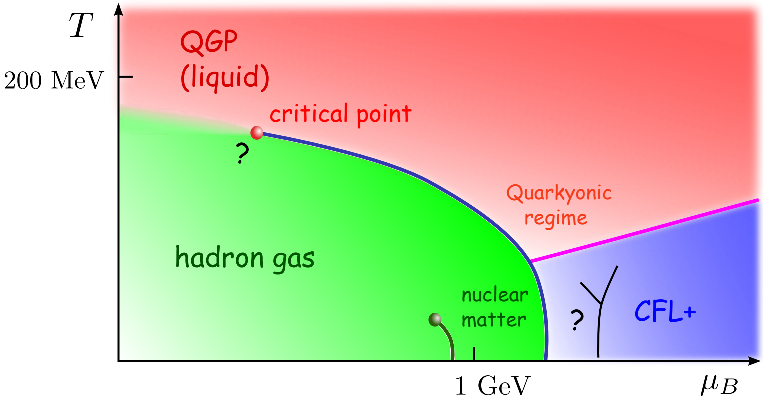
QCD critical point, fluctuations and hydrodynamics

M. Stephanov

QCD Phase Diagram *(a theorist's view)*



Outline

- Equilibrium
- Non-equilibrium

Why fluctuations are large at a critical point?

- The key equation:

$$P(\sigma) \sim e^{S(\sigma)} \qquad \textit{(Einstein 1910)}$$

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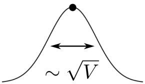

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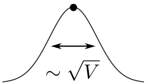
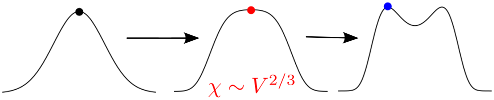


- At the critical point $S(\sigma)$ “flattens”. And $\chi \equiv \langle\sigma^2\rangle/V \to \infty$.



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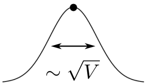

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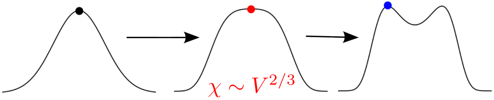
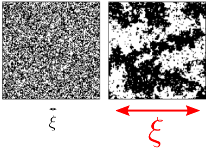
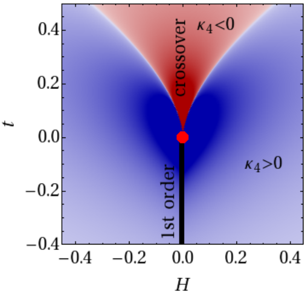


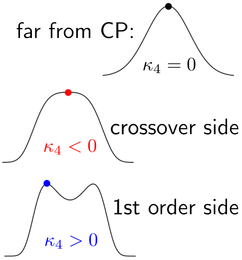


CLT? $\boxed{\sigma \text{ is not a sum of } \infty \text{ many } \textit{uncorrelated} \text{ contributions: } \xi \to \infty}$

Higher order cumulants

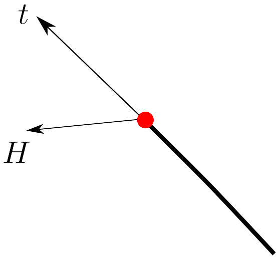
- Higher cumulants (shape of $P(\sigma)$) depend stronger on ξ.
  E.g., $\langle\sigma^2\rangle \sim V\xi^2$ while $\langle\sigma^4\rangle_c \sim V\xi^7$
- Higher moments also depend on which side of the CP we are.
  This dependence is also universal.
- Using Ising model variables:





Mapping Ising to QCD phase diagram

T vs μ_B:



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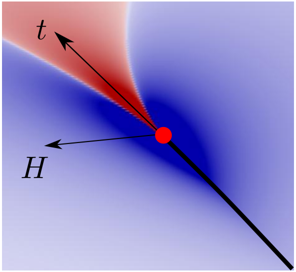

