Theory Overview of TMDs

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- Transverse Momentum and Large x.
- General properties of large b_T evolution.
- Collins-Soper evolution at large b_T.
  (Recent work with John Collins)

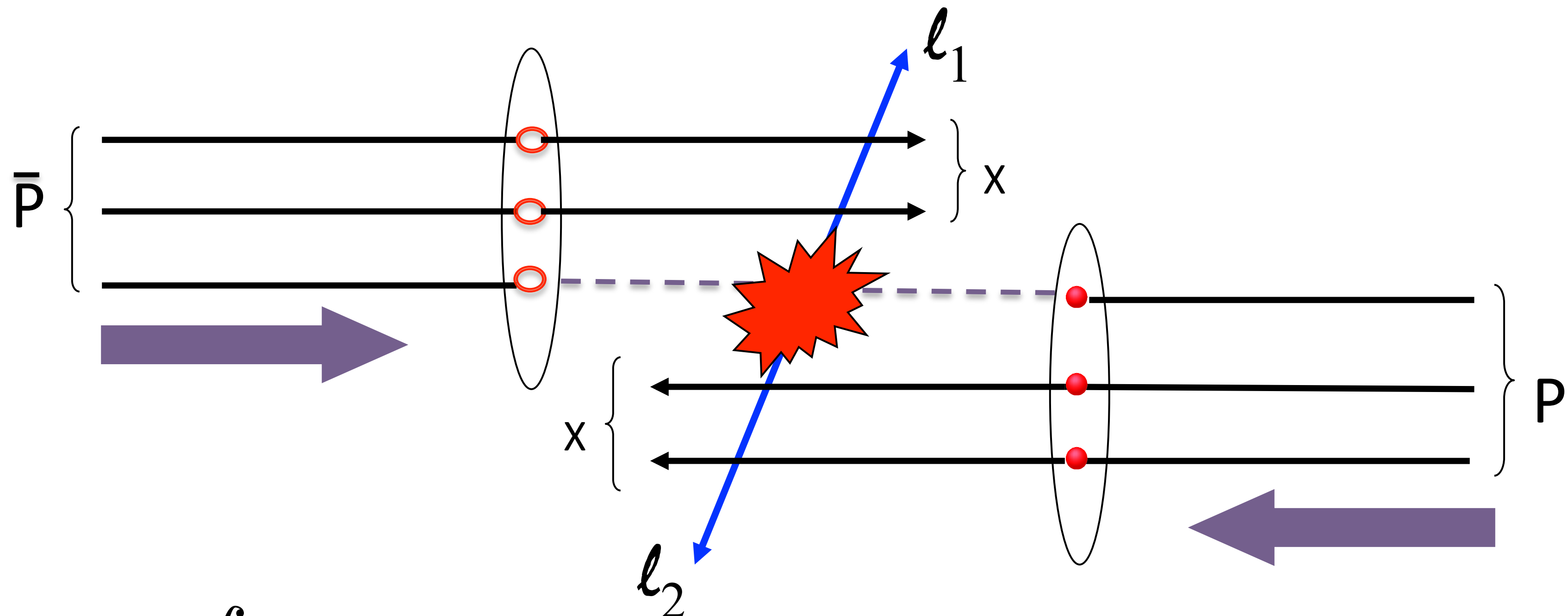
HiX2014 – Frascati, Italy: November 20, 2014

Factorization

- Red: Hard Physics, Small Coupling.

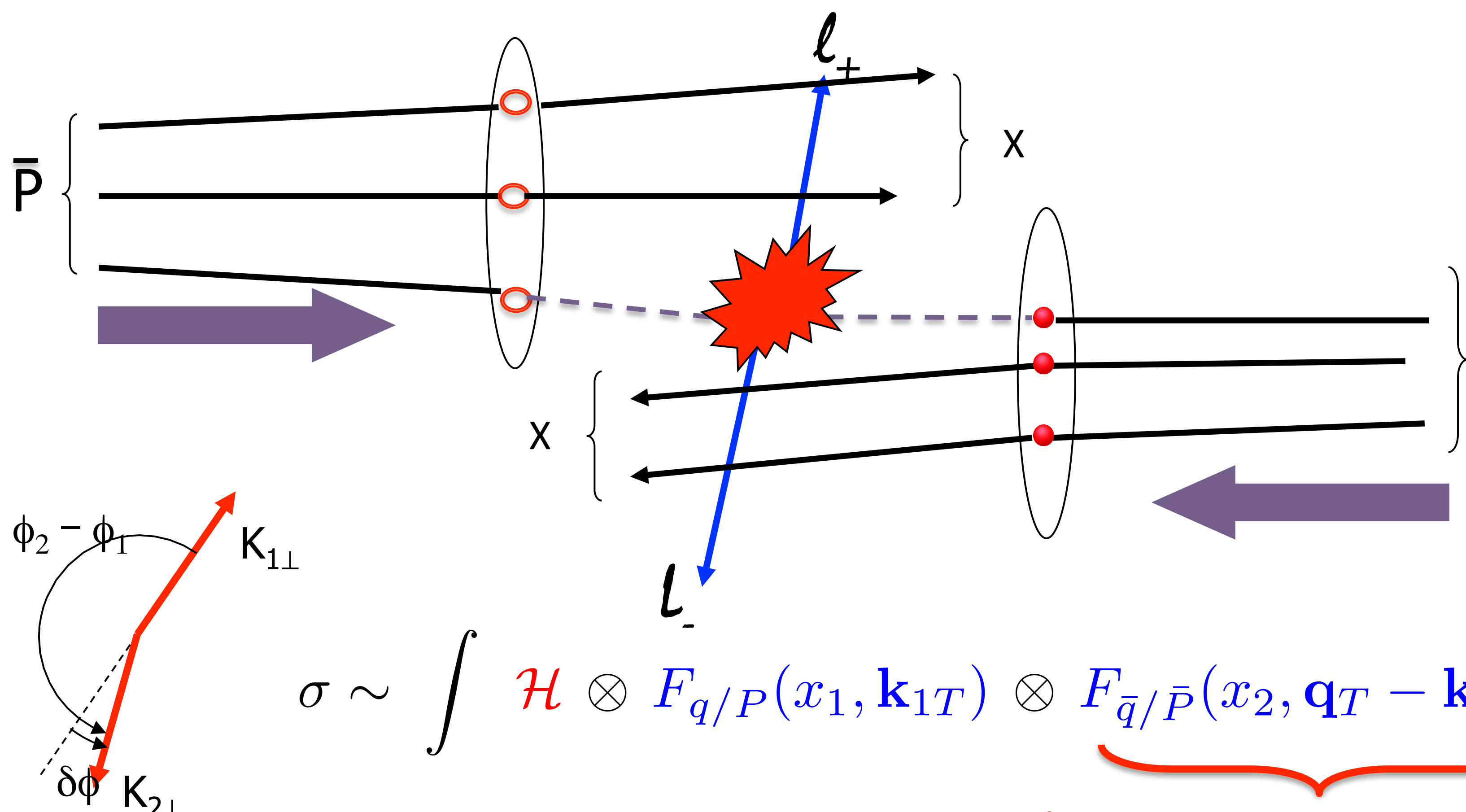
- Blue: Includes non-perturbative physics (but usually with universal properties) .

Example: Drell-Yan



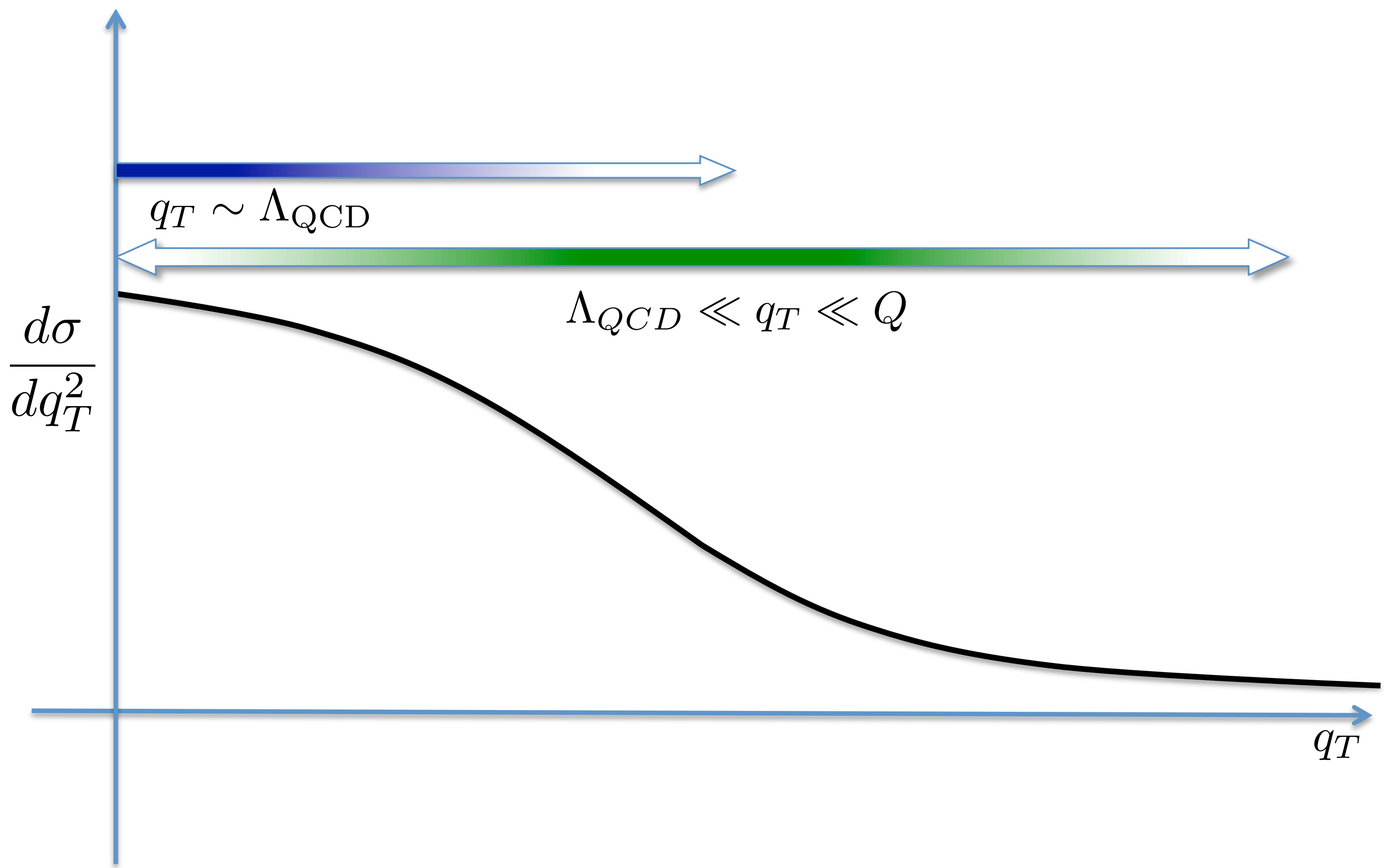
$$\sigma \sim \int \mathcal{H} \otimes f_{q/P}(x_1) \otimes \underbrace{f_{\bar{q}/\bar{P}}(x_2)}_{\text{(Scale Dependence: DGLAP)}}$$

Example: Drell-Yan



$$\sigma \sim \int \mathcal{H} \otimes F_{q/P}(x_1, \mathbf{k}_{1T}) \otimes \underbrace{F_{\bar{q}/\bar{P}}(x_2, \mathbf{q}_T - \mathbf{k}_{1T})}_{\text{(Scale Dependence: ??)}}$$

Small Transverse Momentum



Taxonomy

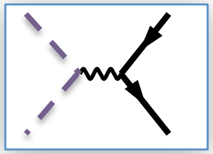
Sivers

Quark \ Proton	Unpolarized	Longitudinally polarized	Transversely polarized
Unpolarized	$f_1(x, k_T)$		$f_{1T}^{\perp}(x, k_T)$
Longitudinally polarized		$g_{1L}(x, k_T)$	$g_{1T}(x, k_T)$
Transversely polarized	$h_1^{\perp}(x, k_T)$	$h_{1L}(x, k_T)$	$h_{1T}(x, k_T)$ $h_{1T}^{\perp}(x, k_T)$

Boer-Mulders

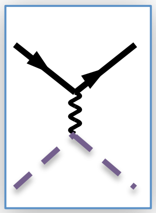
"Worm Gear"

"Pretzelocity"



Drell-Yan $\sigma \sim \int \mathcal{H}(Q) \otimes \underline{F_{q/P}(x_1, \mathbf{k}_{1T}, S_1)} \otimes F_{\bar{q}/\bar{P}}(x_2, \mathbf{q}_T - \mathbf{k}_{1T}, S_2)$

SIDIS $\sigma \sim \int \mathcal{H}(Q) \otimes \underline{F_{q/P}(x_1, \mathbf{k}_{1T}, S_1)} \otimes D_{H/q}(z, \mathbf{q}_T + \mathbf{k}_{1T})$



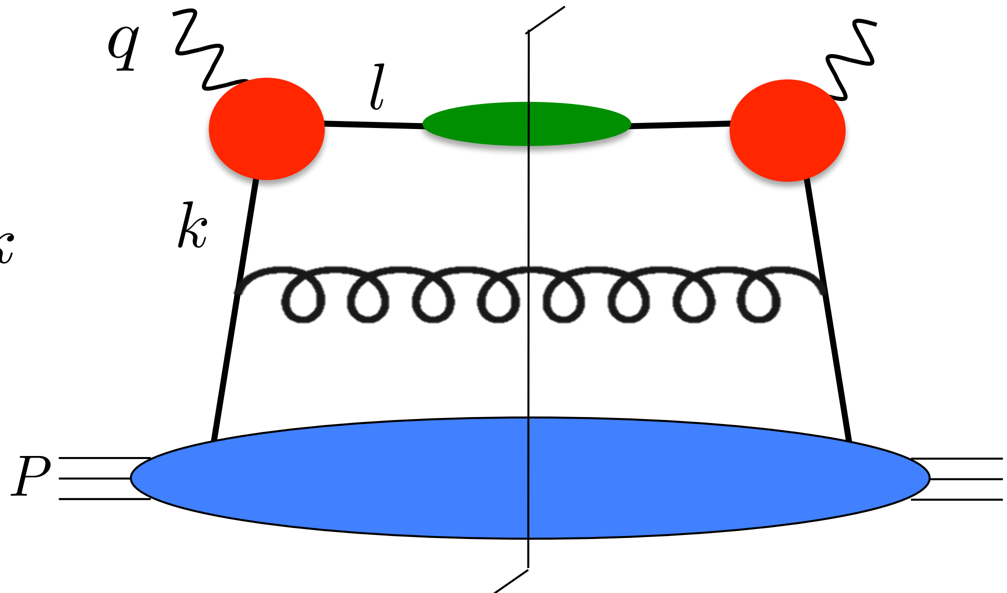
Quark \ Proton	Unpolarized	Longitudinally polarized	Transversely polarized
Unpolarized	$f_1(x, k_T)$		- $f_{1T}^{\perp}(x, k_T)$ + $f_{1T}^{\perp}(x, k_T)$
Longitudinally polarized		$g_{1L}(x, k_T)$	$g_{1T}(x, k_T)$
Transversely polarized	$h_1^{\perp}(x, k_T)$	$h_{1L}(x, k_T)$	$h_{1T}(x, k_T)$ $h_{1T}^{\perp}(x, k_T)$

Non-Zero!

