Low-energy constants from ALEPH hadronic tau decay data

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Aim: determination of LECs appearing in *VV* and *AA* correlators

- Input:
  - 2013 revised ALEPH spectral data from non-strange tau decays (Davier *et al*. '14, original data had incomplete correlation matrix (Boito *et al*. '11); previously: OPAL data, Boito *et al*. '13; MG, Maltman & Peris '14)
  - B-factory strange mode distributions for main exclusive modes
  - RBC/UKQCD lattice data (Boyle *et al*. '14)
  - fit to revised ALEPH non-strange spectral data (Boito *et al*. '14)

- Ingredients:
  relate spectral data to vacuum polarizations:
  - *V-A* finite-energy sum rules (FESRs)
  - flavor-breaking *V* and *V*±*A* inverse-moment FESRs (Dürr & Kambor '99)

  connect with low-energy constants (LECs):
  - NNLO ChPT (Amoros, Bijnens & Talavera '99)

Non-strange *V-A* vacuum polarization sum rules

Define $\overline{\Pi}^{(w)}_{V-A}(Q^2) = \int_0^\infty ds\, w(s/s_0)\, \frac{\rho_V(s) - \overline{\rho}_A(s)}{s+Q^2}\,, \qquad 0 < s_0 \le m_\tau^2$

(bars indicate that the pion pole has been subtracted)

then (using $D = 2,\ 4$ OPE coefficients; Boito *et al*. '13)

$$-8L_{10}^{\mathrm{eff}} \equiv \overline{\Pi}_{V-A}(0)$$
$$= \overline{\Pi}^{(w_2)}_{V-A}(0) + \frac{4f_\pi^2}{s_0}\Bigg(1 - \underbrace{\frac{17\alpha_s^2(s_0)m_{u,d}^2(s_0)}{16\pi^4 f_\pi^2} - \frac{m_\pi^2}{2s_0}\left(1+O(\alpha_s)\right)}_{\text{numerically negligible}}\Bigg)$$

$$w_2(x) = (1-x)^2$$

$$-16C_{87}^{\mathrm{eff}} = \overline{\Pi}'_{V-A}(0)$$

determination of L_{10}^{eff} with weight w_2 leads to smaller errors

Evaluation of *V-A* vacuum polarization

Split up integral into two parts:

$$\overline{\Pi}^{(w)}_{V-A}(Q^2) = \sum_{\text{bins}<s_{\text{sw}}} w_{\text{av}}\left(\frac{s_{\text{bin}}}{s_{\text{sw}}}\right)\frac{\rho_V(s_{\text{bin}}) - \overline{\rho}_A(s_{\text{bin}})}{s_{\text{bin}} + Q^2}$$

$$+ \int_{s_{\text{sw}}}^{\infty} ds\, w_{\text{av}}\left(\frac{s}{s_{\text{sw}}}\right)\frac{\rho_V^{\text{DV}}(s) - \rho_A^{\text{DV}}(s)}{s + Q^2}$$

sum over bins with data, integral using

$$\rho_T(s) = e^{-\delta_T - \gamma_T s}\sin(\alpha_T + \beta_T s)\ , \qquad T \in \{V,\ A\}$$

with DV parameters from fits to weighted moments of ALEPH spectral functions (Boito *et al*. '14);
switch point above at $s_{\text{sw}} = 1.55\ \text{GeV}^2$ (at $Q^2 = 0$ insensitive to switch point)

Flavor-breaking IMFESRs (Dürr & Kambor '99)

Define $\Delta\Pi_T(Q^2) \equiv \Pi_{ud;T}(Q^2) - \Pi_{us;T}(Q^2)\ ,\qquad T \in \{V,\ A,\ V \pm A\}$

$$\Delta\Pi_V(0) = \int_{4m_\pi^2}^{s_0} ds\, \frac{w(s/s_0)}{s}\, \Delta\rho_V(s) + \mathrm{OPE}$$

$$\Delta\overline{\Pi}_{V\pm A}(0) = \int_{4m_\pi^2}^{s_0} ds\, \frac{w(s/s_0)}{s}\, \Delta\bar{\rho}_{V\pm A}(s) \pm \left(\frac{2f_K^2}{m_K^2}(1 - w\left(\frac{m_K^2}{s_0}\right)) - (K \to \pi)\right)$$
$$+\ \mathrm{OPE}$$

$$w(x) = \begin{cases} (1-x)^3 \\ (1-x)^3(1 + x + \frac{1}{2}x^2) \qquad \text{(DK weight)} \end{cases}$$

where we neglected DV contributions (triple pinch, $1/s$),

self-consistency checks: $w(x)$ independence, s_0 independence of sum on right-hand side; both satisfied on interval $2\ \mathrm{GeV}^2 < s_0 < m_\tau^2$

