Inclusive determination of $|V_{ub}|$

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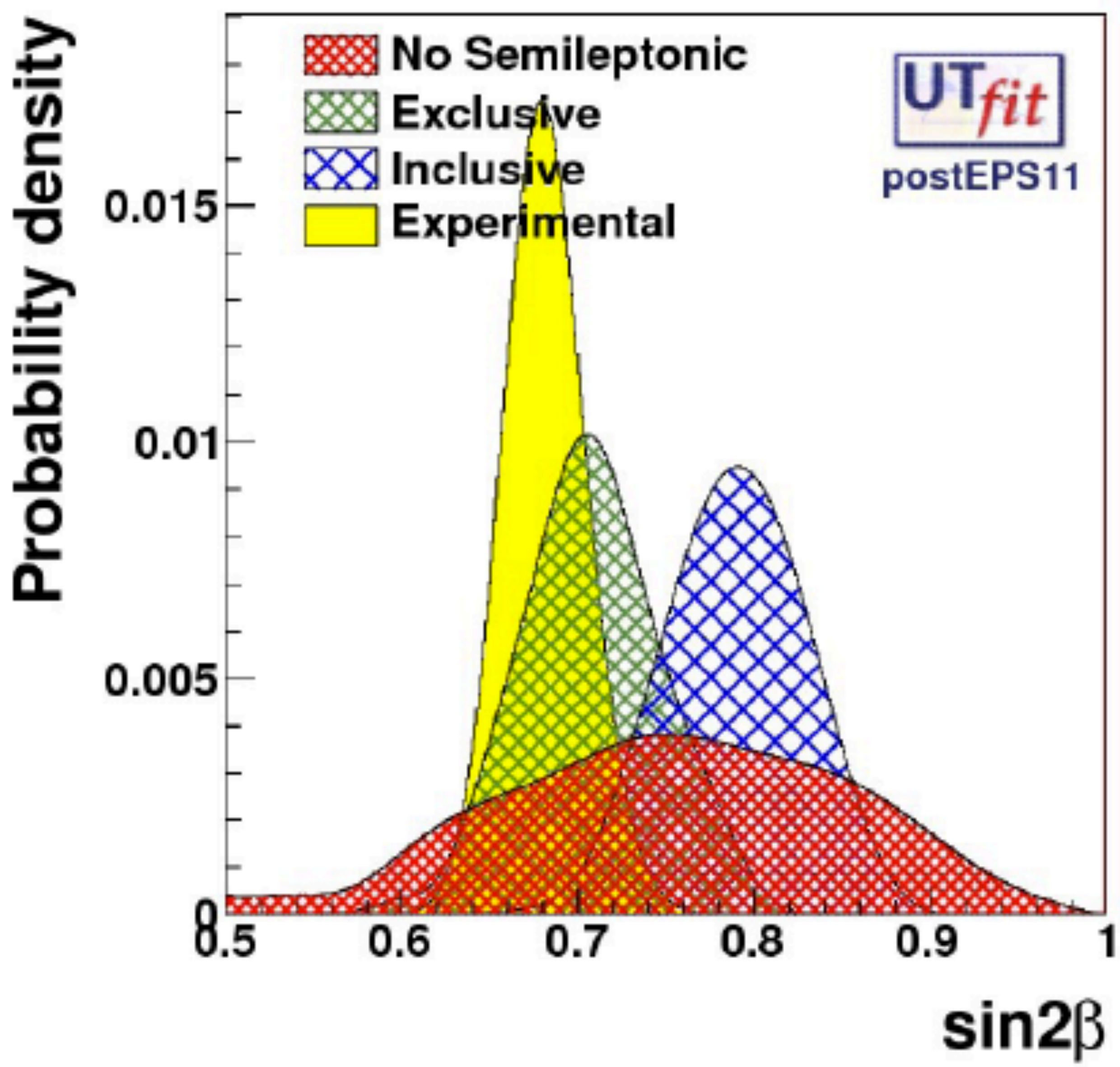
Our world is full of tensions

A puzzling tension

- The results we have for V_{ub} are contradictory, inclusive result is the *odd man out.*
- There is a 2-3σ discrepancy between the inclusive and exclusive determinations. This could signal New Physics in semileptonic B decays, mostly affecting the exclusive determination.
- There is a 3σ tension between the inclusive V_{ub} and its indirect determination of the UT fit. This could be explained by sizeable shift in sin2β.

sin2β predictions:



$$\left|\frac{V_{ub}}{V_{cb}}\right|_{incl} = 0.101 \pm 0.006 \rightarrow \sin 2\beta_{UTfit} = 0.791 \pm 0.041$$

$$\left|\frac{V_{ub}}{V_{cb}}\right|_{excl} = 0.084 \pm 0.008 \rightarrow \sin 2\beta_{UTfit} = 0.706 \pm 0.041$$

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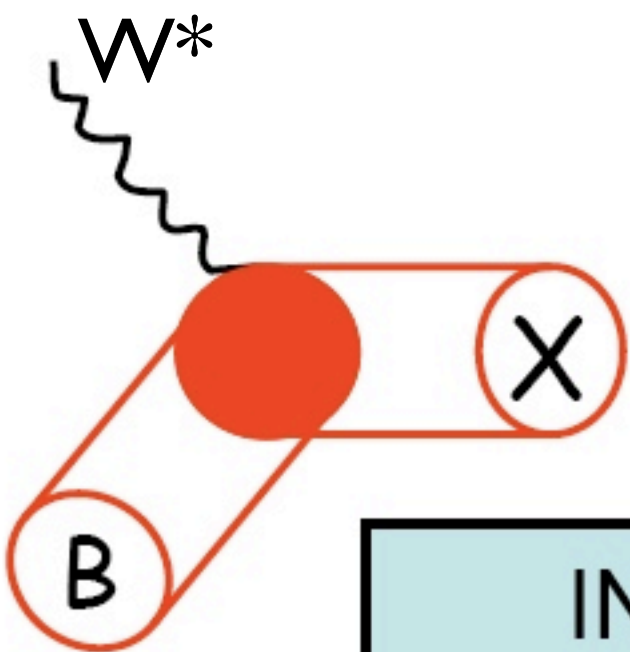
- There is a 3σ tension between the inclusive V_{ub} and its indirect determination of the UT fit. This could be explained by sizeable shift in sin2β.
- *Are we confident in inclusive results? they point to high $|V_{ub}|$ but experimental and theoretical results are quite consistent.*
- *The inclusive destiny of V_{ub} is intertwined with that of V_{cb}*

Inclusive vs exclusive B decays



Simplicity: ew (or em) currents probe the B dynamics

INCLUSIVE	EXCLUSIVE
OPE: non-pert physics described by B matrix elemnts of local operators can be extracted by exp suppressed by $1/m_b^2$	Form factors: in general computed by non pert methods (lattice, sum rules,...) symmetry can provide normalization

As we aim at high precision, both methods are challenging

Inclusive semileptonic B decays: basic features

- ***Simple idea:*** inclusive decay do not depend on final state, factorize long distance dynamics of the meson. OPE allows to express it in terms of matrix elements of local operators

$$T\, J(x)\, J(0) \approx c_1 \bar{b}b + c_2 \bar{b}\, \vec{D}^2\, b + c_3 \bar{b}\sigma\cdot G b + \ldots$$

- The Wilson coefficients are perturbative, matrix elements of local ops parameterize non-pert physics: ***double series in α_s, Λ/m_b***

- Lowest order: decay of a free *b*, linear Λ/m_b absent. Depends on $m_{b,c}$, 2 parameters at $O(1/m_b^2)$, 2 more at $O(1/m_b^3)$...

$$\mu_\pi^2(\mu) = \frac{1}{2M_B}\left\langle B\left|\bar{b}\left(i\vec{D}\right)^2 b\right|B\right\rangle_\mu \qquad \mu_G^2(\mu) = \frac{1}{2M_B}\left\langle B\left|\bar{b}\frac{i}{2}\sigma_{\mu\nu}G^{\mu\nu}b\right|B\right\rangle_\mu$$

The total s.l. width in the OPE

$$\Gamma[B \to X_c l \bar{\nu}] = \Gamma_0 \; g(r) \Big[1 + \frac{\alpha_s}{\pi} c_1(r) + \frac{\alpha_s^2}{\pi^2} c_2(r) - \frac{\mu_\pi^2}{2m_b^2} + c_G(r) \frac{\mu_G^2}{m_b^2} + c_D(r) \frac{\rho_D^3}{m_b^3} + c_{LS}(r) \frac{\rho_{LS}^3}{m_b^3} + O\Big(\alpha_s \frac{\mu_{\pi,G}^2}{m_b^2}\Big) + O\Big(\frac{1}{m_b^4}\Big) \Big]$$

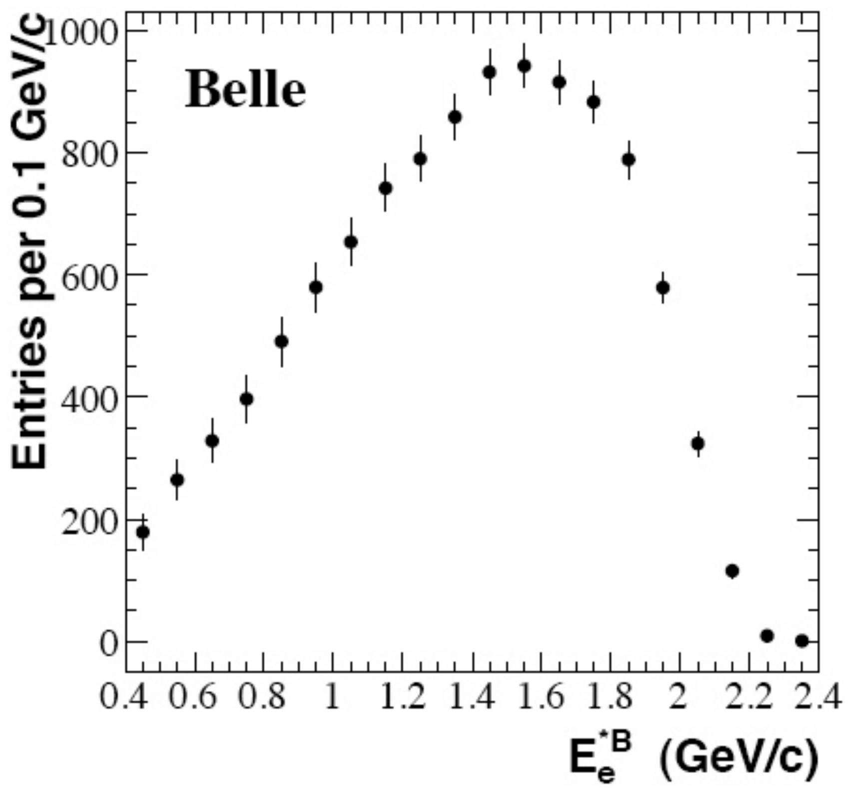
$$r = \frac{m_c^2}{m_b^2} \qquad \Gamma_0 = \frac{G_F^2 |V_{cb}|^2 m_b^5}{192\pi^3}$$

OPE valid for inclusive enough measurements, away from perturbative singularities ➠ moments

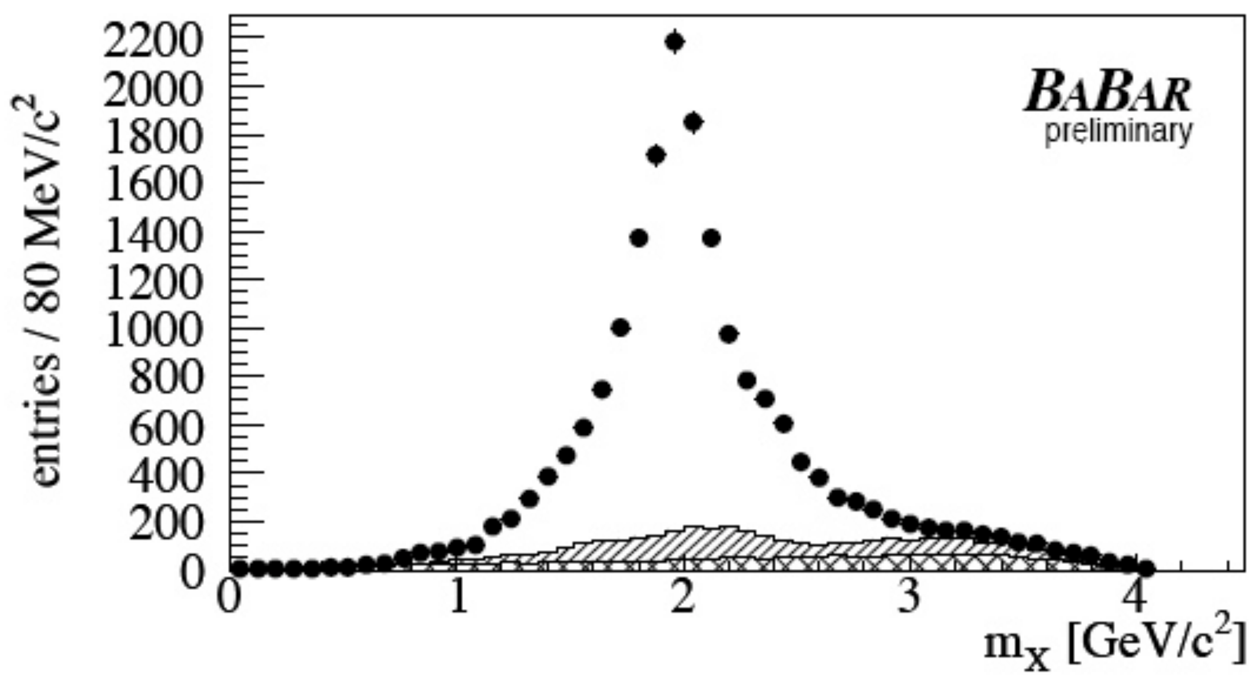
Present implementations include all terms through $O(\alpha_s^2, 1/m_b^3)$: $m_{b,c}$, $\mu^2_{\pi,G}$, $\rho^3_{D,LS}$ 6 parameters

