QUANTUM JUMPS AND SPIKES

A mathematical curiosity ?

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DISCLAIMER

Work done in collaboration with **Denis Bernard** and **Michel Bauer**.

Ongoing work: some results of this talk are buried in a (relatively bad) preprint: arXiv:1410.7231, the rest is **new**

Mathematical curiosity or interesting physical effect ? Glad to have your point of view at the end.

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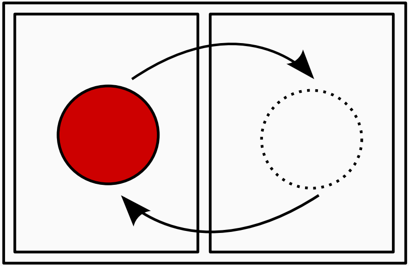
OUTLINE

A SIMPLE CLASSICAL MODEL

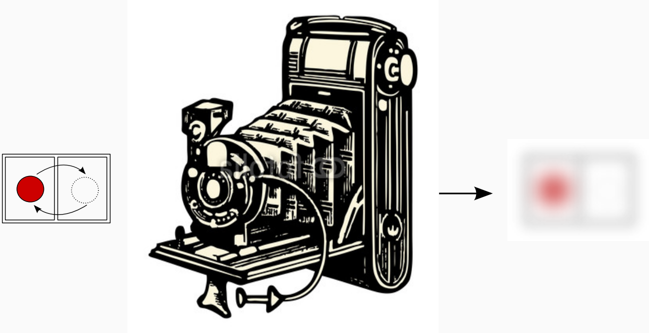
Repeated fuzzy measurements

Consider a classical 2-state Markov process. Typically a particle randomly jumping between two compartments of a box.



Repeated fuzzy measurements

The box is very small and we take fuzzy pictures from far away.



Every picture gives a tiny bit of information about where the particle is.

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Mathematical Model

- The particle jump rate is λ, the number of jumps is a Poisson process.

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$$\mathbb{P}(\delta_n = 1 | \text{particle on the left}) = \frac{1+\epsilon}{2}$$
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- We are interested in:

$$Q_n = \mathbb{P}(\text{particle on the left at time } n|\text{all the pictures before } n)$$

Mathematical Model

To compute Q_{n+1} knowing Q_n we need to:

- Incorporate the measurement result δ_n using **Bayes rule**:

$$\begin{aligned} Q_{n+1} &= \mathbb{P}(\text{left at n+1} | Q_n \& \delta_{n+1}) \\ &= \frac{\mathbb{P}(\delta_{n+1} | \text{left at n+1})\, \mathbb{P}(\text{left at n+1} | Q_n)}{\mathbb{P}(\delta_{n+1} | Q_n)} \end{aligned}$$

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- Incorporate the fact that **we know that the particle tends to jump** during the time interval:

$$\mathbb{P}(\text{left at n+1}|Q_n) = (1-\lambda)Q_n + \lambda(1-Q_n)$$

Mathematical Model

At the continuous limit, for extremely fuzzy pictures (and a proper rescaling) we have:

$$dQ_t = \lambda\left(\frac{1}{2} - Q_t\right)dt + \gamma\, Q_t(1 - Q_t)dW_t \tag{1}$$

with γ the rate at which pictures are taken.

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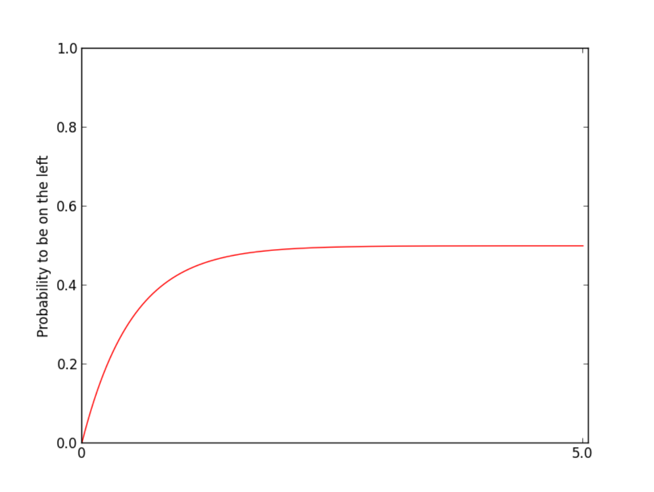
The continuous limit is only needed for the closed form results, all the rest is true in the discrete case. **What follows is not an artifact of the diffusive limit.**

