Axial U(1) symmetry in the chiral symmetric phase of 2-flavor QCD at finite temperature

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1. Introduction

Chiral symmetry of QCD

phase transition

low T $U(1)_B \otimes SU(N_f)_V$ high T $U(1)_B \otimes SU(N_f)_L \otimes SU(N_f)_R$

restoration of chiral symmetry

Some questions

1. Recovery of U(1)_A symmetry at high T ?

relation ?

2. Eigenvalue distribution of Dirac operator $\rho(\lambda)$ λ: eigenvalue of Dirac operator

Eigenvalue density

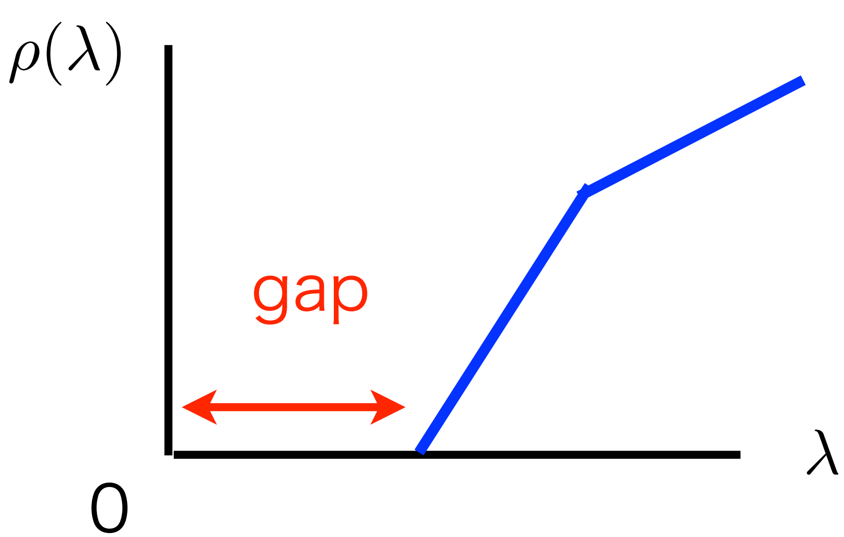
$$\rho(\lambda) = \sum_n \rho_n \frac{\lambda^n}{n!}$$

$$\lim_{m\to 0} \langle \bar{\psi}\psi \rangle = \pi\rho(0)$$

Banks-Casher relation

$\Rightarrow$ $\rho(0) = \rho_0 = 0$ if chiral symmetry is restored.

If $\rho(\lambda)$ has a gap $\Rightarrow$ Anomalous $U(1)_A$ symmetry is fully restored.



Susceptibility

color　　　　　　　　　　flavor

$$M_\Gamma^A(x) = \bar{\psi}^a(x)_\alpha^f (\Gamma \otimes T^A)_{\alpha\beta}^{fg} \psi^a(x)_\beta^g$$ Meson operator

Dirac

$$\begin{array}{ccc} \sigma \text{ meson: } \mathbf{1}\otimes\mathbf{1} & \xleftrightarrow{\text{chiral } SU(2)} & \pi \text{ meson: } \gamma_5 \otimes \tau^a \\ \updownarrow U(1)_A & & \updownarrow U(1)_A \\ \eta \text{ meson: } \gamma_5\otimes\mathbf{1} & \xleftrightarrow{\text{chiral } SU(2)} & \delta \text{ meson: } \mathbf{1}\otimes\tau^a \end{array}$$

$U(1)_A$ susceptibilities

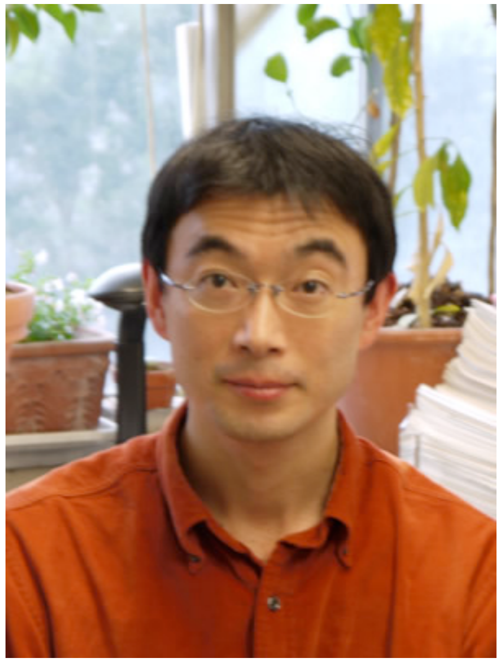
$$\chi_\Gamma^A = \frac{1}{V}\int d^4x \langle M_\Gamma^A(x) M_\Gamma^A(0)\rangle$$

$$\chi^{\sigma-\eta} \equiv \chi^\sigma - \chi^\eta \qquad \chi^{\pi-\delta} \equiv \chi^\pi - \chi^\delta \qquad \chi^{\pi-\eta} \equiv \chi^\pi - \chi^\eta$$

If $U(1)_A$ is recovered, $\chi^{\sigma-\eta} = \chi^{\pi-\delta} = \chi^{\pi-\eta} = 0.$

2. Previous Theoretical Investigation

S.A, H. Fukaya, Y. Taniguchi,
"Chiral symmetry restoration, eigenvalue density of Dirac operator and axial U(1) anomaly at finite temperature",
Phys. Rev D86(2012)114512.

