Quantitative theory of channeling particle diffusion in transverse energy and direct evaluation of dechanneling length

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Plan

- Diffusion theory *vs* Monte Carlo simulations
- Diffusion equation and dechanneling length introduction
- Old diffusion theory restrictions and their overcoming. Channleing definition
- New diffusion equation, its solution and dechanneling length values
- Sone peculiarities of dechanneling process

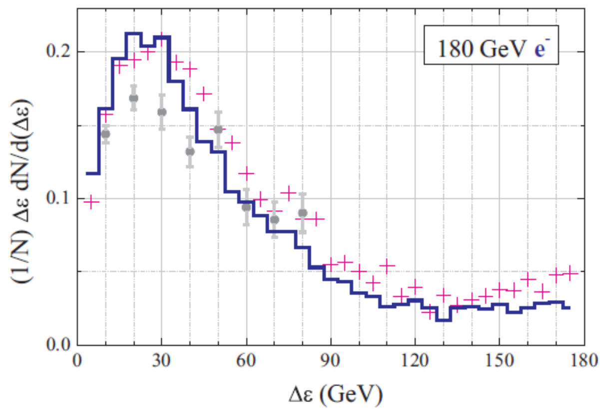
MC simulations can reproduce **any experiment**

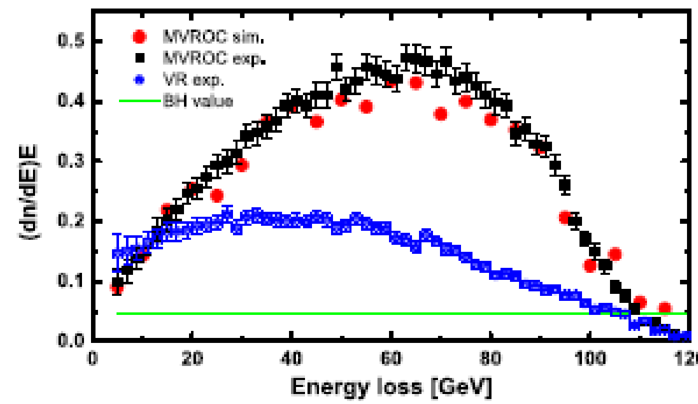
V. Guidi, L. Bandiera, V.V. Tikhomirov. Phys. Rev.A. 86 (2012) 042903

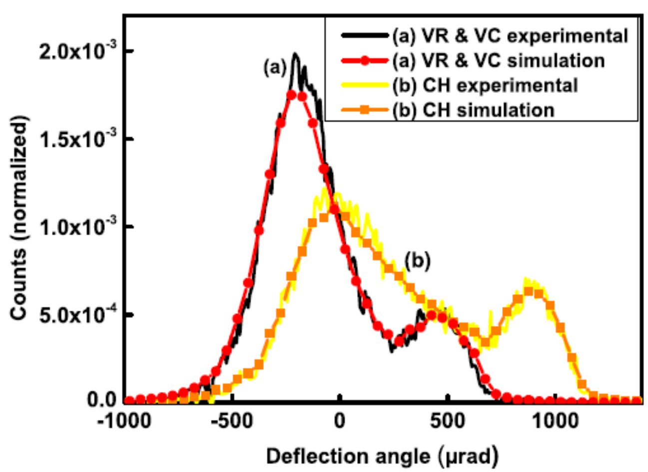
L. Bandiera ... V. Guidi,.. V.V. Tikhomirov , Phys. Rev. Lett. 111 (2013) 255502 .

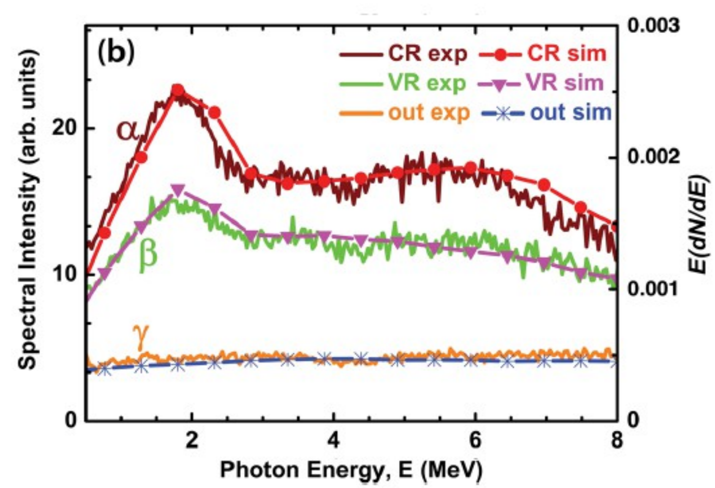
A. Mazzolari ... V. Guidi, ..V.V. Tikhomirov , Phys. Rev. Lett. 112 (2014) 135503.

L. Bandiera ... V. Guidi,.. V.V. Tikhomirov , Phys. Rev. Lett. 115 (2015) 025504.









Can (improved) diffusion theory say more?

– *dechanneling length*

is determined by doubly

averaged behavior

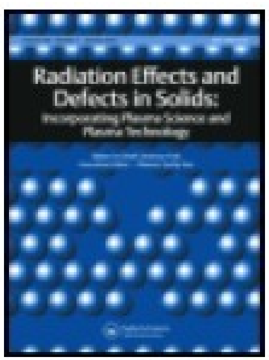
described better

by diffusion theory

Diffusion theory can also give considerable "economy" when $l_{dech} >$ *1÷10 meters*

Dechanneling length introduction in diffusion theory

Radiation Effects



Multiple scattering of channeling ions in crystals-II. Planar channeling

V. V. Beloshitsky , M. A. Kumakhov & V. A. Muralev

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Diffusion equation and distribution function in transverse energy

$$\frac{\partial F}{\partial t} = \frac{1}{2}\frac{\partial}{\partial E_y}\left[\left\langle\frac{\overline{\Delta E_y{}^2}}{\Delta t}\right\rangle T\frac{\partial}{\partial E_y}\left(\frac{F}{T}\right)\right] \quad (7)$$

$$f(p, p_y, y)p\,dp\,dp_y = F(E, E_y, y)\,dE\,dE_y$$

$$f(p, p_y, y) = \underline{F(E, E_y, y)}M^{-2}\sqrt{2M(E_y - Y)}$$

$$F \equiv C\frac{dN}{dE_y} = C\frac{dN}{d\varepsilon_\perp}$$

The approximations

In many cases the planar potential can be approximated by an oscillator potential

$$Y(y) = \kappa(d_p/2 - y)^2 \tag{25}$$

where κ is a proportionality coefficient. In this case the diffusion coefficient for electron and nuclear scattering may be obtained in the explicit form

$$D_e = \frac{1}{2}\left\langle \frac{\overline{\Delta E_y^2}}{\Delta z} \right\rangle_e = \frac{m}{4M} E_\perp \left(\frac{\overline{\Delta E}}{\Delta z} \right)_c \tag{26}$$

where $(\Delta E/\Delta z)_c$ is the energy loss in a channel which is considered to be independent of E_y,

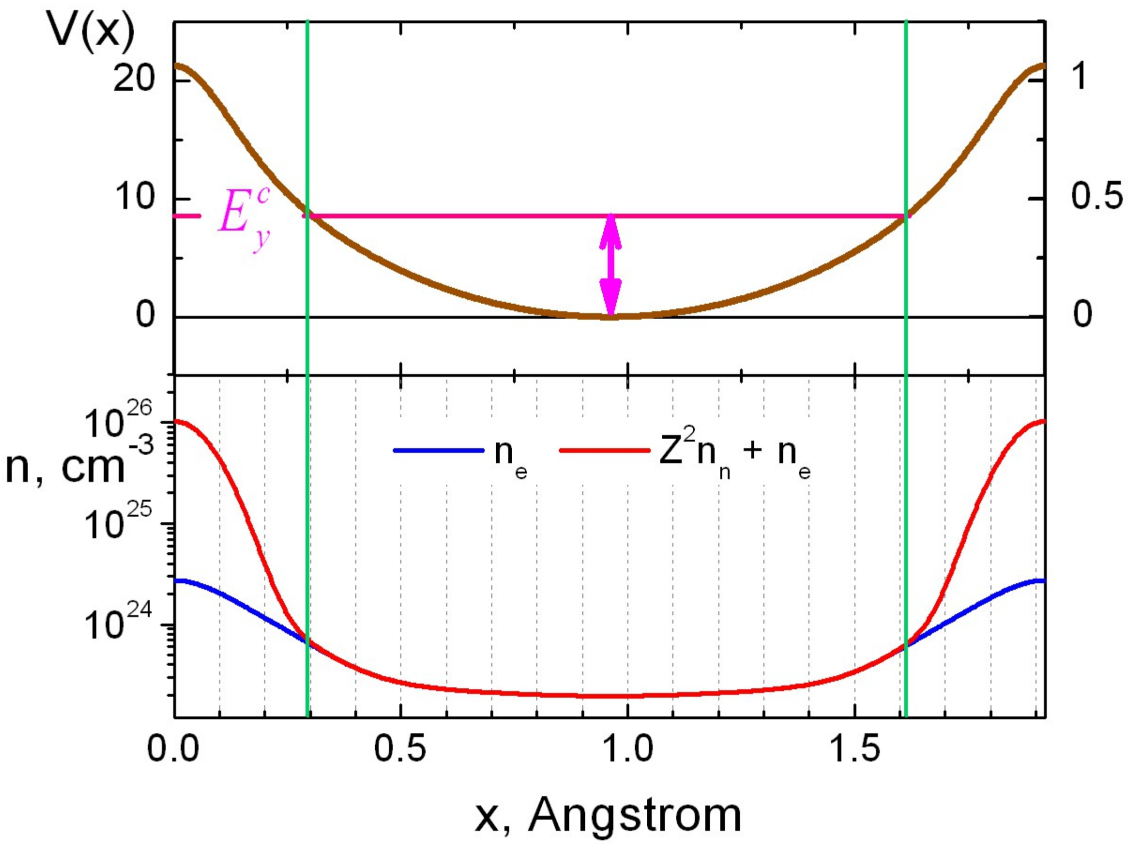
The boundary condition, equation solution and *dechanneling length* introduction

Equation (7) is readily solved with the electron diffusion coefficient given by Eq. (26). Then we obtain for the boundary condition $F(t, E_y{}^c) = 0$ ($E_y{}^c$ is the critical transverse energy) the following solution

$$F = \sum_{n=1}^{\infty} a_n J_0\left(\mu_{0,n}\sqrt{Ey/E_y{}^c}\right) \times \exp\left[-\frac{\mu_0{}^2{}_{,n}}{16}\frac{m}{M}\frac{|\Delta E|}{E_y{}^c}\right] \quad (28)$$

$$x_{1/2} = \Delta E_{1/2}(dE/dx)_c^{-1}$$

Both transverse coordinate and energy applicability regions are severely limited



Nuclear dechanneling can not be touched upon at all

The predictions:

TABLE I
"The halflife length" $x_{1/2}$ of a channeled proton beam in a tungsten crystal.