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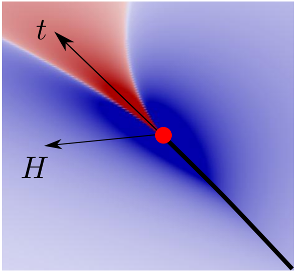
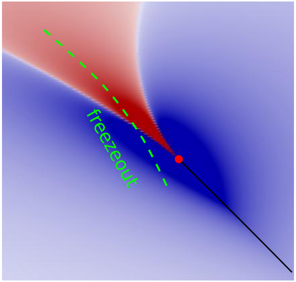
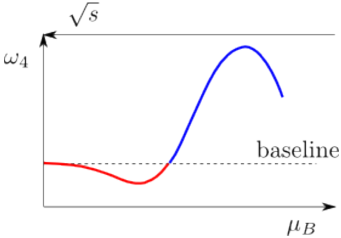


- In QCD $(t, H) \to (\mu - \mu_{\mathrm{CP}}, T - T_{\mathrm{CP}})$
- $\kappa_n(N) = N + \mathcal{O}(\kappa_n(\sigma))$

Beam Energy Scan





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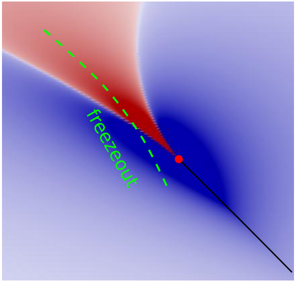

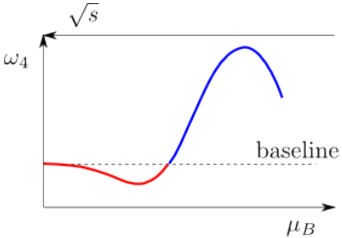

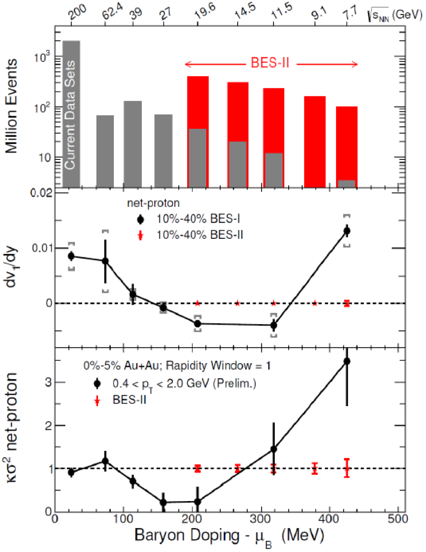

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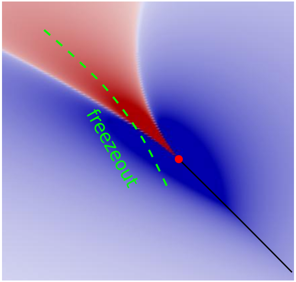

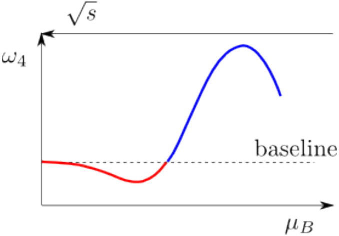

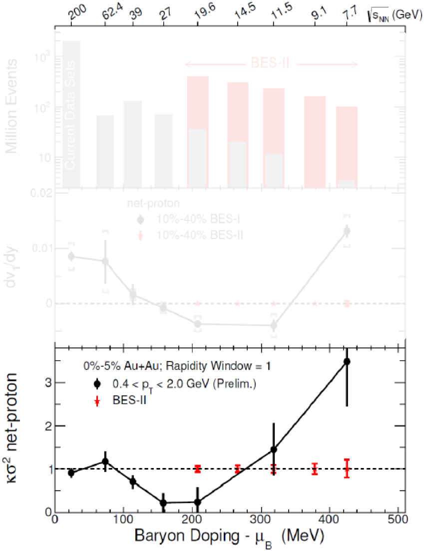

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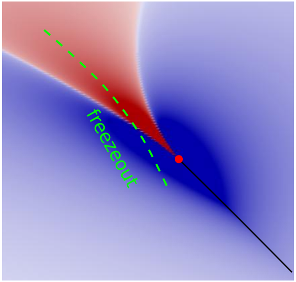