Fitting OPE parameters to the moments

E_l spectrum



m_X spectrum

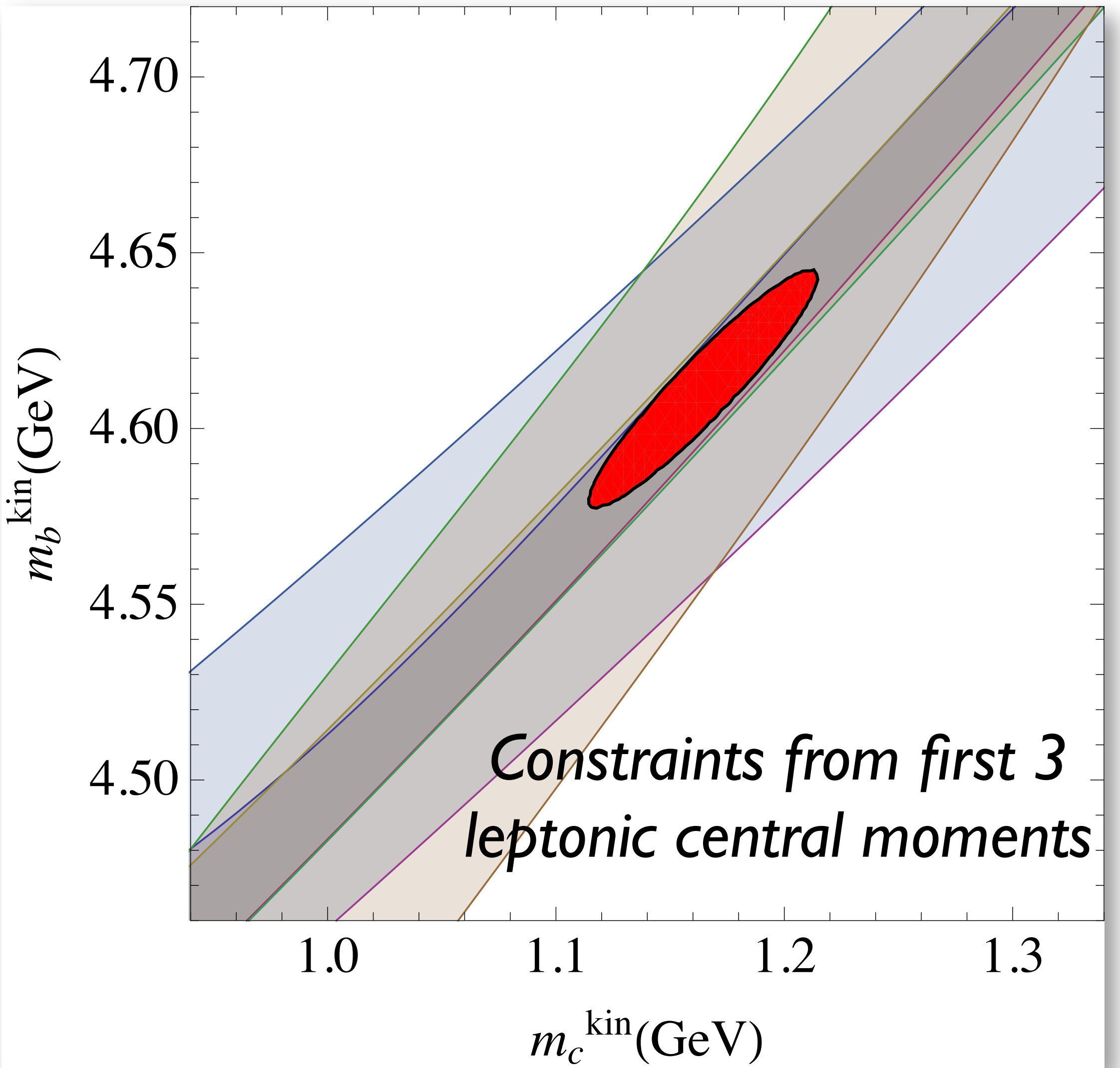


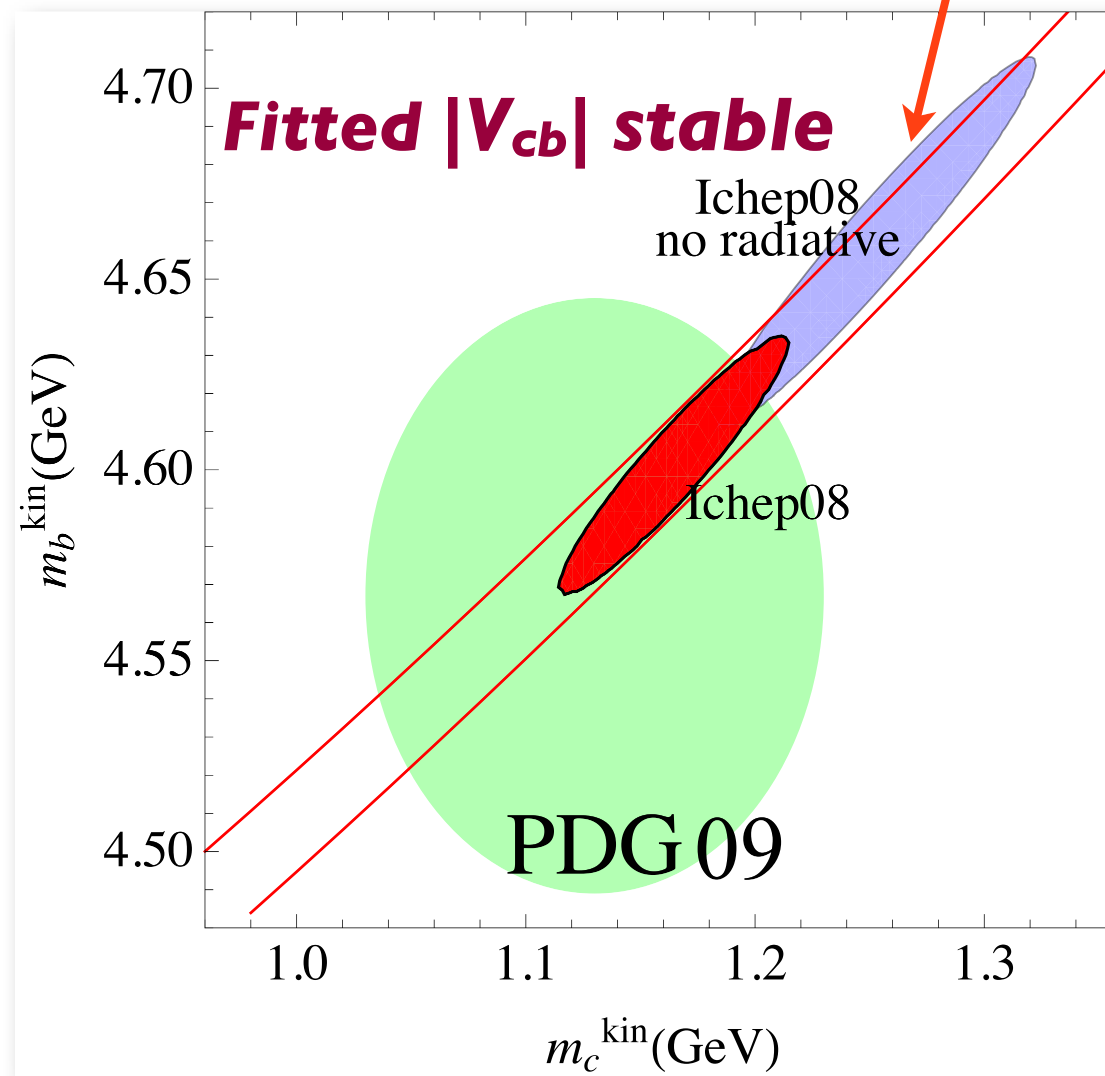
*Total **rate** gives $|V_{cb}|$, global **shape** parameters (first few moments of distributions) tell us about B structure, m_b and m_c*

OPE parameters describe universal properties of the B meson and of the quarks → useful in many applications

A strip in the m_b-m_c plane

Constant values of s.l. width at fixed V_{cb}

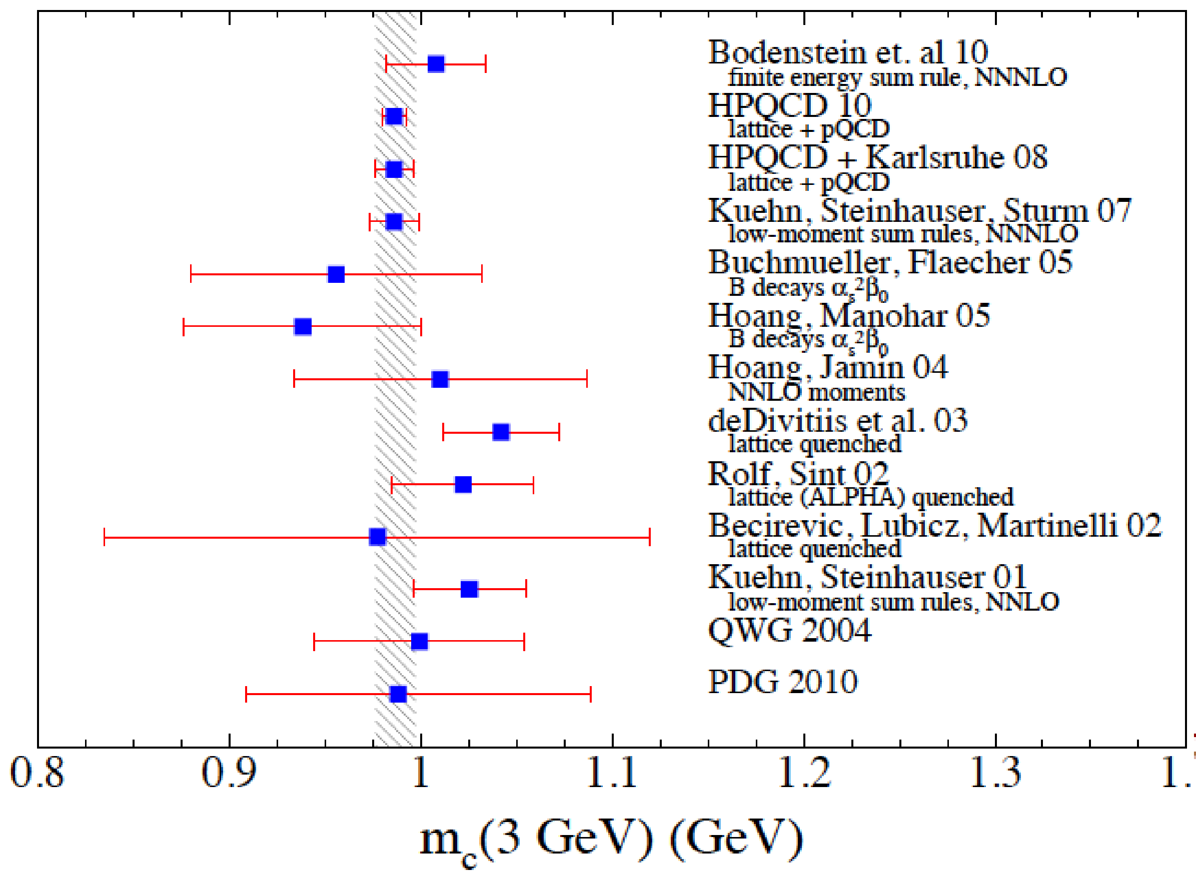




Semileptonic moments do not measure m_b well. They rather identify a strip in (m_b,m_c) plane along which the minimum is shallow.