Direction (plane)	Energy (MeV)	Harmonic potential $x_{1/2}$ (μm)	Rectangular potential $x_{1/2}$ (μm)	Experimental value $x_{1/2}$ (μm)[12]
{100}	2	1.3	1.5	1.3
	3	1.9	2.2	2.8
	6	3.8	4.4	4.0
{110}	2	2.3	2.6	2.7
	3	3.4	3.9	4.1
	6	6.8	7.8	8.8

Further development by T. Waho

PHYSICAL REVIEW B VOLUME 14, NUMBER 11 1 DECEMBER 1976

Planar dechanneling of protons in Si and Ge†

T. Waho

Musashino Electrical Communication Laboratory, Nippon Telegraph and Telephone Public Corporation, Midori-cho 3-9-11, Musashino-shi, Tokyo, 180, Japan

(Received 12 August 1975)

Using the diffusion coefficient previously obtained from the inelastic scattering probability and the diffusion equation, half thicknesses for escape of MeV protons from the (110) planar channels of Si and Ge are calculated in detail. The agreement with the experiment is satisfactory. The difference between the diffusion coefficients along and across the channel plane is also discussed.

$$\frac{\partial f}{\partial t}=\frac{\partial}{\partial E_y}\left(D(E_y)\frac{\partial}{\partial E_y}f\right), \qquad (1)$$

$$D(E_y)=\frac{1}{2}\left\langle\frac{\langle\Delta E_y^2\rangle_{sc}}{\Delta t}\right\rangle_A .$$

$$D(E_y)=D_0E_y^{l} .$$

With the boundary conditions

$$f(E_y, 0) = F(E_y) \quad [F(E_y): \text{ initial distribution}],$$

$$f(E_y^c, t) = 0 \quad (t > 0),$$

we can obtain the following solution:

$$f(E_y, t) = \sum_{k=1}^{\infty} C_k E_y^{(1-l)/2} J_p\left(j_{p,k}(E_y/E_y^c)^{(2-l)/2}\right) \times \exp\left[-\frac{D_0 E_y^{cl}}{E_y^{c2}}\left(\frac{2-l}{2} j_{p,k}\right)^2 t\right], \quad (3)$$

where

$$C_k = \frac{2-l}{E_y^{c\,2-l} J_{p+1}^2(j_{p,k})} \int_0^{E_y^c} E_y^{(1-l)/2} F(E_y) \times J_p\left(j_{p,k}(E_y/E_y^c)^{(2-l)/2}\right) dE_y,$$

$$p = \left|(1-l)/(2-l)\right|, \qquad 0 < l < 2$$

Furthermore, we assume $l = 1$

$$N(t) = \int_0^{E_y^c} f(E_y, t)\, dE_y$$

Diffusion theory application to high energy

Nuclear Instruments and Methods in Physics Research B 86 (1994) 245–250
North-Holland

NIM B
Beam Interactions with Materials & Atoms

On measuring 70 GeV proton dechanneling lengths in silicon crystals (110) and (111)

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Sanct-Petersburg Institute of Nuclear Physics, St. Petersburg, Russian Federation

Received 22 October 1993 and in revised form 21 December 1993

Measurements of 70 GeV proton dechanneling lengths in silicon crystals (110) and (111) are described. The agreement of the obtained results with the predictions of diffusion theory, Monte Carlo simulations and experimental data at other energies is shown.

Dechanneling length is introduced in the same way

$$\frac{\partial f}{\partial z} = \frac{1}{2}\frac{\partial}{\partial E_x}\left\{D(E_x)\frac{\partial f}{\partial E_x}\right\}, \qquad D = D_0 E_x^q, \quad q = 1$$

$$f = \sum_{k=1}^{\infty} C_k J_0(j_{0,k}\sqrt{E_x/E_c}) \exp\left(-\frac{D_0 j_{0,k}^2 z}{4E_c}\right), \tag{4}$$

$$C_k = \frac{1}{E_c J_1^2(j_{0,k})} \int_0^{\infty} f_0(E_x) J_0(j_{0,k}\sqrt{E_x/E_c})\, \mathrm{d}E_x, \tag{5}$$

$$f = C_1 J_0(j_{0,1}\sqrt{E_x/E_c}) \underline{\exp}\left(-\frac{D_0 j_{0,1}^2 \underline{z}}{4E_c}\right), \tag{6}$$

$$\underline{L_D} = \frac{4E_c}{j_{0,1}^2 D_0}. \tag{7}$$

$$\underline{L_{rel.}} = \left(\frac{j_{0,1}}{j_{0,2}}\right)^2 L_D \simeq 0.2 L_D$$

Biryukov et al's results

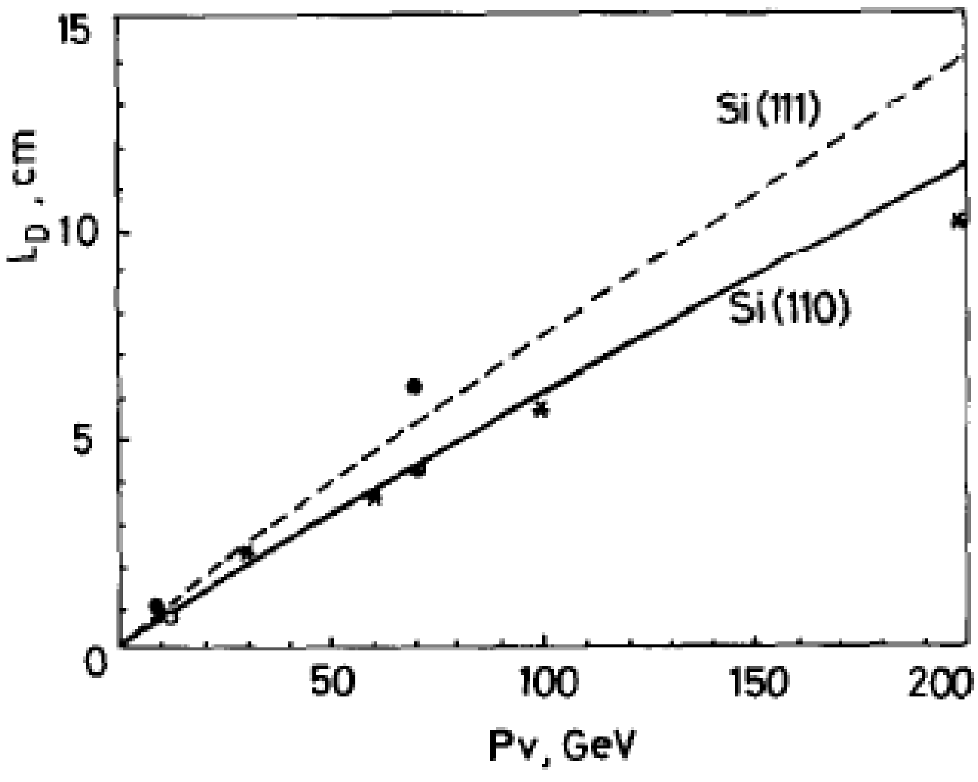


Fig. 4. Calculated functions L_D in channels Si(110) and Si(111) together with experimental data: • – JINR [8]; ○ – CERN [9]; ⋆ – FNAL [1]; ⊗ – IHEP (the data for the plane (111) are presented only for JINR and IHEP).

This old diffusion theory

should be upgraded

at high energies

The main new feature

$$\varepsilon_\perp = \frac{\varepsilon v_x^2}{2} + V(x) = \frac{p_x^2}{2\varepsilon} + V(x), \qquad \varepsilon'_\perp = \varepsilon\left[v_x(x) + \theta_x\right]^2/2 + V(x) = \varepsilon_\perp + \varepsilon v_x(x)\theta_x + \underline{\varepsilon\theta_x^2/2}$$

$$\left\langle(\varepsilon'_\perp - \varepsilon_\perp)^2\right\rangle = \varepsilon^2\left\langle v_x^2(x)\theta_x^{\,2}\right\rangle + \varepsilon^2\left\langle\theta_x^{\,4}\right\rangle/4 = 2\varepsilon\left\langle\underbrace{\left[\varepsilon_\perp - V(x)\right]\theta_x^2}\right\rangle + \varepsilon^2\left\langle\theta_x^{\,4}\right\rangle/4$$

$$\overline{\Delta E_y}/\Delta t = \tfrac{1}{2}E\,\overline{\Delta\theta^2}/\Delta t, \qquad \frac{1}{2}\left\langle\frac{\overline{\Delta E_y^2}}{\Delta t}\right\rangle = \left\langle 2\,\overbrace{\frac{\Delta E_y}{\Delta t}(E_y - Y)}\right\rangle \qquad 0$$

$$\text{MeV ions}: \qquad \theta_{ch} = \sqrt{\frac{V_0}{E_{ion}}} \gg \frac{m_e}{M_{ion}} > \theta_x$$

$$\text{GeV/TeV protons}: \quad \theta_{ch} = \sqrt{\frac{2V_0}{E_p}} \ll \theta_{x\max} = \frac{m_e}{m_p}$$

is the θ_x^4 term consideration

however

$$\left\langle \theta_x{}^4 \right\rangle \mu \int^{\theta} \theta d\theta \mu \ \theta^2$$ diverges, while

$$\left\langle w \right\rangle \mu \int^{\theta} \frac{\theta d\theta}{\theta^4} \mu \ \frac{1}{\theta^2}$$ does not