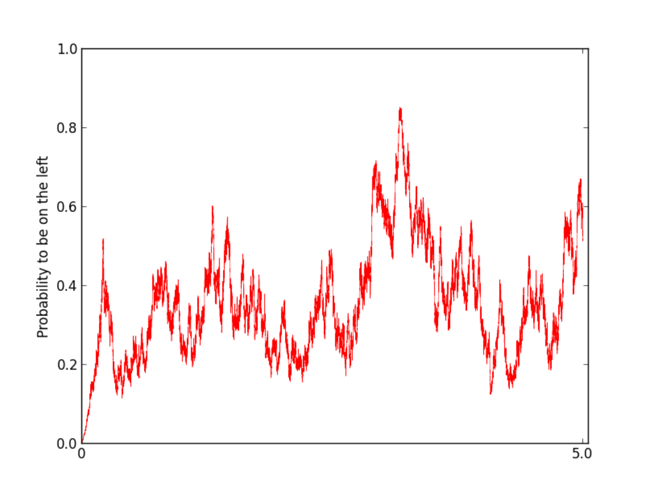
A SIMPLE CLASSICAL MODEL

Results



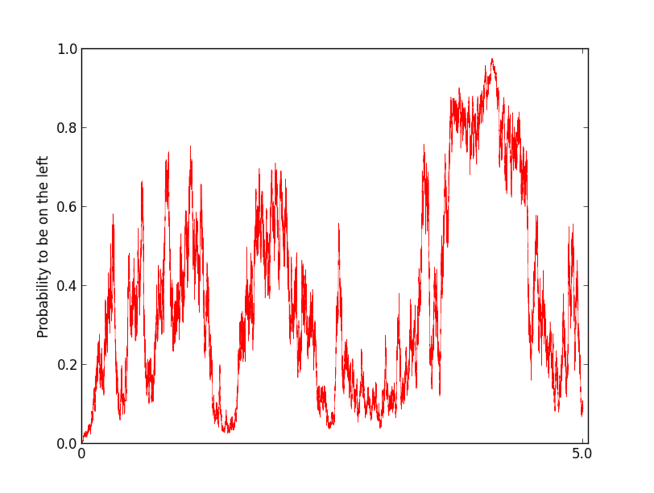
No Measurements

Results



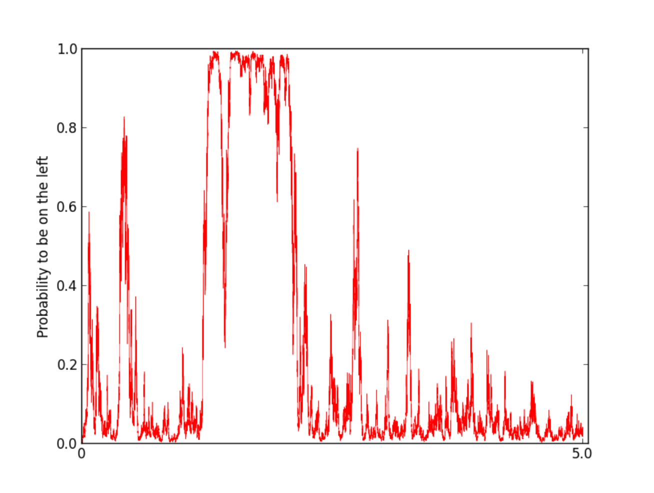
$\gamma = 0.5$

Results



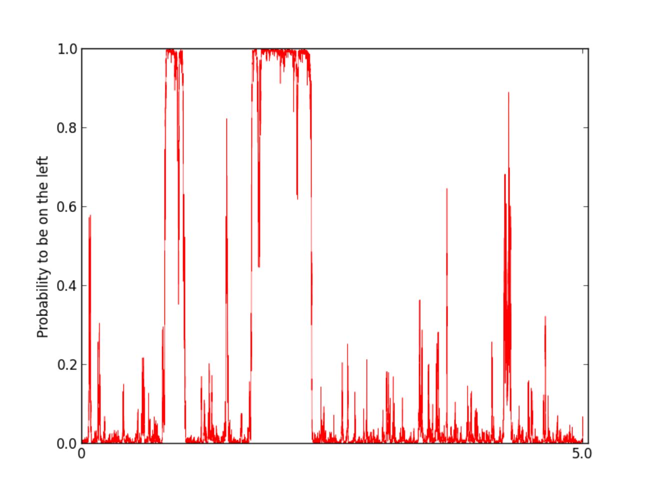
$\gamma = 1.0$

Results



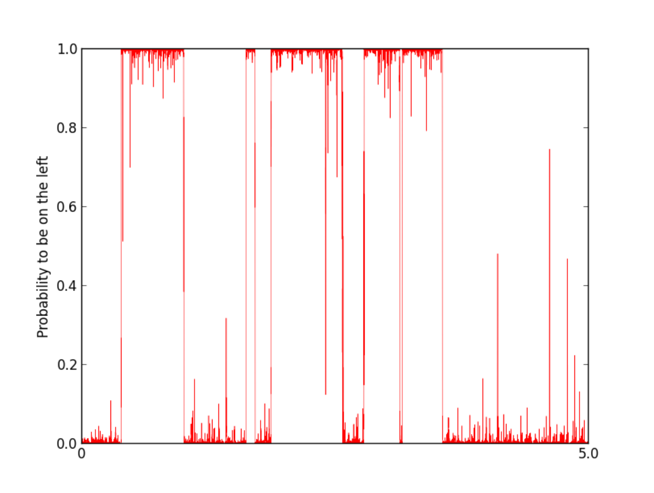
$\gamma = 2.0$

Results