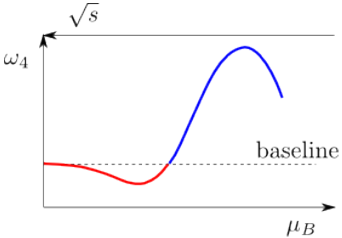

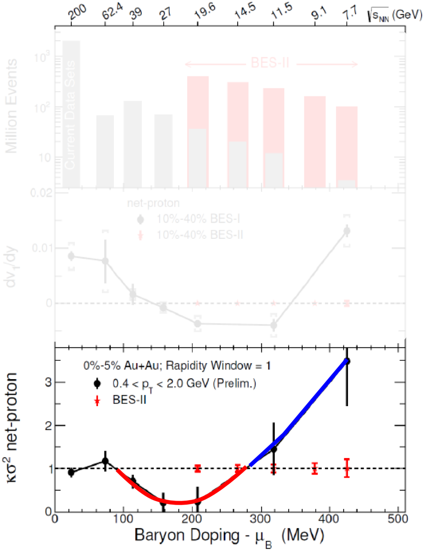

"intriguing hint" (2015 LRPNS)

Non-equilibrium physics is essential near the critical point.

The goal for BEST COLLABORATION

Why ξ is finite

$\boxed{\text{System expands and is } \textit{out of equilibrium}}$

Kibble-Zurek mechanism:

Critical slowing down means $\tau_{\mathrm{relax}} \sim \xi^z$.
Given $\tau_{\mathrm{relax}} \lesssim \tau$ (expansion time scale):

$$\xi \lesssim \tau^{1/z},$$

$z \approx 3$ (universal).

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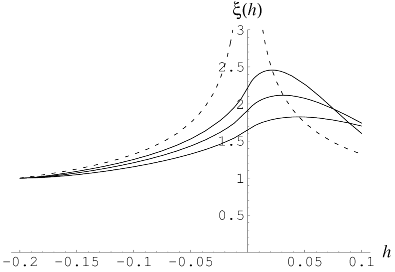
Estimates: $\xi \sim 2-3$ fm
(Berdnikov-Rajagopal)

KZ scaling for $\xi(t)$
and cumulants
(Mukherjee-Venugopalan-Yin)



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  $\longrightarrow$ Observable critical fluctuations
- Can we get *critical* fluctuations from hydrodynamics *directly*?

Hydrodynamics breaks down at CP

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When $k \sim \xi^{-3}$ hydrodynamics breaks down, i.e., while $k \ll \xi^{-1}$ still.

(For simplicity, measure dim-ful quantities in units of T.)

Why does it break at so small k?

Critical slowing down and bulk viscosity

Bulk viscosity is the effect of system taking time to adjust to local equilibrium (Khalatnikov-Landau).

$$p_{\text{hydro}} = p_{\text{equilibrium}} - \zeta\, \boldsymbol{\nabla} \cdot \boldsymbol{v}$$

$\boldsymbol{\nabla} \cdot \boldsymbol{v}$ – expansion rate

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Hydrodynamics breaks down because of large relaxation time (critical slowing down).

Similar to breakdown of an effective theory due to a low-energy mode which should not have been integrated out.

Critical slowing down and Hydro+

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(MS-Yin 1704.07396, in preparation)

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  Crossover occurs when $\Gamma_{\rm hydro} \sim \Gamma_\phi$, or $k \sim \xi^{-3}$.
- Regime II: $k > \xi^{-3}$ – "Hydro+" regime.

Advantages/motivation of Hydro+

- Extends the range of validity of "vanilla" hydro near CP
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- Extends the range of validity of “vanilla” hydro near CP
  to length/time scales shorter than $\mathcal{O}(\xi^3)$.
- No kinetic coefficients diverging as ξ^3.
  (Since noise $\sim \zeta$, also the noise is not large.)

Ingredients of “Hydro+”

- Nonequilibrium entropy, or quasistatic EOS:

$$s^*(\varepsilon, n, \phi)$$

Equilibrium entropy is the maximum of s^*:

$$s(\varepsilon, n) = \max_{\phi} s^*(\varepsilon, n, \phi)$$

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- The 6th equation (constrained by 2nd law):

$$(u \cdot \partial)\phi = -\gamma_\phi \pi - G_\phi(\partial \cdot u), \qquad \text{where } \pi = \frac{\partial s^*}{\partial \phi}$$

Linearized Hydro+

Linearized Hydro+ has 4 longitudinal modes (sound$\times 2$ + density + ϕ).

