50 Years of the Veneziano Model

Heterotic- and Bosonic-String Amplitudes from Field Theories

Oliver Schlotterer (Perimeter Institute & AEI Potsdam)

based on arXiv:1106.2645 with C. Mafra and S. Stieberger

arXiv:1803.05452 with T. Azevedo, M. Choidaroli and H. Johansson

15.05.2018

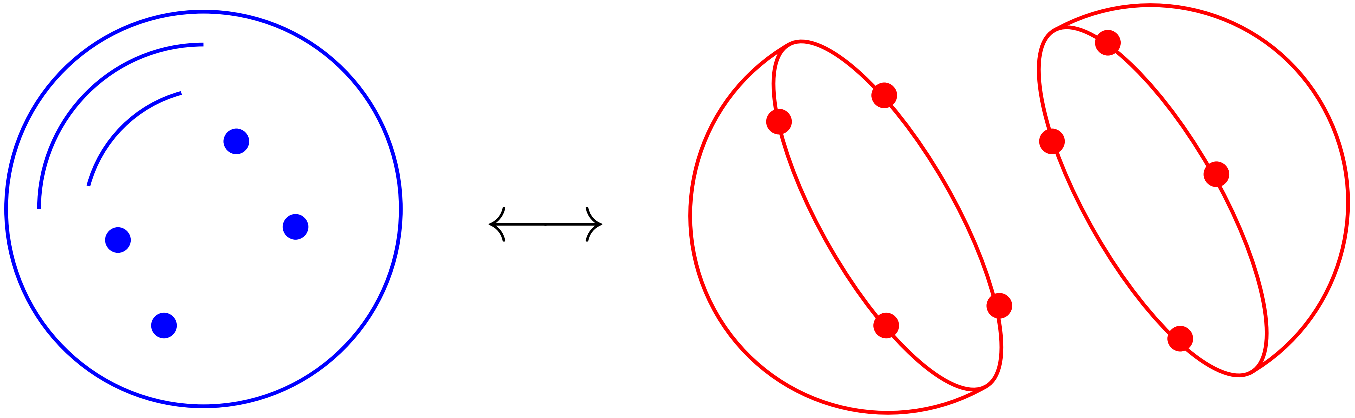
Introduction: Double copy – from string to field theory

Birth of double copy: KLT relations among string amplitudes at tree-level

$$\mathcal{M}^{4\,\mathrm{pt}}_{\mathrm{closed}}(\alpha') \;=\; \bar{\mathcal{A}}_{\mathrm{open}}(1,2,4,3;\alpha')\,\sin\left(\frac{\pi\alpha'}{2}k_1\cdot k_2\right)\mathcal{A}_{\mathrm{open}}(1,2,3,4;\alpha')\;.$$

closed strings:

sphere



open strings:

disk

[Kawai, Lewellen, Tye 1986; see Zvi Bern's talk]

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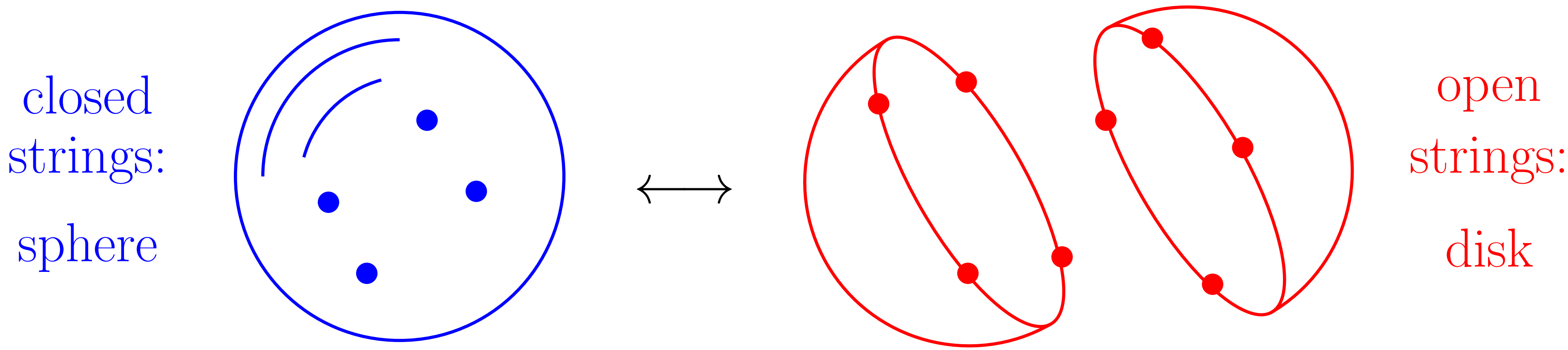