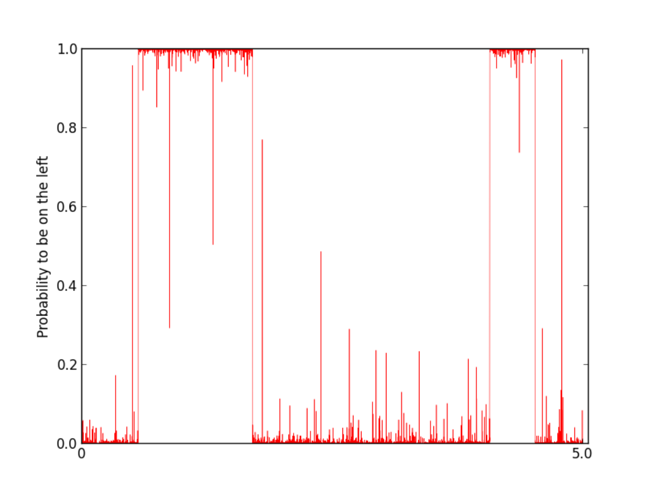
$\gamma = 5.0$

Results



$\gamma = 20.0$

Results



$\gamma = 100.0$, no difference with $\gamma = +\infty$

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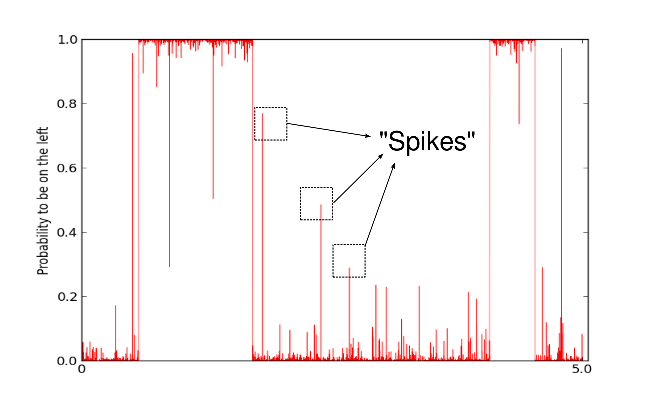
What we call spikes:



Spikes do not disappear when $\gamma \to +\infty$! They just become sharper and sharper.