Angular momentum etc, See M. Burkardt, S. Brodsky talks

Why Transverse Momentum Dependent Functions at Large x?

$$\int d^4k$$



$$k^2_{T,\max} = \frac{\hat{s}}{4} \qquad \hat{s} = (q + \hat{k})^2 = \frac{Q^2(1 - x/\xi)}{(x/\xi)} = \frac{Q^2(1-z)}{z}$$

$$x \to 1.0$$

$$H(x;\mu) \otimes f(x;\mu) \qquad \text{Or?} \qquad \int^{k^2_{T,\max}} dk_T^2 \, \frac{d\hat{\sigma}}{dx\; dk_T^2}$$

Collinear Factorization

TMD factorization

Transverse Momentum Dependent Factorization

- Unified Formalism ⟷ Parton Model

$$\frac{d\sigma}{d^2\mathbf{P}_{hT}} \sim \int \mathcal{H}(\mu/Q, \alpha_s(\mu)) \otimes F_{q/P}(x, \mathbf{k}_T, \mu, \zeta_1) \otimes D_{H/q}(z, \mathbf{p}_T + \mathbf{k}_T, \mu, \zeta_2)$$

Pert. QCD

Small Coupling: Perturbation Theory

+ Large transverse momentum (Y-term)

2 Auxiliary parameters: Arbitrary Operator Definitions

$$\mu \sim \sqrt{\zeta_1} \sim \sqrt{\zeta_2} \sim Q$$

$$\zeta_1 \zeta_2 \sim Q^4$$

(Collins-Soper-Sterman: 1981-1985)

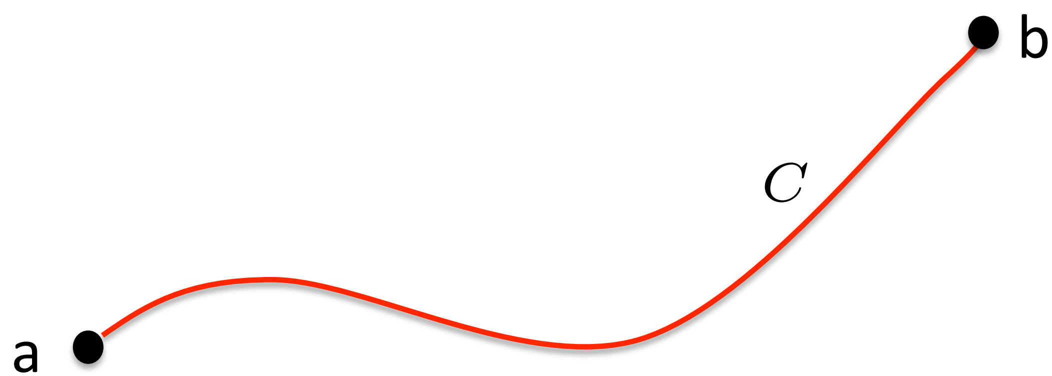
(J.C. Collins: (Book, 2011), Chaps. 10,13,14)

Parton Density Definition: Wilson Lines

- Collinear

$$\mathrm{F.T.}\ \langle P|\,\bar{\psi}(0,w^-,\mathbf{0}_t) \quad ?? \quad \psi(0,0,\mathbf{0}_t)\,|P\rangle$$

Wilson Lines

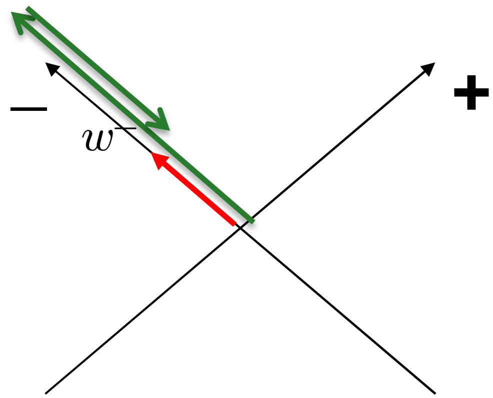


$$WL_C[a,b] = P\left\{\exp\left[-ig_0\int_C dx^\mu A^\alpha_{(0)\mu}(x)t_\alpha\right]\right\}$$

$$WL_C[a,b] \to e^{-ig_0 t^a \omega(a)}\, WL_C[a,b]\, e^{ig_0 t^a \omega(b)}$$

Parton Density Definition: Wilson Lines

- Paths of Wilson lines in coordinate space:



Standard (Integrated)

Parton Density Definition: Wilson Lines

- Collinear

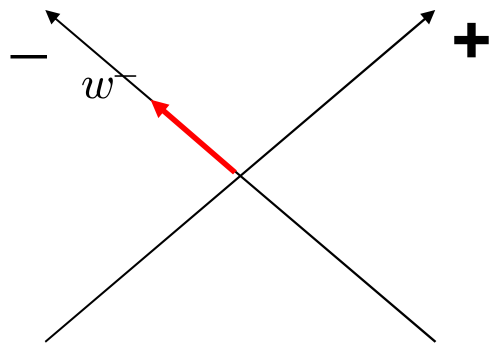
$$\text{F.T.}\ \langle P|\,\bar{\psi}(0,w^-,\mathbf{0}_t)\quad ??\quad \psi(0,0,\mathbf{0}_t)\,|P\rangle$$

- TMD

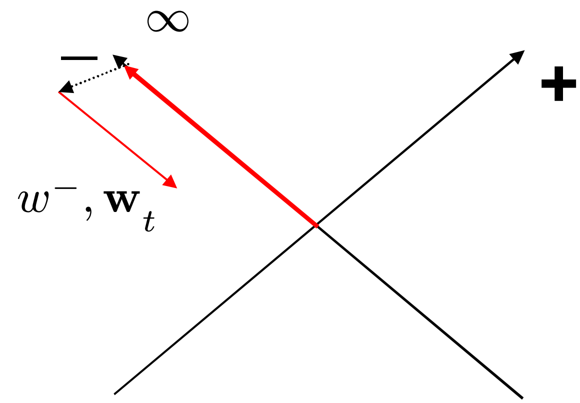
$$\text{F.T.}\ \langle P|\,\bar{\psi}(0,w^-,\mathbf{w}_t)\quad ??\quad \psi(0,0,\mathbf{0}_t)\,|P\rangle$$

Parton Density Definition: Wilson Lines

- Paths of Wilson lines in <u>coordinate space</u>:



Standard (Integrated)

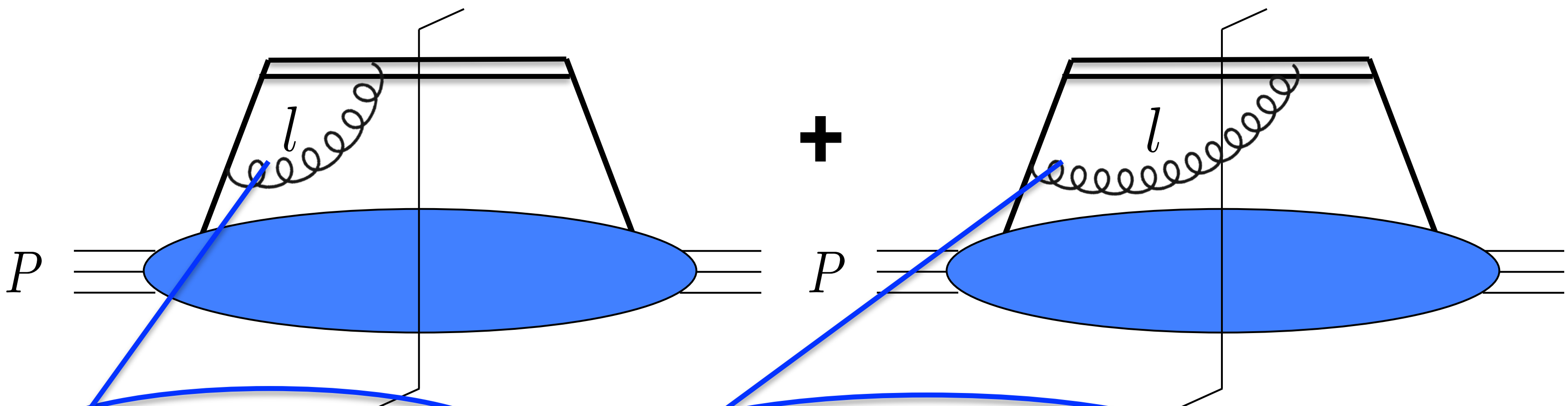


Unintegrated First Try

Transverse Momentum Dependence and Large x: Evolution

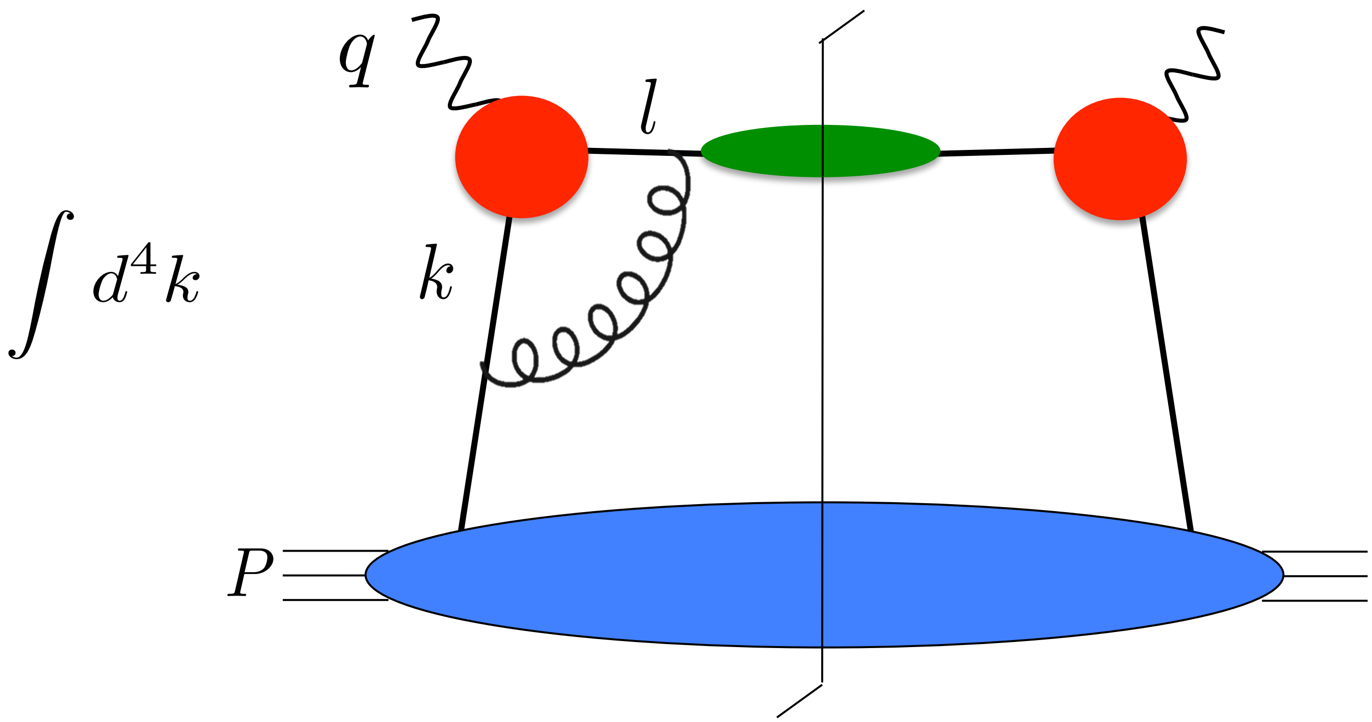
TMD kinematics/Approximation

$$\int_0^\infty dl^+$$



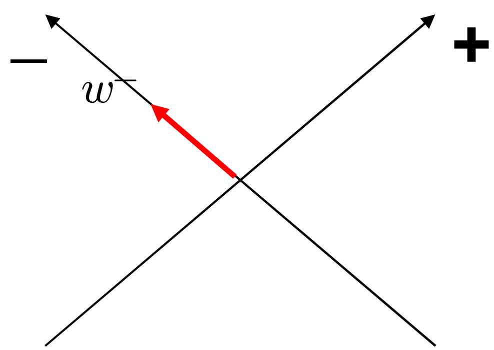
- UV Renormalization. But, no l_T divergence here.
- No cancelation of light-cone divergence at $l^+ = 0$.
- CSS evolution needed.

Why Transverse Momentum Dependent Functions at Large x?