[Kawai, Lewellen, Tye 1986; see Zvi Bern's talk]

Field-theory limit $\alpha' \to 0$: relate gravity to double copy of gauge theories:

$$M^{4\,\mathrm{pt}}_{\mathrm{SUGRA}} \;=\; \bar{A}_{\mathrm{SYM}}(1,2,4,3)\,k_1\cdot k_2\,A_{\mathrm{SYM}}(1,2,3,4) \;\equiv\; \bar{A}_{\mathrm{SYM}} \otimes_{\mathrm{KLT}} A_{\mathrm{SYM}}\,.$$

Will refer to operation $\otimes_{\mathrm{KLT}}$ at $\alpha' \to 0$ as **field-theory double copy**.

Introduction: Double copy – from string to field theory

At n points, more combinatorics and $(n-3)!$-element BCJ bases

$$\Big\{ A_{\rm SYM}(1,\rho(2,3,\ldots,n-2),n-1,n) \ , \quad \text{permutation } \rho \in S_{n-3} \Big\}$$

[Bern, Carrasco, Johansson 0805.3993; see Zvi Bern's talk]

$$M^{n \text{ pt}}_{\rm SUGRA} = \sum_{\rho,\tau \in S_{n-3}} \bar A_{\rm SYM}(1,\rho,n,n-1)\, \underbrace{S(\rho|\tau)_1}_{(n-3)! \times (n-3)! \text{ KLT matrix, entries are } \sim (k_i \cdot k_j)^{n-3}} A_{\rm SYM}(1,\tau,n-1,n)$$

[Bern, Dixon, Perelstein, Rozowsky 1998]

[Bjerrum-Bohr, Damgaard, Feng, Sondergaard, Vanhove 2010]

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e.g. 2×2 terms at 5 points with

$$S(\rho(2,3)|\tau(2,3))_1 = \begin{pmatrix} (k_1\cdot k_2)(k_{1+2}\cdot k_3) & (k_1\cdot k_2)(k_1\cdot k_3) \\ (k_1\cdot k_2)(k_1\cdot k_3) & (k_1\cdot k_3)(k_{1+3}\cdot k_2) \end{pmatrix}$$

Shorthand for KLT formulae: $\boxed{M_{\rm SUGRA} = \bar A_{\rm SYM} \otimes_{\rm KLT} A_{\rm SYM}}$

Key result: Massless tree amplitudes in various string theories

Field-theory double copy $\otimes_{\mathrm{KLT}} \Rightarrow$ web of relations for string amplitudes

$\rightarrow$ representations of the flavour (field theory) $\otimes_{\mathrm{KLT}}$ (stringy building block)

$\otimes_{\mathrm{KLT}}$	SYM		
Z-theory	open superstring		

- "Z-theory" $\leftrightarrow$ α'-dependent disk integrals (over moduli space $\mathcal{M}_{0,n}$)

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Outline

I. Open superstrings as a field-theory double copy

[Mafra, OS, Stieberger 1106.2645; Broedel, OS, Stieberger 1304.7267]

II. Closed superstrings from single-valued open superstrings

[OS, Stieberger 1205.1516; Stieberger 1310.3259]

III. Bosonic strings from $(DF)^2 + \mathrm{YM}$ field theory

[Azevedo, Chiodaroli, Johansson, OS 1803.05452]

IV. Heterotic strings and $(DF)^2 + \mathrm{YM} + \phi^3$ field theory

[Azevedo, Chiodaroli, Johansson, OS 1803.05452]

