Proton radius and γZ-contributions : a survey

Rome, September V - IX, MMXI

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Proton

charge radius

size of proton : electric charge radius

Lamb shift in muonic H (PSI)

Pohl et al.
Nature 466 (2010) 213

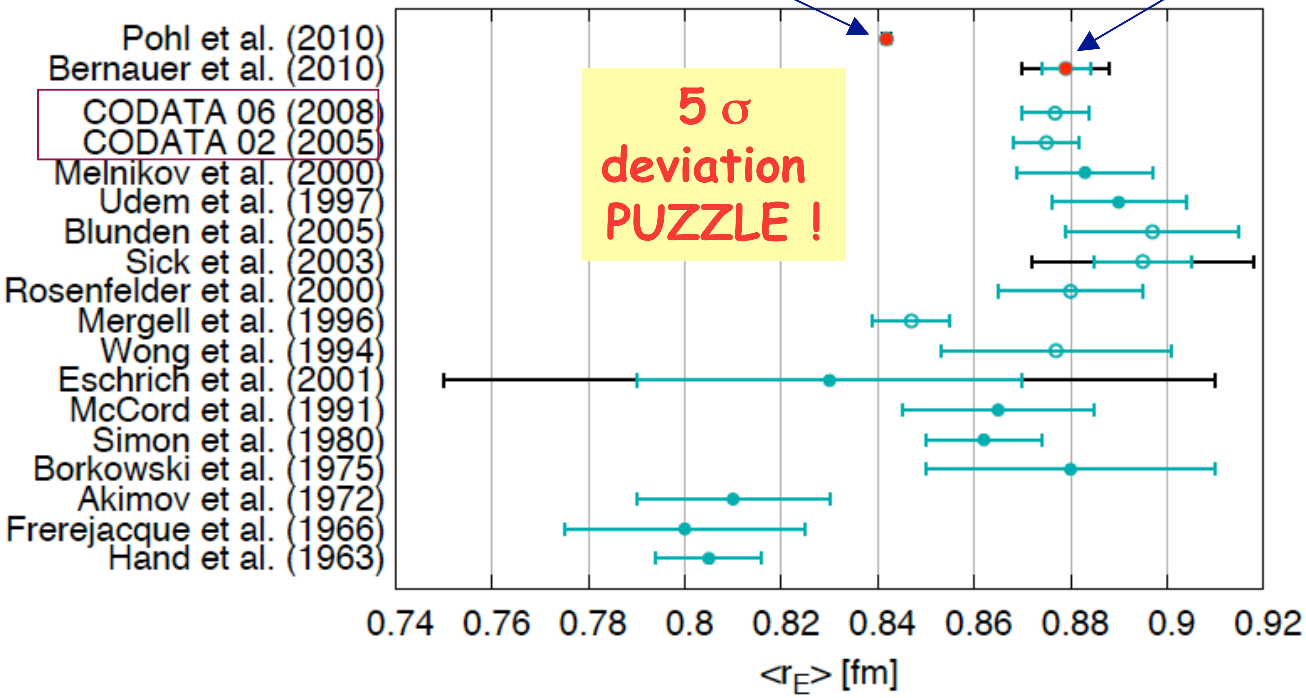
R_E = 0.84184 (67) fm

ep-scattering (MAMI)

Bernauer et al.
PRL 105 (2010) 242001

R_E = 0.879 (8) fm

R_E :
rms radius $\sqrt{\langle r_E^2 \rangle}$



5 σ
deviation
PUZZLE !

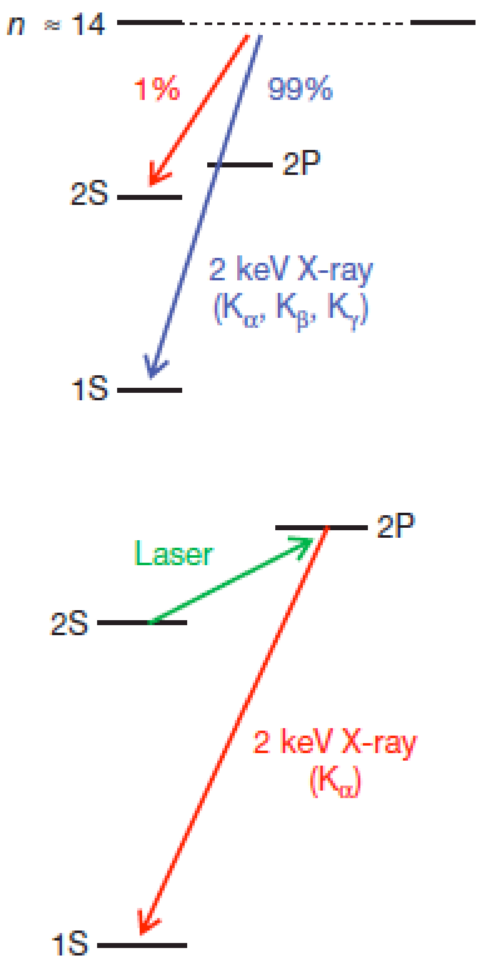
corrections to Lamb shift are 300 μeV below expectation