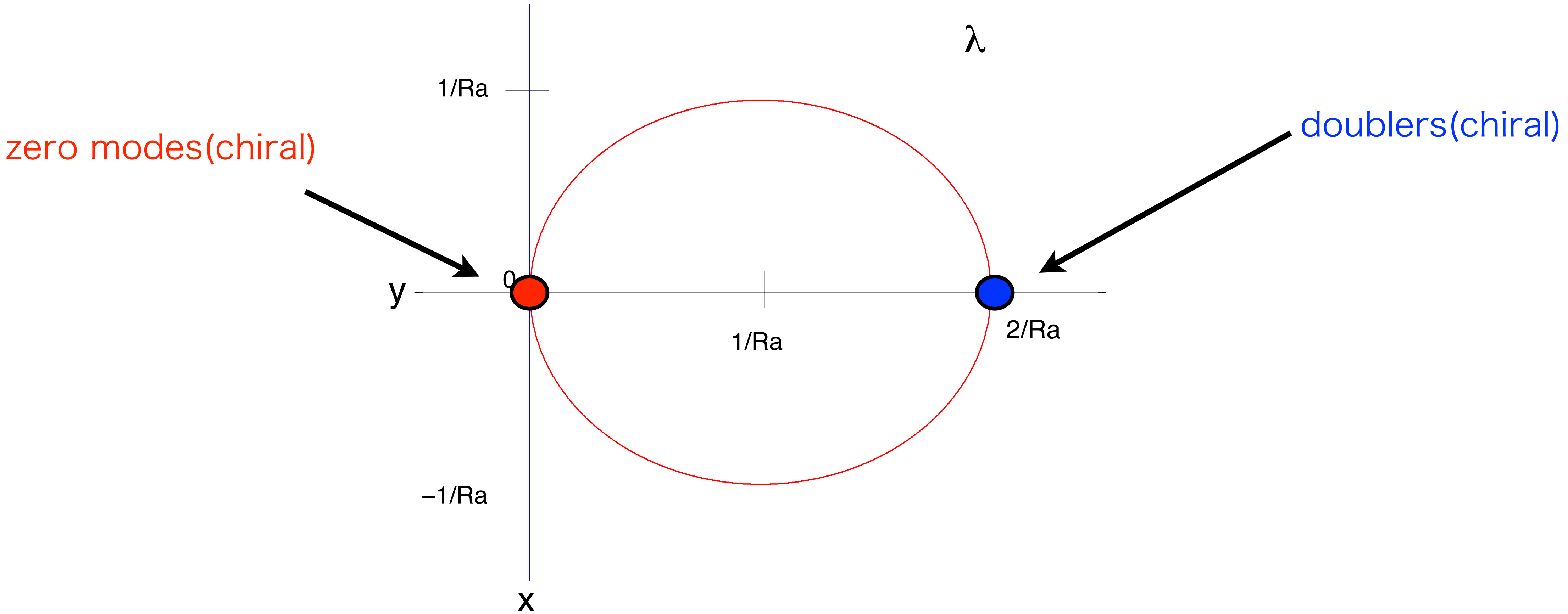


Set up

Lattice regularization with Overlap fermion, 2-flavor

Ginsparg-Wilson relation $D\gamma_5 + \gamma_5 D = aDR\gamma_5 D$ Exact “chiral” symmetry

$\boxed{\text{Eigenvalue spectrum}}$ $\lambda_n^A + \bar{\lambda}_n^A = aR\bar{\lambda}_n^A\lambda_n^A$ A: gauge configuration



Some assumptions

Assumption 1 non-singlet chiral symmetry is restored.

Assumption 2 if $\mathcal{O}(A)$ is m-independent A: gauge configuration

$$\langle \mathcal{O}(A) \rangle_m = f(m^2)$$

$f(x)$ is analytic at $x = 0$

Note that this does not hold if the chiral symmetry is spontaneously broken.

Ex.

$$\lim_{V \to \infty} \frac{1}{V} \langle Q(A)^2 \rangle_m = m \frac{\Sigma}{N_f} + O(m^2)$$

topological charge

Results

Non-singlet chiral Ward-Takahashi identities

$$\rho^A(\lambda) \equiv \lim_{V\to\infty} \frac{1}{V} \sum_n \delta\left(\lambda - \sqrt{\bar{\lambda}_n^A \lambda_n^A}\right) = \sum_{n=0}^{\infty} \rho_n^A \frac{\lambda^n}{n!}$$

eigenvalues density

$$\boxed{\lim_{m\to 0} \langle \rho^A(\lambda) \rangle_m = \lim_{m\to 0} \langle \rho_3^A \rangle_m \frac{|\lambda|^3}{3!} + O(\lambda^4)}$$

No constraints to higher $\langle \rho_n^A \rangle_m$

$\langle \rho_3^A \rangle_m \neq 0$ even for "free" theory.

$$\langle \rho_0^A \rangle_m = 0$$

$$\lim_{V\to\infty} \frac{1}{V^k} \langle (N_{R+L}^A)^k \rangle_m = 0, \quad \lim_{V\to\infty} \frac{1}{V^k} \langle Q(A)^{2k} \rangle_m = 0$$

total number of zero modes $\quad N_{R+L}^A = N_R^A + N_L^A$

topological charge $\quad Q(A) = N_R^A - N_L^A$

N_R^A a number of right-handed zero modes

N_L^A a number of left-handed zero modes

Consequences

Singlet susceptibility at high T

$$\lim_{V\to 0} \chi^{\pi-\eta} = \lim_{m\to 0}\lim_{V\to\infty} \frac{N_f^2}{m^2 V}\langle Q(A)^2\rangle_m = 0$$

This, however, does not mean U(1)_A symmetry is recovered at high T.

$$\lim_{m\to 0} \chi^{\pi-\eta} = 0 \quad \Longrightarrow \quad ”m_\pi = m_\eta”$$

is necessary but NOT “sufficient” for the recovery of U(1)_A .

Effective symmetry at hight T

full $U(1)_A$ is not recovered.

$SU(2)_L\otimes SU(2)_R\otimes Z_4$ not $SU(2)_L\otimes SU(2)_R\otimes U(1)_A$

What is the order of chiral phase transition in 2-flavor QCD ?
1st or 2nd ?

