_b \sim 30 \cdot 10^{-15}$ s (SwissFEL parameters) instantaneous radiation power in case $k_\rho R \approx 1$ will be $P \approx 18$ MW.

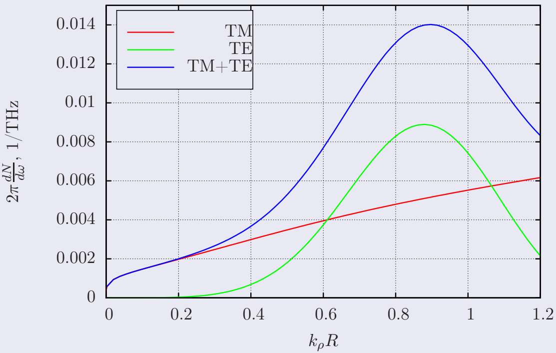
Intensity of Parametric Radiation



The radiation power grows with the increase of the crystal length L. For example, at $k_\rho R \approx 1$ and $L = 1$ m for the same bunches it will be about 160 MW.

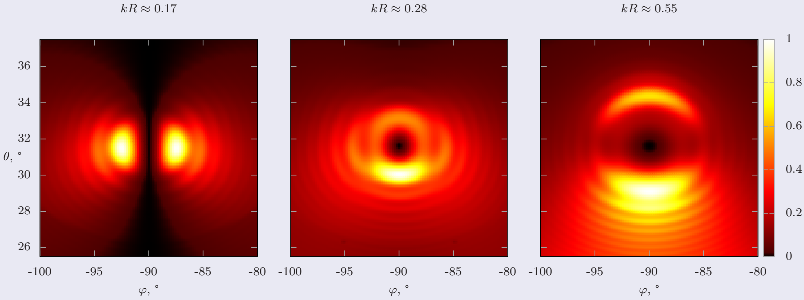
Intensity of Cherenkov and Transition Radiation

$d = 2$ mm, $L = 10$ cm, $\gamma = 100$, $f \approx 0.85$ THz



Similar results were obtained for Cherenkov and transition radiation, although their intensity turns out to be smaller than that for parametric radiation ($\sim$ 14 MW in the same frequency range for considered bunches).

Angular Distributions



$L = 50$ cm, $\gamma = 100$ ($E = 50.5$ MeV).

For large $k_\rho R$ contributions of the TE- and TM-polarized waves to the total intensity of radiation can be comparable (in considered case $k_\rho R \sim 1$ contribution of TE-wave is dominant), whereas for thin wires ($k_\rho R \ll 1$) only TM-polarized wave contribute noticeably to the total radiation intensity.

Summary

- The dispersion equation that describes diffraction and refraction of waves in crystals built from metallic wires in the range $0 < kR \lesssim 1$ is derived.
- The intensity of Cherenkov and parametric radiation increases as the wire radius is increased and achieves its maximal value in the range $kR \sim 1$.
- The case when the condition $kR \sim 1$ is fulfilled in the THz frequency range is considered in detail. The calculations show that the instantaneous power of Cherenkov and parametric radiations from electron bunches in the crystal studied can be tens – hundreds megawatts.

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Thank you for your attention!

For Further Reading I

V.G Baryshevsky, I.D. Feranchuk, A.P. Ulyanenkov
Parametric X-Ray Radiation in Crystals: Theory, Experiment and Applications.
Springer, 2005.

V.G. Baryshevsky, E.A. Gurnevich
Dynamical diffraction theory of waves in photonic crystals built from anisotropically scattering elements
Journal of Nanophotonics, 6, 061713, 2012.

V.G. Baryshevsky, A.A. Gurinovich
Quasi-Cherenkov parametric radiation from relativistic particles passing through a photonic crystal
Nucl. Instr. Meth. B, 355, 69–75, 2015.

For Further Reading II

V.G. Baryshevsky, E.A. Gurnevich
Cherenkov and parametric (quasi-Cherenkov) radiation from relativistic charged particles moving in crystals formed by metallic wires
LANL e-print arxiv:1609.05689, 2016.