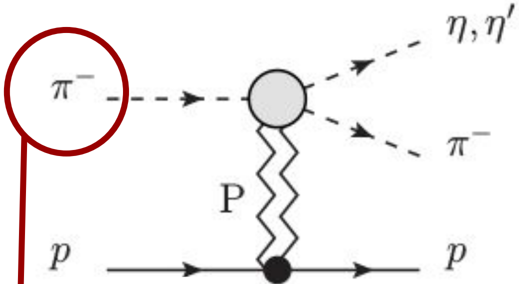
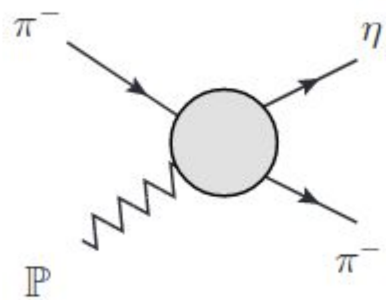
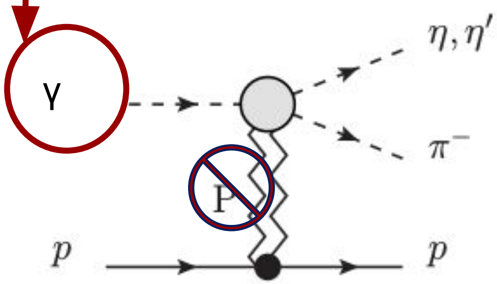
Constraining meson production processes

Jannes Nys

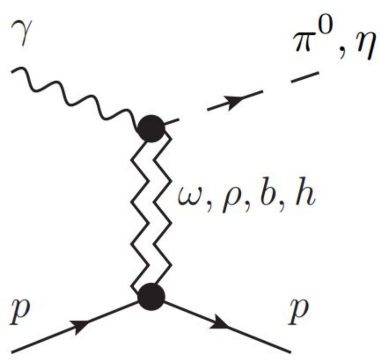
Production process



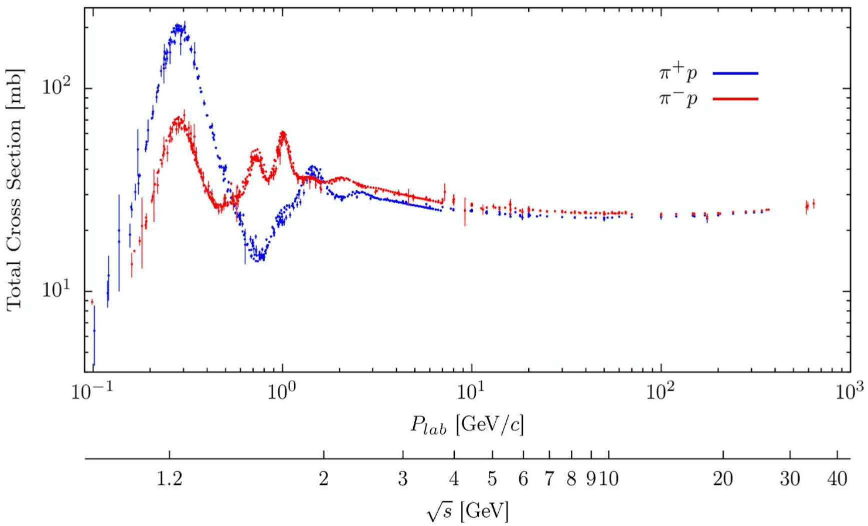




$\pi^0, \eta(0^{-+})$ **have same production as** $\pi_1^0, \eta_1(1^{-+})$



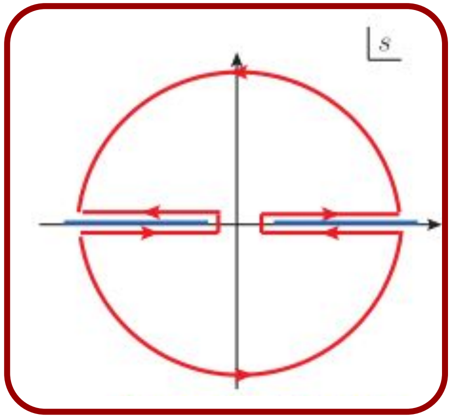
S-matrix theory



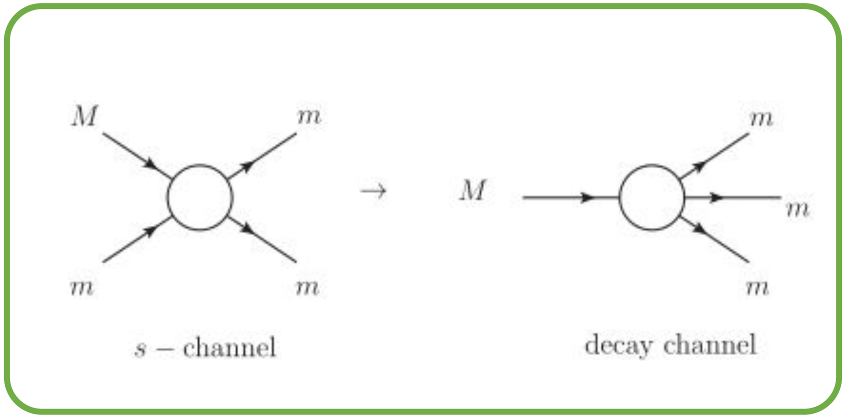
S-matrix theory

Build models: general principles

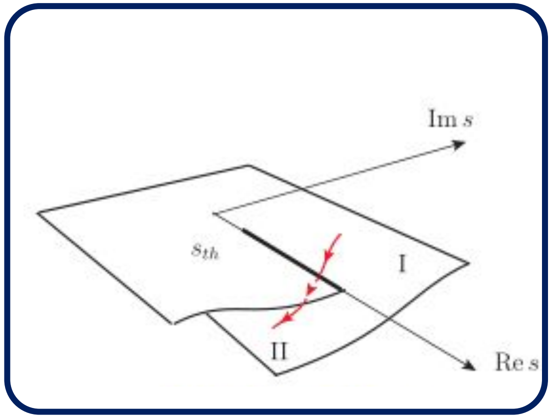
- Analyticity
- Crossing symmetry
- Unitarity
- Lorentz symmetries
- Global symmetries of QCD



ANALYTICITY

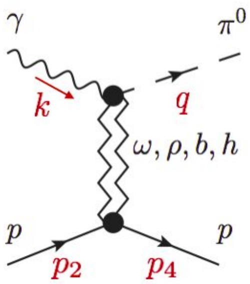


CROSSING SYMMETRY



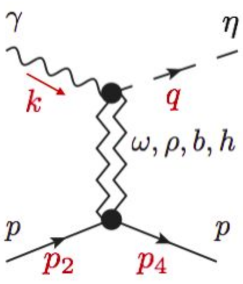
UNITARITY

Neutral meson production

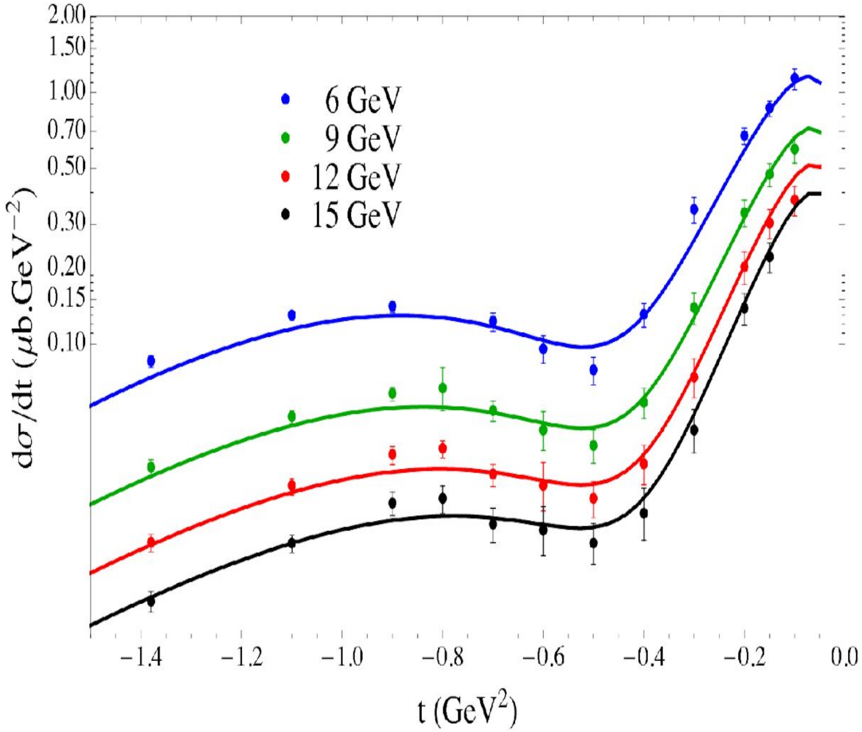


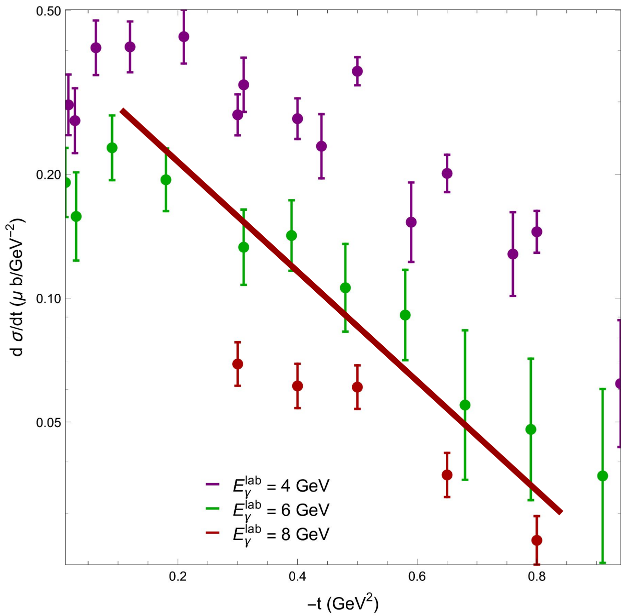
$$\gamma N \to \pi^0 N$$

$$\gamma N \to \eta N$$



Need information on t-dependence

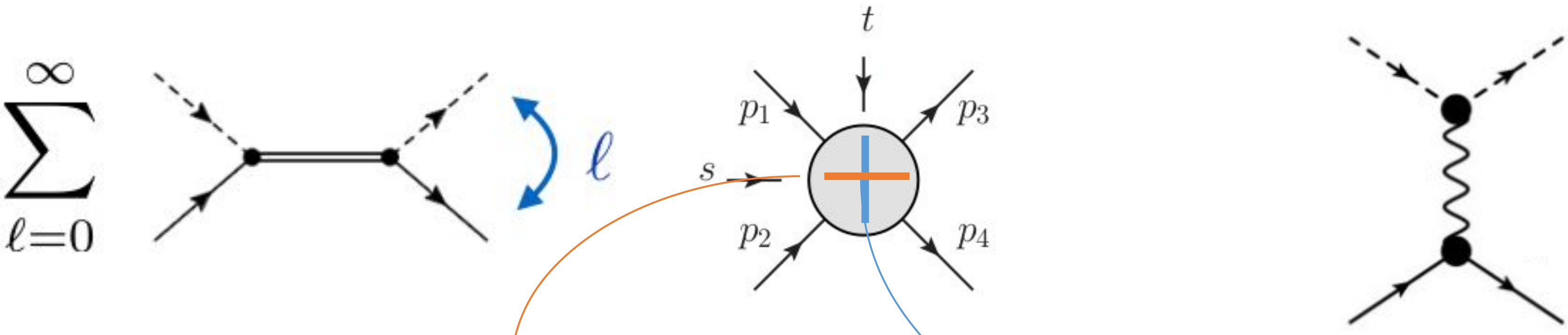


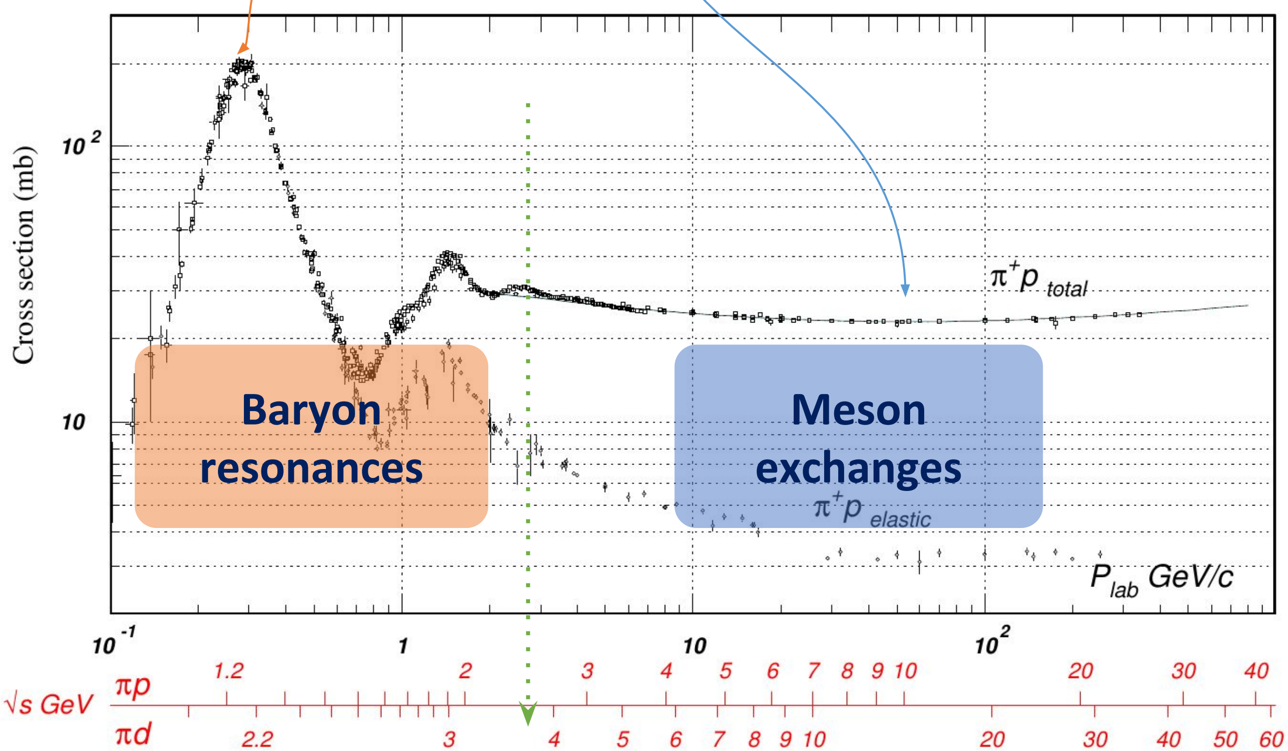


[V. Mathieu *et al.*, PRD92 (2015) 074013]

[J.N. *et al.*, PRD95 (2017) 034014]

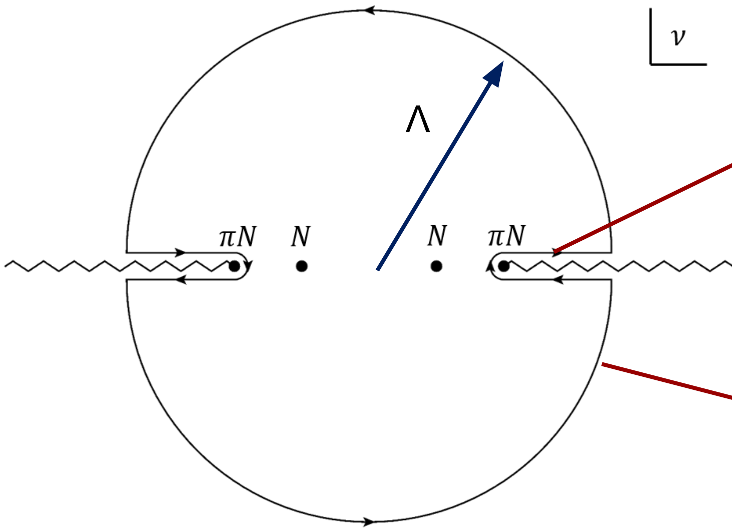
Analytic constraints





Connect low- and high-energy dynamics.

Finite-Energy Sum Rules

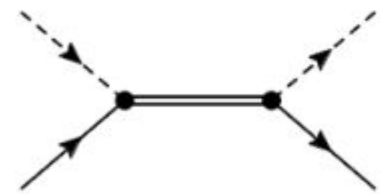


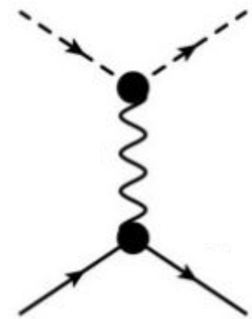
$$\mathrm{Im}\, A_i^{\mathrm{PWA}}(\nu, t)$$

INPUT

$$\beta_i(t)\,\nu^{\alpha_i(t)}$$ ($\nu \approx s$)

OUTPUT





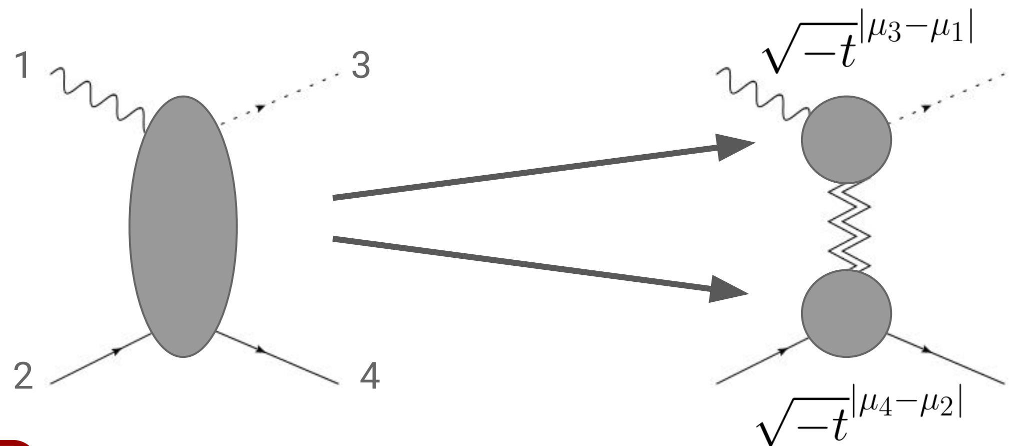
High energies: Regge theory (meson exchanges)

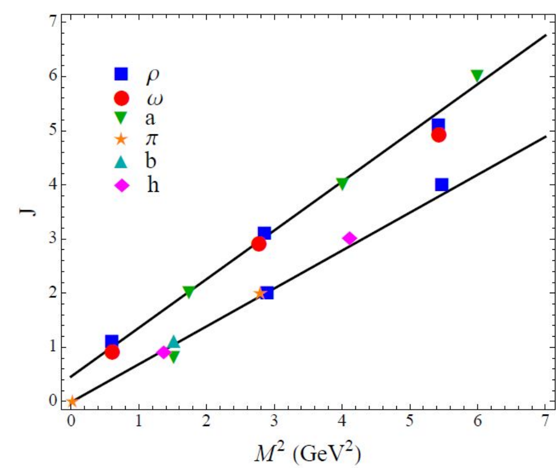
Low energies: partial-wave analyses (baryon resonances)

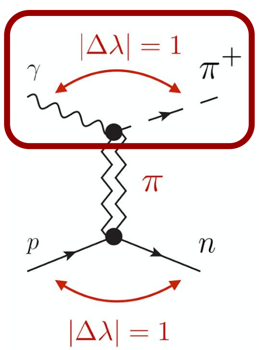
- SAID, MAID, Bonn-Gatchina, Julich-Bonn,...