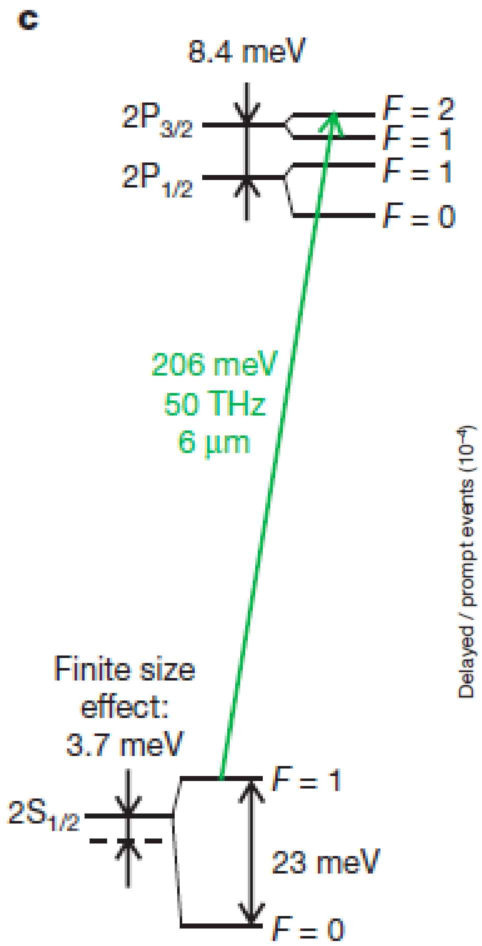
proton charge radius

from

muonic hydrogen Lamb shift

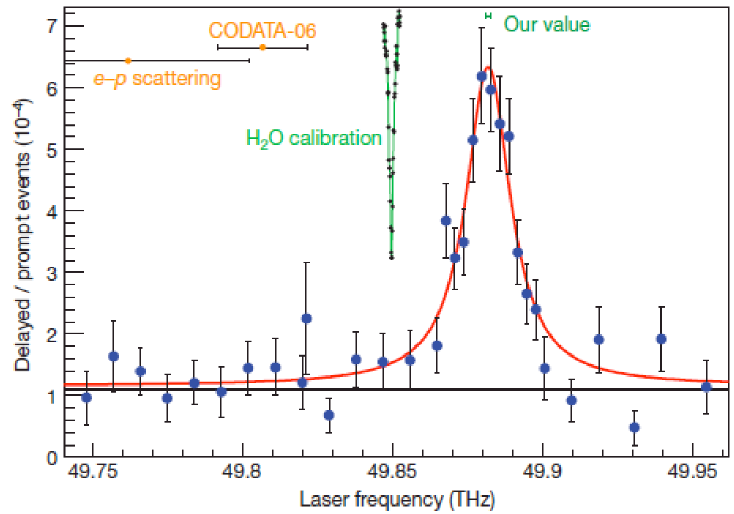
Lamb shift measurement in muonic H



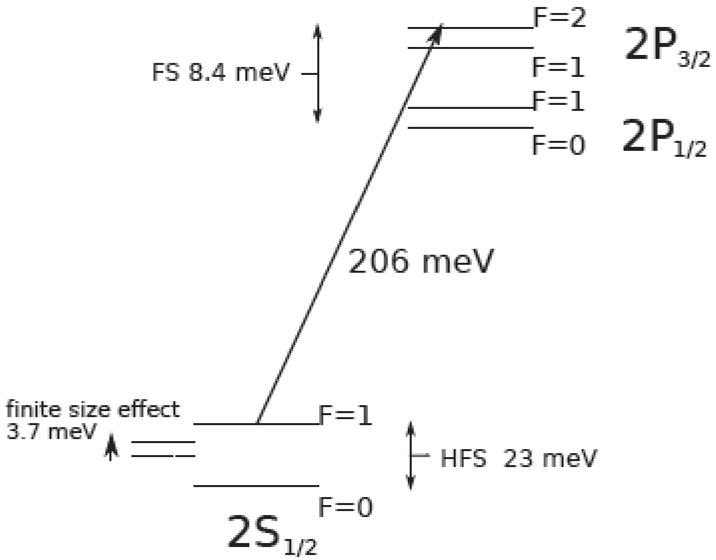


Experiment at PSI

Pohl et al.
Nature 466 (2010) 213



extraction of R_E from μH Lamb shift



Lamb Shift

- Lamb shift is dominated by vacuum polarization :
  drops 2S state by a lot
- Experiment measures 2S F=1 to $2P_{3/2}$ F = 2 state
  (F is total angular momentum)
- Finite size effect on s-wave states (l = 0)
  Non-relativistic 1γ-exchange calculation

  Karplus, Klein, Schwinger (1952)

$$\Delta E = \frac{2\pi\alpha}{3} R_E^2 \, \phi_n^2(0)$$

- Leading term of order **O(α⁴)** : $\phi_n^2(0) = m_r^3 \alpha^3 / (\pi n^3)$

$$\boxed{\Delta E_{LS} = 209.9779\,(49) - 5.2262\ R_E^2 + 0.00913\ R^3_{(2)}\ \text{meV}}$$

3.70 meV   0.026 meV

$R^3_{(2)}$: $O(\alpha^5)$ correction term

Difference between R_E = 0.84184 fm and R_E from ep is equivalent to an additional correction on 2S state of around -300 μeV

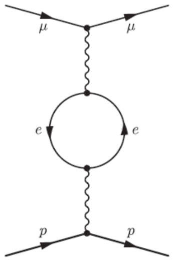
Lamb shift : QED corrections

- Calculated by several groups

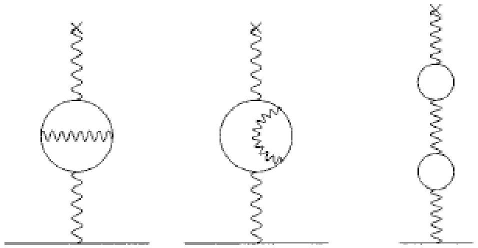
Pachucki (1996, 1999)

Borie (1976, 2005)