In addition to the usual c_s, D, etc. Hydro+ has two more parameters $\Delta c^2 = c_*^2 - c_s^2$ and $\Gamma = \Gamma_\phi$.

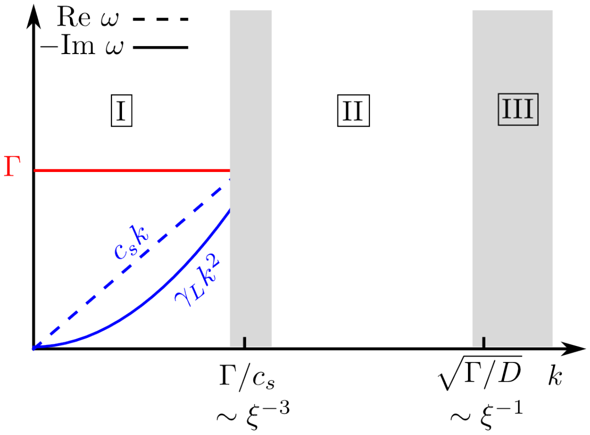
The sound velocities are different in Regime I ($c_s k \ll \Gamma$) and II:

$$c_s^2 = \left(\frac{\partial p}{\partial \varepsilon}\right)_{s/n,\pi=0} \quad \text{and} \quad c_*^2 = \left(\frac{\partial p^*}{\partial \varepsilon}\right)_{s/n,\phi}$$

The bulk viscosity receives large contribution from the slow mode given by Landau-Khalatnikov formula

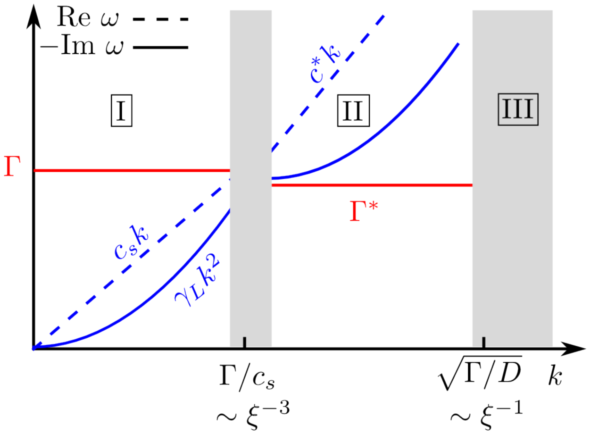
$$\Delta\zeta = w\Delta c^2/\Gamma$$

Modes



Re ω
$-$Im ω
I
II
III
Γ
$c_s k$
$\gamma_L k^2$
Γ/c_s
$\sim \xi^{-3}$
$\sqrt{\Gamma/D}$
$\sim \xi^{-1}$
k

Modes



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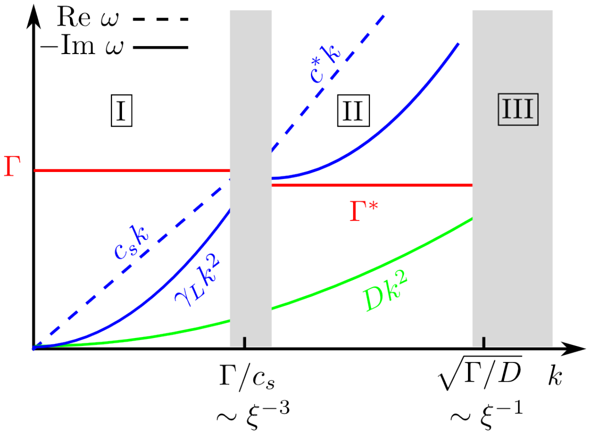

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$\sqrt{\Gamma/D}$
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k

Microscopic origins of Hydro+

Understanding the microscopic origin of the slow mode:

The fluctuations around equilibrium are controlled by the entropy functional $P \sim e^{S}$.