High-energy model

- Contribution from photon and baryon vertex
- Suppresses amplitudes in forward direction (t=0)







$$A_{\mu_4\mu_3\mu_2\mu_1}(s,t) = \beta^{\text{top}}_{\mu_3\mu_1}(t)\beta^{\text{bottom}}_{\mu_4\mu_2}(t)R(s,t)$$

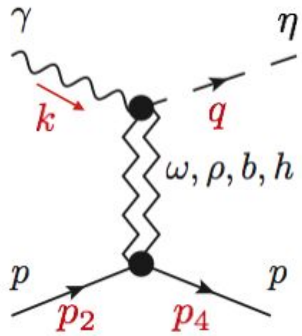
$s^{\alpha(t)}$

Choice of amplitudes

$$A_{\lambda';\lambda\lambda_\gamma}(s,t) = \overline{u}_{\lambda'}(p')\left(\sum_{k=1}^{4} A_k(s,t) M_k\right) u_\lambda(p)$$

- No kinematic singularities
- No kinematic zeros
- Discontinuities:
    - Unitarity cut
    - Nucleon pole

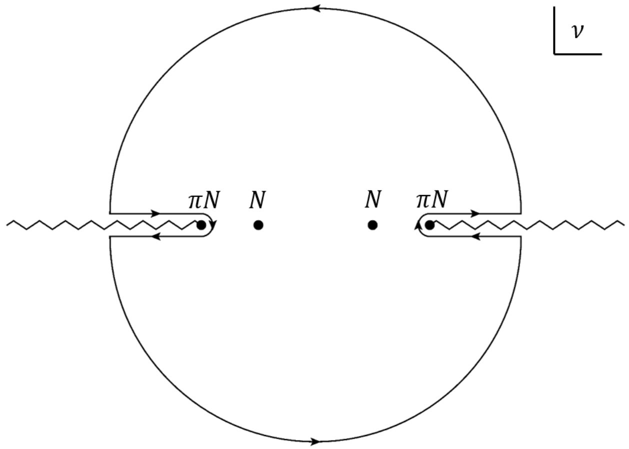
$$M_1 = \frac{1}{2}\gamma_5\gamma_\mu\gamma_\nu F^{\mu\nu}\,,$$
$$M_2 = 2\gamma_5 q_\mu P_\nu F^{\mu\nu}\,,$$
$$M_3 = \gamma_5\gamma_\mu q_\nu F^{\mu\nu}\,,$$
$$M_4 = \frac{i}{2}\epsilon_{\alpha\beta\mu\nu}\gamma^\alpha q^\beta F^{\mu\nu}\,.$$

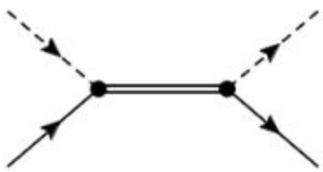


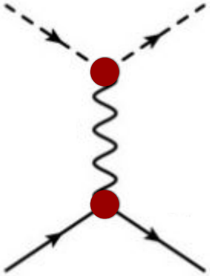
A_i	I^G	J^{PC}	η	Leading exchanges
A_1	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$
A_2'	$0^-, 1^+$	$(1,3,5,...)^{+-}$	-1	$h_1(1170), b_1(1235)$
A_3	$0^-, 1^+$	$(2,4,...)^{--}$	-1	$\rho_2(??), \omega_2(??)$
A_4	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$

$$\gamma p \to \eta p\,, \qquad A = (\omega + h + \omega_2) + (\rho + b + \rho_2)$$
$$\gamma n \to \eta n\,, \qquad A = (\omega + h + \omega_2) - (\rho + b + \rho_2)$$

Finite-Energy Sum Rules

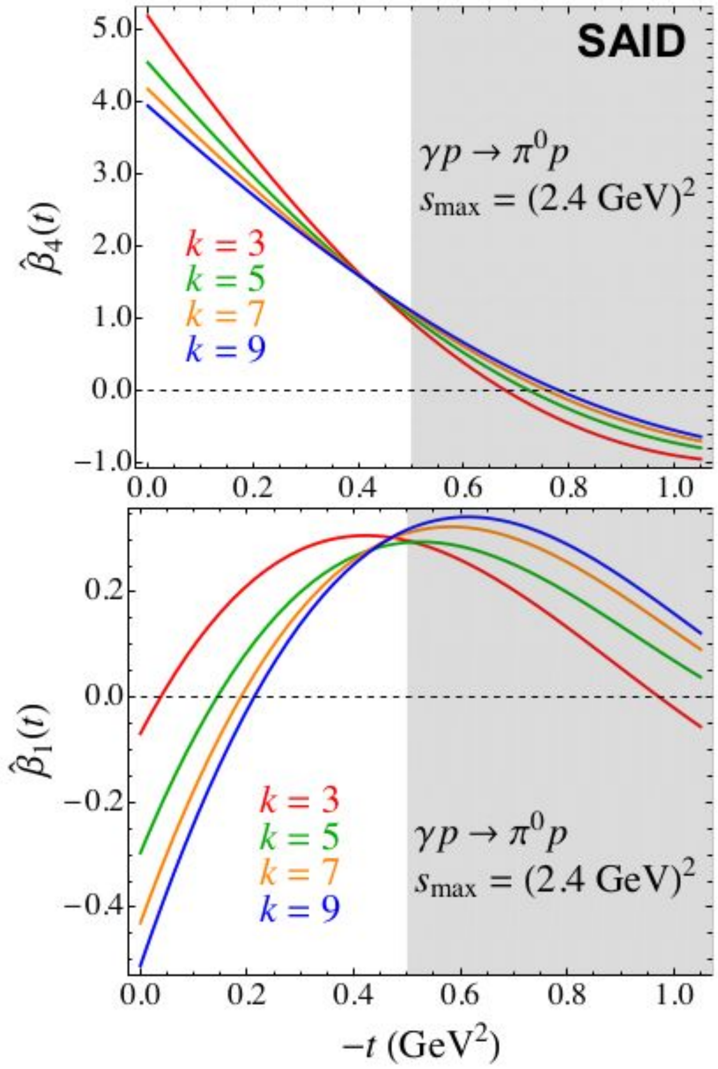




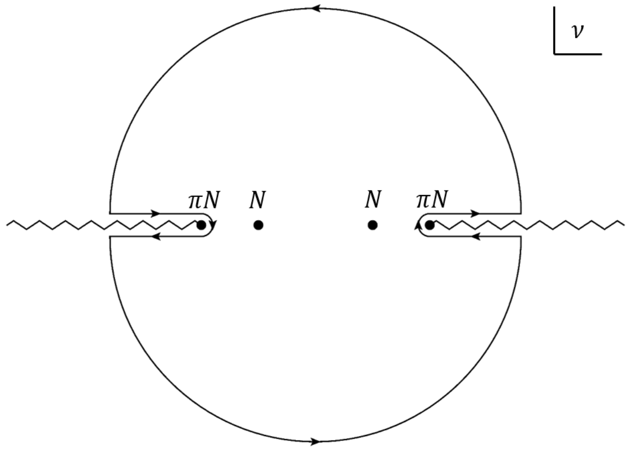


$$\int_0^\Lambda \mathrm{Im}\ A_i(\nu,t)\nu^k d\nu = \beta_i(t)\frac{\Lambda^{\alpha(t)+k}}{\alpha(t)+k}$$

$$\boxed{\beta_i(t) = \frac{\alpha(t)+k}{\Lambda^{\alpha(t)+k}}\int_0^\Lambda \mathrm{Im}\ A_i(\nu,t)\nu^k d\nu}$$



Finite-Energy Sum Rules



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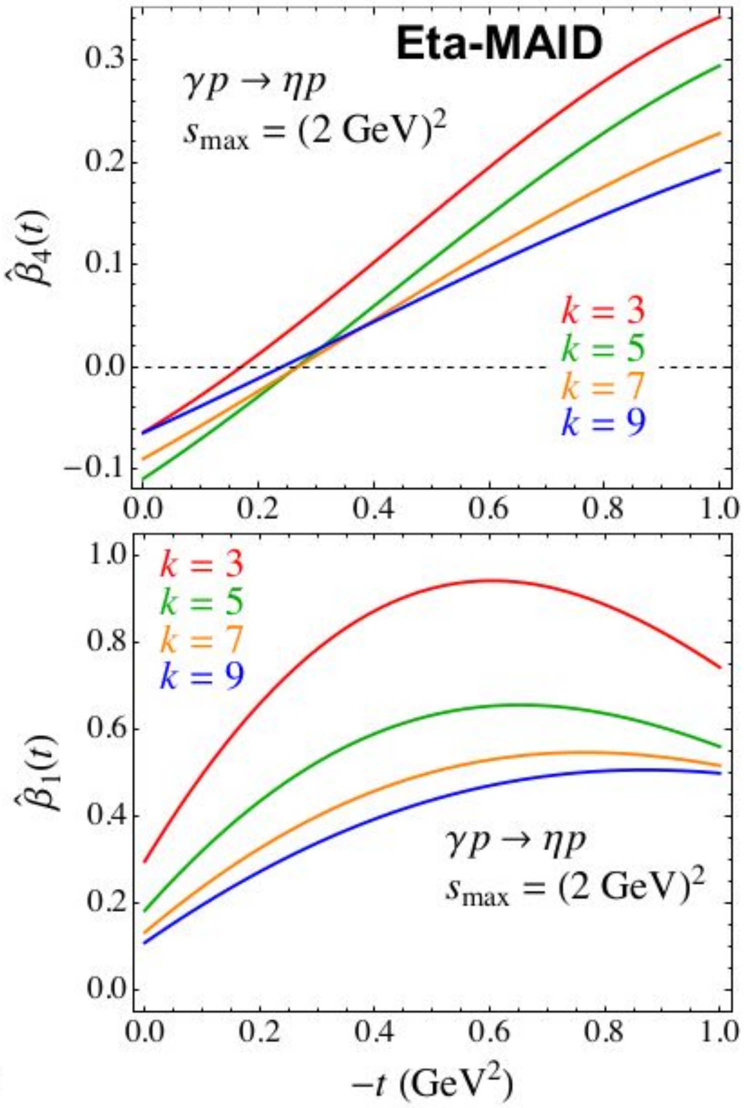
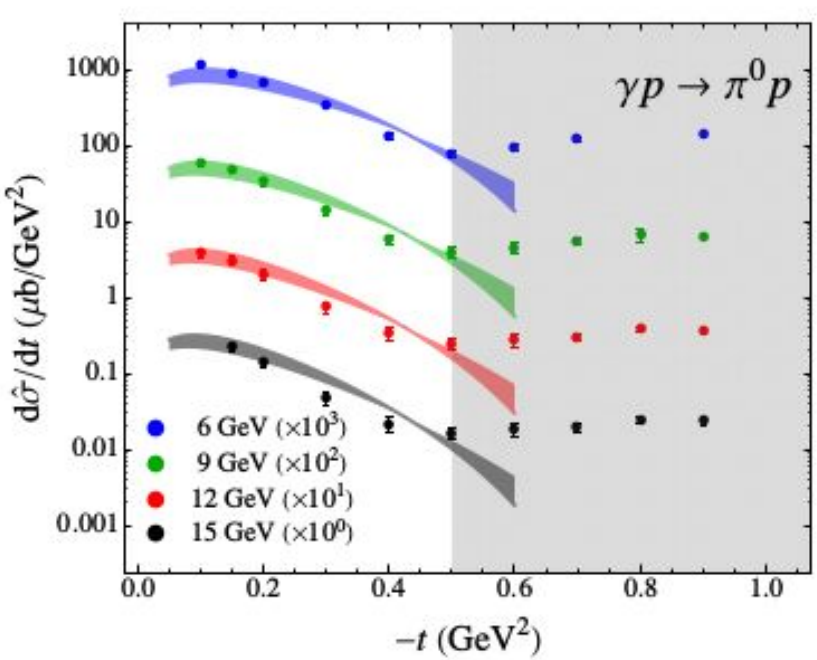
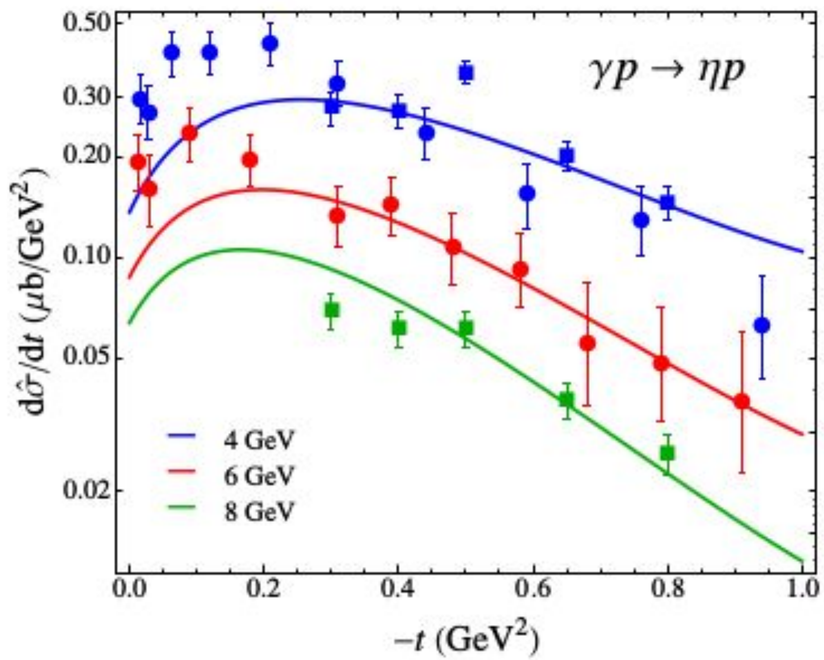


Finite-Energy Sum Rules

[V. Mathieu, J.N. *et al.* (JPAC) 1708.07779 (2017)]





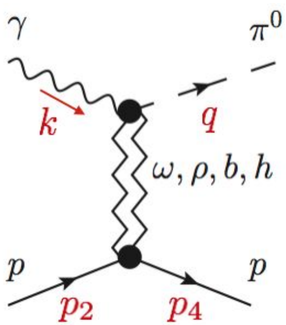
Combine energy regimes

- Low-energy model
- Predict high-energy observables

Two applications

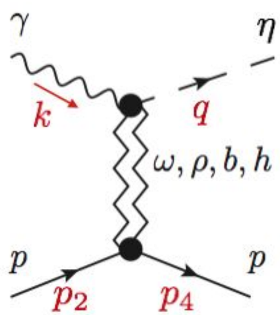
- Understand high-energy dynamics
- Constraining low-energy models

Photoproduction of neutral mesons

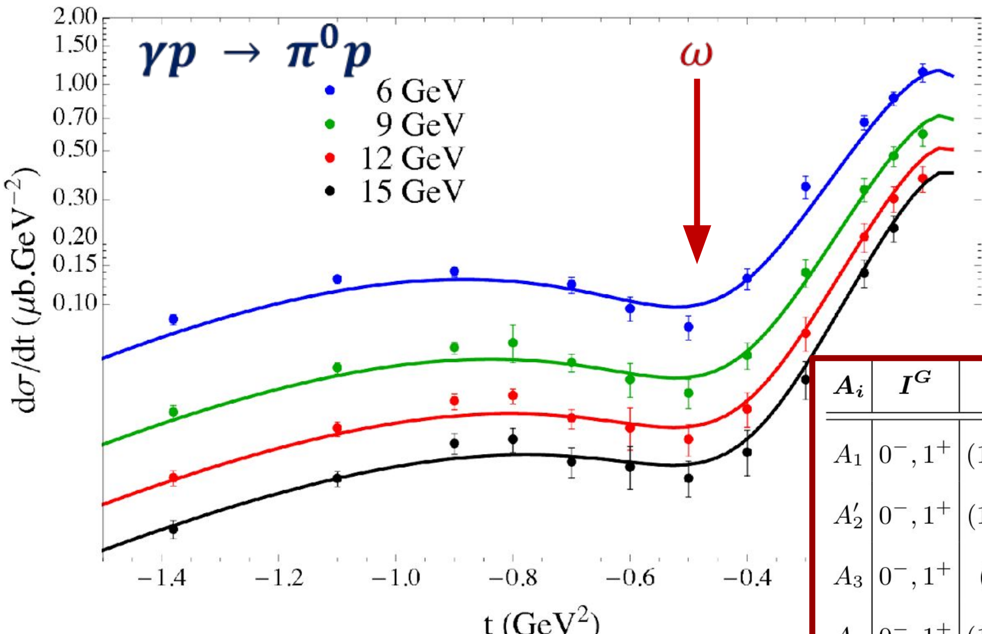


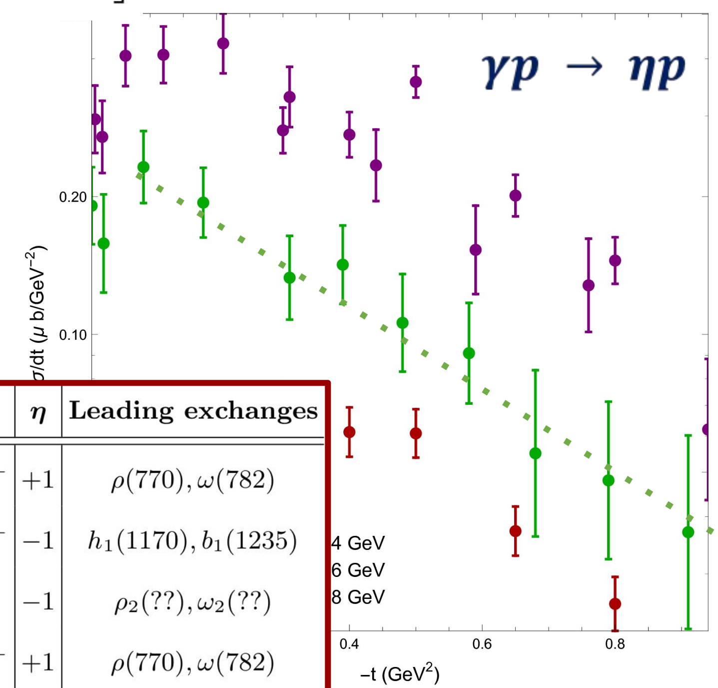
π^0 and η photoproduction

- Same exchanges
  - *isovector* more important for η
- Very different cross section
  - π^0 : dip (ω)
  - η : featureless (ρ)



$$A(\eta) = \sqrt{3}\mathcal{A}\left[A_\rho(\pi^0) + A_b(\pi^0) + \frac{1}{9}\left(A_\omega(\pi^0) + A_h(\pi^0)\right)\right]$$

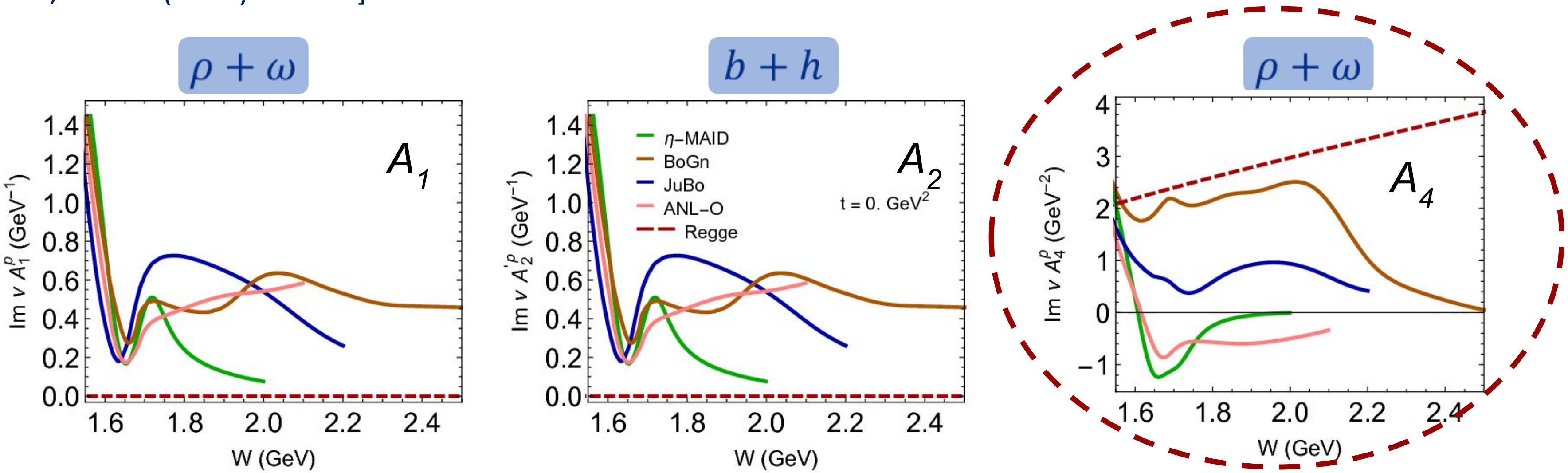




A_i	I^G	J^{PC}	η	Leading exchanges
A_1	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$
A'_2	$0^-, 1^+$	$(1,3,5,...)^{+-}$	-1	$h_1(1170), b_1(1235)$
A_3	$0^-, 1^+$	$(2,4,...)^{--}$	-1	$\rho_2(??), \omega_2(??)$
A_4	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$

Low-energy models (η)

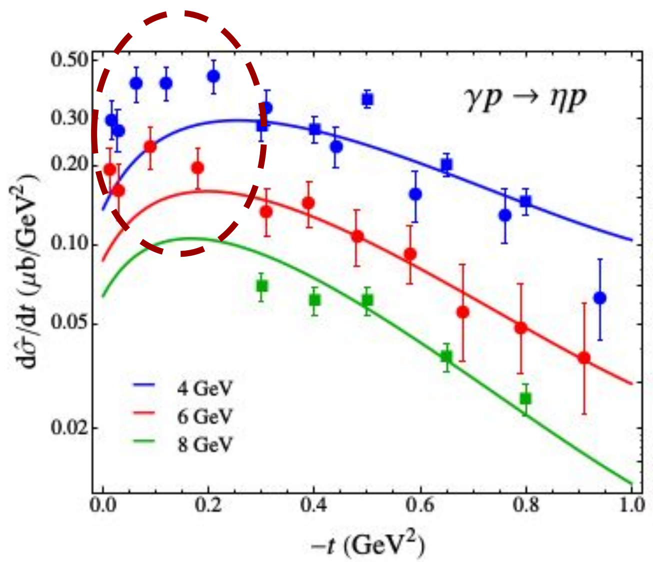
[J.N. *et al.*, PRD95 (2017) 034014]



Ambiguities in the low-energy model (η-MAID)
⟶ Mismatch with high-energy data

Possibilities

- Low-energy model inconsistent
- Cut-off not high enough
  - High mass resonances!