Both "catastrophic" single scattering

$$w = w(\varepsilon_\perp, x) = \int d\Sigma^C \qquad d\Sigma = \frac{4\alpha^2\left[Z^2 n_n(x) + n_e(x)\right]}{\beta^2 p^2\left(\theta^2+\theta_1^2\right)^2} d\theta_x d\theta_y$$

and revised diffusion equation

$$\frac{\partial F}{\partial z} = -\frac{\partial}{\partial \varepsilon_\perp}\left(\frac{\Delta\varepsilon_\perp}{\Delta z}F\right) + \frac{1}{2}\frac{\partial^2}{\partial\varepsilon_\perp^2}\left(\frac{(\Delta\varepsilon_\perp)^2}{\Delta z}F\right) - wF,$$

should to be introduced

a *key technical point* is specially determined integration limits

$$d\Sigma = \frac{4\alpha^2\left[Z^2 n_n(x) + n_e(x)\right]}{\beta^2 p^2\left(\theta^2 + \theta_1^2\right)^2} d\theta_x d\theta_y,$$

$$\theta_\pm(\varepsilon_\perp, x) = -v_x(\varepsilon_\perp, x) \pm \sqrt{2\left(V_{\max} - V(x)\right)/\varepsilon},$$

$$\theta_-(\varepsilon_\perp, x) \le \theta_x \le \theta_+(\varepsilon_\perp, x)$$

$$\frac{\Delta\varepsilon_\perp(\varepsilon_\perp, x)}{\Delta z} = \frac{\pi\alpha^2}{\beta^3 p}\left[Z^2 n_n(x) + n_e(x)\right]\left\{\ln\left[\frac{\theta_+(x) + \sqrt{\theta_+^2(x) + \theta_1^2}}{\theta_-(x) + \sqrt{\theta_-^2(x) + \theta_1^2}}\right] + \frac{\theta_-(x)}{\sqrt{\theta_-^2(x) + \theta_1^2}} - \frac{\theta_+(x)}{\sqrt{\theta_+^2(x) + \theta_1^2}}\right\};$$

More intermediate values employing specially determined **integration limits**

$$\theta_-(\varepsilon_\perp, x) \le \theta_x \le \theta_+(\varepsilon_\perp, x)$$

$$\frac{(\Delta\varepsilon_\perp)^2(\varepsilon_\perp, x)}{\Delta z} = a(\varepsilon_\perp, x) + b(\varepsilon_\perp, x);$$

$$a(\varepsilon_\perp, x) = 4\left[\varepsilon_\perp - V(x)\right]\frac{\Delta\varepsilon_\perp}{\Delta z};$$

$$b(\varepsilon_\perp, x) = \frac{\pi\alpha^2}{4}\left[Z^2 n_n(x) + n_e(x)\right]\Bigg\{\theta_+(x)\sqrt{\theta_+^2(x) + \theta_1^2} - \theta_-(x)\sqrt{\theta_-^2(x) + \theta_1^2} + \frac{2\theta_1^2\theta_+(x)}{\sqrt{\theta_+^2(x) + \theta_1^2}} - \frac{2\theta_1^2\theta_-(x)}{\sqrt{\theta_-^2(x) + \theta_1^2}} - 3\theta_1^2 \ln\left[\frac{\theta_+(x) + \sqrt{\theta_+^2(x) + \theta_1^2}}{\theta_-(x) + \sqrt{\theta_-^2(x) + \theta_1^2}}\right]\Bigg\}.$$

The integration limits use in w evaluation

$$d\Sigma = \frac{4\alpha^2\left[Z^2 n_n(x) + n_e(x)\right]}{\beta^2 p^2\left(\theta^2 + \theta_1^2\right)^2} d\theta_x d\theta_y,$$

$$\theta_\pm(\varepsilon_\perp, x) = -v_x(\varepsilon_\perp, x) \pm \sqrt{2\left(V_{\max} - V(x)\right)/\varepsilon},$$

$$\theta_x > \theta_+(\varepsilon_\perp, x),\;\; \theta_x < \theta_-(\varepsilon_\perp, x)$$

$$w(\varepsilon_\perp, x) = \int d\Sigma^C =$$

$$= \frac{\pi\alpha^2}{\beta^2 p^2 \theta_1^2}\left[Z^2 n_n(x) + n_e(x)\right]\left\{2 + \frac{\theta_-(x)}{\left(\theta_-^2(x) + \theta_1^2\right)^{1/2}} - \frac{\theta_+(x)}{\left(\theta_+^2(x) + \theta_1^2\right)^{1/2}}\right\}.$$

Averaging over the channeling period

$$\frac{\partial F}{\partial z} = -\frac{\partial}{\partial \varepsilon_\perp}\left(\frac{\Delta\varepsilon_\perp}{\Delta z} F\right) + \frac{1}{2}\frac{\partial^2}{\partial \varepsilon_\perp^2}\left(\frac{(\Delta\varepsilon_\perp)^2}{\Delta z} F\right) - wF,$$

$$\langle \Phi(\varepsilon_\perp, x)\rangle = \int_{x_l(\varepsilon_\perp)}^{x_r(\varepsilon_\perp)} \Phi(\varepsilon_\perp, x) f_{\varepsilon_\perp}(x)\, dx$$

$$F(\varepsilon_\perp, x, z) = \varphi(\varepsilon_\perp, z) f_{\varepsilon_\perp}(x), \qquad \varphi(\varepsilon_\perp, z) \equiv \frac{1}{N}\frac{dN}{d\varepsilon_\perp}.$$

$$\frac{\partial \varphi(\varepsilon_\perp, z)}{\partial z} = -\frac{\partial}{\partial \varepsilon_\perp}\left(A(\varepsilon_\perp)\frac{\partial}{\partial \varepsilon_\perp}\frac{\varphi(\varepsilon_\perp, z)}{T(\varepsilon_\perp)}\right) + \frac{\partial^2}{\partial \varepsilon_\perp^2}\left(B(\varepsilon_\perp)\frac{\varphi(\varepsilon_\perp, z)}{T(\varepsilon_\perp)}\right) - W(\varepsilon_\perp)\varphi(\varepsilon_\perp, z)$$

$$A(\varepsilon_\perp) = \left\langle \frac{\Delta\varepsilon_\perp(\varepsilon_\perp, x)}{\Delta z}\right\rangle, \quad B(\varepsilon_\perp) = \langle b(\varepsilon_\perp, x)\rangle, \quad W(\varepsilon_\perp) = \langle w(\varepsilon_\perp, x)\rangle$$

Reduction to canonical Sturm–Liouville form

$$u(\xi) = \frac{\varphi(\varepsilon_\perp)}{T(\varepsilon_\perp)} = \frac{1}{N}\frac{dN}{T(\varepsilon_\perp)d\varepsilon_\perp} = \frac{1}{N}\frac{dN}{d\xi}, \qquad \xi(\varepsilon_\perp) = \int_0^{\varepsilon_\perp} T(\varepsilon_\perp)d\varepsilon_\perp,$$

$$p(\xi) = \left[B(\varepsilon_\perp(\xi)) + A(\varepsilon_\perp(\xi))\right]T(\varepsilon_\perp(\xi))r(\xi);$$

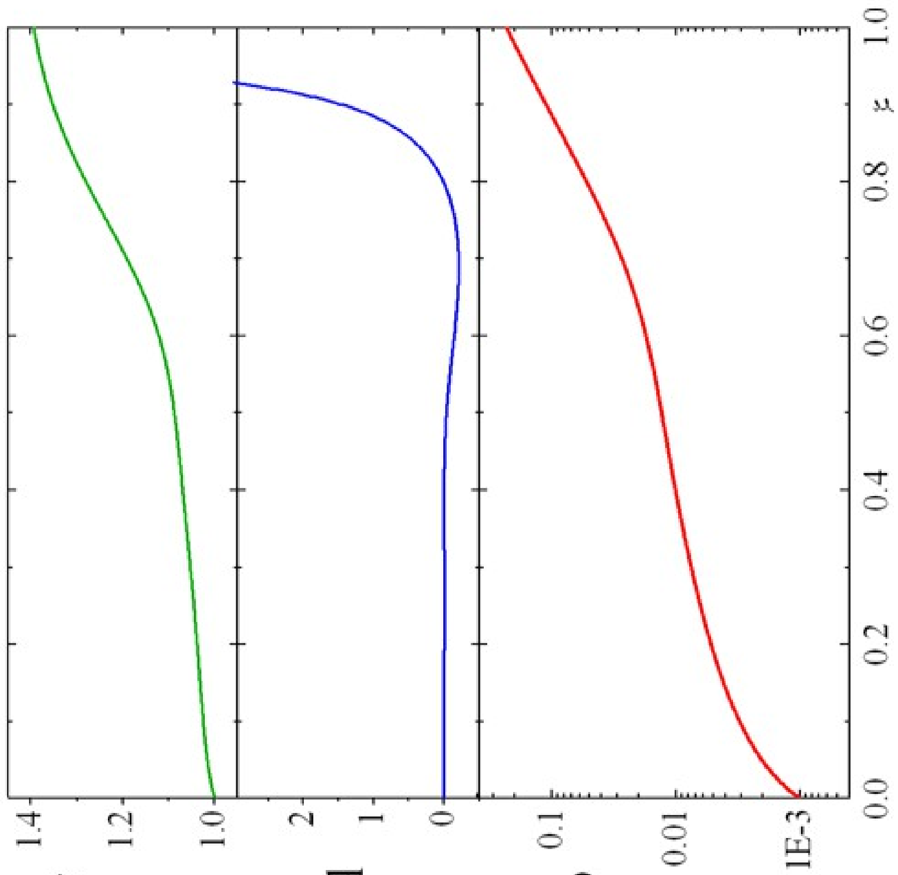
$$q(\xi) = \left[W(\varepsilon_\perp(\xi)) - B''(\varepsilon_\perp(\xi))\right]T^{-1}(\varepsilon_\perp(\xi))r(\xi);$$