Theorem

When $\gamma \to \infty$, on a time interval without jumps, the process giving the top of the spikes $\tilde{Q}_t$ is a Poisson process of intensity:

$$\mathrm{d}\nu = \lambda \mathrm{d}t \, \frac{\mathrm{d}Q}{Q^2}$$

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$$d\nu = \lambda dt \, \frac{dQ}{Q^2}$$

- Infinitely many small spikes on any time interval
- The proof is general in the sense that it relies only on the fact that a stopped martingale is a martingale.

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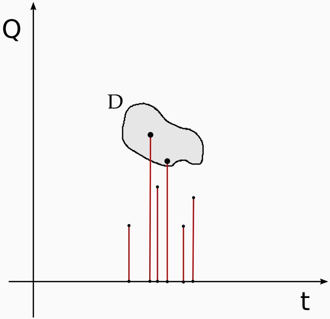
General Poisson Process



The number N of spikes in the domain D is a standard Poisson process of intensity μ, i.e. $\mathrm{P(N)} = \frac{\mu^{\mathrm{N}}}{\mathrm{N!}} e^{-\mu}$ with:

$$\mu = \int_{\mathrm{D}} \mathrm{d}\nu$$

Classical meaning of spikes

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- Spikes can be seen as a **artifact** of Bayesian inference, nothing real is intrinsically ”spiky”.

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- Spikes can be seen as a **artifact** of Bayesian inference, nothing real is intrinsically ”spiky”.
- Spikes can be removed by forward-backward estimation (smoothing) or more brutally by low-pass filtering.

BACK TO THE QUANTUM

A quantum system with spikes

Consider a two level system (a qubit) with Hamiltonian $H = \frac{\omega}{2}\sigma_x$ with σ_z continuously monitored at a rate γ.

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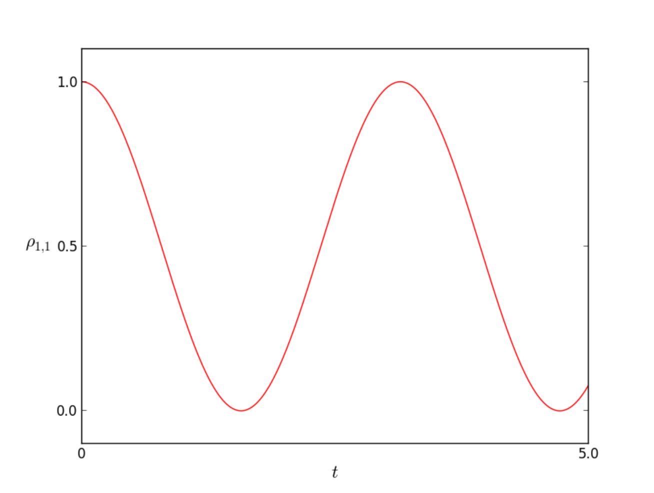
The evolution is given by the stochastic master equation:

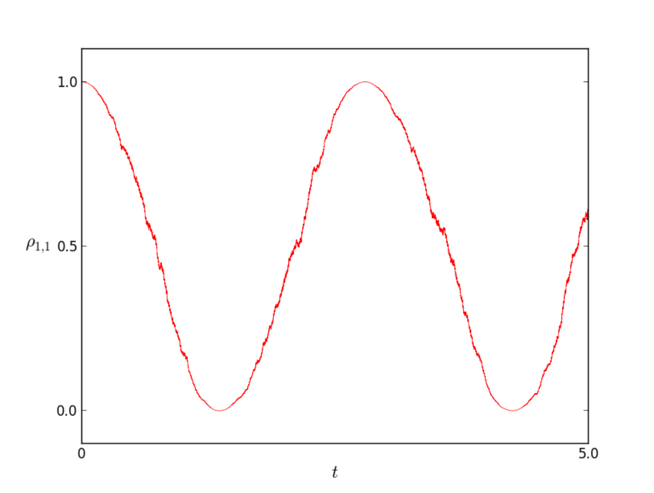
$$d\rho_t = -i\frac{\omega}{2}[\sigma_x, \rho_t]dt - \frac{\gamma^2}{2}\left[\sigma_z\left[\sigma_z, \rho_t\right]\right]dt + \gamma\left(\sigma_z\rho_t + \rho_t\sigma_z - 2\mathrm{tr}\sigma_z\rho_t\right)dW_t$$