V. Conclusions & Outlook

I. Open superstrings as a field-theory double copy

Color ordered n-point trees of open superstring (with $\sigma = \sigma(2, 3, \ldots, n-2)$)

$$\boxed{\mathcal{A}^{\text{open}}_{\text{super}}(1, \sigma, n-1, n; \alpha') = \sum_{\tau \in S_{n-3}} F_\sigma{}^\tau(\alpha')\, A_{\text{SYM}}(1, \tau, n-1, n)}$$

[Mafra, OS, Stieberger 1106.2645, 1106.2646]

- all polarizations in BCJ basis of 10-dim SYM amplitudes $A_{\text{SYM}}(\ldots)$

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- all polarizations in BCJ basis of 10-dim SYM amplitudes $A_{\text{SYM}}(\ldots)$
- all the α' dependence in $(n{-}3)! \times (n{-}3)!$ basis of disk integrals $F_\sigma{}^\tau(\alpha')$

$$F_\sigma{}^\tau(\alpha') \ = \ \int\limits_{0<z_{\sigma(2)}<z_{\sigma(3)}<\ldots<z_{\sigma(n-2)}<1} \mathrm{d}z_2 \ \ldots \ \mathrm{d}z_{n-2} \prod_{i<j}^{n-1} |z_i - z_j|^{2\alpha' k_i \cdot k_j} \, \tau \Big\{ \prod_{l=2}^{n-2} \sum_{m=1}^{l-1} \frac{2\alpha' k_l \cdot k_m}{z_l - z_m} \Big\}$$

- $\tau \in S_{n-3}$ acts on k_j and z_j enclosed in $\{\ldots\}$ with $j = 2,3,\ldots,n{-}2$
- recover $\mathcal{A}^{\text{open}}_{\text{super}} \to A_{\text{SYM}}$ from field-theory limit $F_\sigma{}^\tau(\alpha') \ = \ \delta^\tau_\sigma + \mathcal{O}(\alpha'^2)$

I. Open superstrings as a field-theory double copy

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A KLT formula in disguise involving disk integrals (with $z_{ij} \equiv z_i - z_j$)

$$Z_\sigma\big(\rho(1,2,\ldots,n)\big) \equiv (2\alpha')^{n-3} \int_{z_{\sigma(i)}<z_{\sigma(i+1)}} \frac{\mathrm{d}z_1\,\ldots\,\mathrm{d}z_n}{\mathrm{vol}\,\mathrm{SL}_2(\mathbb{R})} \frac{\prod_{i<j}^n |z_{ij}|^{2\alpha' k_i\cdot k_j}}{\rho(z_{12}z_{23}\ldots z_{n-1,n}z_{n,1})}$$

Permutation $\rho = \rho(1,2,\ldots,n)$ acts on cyclic denominator $(z_{12}z_{23}\ldots z_{n,1})^{-1}$

... and integrand of $F_\sigma{}^\tau$ is $\sum_\rho S(\rho|\tau)_1 \rho(z_{12}z_{23}\ldots z_{n,1})^{-1}$ w. KLT matrix

[Brödel, OS, Stieberger 1304.7267]

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$$\boxed{\mathcal{A}^{\rm open}_{\rm super}(1,\sigma,n{-}1,n;\alpha') = \sum_{\rho,\tau\in S_{n-3}} Z_\sigma(1,\rho,n,n{-}1)\, S(\rho|\tau)_1\, A_{\rm SYM}(1,\tau,n{-}1,n)}$$

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$\Longrightarrow$ field-theory double copy $\boxed{\mathcal{A}^{\rm open}_{\rm super}(\sigma) = Z_\sigma \otimes_{\rm KLT} A_{\rm SYM}}$

Integrals Z_σ dubbed "Z-theory amplitudes" [Carrasco, Mafra, OS 1608.02569]

II. Closed superstrings from single-valued open superstrings

α'-expansion of $F_\sigma{}^\tau$ & $\mathcal{A}^{\text{open}}_{\text{super}}$ involves multiple zeta values (MZVs)

$$\zeta_{n_1,n_2,\ldots,n_r} \equiv \sum_{0<k_1<k_2<\ldots<k_r}^{\infty} k_1^{-n_1} k_2^{-n_2} \ldots k_r^{-n_r} \, , \qquad n_r \geq 2$$

[Terasoma 2002 & Brown 2006]