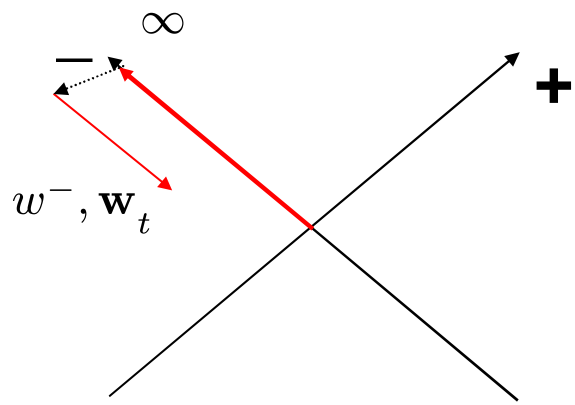


Parton Density Definition: Wilson Lines

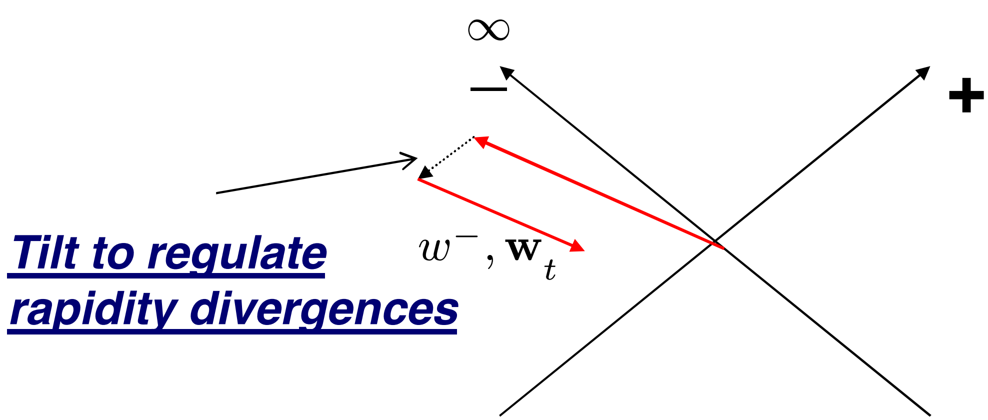
- Paths of Wilson lines in coordinate space:



Standard (Integrated)



Unintegrated First Try

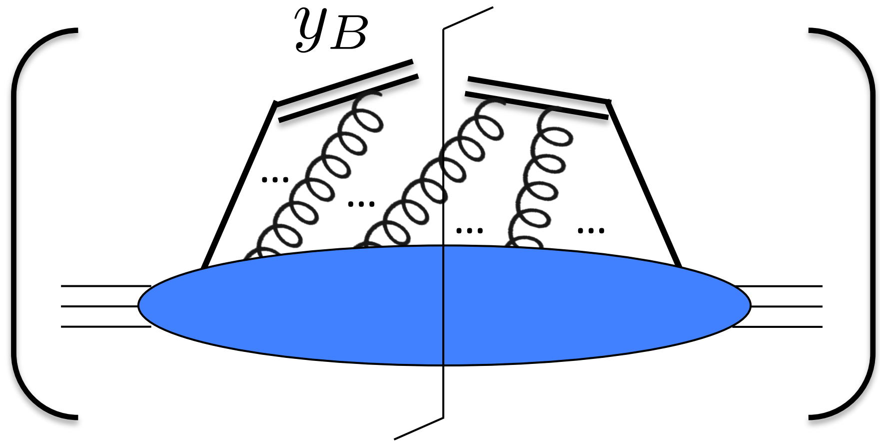


Unintegrated "tilted" Wilson lines

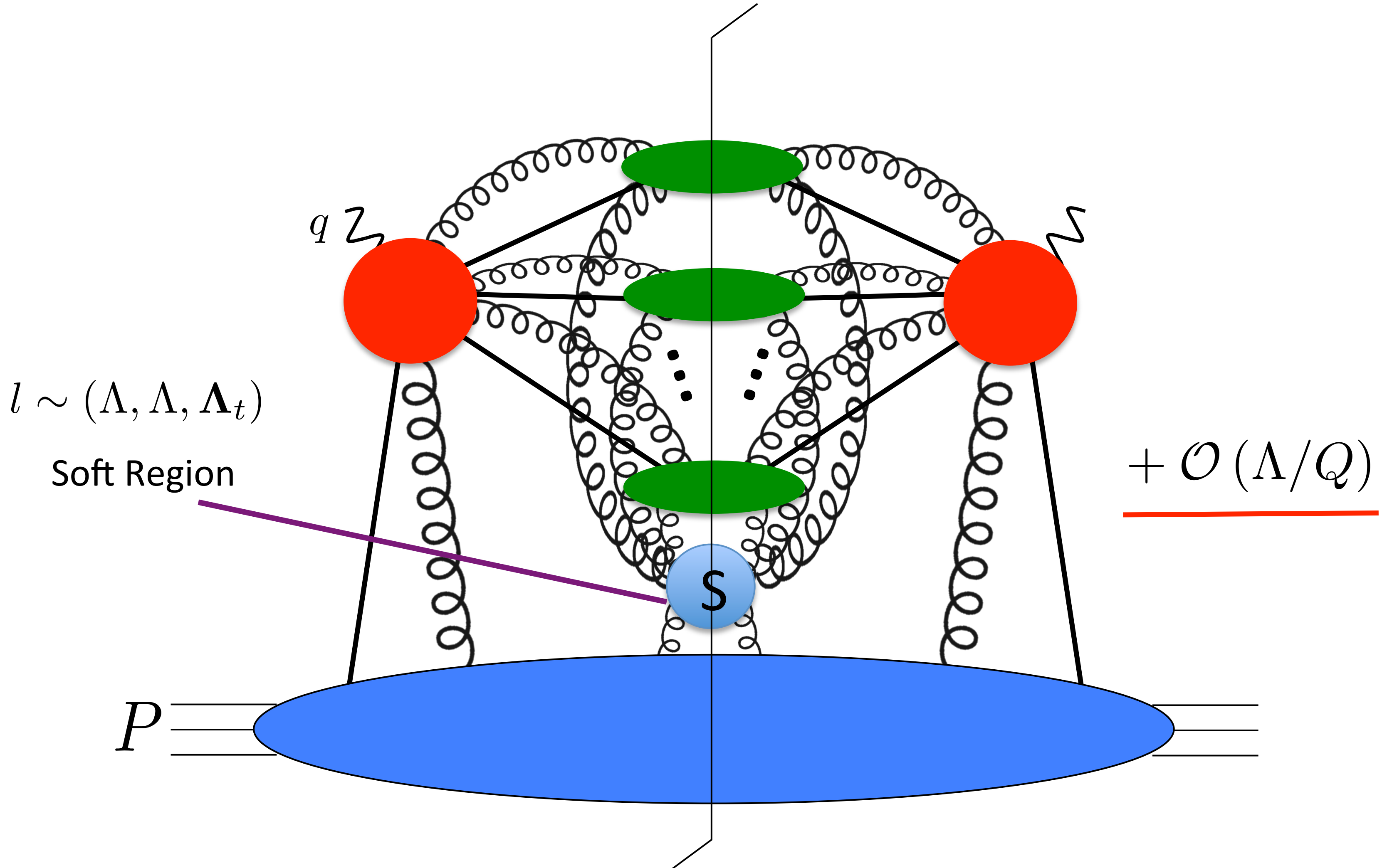
Parton Density Definition

- Unsubtracted

$$\tilde{F}^{\rm unsub}_{f/P}(\mathbf{b}_T, x; \mu; y_P - y_B) \sim \mathrm{F.T.}$$

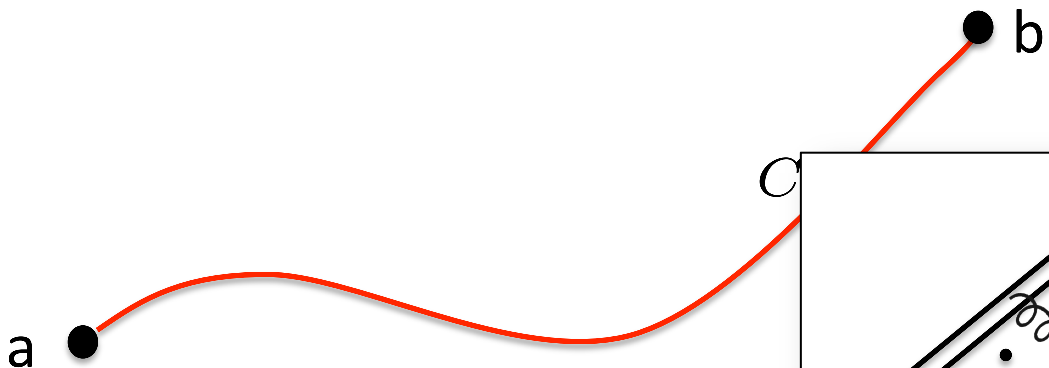


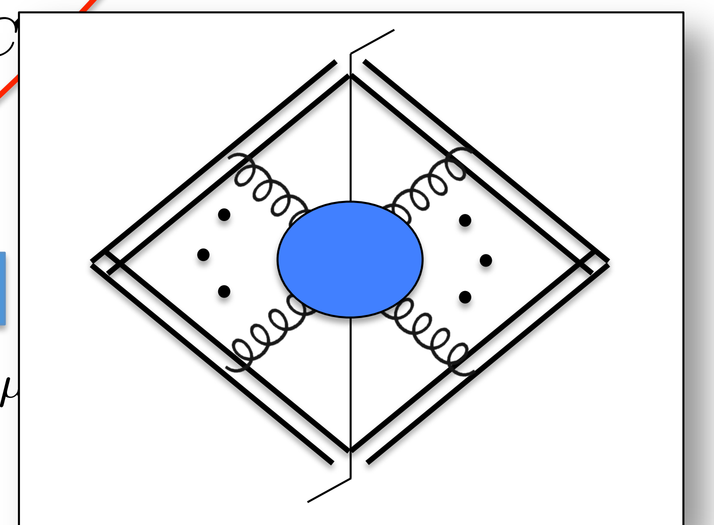
Factorization



Reduced Diagrams

Wilson Lines





$$WL_C[a,b] = P\left\{\exp\left[-ig_0 \int_C dx^\mu \right.\right.$$

$$WL_C[a,b] \to e^{-ig_0 t^a \omega(a)}\, WL_C[a,b]\, e^{ig_0 t^a \omega(b)}$$

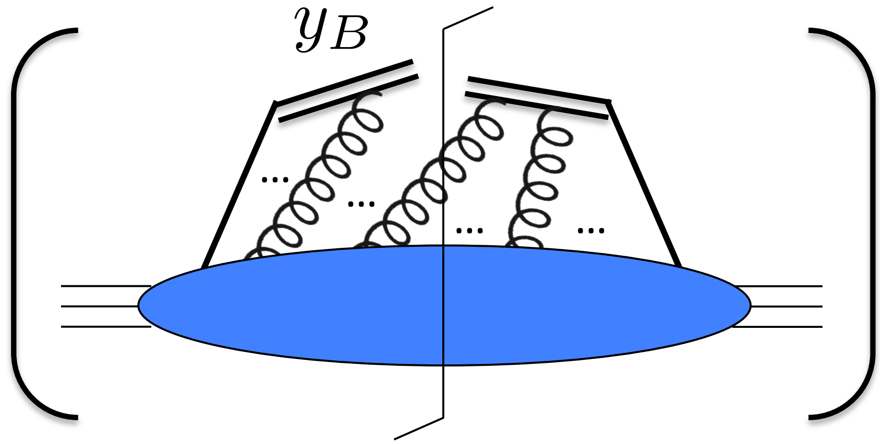
VEV $\langle 0 \,|\, WL_C[a,a] \,|\, 0 \rangle$

No Hadrons

Parton Density Definition

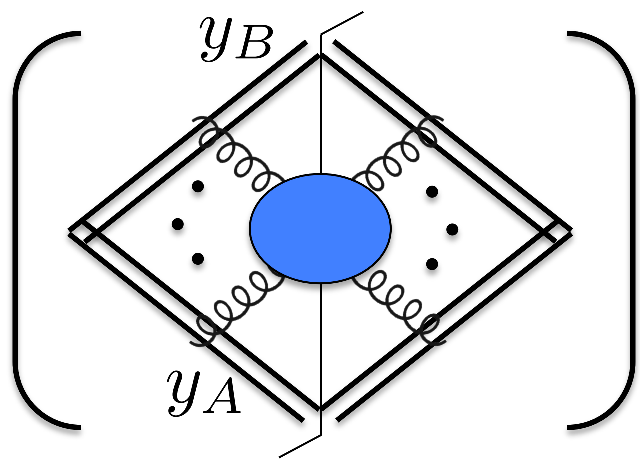
- Unsubtracted

$$\tilde{F}^{\text{unsub}}_{f/P}(\mathbf{b}_T, x; \mu; y_P - y_B) \sim \text{F.T.}$$



- Soft Factor

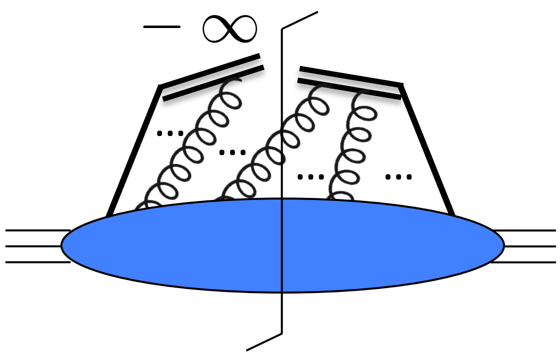
$$\tilde{S}(\mathbf{b}_T; y_A; y_B) \sim \text{F.T.}$$