Near the critical point convenient to "rotate" the basis of variables to "Ising"-like critical variables $\mathcal{E}$ and $\mathcal{M}$.

$$\delta\mathcal{S}[\delta\mathcal{E},\delta\mathcal{M}] = \left[\frac{1}{2}\,a_{\mathcal{M}}\,\left(\delta\mathcal{M}\right)^2 + \frac{1}{2}\,a_{\mathcal{E}}\,\left(\delta\mathcal{E}\right)^2 + b\,\delta\mathcal{E}\,\delta\mathcal{M}^2 + \ldots\right].$$

Since $a_{\mathcal{M}} \ll a_{\mathcal{E}}$ fluctuations of $\mathcal{M}$ are large and are slow to equilibrate.

Their magnitude is related to the slow relaxation mode ϕ.

Hydro + mode distribution

Separate "hard" $k > \xi^{-1}$ and "soft" $k \ll \xi^{-1}$ modes.

The new variable, "mode distribution function":

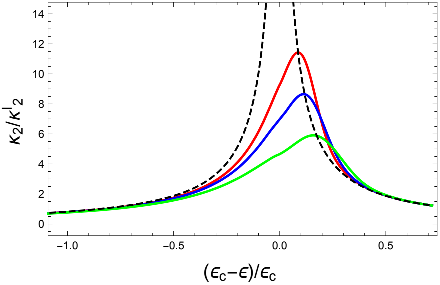
$$n_{\mathcal{M}}(t, \boldsymbol{x}, \boldsymbol{Q}) = \int_{\boldsymbol{y}} \langle\, \delta\mathcal{M}(t, \boldsymbol{x} + \boldsymbol{y}/2)\, \delta\mathcal{M}(t, \boldsymbol{x} - \boldsymbol{y}/2) \,\rangle\; e^{-i\boldsymbol{Q}\cdot\boldsymbol{y}}$$

The additional mode distribution function relaxation equation:

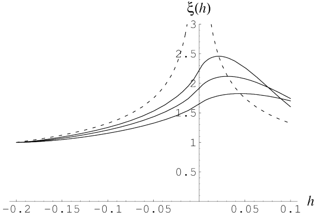
$$(u \cdot \partial) n_{\mathcal{M}}(t, \boldsymbol{x}, \boldsymbol{Q}) = 2\Gamma_{\mathcal{M}}(\boldsymbol{Q}) \left[a_{\mathcal{M}}^{-1} - n_{\mathcal{M}}(t, \boldsymbol{x}, \boldsymbol{Q}) \right]$$

where $\Gamma_{\mathcal{M}}(\boldsymbol{Q})$ is known from model H (Kawasaki).

Simple model. Bjorken expansion.



Berdnikov-Rajagopal



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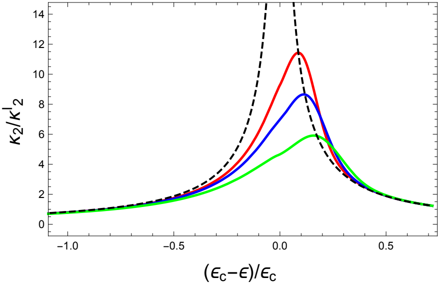
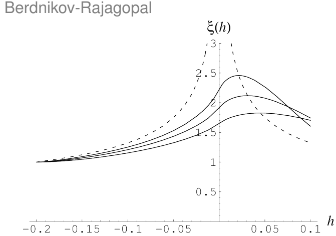
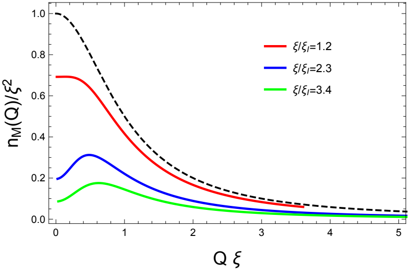




mode distribution function falls out of equilibrium



Summary

- A fundamental question for Heavy-Ion collision experiments:

  Is there a critical point on the boundary between QGP and hadron gas phases?

  Theoretical framework is needed – the goal for BEST COLLABORATION.

- Large (non-gaussian) fluctuations – universal signature of a critical point.

- In H.I.C., the magnitude of the signatures is controlled by dynamical non-equilibrium effects. The physics of the interplay of critical and dynamical phenomena can be captured by hydrodynamics with a critically slow mode(s) – Hydro+.