Order of phase transition at N_f=2

U(1)$_A$ is still broken at T > T$_c$

$SU(2)_L \otimes SU(2)_R$

2nd order

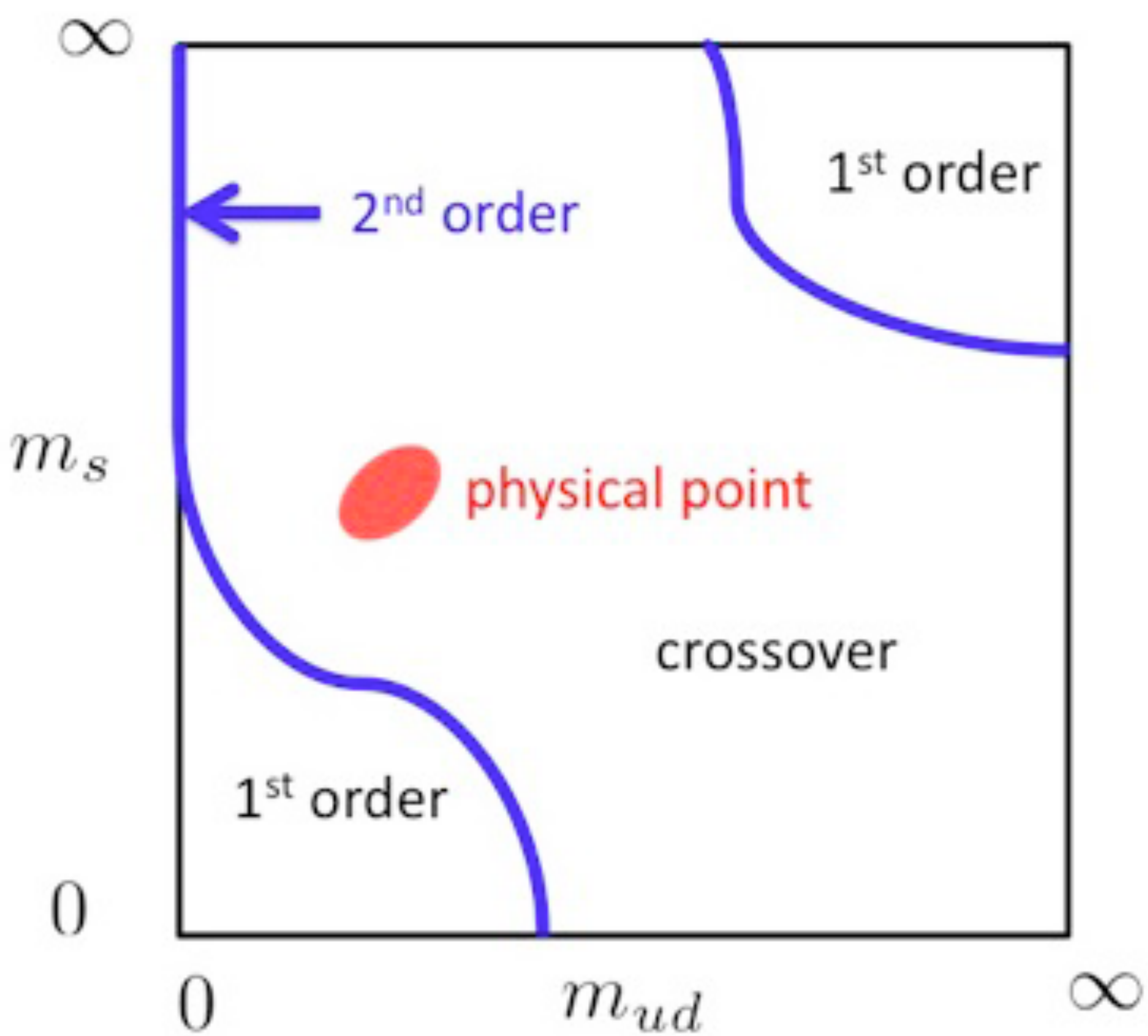
$SU(2)_L \otimes SU(2)_R \otimes Z_4$

U(1)$_A$ is restored at T > T$_c$

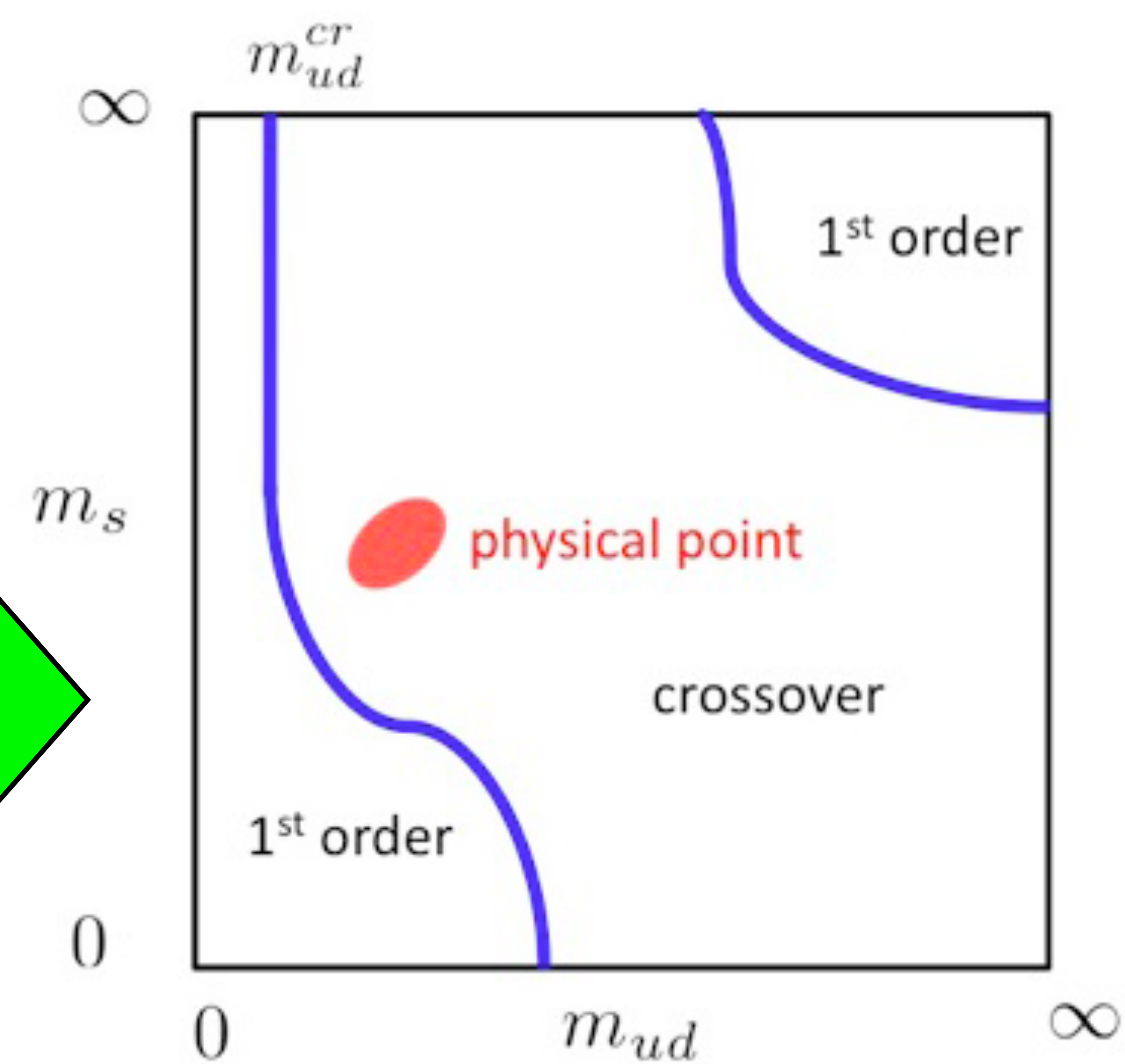
$SU(2)_L \otimes SU(2)_R \otimes U(1)$

1st order ?