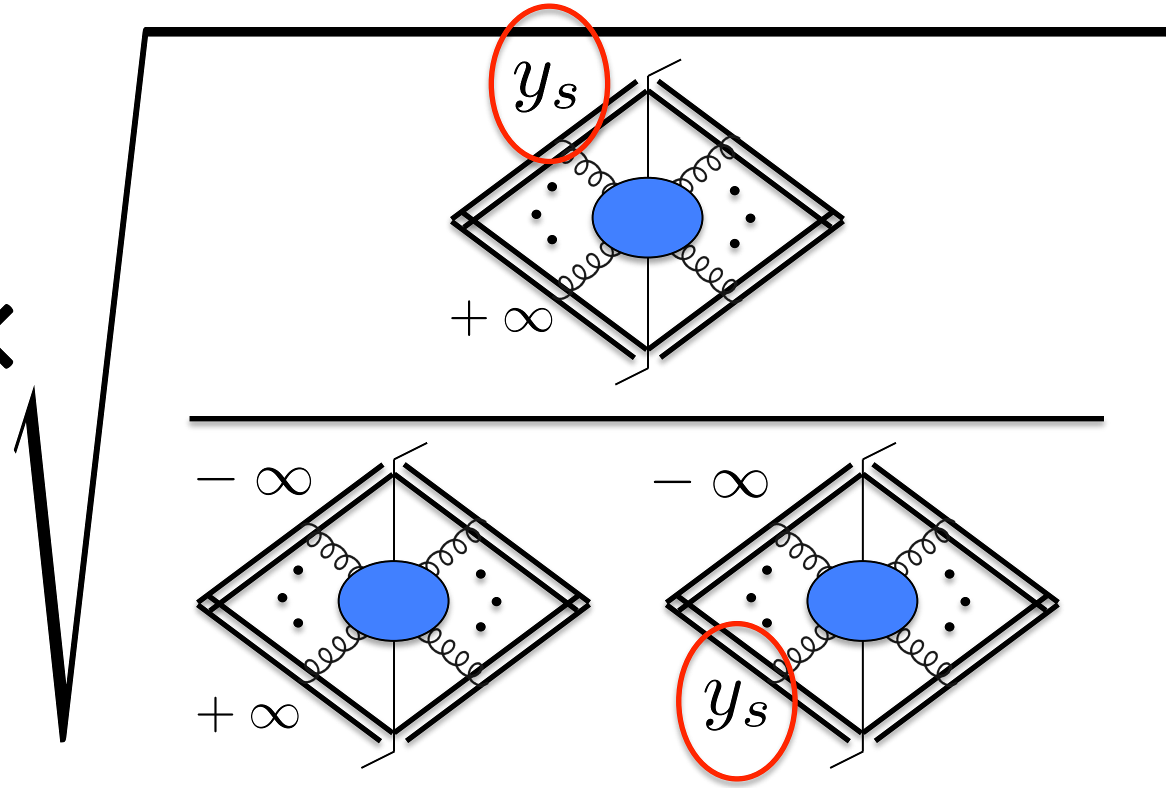
Parton Density Definition

$$\zeta_1 = x^2 M_p^2 e^{2(y_P - y_s)}$$

$$\tilde{F}_{f/P}(\mathbf{b}_T, x; \mu; \zeta_1) \; =$$



$\times$



(UV and rapidity renormalization needed)

(Collins (2011), chapt. 13)
Generalized Lightcone Renormalization Factor

Parton Density Definition and Evolution

- Renormalization Analogy

$$A_0^\mu \to Z_3^{1/2} A^\mu \qquad \psi_0 \to Z_2^{1/2}\psi$$

- DGLAP evolution:

$$f_{j/p}(\xi;\mu) = \sum_i \int \frac{dz}{z} Z_{ji}(z, g_s(\mu))\, f_{0,i/p}(\xi/z)$$
$$= Z_{ji} \otimes f_{0,i/p}$$

Collins-Soper / Light-cone Renormalization

- Collinear PDFs:

$$f_{j/p}(\xi;\mu) = \sum_i \int \frac{dz}{z}\, Z_{ji}(z, \alpha_s(\mu))\, f_{0,i/p}(\xi/z) \quad = \overbrace{Z_{ji}}^{\textit{Independent of hadron}} \otimes f_{0,i/p}$$

$$f_{0,i/p}(\xi) = \int \frac{dw^-}{(2\pi)}\, e^{-i\xi P^+ w^-} \, \langle P|\, \bar{\psi}_{0,j}(0, w^-, \mathbf{0}_t) U^{[+]}(w^-, 0) \frac{\gamma^+}{2} \psi_{0,j}(0, 0, \mathbf{0}_t)\, |P\rangle$$

- TMD PDFs, CS Equation:

$$\tilde{F}^{\text{unsub.}}_{f/P}(x_1, \mathbf{b}_T; \mu, y_s) = \tilde{F}^{\text{unsub.}}_{f/P}(x_1, \mathbf{b}_T; \mu, -\infty) \times Z_{\text{CS}}(\mathbf{b}_T; y_s, +\infty, -\infty)$$

$$\tilde{F}^{\text{unsub.}}_{f/P}(x_1, \mathbf{b}_T; \mu, y_s) = \lim_{\text{WL Raps}\to\infty} \left(\tilde{F}^{\text{unsub.}}_{f/P}(x_1, \mathbf{b}_T; \mu) \times \underbrace{Z_{\text{CS}}(\mathbf{b}_T; y_s)}_{\textit{Independent of hadron}} \right)$$

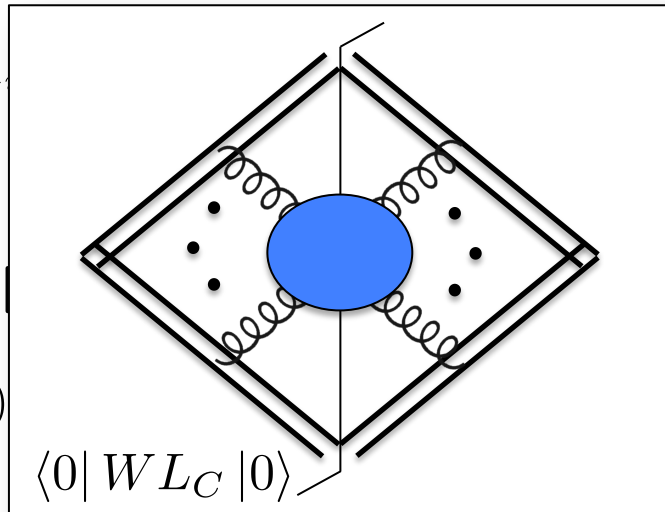
Collins-Soper / Light-cone Renormalization

- Collinear PDFs:

Independent of hadron

$$f_{j/p}(\xi;\mu) = \sum_i \int \frac{dz}{z}\, Z_{ji}(z,\alpha_s(\mu))\, f_{0,i/p}(\xi/z) \;\; = \underbrace{Z_{ji}}\otimes f_{0,i/p}$$

$$f_{0,i/p}(\xi) = \int \frac{dw^-}{(2\pi)}\, e^{-} \quad \cdots \quad 0)\, \frac{\gamma^+}{2}\, \psi_{0,j}(0,0,\mathbf{0}_t)\, |P\rangle$$



- TMD PDFs, CS

$$\tilde{F}^{\text{unsub.}}_{f/P}(x_1,\mathbf{b}_T;\mu,y_s) \quad \cdots \quad (\mathbf{b}_T; y_s, +\infty, -\infty)$$

$$\tilde{F}^{\text{unsub.}}_{f/P}(x_1,\mathbf{b}_T;\mu,y_s) \quad \sqrt{\frac{\tilde{S}(b_{\rm T};+\infty,y_s)}{\tilde{S}(b_{\rm T};+\infty,-\infty)\,\tilde{S}(b_{\rm T};y_s,-\infty)}} \quad \underbrace{Z_{\rm CS}(\mathbf{b}_T;y_s)}\Big)$$

t of hadron

Transverse Momentum Dependent Evolution

- Collinear / DGLAP, Evolution with Scale:

$$\frac{d}{d\ln\mu} f_{j/P}(x;\mu) = 2\int P_{jj'}(x') \otimes f_{j'/P}(x/x';\mu)$$

- TMD Case:

$$\frac{\partial \ln \tilde{F}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T;\mu)$$

Large b_T:
Non-perturbative
$-g_K(b_{\rm T}; b_{\rm max})$

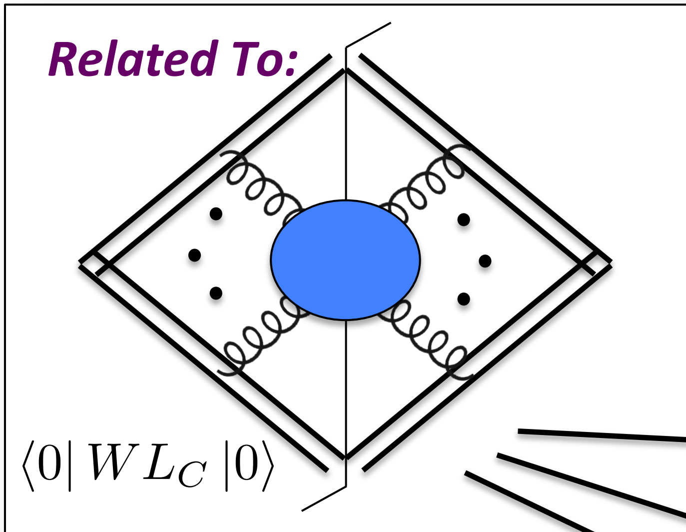
Small b_T:
perturbative

$$\frac{d\tilde{K}(b_T;\mu)}{d\ln\mu} = -\gamma_K(g(\mu))$$

$$\frac{d\ln \tilde{F}(x, b_T; \mu, \zeta)}{d\ln\mu} = \gamma_F(g(\mu); \zeta/\mu^2)$$

(Collins: (2011), Chapts. 10,13,14)

Transverse Momentum Dependent Evolution



volution with Scale:

$$= 2 \int P_{jj'}(x') \otimes f_{j'/P}(x/x';\mu)$$

- TMD Case:

$$\frac{\partial \ln \tilde{F}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T;\mu)$$

Large b_T:
Non-perturbative
$-g_K(b_{\mathrm{T}}; b_{\max})$

Small b_T:
perturbative

$$\frac{d\tilde{K}(b_T;\mu)}{d\ln\mu} = -\gamma_K(g(\mu))$$

$$\frac{d \ln \tilde{F}(x, b_T; \mu, \zeta)}{d\ln\mu} = \gamma_F(g(\mu); \zeta/\mu^2)$$

(Collins: (2011), Chapts. 10,13,14)

One TMD PDF: Solution to Evolution Equations

- Version 1:

$$\tilde{F}_{f/P}(x, \mathbf{b}_{\mathrm{T}}; Q, Q^2) =$$
$$\tilde{F}_{f/P}(x, \mathbf{b}_{\mathrm{T}}; \mu_0, Q_0^2) \ \exp\left\{ \tilde{K}(b_T; \mu_0) \ln \frac{Q}{Q_0} + \int_{\mu_0}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right\}$$

- Operator product expansion:

$$\tilde{F}_{j/P}(x, \mathbf{b}_T; \mu, \zeta_1) = \sum_k \int_x^1 \frac{d\xi}{\xi} \tilde{C}_{j/k}(x/\xi, b_{\mathrm{T}}; \zeta_1, \mu, \alpha_s(\mu)) f_{k/P}(\xi, \mu) \ + \ O((b_T m_q)^a)$$

One TMD PDF: Solution to Evolution Equations

Ex: Matching Prescription:

$$\mathbf{b}_*(\mathbf{b}_\mathrm{T}) \equiv \frac{\mathbf{b}_\mathrm{T}}{\sqrt{1 + b_T^2/b_{\mathrm{max}}^2}}$$

$$\mu_b \equiv C_1/|\mathbf{b}_*(b_T)|$$

Collinear PDFs

$$\tilde{F}_{f/P}(x, \mathbf{b}_\mathrm{T}; Q, Q^2) =$$

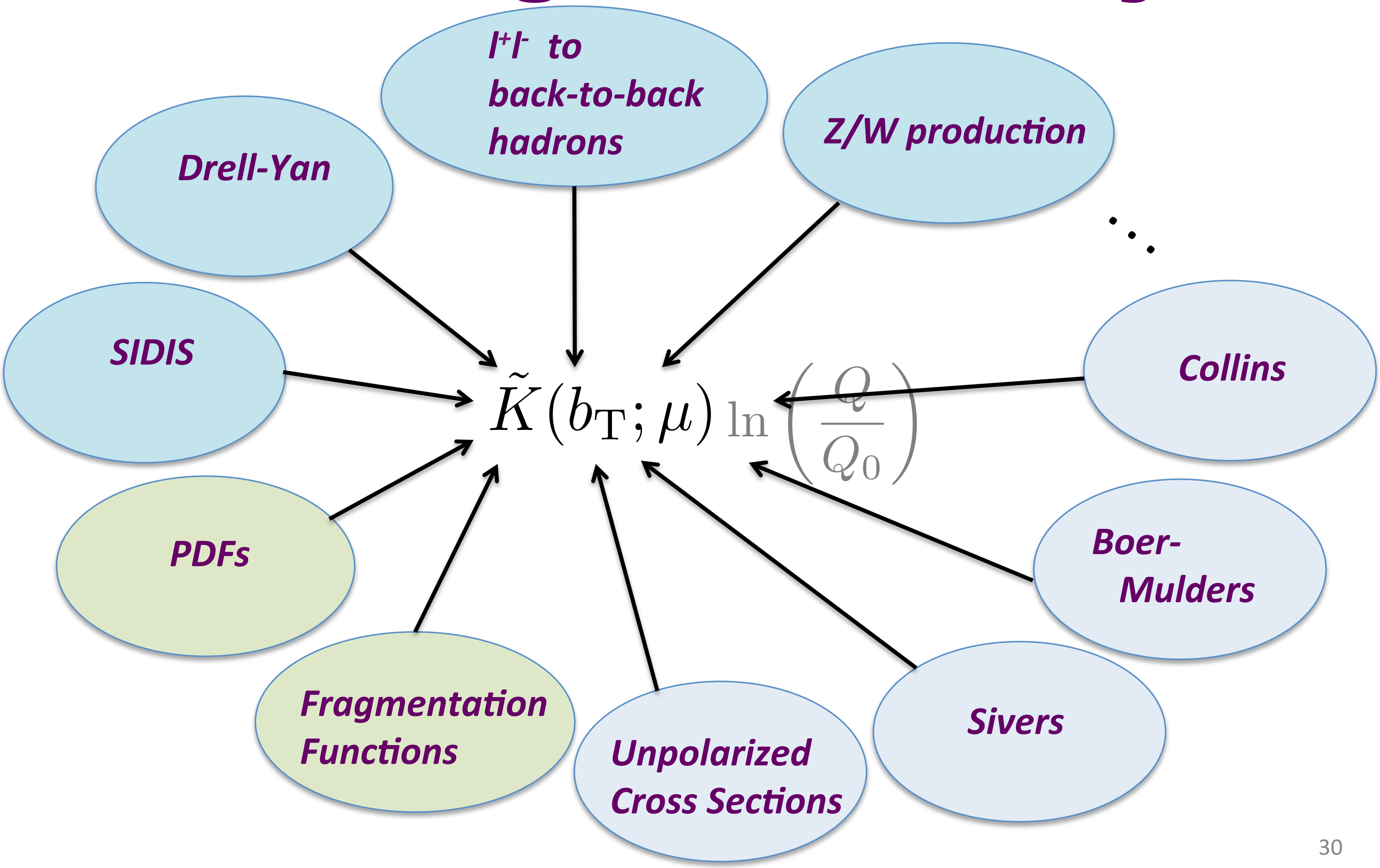
$$\sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{f/j}(x/\hat{x}, b_*; \mu_b^2, \mu_b, g(\mu_b)) f_{j/P}(\hat{x}, \mu_b) \times$$

$$\times \exp\left\{ \ln \frac{Q}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{Q} \frac{d\mu'}{\mu'} \left[\gamma_F(g(\mu'); 1) - \ln \frac{Q}{\mu'} \gamma_K(g(\mu')) \right] \right\} \times$$