- 1 loop electron

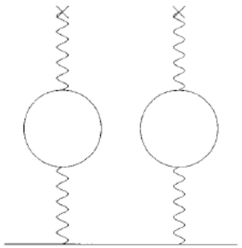


$\Delta E = 205.0282$ meV



$\Delta E = 1.5081$ meV

- 2 loop electron



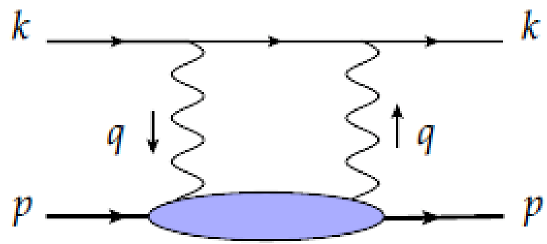
$\Delta E = 0.1509$ meV

- Muon self-energy, vacuum polarization $\Delta E = -0.6677$ meV

- Many other QED corrections calculated : all of size 0.005 meV or smaller << 0.3 meV

Lamb shift : hadronic corrections (I)

- $O(\alpha^5)$ **finite-size correction :**
  $\gamma\gamma$ **box diagram**



- **"3rd Zemach moment"**
  non-rel. calculation
  Friar(1979)

$$R^3_{(2)} = \int d^3\vec{r}_1\, d^3\vec{r}_2\, |\vec{r}_1 - \vec{r}_2|^3\, \rho_E(r_1)\, \rho_E(r_2)$$

$$= \frac{48}{\pi} \int_0^\infty \frac{dQ}{Q^4} \left[G_E^2(Q) - 1 - 2Q^2 G_E(0) \frac{dG_E}{dQ^2}(0) \right]$$

  recent evaluation
  Distler, Bernauer, Walcher(2011)

  $R^3_{(2)} = 2.85\ (8)\ \mathrm{fm}^3 \longrightarrow \Delta E \approx -0.026$ meV

- What do we know **model independently** ?

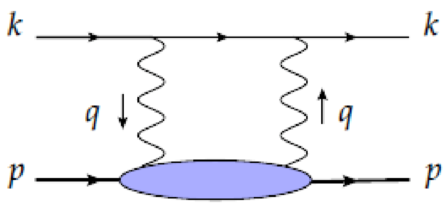
  Lower blob contains both elastic (nucleon) and in-elastic states

  Information is contained in forward, double virtual Compton scattering

- For **model** estimates, see e.g. recent work of Miller, Thomas, Caroll, Rafelski (2011)

Lamb shift : hadronic corrections (II)

- forward, doubly virtual Compton scattering (unpolarized)



$$T^{\mu\nu}(p,q) = \frac{i}{8\pi M}\int d^4x\, e^{iqx}\langle p|Tj^\mu(x)j^\nu(0)|p\rangle$$
$$= \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)T_1(\nu,Q^2)$$
$$+ \frac{1}{M^2}\left(p^\mu-\frac{p\cdot q}{q^2}q^\mu\right)\left(p^\nu-\frac{p\cdot q}{q^2}q^\nu\right)T_2(\nu,Q^2)$$

- $\mathrm{Im}\,T_1(\nu,Q^2) = \frac{1}{4M}F_1(\nu,Q^2)$
  $\mathrm{Im}\,T_2(\nu,Q^2) = \frac{1}{4\nu}F_2(\nu,Q^2)$ → Unpolarized forward structure functions

- ΔE evaluated through an integral over Q^2 and ν

Pachucki (1996, 1999)
Faustov, Martynenko (2000)

$$\Delta E = \Delta E^{el}$$ → elastic, nucleon pole **only** (non-pole part had been included in previous works)

$$+ \Delta E^{subtr}$$ → subtraction , required for the amplitude T_1 → $T_1(0,Q^2)$

$$+ \Delta E^{inel}$$ → inelastic, dispersion integrals evaluated using most recent F_1, F_2 information from Jlab Christy, Bosted (2010)

Lamb shift : hadronic corrections (III)

- Low-energy expansion of forward, doubly virtual Compton scattering constrains subtraction term $T_1(0,Q^2)$ (as well as models)

effective Hamiltonian : $\mathcal{H} = -\frac{1}{2} 4\pi \alpha_E \vec{E}^{\,2} - \frac{1}{2} 4\pi \beta_M \vec{B}^{\,2}$

electric magnetic polarizabilities

$$\lim_{\nu^2, Q^2 \to 0} T_1^{\mathrm{non-Born}}(\nu, Q^2) = \frac{\nu^2}{e^2}(\alpha_E + \beta_M) + \frac{Q^2}{e^2}\beta_M$$

$$\lim_{\nu^2, Q^2 \to 0} T_2^{\mathrm{non-Born}}(\nu, Q^2) = \frac{Q^2}{e^2}(\alpha_E + \beta_M)$$

subtraction term for T_1

- Numerical evaluations :

$(\mu\mathrm{eV})$	Carlson,Vdh (2011)	Pachucki (1999)	Martynenko (2006)
ΔE^{subt}	5.3 ± 1.9	1.8	2.3
ΔE^{inel}	-12.7 ± 0.5	-13.9	-13.8
ΔE^{el}	-29.5 ± 1.3	-23.0	-23.0
ΔE	-36.9 ± 2.4	-35.1	-34.5

PRA 84 (2011) 020102 (R) : $\Delta E = (-\,36.9 \pm 2.4)\ \mu\mathrm{eV}$ or about **12%** of the needed correction ...