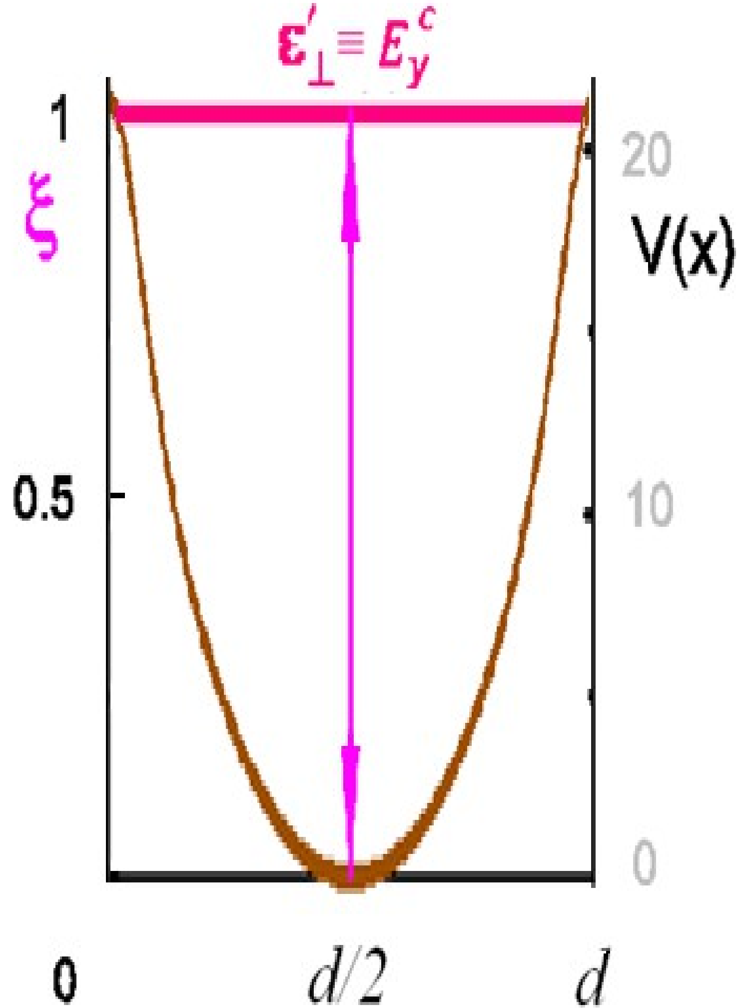
$$r(\xi) = \exp\int_0^{\varepsilon_\perp(\xi)} \frac{B'(\varepsilon_\perp)d\varepsilon_\perp}{A(\varepsilon_\perp) + B(\varepsilon_\perp)}; \quad \varepsilon_\perp(\xi) = \int_0^{\xi}\frac{d\xi}{T(\varepsilon_\perp)},$$

$$-\frac{\partial}{\partial\xi}\left[p(\xi)\frac{\partial}{\partial\xi}u_n(\xi)\right] + q(\xi)u_n(\xi) = \lambda_n r(\xi)u_n(\xi), \quad \partial u_n(0)/\partial\xi = 0, \quad u_n(1) = 0,$$

Eq. $r(\xi)\frac{\partial u(\xi, z)}{\partial z} = \frac{\partial}{\partial \xi}\left(p(\xi)\frac{\partial u(\xi, z)}{\partial \xi} \right) - q(\xi)u(\xi, z)$

coefficients' behavior



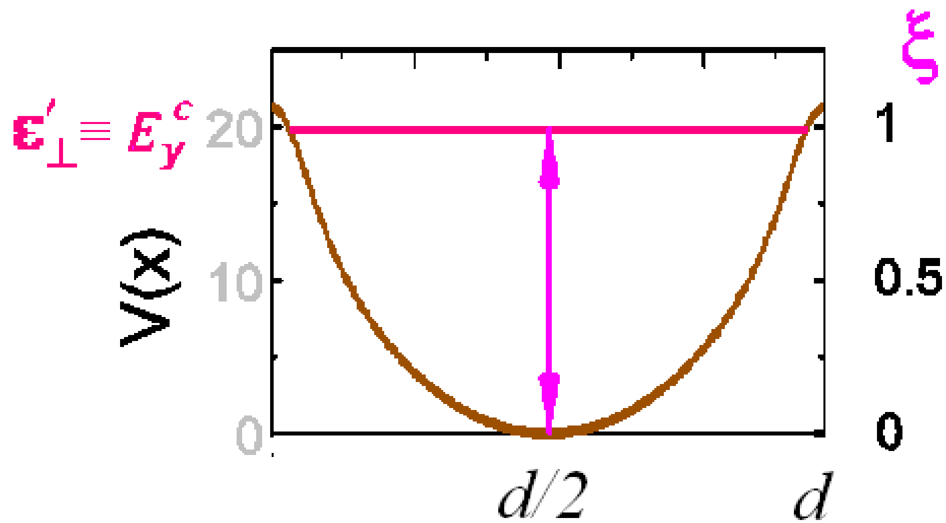


The boundary condition
(what is channeling?)

boundary condition:

KWB: $F(t, E_y{}^c) = 0$

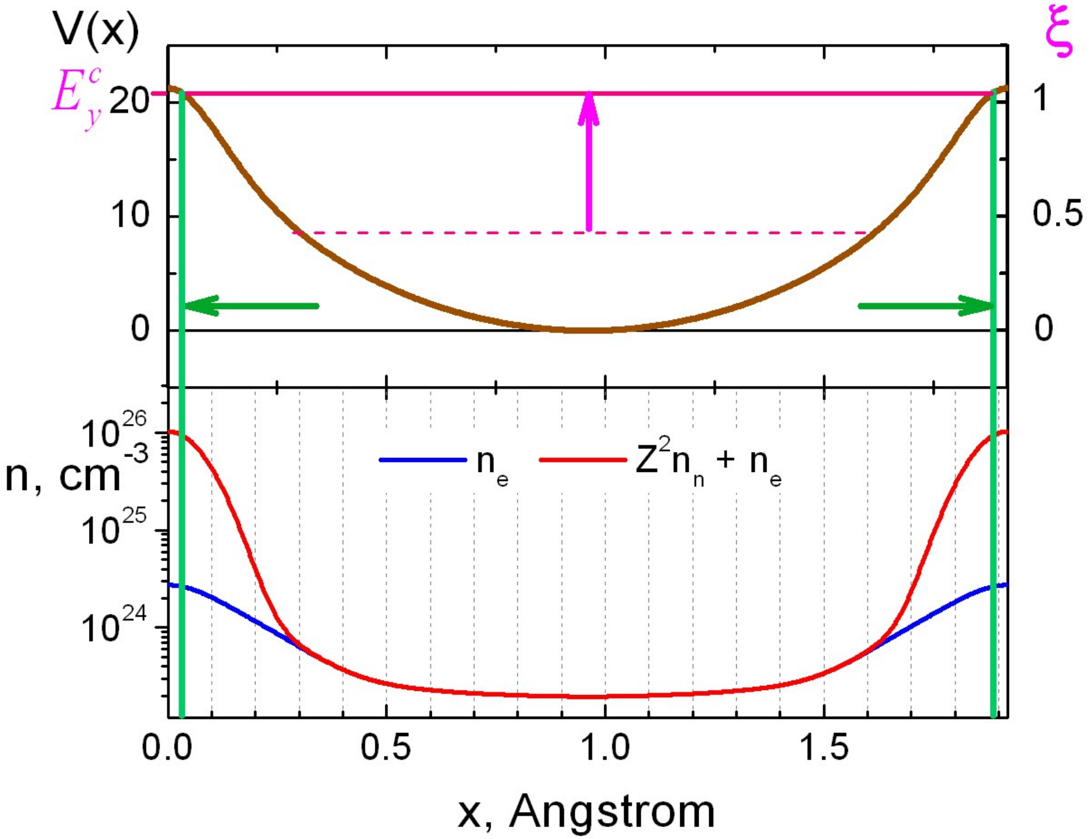
tt: $u_n(1) = 0$



$$\delta\varepsilon_\perp(\varepsilon') = \left[\left\langle \frac{(\Delta\varepsilon_\perp(\varepsilon', x))^2}{\Delta z} \right\rangle T(\varepsilon') - \left\langle \frac{\Delta\varepsilon_\perp(\varepsilon', x)}{\Delta z} \right\rangle^2 T^2(\varepsilon')\right]^{1/2}$$

$$\frac{\delta\varepsilon_\perp(\varepsilon'_\perp)}{V_{\max} - \varepsilon'_\perp} \leq 1$$

Both transverse coordinate and energy applicability regions are drastically **widened**