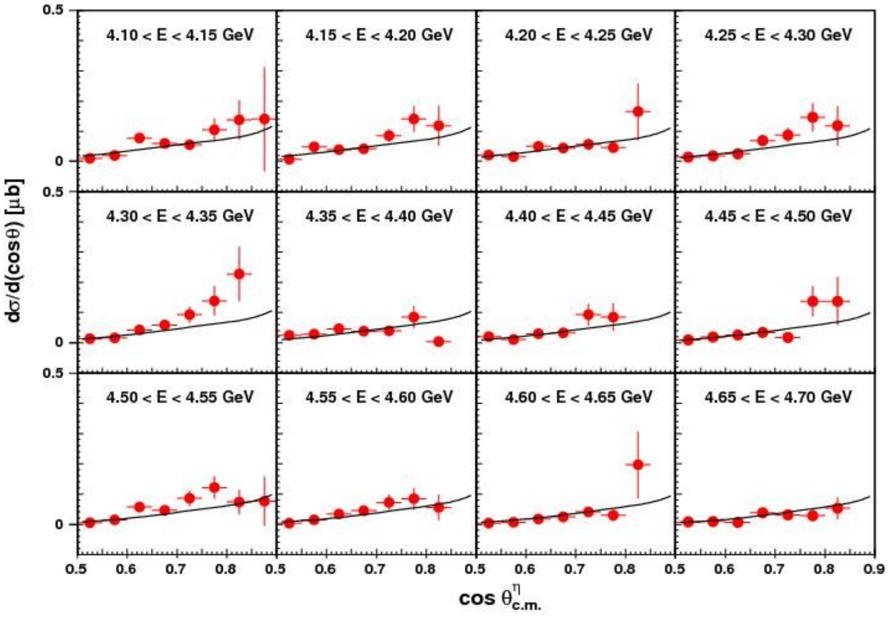


*Pre*dictions for GlueX & CLAS

$$\Sigma = \frac{\sigma_\perp - \sigma_\parallel}{\sigma_\perp + \sigma_\parallel} = \frac{|\rho+\omega|^2 - |b+h|^2}{|\rho+\omega|^2 + |b+h|^2}$$

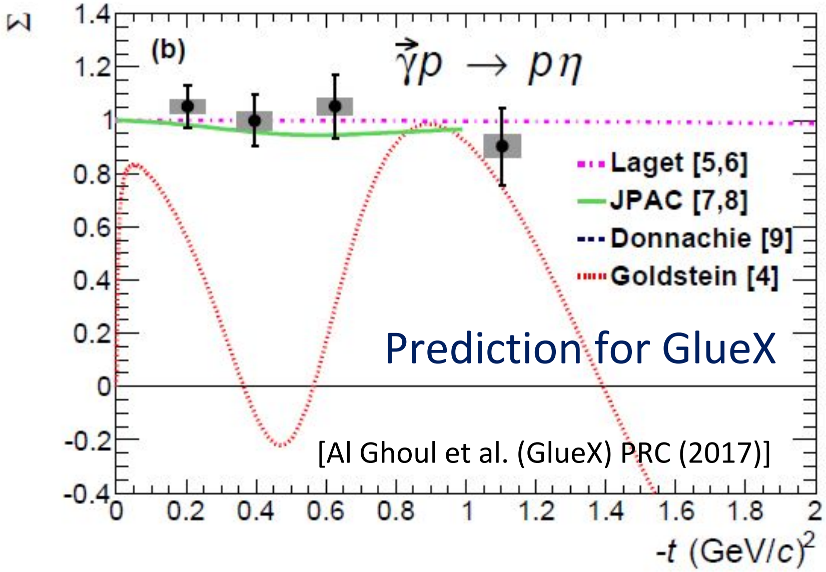
$$\Sigma = +1 \quad : \quad \rho, \omega$$

$$\Sigma = -1 \quad : \quad b, h$$



Preliminary (transition region)
[Courtesy of Zulkaida Akbar (**CLAS**)]

Natural dominant: $\Sigma = +1$
Unnatural dominant: $\Sigma = -1$



Fill up the dip with natural contribution: ρ

η’ photoproduction

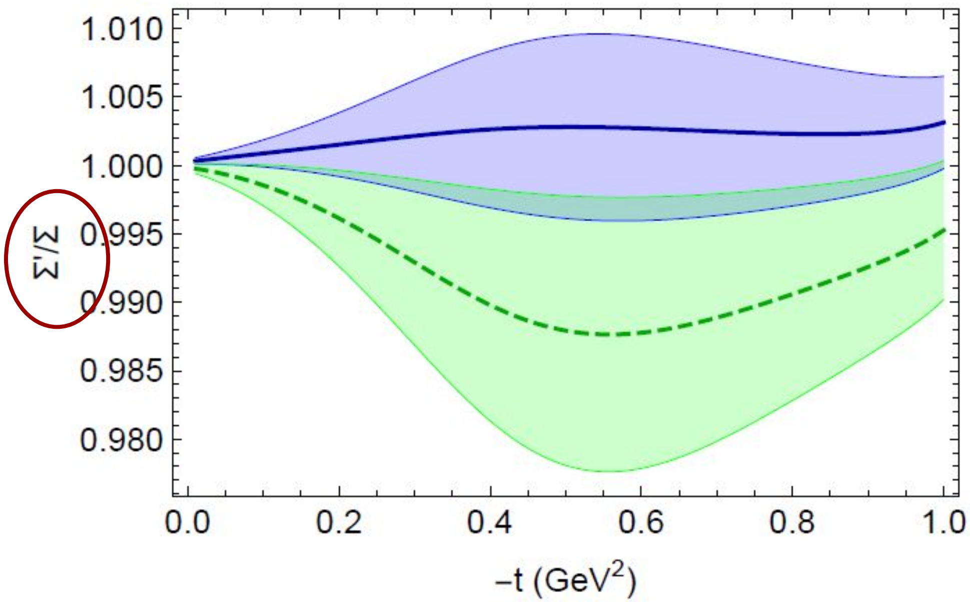
$$\Sigma = \frac{d\sigma_\perp - d\sigma_\parallel}{d\sigma_\perp + d\sigma_\parallel} \qquad (\gamma p \to \eta p)$$

$$\Sigma' = \frac{d\sigma'_\perp - d\sigma'_\parallel}{d\sigma'_\perp + d\sigma'_\parallel} \qquad (\gamma p \to \eta' p)$$

$$\Sigma = \frac{|\rho+\omega|^2 - |b+h|^2}{|\rho+\omega|^2 + |b+h|^2} = \Sigma'$$

$$\Sigma = \frac{|\rho+\omega+\boxed{\phi}|^2 - |b+h+\boxed{h'}|^2}{|\rho+\omega+\phi|^2 + |b+h+h'|^2} \neq \Sigma'$$

V.Mathieu, J.N. *et al.* (JPAC) [PLB774 (2017) 362]



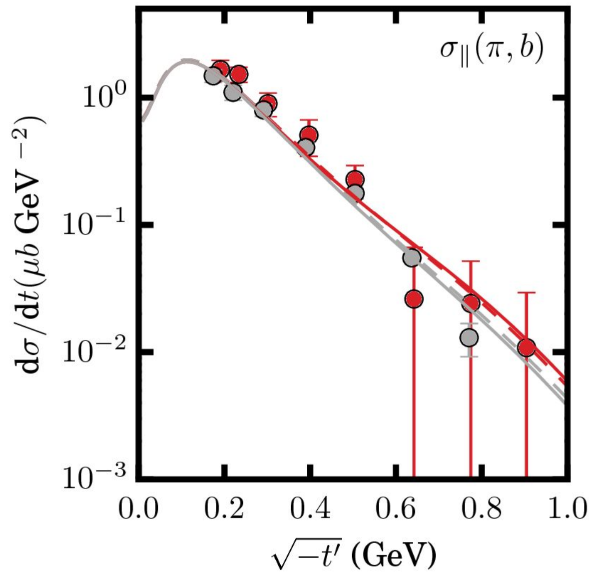
Based on the FESR for η:

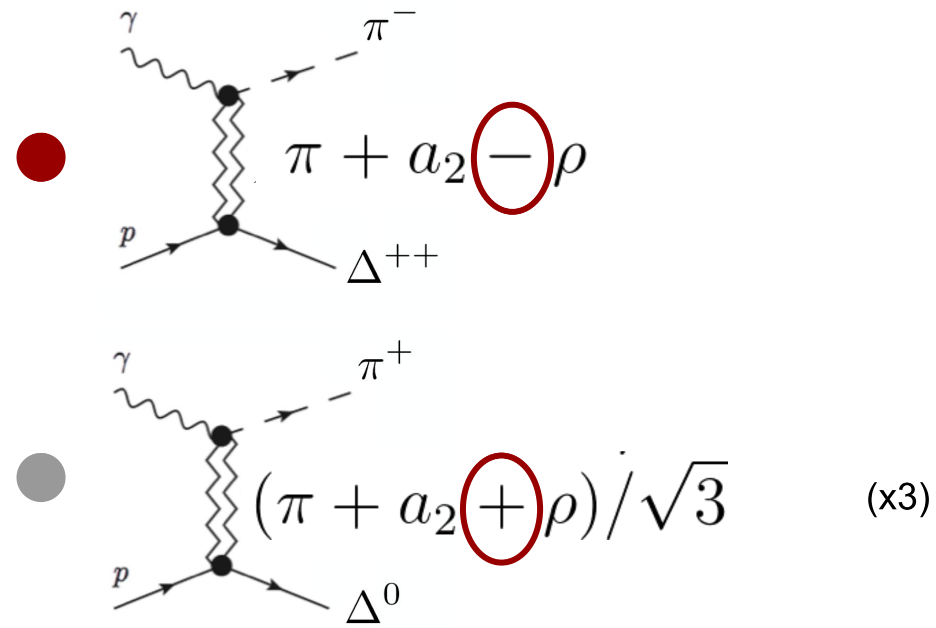
predict beam asymmetry for η’

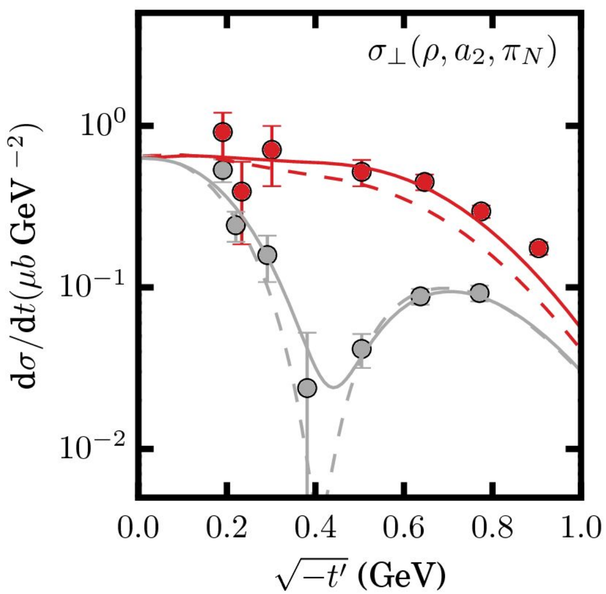
- Same exchanges
- Natural exchanges (ρ,ω) dominant
    - Couplings from radiative decays
    - Mixing angle cancels in ratio
- Unknown behavior of
    - ϕ exchange
    - unnatural exchanges (b,h)

Prediction: **≈ same beam asymmetry**

πΔ photoproduction







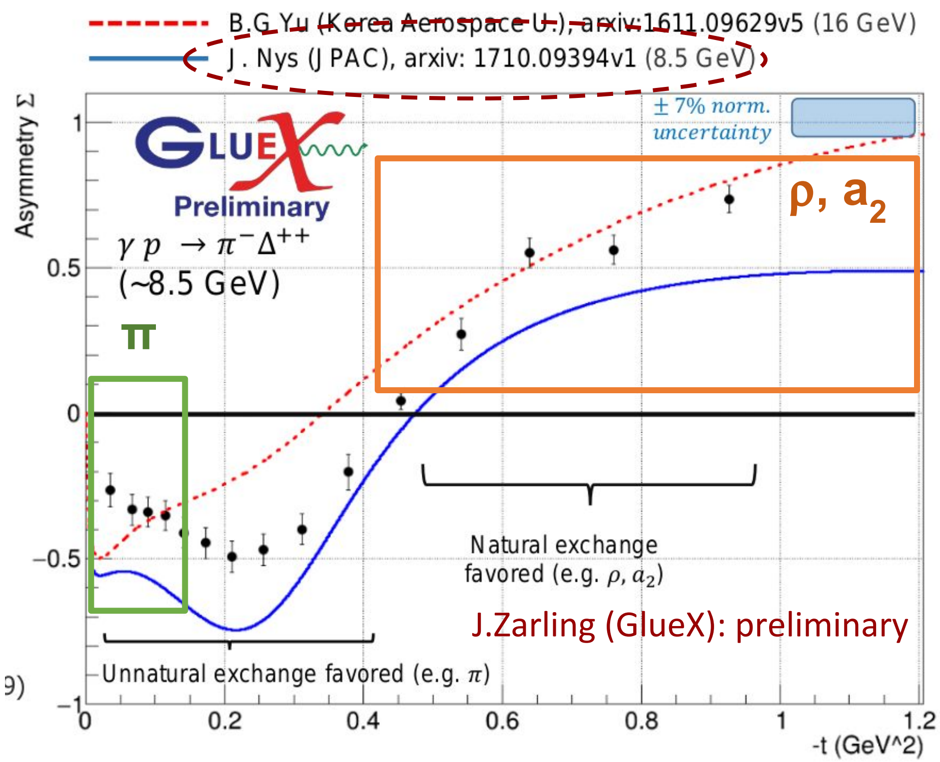
Data available at **16 GeV**

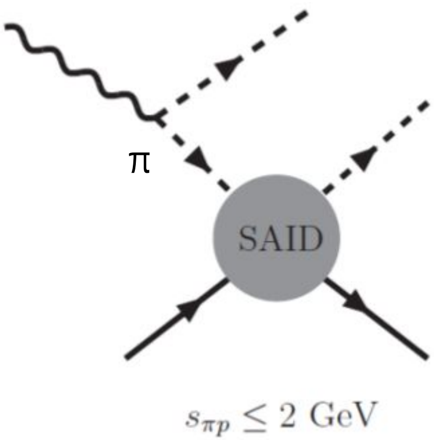
- π-exchange is featureless and entirely fixed
- Strong interference pattern in natural exchange sector
- Negligible role of b exchange

Fix t-dependence and extrapolate to JLab energies (**9 GeV**)

[J.N. *et al.*, PLB779 (2018)]

πΔ photoproduction





Łukasz Bibrzycki (Cracow)

Comparison to GlueX data

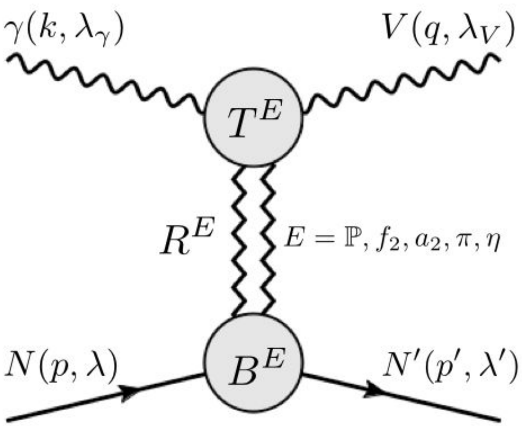
- Confirmation of interference pattern
- High -t: natural, low -t: unnatural
- Mismatch: oddly behaved π exchange
  - Ongoing analysis
  - Experimental or theoretical?