proton charge radius

from

elastic

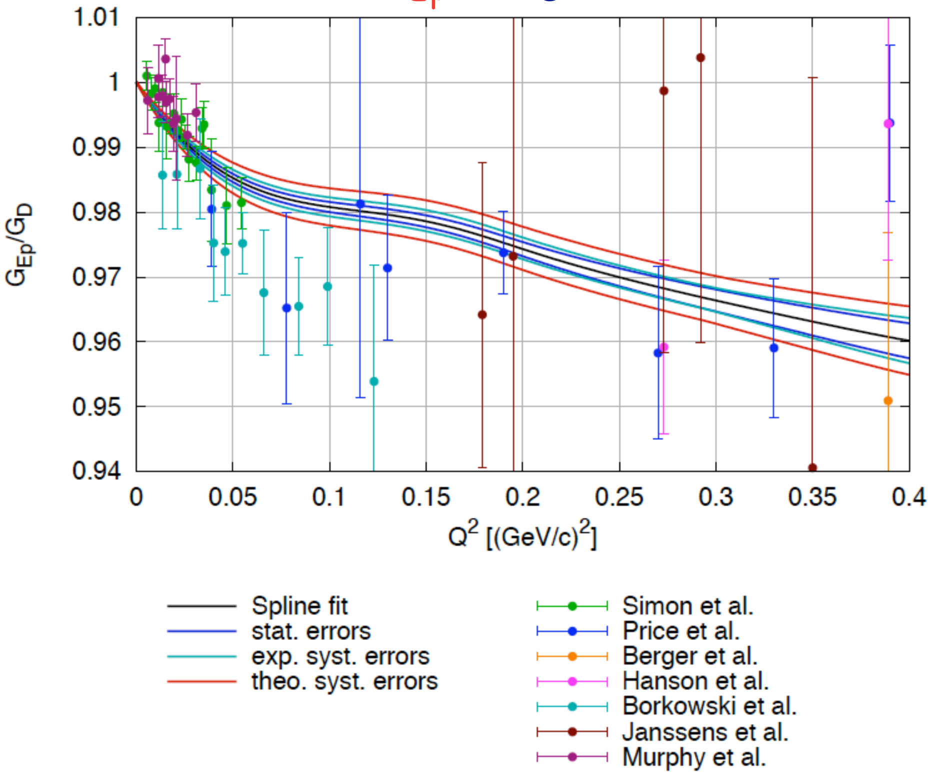
ep - scattering

proton electromagnetic form factors

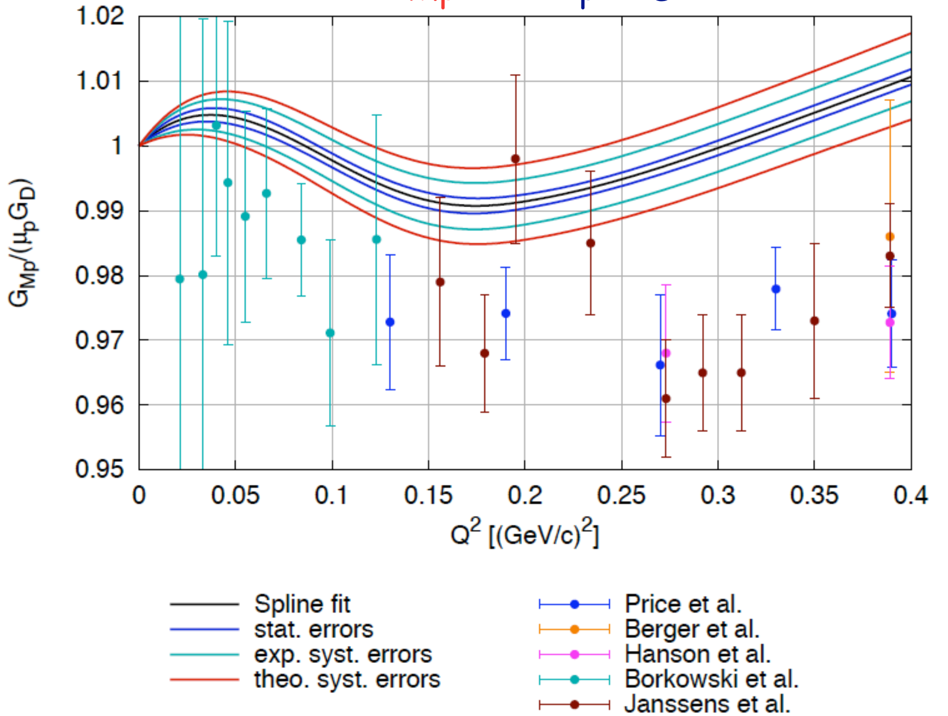
recent cross section data @ MAMI — Bernauer et al. — PRL 105 (2010) 242001

in range Q^2 = 0.004 - 1 GeV^2

G_{Ep} / G_D



$G_{Mp} / (\mu_p G_D)$



R_E = 0.879 ± 0.005_{stat} ± 0.004_{syst} ± 0.005_{model} fm

R_M = 0.777 ± 0.013_{stat} ± 0.009_{syst} ± 0.005_{model} fm

➡ see talk : M. Distler

proton electromagnetic form factors

recent data at low Q^2 @ JLAB for G_{Ep}/G_{Mp} measuring P_t/P_l Zhan et al.