Unknown non-pert $O(\alpha_s/m_b)$ effects in radiative moments. Possibly irrelevant here but must be studied. But role of radiative moments in the fits is equivalent to using a bound on m_b

Using mass determinations

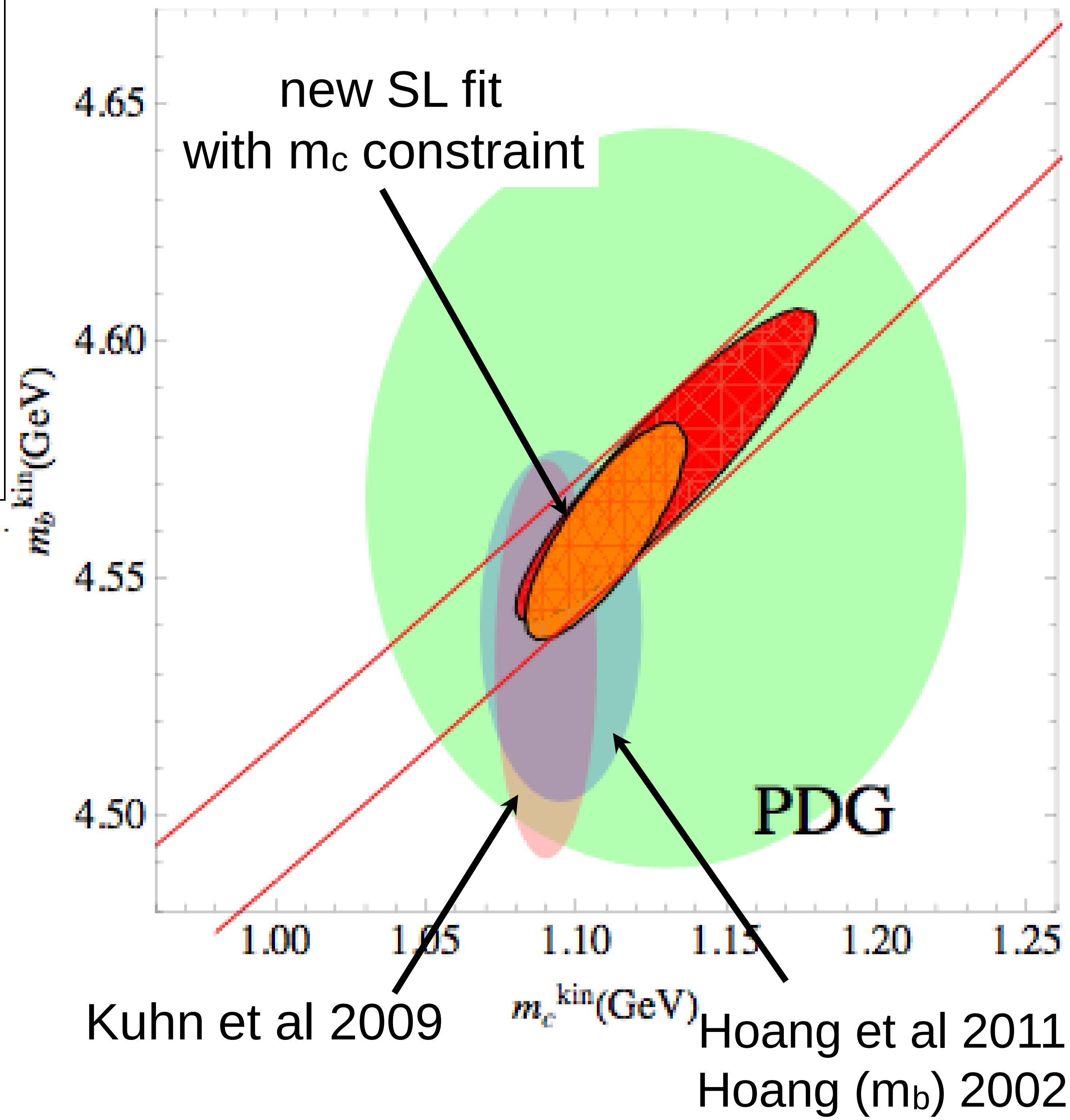


Comparison and combination of $m_{b,c}$ penalized by changes of scheme.

New fit with Hoang et al $m_c(3GeV)=0.998(29)GeV$ leads to

$m_b^{kin}=4.56(2)GeV$ ➨ $m_b(m_b)=4.19(4)GeV$

Recent sum rules determinations converted to kin scheme

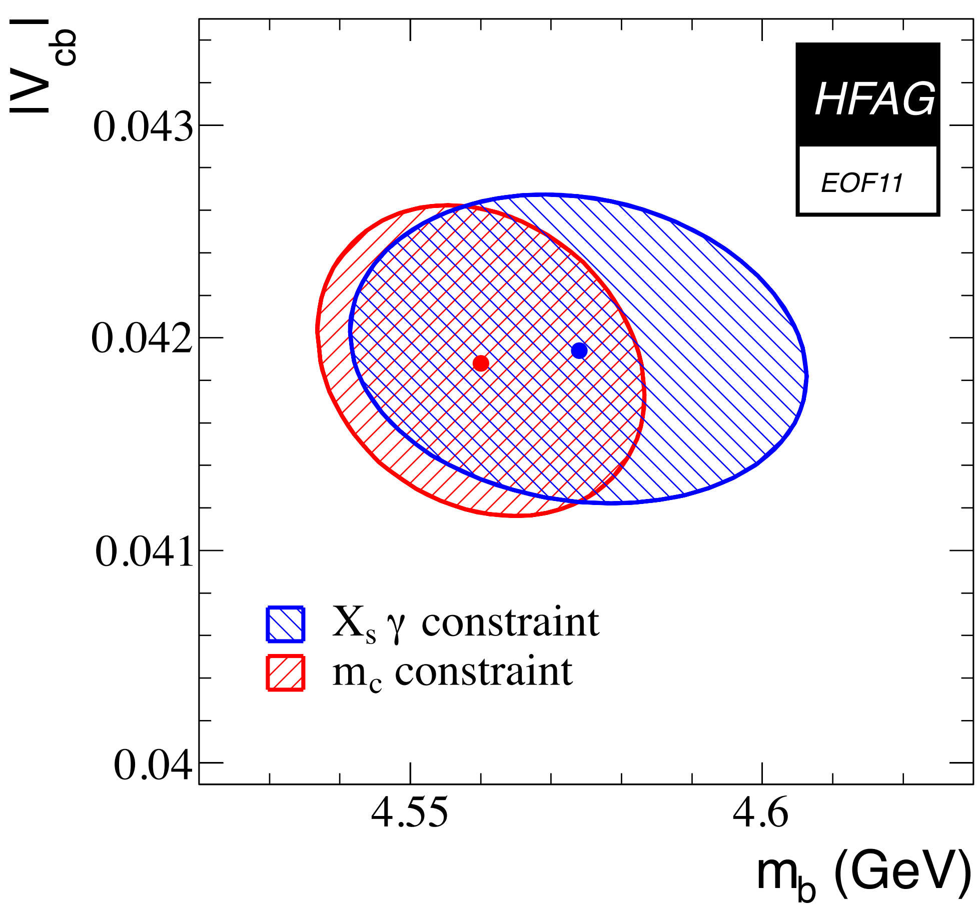


New Global HFAG fit 2012

Inputs	$\lvert V_{cb}\rvert\ 10^3$	m_b^{kin}	χ^2/ndf
$b\to c$ & $b\to s\gamma$	41.94(43)(58)	4.574(32)	29.7/59
$b\to c$ & m_c	41.88(44)(58)	4.560(23)	24.2/48

Based on PG, Uraltsev, Benson et al

These results refer to the kinetic scheme, *where the contributions of gluons with energy below $\mu\approx 1$ GeV are absorbed in the OPE parameters*



A number of different assumptions are also important: which data are included, how theory errors are computed...

Similar NLO result for $|V_{cb}|$ in 1S scheme

Bauer Ligeti Luke Manohar Trott

Open problems

- *Theoretical errors are dominant. Need to understand (not only compute) higher order contributions*
- Perturbative $O(\alpha_s)$ corrections to power suppressed contributions: partially known, the rest is in the pipeline
- Non-perturbative $1/m^4$, $1/m^5$ (Mannel, Turczyk, Uraltsev) seem to mostly shift the OPE parameters, need to be studied.
- Role of theoretical correlations
- *Quark-hadron duality violation*

Exclusive decay $B \to D^* l \nu$

At zero recoil, where rate vanishes, the ff is

$$\mathcal{F}(1) = \eta_A \left[1 + O\left(\frac{1}{m_c^2}\right) + \ldots\right]$$

Recent progress in measurement of slopes and shape parameters, exp error only ~2%

The ff F(1) cannot be experimentally determined or constrained

Unquenched Lattice QCD (only group): F(1) =0.902(17) Laiho et al 2010

$$\boxed{|V_{cb}|=39.6(0.7)(0.6)\times 10^{-3}}$$

~1.9σ from inclusive determination ***2.1% error***

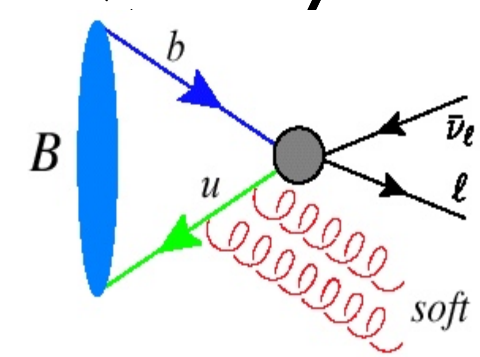
Heavy quark sum rules imply a lower F(1)~0.86, in agreement with inclusive V_{cb} PG, Mannel, Uraltsev

$B \to D l \nu$ has larger errors $|V_{cb}|=39.1(1.4)(1.3)\times 10^{-3}$

The total B→X_ulν width in the OPE

$$
\begin{aligned}
\Gamma[\bar{B} \to X_u e \bar{\nu}] \;=\; & \frac{G_F^2 m_b^5}{192\pi^3}|V_{ub}|^2 \Bigg[1 + \frac{\alpha_s}{\pi} p_u^{(1)}(\mu) + \frac{\alpha_s^2}{\pi^2} p_u^{(2)}(r,\mu) - \frac{\mu_\pi^2}{2m_b^2} - \frac{3\mu_G^2}{2m_b^2} \\
& + \left(\frac{77}{6} + 8\ln\frac{\mu_{\rm WA}^2}{m_b^2}\right)\frac{\rho_D^3}{m_b^3} + \frac{3\rho_{LS}^3}{2m_b^3} + \frac{32\pi^2}{m_b^3} B_{\rm WA}(\mu_{\rm WA})\Bigg] \\
& + \; O\left(\alpha_s \frac{\mu_{\pi,G}^2}{m_b^2}\right) \; + O\left(\frac{1}{m_b^4}\right)
\end{aligned}
$$

Using the results of the fit, life would be relatively easy if we had the total width...



Weak Annihilation, severely constrained from D decays,
see Kamenik, PG, arXiv:1004.0114

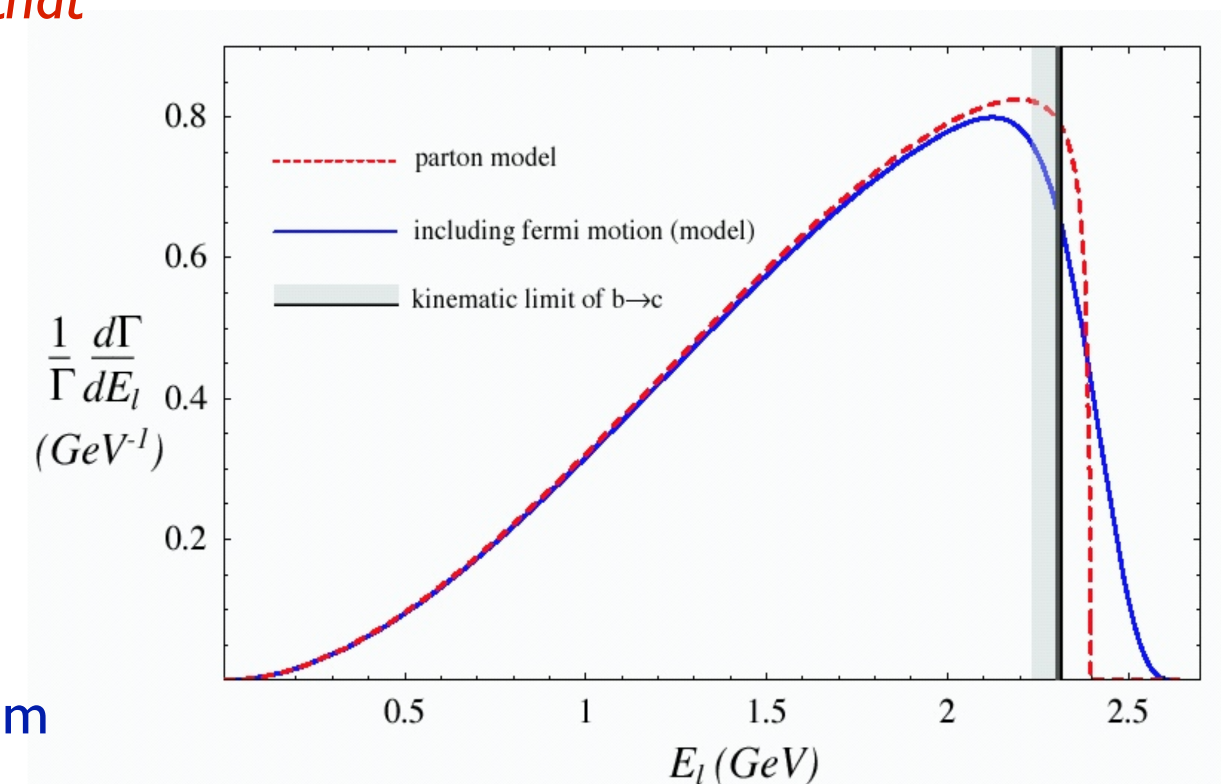
The problems with cuts

Experiments often use kinematic cuts to avoid the ~100x larger $b \to cl\nu$ background:

$m_X < M_D$ $\qquad$ $E_l > (M_B^2 - M_D^2)/2M_B$ $\qquad$ $q^2 > (M_B - M_D)^2$ ...