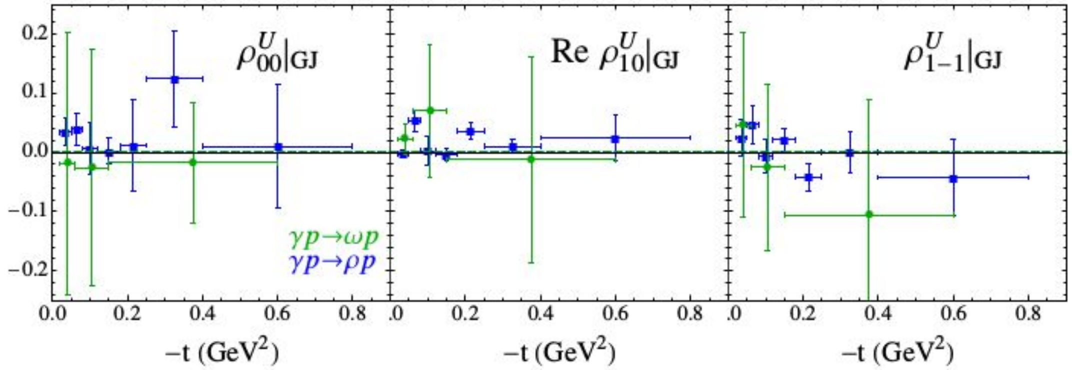
Neutral vector mesons

- Pomeron dominates at high energies
- Isoscalar exchanges dominantly helicity non-flip (λ=λ’)
- Unnatural exchanges: only helicity flip (|λ-λ’|=1)

$$\mathcal{M}_{\substack{\lambda_V,\lambda_\gamma \\ \lambda',\lambda}}(s,t) = \sum_{E=\pi,\eta,\mathbb{P},f_2,a_2} \mathcal{M}^{E}_{\substack{\lambda_V,\lambda_\gamma \\ \lambda',\lambda}}(s,t)$$

$$\mathcal{M}^{\substack{N \\ U}}_{\substack{-\lambda_\gamma,-\lambda_V \\ \lambda,\lambda'}} = \pm(-1)^{\lambda_\gamma-\lambda_V}\mathcal{M}^{\substack{N \\ U}}_{\substack{\lambda_\gamma,\lambda_V \\ \lambda,\lambda'}}$$





$$\rho^{\substack{N \\ U}}_{00} = \frac{1}{2}\left(\rho^0_{00} \mp \rho^1_{00}\right),$$

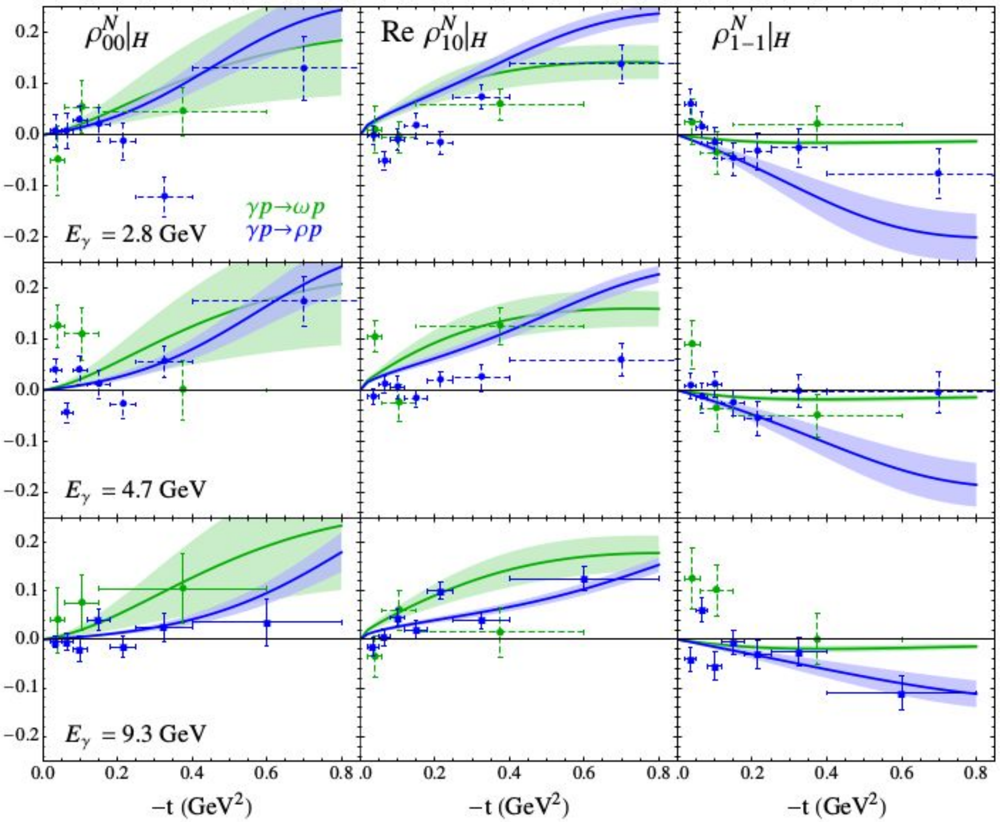
$$\mathrm{Re}\,\rho^{\substack{N \\ U}}_{10} = \frac{1}{2}\left(\mathrm{Re}\,\rho^0_{10} \mp \mathrm{Re}\,\rho^1_{10}\right),$$

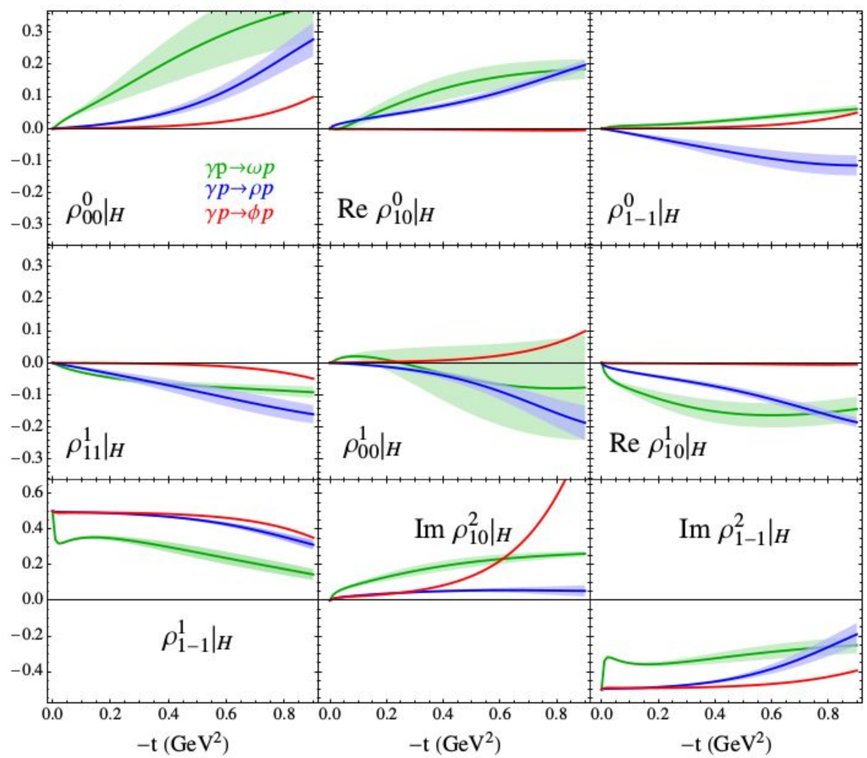
$$\rho^{\substack{N \\ U}}_{1-1} = \frac{1}{2}\left(\rho^1_{1-1} \pm \rho^1_{11}\right).$$

[V.Mathieu, J.N. *et al.*, (2018) arXiv:1802.09403]

Neutral vector mesons

$$\mathcal{M}_{\lambda_V,\lambda_\gamma \atop \lambda',\lambda}(s,t) = \sum_{E=\pi,\eta,\mathbb{P},f_2,a_2} \mathcal{M}^E_{\lambda_V,\lambda_\gamma \atop \lambda',\lambda}(s,t)$$





[V.Mathieu, J.N. *et al.*, (2018) arXiv:1802.09403]

Summary

Theory support for GlueX and CLAS with JPAC

- *Various photoproduction reactions analyzed*
  - πN, $\pi\Delta$, ηN, $\eta' N$ + many more
  - Comparison to first GlueX data
    - Unnatural exchanges negligible
    - Natural exchanges dominate
  - Importance of analytic constraints (FESR)
  - Connection between baryon spectroscopy and high-energy data
  - SDME predictions for neutral meson prediction (Pomeron dominated)

http://www.indiana.edu/~jpac/

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JPAC acknowledges support from DOE and NSF

NEWS

Photoproduction:

1. High energy model for $\pi\Delta$ photoproduction beam asymmetry: (in construction)
2. High energy model for ρ^0, ω, ϕ spin density matrix elements: $\gamma p \to Vp$ page
3. High energy model for η' photoproduction beam asymmetry: $\gamma p \to \eta^{(')} p$ page
4. High energy model for η photoproduction: $\gamma p \to \eta p$ page
5. High energy model for π^0 photoproduction: $\gamma p \to \pi^0 p$ page
6. High energy model for J/ψ photoproduction: $\gamma p \to J/\psi p$ page

Backup

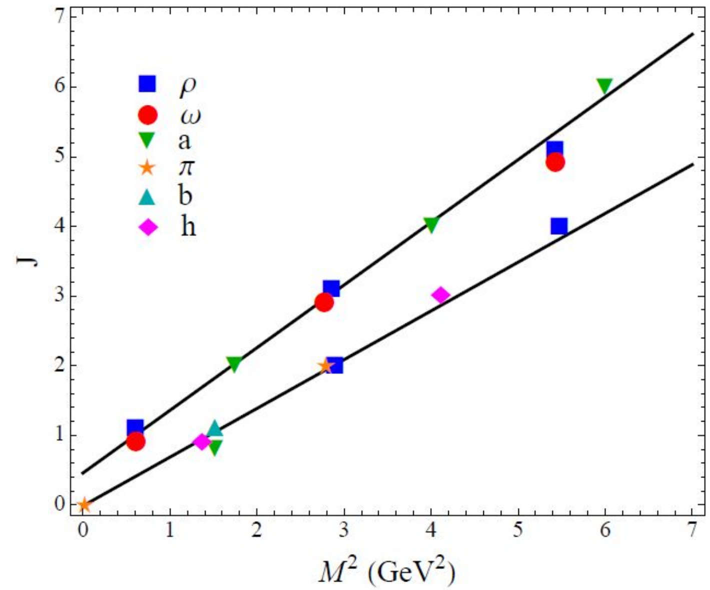
High-energy model

Regge pole model

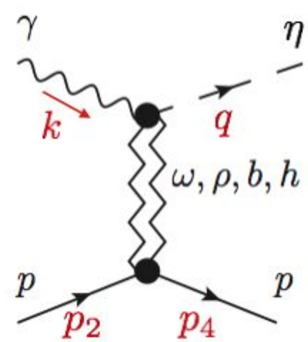
$J(m^2)$

$$A_i(\nu,t) = -\beta_i(t)\frac{\pm 1 + e^{-i\pi\alpha(t)}}{\sin \pi\alpha(t)}\nu^{\alpha(t)}$$

Dominant: vector exchanges



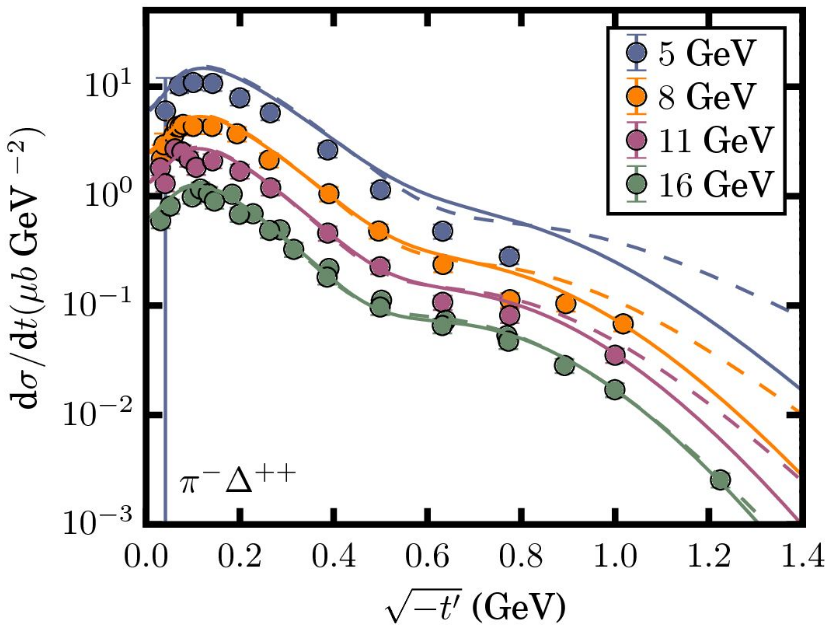
A_i	I^G	J^{PC}	η	Leading exchanges
A_1	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$
A_2'	$0^-, 1^+$	$(1,3,5,...)^{+-}$	-1	$h_1(1170), b_1(1235)$
A_3	$0^-, 1^+$	$(2,4,...)^{--}$	-1	$\rho_2(??), \omega_2(??)$
A_4	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$

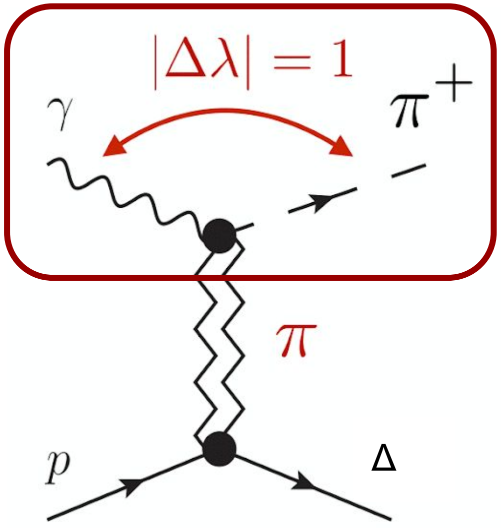


$$\gamma p \to \eta p\,, \qquad A = (\omega + h + \omega_2) + (\rho + b + \rho_2)$$
$$\gamma n \to \eta n\,, \qquad A = (\omega + h + \omega_2) - (\rho + b + \rho_2)$$

πΔ photoproduction

J.N *et al.* (JPAC) [arXiv:1710.09394]



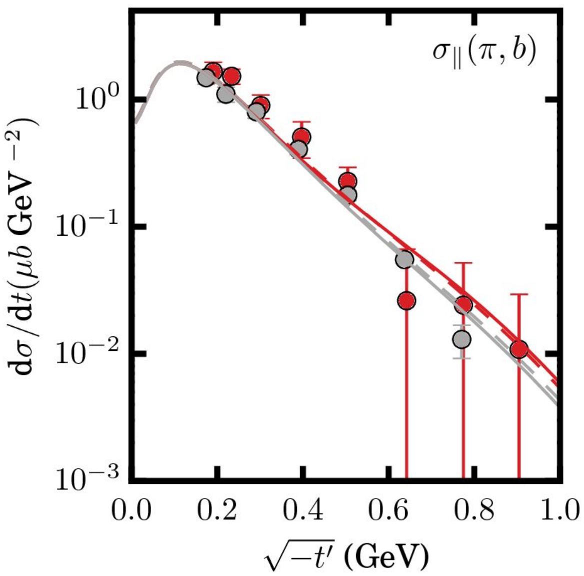


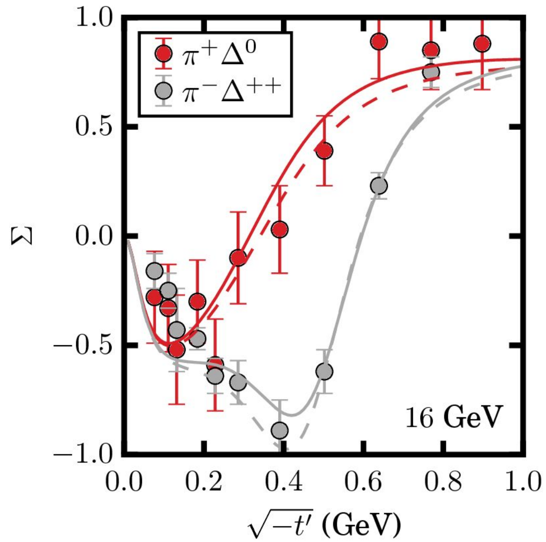
$$A^{10}_{-\frac{1}{2}\frac{1}{2}} \propto \frac{\boxed{-t}}{m_\pi^2 - t} \quad \rightarrow \frac{-m_\pi^2}{m_\pi^2 - t}$$

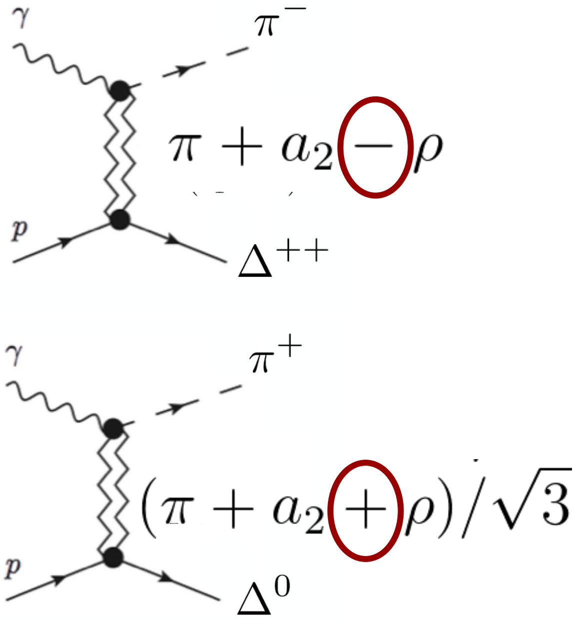
From residue factorization:
$\sqrt{-t}$ **for each helicity flip**
Not seen in data → contact term

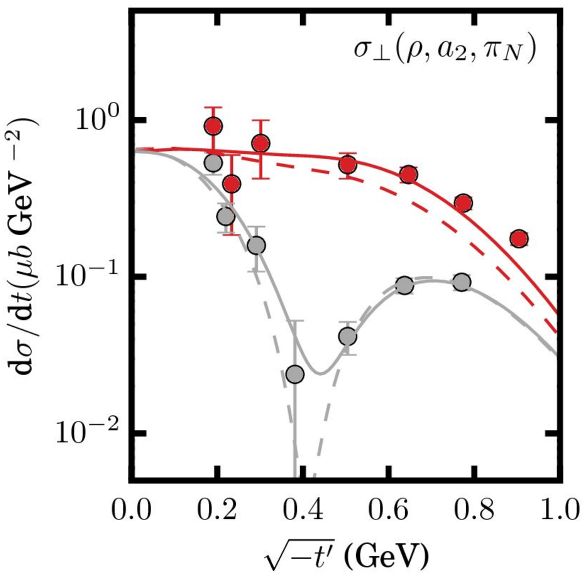
πΔ photoproduction

J.N. *et al.* (JPAC) [arXiv:1710.09394]







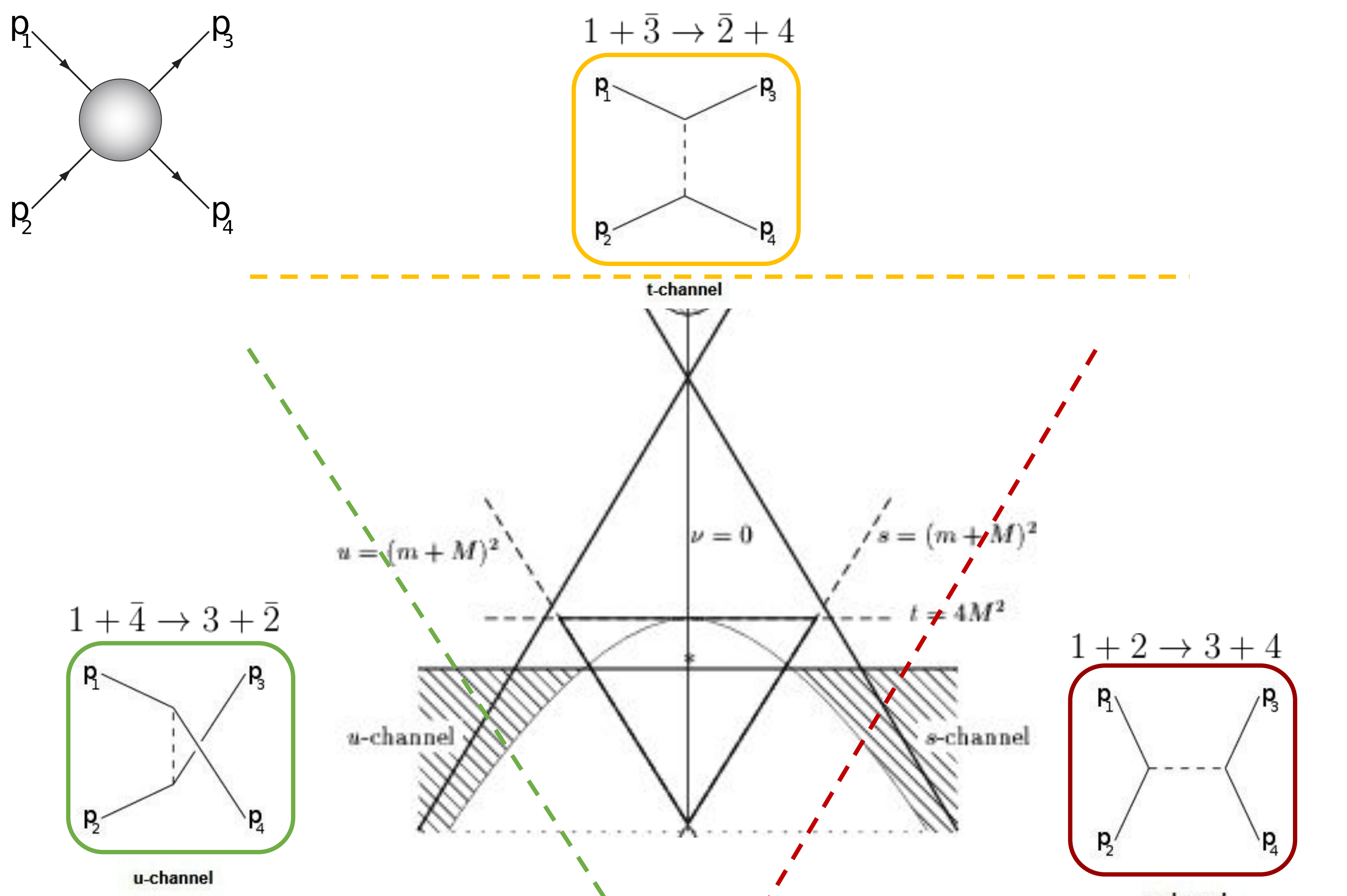


Data available at **16 GeV**

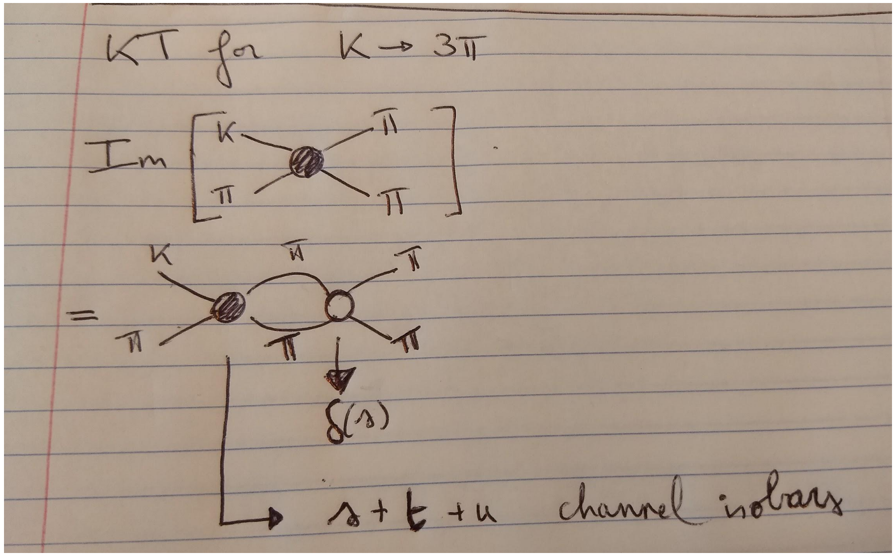
- π-exchange is featureless and entirely fixed
- Strong interference pattern in natural exchange sector
- Negligible role of b exchange