Results from data

We find

$$L_{10}^{\mathrm{eff}} = -6.446(50) \times 10^{-3}$$
$$C_{87}^{\mathrm{eff}} = \quad 8.38(18) \times 10^{-3}\ \mathrm{GeV}^{-2}$$

from non-strange ALEPH spectral data, and

$$\Delta\Pi_V(0) = 0.0224(9)$$
$$\Delta\overline{\Pi}_A(0) = 0.113(8)$$
$$\Delta\overline{\Pi}_{V+A}(0) = 0.0338(10)$$
$$\Delta\overline{\Pi}_{V-A}(0) = 0.0111(11)$$

from ALEPH non-strange and strange spectral data

ChPT connects to LECs (ABT '99)

V-*A* to NNLO:

$$L_{10}^{\rm eff} = L_{10}^r\left(1 - 4(2\mu_\pi + \mu_K)\right) - 2(2\mu_\pi + \mu_K)L_9^r - \frac{1}{8}\hat{R}_{\pi K}(\mu, 0)$$
$$- 4m_\pi^2(C_{12}^r - C_{61}^r + C_{80}^r) - 4(2m_K^2 + m_\pi^2)(C_{13}^r - C_{62}^r + C_{81}^r)$$
$$C_{87}^{\rm eff} = C_{87}^r - \frac{1}{64\pi^2}\left(1 - \log\frac{\mu^2}{m_\pi^2} + \frac{1}{3}\log\frac{m_K^2}{m_\pi^2}\right)L_9^r - \frac{1}{16}\hat{R}'_{\pi K}(\mu, 0)$$

use $L_9^r = 5.93(43)\times 10^{-3}$ from Bijnens & Talavera '02;

(all values at $\mu = 770\ \mathrm{MeV}$)

lattice results to disentangle $C_{12}^r - C_{61}^r + C_{80}^r$, $C_{13}^r - C_{62}^r + C_{81}^r$ and L_{10}^r

use 3 RBC/UKQCD ensembles: $1/a = 1.379(7)\ \mathrm{GeV},\ m_\pi = 172,\ 250\ \mathrm{MeV}$

$1/a = 1.785(5)\ \mathrm{GeV},\ m_\pi = 340\ \mathrm{MeV}$

ChPT for flavor-breaking case

using physical meson masses and decay constants, and $\mu = 770\ \mathrm{MeV}$

$$\Delta\Pi_V(0) = 0.00775 - 0.7218 L_5^r + 1.423 L_9^r + 1.062 L_{10}^r + 3.740 C_{61}^r$$

$$\Delta\overline{\Pi}_{V+A}(0) = 0.00880 - 0.7218 L_5^r + 1.423 L_9^r + 3.740 (C_{12}^r + C_{61}^r + C_{80}^r)$$

$$\Delta\overline{\Pi}_{V-A}(0) = 0.00670 - 0.7218 L_5^r + 1.423 L_9^r + 2.125 L_{10}^r - 3.740 (C_{12}^r - C_{61}^r + C_{80}^r)$$

use further $L_5^r = 0.84(38) \times 10^{-3}$ (MILC '09) and L_9^r from BT '02

- get $C_{12}^r + C_{61}^r + C_{80}^r$ from $\Delta\overline{\Pi}_{V+A}(0)$
- from $\Delta\overline{\Pi}_{V-A}(0)$, L_{10}^{eff} and the lattice, get L_{10}^r , $C_{12}^r - C_{61}^r + C_{80}^r$ and $C_{13}^r - C_{62}^r + C_{81}^r$
- $\Delta\Pi_V(0)$ then directly yields C_{61}^r

Results

Taking correlations completely into account, we find

$$
\begin{aligned}
L_{10}^r &= -3.50(17) \times 10^{-3} \\
C_{12}^r + C_{61}^r + C_{80}^r &= 2.37(16) \times 10^{-3}\ \mathrm{GeV}^{-2} \\
C_{12}^r - C_{61}^r + C_{80}^r &= -0.56(15) \times 10^{-3}\ \mathrm{GeV}^{-2} \\
C_{13}^r - C_{62}^r + C_{81}^r &= 0.46(9) \times 10^{-3}\ \mathrm{GeV}^{-2} \\
C_{61}^r &= 1.46(15) \times 10^{-3}\ \mathrm{GeV}^{-2} \\
C_{12}^r + C_{80}^r &= 0.90(9) \times 10^{-3}\ \mathrm{GeV}^{-2} \\
C_{87}^r &= 5.10(22) \times 10^{-3}\ \mathrm{GeV}^{-2}
\end{aligned}
$$

(the latter directly from C_{87}^{eff})

Comments

- Fit errors improved compared to our previous, OPAL-based, analysis (ALEPH data has smaller errors)

- Errors have reached the level of the expected systematic uncertainties from the neglected NNNLO terms (about 6% for L_{10}^r , and about 25% for NNLO LECs):

  precision attainable with an NNLO analysis has been reached

- Large-*N* suppressed LEC combination $C_{13}^r - C_{62}^r + C_{81}^r$ not smaller in size than $C_{12}^r - C_{61}^r + C_{80}^r$ (significant cancellations in the latter)

- Also determined D = 6, 8 OPE condensates, ~2.4 σ different from values found based on OPAL data