Schematically,

$$\mathcal{A}^{\text{open}}_{\text{super}}(\sigma) \, = \, \overbrace{\Big(\mathbb{1} \, + \, \zeta_2 \, (\alpha' k_i \cdot k_j)^2 \, + \, \zeta_3 \, (\alpha' k_i \cdot k_j)^3 \, + \, \mathcal{O}(\alpha'^4)\Big)_\sigma{}^\tau}^{F_\sigma{}^\tau(\alpha')} \, A_{\text{SYM}}(\tau)$$

Polynomial structure in $\alpha' k_i \cdot k_j$ at n points can be determined to any order

[Brödel, OS, Stieberger, Terasoma 1304.7304 & Mafra, OS 1609.07078]

Explicit results at $n \leq 7$ points available for download

`http://wwwth.mpp.mpg.de/members/stieberg/mzv/index.html`

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Define single-valued projection **sv** of MZVs via their polylogarithm origin

[**Schnetz 1302.6445 & Brown 1309.5309**]

$$\begin{tikzcd}
\text{MZVs} \arrow[rr, dashed, "\textbf{sv}"'] & & \text{single-valued MZVs} \\
\begin{array}{c}\text{polylogarithms} \\ G(\{\text{word in } 0,1\};z)\end{array} \arrow[u, "z\to 1"] \arrow[rr, "\begin{array}{c}\text{Francis Brown 2004:} \\ \exists \text{ unique procedure to}\end{array}", "\text{cancel monodromies}"'] & & \begin{array}{c}\text{single-valued polylog's} \\ G_{\rm sv}(\{\text{word in } 0,1\};z)\end{array} \arrow[u, "z\to 1"']
\end{tikzcd}$$

e.g. $G(1;z) \ = \ \log(1-z) \quad \longrightarrow \quad G_{\rm sv}(1;z) = \log|1-z|^2$

II. Closed superstrings from single-valued open superstrings

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$$\mathbf{sv}(\zeta_{2k}) \ = \ 0 \, , \quad \mathbf{sv}(\zeta_{2k+1}) \ = \ 2\,\zeta_{2k+1} \, , \quad \mathbf{sv}(\zeta_{3,5}) \ = \ -10\,\zeta_3\,\zeta_5 \, , \quad \text{etc.}$$

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In principle, **[Kawai, Lewellen, Tye 1986]** determine closed-superstring trees as

$$\mathcal{M}^{\text{closed}}_{\text{super}}(\alpha') \sim \bar{\mathcal{A}}^{\text{open}}_{\text{super}}(\ldots;\alpha') \prod \sin\left(\frac{\pi\alpha'}{2}k_i \cdot k_j\right) \mathcal{A}^{\text{open}}_{\text{super}}(\ldots;\alpha') \ ,$$

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$$\boxed{\mathcal{M}^{\rm closed}_{\rm super}(\alpha') = \bar{A}_{\rm SYM} \otimes_{\rm KLT} \textbf{sv}\, \mathcal{A}^{\rm open}_{\rm super}(\alpha')}$$

[OS, Stieberger 1205.1516 & Stieberger 1310.3259]

Schematically, by $\textbf{sv}(\zeta_{2k}) = 0$ and $\textbf{sv}(\zeta_{2k+1}) = 2\,\zeta_{2k+1}$,

$$\mathcal{A}^{\rm open}_{\rm super}(\sigma) = \Big(\mathbb{1} + \zeta_2(\alpha' k_i\cdot k_j)^2 + \zeta_3(\alpha' k_i\cdot k_j)^3 + \mathcal{O}(\alpha'^4)\Big)_\sigma{}^\tau A_{\rm SYM}(\tau)$$

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- emergence of $\mathbf{sv}\, \mathcal{A}^{\text{open}}_{\text{super}}(\alpha')$ still conjectural (tested to high orders in α')

- $\exists$ first one-loop echos of **sv**-map from open to closed strings

[D'Hoker, Green, Gürdogan, Vanhove 1512.06779 & Brödel, OS, Zerbini 1803.00527]

III. Bosonic strings from $(DF)^2 + \mathrm{YM}$ field theory

Open **bosonic** string: can still expand n-point trees via integrals $F_\sigma{}^\tau$