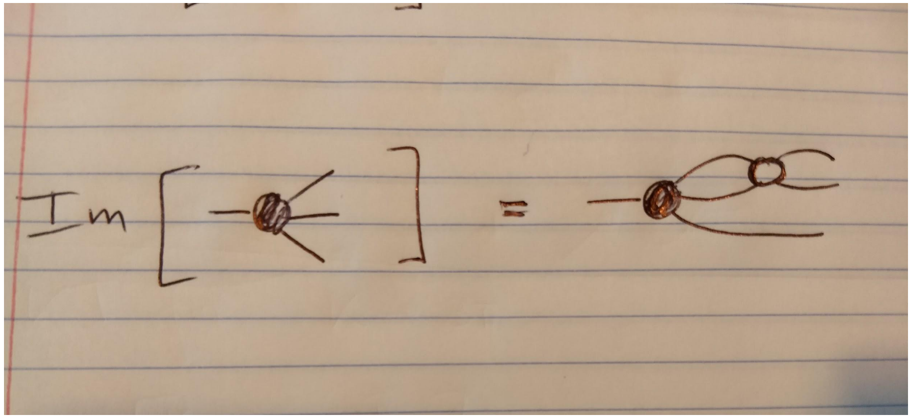
Fix t-dependence and extrapolate to JLab energies (**9 GeV**)

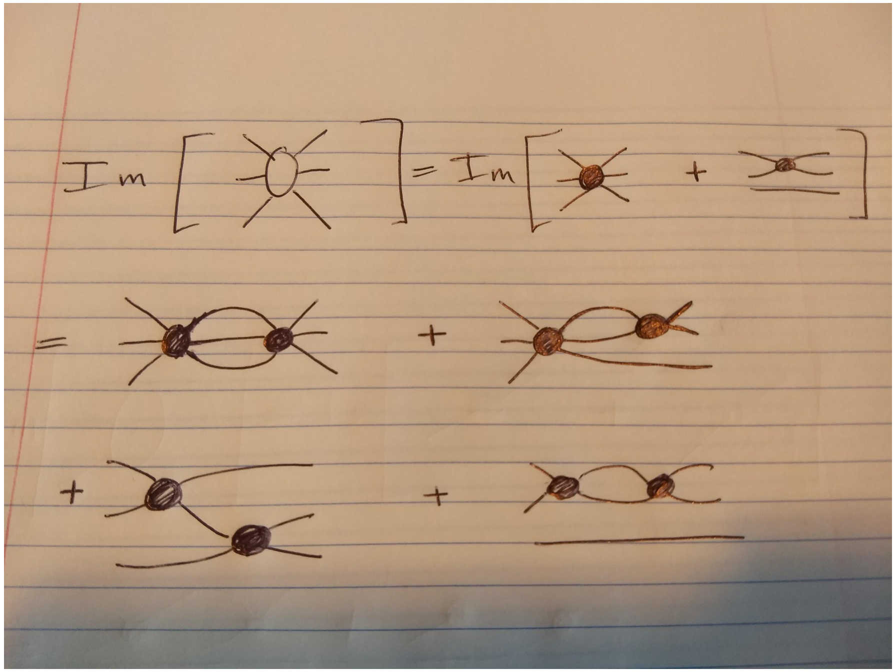
Using the right degrees of freedom



KT approach







•

$$\frac{1}{\sqrt{2s}}\left(A_{+,+1}+A_{-,-1}\right)=\sqrt{-t}A_4 \tag{19}$$

$$\frac{1}{\sqrt{2s}}\left(A_{+,-1}-A_{-,+1}\right)=A_1 \tag{20}$$

$$\frac{1}{\sqrt{2s}}\left(A_{+,+1}-A_{-,-1}\right)=\sqrt{-t}A_3 \tag{21}$$

$$\frac{1}{\sqrt{2s}}\left(A_{+,-1}+A_{-,+1}\right)=-A_2'=-(A_1+tA_2) \tag{22}$$

Thus, at high energies the invariants A_3 and A_4 (A_1 and A_2') correspond to the s-channel nucleon-helicity non-flip (flip), respectively. Combining Eqs. (20) and (22) we obtain

$$A_{-,+1}=-\frac{s}{\sqrt{2}}\left(A_2'+A_1\right) . \tag{23}$$

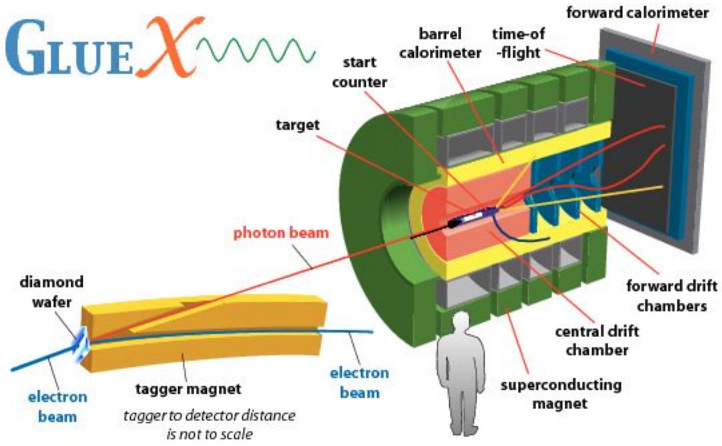
$$A_{\mu_f,\mu_i\,\mu_\gamma} \underset{t\to 0}{\sim} (-t)^{n/2} , \tag{17}$$

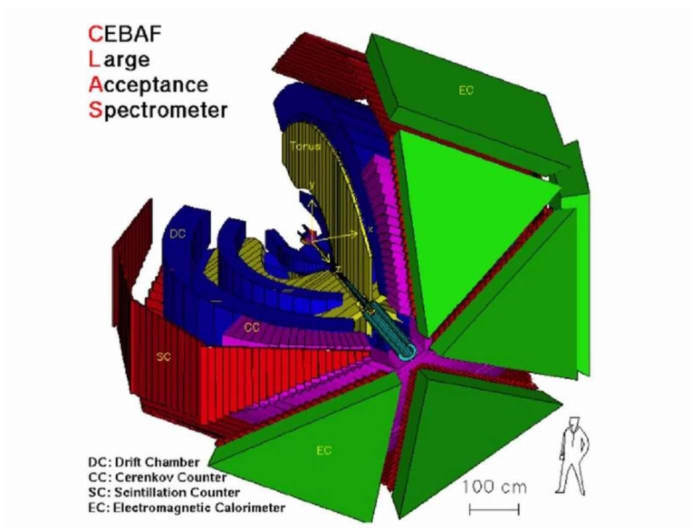
where $n=|(\mu_\gamma-\mu_i)-(-\mu_f)|\geq 0$ is the net s-channel helicity flip. This is a weaker condition than the one imposed by angular-momentum conservation on factorizable Regge amplitudes,

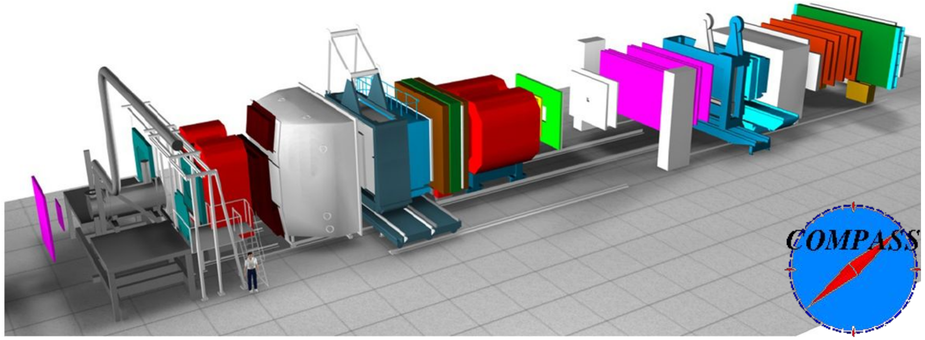
$$A_{\mu_f,\mu_i\,\mu_\gamma} \underset{t\to 0}{\sim} (-t)^{(n+x)/2} , \tag{18}$$

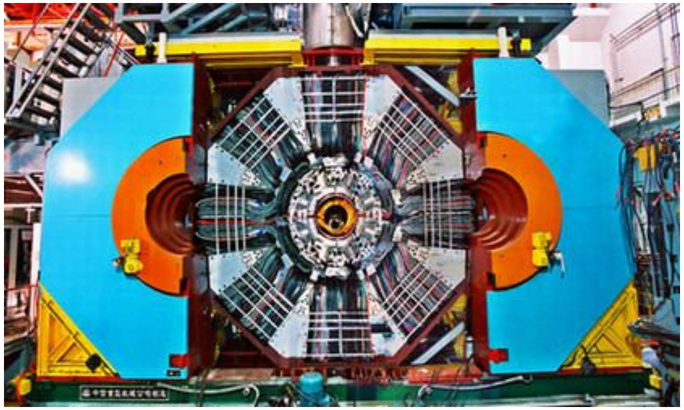
where $n+x=|\mu_\gamma|+|\mu_i-\mu_f|\geq 1$. We summarize the

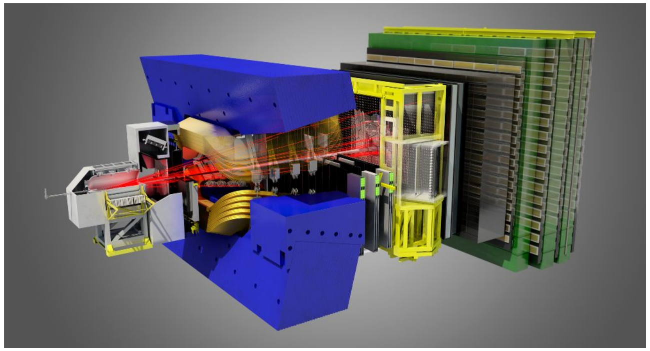
Spectroscopy (experiment)





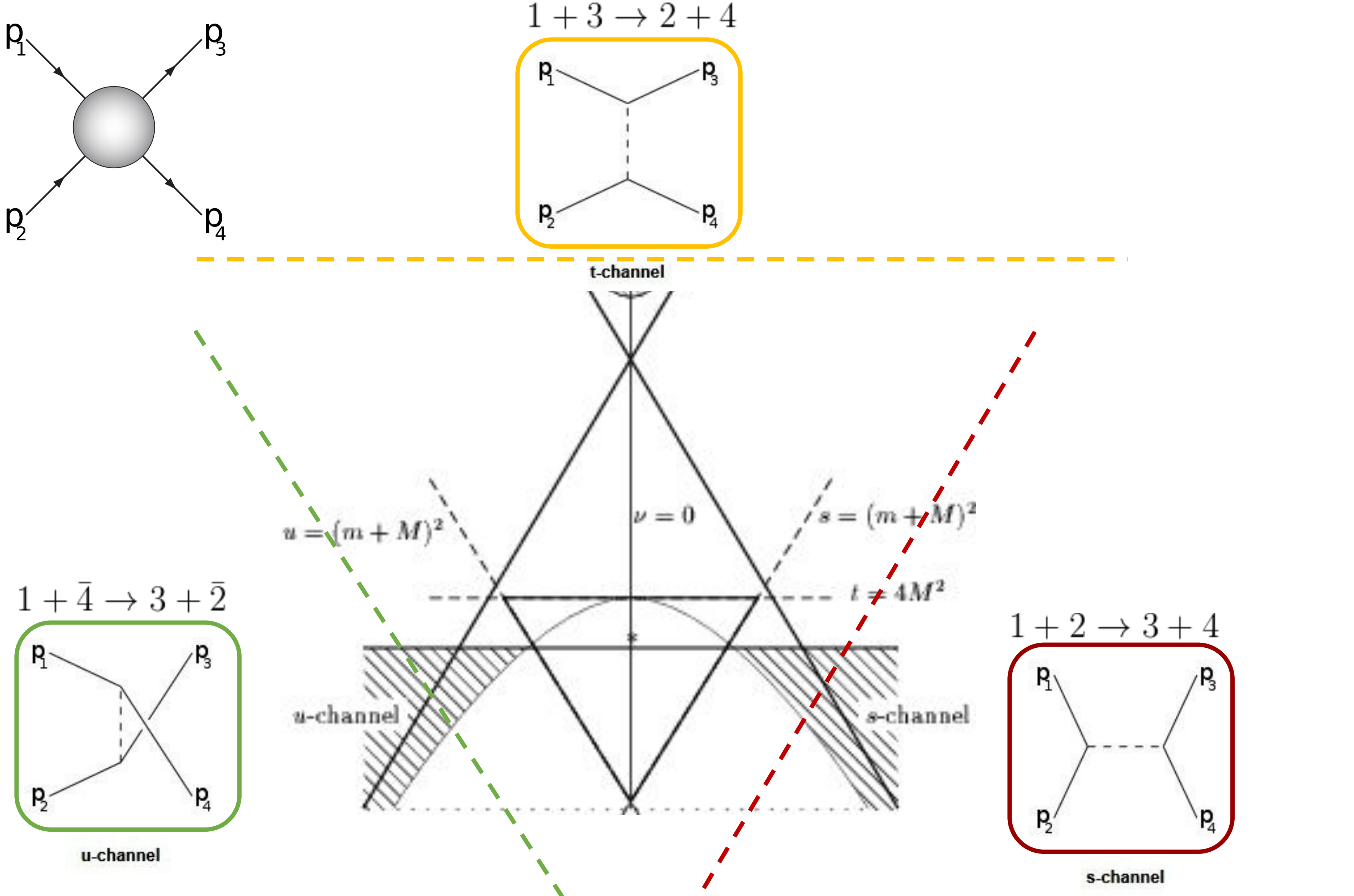




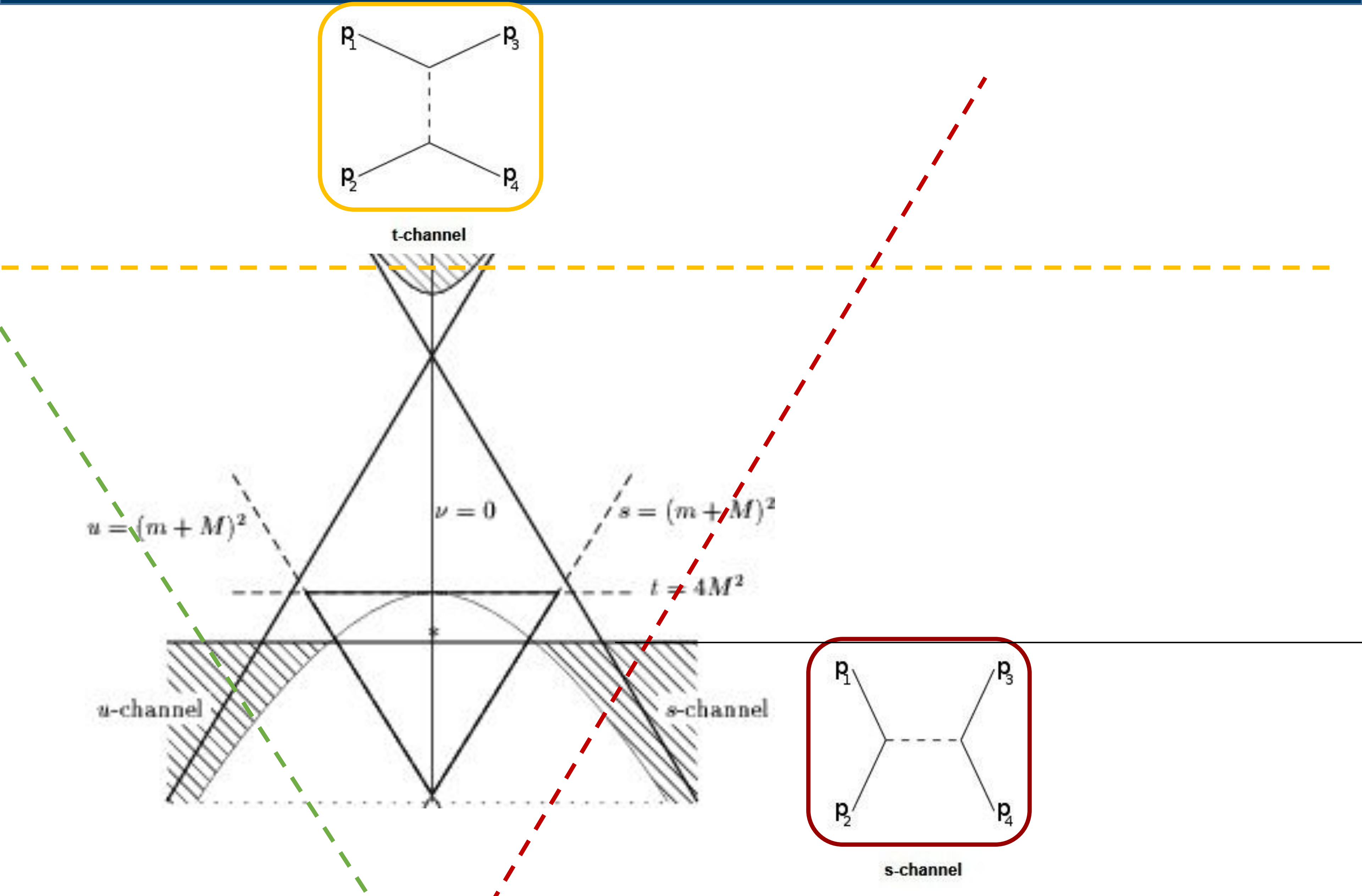


$$\alpha_{1,4}^{(\sigma)} \equiv \alpha_N(t) = 0.9(t - m_\rho^2) + 1$$
$$\alpha_{2,3}^{(\sigma)} \equiv \alpha_U(t) = 0.7(t - m_\pi^2) + 0$$