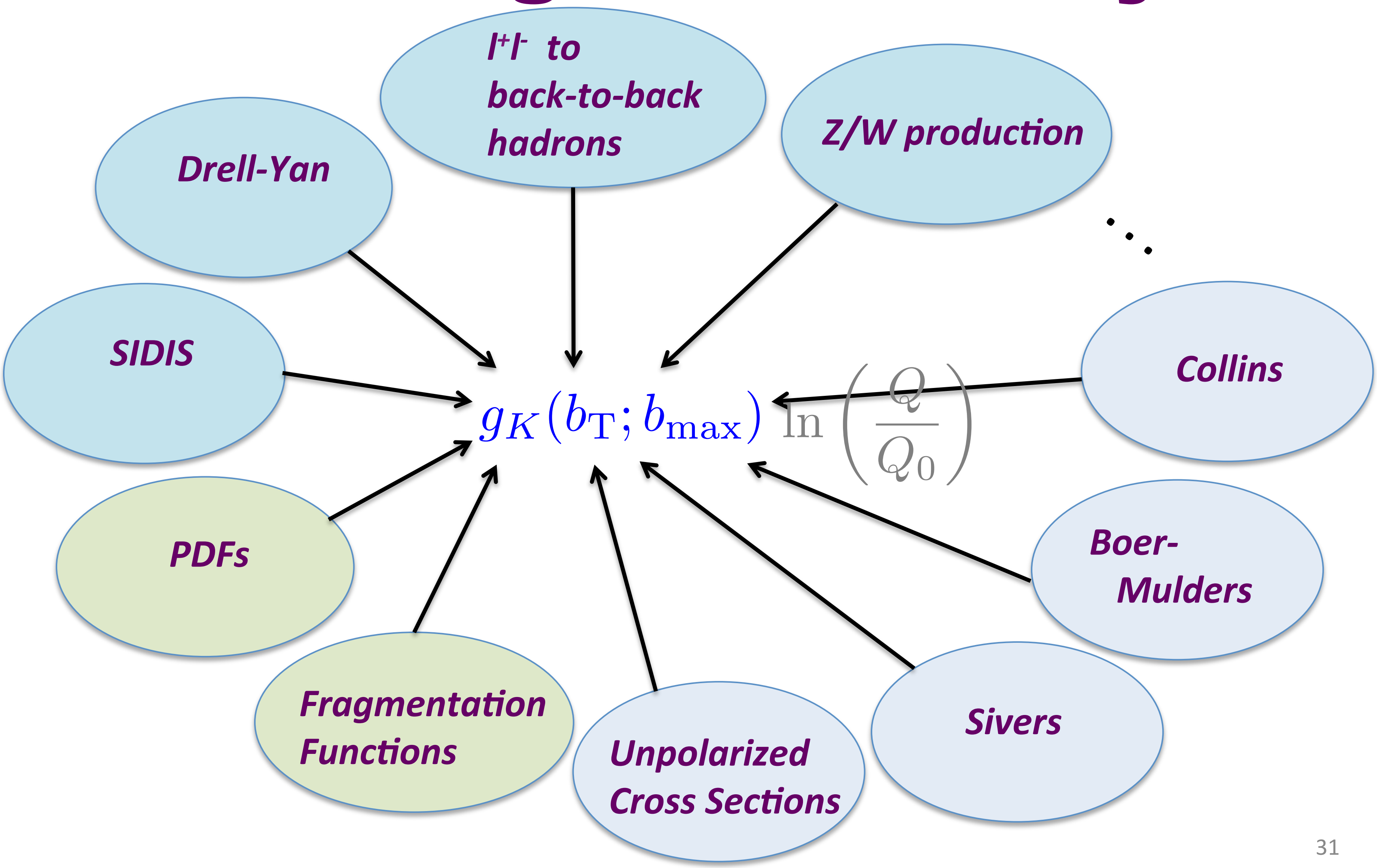
$$\times \exp\left\{ \underline{-g_{f/P}(x, b_T; b_{\mathrm{max}})} \underline{-g_K(b_T; b_{\mathrm{max}})} \ln \frac{Q}{Q_0} \right\}$$

Nonperturbative parts large b_T

Strong Universality



Strong Universality



Phenomenology

- ResBos: CSS formalism

$$\exp\left[-g_{j/A}(x_A, b_\mathrm{T}; b_\mathrm{max}) - g_{\bar{j}/B}(x_B, b_\mathrm{T}; b_\mathrm{max}) - \right.$$
$$\left. -g_K(b_\mathrm{T}; b_\mathrm{max}) \ln(Q^2/Q_0^2)\right]$$
$$= \exp\left\{-\left[g_1 + g_2 \ln\frac{Q}{2Q_0} + g_1 g_3 \ln(100 x_A x_B)\right] b_\mathrm{T}^2\right\}$$

$$g_1 = 0.21^{+0.01}_{-0.01}\,\mathrm{GeV}^2,$$
$$g_2 = 0.68^{+0.01}_{-0.02}\,\mathrm{GeV}^2,$$
$$g_3 = -0.6^{+0.05}_{-0.04}.$$
$$(\mathrm{BLNY},\ b_\mathrm{max} = 0.5\,\mathrm{GeV}^{-1} = 0.1\,\mathrm{fm})$$

(Landry, Brock, Nadolsky, Yuan, (2003))

$$g_K(b_\mathrm{T}; b_\mathrm{max}) = \frac{0.184 \pm 0.018}{2} b_\mathrm{T}^2.$$
$$(\mathrm{KN},\ b_\mathrm{max} = 1.5\,\mathrm{GeV}^{-1} = 0.3\,\mathrm{fm},\ C_1 = 2e^{-\gamma_E})$$

(Konychev, Nadolsky (2006))

Phenomenology

- ResBos: CSS formalism

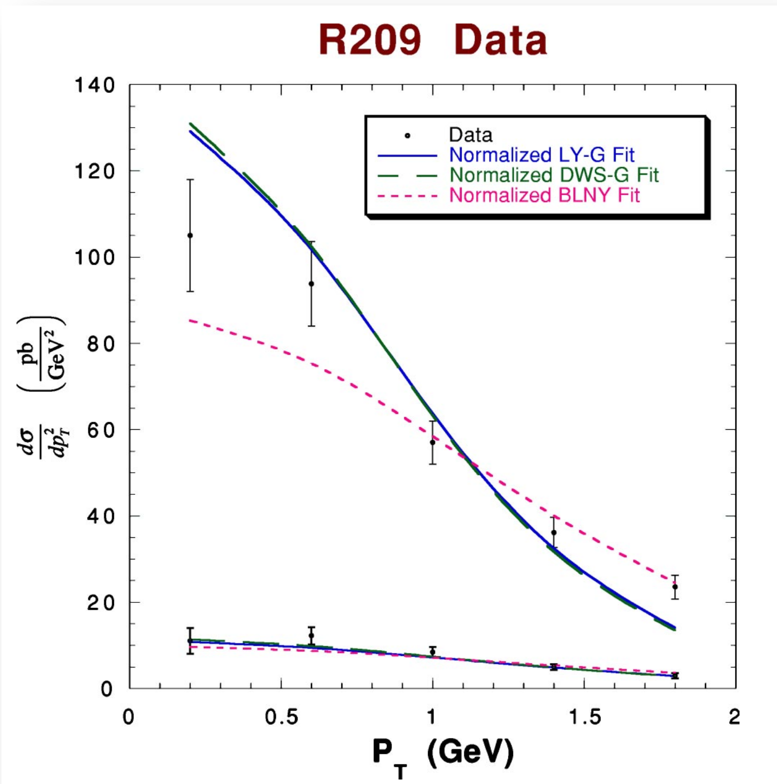
http://hep.pa.msu.edu/resum/

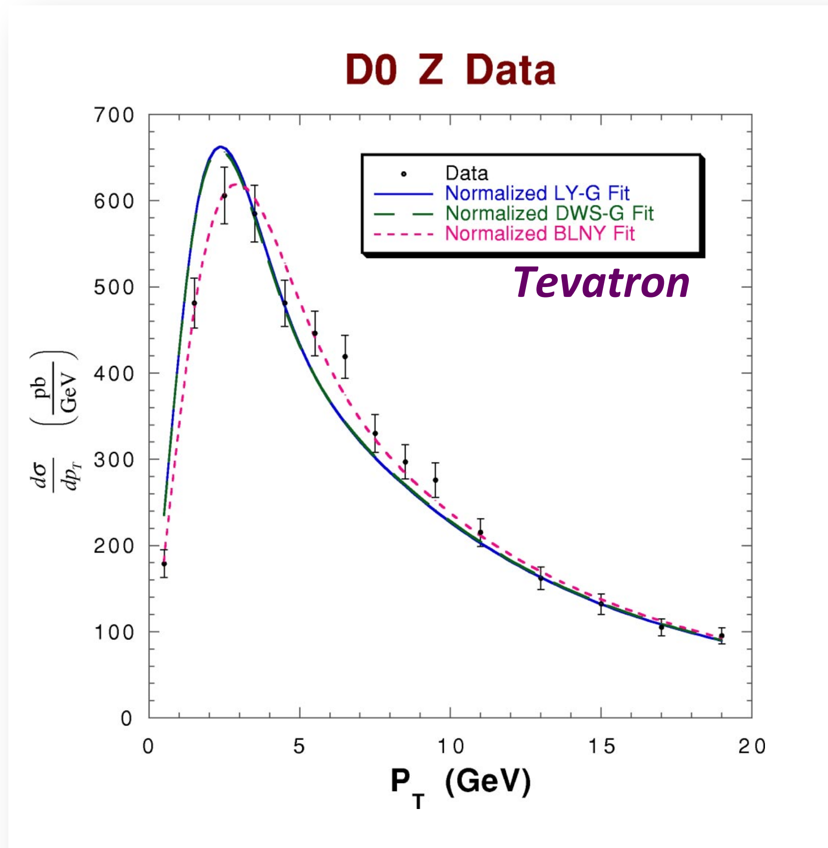
(Landry, Brock, Nadolsky, Yuan, (2003))

$$g_K(b_T)\ln\left(\frac{Q}{Q_0}\right) = -g_2\frac{1}{2}b_T^2\ln\left(\frac{Q}{Q_0}\right)$$

Gaussian ansatz

$g_2 = .68\ \mathrm{GeV}^2$ $\qquad b_{\max} = .5\ \mathrm{GeV}^{-1}$





Transverse Momentum Dependent Evolution

- Successful phenomenology:
  - Brock, Landry, Nadolsky, Yuan, Nadolsky, Konychev (hep-ph/0212159, hep-ph/0506225)
  - Qiu, Zhang (hep-ph/0012348)
  - Anselmino, et al (arXiv:1304.7691) ***(Recall S. Melis Talk, Tuesday)***
- Functions analogous to $\tilde{K}(b_T; \mu)$:
  - Soft-Collinear Effective Theory (SCET)
    - Echevarria, Idilbi, Schafer, Scimemi (arXiv:1111.4996 [hep-ph], arXiv:1208.1281 [hep- ph])
      Called -2D
    - Mantry, Petriello (arXiv:0911.4135 [hep-ph], arXiv:1011.0757 [hep-ph])
    - Becher, Neubert (arXiv:1007.4005 [hep-ph])
      Called $-F_{qq}$
  - Other
    - Boer (arXiv:0804.2408 [hep-ph], arXiv:1304.5387 [hep-ph])
    - Sun, Yuan (arXiv:1304.5037 [hep-ph], arXiv:1308.5003 [hep-ph])
  - Others….

A Standard Function, $A(b_T)$

(Recent work with John Collins)

Drell-Yan Cross Section

- TMD factorization cross section:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4 q\,\mathrm{d}\Omega} = \frac{2}{s}\sum_j \frac{\mathrm{d}\hat{\sigma}_{j\bar{j}}(Q,\mu,\alpha_s(\mu))}{\mathrm{d}\Omega}\int \mathrm{d}^2\mathbf{k}_{1T}\,\mathrm{d}^2\mathbf{k}_{2T}\;F_{j/A}(x_A,\mathbf{k}_{1T};\zeta_A,\mu)\;F_{\bar{j}/B}(x_B,\mathbf{k}_{2T};Q^4/\zeta_A,\mu)\,\delta^{(2)}(\mathbf{q}_T-\mathbf{k}_{1T}-\mathbf{k}_{2T})$$

+ Large transverse momentum (Y-term)

Drell-Yan Cross Section

- TMD factorization cross section:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\,\mathrm{d}\Omega} = \frac{2}{s}\sum_j \frac{\mathrm{d}\hat{\sigma}_{j\bar{\jmath}}(Q,\mu,\alpha_s(\mu))}{\mathrm{d}\Omega} \int \mathrm{d}^2\boldsymbol{b}_\mathrm{T}\; e^{i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\; \tilde{F}_{j/A}(x_A,\boldsymbol{b}_\mathrm{T};\zeta_A,\mu)\; \tilde{F}_{\bar{\jmath}/B}(x_B,\boldsymbol{b}_\mathrm{T};Q^4/\zeta_A,\mu)$$

+ Large transverse momentum (Y-term)

- W term.

$$\tilde{W}(\boldsymbol{b}_\mathrm{T};Q)_j \equiv Q^2\frac{\mathrm{d}\hat{\sigma}_{j\bar{\jmath}}(Q,\mu,\alpha_s(\mu))}{\mathrm{d}\Omega}\tilde{F}_{j/A}(x_A,\boldsymbol{b}_\mathrm{T};\zeta_A,\mu)\,\tilde{F}_{\bar{\jmath}/B}(x_B,\boldsymbol{b}_\mathrm{T};Q^2/\zeta_A,\mu)$$