The cuts destroy convergence of the OPE that works so well in $b \to c$. OPE expected to work only away from pert singularities

Rate becomes sensitive to *local* b-quark wave function properties like Fermi motion. Dominant non-pert contributions can be resummed into a **SHAPE FUNCTION** f(k+). Equivalently the SF is seen to emerge from soft gluon resummation



Luke

How to access the SF?

$$\frac{d^3\Gamma}{dp_+ dp_- dE_\ell} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3} \int dk C(E_\ell, p_+, p_-, k) F(k) + O\left(\frac{\Lambda}{m_b}\right)$$

Subleading SFs

Prediction *based* on resumed pQCD DGE, ADFR	OPE constraints + parameterization without/with resummation GGOU, BLNP

Fit radiative data (and $b \to ul\nu$)
SIMBA

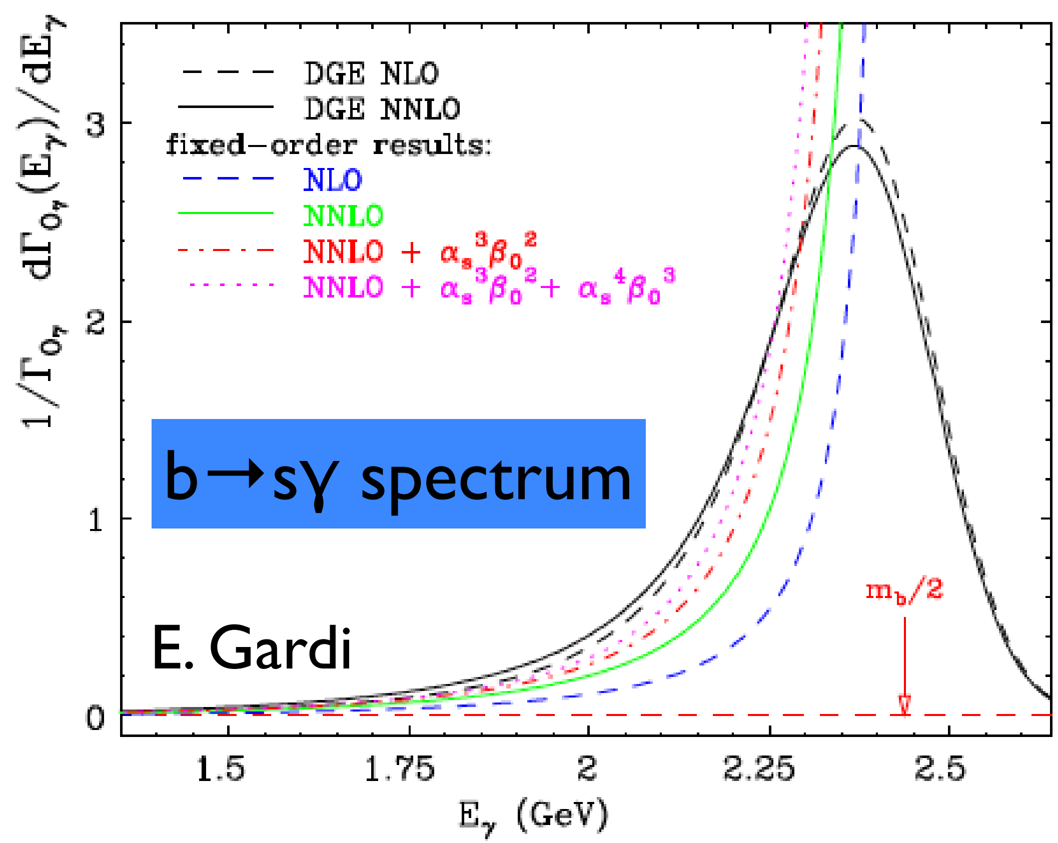
SF from perturbation theory

Resummed perturbation theory is qualitatively different: Support properties; stability! *(E. Gardi)*

b *quark* SF emerges from resummed pQCD but needs an IR prescription and power corrections for b →B

Dressed Gluon Exponentiation (DGE) by Gardi et al employs renormalon resummation to define Fermi motion. Power corrections can be partly accomodated.

Aglietti et al (ADFR) use Analytic Coupling in the IR, a model



The SF in the OPE

Local OPE has also threshold singularities and SF can be equivalently introduced resumming dominant singularities *Bigi et al, Neubert*

Fermi motion can be parameterized within the OPE like PDFs in DIS. At leading order in m_b only a single universal function of one parameter enters (SF).

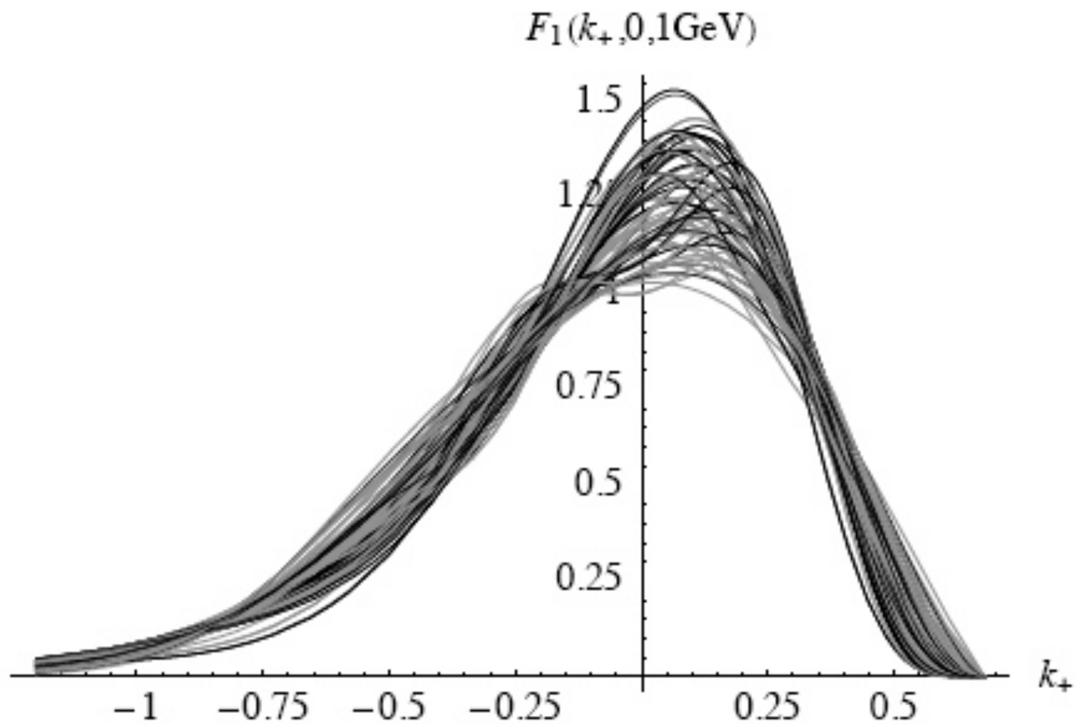
Unlike resumed pQCD, ***the OPE does not predict the SF****, only its first few moments. One then* ***needs an ansatz for its functional form****.*

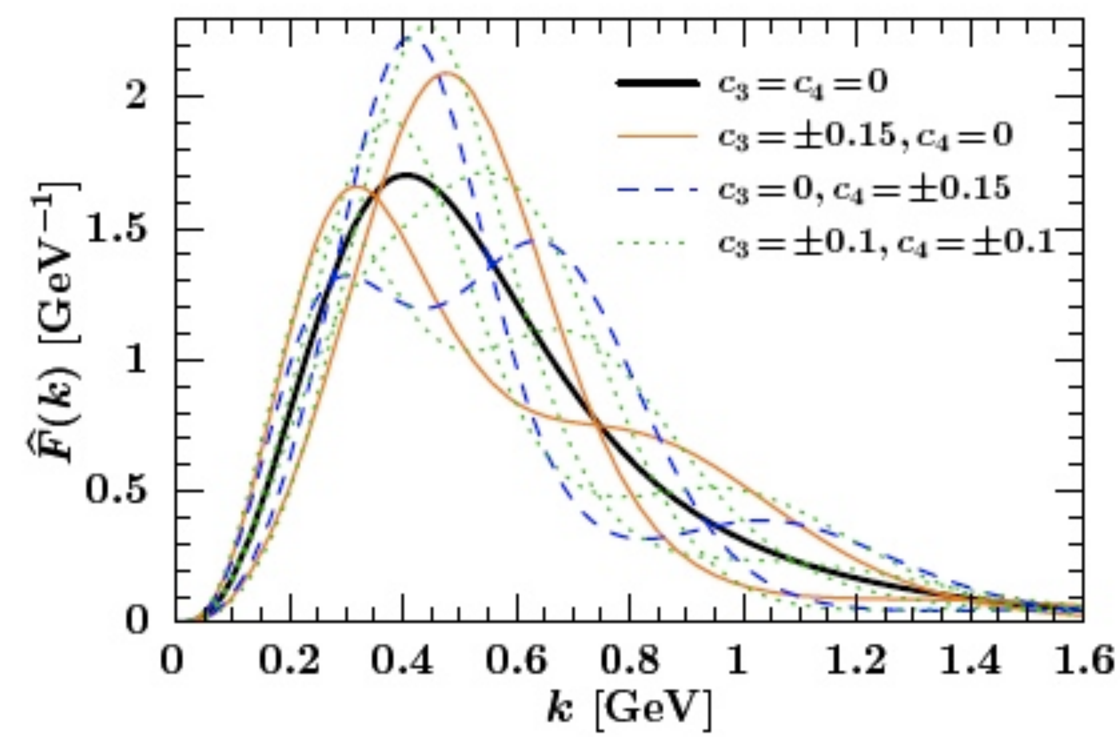
$$\int dk_+ \, k_+^n \, F_i(k_+, q^2) =$$ local OPE prediction $\Leftarrow$ moments fits

Two very different implementations:
PG,Giordano,Ossola,Uraltsev (GGOU)
Bosch,Lampe,Neubert,Paz (BLNP)