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$$\mathcal{A}^{\rm open}_{\rm bos}(1,\sigma,n{-}1,n;\alpha') \ = \sum_{\tau \in S_{n-3}} F_\sigma{}^\tau(\alpha') \ \underbrace{B(1,\tau,n{-}1,n;\alpha')}_{\text{kin. factor with BCJ rel's}}$$

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α'-dependent kinematic factors $B(\ldots;\alpha')$, e.g.

tachyon pole

$$B(1,2,3;\alpha') \;=\; A_{\mathrm{YM}}(1,2,3) \;-\; 4\alpha'\,(e_1\cdot k_2)(e_2\cdot k_3)(e_3\cdot k_1)$$

$$B(1,2,3,4;\alpha') \;=\; A_{\mathrm{YM}}(1,2,3,4) \;-\; 4\alpha' s_{13}\Big\{\Big[\frac{f_{12}f_{34}}{s_{12}^2(1-2\alpha' s_{12})} + \mathrm{cyc}(2,3,4)\Big] - \frac{g_1g_2g_3g_4}{s_{12}^2 s_{13}^2 s_{23}^2}\Big\}$$

with $s_{ij}\equiv k_i{\cdot}k_j$ and $f_{ij}\equiv s_{ij}(e_i{\cdot}e_j) - (k_i{\cdot}e_j)(k_j{\cdot}e_i)$ and $g_i \equiv (k_{i-1}{\cdot}e_i)s_{i,i+1} - (k_{i+1}{\cdot}e_i)s_{i-1,i}$.

[Huang, OS, Wen 1602.01674]

Is there a field-theory interpretation of these $B(\ldots;\alpha')$ with BCJ rel's?

Open **bosonic** string: can still expand n-point trees via integrals $F_\sigma{}^\tau$

$$\mathcal{A}^{\rm open}_{\rm bos}(1,\sigma,n{-}1,n;\alpha') \ = \ \frac{\alpha'}{2} \sum_{\tau \in S_{n-3}} F_\sigma{}^\tau(\alpha')\, A_{(DF)^2+{\rm YM}}(1,\tau,n{-}1,n;\alpha')$$

"$(DF)^2+\mathrm{YM}$" gauge theory $\Longrightarrow$ kin. factors $B \rightarrow \frac{\alpha'}{2} A_{(DF)^2+{\rm YM}}$

[Azevedo, Chiodaroli, Johansson, OS 1803.05452]

$$\mathcal{L}_{(DF)^2+{\rm YM}} \equiv \frac{1}{2}(D_\mu F^{a\,\mu\nu})^2 \ - \ \frac{1}{4} m^2 (F^a_{\mu\nu})^2 \ - \ \frac{1}{3} f^{abc} F^{a\nu}_\mu F^{b\lambda}_\nu F^{c\mu}_\lambda$$
$$+ \frac{1}{2}(D_\mu \varphi^\alpha)^2 \ - \ \frac{1}{2} m^2 (\varphi^\alpha)^2 \ + \ \frac{1}{2} C^{\alpha a b} \varphi^\alpha F^a_{\mu\nu} F^{b\,\mu\nu} \ + \ \frac{1}{3!} d^{\alpha\beta\gamma} \varphi^\alpha \varphi^\beta \varphi^\gamma \, \Big|_{m^2 = -\frac{1}{\alpha'}}$$

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- kin. operator $(\partial^4 - m^2 \partial^2) A^2 \ \Rightarrow$ 2 gluon modes: $\left({\text{massless} \atop \text{physical}} \right) \oplus \left({\text{massive} \atop \text{ghost}} \right)$

- massive-ghost scalar φ^α: index $\alpha \leftrightarrow$ real representation of gauge group

$\Rightarrow \ m^2 = -\dfrac{1}{\alpha'}$ gives required tachyon poles $A_{(DF)^2+{\rm YM}} \sim \dfrac{1}{1-2\alpha' k_i \cdot k_j}$

Open **bosonic** string: can still expand n-point trees via integrals $F_\sigma{}^\tau$

$$\mathcal{A}^{\rm open}_{\rm bos}(1,\sigma,n{-}1,n;\alpha') \ = \ \frac{\alpha'}{2} \sum_{\tau \in S_{n-3}} F_\sigma{}^\tau(\alpha')\, A_{(DF)^2+{\rm YM}}(1,\tau,n{-}1,n;\alpha')$$