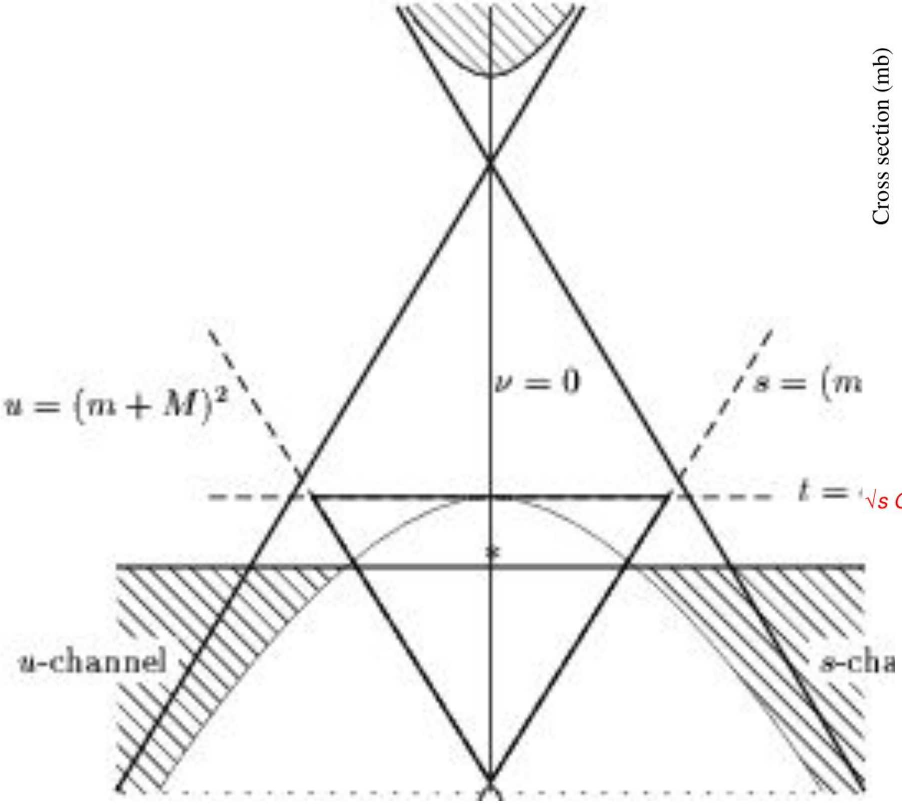
Using the right degrees of freedom

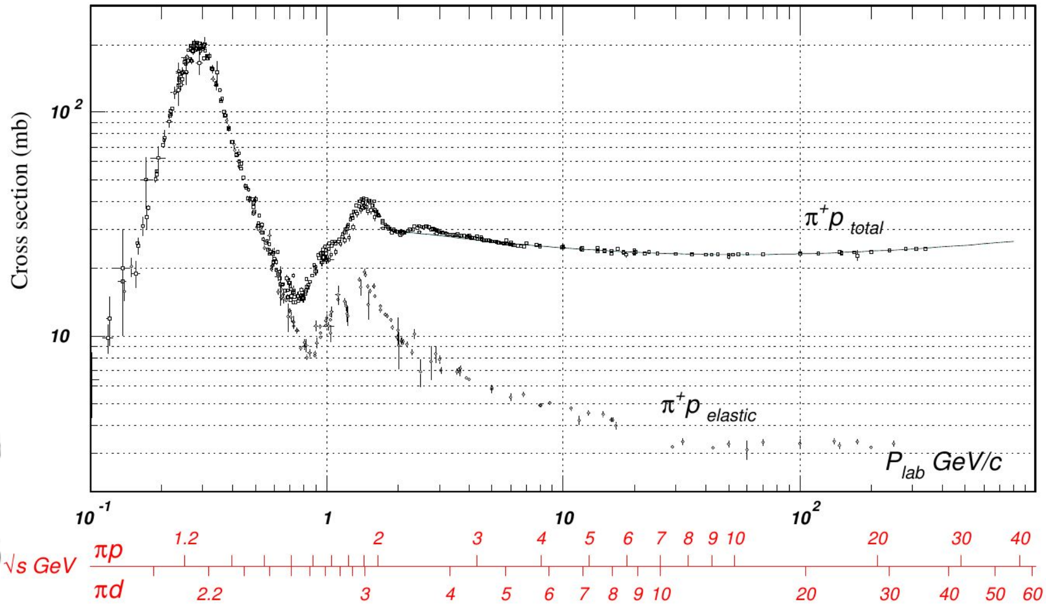


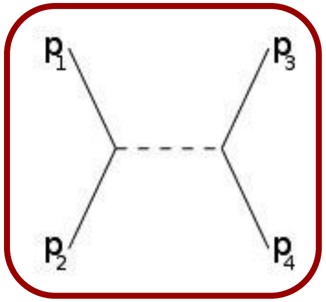
Using the right degrees of freedom



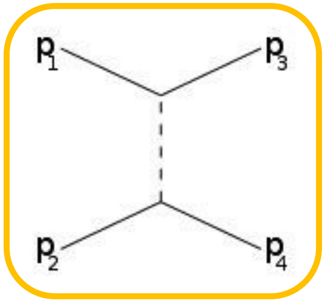
Using the right degrees of freedom







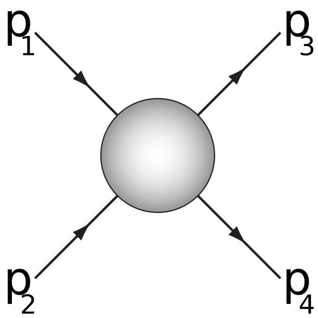
s-channel

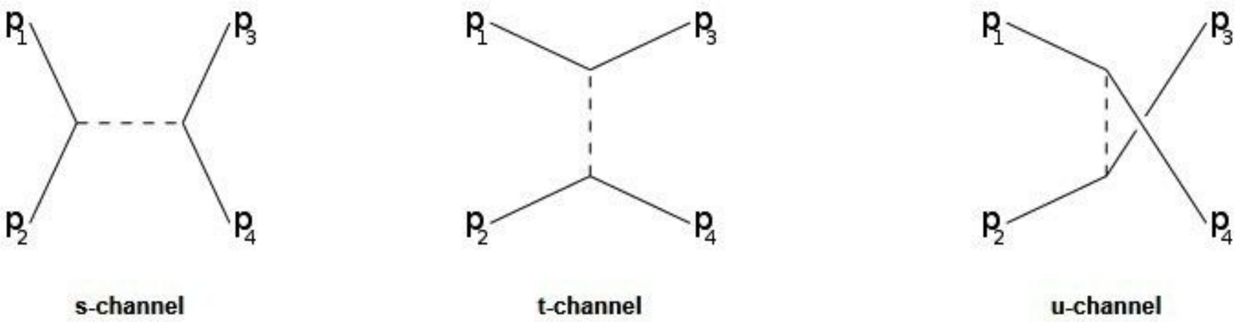


t-channel

Partial-wave expansion in any channel

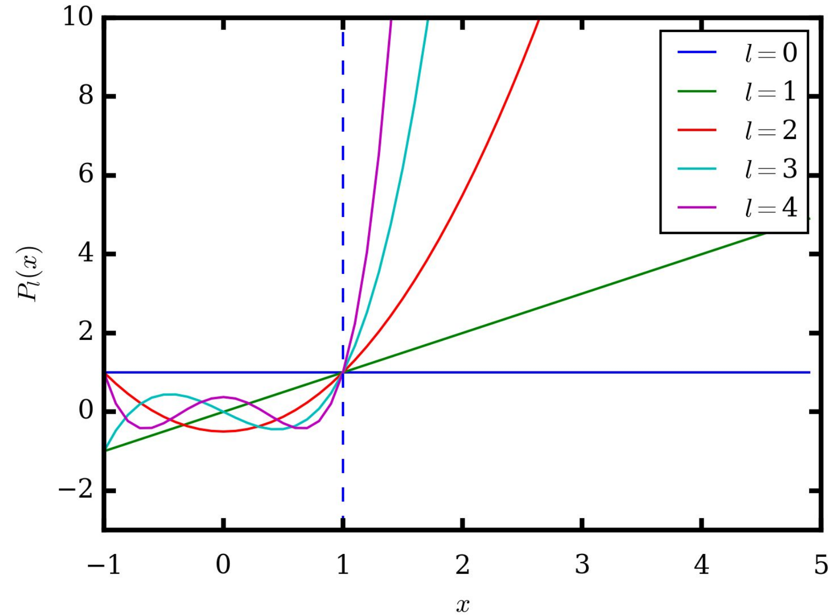
$$A(s,t) = \sum_{l=0}^{\infty} A_l(s) P_l(z_s) = \sum_{l=0}^{\infty} A_l(t) P_l(z_t) = \sum_{l=0}^{\infty} A_l(u) P_l(z_u)$$

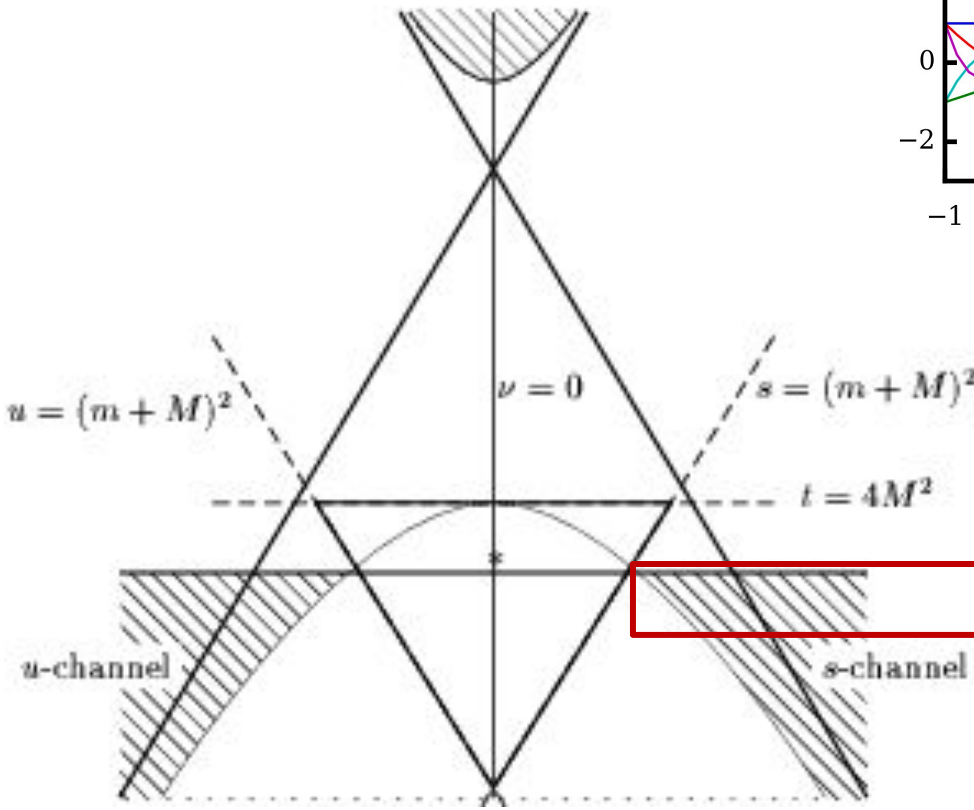




$$A_l(s) \sim q_s^l \qquad (q_s \to 0 \text{ for } s \to s_{thr})$$

Truncated partial-wave expansion





$$\sum_{l=0}^{\cancel{\infty}} A_l(s)P_l(z_s)$$

$$\sum_{l=0}^{\infty} A_l(t)P_l(z_t)$$

s-channel: *truncated* partial-wave analysis

$$\sum_{l=0}^{\cancel{\infty}} A_l(s) P_l(z_s)$$

- Various models available for extracting baryon resonances (**W < 2 GeV**)
  - SAID
  - MAID
  - Bonn-Gatchina
  - Julich-Bonn
  - ...

s-channel: *truncated* partial-wave analysis

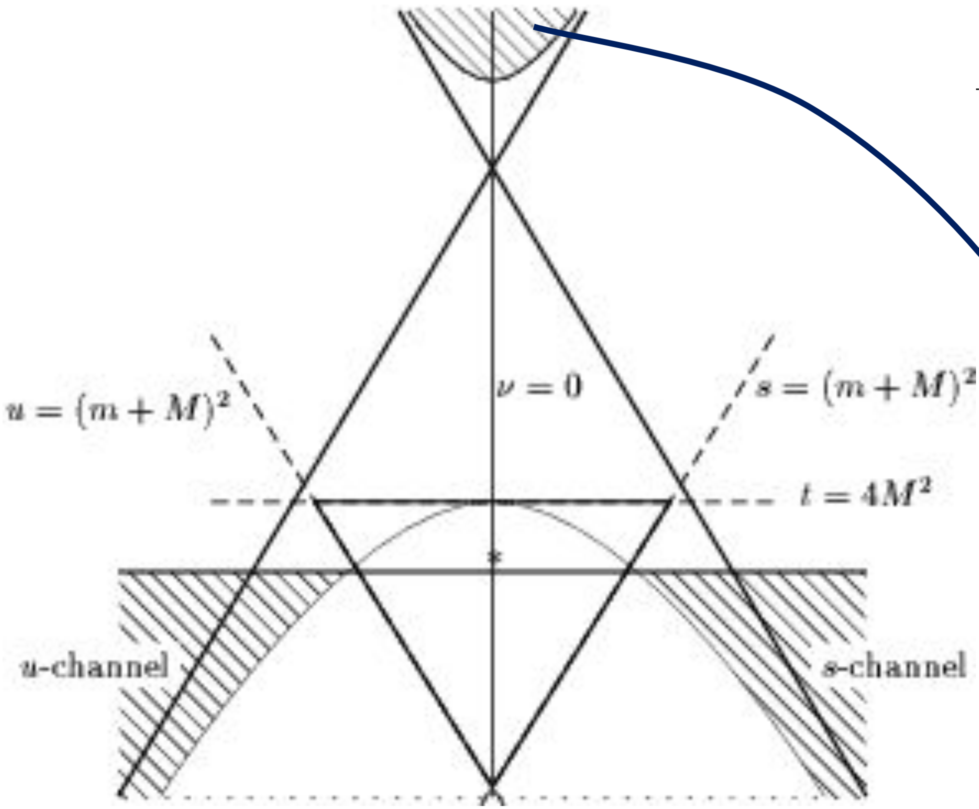
$$\sum_{l=0}^{\cancel{\infty}} A_l(s) P_l(z_s)$$

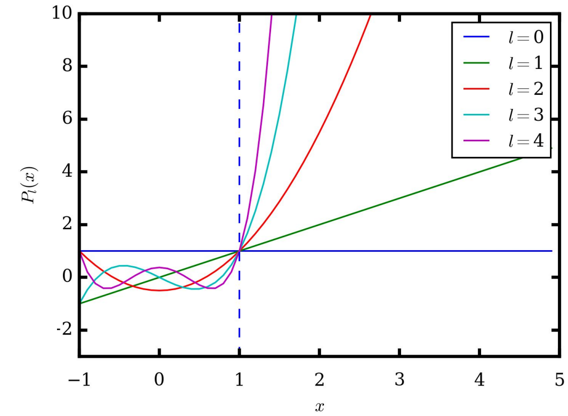
- Analyticity, Unitarity, Crossing symmetry
- Look for poles on the second Riemann sheet
- Cutoff L increases as s increases

t-channel: *no truncation possible*

$$\sum_{l=0}^{\infty} A_l(t) P_l(z_t)$$

Convergent here





Not convergent here

t-channel: *no truncation possible*

$$\sum_{l=0}^{\infty} A_l(t)P_l(z_t)$$

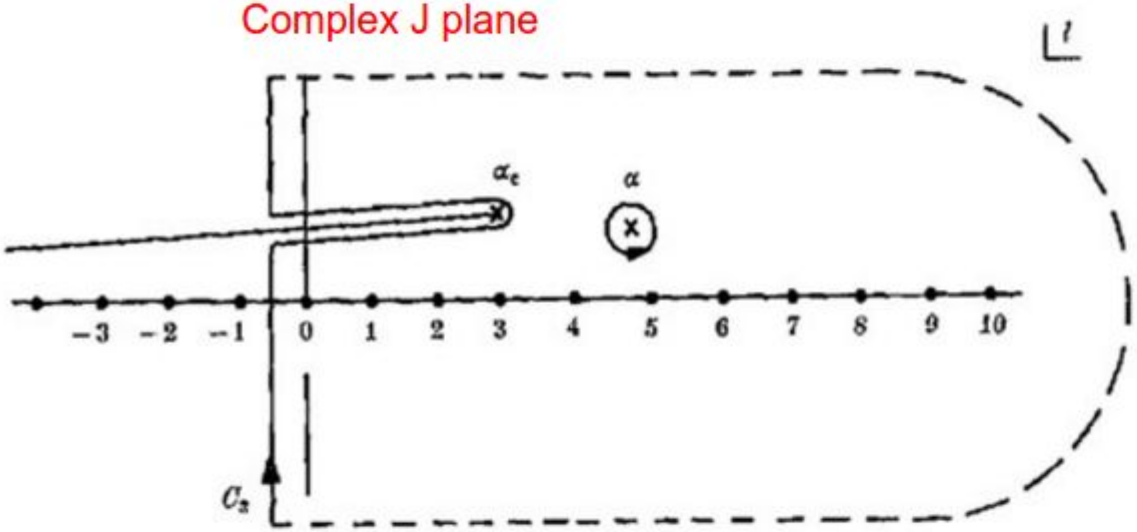
$$A(s,t) = -\frac{1}{2i}\oint_C \frac{A(t,J)P_J(-z_t)}{\sin \pi J} dJ$$

$$A(t,J) = \frac{\beta(t)}{J-\alpha(t)}$$

$$\oint_C A(z)dz = 2\pi i \sum_i \mathrm{Res}_{z\to z_i} A(z)$$

but z=J

Complex J plane



$$\frac{1}{\sin \pi J}$$

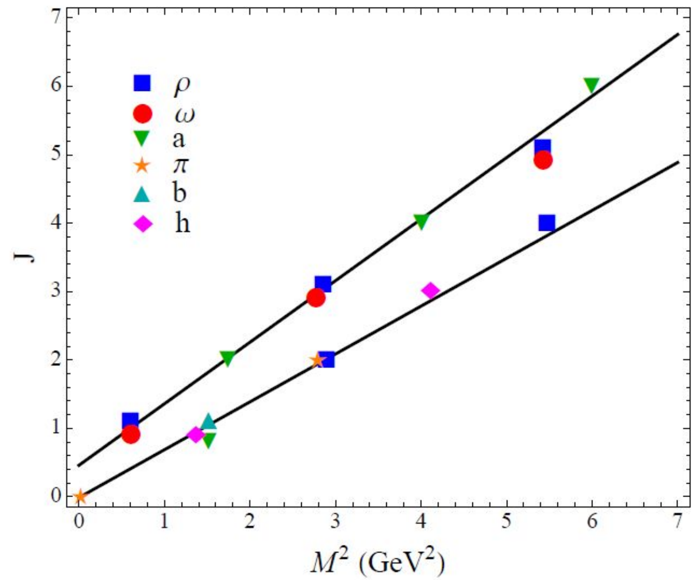
t-channel: *no truncation possible*

$$\sum_{l=0}^{\infty} A_l(t) P_l(z_t)$$

Using $A(t,J) = \dfrac{\beta(t)}{J-\alpha(t)}$

so

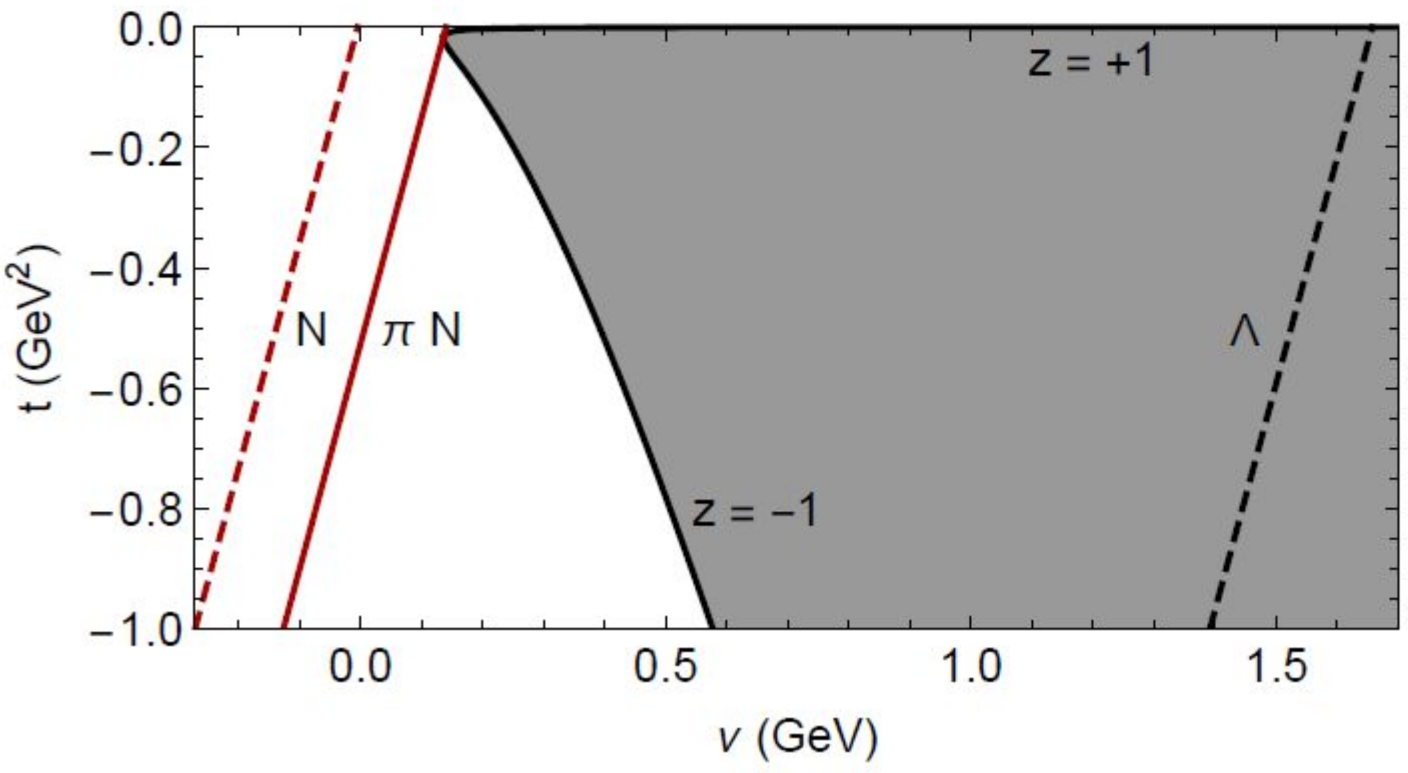
$$A_0(t) \sim \frac{1}{m_0^2 - t}$$
$$A_2(t) \sim \frac{1}{m_2^2 - t}$$
$$A_4(t) \sim \frac{1}{m_4^2 - t}$$