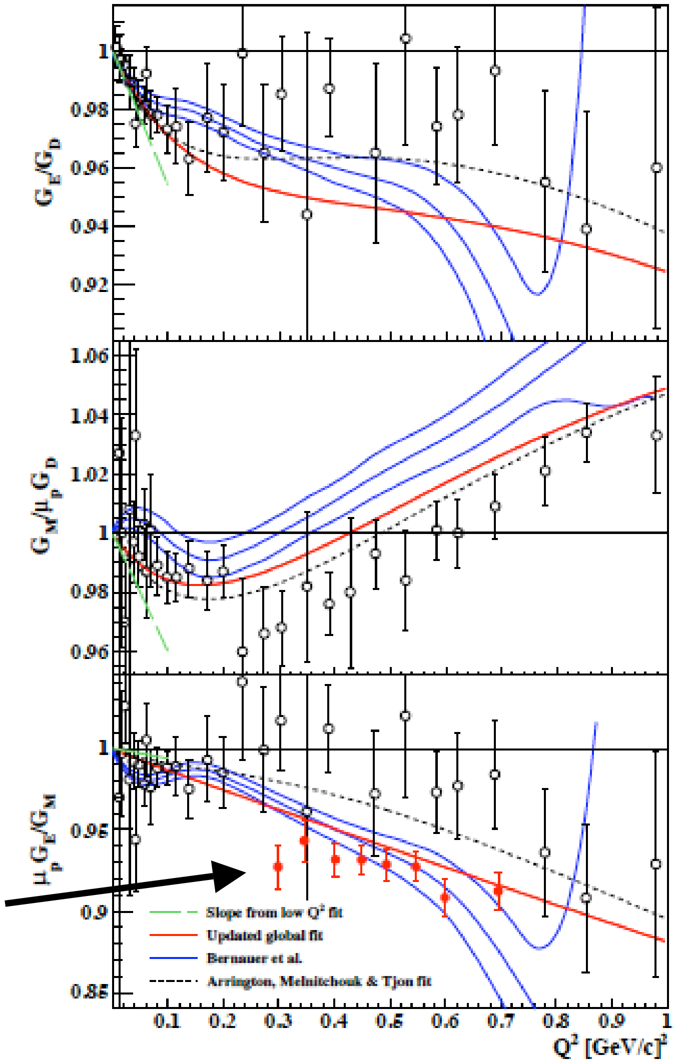
arXiv:1102.0318 [nucl-ex]

G_{Ep}/G_D

$G_{Mp}/(\mu_p G_D)$

$G_{Ep}/(\mu_p G_{Mp})$

JLAB/Hall A data



+ re-analysis of world data NOT including new MAMI data yields

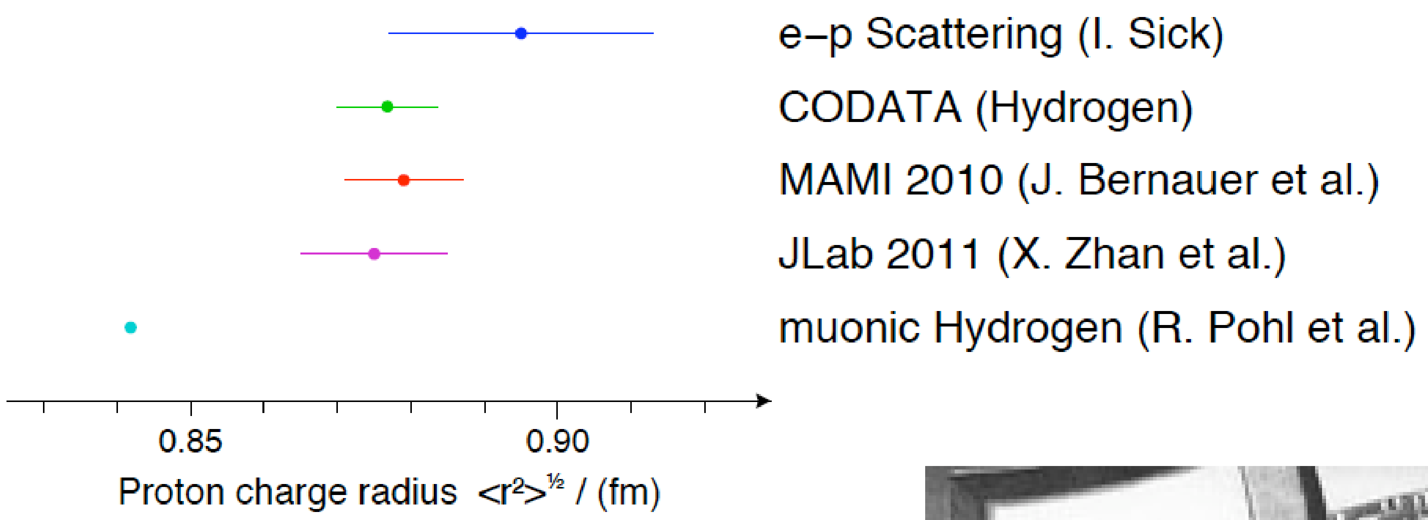
R_E = 0.875 ± 0.010 fm

R_M = 0.867 ± 0.020 fm

R_E : agreement between Bernauer et al. and Zhan et al.

R_M : disagreement between both Bernauer G_M data 1-1.5 % larger than global fit of Zhan et al.

proton electric charge radius : status



Combination of ep-data :
CODATA, Bernauer et al., Zhan et al.

R_E = 0.8772 ± 0.0046 fm

7.7σ
difference !?

μH data : Pohl et al.

R_E = 0.8418 ± 0.0007 fm

R_E puzzle : what could it mean ?

- **unknown correction ?** …after known constraints have been built in !

- **Change in Rydberg constant ?**

  In absence of further (sizeable) corrections, use of muonic extraction of R_E plugged into electron H Lamb shift yields R_∞ which is 4.9σ away from CODATA value (and factor 4.6 more precise) Pohl et al. (2010)