Nuclear dechanneling can be readilly studied

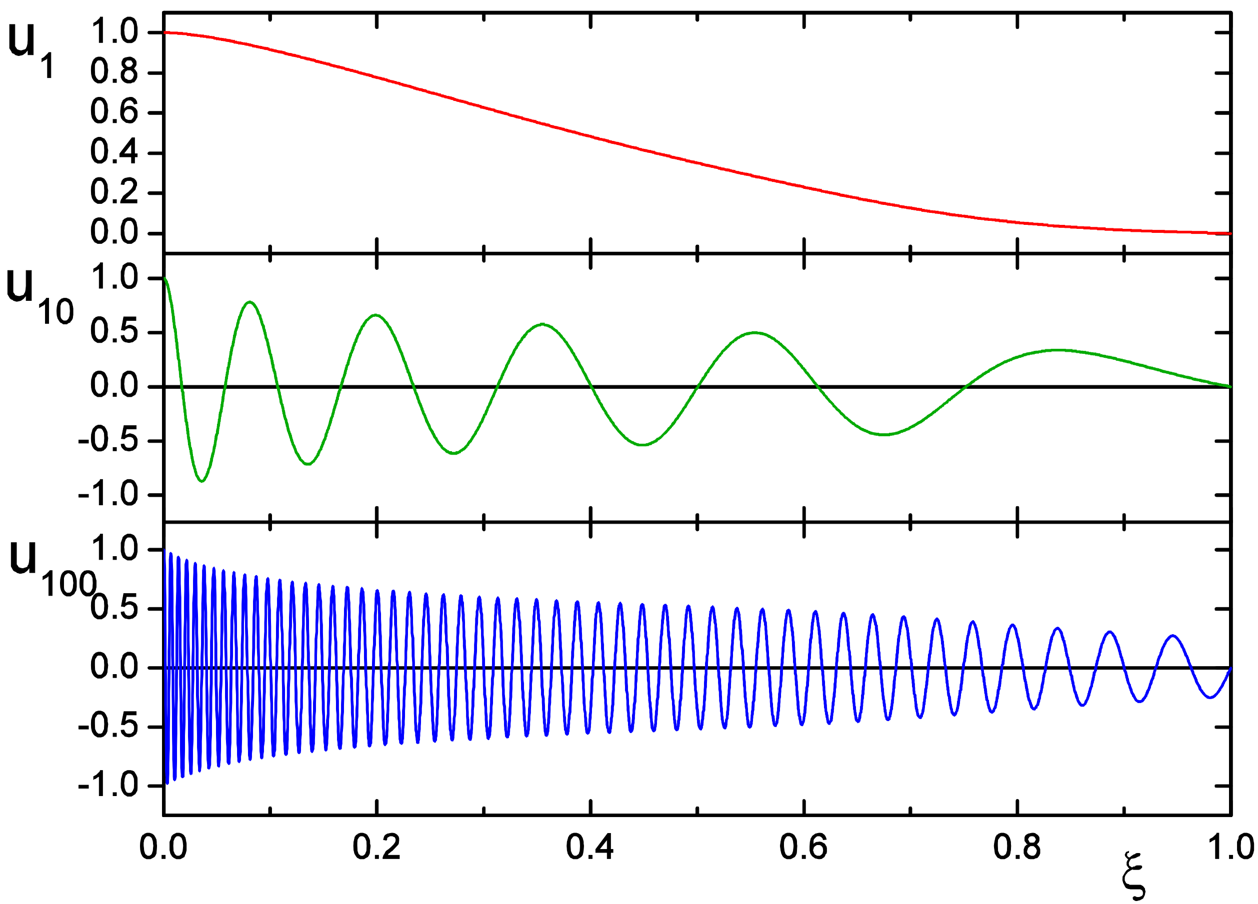
400 GeV example

Dechanneling length evaluation precision

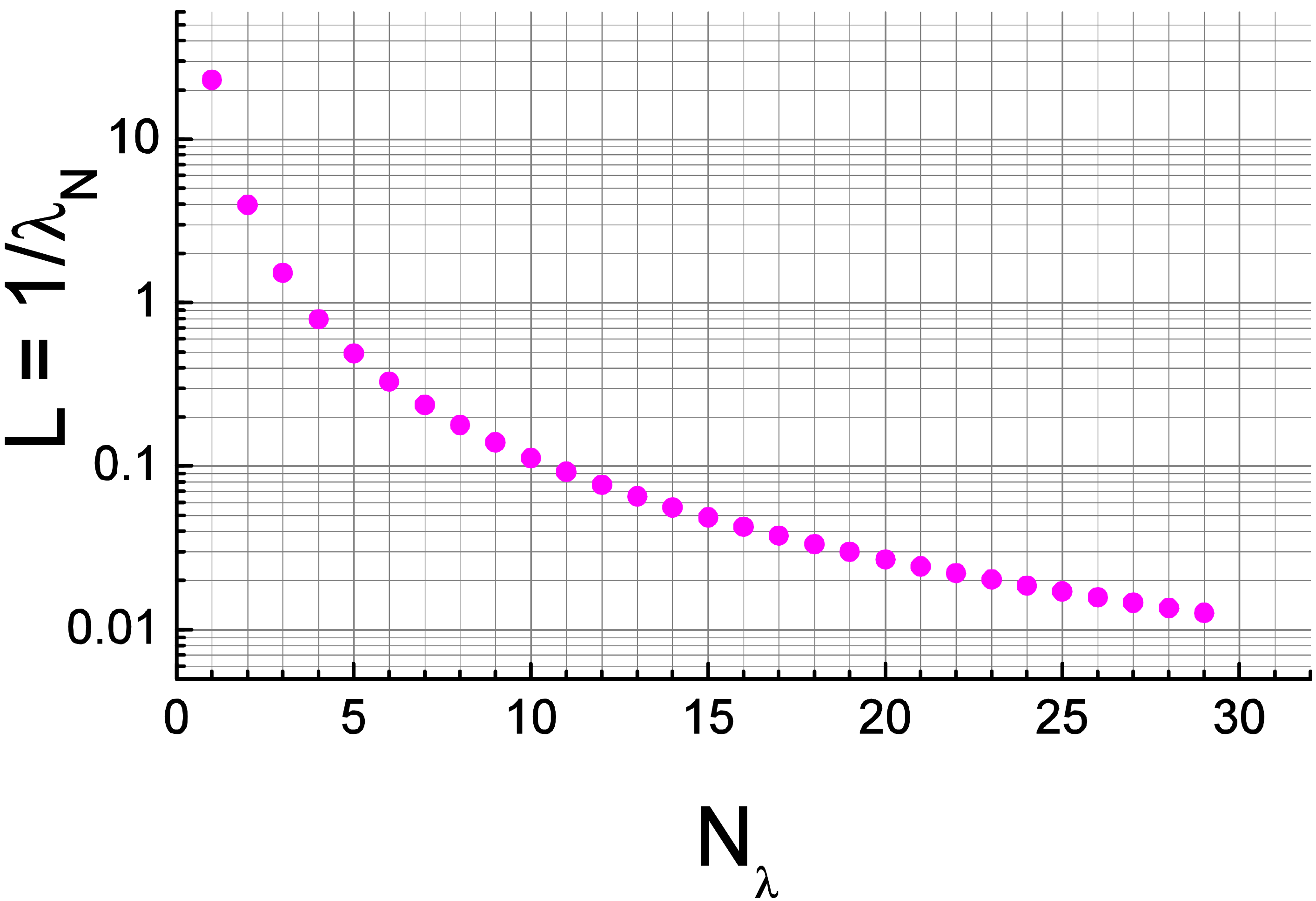
$$\delta\varepsilon_\perp(\varepsilon') = \left[\left\langle \frac{(\Delta\varepsilon_\perp)^2(\varepsilon', x)}{\Delta z} \right\rangle T(\varepsilon') - \left\langle \frac{\Delta\varepsilon_\perp(\varepsilon', x)}{\Delta z} \right\rangle^2 T^2(\varepsilon') \right]^{1/2}$$

Potential model	$\dfrac{\delta\varepsilon_\perp(\varepsilon'_\perp)}{V_{max} - \varepsilon'_\perp}$	l_{dech}, cm	Δl_1,%	Δl_2,%
Tobiyama	1	**23.1**	+ 0.61	**0**
Tobiyama	0.5	**22.9**	+1.3	**– 0.81**
Tobiyama	2	**23.2**	+ 0.30	**+ 0.37**
DT	1	**23.3**	+ 0.65	**+ 0.61**
Moliere	1	21.4	+ 0.61	– 7.255

Eigen functions for n = 1, 10 and 100



Inverse eigen numbers



Diffusion equation solution & **dechanneling length** strict introduction

$$-\frac{\partial}{\partial \xi}\left[p(\xi)\frac{\partial}{\partial \xi}u_n(\xi)\right]+q(\xi)u_n(\xi)=\lambda_n r(\xi)u_n(\xi),\quad \partial u_n(0)/\partial\xi=0,\quad u_n(1)=0,$$

$$u(\xi,z)=\sum_{n=0}^{\infty}c_n\exp(-\lambda_n z)u_n(\xi)\xrightarrow{z\lambda_1\gg 1}c_0\,\underline{\exp(-\lambda_0 z)}\,u_0(\xi),$$

$$c_n=\int_0^1 u(\xi,0)u_n(\xi)r(\xi)d\xi\left(\int_0^1 u_n^2(\xi)r(\xi)d\xi\right)^{-1}$$

Proton dechanneling lengths at major collider energies

acc-r	ε, GeV	$\boldsymbol{l}_{\mathbf{dech}}$	Δl_{dech}, %	λ_2/λ_1	N_{ch0}/ N_{inc}
SPS	400	**23.**1 **cm**	0.61	6.0	0.895
LHC	6500	**3 m** 3.6cm	0.34	5.7	0.895
FFC	10^5	**39 m** 36 cm	0.18	5.6	0.895