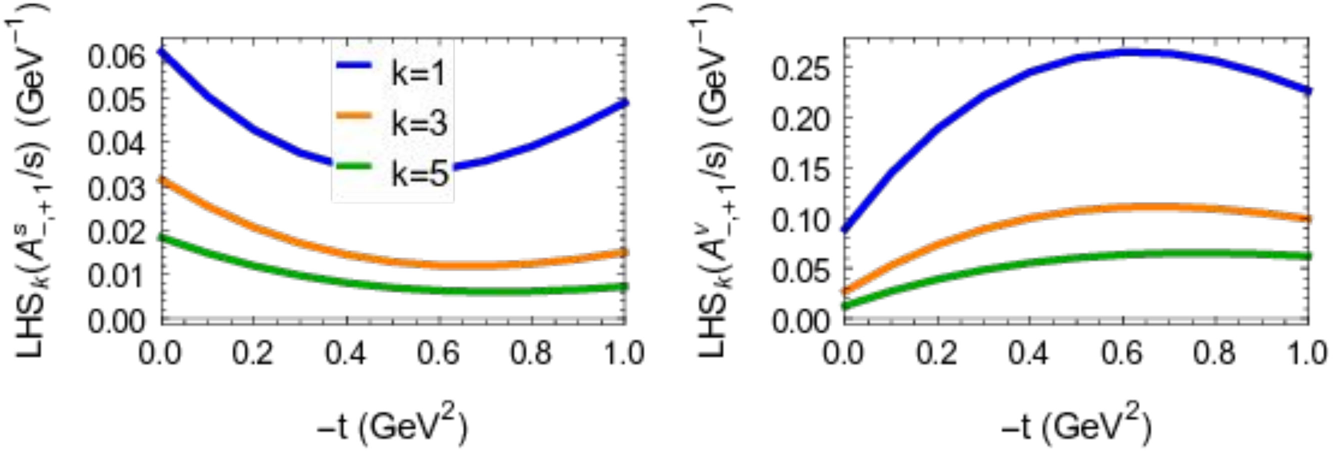


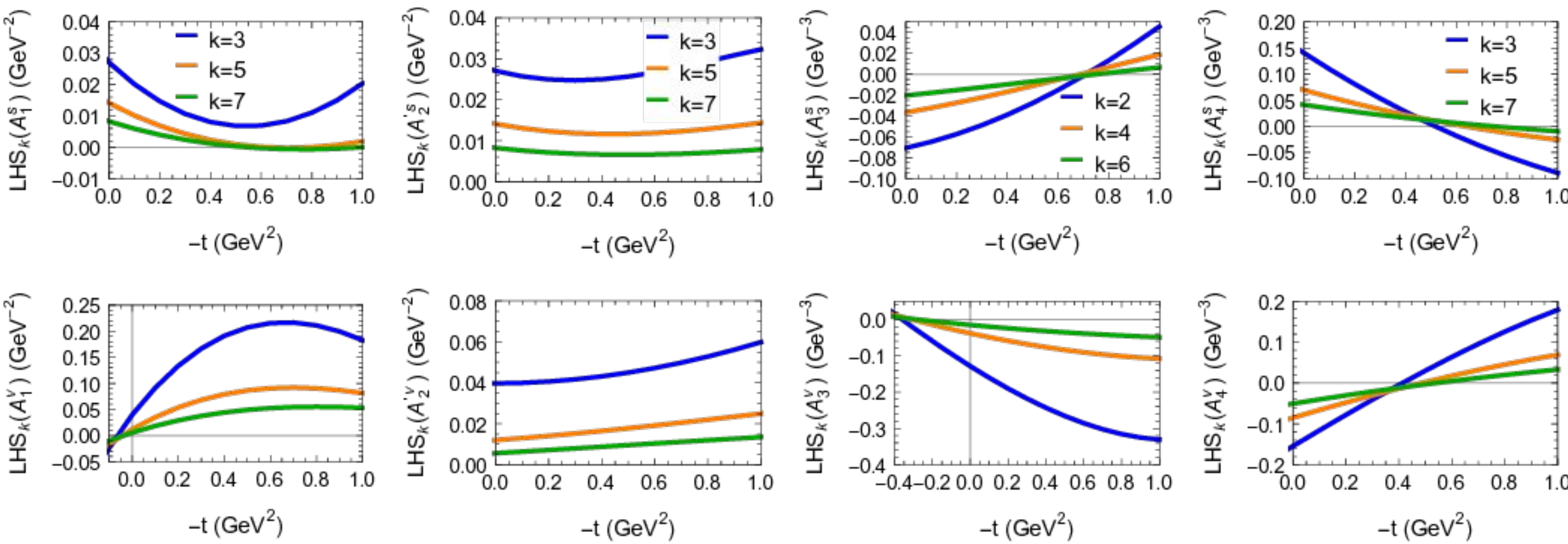
Solution $\alpha(t) = \alpha'(t - m_0^2)$

$$P_J(z_t \to +\infty) \to z_t^J$$

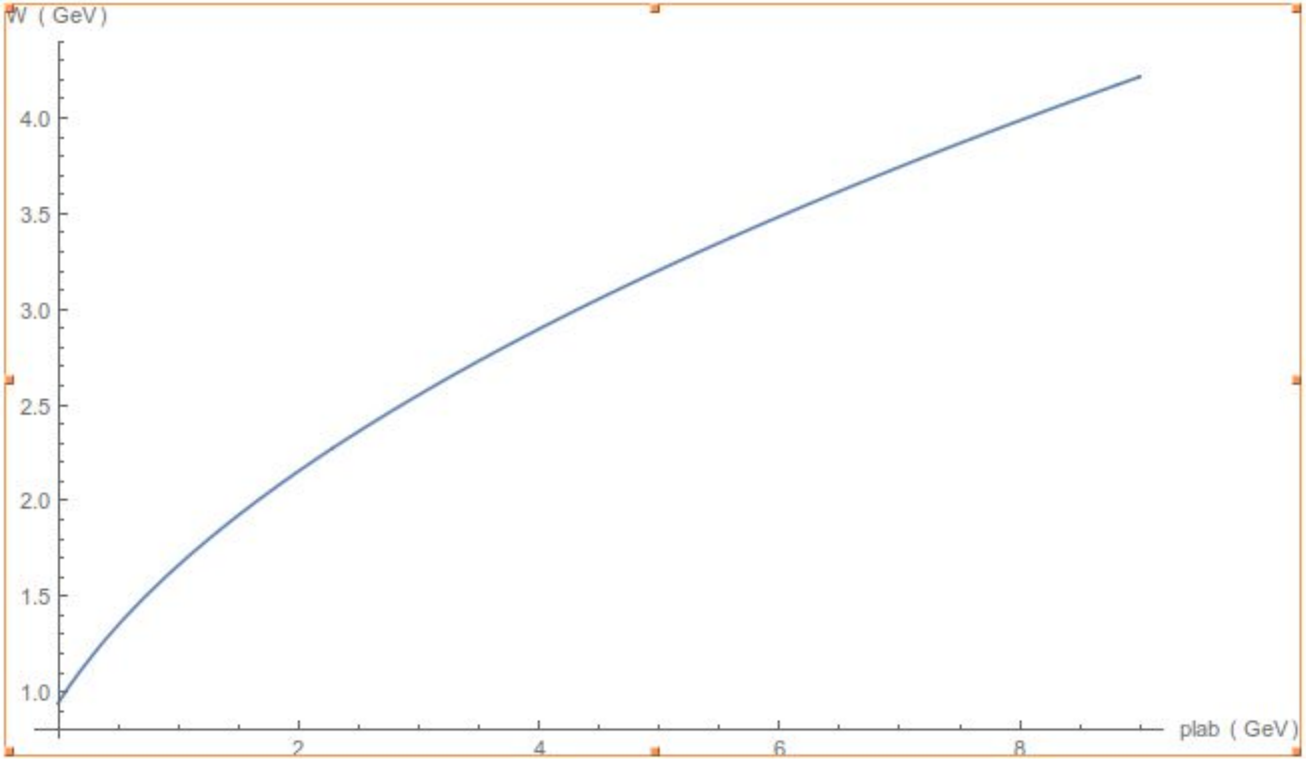
$$\boxed{A(s,t) = \beta(t)\frac{\tau + e^{-i\pi\alpha(t)}}{\sin \pi\alpha(t)} s^{\alpha(t)}}$$

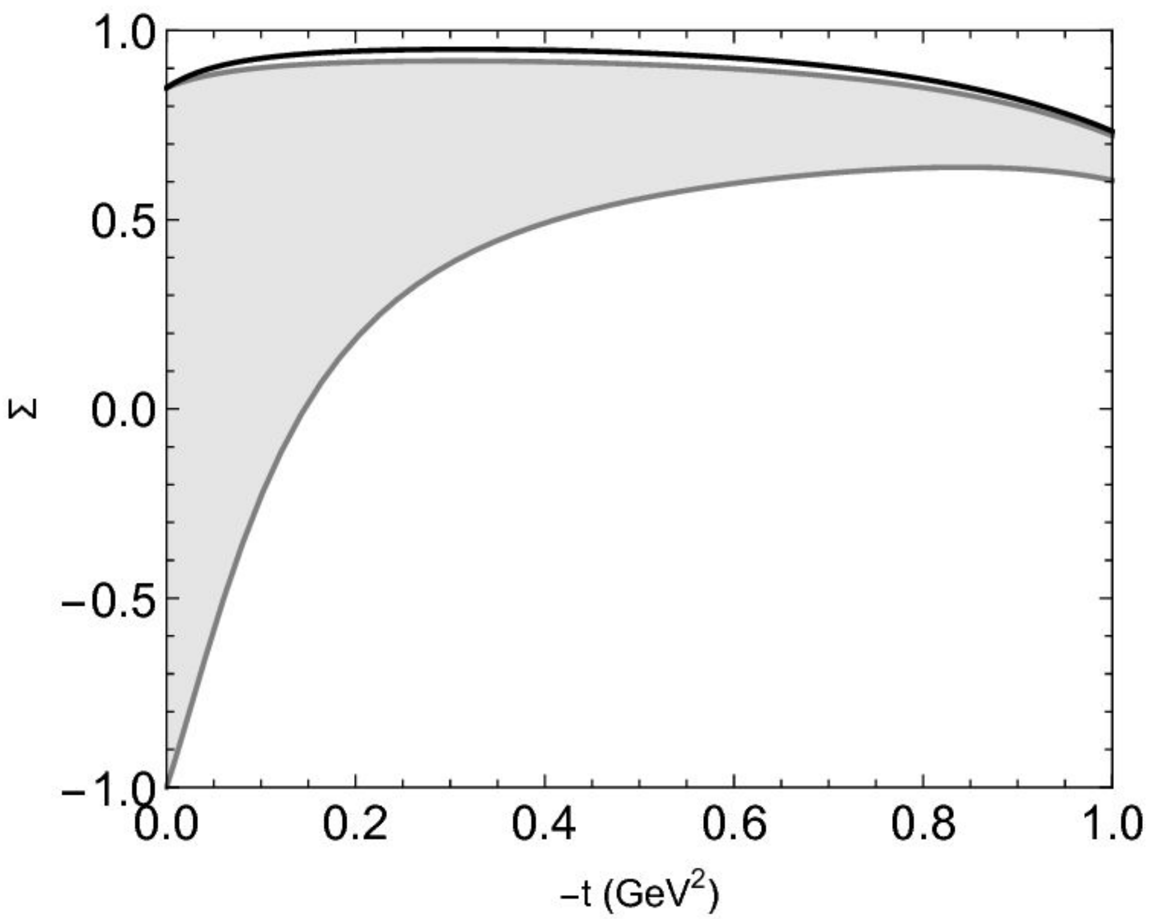

0.0
−0.2
−0.4
−0.6
−0.8
−1.0
t (GeV^2)
0.0
0.5
1.0
1.5
ν (GeV)
N
π N
z = +1
z = −1
Λ


k=1
k=3
k=5
LHS$_k$($A^s_{-,+1}$/s) (GeV^{-1})
0.06
0.05
0.04
0.03
0.02
0.01
0.00
0.0
0.2
0.4
0.6
0.8
1.0
−t (GeV^2)
LHS$_k$($A^v_{-,+1}$/s) (GeV^{-1})
0.25
0.20
0.15
0.10
0.05
0.00
0.0
0.2
0.4
0.6
0.8
1.0
−t (GeV^2)

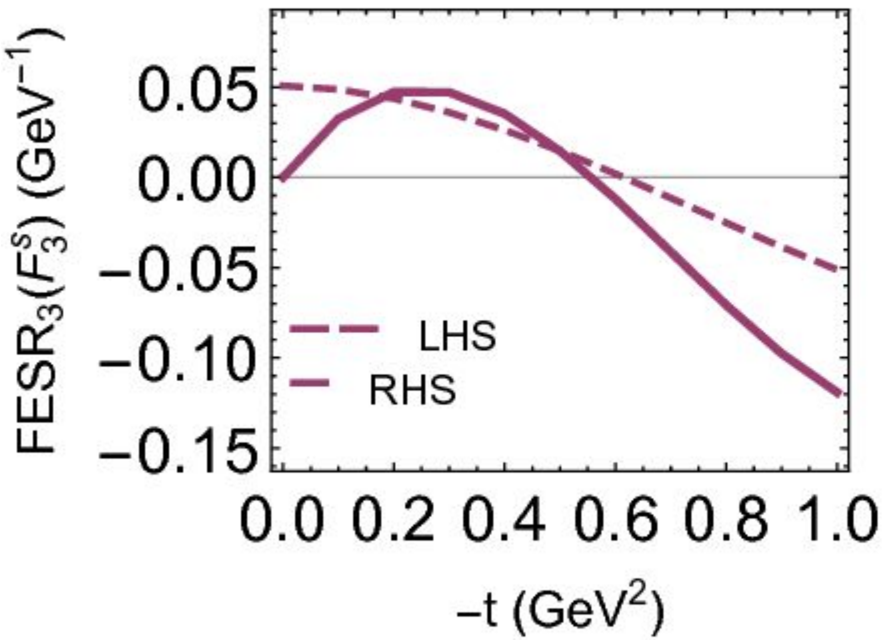


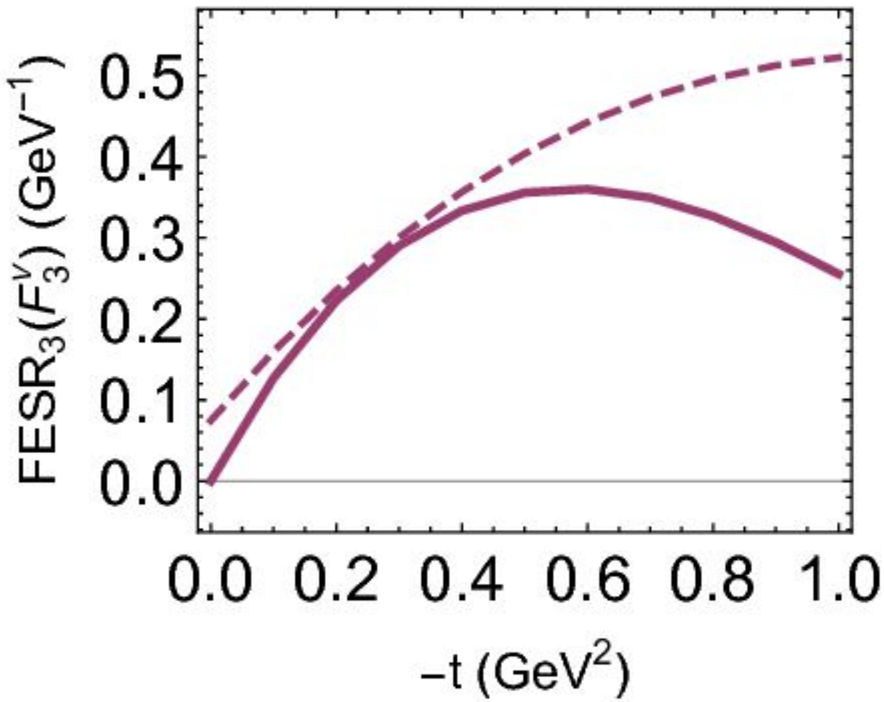
$\mathrm{LHS}_k(A_1^s)$ (GeV^{-2})
k=3
k=5
k=7
$-t$ (GeV^2)
$\mathrm{LHS}_k(A_2'^s)$ (GeV^{-2})
$\mathrm{LHS}_k(A_3^s)$ (GeV^{-3})
k=2
k=4
k=6
$\mathrm{LHS}_k(A_4^s)$ (GeV^{-3})
$\mathrm{LHS}_k(A_1^v)$ (GeV^{-2})
$\mathrm{LHS}_k(A_2'^v)$ (GeV^{-2})
$\mathrm{LHS}_k(A_3^v)$ (GeV^{-3})
$\mathrm{LHS}_k(A_4^v)$ (GeV^{-3})


W (GeV)
4.0
3.5
3.0
2.5
2.0
1.5
1.0
plab (GeV)
2
4
6
8



1.0
0.5
0.0
−0.5
−1.0
Σ
0.0
0.2
0.4
0.6
0.8
1.0
−t (GeV^2)





$$F_3 = 2M_N A_1 - tA_4$$
$$F_4 = A_3\,.$$

Overview

- Intro
- Dispersion relations
- Low-energy amplitudes (PWA)
- High-energy amplitudes
- Applications to π,η photoproduction:
  Finite-Energy Sum Rules