- **New physics ?**

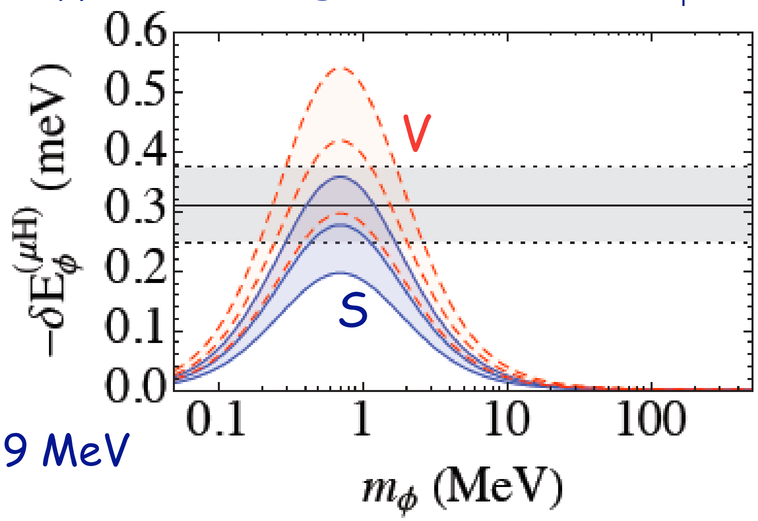
  Example : explain 3.6σ $(g-2)_\mu$ discrepancy AND 7.7σ R_E discrepancy from μH Lamb shift simultaneously invoking a correction by a hypothetical light boson (mass m_ϕ)

Tucker, Smith (2010)

Scalar (**S**)

Vector (**V**)

e.g. dark gauge boson scenario



correction largest for $m_\phi \approx a_\mu^{-1} = \alpha\, m_r \approx 0.69$ MeV

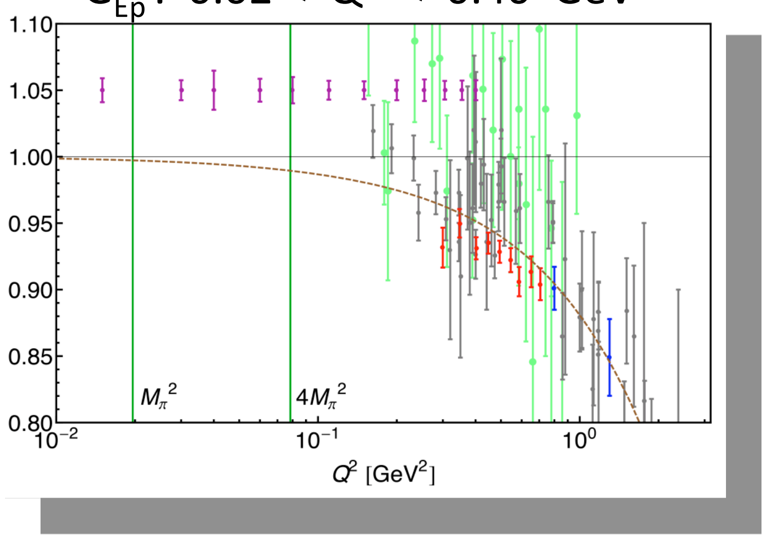
$(g-2)_e$ puts strong limit on coupling to e -> much smaller

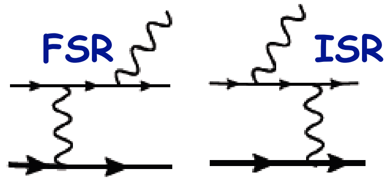
R_E puzzle : what's next ?

- Muonic Lamb shift : muonic D, muonic ^{3}He measurements planned
- Electronic Lamb shift : higher accuracy measurement very timely
- G_{Ep} measurements at very low Q^2

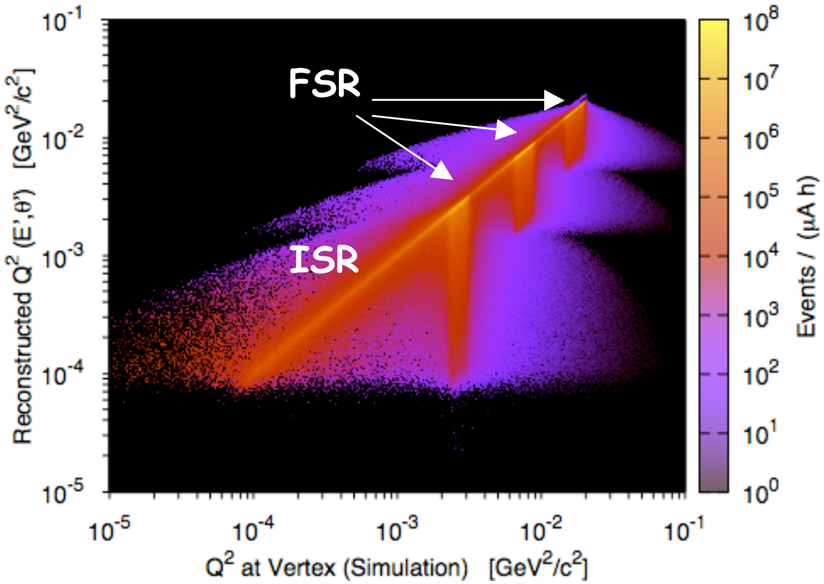
JLAB/Hall A : Nov 2011 - May 2012

G_{Ep} : $0.02 < Q^2 < 0.40$ GeV^2





MAMI : also use initial state radiation



- JLAB/Hall B **proposal** : magnetic-spectrometer-free experiment (HyCal)
  $Q^2 = 2\times10^{-4}$ - 2×10^{-2} GeV^2 $ep \rightarrow ep$ cross sections normalized to Moller scattering

γZ boxes

forward PV ep scattering

- **PV ep Asymmetry**

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = \frac{G_F}{4\pi\alpha\sqrt{2}} t Q_W^p$$

$$Q_W^{p,LO} = 1 - 4\sin^2\theta_W(0)$$

including corrections

$$\boxed{\begin{aligned} Q_W^p = {} & (1 + \Delta\rho + \Delta_e)(Q_W^{p,\mathrm{LO}} + \Delta'_e) \\ & + \Box_{WW} + \Box_{ZZ} + \mathrm{Re}\Box_{\gamma Z}. \end{aligned}}$$