phase diagram of 2+1 flavor QCD



?



Remarks

Important conditions

Large volume limit $V \to \infty$

chiral limit $m \to 0$

lattice chiral symmetry Ginsparg-Wilson relation $D\gamma_5 + \gamma_5 D = aDR\gamma_5 D$

Fractional power for the eigenvalue density

$\rho^A(\lambda) \simeq c_A \lambda^\gamma, \ \gamma > 0$ non-singlet chiral symmetry is recovered.

⇒ $\gamma \leq 2$ is excluded. ⇒ $\gamma > 2$

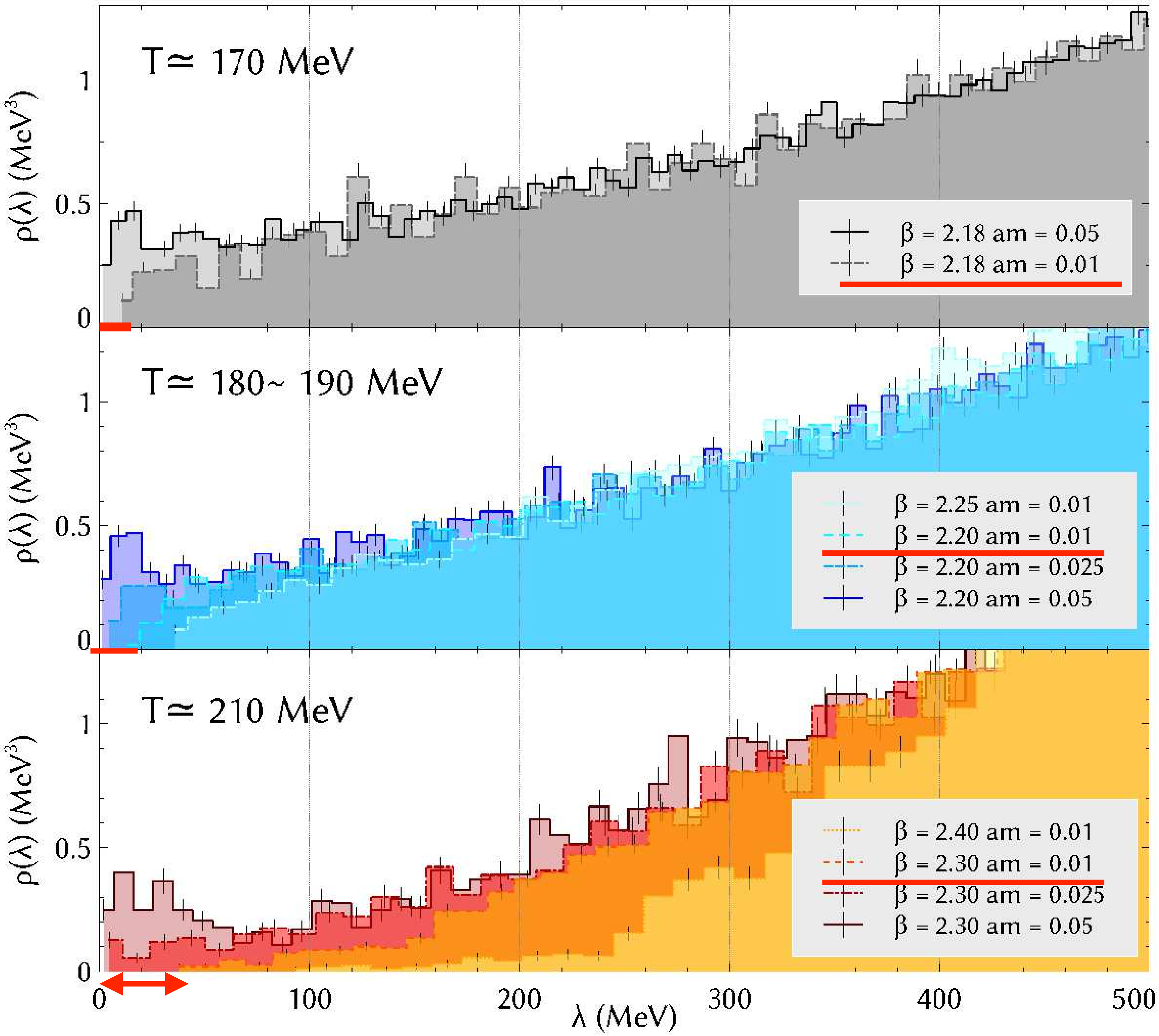
consistent with the integer case (n > 2)

3. Recent Numerical Results

Eigenvalue densities

$$\rho(\lambda) = \lim_{V\to\infty} \frac{1}{V} \sum_n \delta(\lambda - \lambda_n)$$