Several new subleading SFs appear at $O(\Lambda/m_b)$

Functional forms

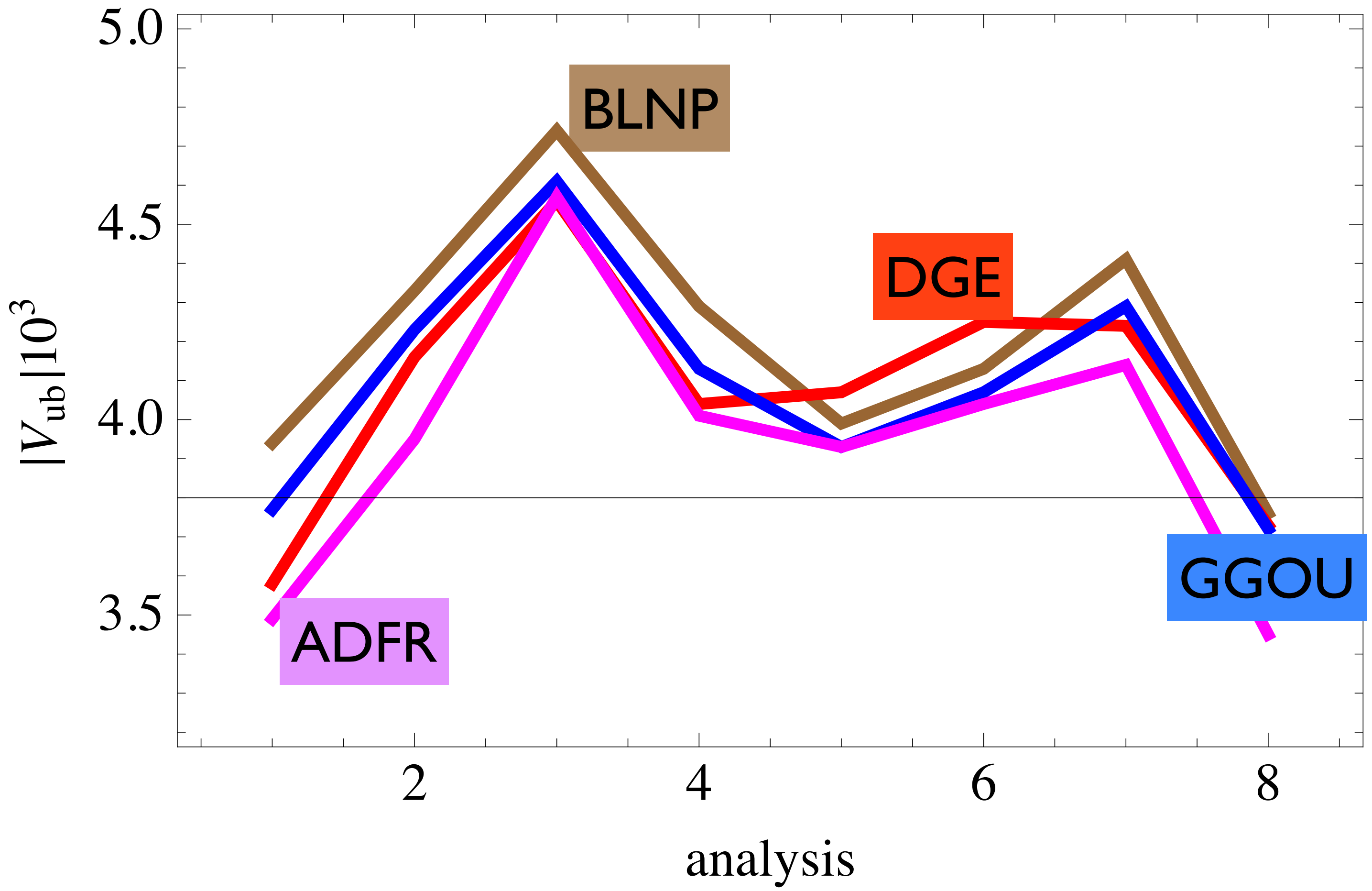




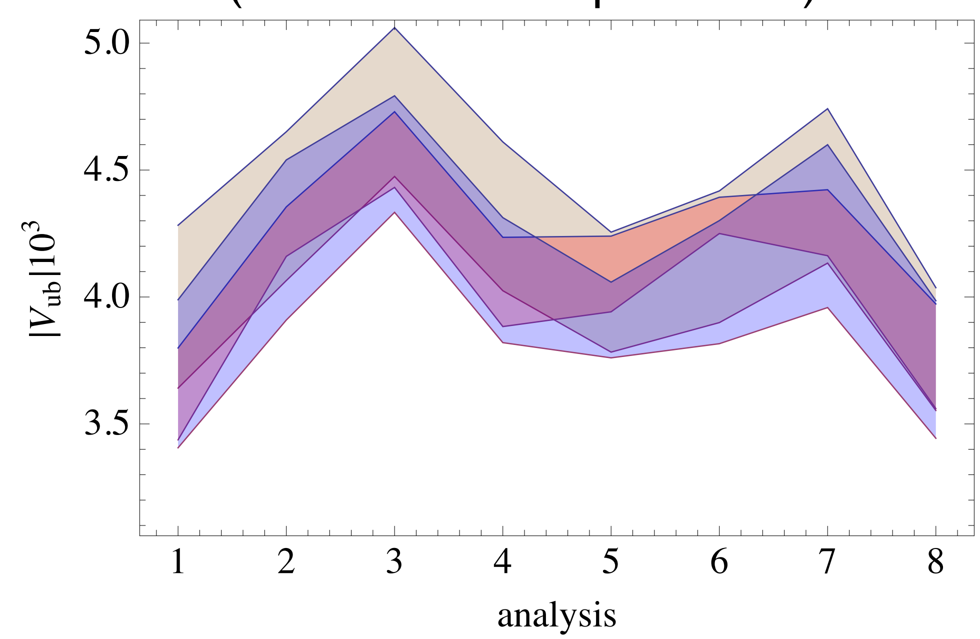
About 100 forms considered in GGOU, large variety, double max discarded. Small uncertainty (1-2%) on V_{ub}

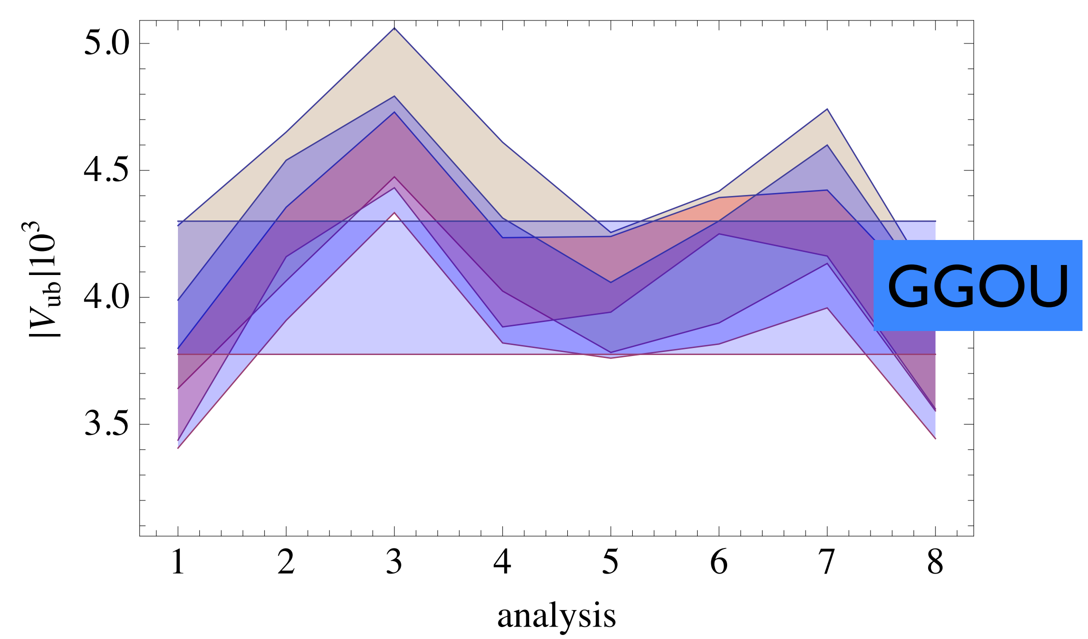
A more systematic method by Ligeti et al. arXiv:0807.1926 Plot shows 9 SFs that satisfy all the first three moments

A global comparison 0907.5386, Phys Rept



only theory errors
(without common parametric)





- ✴ common inputs (except ADFR)
- ✴ Overall good agreement **SPREAD WITHIN THEORY ERRORS**
- ✴ NNLO BLNP still missing: will push it up a bit
- ✴ Systematic offset of central values: normalization? to be investigated

$|V_{ub}|$ in the kinetic scheme - GGOU

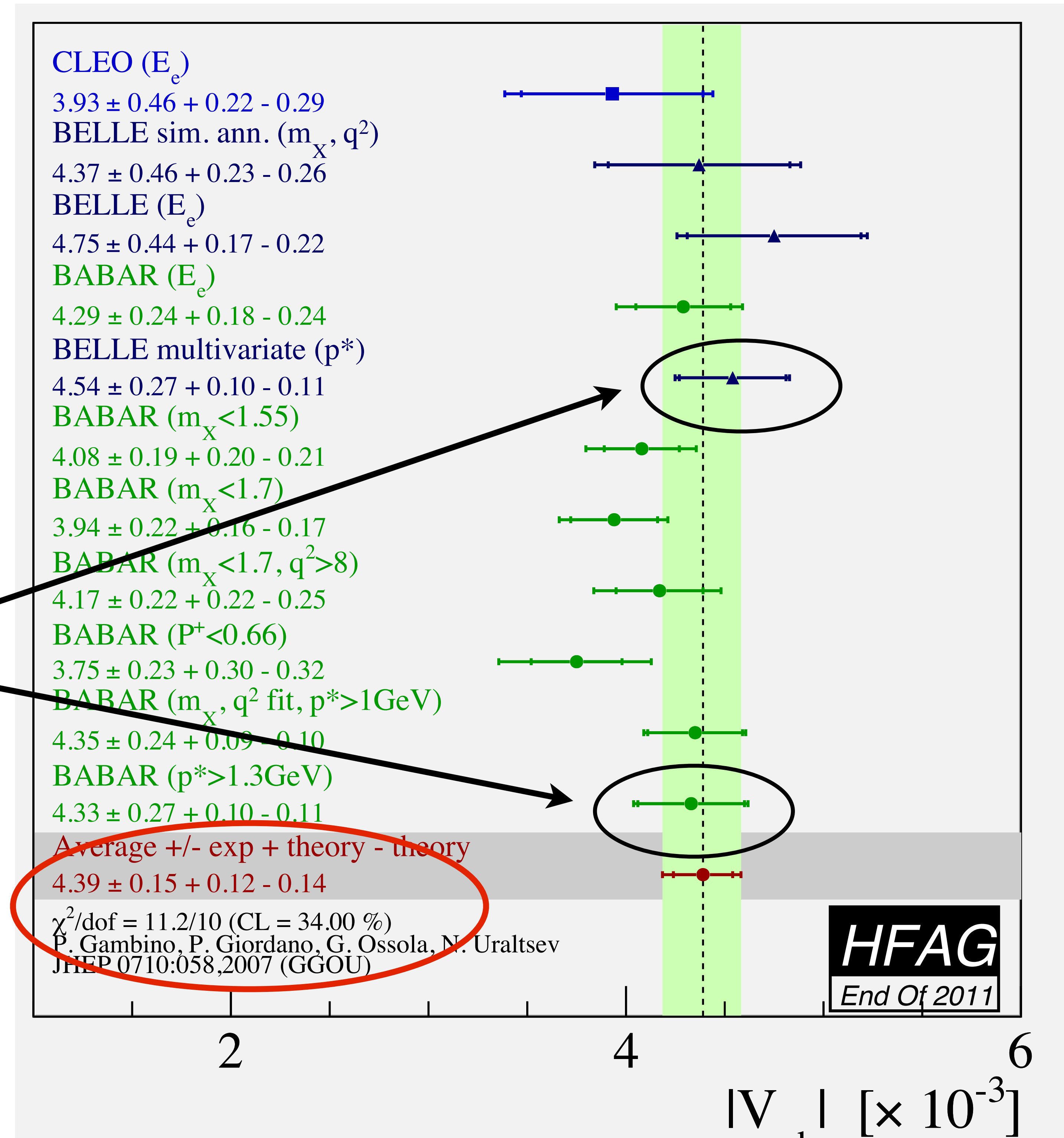
PG,Giordano,Ossola,Uraltsev

Good consistency & small th error.