Results



Without measurement $\gamma = 0.0$

Results



$\gamma = 0.1$

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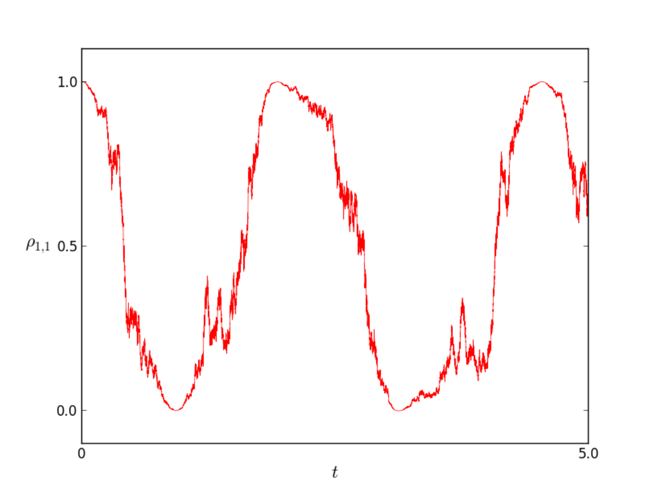
Results



$\gamma = 0.5$

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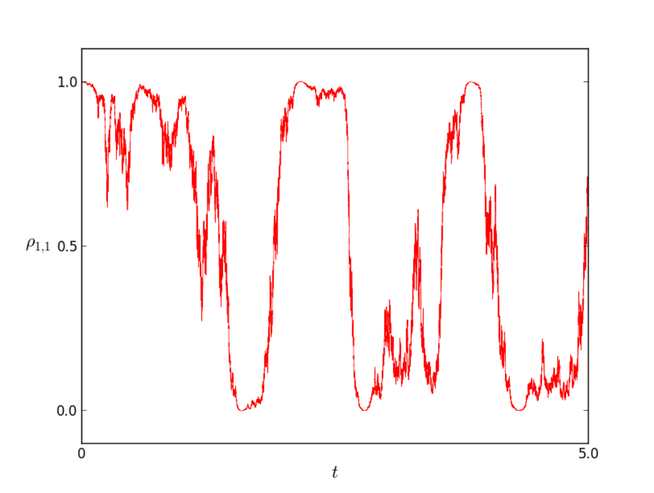
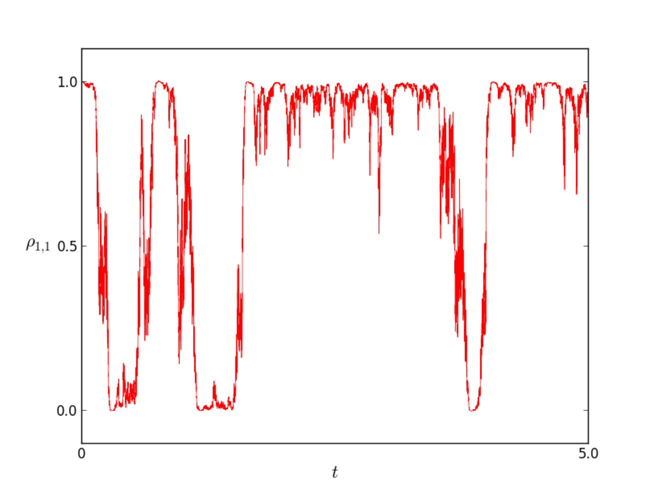
Results