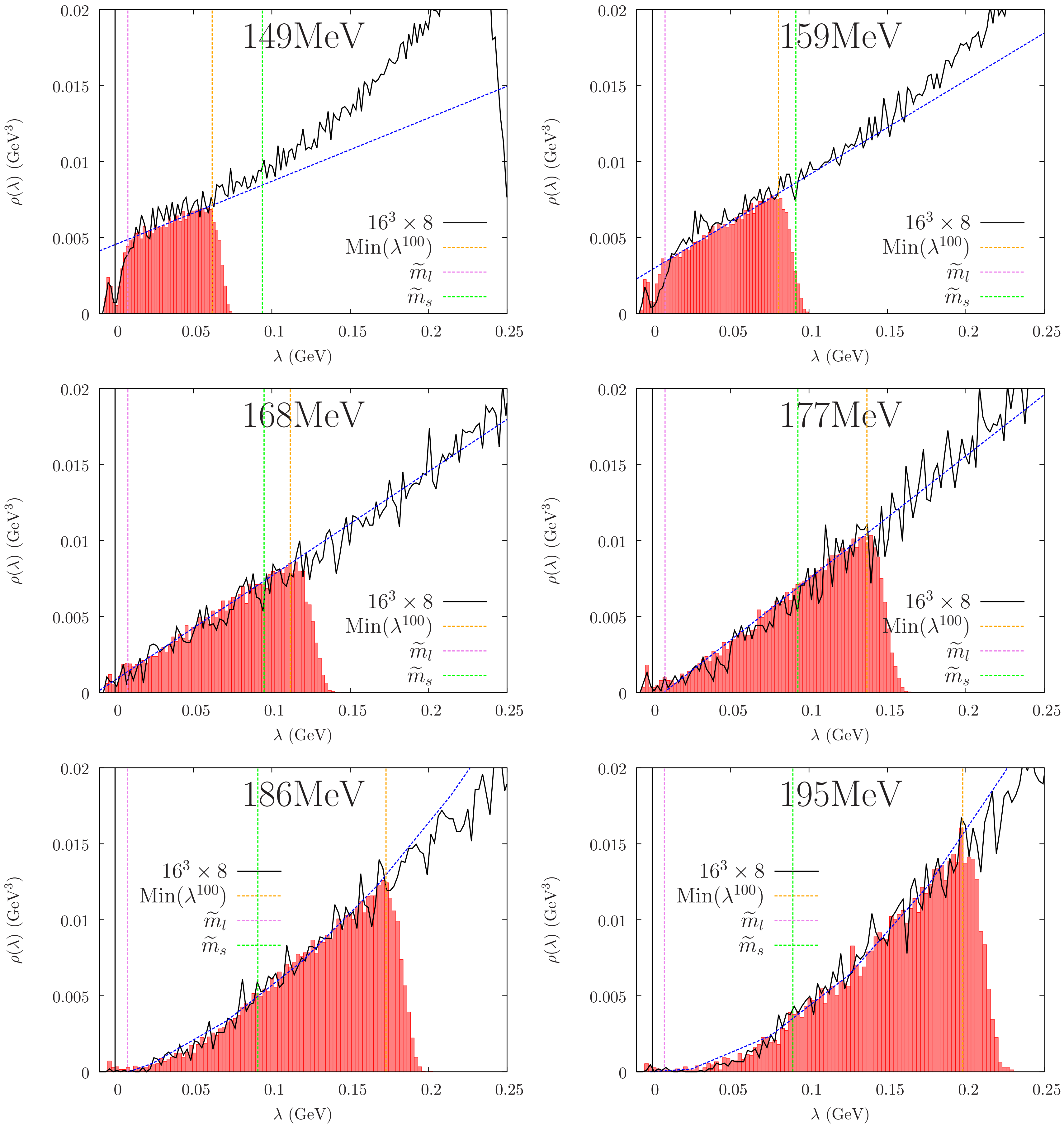
Cossu *et al.* (JLQCD), Overlap
Phys. Rev. D87 (2013) 114514



Gap seems to open at smaller quark mass.

$T_c \simeq 180$ MeV

Buchoff *et al.* (LLNL/RBC), DomainWall, Phys. Rev. D89 (2014) 5,054514



Small eigenvalues appear.

Gap seems to close at or above critical temperature

$$T_c \simeq 180 \text{ MeV}$$

What causes this difference ?

volume ? quark mass ? lattice chiral symmetry ?

JLQCD collaboration

Overlap: exact GW relation

LLNL/RBC collaborations

DomainWall: approximated GW relation

Recent study by A. Tomiya et al. for JLQCD collaboration

Preliminary

generate gauge configurations with an improved DomainWall quarks

very small violation of GW relation

(1)calculate eigenvalue distrubution of overlap operator on these configurations

partially quenched

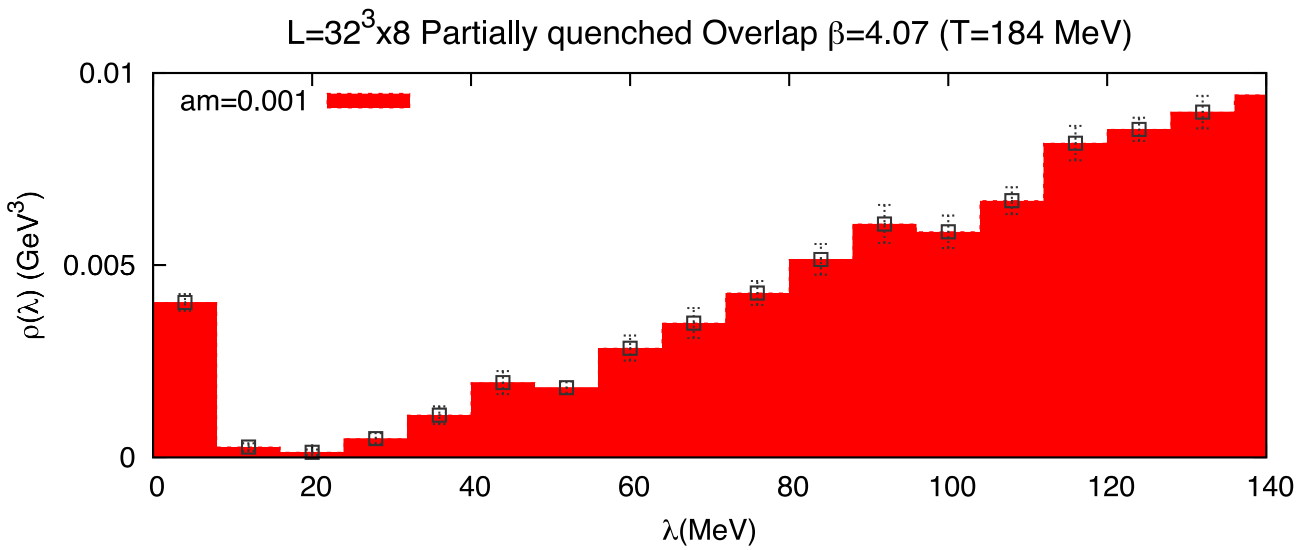
(2)reweighting factor from the improved DW to Overlap is intorudced to obtain the full eigenvalue distribution