Dispersion relations - FESR

$A_i(\nu, t)$

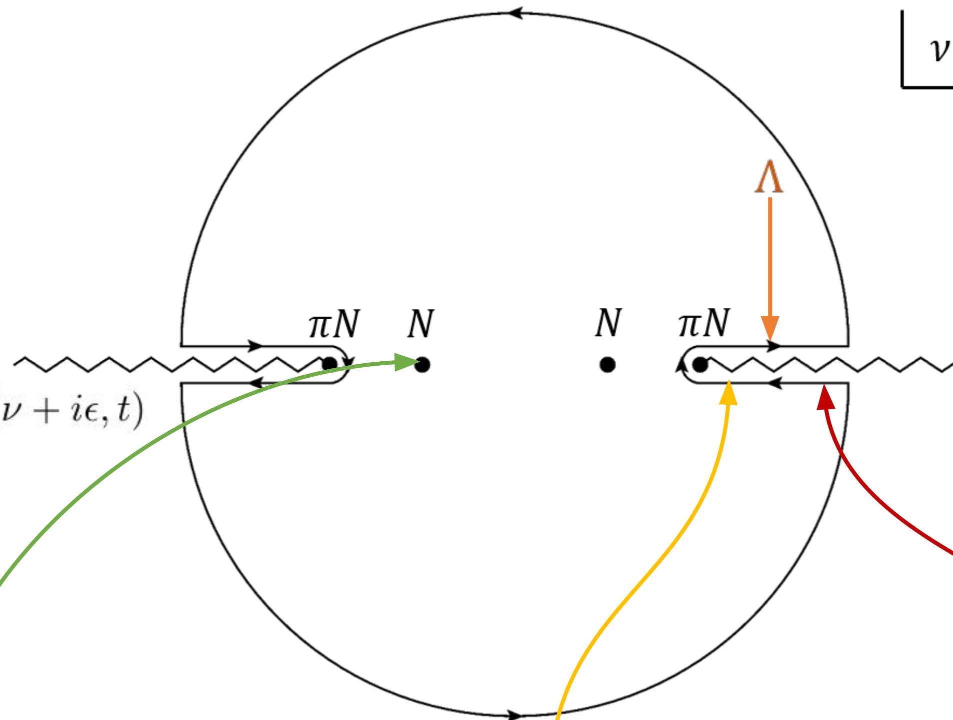
ν

Fixed t

Crossing variable: $\nu = \frac{s-u}{4\,M}$

s-channel: $+\,\nu$

u-channel: $-\,\nu$



$A_i^\sigma(-\nu - i\epsilon, t) = \xi_i A_i^\sigma(\nu + i\epsilon, t)$

$-\beta_i(t)\dfrac{\tau + e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}(r_i\nu)^{\alpha(t)-1}$

$$\boxed{\pi B_i^\sigma(t)\left(\frac{\nu_M}{\Lambda}\right)^k} + \boxed{\int_{\nu_\pi}^{\Lambda} \mathrm{Im}\ A_i^\sigma(\nu', t)\left(\frac{\nu'}{\Lambda}\right)^k d\nu'} = \boxed{\beta_i^\sigma(t)\frac{(\Lambda/\nu_0)^{\alpha(t)}}{\alpha(t)+k}\nu_0}$$

Nucleon pole

Low-energy model

Regge pole

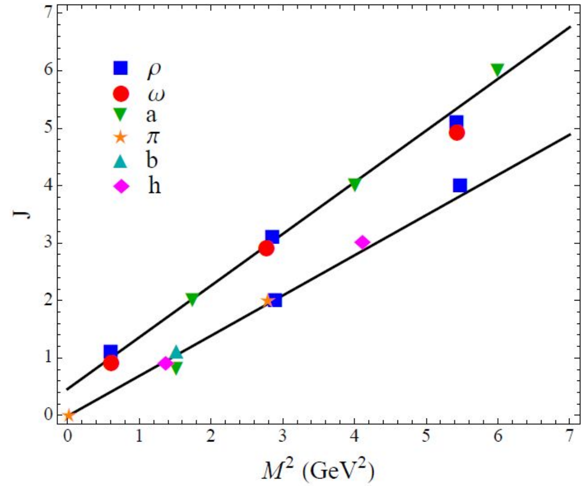
Analyticity results in Finite-Energy Sum Rules.

High energies

Regge pole model

$$A_{i,R}(\nu,t) = -\beta_i(t)\frac{\tau + e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}(r_i\nu)^{\alpha(t)-1}$$

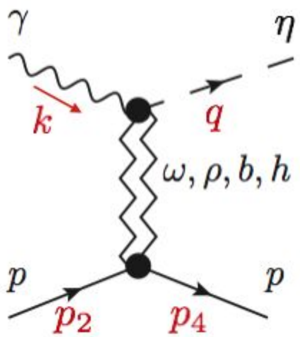
$J(m^2)$



Dominant: vector exchanges

A_i	I^G	J^{PC}	η	Leading exchanges
A_1	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$
A_2'	$0^-, 1^+$	$(1,3,5,...)^{+-}$	-1	$h_1(1170), b_1(1235)$
A_3	$0^-, 1^+$	$(2,4,...)^{--}$	-1	$\rho_2(??), \omega_2(??)$
A_4	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$

$A_2' = A_1 + tA_2$

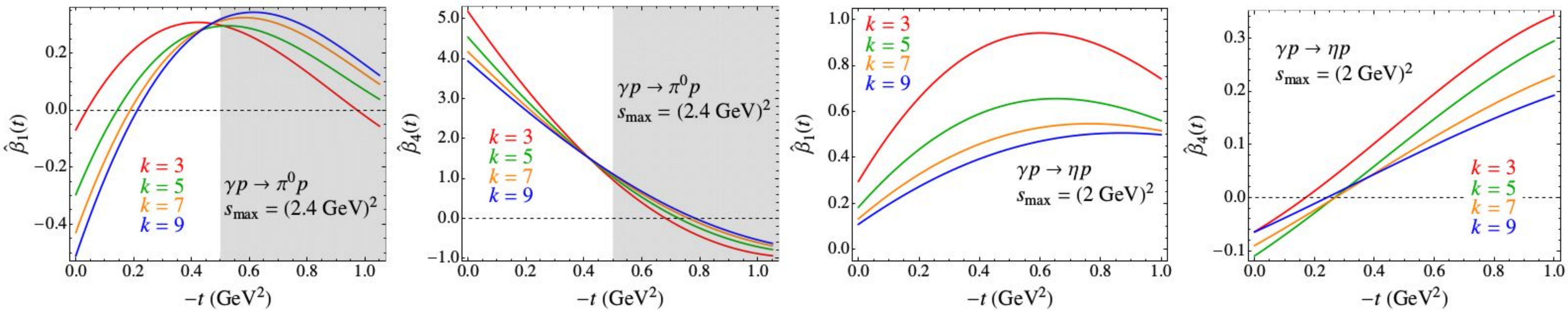


$$\gamma p \to \eta p\,, \qquad A = (\omega + h + \omega_2) + (\rho + b + \rho_2)$$
$$\gamma n \to \eta n\,, \qquad A = (\omega + h + \omega_2) - (\rho + b + \rho_2)$$

Sensitivity to k

$$\pi B_i^\sigma(t)\left(\frac{\nu_M}{\Lambda}\right)^k + \int_{\nu_\pi}^{\Lambda} \mathrm{Im}\ A_i^\sigma(\nu',t)\left(\frac{\nu'}{\Lambda}\right)^k d\nu' = \beta_i^\sigma(t)\frac{(\Lambda/\nu_0)^{\alpha(t)}}{\alpha(t)+k}\nu_0$$

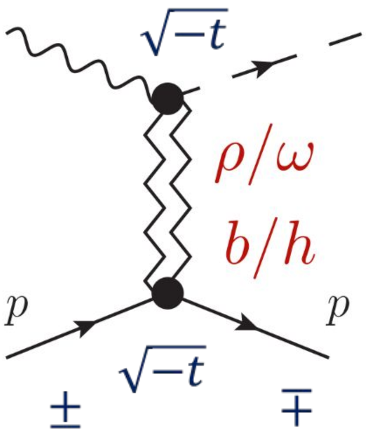
$$\widehat{\beta}_i(t) = \frac{\alpha(t)+k}{\Lambda^{\alpha(t)+k}}\int_0^\Lambda \mathrm{Im}\, A_i^{\mathrm{PWA}}(\nu,t)\,\nu^k\,\mathrm{d}\nu$$



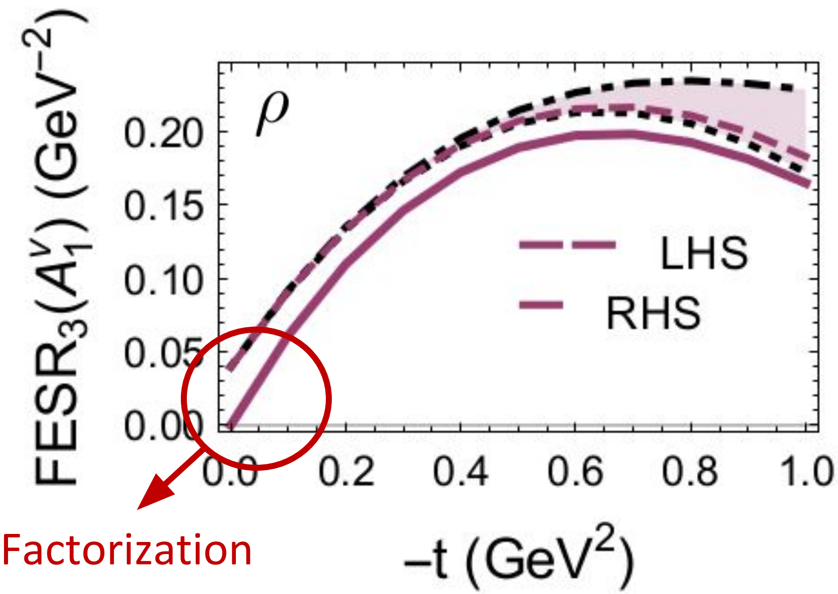
Matching: natural exchanges

$$\pi B_i^\sigma(t)\left(\frac{\nu_M}{\Lambda}\right)^k + \int_{\nu_\pi}^{\Lambda} \mathrm{Im}\, A_i^\sigma(\nu',t)\left(\frac{\nu'}{\Lambda}\right)^k d\nu' = \beta_i^\sigma(t)\frac{(\Lambda/\nu_0)^{\alpha(t)}}{\alpha(t)+k}\nu_0$$

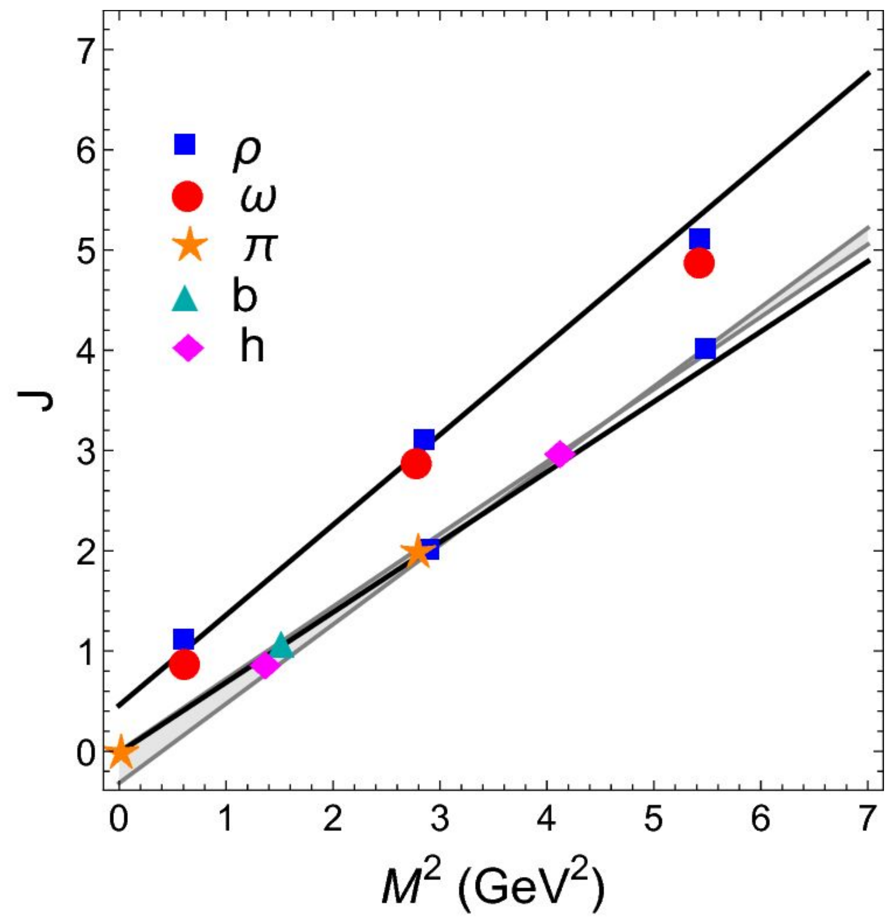
Nucleon pole | Low-energy model | Regge pole



$$\text{ang. mom. : } A_1 \sim 1$$
$$\text{single pole : } A_1 \sim t$$



$F_3 = 2\, M_N A_1 - t A_4$



ρ
ω
π
b
h
J
0
1
2
3
4
5
6
7
M^2 (GeV^2)

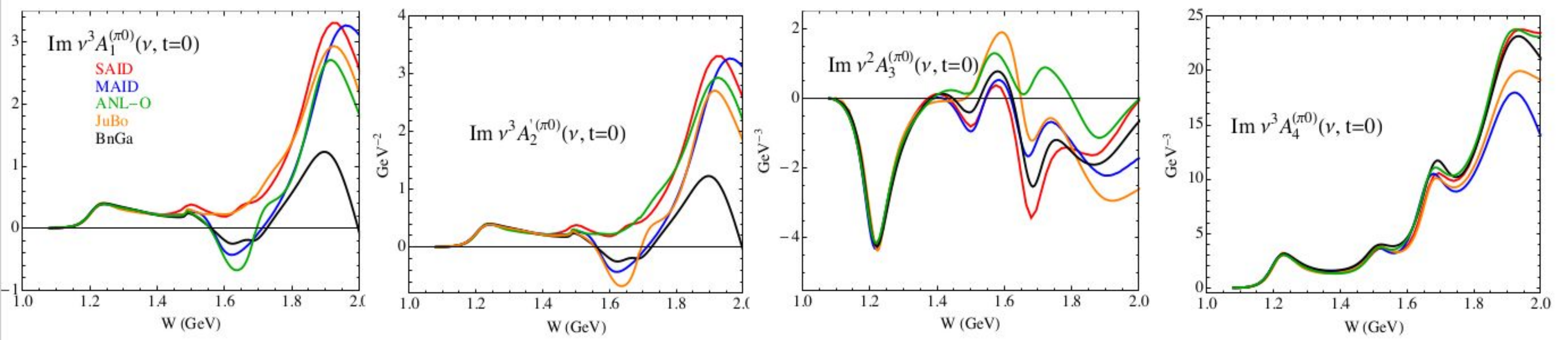
Low energies

$$\int_{\nu_\pi}^{\Lambda} \mathrm{Im}\ A_i^\sigma(\nu', t) \left(\frac{\nu'}{\Lambda}\right)^k d\nu'$$

Low energy models

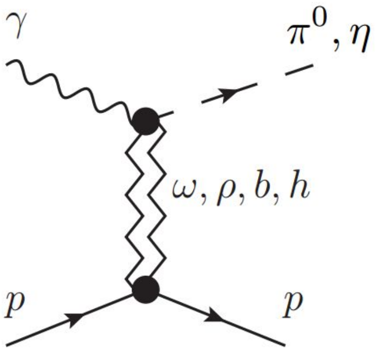
- BnGa, Julich-Bonn, ANL-Osaka, SAID, MAID,...

$$\gamma N \to \pi^0 N$$



Formalism

$\pi^0, \eta(0^{-+})$ **have same production as** $\pi_1^0, \eta_1(1^{-+})$



$$M_k \equiv M_k(s, t, \lambda_\gamma)$$

$$A_{\lambda';\lambda\lambda_\gamma}(s,t) = \overline{u}_{\lambda'}(p') \left(\sum_{k=1}^{4} A_k(s,t) M_k \right) u_\lambda(p)$$

$$M_1 = \frac{1}{2}\gamma_5\gamma_\mu\gamma_\nu F^{\mu\nu},$$

$$M_2 = 2\gamma_5 q_\mu P_\nu F^{\mu\nu},$$

$$M_3 = \gamma_5\gamma_\mu q_\nu F^{\mu\nu},$$

$$M_4 = \frac{i}{2}\epsilon_{\alpha\beta\mu\nu}\gamma^\alpha q^\beta F^{\mu\nu}.$$

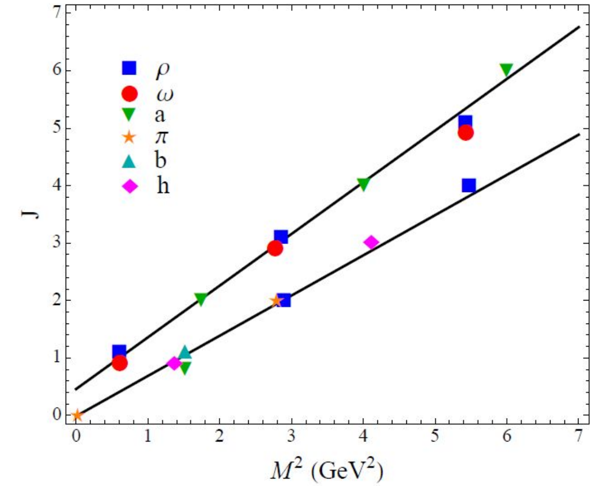
- No kinematic singularities
- No kinematic zeros
- Discontinuities:
  - Unitarity cut
  - Nucleon pole

High energies

Regge pole model

$J(m^2)$

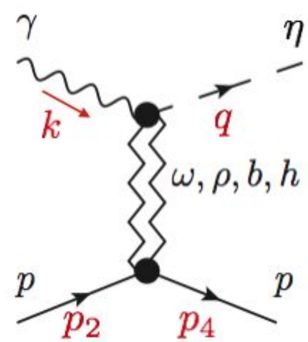
$$A_{i,R}(\nu,t) = -\beta_i(t)\frac{\tau + e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}(r_i\nu)^{\alpha(t)-1}$$



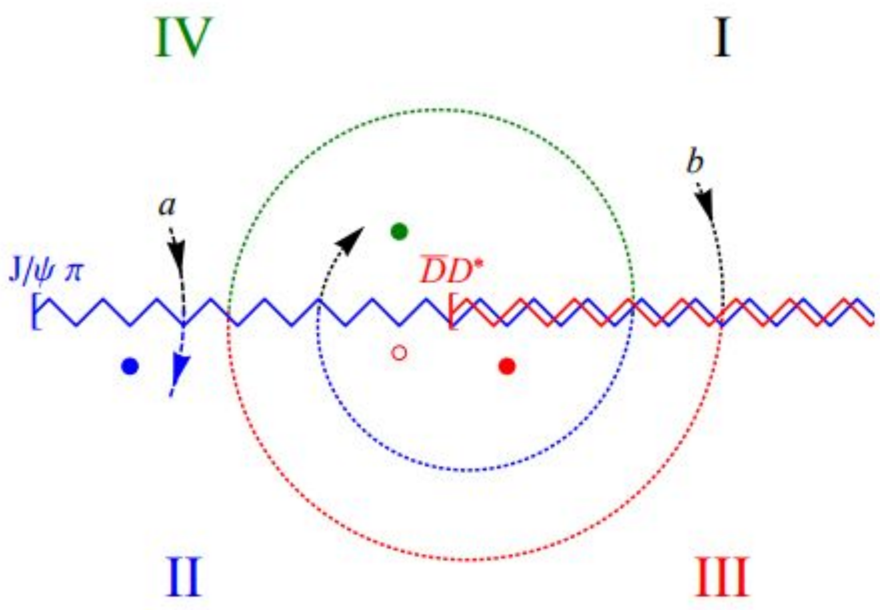
Dominant: vector exchanges

A_i	I^G	J^{PC}	η	Leading exchanges
A_1	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$
A_2'	$0^-, 1^+$	$(1,3,5,...)^{+-}$	-1	$h_1(1170), b_1(1235)$
A_3	$0^-, 1^+$	$(2,4,...)^{--}$	-1	$\rho_2(??), \omega_2(??)$
A_4	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$

$A_2' = A_1 + tA_2$



$$\gamma p \to \eta p\,, \qquad A = (\omega + h + \omega_2) + (\rho + b + \rho_2)$$
$$\gamma n \to \eta n\,, \qquad A = (\omega + h + \omega_2) - (\rho + b + \rho_2)$$


IV
I
b
a
J/ψ π
$\bar{D}D^*$
II
III