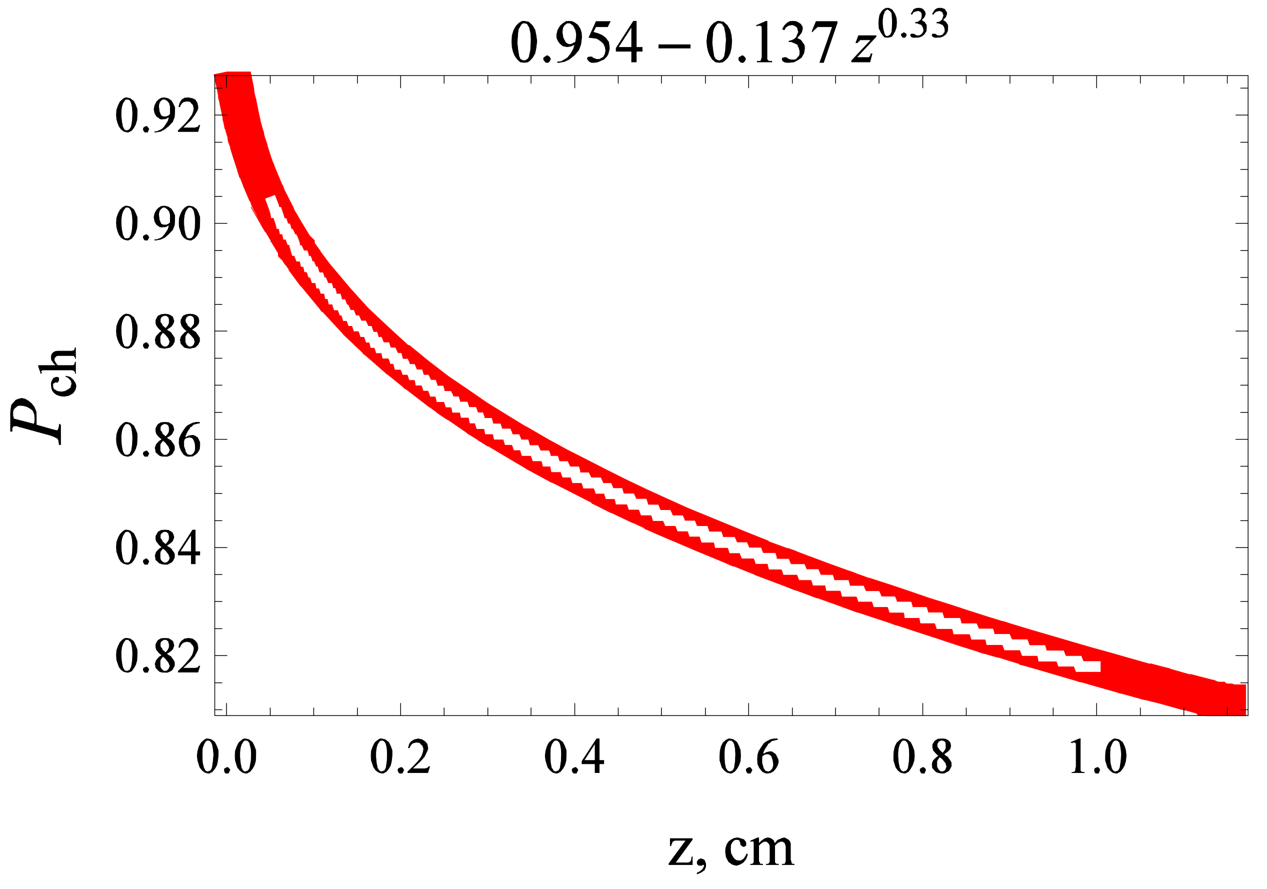
Dechanneling lengths uncertainly is about **one percent** and is more limited by potential evaluation precision

Some peculiarities of channeling fraction evolution from

$$u(\xi, z) = \sum_{n=0}^{\infty} c_n \exp(-\lambda_n z) u_n(\xi),$$

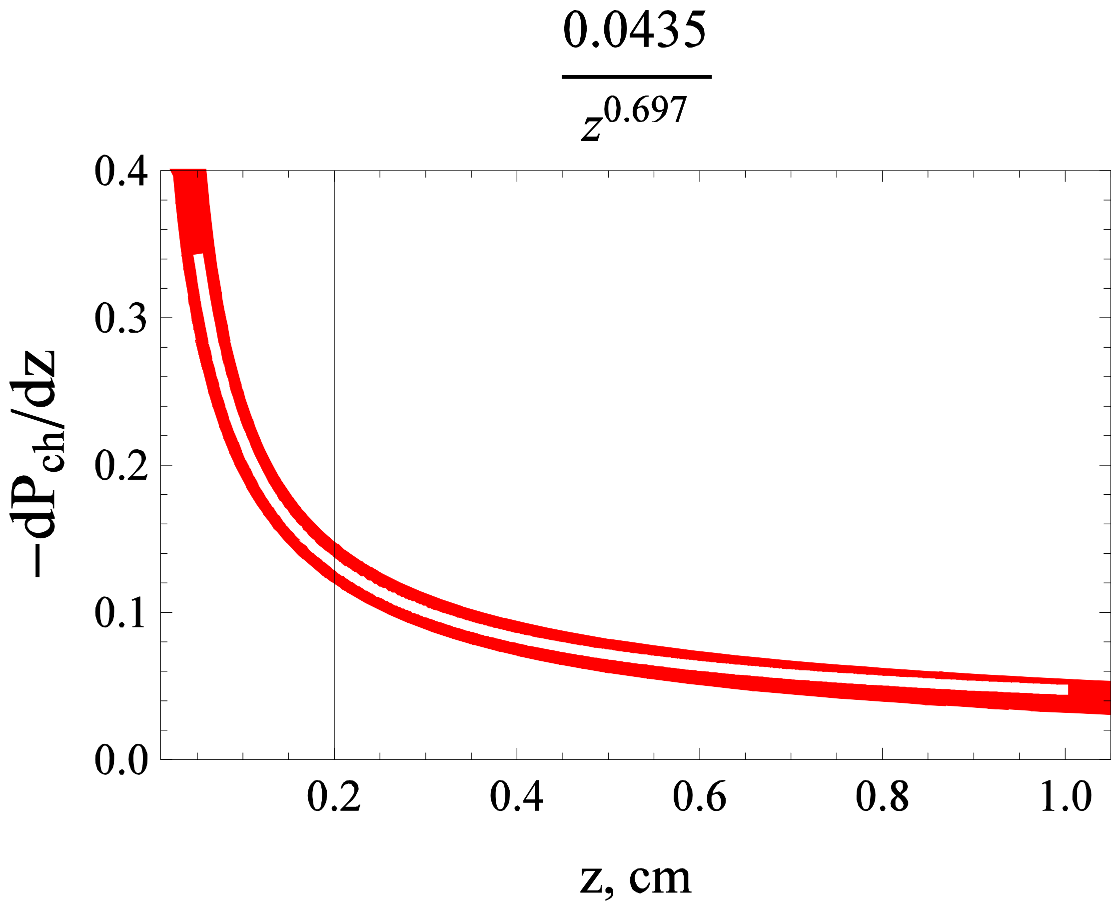
$$c_n = \int_0^1 u(\xi, 0) u_n(\xi) r(\xi) d\xi \left(\int_0^1 u_n^2(\xi) r(\xi) d\xi \right)^{-1}$$

Channeling probability vs crystal thickness for 400 GeV Si(110)



The formula − in red, approximation − in white

Channeling probability change rate vs crystal thickness for 400 GeV Si(110)



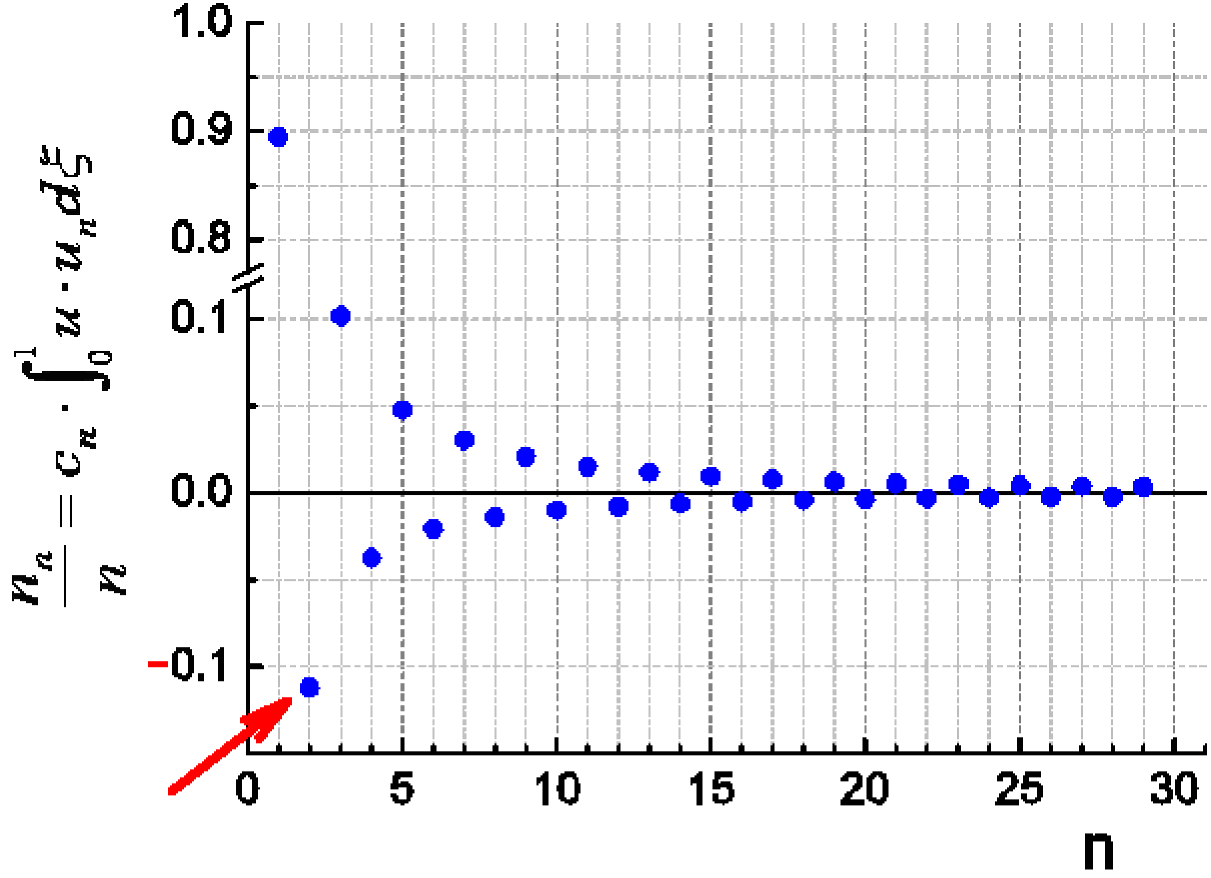
The formula – in red, approximation – in white