- Small q_T cross section:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\,\mathrm{d}\Omega} \overset{q_T\ll Q}{\simeq} \frac{2}{sQ^2}\int \mathrm{d}^2\boldsymbol{b}_\mathrm{T}\, e^{i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\tilde{W}(\boldsymbol{b}_\mathrm{T};Q)$$

More on CS kernel

- Small q_T cross section:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4 q\, \mathrm{d}\Omega} \overset{q_T \ll Q}{\simeq} \frac{2}{sQ^2} \int \mathrm{d}^2 \boldsymbol{b}_\mathrm{T}\, e^{i \boldsymbol{q}_\mathrm{T} \cdot \boldsymbol{b}_\mathrm{T}} \tilde{W}(\boldsymbol{b}_\mathrm{T}; Q)$$

$$\frac{\partial \ln \tilde{W}(\boldsymbol{b}_\mathrm{T}, Q, x_A, x_B)}{\partial \ln Q^2} = \underbrace{\tilde{K}(b_\mathrm{T}; \mu)}_{\text{Collins-Soper Kernel}} + b_T \text{ Independent Terms}$$

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}\ln\mu} \tilde{K}(b_\mathrm{T}; \mu) = -\gamma_K\left(\alpha_s(\mu)\right) \quad \text{Renormalization Group}}$$

$$\tilde{K}(b_\mathrm{T}; \mu) \sim \alpha_s(\mu) \ln\left(\mu b_\mathrm{T}\right) + \cdots + O\left(\alpha_s(\mu)^n \ln\left(\mu b_\mathrm{T}\right)^m\right)$$

Use $\mu_b = C_1 / b_\mathrm{T}$ *to Exploit* ***Asymptotic Freedom*** *&*
Renormalization Group Improvement at Small b_T

CS Kernel: Perturbative and Non-Perturbative Behavior

- Separate large and small b_T.

$$\mathbf{b}_*(\mathbf{b}_\mathrm{T}) \equiv \frac{\mathbf{b}_\mathrm{T}}{\sqrt{1 + b_T^2/b_\mathrm{max}^2}}$$

$$\mu_b = C_1/b_\mathrm{T} \qquad \mu_{b^*} = C_1/b_*$$

$$\tilde{K}(b_\mathrm{T};\mu_0;\alpha_s(\mu_0)) = \underbrace{\tilde{K}(b_*;\mu_0;\alpha_s(\mu_0))} + \underbrace{\tilde{K}(b_\mathrm{T};\mu_0;\alpha_s(\mu_0)) - \tilde{K}(b_*;\mu_0;\alpha_s(\mu_0))}_{-g_K(b_\mathrm{T};b_\mathrm{max})}$$

$$\tilde{K}(b_*;\mu_0;\alpha_s(\mu_0)) = \tilde{K}(b_*;\mu_{b_*};\alpha_s(\mu_{b_*})) + \int_{\mu_{b_*}}^{\mu_0} \frac{d\mu'}{\mu'}\gamma_K(\alpha_s(\mu'))$$

CS Kernel: Perturbative and Non-Perturbative Behavior

- Separate large and small b_T.

$$\tilde{K}(b_{\mathrm{T}};Q;\alpha_s(Q)) = \underbrace{\underbrace{\tilde{K}(b_*;\mu_{b_*};\alpha_s(\mu_{b_*})) + \int_{\mu_{b_*}}^{Q} \frac{d\mu'}{\mu'}\gamma_K(\alpha_s(\mu'))} \underbrace{- g_K(b_{\mathrm{T}};b_{\max})}}_{b_{max}\text{ dependence cancels}}$$

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\tilde{K}(b_{\mathrm{T}};\mu) = -\gamma_K\left(\alpha_s(\mu)\right)$$

Q dependence is exactly independent of b_T

$\mathbf{A(b_T)}$

$$-\frac{\partial}{\partial \ln b_{\rm T}^2}\frac{\partial}{\partial \ln Q^2}\ln \tilde{W}(\boldsymbol{b}_{\rm T}, Q, x_A, x_B) \equiv A(b_{\rm T})$$

- *Same as Collins-Soper-Sterman $A(b_T)$ (1985)*
- Scale/Scheme independent

$$= -\frac{\partial}{\partial \ln b_{\rm T}^2}\tilde{K}(b_{\rm T};\mu;\alpha_s(\mu))$$

$$= \frac{\partial}{\partial \ln b_{\rm T}^2} g_K(b_{\rm T};b_{\max}) + \frac{b_*^2}{b_{\rm T}^2}A(b_*;\mu_{b_*})$$

$A(b_T)$

- 1985 Collins-Soper-Sterman Drell-Yan Notation:

$$\tilde{W}_j = (Q\text{-independent factor}) \times \exp\left\{ - \int_{C_1^2/b_T^2}^{C_2^2 Q^2} \frac{d\mu'^2}{\mu'^2} \left[\underline{A_{\rm CSS}(\alpha_s(\mu'); C_1)} \ln\left(\frac{C_2^2 Q^2}{\mu'^2} \right) + B_{\rm CSS}(\alpha_s(\mu'); C_1, C_2) \right] \right\}$$

- "$A(b_T)$" function:

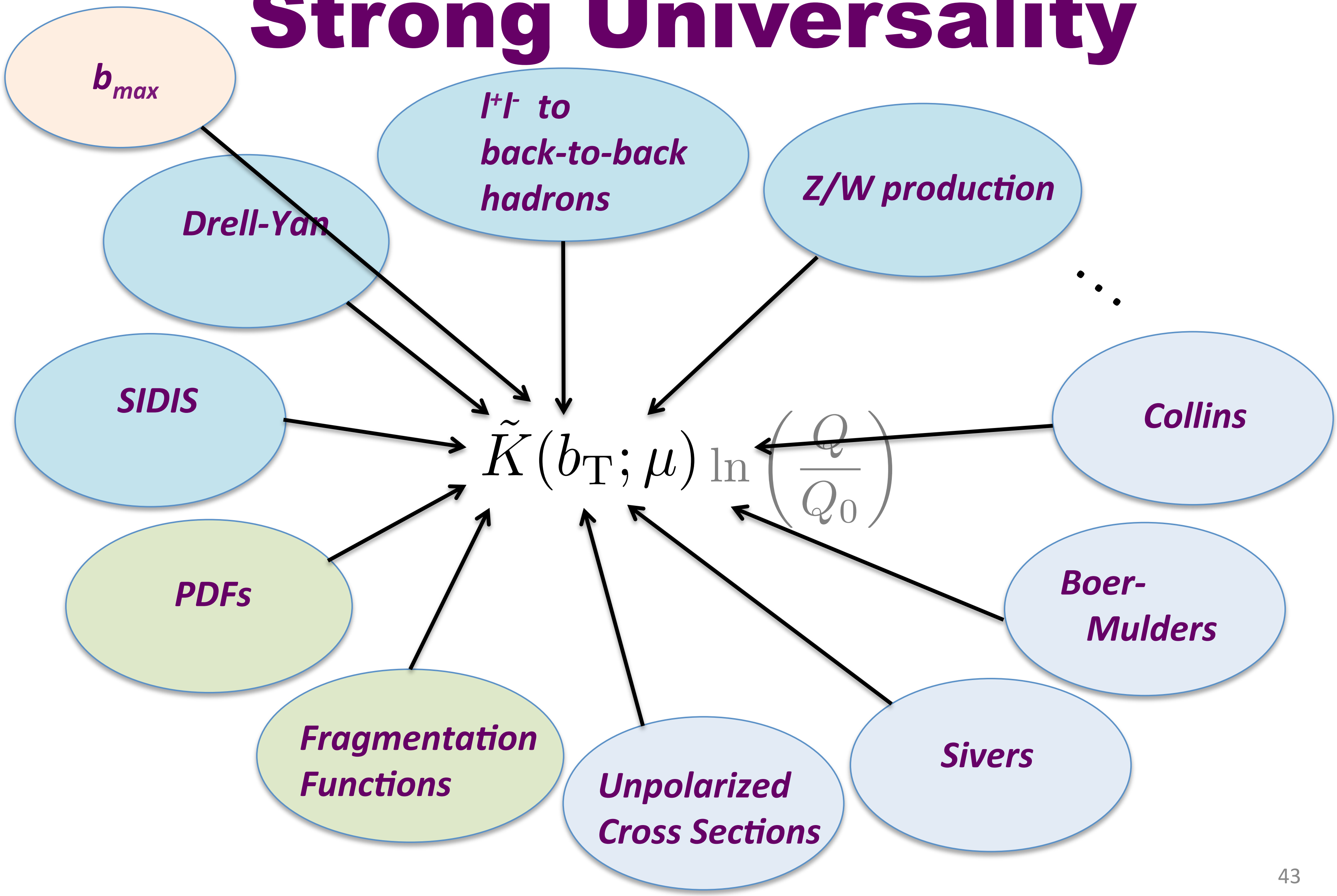
$$A(b_{\rm T}; \mu) = A_{\rm CSS}(\alpha_s(C_1/b_{\rm T}); C_1)$$

- Calculations:

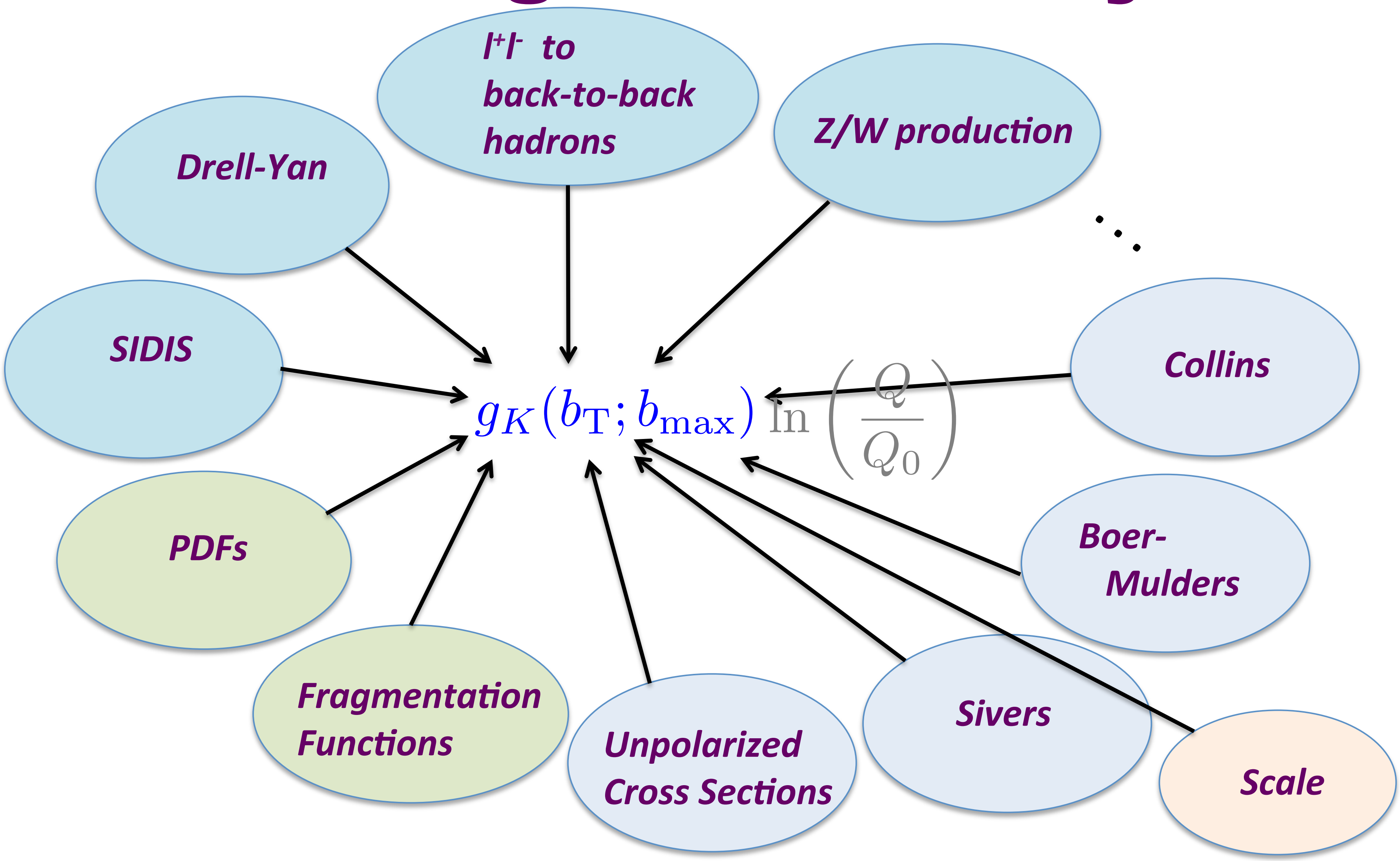
$$A(b_{\rm T}) = \frac{\alpha_s(\mu) C_F}{\pi} + \left(\frac{\alpha_s(\mu)}{\pi} \right)^2 C_F \left[\left(\frac{67}{36} - \frac{\pi^2}{12} \right) C_A - \frac{5}{9} T_R n_f + \left(\frac{11}{12} C_A - \frac{1}{3} T_R n_f \right) \ln\left(\frac{\mu^2 b_{\rm T}^2}{4 e^{-2\gamma_E}} \right) \right] + O(\alpha_s(\mu)^3)$$

(Kodaira, Trentadue (1982), Davies, Stirling (1984))