"$(DF)^2 + \text{YM}$" gauge theory $\Longrightarrow$ kin. factors $B \rightarrow \frac{\alpha'}{2} A_{(DF)^2+{\rm YM}}$

[Azevedo, Chiodaroli, Johansson, OS 1803.05452]

$$\mathcal{L}_{(DF)^2+{\rm YM}} \equiv \frac{1}{2}(D_\mu F^{a\,\mu\nu})^2 \ - \ \frac{1}{4} m^2 (F^a_{\mu\nu})^2 \ - \ \frac{1}{3} f^{abc} F^{a\nu}_\mu F^{b\lambda}_\nu F^{c\mu}_\lambda \\ + \frac{1}{2}(D_\mu \varphi^\alpha)^2 \ - \ \frac{1}{2} m^2 (\varphi^\alpha)^2 \ + \ \frac{1}{2} C^{\alpha a b} \varphi^\alpha F^a_{\mu\nu} F^{b\,\mu\nu} \ + \ \frac{1}{3!} d^{\alpha\beta\gamma} \varphi^\alpha \varphi^\beta \varphi^\gamma \Big|_{m^2 = -\frac{1}{\alpha'}}$$

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- for external gluons, Clebsch Gordans $C^{\alpha a b}$ & $d^{\alpha\beta\gamma}$ conspire to $\prod f^{abc}$
- initial construction of $\mathcal{L}_{(DF)^2+{\rm YM}}$ guided by imposing BCJ relations

[Johansson, Nohle 1707.02965]

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Since $F_\sigma{}^\tau = \sum_\rho Z_\sigma(\rho) S(\rho|\tau)$ signal KLT formula in disguise,

$\Longrightarrow$ field-theory double copy $\boxed{\mathcal{A}^{\text{open}}_{\text{bos}}(\sigma) \;=\; Z_\sigma \otimes_{\text{KLT}} A_{(DF)^2+\text{YM}}}$

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Corollaries by recycling the underlying open-string CFT correlator:

- via **sv** projection of MZVs: $\boxed{\mathcal{M}^{\rm closed}_{\rm bos} \ = \ \textbf{sv}\, \bar{\mathcal{A}}^{\rm open}_{\rm bos} \otimes_{\rm KLT} A_{(DF)^2+{\rm YM}}}$
- heterotic strings (gravity): can put SUSY on either side of double copy

$$\mathcal{M}^{\rm het}_{\rm grav} \ = \ \begin{cases} \textbf{sv}\, \bar{\mathcal{A}}^{\rm open}_{\rm super} \otimes_{\rm KLT} A_{(DF)^2+{\rm YM}} & : \ \text{SUSY on } \textbf{sv}\text{(string) side} \\ \textbf{sv}\, \bar{\mathcal{A}}^{\rm open}_{\rm bos} \otimes_{\rm KLT} A_{\rm SYM} & : \ \text{SUSY on field-theory side} \end{cases}$$

[Azevedo, Chiodaroli, Johansson, OS 1803.05452]

IV. Heterotic strings and $(DF)^2 + \mathrm{YM} + \phi^3$ field theory

Now incorporate gauge sector & gauge/gravity couplings of heterotic string

- single-trace gluon amplitudes: $\mathcal{A}^{\rm het}_{\rm s.tr.}(1,\ldots,n) = \mathbf{sv}\, \mathcal{A}^{\rm open}_{\rm super}(1,\ldots,n)$

[Stieberger, Taylor 1401.1218]

- various mixed gauge/gravity & double-trace amplitudes reduced to $\mathcal{A}^{\rm het}_{\rm s.tr.}$

[OS 1608.00130]

Suggests double-copy structure in color-dressed het. amplitudes $\mathcal{M}^{\rm het}_{\substack{\rm gauge \\ \oplus \rm grav}}$,

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n gravitons
k gluons

n gluons
k scalars

adds spin one and
carries the SUSY

As an amplitude $A_{??} \supset$ colored scalars "s" & uncolored vectors "g" e.g.