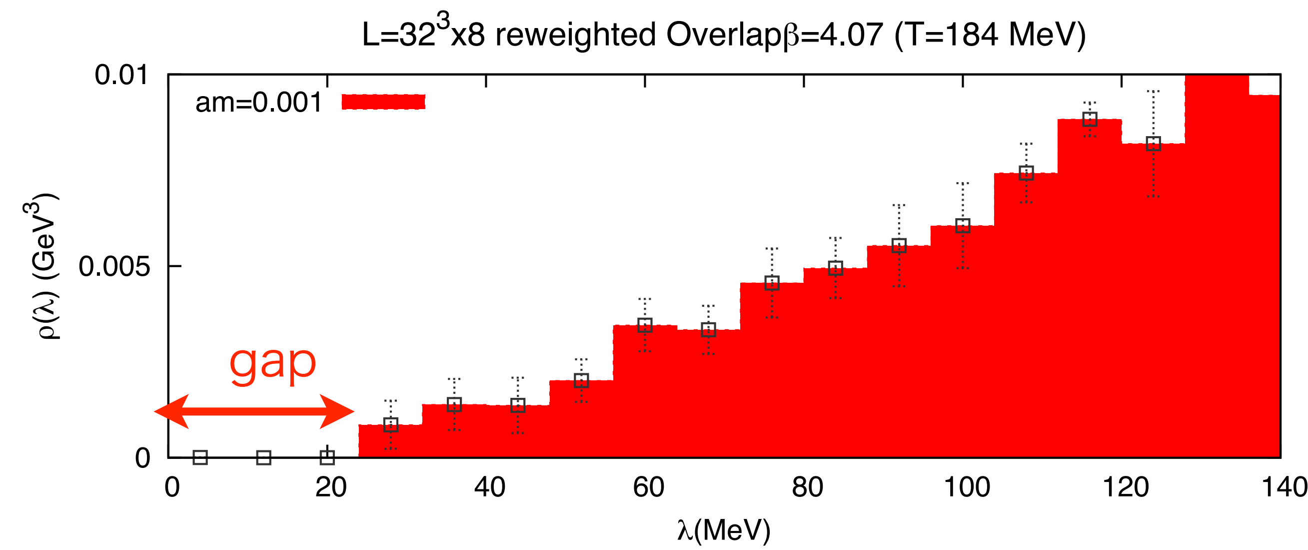
full Overlap

partially quenched(PQ)

Preliminary

full Overlap





small eigenvalue

$$T \simeq 184 \text{ MeV}$$

After the reweighting, small eigenvalues in PQ disappear, and the gap seems to open in full Overlap.

An exact lattice chiral symmetry is essential to obtain the correct result.
A tiny violation of the chiral symmetry may destroy the theoretically expected relation.

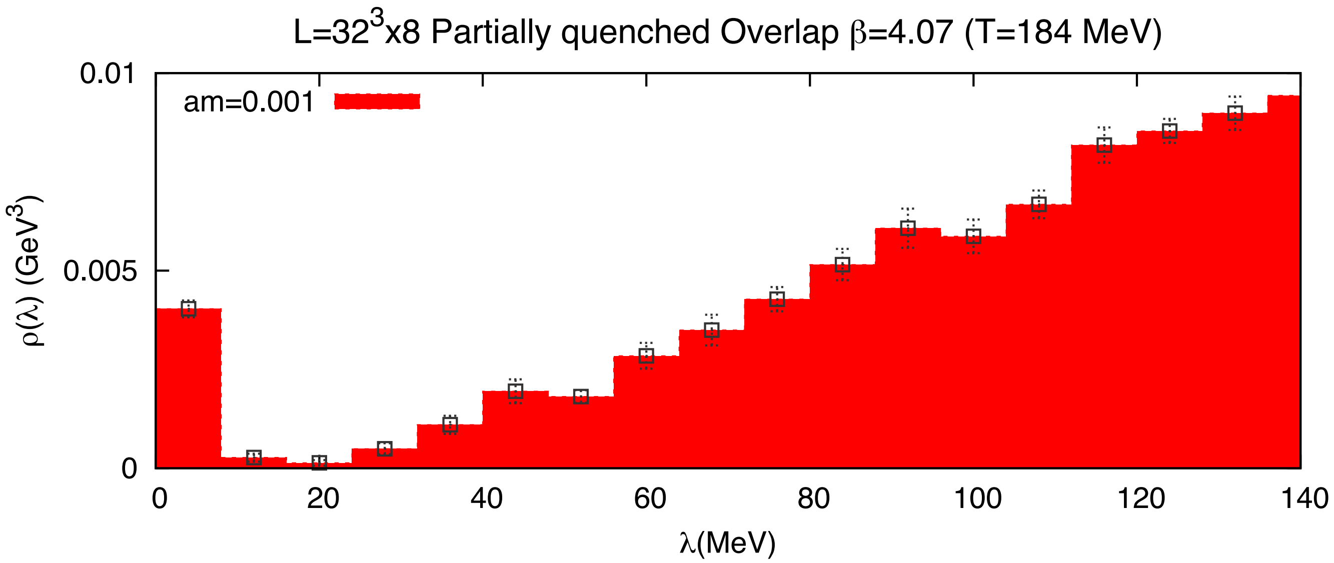
Does the gap really open at critical temparutue?