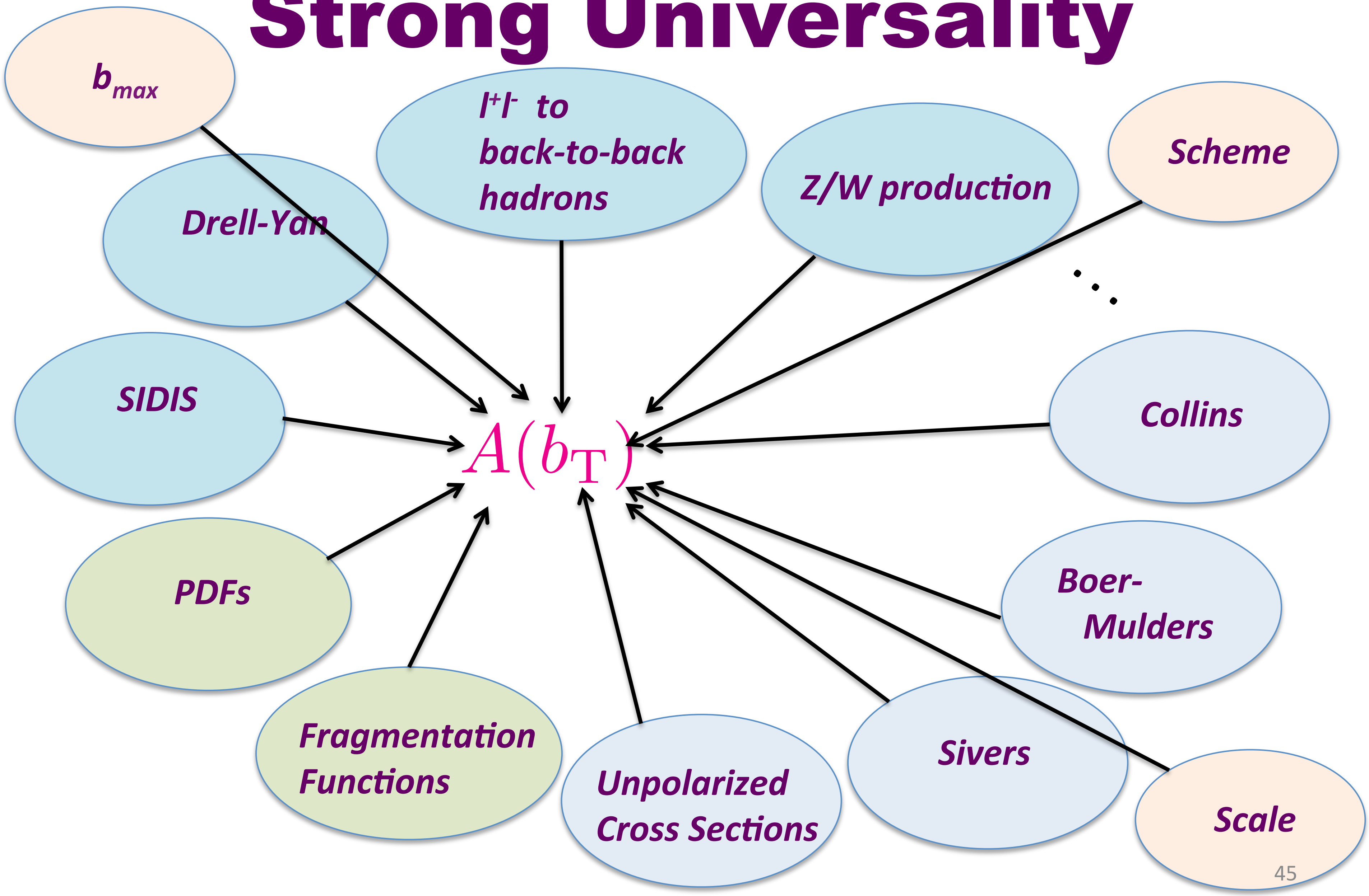
Strong Universality



Strong Universality



Strong Universality



Properties of $\tilde{K}(b_T;\mu)$

1) $\tilde{K}(b_T;\mu)$ goes to a (μ-dependent) constant at large b_T.

   – The constant has perturbatively calculable Q dependence.
   – The constant is negative (thus, the large b_T tail decreases with Q).

2) At small b_T, $\tilde{K}(b_T;1/b_T)$ is perturbatively calculable to fixed order.

3) At large b_T, $g_K(b_T;b_{max})$ goes to a constant such that $\tilde{K}(b_T;\mu)$ goes to a negative constant.

4) At small b_T, $g_K(b_T;b_{max})$ is a power series in $(b_T/b_{max})^2$ with perturbatively calculable coefficients.

Further Properties of $g_K(b_T;b_{max})$

5)

$$\operatorname*{asy}_{b_{\mathrm{T}} \ll b_{\max}} \left. \frac{\mathrm{d}}{\mathrm{d}b_{\max}} g_K(b_{\mathrm{T}}; b_{\max}) \right|_{\text{parametrized}} = \operatorname*{asy}_{b_{\mathrm{T}} \ll b_{\max}} \left. \frac{\mathrm{d}}{\mathrm{d}b_{\max}} g_K(b_{\mathrm{T}}; b_{\max}) \right|_{\text{truncated PT}}$$

6)

$$\frac{\mathrm{d}}{\mathrm{d}\ln b_{\max}} g_K(b_{\mathrm{T}} = \infty; b_{\max}) = \left[\frac{\mathrm{d}\tilde{K}(b_{\max}; C_1/b_{\max})}{\mathrm{d}\ln b_{\max}} - \gamma_K(\alpha_s(C_1/b_{\max})) \right]_{\text{truncated PT}}$$

Sample 1-parameter $g_K(b_T;b_{max})$

- $$g_K(b_{\rm T}; b_{\max}) = g_0(b_{\max}) \left(1 - \exp\left[- \frac{C_F \alpha_s(\mu_{b_*}) b_{\rm T}^2}{\pi g_0(b_{\max})\, b_{\max}^2} \right]\right)$$

- $$g_0(b_{\max}) = g_0(b_{\max,0}) + \frac{2C_F}{\pi} \int_{C_1/b_{\max,0}}^{C_1/b_{\max}} \frac{d\mu'}{\mu'} \alpha_s(\mu')$$

Sample 1-parameter $g_K(b_T;b_{max})$

- Actual g_K

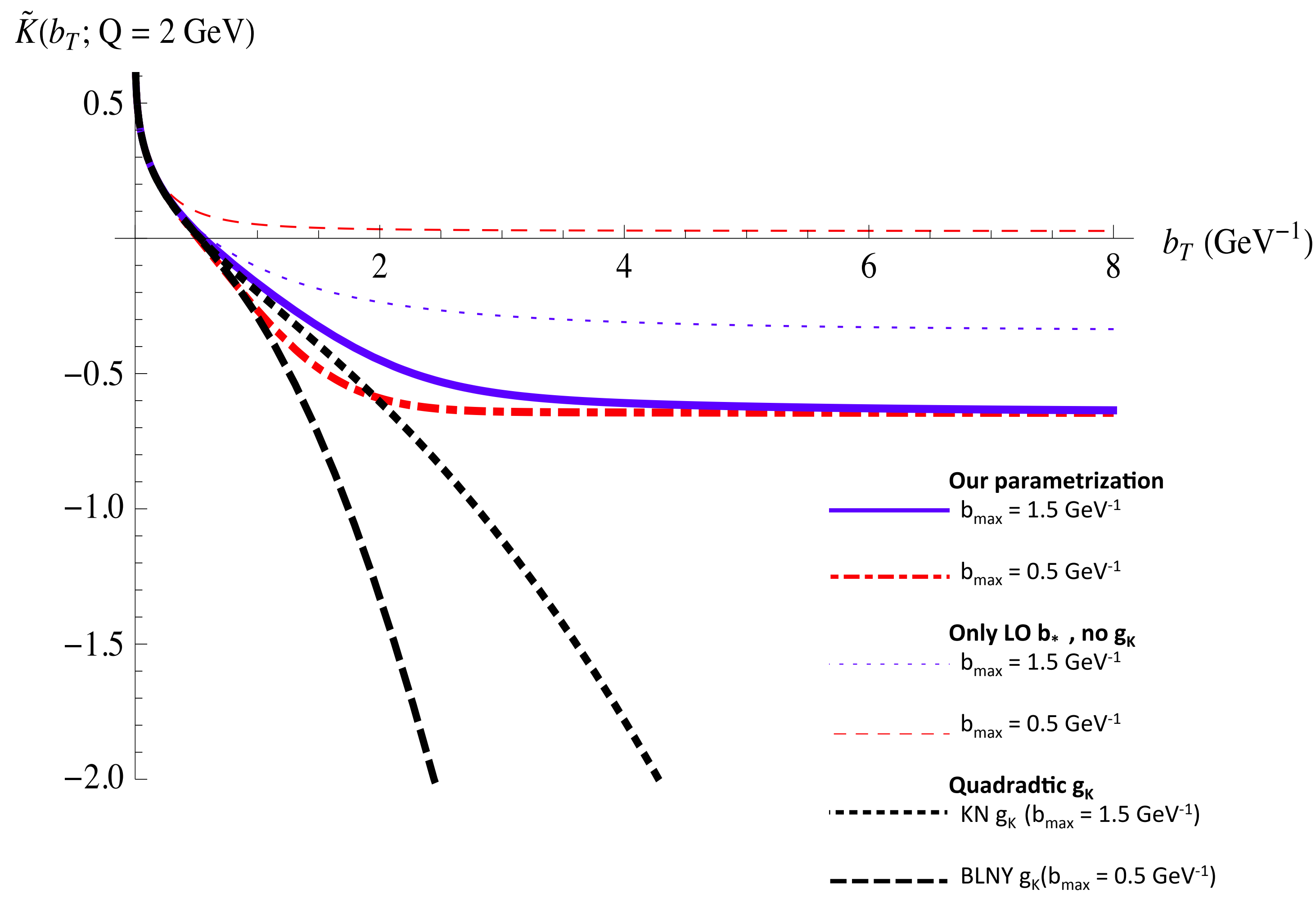
$$g_K(b_{\rm T}; b_{\rm max}) \equiv -\tilde{K}(b_{\rm T}; \mu_{b_*}; \alpha_s(\mu_{b_*})) + \tilde{K}(b_*; \mu_{b_*}; \alpha_s(\mu_{b_*}))$$

$$= \frac{C_F}{\pi} \frac{b_{\rm T}^2}{b_{\rm max}^2} \alpha_s(\mu_{b_*}) + O\left(\frac{b_{\rm T}^4}{\pi^2 b_{\rm max}^4} \alpha_s(\mu_{b_*})^2 \right)$$

- Parametrization

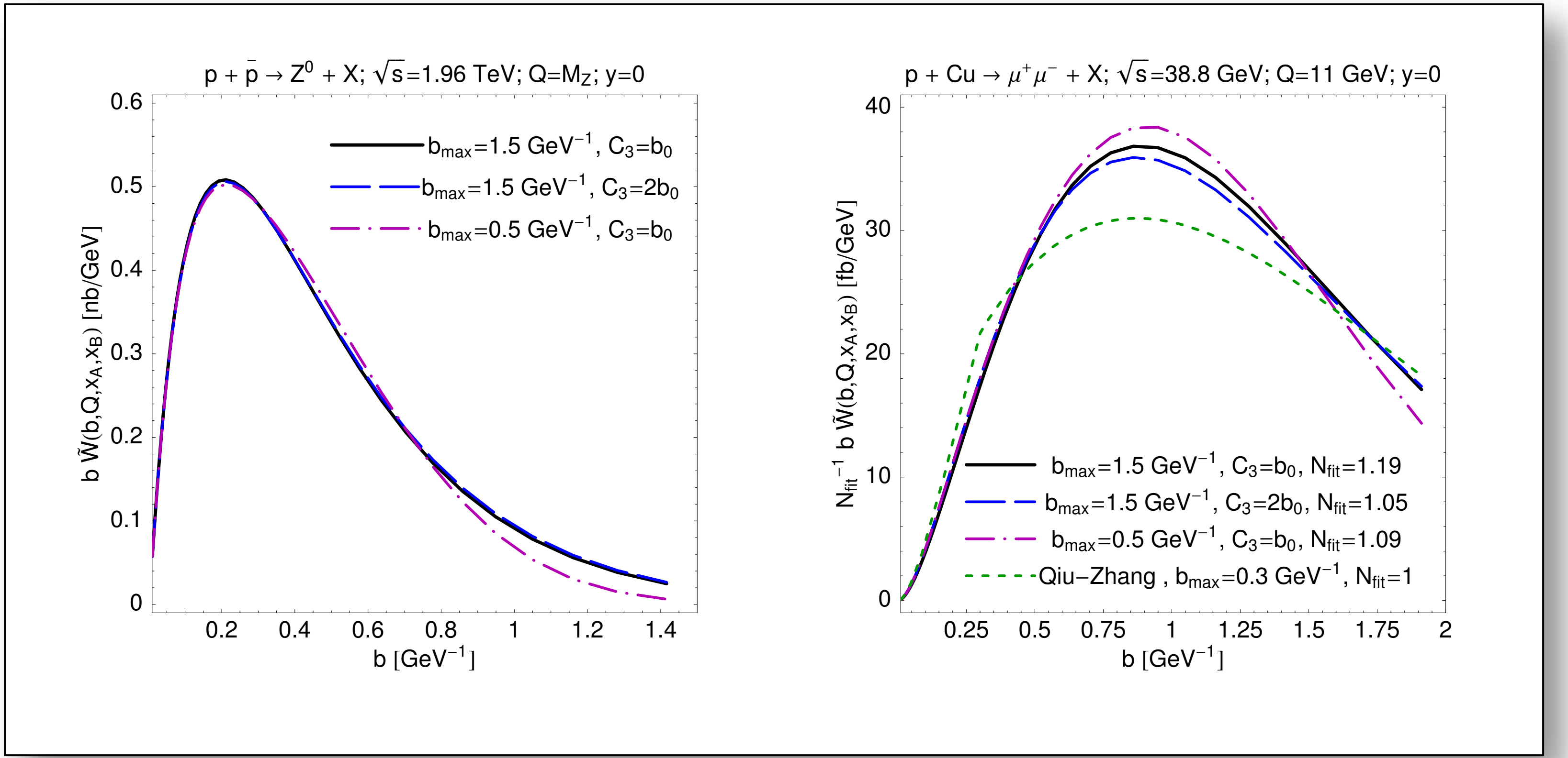
$$g_K(b_{\rm T}; b_{\rm max}) = \frac{C_F}{\pi} \frac{b_{\rm T}^2}{b_{\rm max}^2} \alpha_s(\mu_{b_*}) + O\left(\frac{b_{\rm T}^4 C_F^2 \alpha_s(\mu_{b_*})^2}{b_{\rm max}^4 \pi^2 g_0(b_{\rm max})} \right)$$