$$A_{??}(1_s, 2_s, 3_g) \ \sim \ \delta^{A_1 A_2} (e_3 \cdot k_1) \ .$$

Further examples of $A_{??}$ (with colored scalar "s" & uncolored vectors "g")

$$A_{??}(1_s, 2_s, 3_s, 4_s) \;=\; \overbrace{\frac{\bar{f}^{A_1 A_2 B}\,\bar{f}^{B A_3 A_4}}{2 s_{12}} + \frac{\bar{f}^{A_2 A_3 B}\,\bar{f}^{B A_4 A_1}}{2 s_{23}}}^{\text{single trace}} + \overbrace{s_{13}\Big\{ \frac{2\alpha' \delta^{A_1 A_2} \delta^{A_3 A_4}}{s_{12}(1 - 2\alpha' s_{12})} + \mathrm{cyc}(2,3,4) \Big\}}^{\text{double trace}}$$

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$(DF)^2$+YM+ϕ^3

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suggest extension of $(DF)^2$+YM by bi-adjoint scalars $\phi = \sum_{a,A} \phi^{aA} t^a \otimes \bar t^A$

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- only A^a_μ,φ^α have tachyonic mode @ $m^2=-\alpha'^{-1}$, the ϕ^{aA} are massless

- only color order w.r.t. $\mathrm{Tr}(t^at^b\ldots)$, not w.r.t. $\mathrm{Tr}(\bar t^A\bar t^B\ldots)$

$$A_{(DF)^2+\mathrm{YM}+\phi^3}(1,2,\ldots,n)\;\equiv\;M_{(DF)^2+\mathrm{YM}+\phi^3}\Big|_{\mathrm{Tr}(t^1t^2\ldots t^n)}$$

All mixed gluon/graviton amplitudes of heterotic string from $(DF)^2 + \mathrm{YM} + \phi^3$,

$$\underbrace{\mathcal{M}^{\text{het}}_{\substack{\text{gauge}\\ \oplus\text{grav}}}}_{\substack{n \text{ gravitons}\\ k \text{ gluons}}} \sim \underbrace{A_{(DF)^2+\mathrm{YM}+\phi^3}}_{\substack{n \text{ gluons}\\ k \text{ scalars}}} \otimes_{\mathrm{KLT}} \underbrace{\mathbf{sv}\,\mathcal{A}^{\text{open}}_{\text{super}}}_{\substack{\text{adds spin one and}\\ \text{carries the SUSY}}}$$

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$\mathcal{M}^{\text{het}}_{\text{gauge}\oplus\text{grav}}$	$A_{(DF)^2+\text{YM}+\phi^3}$	$\textbf{sv}\,\mathcal{A}^{\text{open}}_{\text{super}}$
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The $(DF)^2$+YM+ϕ^3-theory is unique once we impose BCJ relations and

- limiting behaviour as $\alpha' \to 0$ and $\alpha' \to \infty$

- relative normalization of het. single-trace vs. double-trace amplitudes

[Azevedo, Chiodaroli, Johansson, OS 1803.05452]

V. Conclusions & Outlook

- generated all massless tree amplitudes of superstrings, bosonic strings and heterotic strings from field-theory double-copy "$\otimes_{\mathrm{KLT}}$"

$$\begin{pmatrix}\text{string}\\ \text{amplitudes}\end{pmatrix} = \begin{pmatrix}\text{gauge theory: possibly}\\ \text{with } (DF)^2/\phi^3 \text{ extension}\end{pmatrix} \otimes_{\mathrm{KLT}} \begin{pmatrix}\text{disk integrals or}\\ \mathbf{sv}(\text{open strings})\end{pmatrix}$$

$\otimes_{\mathrm{KLT}}$	SYM	$(DF)^2$+YM	$(DF)^2$+YM+ϕ^3
Z-theory	open superstring	open bosonic string	comp(open bos. string)
sv(open superstring)	closed superstring	heterotic string (grav)	het. string (gauge/grav)
sv(open bos. string)	heterotic string (grav)	closed bosonic string	comp(closed bos. string)

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- also at one loop: evidence for double-copy structure of open superstrings

[Mafra, OS 1711.09104 & in progress]

$\longrightarrow$ generic property at loop level? simplify higher-genus calculations?

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Thank you for your attention !