Channeling fraction

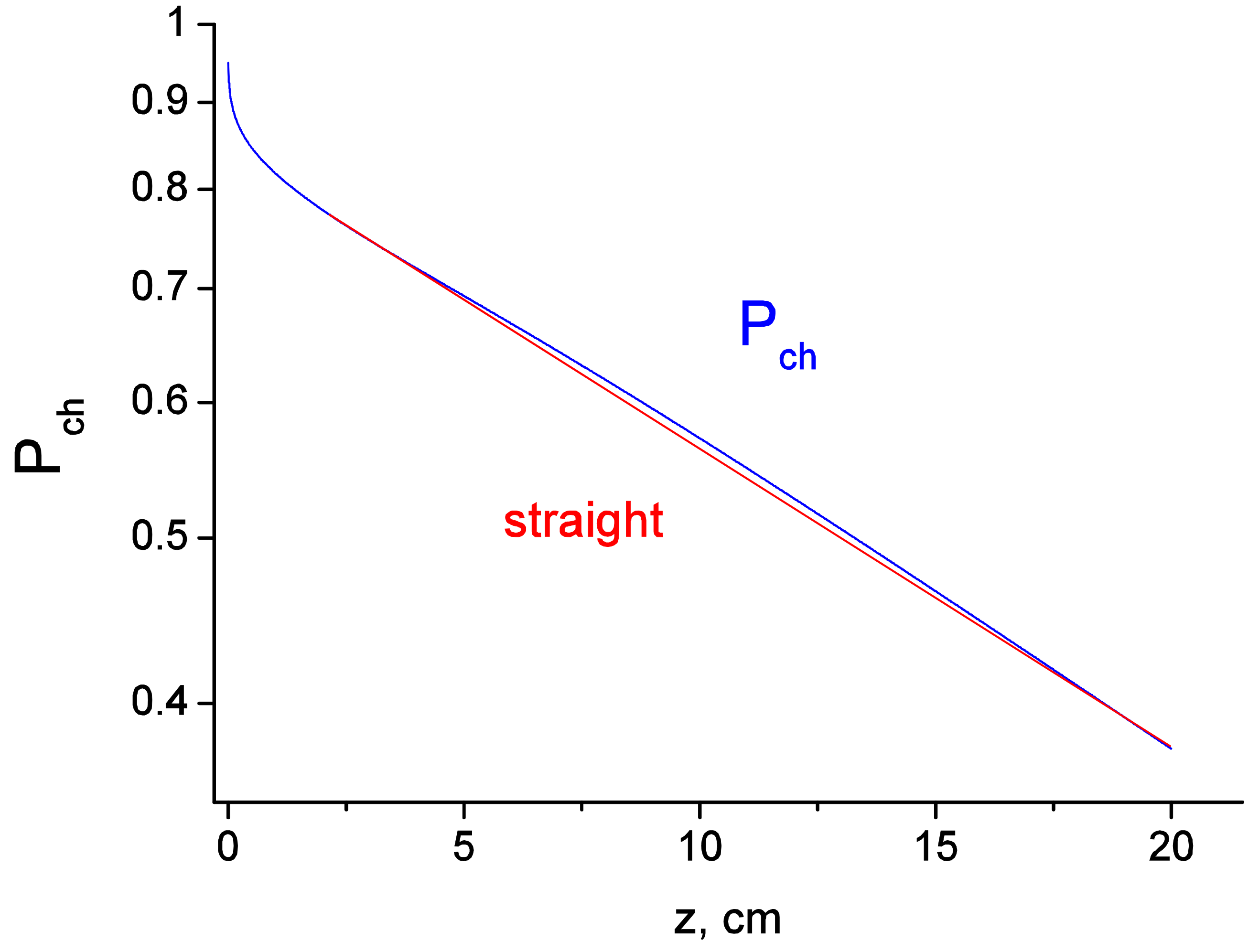
decreases like $z^{1/3}$ in the

nuclear dechanneling region

Eigenvalue contributions are **sign-changing**!



A peculiarity of P_{ch} z-dependence



Can the diffusion theory be applied to **negatively charged** particles?

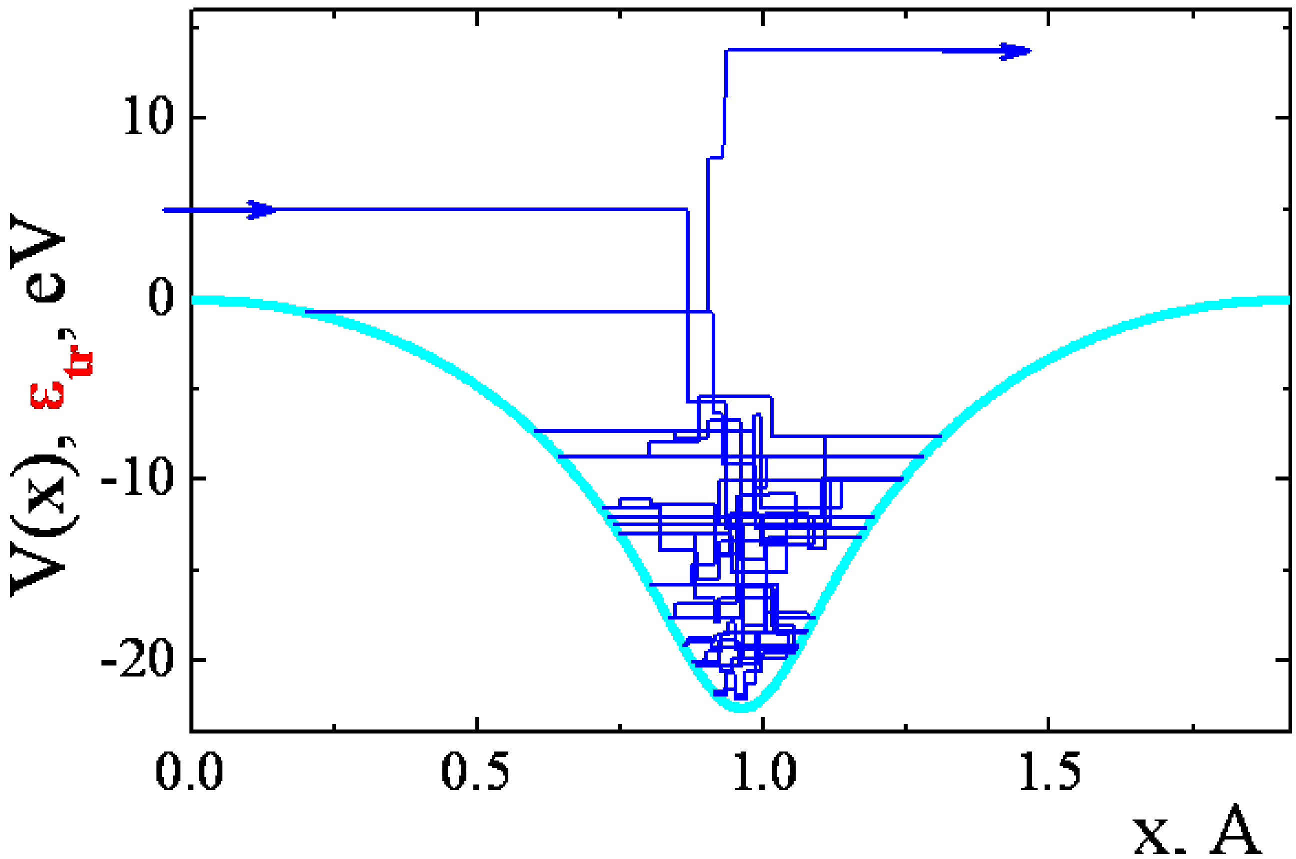
Electron dechanneling lengths and channeling fraction

ε, GeV	l_{dech}	λ_1/λ_0	Δl_1, %	N_{ch0}/ N_{inc}
1	**6.0 μm**	7.8	**130**	**0.33**
10	**50 μm**	6.9	**78**	**0.39**
100	**0.44 mm**	6.4	**46**	**0.44**
1000	**3.8 mm**	6.1	**28.5**	**0.49**

Both high dechanneling lengths uncertainly and high non-channeling fraction make this method **ineffective** for negatively charged particles