4.7% total error

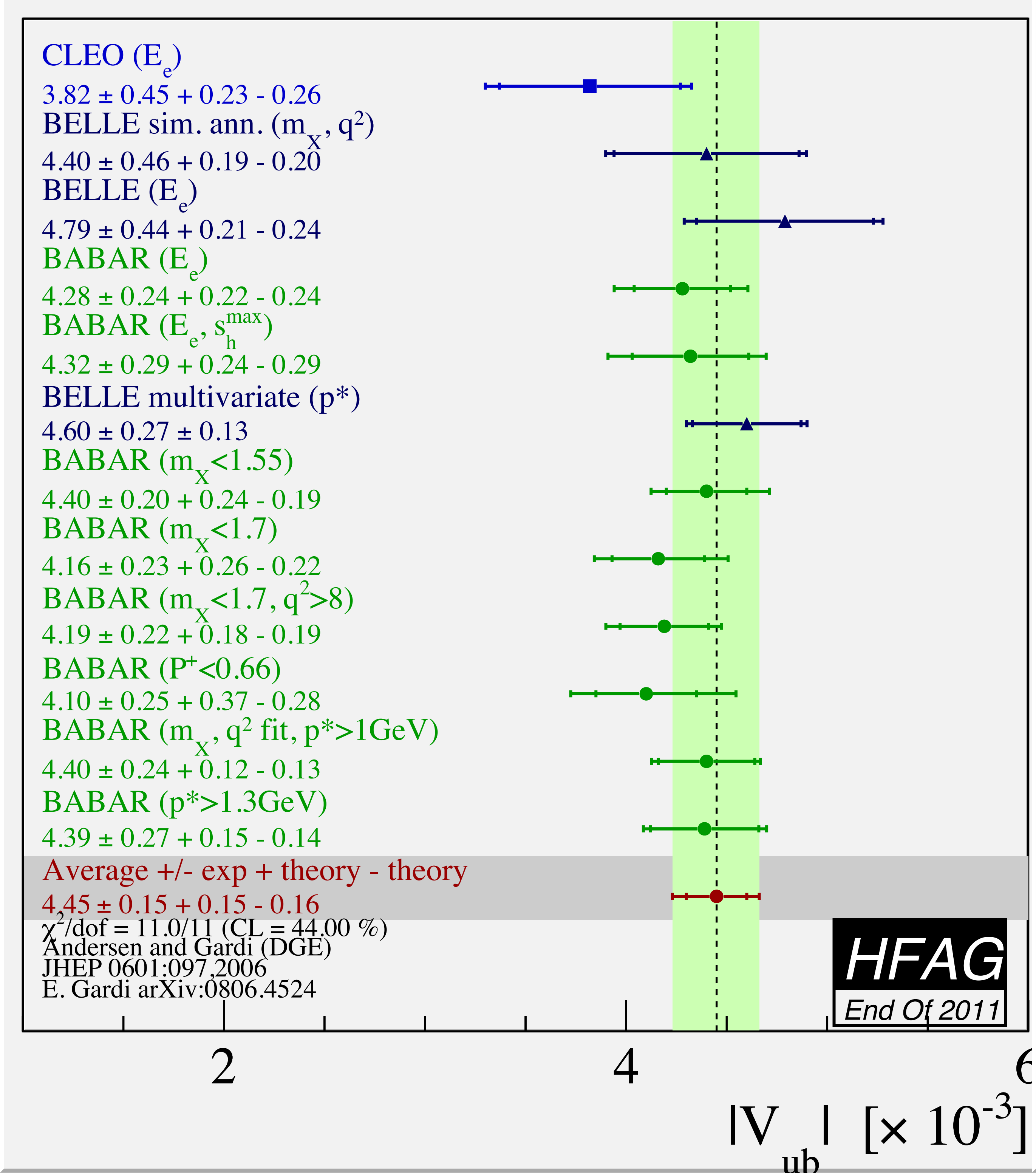
very strong dependence on m_b

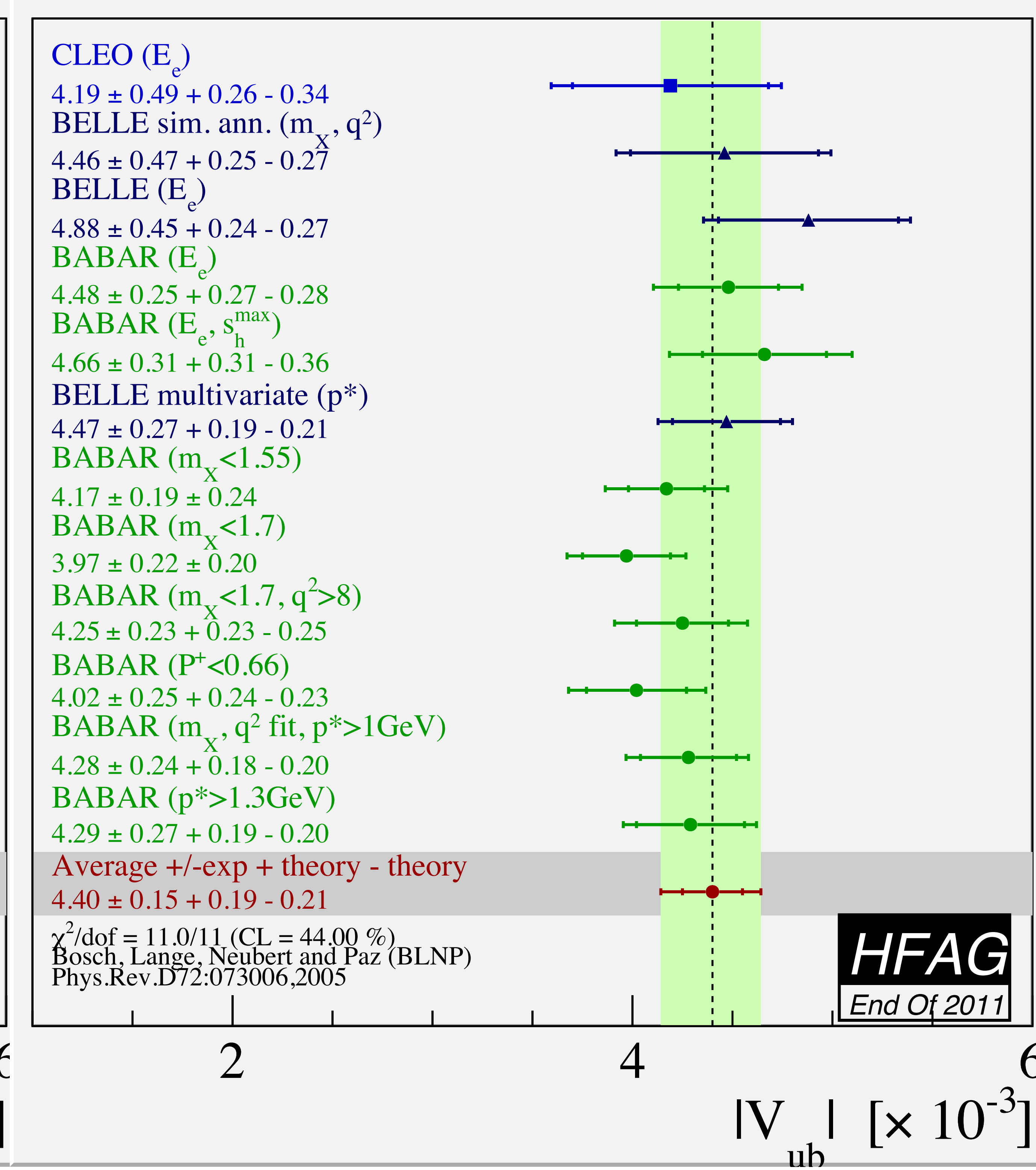
recent *multivariate results* are theoretically cleanest but signal simulation relies on theoretical models



DGE

BLNP


CLEO (E_e)
3.82 ± 0.45 + 0.23 - 0.26
BELLE sim. ann. (m_X, q²)
4.40 ± 0.46 + 0.19 - 0.20
BELLE (E_e)
4.79 ± 0.44 + 0.21 - 0.24
BABAR (E_e)
4.28 ± 0.24 + 0.22 - 0.24
BABAR (E_e, s_h^max)
4.32 ± 0.29 + 0.24 - 0.29
BELLE multivariate (p*)
4.60 ± 0.27 ± 0.13
BABAR (m_X<1.55)
4.40 ± 0.20 + 0.24 - 0.19
BABAR (m_X<1.7)
4.16 ± 0.23 + 0.26 - 0.22
BABAR (m_X<1.7, q²>8)
4.19 ± 0.22 + 0.18 - 0.19
BABAR (P+<0.66)
4.10 ± 0.25 + 0.37 - 0.28
BABAR (m_X, q² fit, p*>1GeV)
4.40 ± 0.24 + 0.12 - 0.13
BABAR (p*>1.3GeV)
4.39 ± 0.27 + 0.15 - 0.14
Average +/- exp + theory - theory
4.45 ± 0.15 + 0.15 - 0.16
χ²/dof = 11.0/11 (CL = 44.00 %)
Andersen and Gardi (DGE)
JHEP 0601:097,2006
E. Gardi arXiv:0806.4524
HFAG
End Of 2011
2
4
6
|V_ub| [× 10^-3]


CLEO (E_e)
4.19 ± 0.49 + 0.26 - 0.34
BELLE sim. ann. (m_X, q²)
4.46 ± 0.47 + 0.25 - 0.27
BELLE (E_e)
4.88 ± 0.45 + 0.24 - 0.27
BABAR (E_e)
4.48 ± 0.25 + 0.27 - 0.28
BABAR (E_e, s_h^max)
4.66 ± 0.31 + 0.31 - 0.36
BELLE multivariate (p*)
4.47 ± 0.27 + 0.19 - 0.21
BABAR (m_X<1.55)
4.17 ± 0.19 ± 0.24
BABAR (m_X<1.7)
3.97 ± 0.22 ± 0.20
BABAR (m_X<1.7, q²>8)
4.25 ± 0.23 + 0.23 - 0.25
BABAR (P+<0.66)
4.02 ± 0.25 + 0.24 - 0.23
BABAR (m_X, q² fit, p*>1GeV)
4.28 ± 0.24 + 0.18 - 0.20
BABAR (p*>1.3GeV)
4.29 ± 0.27 + 0.19 - 0.20
Average +/-exp + theory - theory
4.40 ± 0.15 + 0.19 - 0.21
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Bosch, Lange, Neubert and Paz (BLNP)
Phys.Rev.D72:073006,2005
HFAG
End Of 2011
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|V_ub| [× 10^-3]

Perturbative calculations

$O(\alpha_s)$ implemented by all groups

De Fazio,Neubert

Running coupling $O(\alpha_s^2\beta_0)$ PG,Gardi,Ridolfi

in GGOU, DGE lead to -5% & +2%, resp. in $|V_{ub}|$

- $P_+ < 0.66$ GeV:

	$\Gamma_u^{(0)}$	μ_h	μ_i
NLO	60.37	$^{+3.52}_{-3.37}$	$^{+3.81}_{-6.67}$
NNLO	52.92	$^{+1.46}_{-1.72}$	$^{+0.09}_{-2.79}$

Greub,Neubert,Pecjak arXiv:0909.1609

Not yet included in HFAG averages

complete $O(\alpha_s^2)$ in the SF region (2008)

Asatrian,Greub,Pecjak-Bonciani,Ferroglia-Beneke,Huber, Li - G. Bell

$O(\alpha_s^2)$ in SF region leads to up to 8% increase of V_{ub} in BLNP: <u>most likely an artefact of that approach</u>. $O(\alpha_s^2)$ in the full phase space necessary

- $P_+ < 0.66$ GeV:

Fixed-Order	$\Gamma_u^{(0)}$	μ
NLO	49.11	$^{+5.43}_{-9.41}$
NNLO	49.53	$^{+0.13}_{-4.01}$

Offset appears related to resummation

In summary

HFAG 2012	Average $\|V_{ub}\| \times 10^3$
DGE	$4.45(15)_{ex}{}^{+15}_{-16}$
BLNP	$4.40(15)_{ex}{}^{+19}_{-21}$
GGOU	$4.39(15)_{ex}{}^{+12}_{-14}$

2.7-3σ from $B\to\pi l\nu$ (MILC-FNAL)
2σ from $B\to\pi l\nu$ (LCSR, Siegen)
2.5-3σ from UTFit 2011

- **5-6% total error**

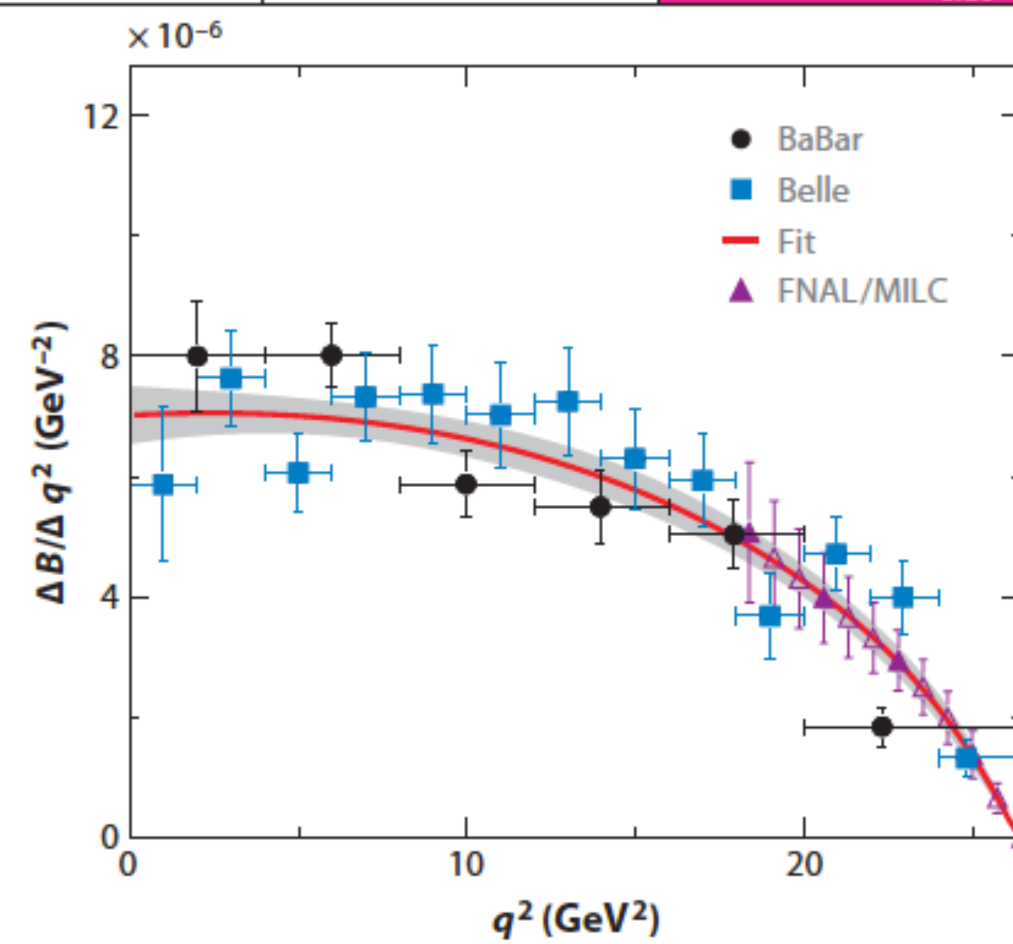
V_{ub} exclusive (B→πlν)

Experimental situation not completely clear at low q^2

	$\mathcal{B}_{tot}$	$\Delta\mathcal{B}(q^2 < 12\,\mathrm{GeV}^2)$	$\Delta\mathcal{B}(q^2 > 16\,\mathrm{GeV}^2)$
Average untagged	$1.44 \pm 0.03 \pm 0.04$	$0.84 \pm 0.02 \pm 0.03$	$0.36 \pm 0.03 \pm 0.02$
Average tagged	$1.32 \pm 0.08 \pm 0.03$	$0.69 \pm 0.05 \pm 0.02$	$0.37 \pm 0.04 \pm 0.01$

Theory	Experiment	q^2 range (GeV^2)	$\Delta\mathcal{B}^a$ (10^{-4})	$\Delta\zeta^b$ (ps^{-1})	$\lvert V_{ub}\rvert^c$ (10^{-3})
LCSR	untagged average	0–12	0.84 ± 0.04	$4.00^{+1.01}_{-0.95}$	$3.69 \pm 0.08^{+0.57}_{-0.37}$
	tagged average	0–12	0.69 ± 0.05	$4.00^{+1.01}_{-0.95}$	$3.37 \pm 0.13^{+0.53}_{-0.36}$
HPQCD	untagged average	16–26.4	0.36 ± 0.04	2.02 ± 0.55	$3.41 \pm 0.10^{+0.59}_{-0.39}$
	tagged average	16–26.4	0.37 ± 0.04	2.02 ± 0.55	$3.47 \pm 0.19^{+0.60}_{-0.39}$