Erler, Kurylov, Ramsey-Musolf (2003)

calculated perturbatively

requires non-perturbative evaluation

➡ see talks : W. Melnitchouk, C.J.Horowitz

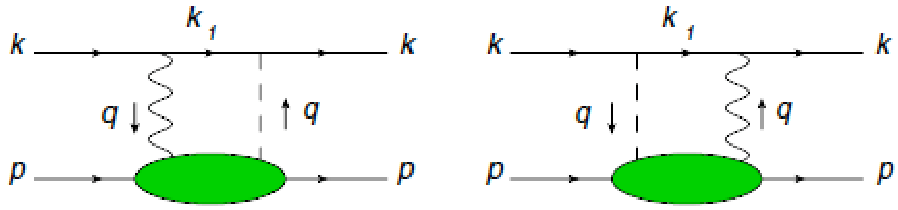
- **Needed accuracy**

		precision of Q_W^p measurement		needed precision on $\mathrm{Re}\Box_{\gamma Z}$
@1.165 GeV	$Q_W^p \approx 0.07$	4 %	➡	0.0028
@137 MeV		2.1 %	➡	0.00064
		$\Delta \sin^2\vartheta_W \approx 0.00037$		

➡ see talk : F. Maas

dispersive framework for γZ box

forward



$$\Box_{\gamma Z}(E) = \Box^A_{\gamma Z}(E) + \Box^V_{\gamma Z}(E)$$

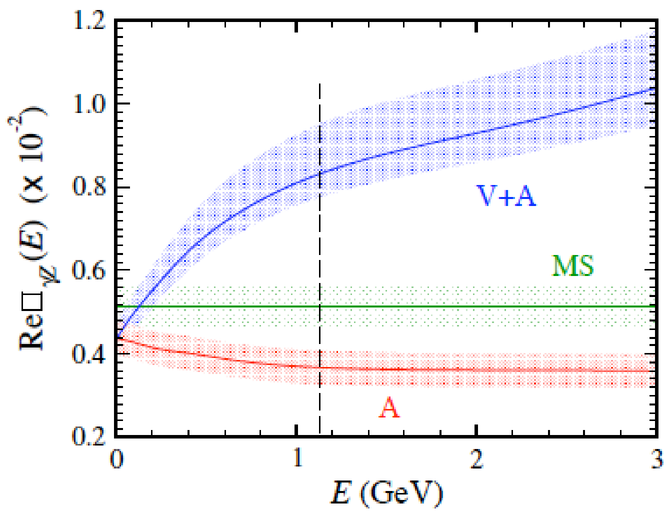
- $\mathrm{Re}\Box^A_{\gamma Z}$ weak energy dependence

  Marciano, Sirlin (1983, 1984, 1985)

  recent re-evaluation :

  Blunden, Melnitchouk, Thomas (2011)

  E=0 : Q_W^p = 0.0713 (8) -> 0.0705 (8)



- $\mathrm{Re}\Box^V_{\gamma Z}$

$$\mathrm{Re}\,\Box^V_{\gamma Z}(E) = \frac{2E}{\pi}\int_{\nu_\pi}^{\infty}\frac{dE'}{E'^2 - E^2}\,\mathrm{Im}\Box^V_{\gamma Z}(E')$$

$$\mathrm{Im}\Box^V_{\gamma Z}(E)$$

$$= \frac{\alpha}{(2ME)^2}\int_{W_\pi^2}^{s} dW^2 \int_0^{Q^2_{max}} dQ^2 \frac{F_1^{\gamma Z}(x,Q^2) + AF_2^{\gamma Z}(x,Q^2)}{1 + Q^2/M_Z^2}$$

γZ structure functions

Gorchtein, Horowitz (2009)

Sibirtsev, Blunden, Melnitchouk, Thomas (2010)

Rislow, Carlson (2011)

structure function (SF) input

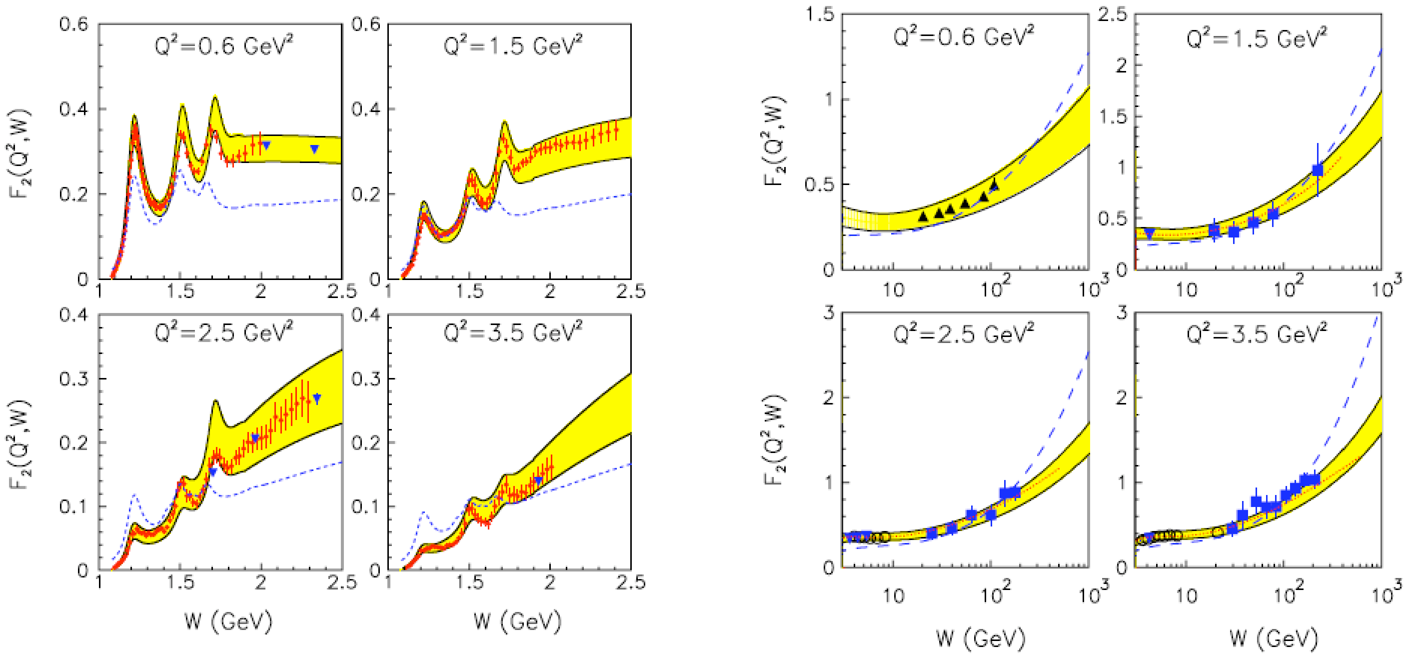
- Comparison forward $\gamma\gamma$ box with forward γZ box