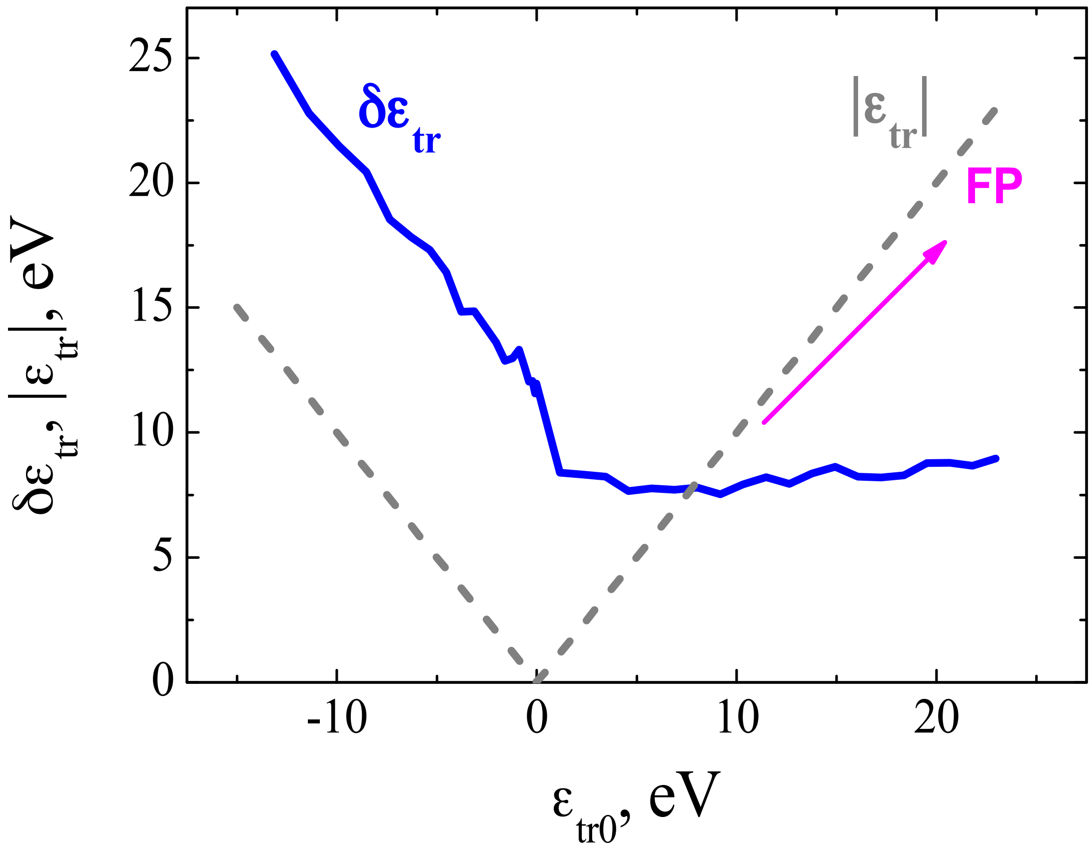
From the
2012 year presentation:

Stochastic evolution of electron transverse energy



electron pass the **highest half** of under-barrier transverse energy region quite **fast**

Transverse energy asquires large dispersion at one channeling period



Fokker-Planck equation has very limited applicability

Monte Carlo simulations are much more productive than the diffusuion theory in the case of **negatively charged** particles

Key simulation points:

Simulation of **incoherent scattering** on both nuclei and electrons

Separate simulation of **single** and **multiple** scattering partly **suppressed** in crystals

Trajectory simulations in the most **realistic potentials**

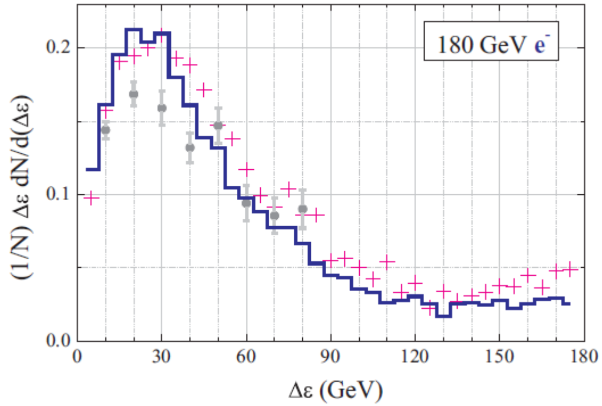
Monte Carlo simulations is the best way to solve all the problems with **negatively charged** channeling particles

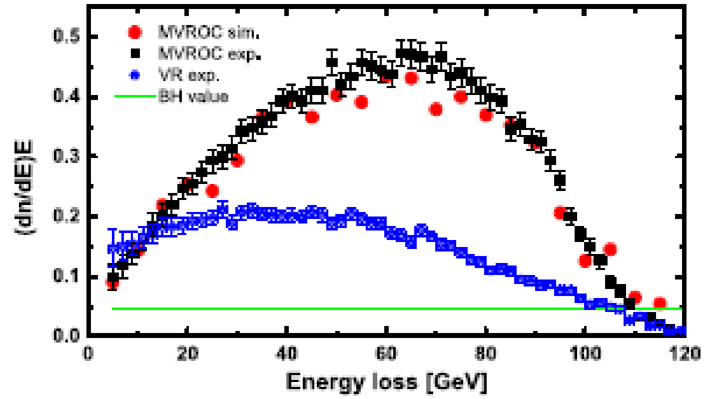
V. Guidi, L. Bandiera, V.V. Tikhomirov. Phys. Rev.A. 86 (2012) 042903

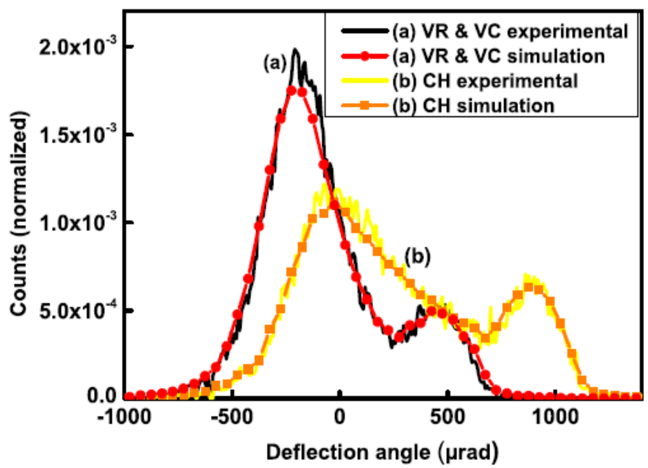
L. Bandiera ... V. Guidi,.. V.V. Tikhomirov , Phys. Rev. Lett. 111 (2013) 255502 .

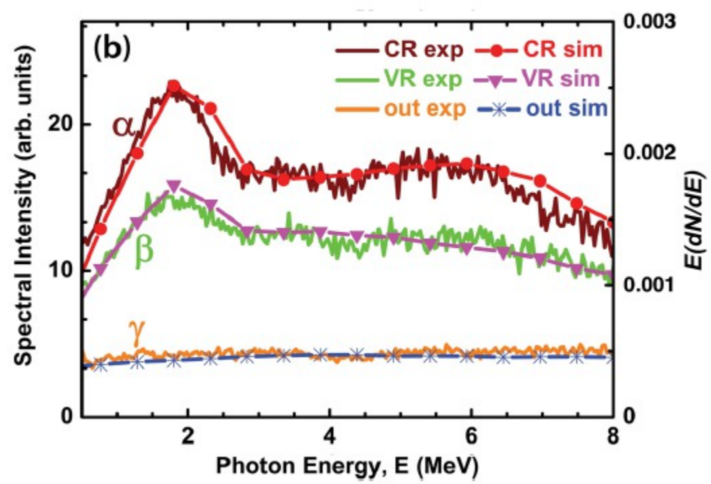
A. Mazzolari ... V. Guidi, ..V.V. Tikhomirov , Phys. Rev. Lett. 112 (2014) 135503.

L. Bandiera ... V. Guidi,.. V.V. Tikhomirov , Phys. Rev. Lett. 115 (2015) 025504.









PRL 112, 135503 (2014) PHYSICAL REVIEW LETTERS week ending 4 APRIL 2014

Steering of a Sub-GeV Electron Beam through Planar Channeling Enhanced by Rechanneling

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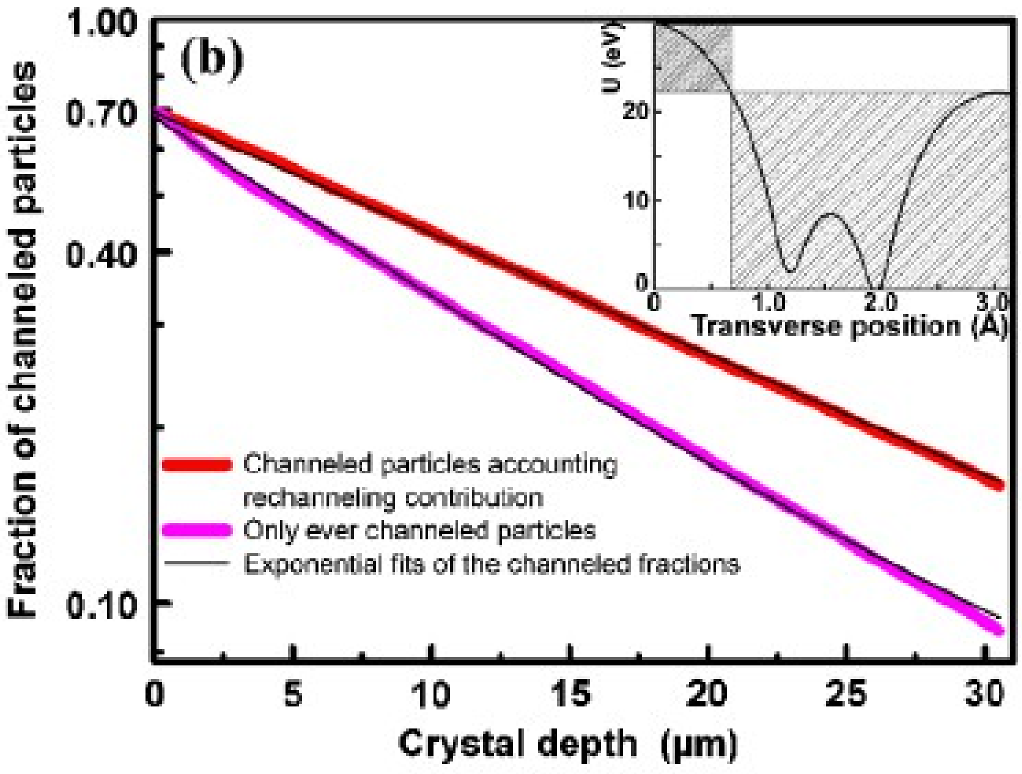
MC simulations demonstrate the **exponential behavior** of both permanently and currently channeled electrons



a real challenge for a new theory

Conclusions

- Diffusion equation has been undated to include large angle nuclear scattering
- Proton dechanneling lengths have been evaluated with high precision
- Power-type proton dechanneling law has been revealed
- Poor applicability of the diffusion equation for negatively charged particles has been demonstrated

Thank you for attention!