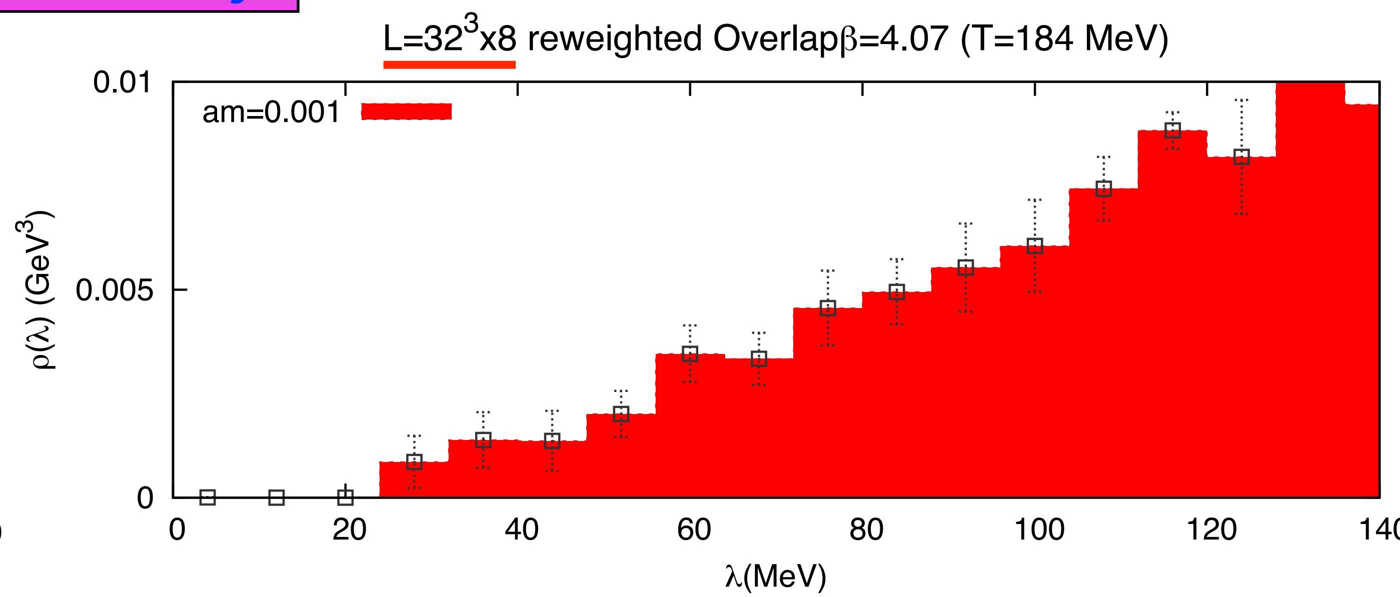
Further inverstigations are needed. a precise determination of T_c

partially quenched(PQ)