  forward $\gamma\gamma$ box : data based evaluation possible using forward e.m. SF input $F_{1,2}^{\gamma\gamma}$

  (modulo one subtraction !)

  forward γZ box : $F_{1,2}^{\gamma Z}$ requires PV inelastic asymmetries in different kinematical regimes

- $F_{1,2}^{\gamma\gamma}$ SF input (Resonance region, DIS region, Regge region)



Sibirtsev, Blunden, Melnitchouk, Thomas (2010)

Isospin dependence of SF

- $F_{1,2}^{\gamma Z}$ in DIS

$$F_2^{\gamma Z} = x\sum 2e_q g_q^V f_q(x, Q^2)$$

e_q^2 for $F_{1,2}^{\gamma\gamma}$

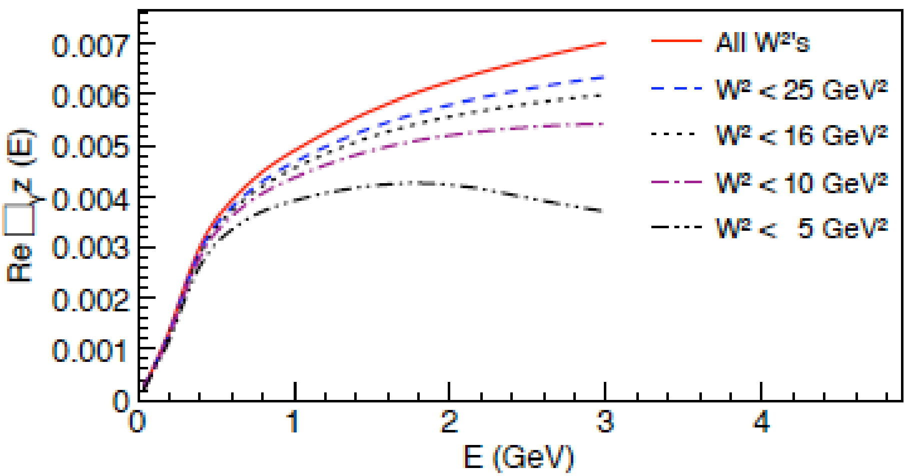
- $F_{1,2}^{\gamma Z}$ in resonance region

I = 3/2 resonance : isovector currents $\longrightarrow$ multiply $F_{1,2}^{\gamma\gamma}$ by $(1 + Q_W^p)$

I = 1/2 resonance : use SU(6) quark model to relate the couplings

background : more modeling needed, difficult to estimate error reliably

models used for isospin rotation : VMD model (ρ, ω, ϕ, ...)



Gorchtein, Horowitz, Ramsey-Musolf (2011)

$\Rightarrow$ Background : responsible for more conservative error estimate (using 2 VMD models)

Results

recent estimates	$\mathrm{Re}\Box^V_{\gamma Z}$ *(x 10^{-3})*	*Error (x 10^{-3})*
Gorchtein, Horowitz, Ramsey-Musolf (2011)	5.39	± 0.27 (mod.av.) ± 1.88 (backgr.) +0.58 / -0.49 (res.) ± 0.07 (t-dep.)
Sibirtsev, Blunden, Melnitchouk, Thomas (2010)	4.7	+ 1.1 - 0.4
Rislow, Carlson (2011)	5.7	± 0.9

accuracy goal : **2.8** (x 10^{-3})

Summary

Proton charge radius

- R_E from μH has 7.7σ difference with determinations based on Lamb shift in electronic H and elastic ep scattering

- Corrections re-visited. Hadronic $\gamma\gamma$ box corrections can be estimated in a dispersive framework : nucleon pole + inelastic ($F_{1,2}$) + subtraction (β_M)
  $\gamma\gamma$ box corrections : around 12% of discrepancy

- ep-scattering : new measurements determination well in agreement with re-analysis of world data including new JLab data.

  difference between both analyses on normalization of G_M at low Q^2 (1% level)

- Next steps : new muonic and electronic Lamb shift measurements underway, measurements of G_E to Q^2 values below 10^{-3} GeV^2

γZ box contributions to Q_W^p

- Sizeable in magnitude (around 7-8 % of Q_W^p for Jlab experiment), dispersive estimate done by several groups

- Error estimate depends largely on model for isospin rotation to extract $F_{1,2}^{\gamma Z}$
  accuracy of model estimates : 1.5 - 3 % of Q_W^p -> OK with 4 % accuracy goal

- Further theory work welcome, corrections less important at lower energies : around 0.4 % of Q_W^p at 137 MeV