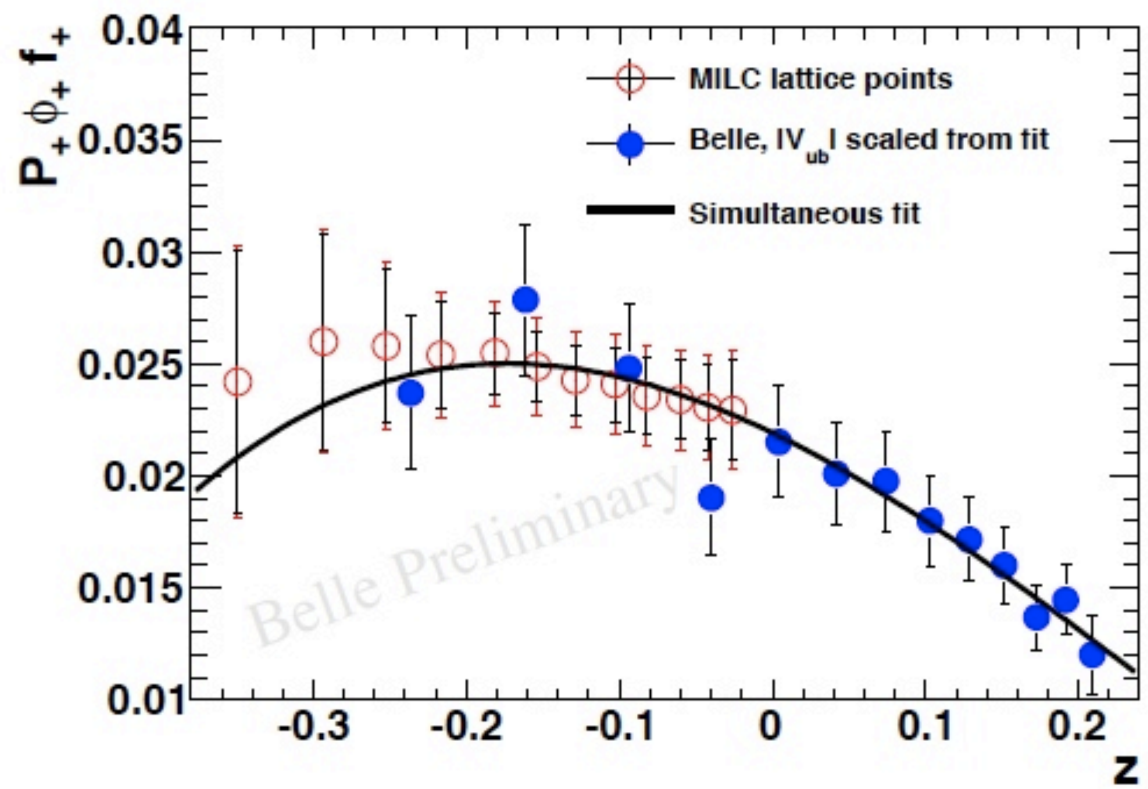
These determinations use the decay rates, but not the information from the q^2 spectrum shape. The situation can be improved by fitting lattice/LCSR together with data



V_{ub} exclusive

Recent method employed in PRD **79** 054507 (2009) (MILC)

- $z = z(q^2)$
- Lattice points:
  $f_+(q^2)$ $q^2>16GeV^2$
- Experiment:
  $|V_{ub}| \times f_+(q^2)$
- Simultaneous fit $\Rightarrow |V_{ub}|$



Belle Result:

$$|V_{ub}| = (3.43 \pm 0.33) \times 10^{-3} \quad (Error\ stat.\ and\ syst.\ combined)$$

Including Babar data with the same lattice points leads to ***<u>3.25(31)x10</u>***

A light-cone sum rule calculation is also possible. Most recent result

$$|V_{ub}| = \left(3.50^{+0.38}_{-0.33}\Big|_{th.} \pm 0.11\Big|_{exp.}\right) \times 10^{-3}$$

Khodjamirian, Mannel, Offen, Wang 2011
see also Ball & Bharucha

Conclusions

- *Semileptonic B decays provide us with a lot of information: V_{cb}, V_{ub}, constraints on $m_{b,c}$ (consistent with sum rules)*
- *New HFAG fit improves m_b determination for V_{ub}*
- *Some tension persists between exclusive and inclusive $|V_{cb}|$*
- *Inclusive V_{ub} ~2-3σ from exclusive one and UT fit*
- *cleanest exp results don't need SF (at least directly)*
- *no sign of inconsistency in the theoretical picture*
- *my favourite M_X cut analyses give $V_{ub} \sim 4.0 \times 10^{-3}$*

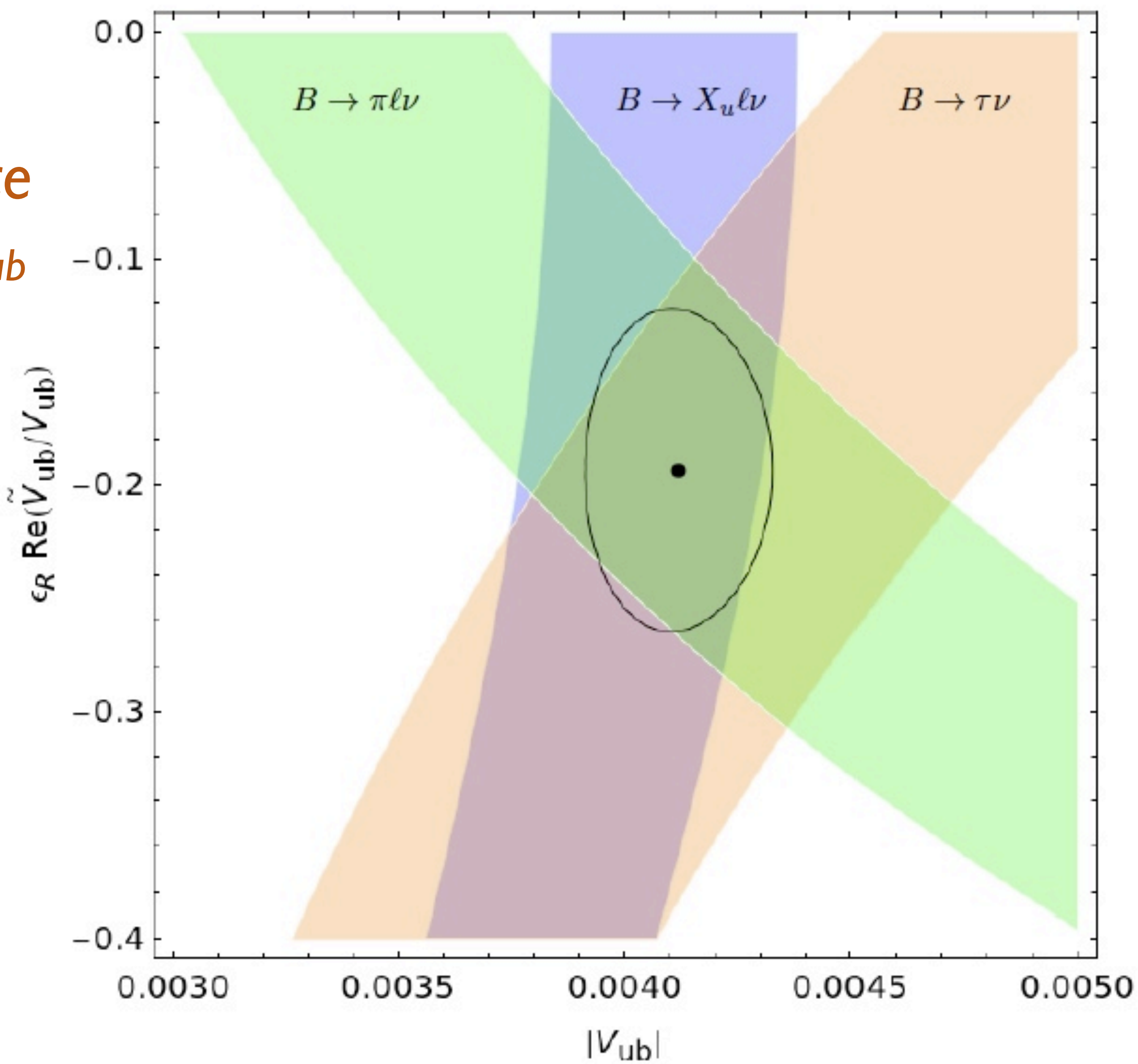
Back-up slides

New physics?

LR models can explain a difference between inclusive and exclusive V_{ub} determinations *Chen,Nam*

Also in MSSM *Crivellin*

BUT the RH currents affect predominantly the exclusive V_{ub}, making the conflict between V_{ub} and sin2β (ψK_S) stronger...



Buras, Gemmler, Isidori 1007.1993

Higher power corrections

Mannel, Turczyk, Uraltsev **1009.4622**

Proliferation of non-pert parameters: for ex at $1/m_b^4$

$$2M_B m_1 = \langle ((\vec{p})^2)^2 \rangle$$
$$2M_B m_2 = g^2 \langle \vec{E}^2 \rangle$$
$$2M_B m_3 = g^2 \langle \vec{B}^2 \rangle$$
$$2M_B m_4 = g \langle \vec{p} \cdot \mathrm{rot}\, \vec{B} \rangle$$

$$2M_B m_5 = g^2 \langle \vec{S} \cdot (\vec{E} \times \vec{E}) \rangle$$
$$2M_B m_6 = g^2 \langle \vec{S} \cdot (\vec{B} \times \vec{B}) \rangle$$
$$2M_B m_7 = g \langle (\vec{S} \cdot \vec{p})(\vec{p} \cdot \vec{B}) \rangle$$
$$2M_B m_8 = g \langle (\vec{S} \cdot \vec{B})(\vec{p})^2 \rangle$$
$$2M_B m_9 = g \langle \Delta(\vec{\sigma} \cdot \vec{B}) \rangle$$

can be estimated by Ground State Saturation

$$\langle \Omega_0 | \bar{Q}\, iD_j iD_k iD_l iD_m\, Q | \Omega_0 \rangle = \langle \Omega_0 | \bar{Q} iD_j iD_k Q | \Omega_0 \rangle\, \langle \Omega_0 | \bar{Q}\, iD_l iD_m Q | \Omega_0 \rangle$$

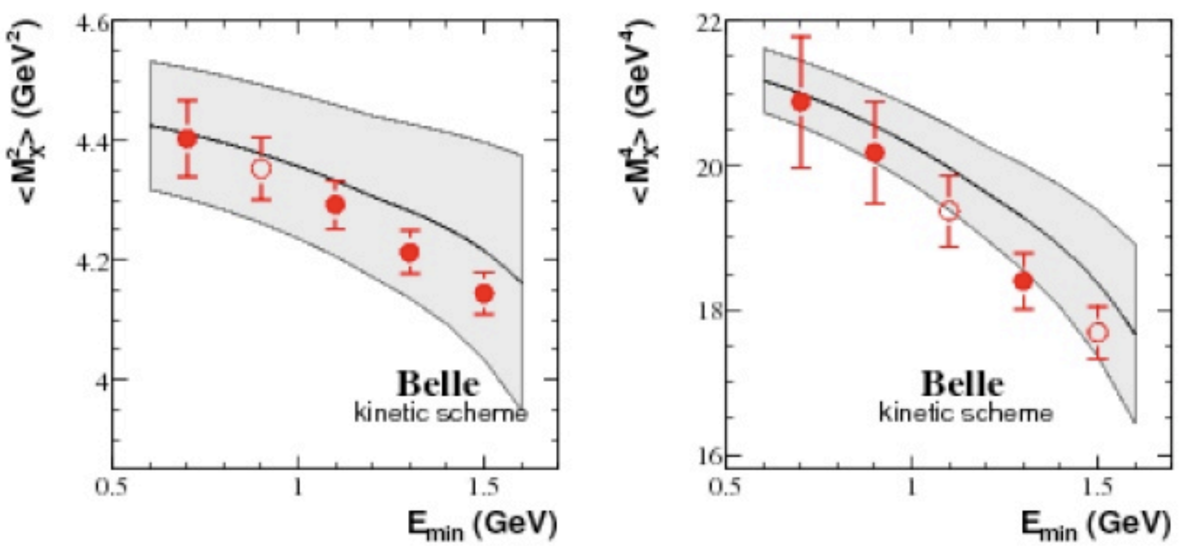
$$\frac{\delta\Gamma_{1/m^4} + \delta\Gamma_{1/m^5}}{\Gamma} \approx 0.013 \qquad \frac{\delta V_{cb}}{V_{cb}} \approx +0.4\%$$

after inclusion of the corrections in the moments. While this might set the scale of effect, not yet clear *how much it depends on assumptions on expectation values.*

How reliable are mass determinations?