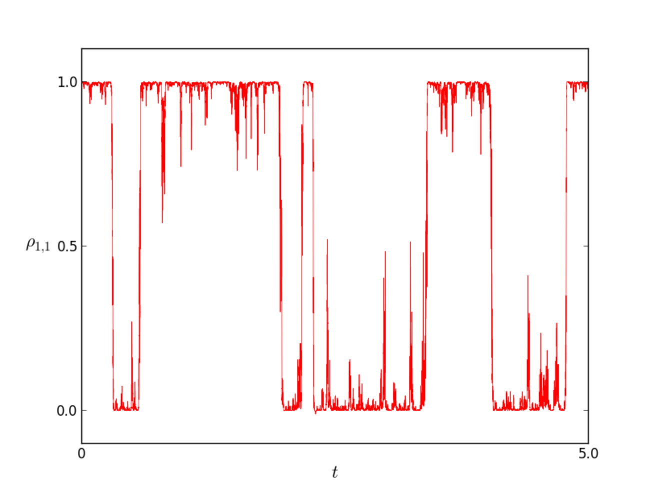
$\gamma = 1.0$

Results



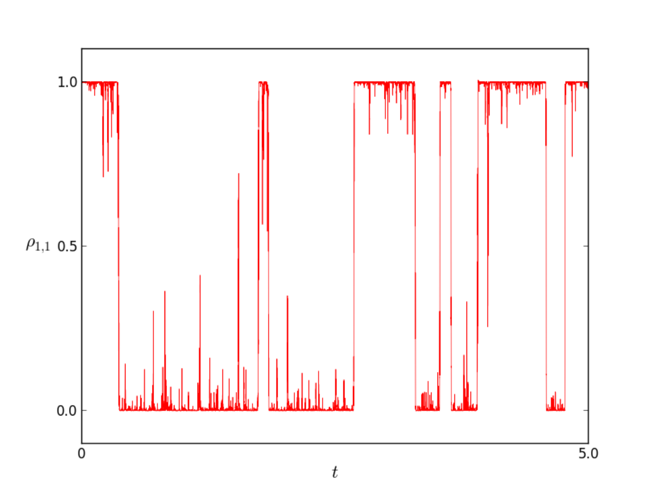
$\gamma = 2.0$

Results



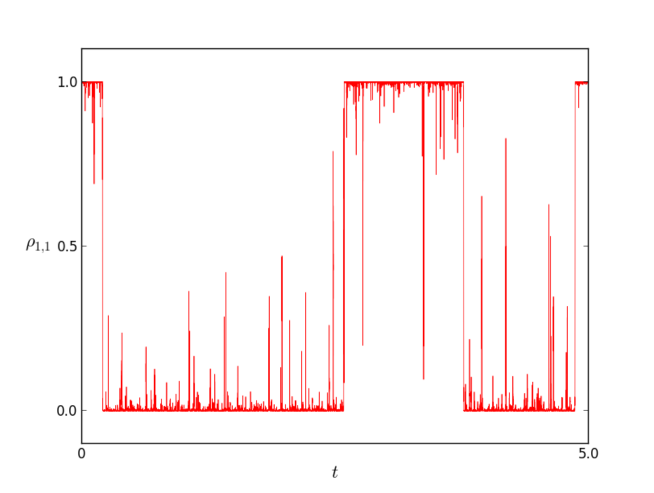
$\gamma = 5.0$

Results



$\gamma = 10$

Results



$\gamma = 20$

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Theorem

With $\omega = \gamma\Omega$ (to avoid complete Zeno freezing), when $\gamma \to \infty$ and on a time interval without jumps, $\rho_{1,1}(t) = Q_t$ is a Poisson process of intensity:

$$d\nu = \Omega dt \, \frac{dQ}{Q^2}$$

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Remark

- The system density matrix stays pure during the spike (a probability spike is compensated by a phase spike)

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Conjecture

Spikes are **universal**, i.e. for any quantum system subjected to a strong measurement and on a time interval without jumps, $\rho_{ii}(t) = Q_i(t)$ has spikes given by a Poisson process of intensity:

$$d\nu \propto dt \frac{dQ_i}{Q_i^2}$$

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"Physicists" arguments but rigorous proofs in two situations

- for any system with an evolution preserving the diagonality of the density matrix
- for a 2-level system with a generic Hamiltonian

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DISCUSSION

A few additional facts

- The spikes disappear with Gammelmark's et al. quantum equivalent of forward-backward filtering, the **past quantum state** –defined in PRL 111, 160401(2013)
- They do not disappear with Guevara & Wiseman's version called **quantum smoothing** –defined in arXiv:1503.02799

An interest for collapse models ?

With an ontic state, spikes become more interesting

- With the matter density ontology, spikes are actual fluctuations of matter, is it a problem or a good thing ?

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With an ontic state, spikes become more interesting

- With the matter density ontology, spikes are actual fluctuations of matter, is it a problem or a good thing ?
- To eliminate spikes from the theory, is it possible to use the past quantum state in some way in collapse models ?

DISCUSSION

Thank you for your time

Erice is beautiful !