$\tilde{K}(b_T;\mu)$



$\tilde{K}(b_T; \mathrm{Q} = 2\ \mathrm{GeV})$
0.5
−0.5
−1.0
−1.5
−2.0
2
4
6
8
$b_T\ (\mathrm{GeV}^{-1})$
Our parametrization
b_{max} = 1.5 GeV^{-1}
b_{max} = 0.5 GeV^{-1}
Only LO b_* , no g_K
b_{max} = 1.5 GeV^{-1}
b_{max} = 0.5 GeV^{-1}
Quadradtic g_K
KN g_K (b_{max} = 1.5 GeV^{-1})
BLNY g_K(b_{max} = 0.5 GeV^{-1})

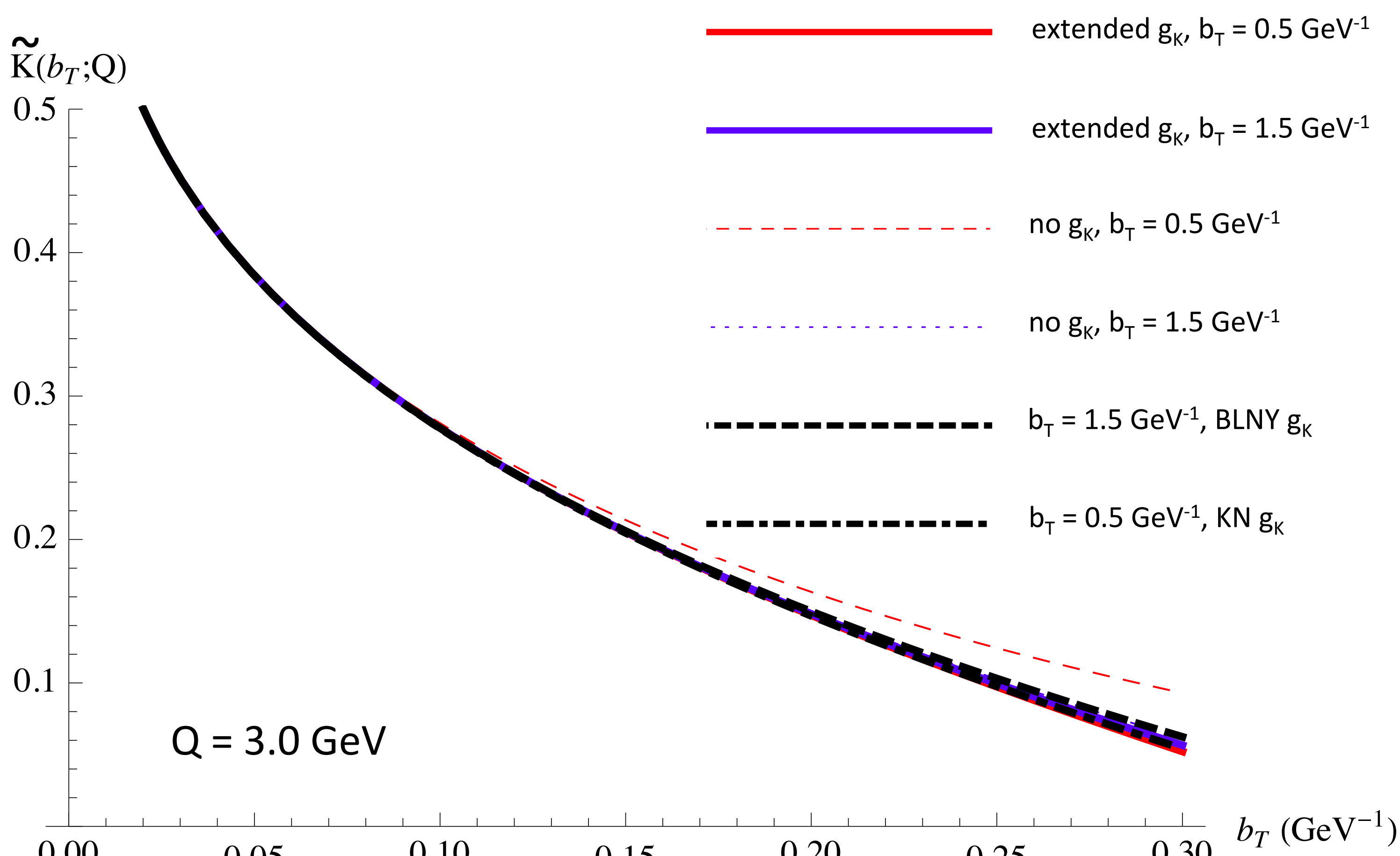
$\tilde{K}(b_T;\mu)$



High Energy

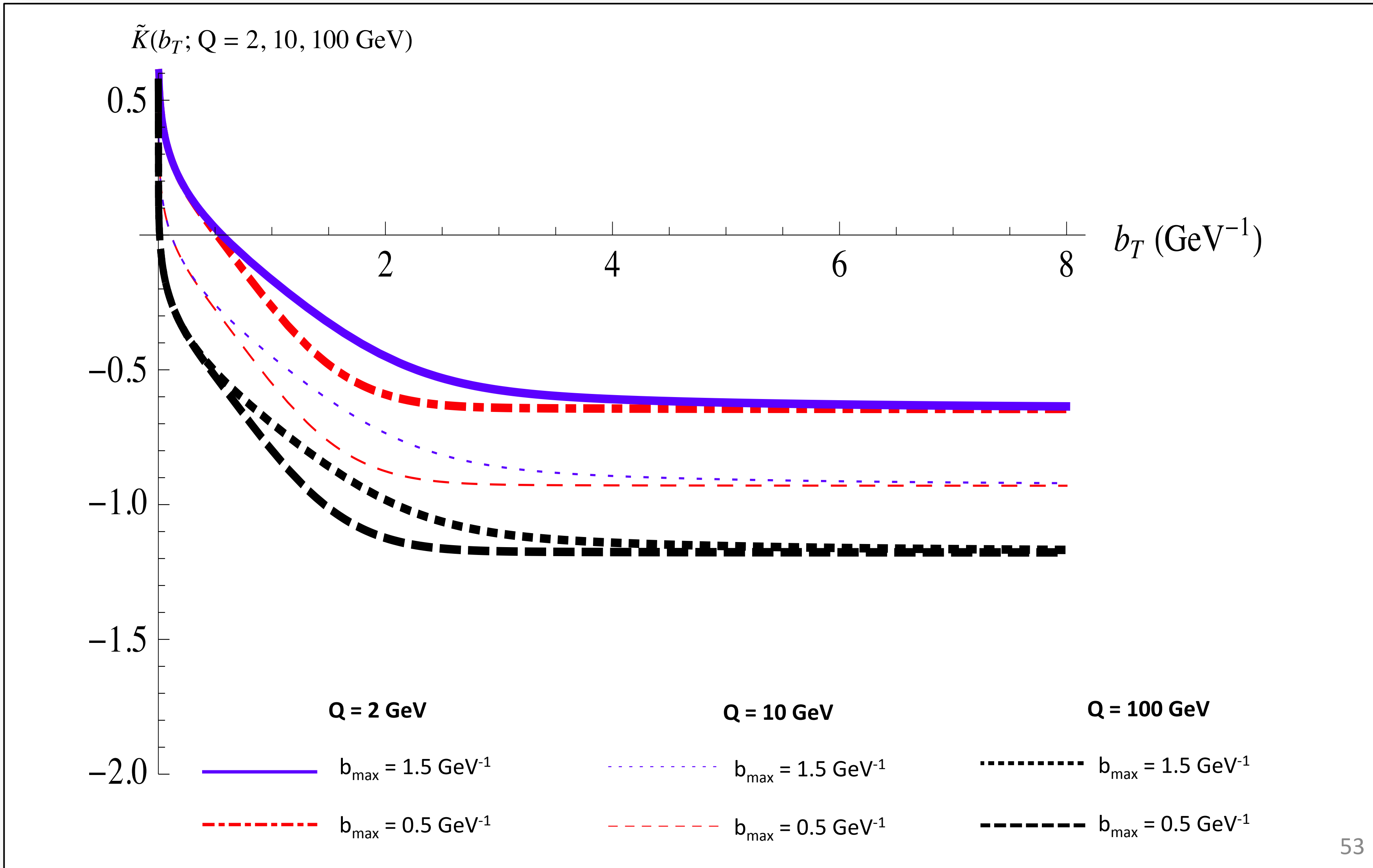
(Konychev, Nadolsky (2006))

$\tilde{K}(b_T;\mu)$



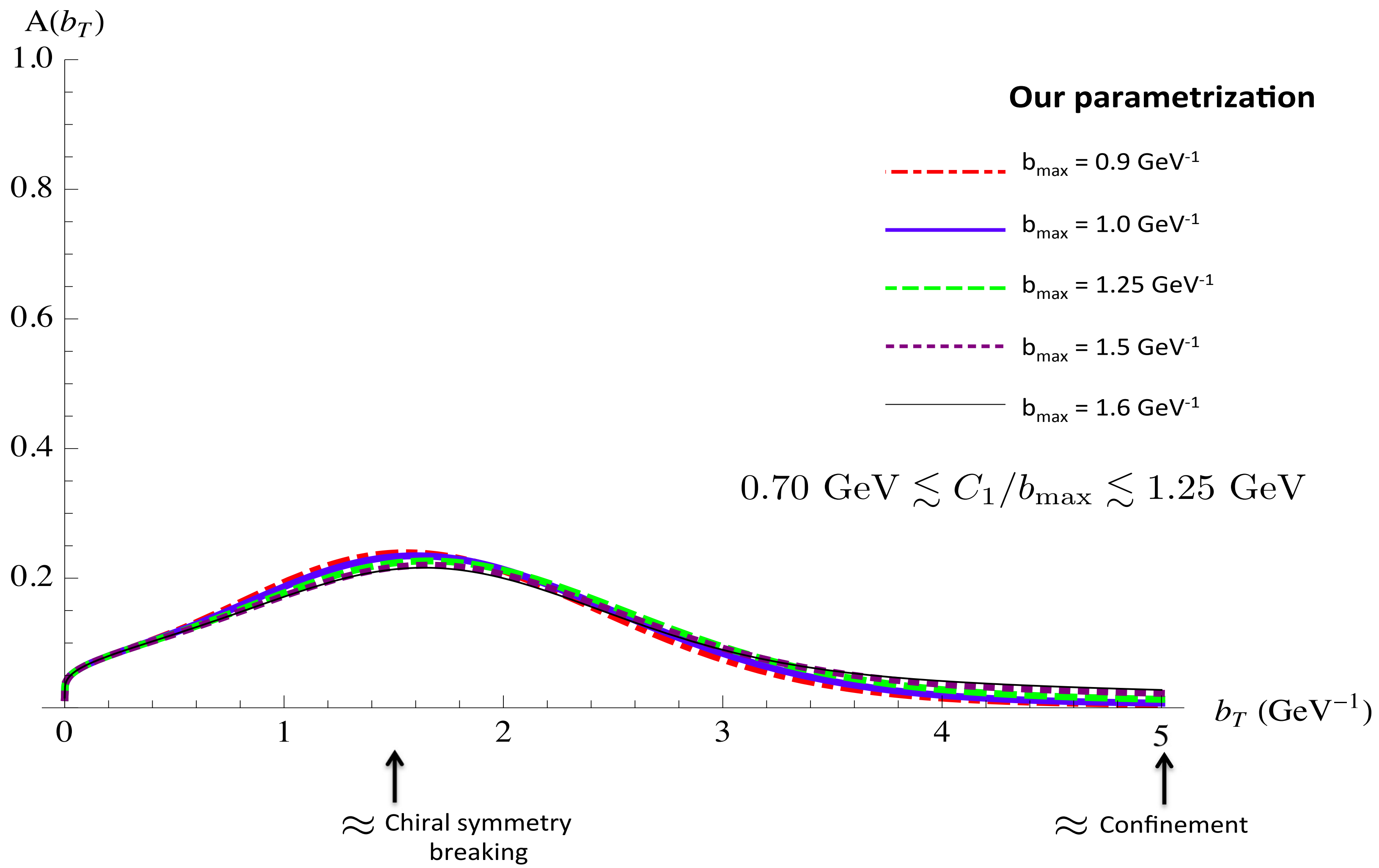
(a)

$\tilde{K}(b_T;\mu)$



$\tilde{K}(b_T; \mathrm{Q} = 2, 10, 100\ \mathrm{GeV})$
0.5
−0.5
−1.0
−1.5
−2.0
2
4
6
8
$b_T\ (\mathrm{GeV}^{-1})$
Q = 2 GeV
b_{max} = 1.5 GeV^{-1}
b_{max} = 0.5 GeV^{-1}
Q = 10 GeV
b_{max} = 1.5 GeV^{-1}
b_{max} = 0.5 GeV^{-1}
Q = 100 GeV
b_{max} = 1.5 GeV^{-1}
b_{max} = 0.5 GeV^{-1}

A(b_T)


A(b_T)
1.0
0.8
0.6
0.4
0.2
0
1
2
3
4
5
b_T (GeV^{-1})
Our parametrization
b_{max} = 0.9 GeV^{-1}
b_{max} = 1.0 GeV^{-1}
b_{max} = 1.25 GeV^{-1}
b_{max} = 1.5 GeV^{-1}
b_{max} = 1.6 GeV^{-1}
$0.70 \text{ GeV} \lesssim C_1/b_{\max} \lesssim 1.25 \text{ GeV}$
≈ Chiral symmetry breaking
≈ Confinement

In Progress

- New fits to HERMES and COMPASS SIDIS using ResBos(SIDIS) generator.

  In progress: With P. Nadolsky and B. Wang

Summary

- Large-x processes and sensitivity to TMD physics?
- Suggestion for fits to TMD functions:
  - Present evolution results as fits to $K(b_T; \mu)$ and $A(b_T)$.
    - Exploit strong soft universality and b_{max} independence.
  - Very large b_T non-perturbative evolution behavior is in $g_K(b_T;b_{max})$. Still strongly universal.
- $A(b_T)$ as a diagnostic tool. Relationship to older CSS nomenclature.
- Related to VEV of Wilson loop. Non-perturbative methods? *(e.g., lattice: Ji, et al (2014), Musch et al (2011-))*
- CSS formalism equivalent to TMD factorization. Components are clearer when organized in term of TMDs.
- Relationship to hadron structure.

Summary

- Large-x processes and sensitivity to TMD physics?
- Suggestion for fits to TMD functions:
  - Present evolution results as fits to $K(b_T; \mu)$ and $A(b_T)$.
    - Exploit strong soft universality and b_{max} independence.
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Thank You!!

- Relationship to hadron structure.