Collaboration with C. Schwanda, in progress

Theoretical correlations

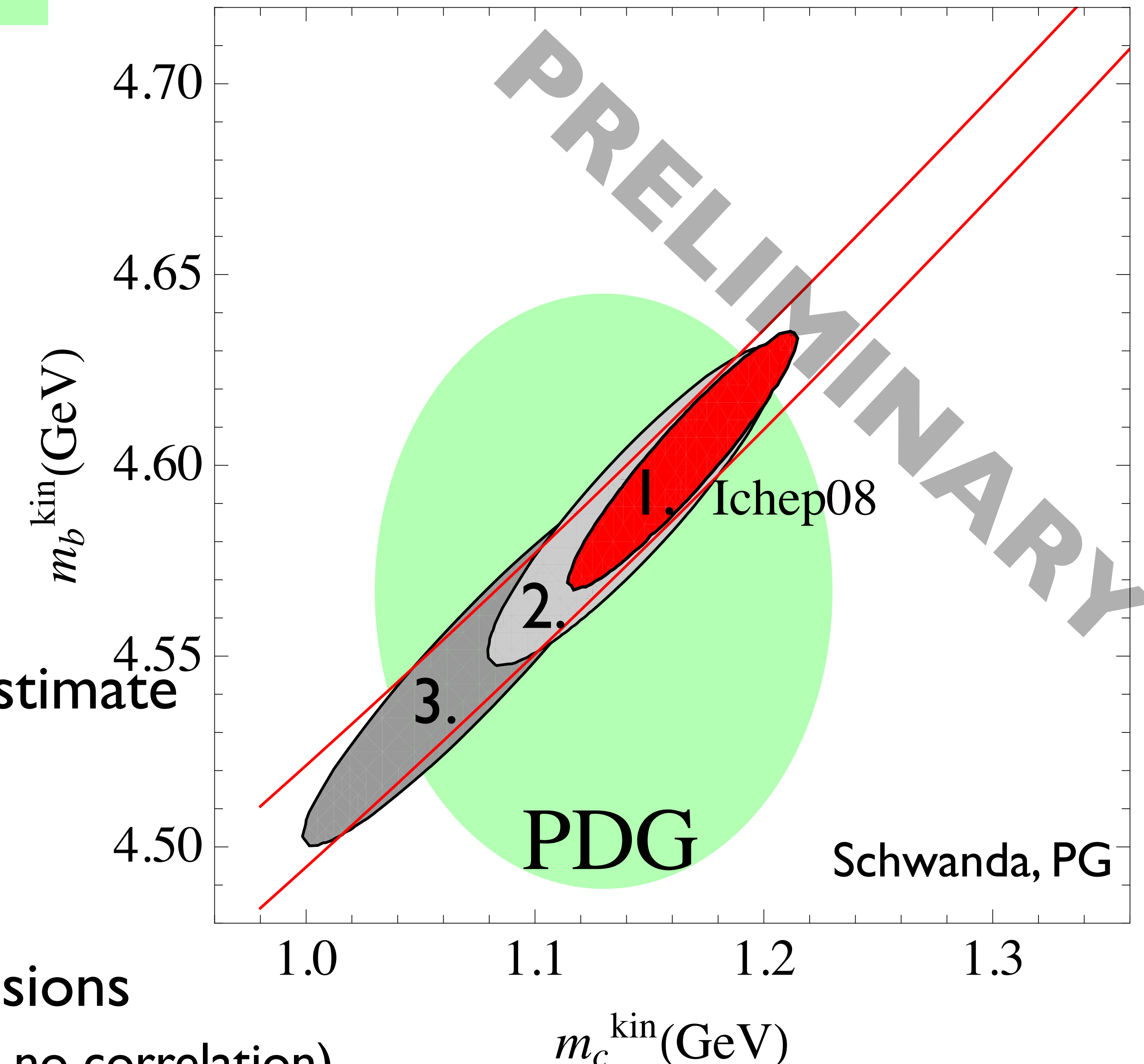


Correlations between theory errors of moments with different cuts difficult to estimate

Examples:

1. 100% correlations
2. corr. computed from low-order expressions
3. experimental correlations (very similar to no correlation)

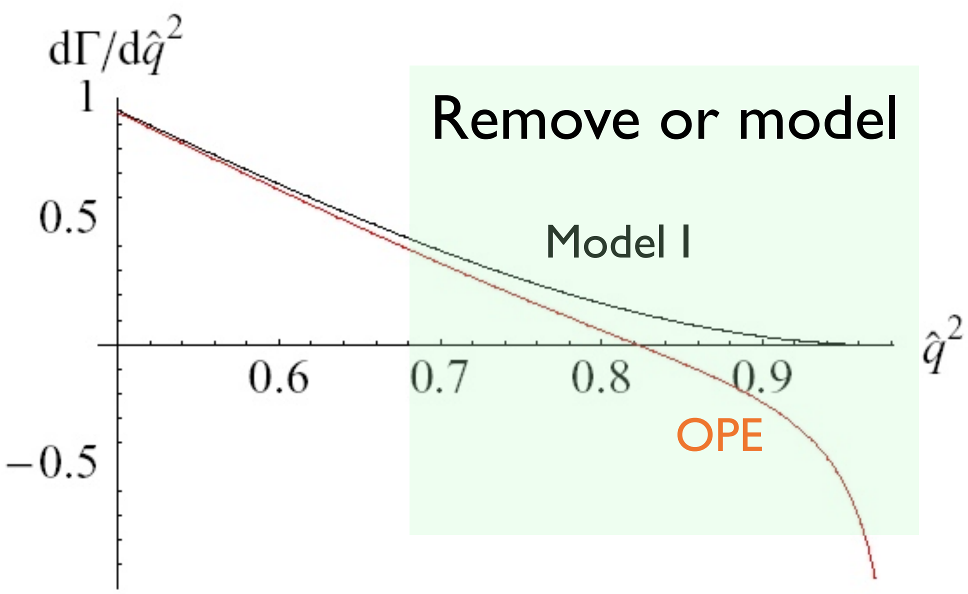
always assume different central moments uncorrelated



The high q^2 tail

At high q^2 higher dimensional operators are not suppressed leading to pathological features. Origin in the non-analytic square root

$$\frac{d\Gamma}{dq_0\, dq^2} \propto \sqrt{q_0^2 - q^2} \quad \Rightarrow \quad \frac{d\Gamma}{dq^2} \sim -\sum_{n=1}^{\infty} \frac{(-1)^n\, b_n(\hat{q}^2)}{(1-\hat{q}^2)^{n-2}} \left(\frac{\bar{\Lambda}}{m_b}\right)^n$$



In the integrated rate the $1/m_b^3$ singularity is removed by the WA operator: needs modelling for q^2 spectrum

$$\delta\Gamma \sim \left[C_{\rm WA}\, B_{\rm WA}(\mu_{\rm WA}) - \left(8 \ln \frac{m_b^2}{\mu_{\rm WA}^2} - \frac{77}{6}\right) \frac{\rho_D^3}{m_b^3} + \mathcal{O}(\alpha_s) \right]$$

WA matrix element B_{WA} parameterizes global properties of the tail, affects V_{ub} depending on cuts, <u>tends to decrease V_{ub}</u>, may pollute all present determinations

Heavy Quark Sum Rule for B→D*lν

The local OPE for inclusive B decays provides a (unitarity) bound on F(1):

$$\underbrace{F_{D^*}^2 + \sum_{f\neq D^*} |F_{B\to f}|^2}_{\text{inelastic }>0} = \xi_A^{\text{pert}} - \frac{\mu_G^2}{3m_c^2} - \underbrace{\frac{\mu_\pi^2 - \mu_G^2}{4}}_{>0}\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_c m_b}\right) - \Delta_{\frac{1}{m_Q^3}} + \Delta_{\frac{1}{m_Q^4}} + \ldots$$

A strict bound follows for zero inelastic contributions.

$$\sqrt{\xi_A^{pert}} = 0.98 \pm .01 \qquad \Delta_{\text{power}} = 0.09 + 0.03 - 0.02 \approx 0.10$$

$$\boxed{F(1) < 0.93}$$

Uraltsev, Mannel, PG arXiv:1004.2859

Also the inelastic piece can be estimated, although with large uncertainty. It typically leads to **F(1)≈0.86**, in agreement with V_{cb} inclusive.

The SF in GGOU

Leading SF resums leading twist effects, $m_b \to \infty$
universal, q^2 indep

Finite m_b distribution functions
include all $1/m_b$ effects, *non-universal*
no need for subleading SFs

$$F(k_+) \longrightarrow F_i(k_+, q^2, \mu)$$

Structure function
(i = 1,2,3)

q^2 dependence

cutoff dependence
(gluons with $E_g<\mu$)

$$\frac{d^3\Gamma}{dq^2\,dq_0\,dE_\ell} = \frac{G_F^2|V_{ub}|^2}{8\pi^3}\left\{q^2W_1 - \left[2E_\ell^2 - 2q_0E_\ell + \frac{q^2}{2}\right]W_2 + q^2(2E_\ell - q_0)W_3\right\}$$

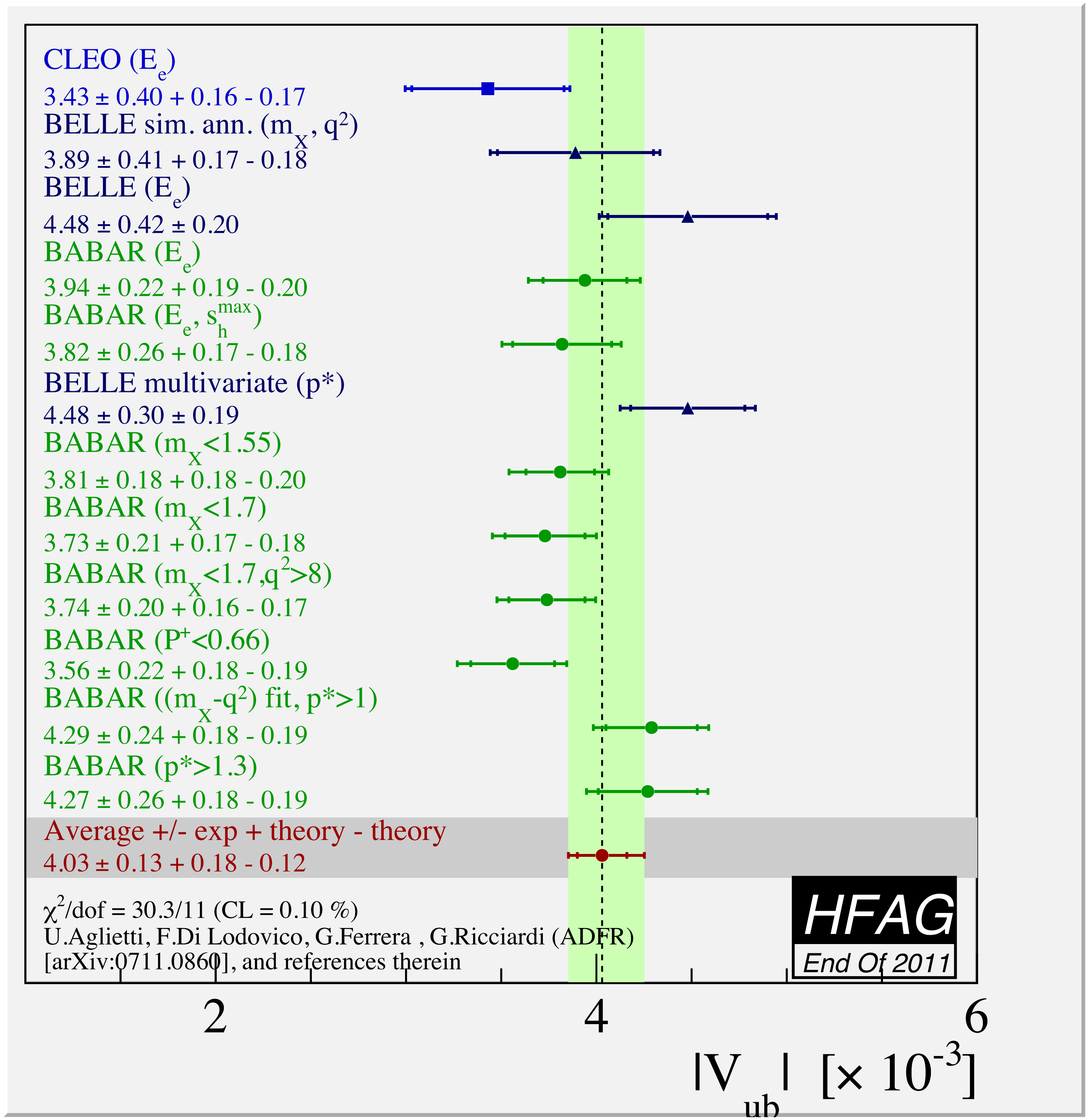
$$W_i(q_0, q^2) = m_b^{n_i}(\mu)\int dk_+\; F_i(k_+, q^2, \mu)\; W_i^{pert}\left[q_0 - \frac{k_+}{2}\left(1 - \frac{q^2}{m_b M_B}\right), q^2, \mu\right]$$

This factorization formula perturbatively defines the distribution functions

see also Benson, Bigi, Uraltsev for bsγ

$$\int dk_+\; k_+^n\; F_i(k_+, q^2) = \quad \text{local OPE}$$

Importance of subleading effects


CLEO (E_e)
3.43 ± 0.40 + 0.16 - 0.17
BELLE sim. ann. (m_X, q²)
3.89 ± 0.41 + 0.17 - 0.18
BELLE (E_e)
4.48 ± 0.42 ± 0.20
BABAR (E_e)
3.94 ± 0.22 + 0.19 - 0.20
BABAR (E_e, s_h^max)
3.82 ± 0.26 + 0.17 - 0.18
BELLE multivariate (p*)
4.48 ± 0.30 ± 0.19
BABAR (m_X<1.55)
3.81 ± 0.18 + 0.18 - 0.20
BABAR (m_X<1.7)
3.73 ± 0.21 + 0.17 - 0.18
BABAR (m_X<1.7,q²>8)
3.74 ± 0.20 + 0.16 - 0.17
BABAR (P⁺<0.66)
3.56 ± 0.22 + 0.18 - 0.19
BABAR ((m_X-q²) fit, p*>1)
4.29 ± 0.24 + 0.18 - 0.19
BABAR (p*>1.3)
4.27 ± 0.26 + 0.18 - 0.19
Average +/- exp + theory - theory
4.03 ± 0.13 + 0.18 - 0.12
χ²/dof = 30.3/11 (CL = 0.10 %)
U.Aglietti, F.Di Lodovico, G.Ferrera , G.Ricciardi (ADFR)
[arXiv:0711.0860], and references therein
HFAG
End Of 2011
2
4
6
|V_ub| [× 10⁻³]