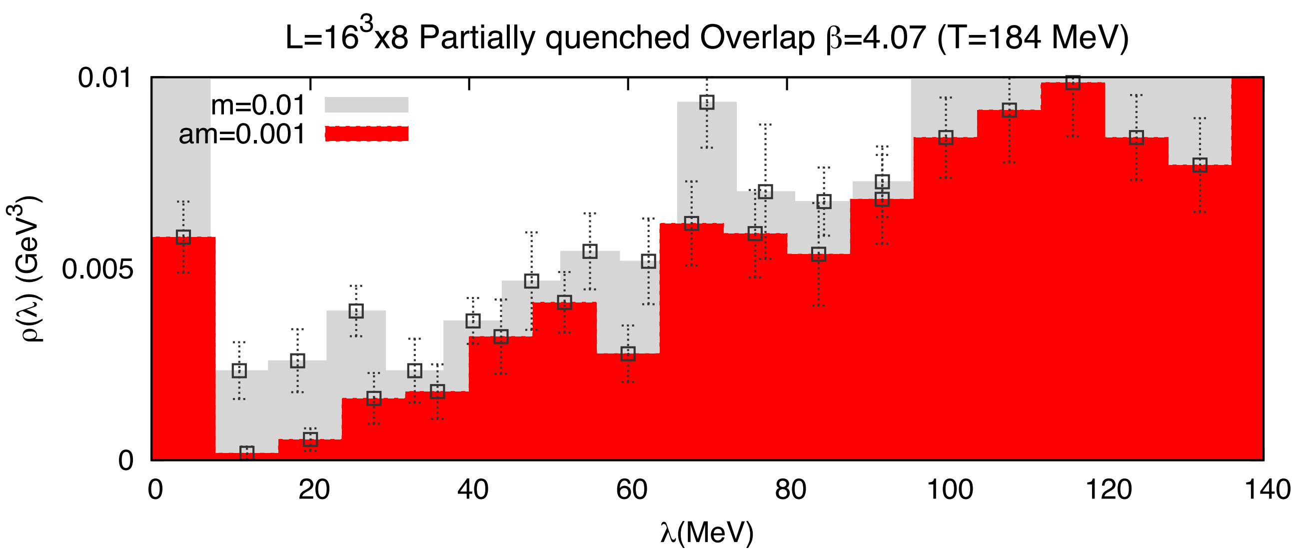
Preliminary

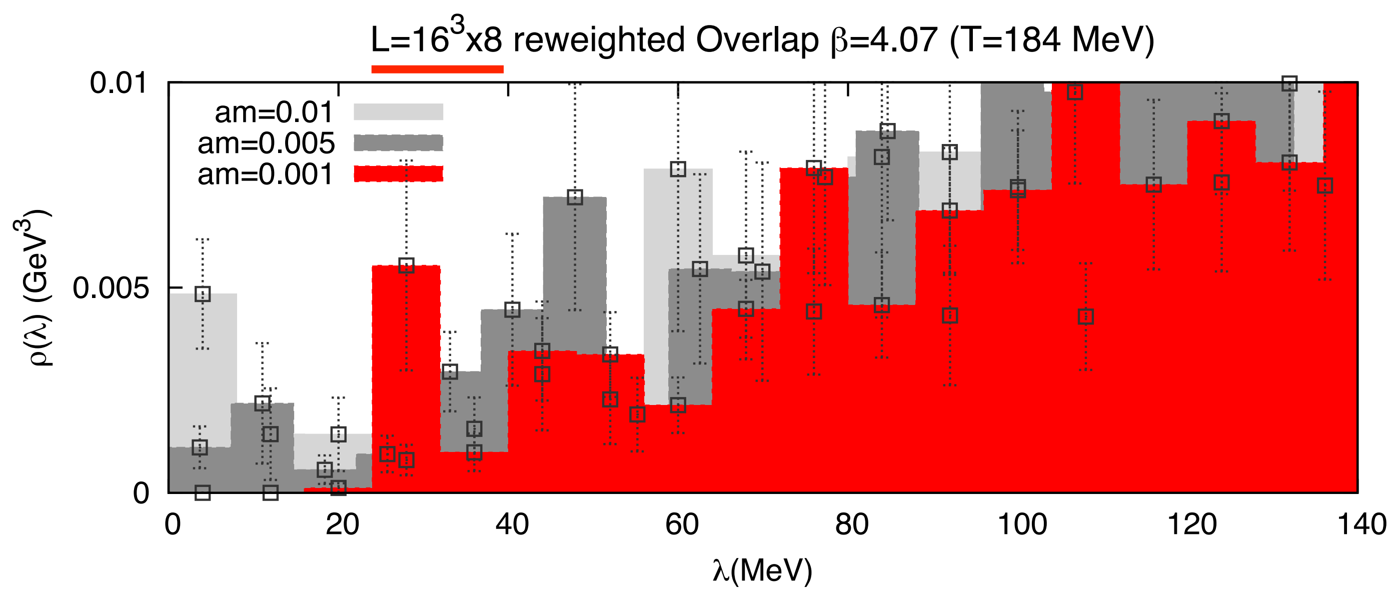
full Overlap





Smaller volume also show a simlira behavior.

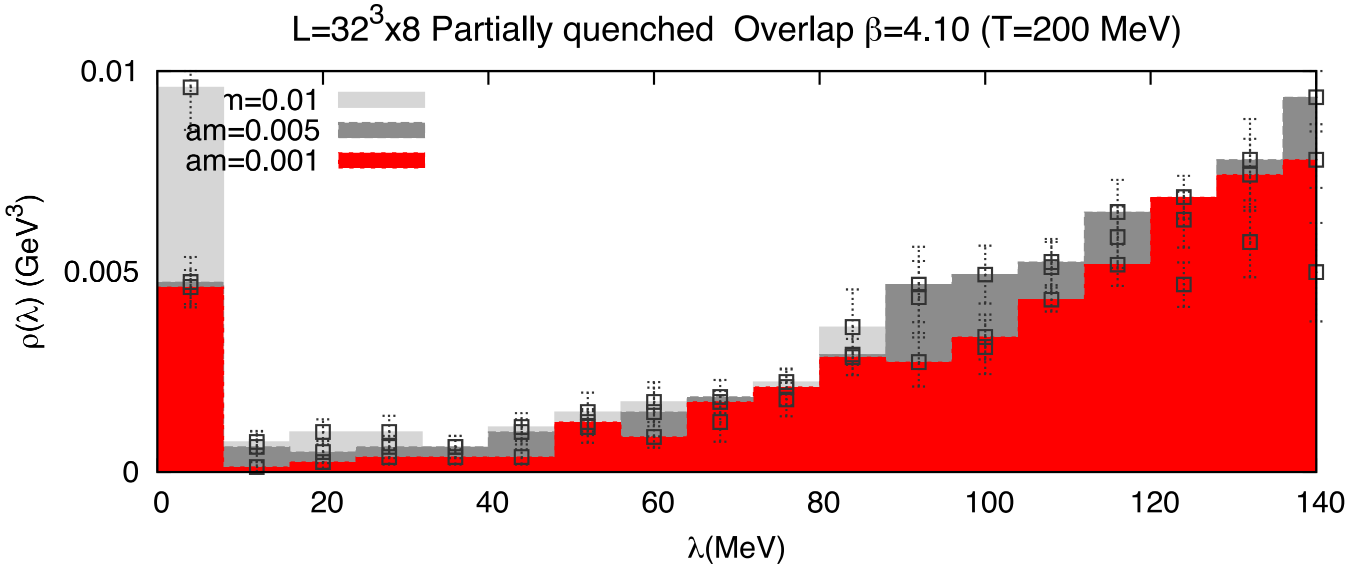


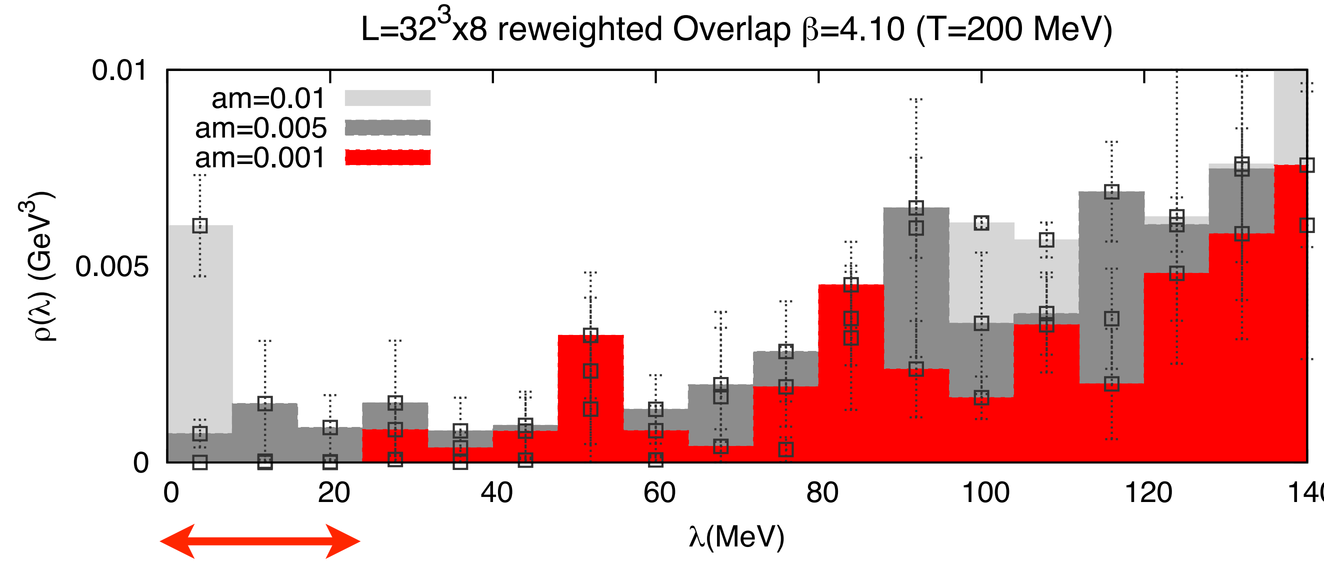


Smaller quark mass is needed to see the gap.

higher temparture

$$T \simeq 200\ \mathrm{MeV}$$

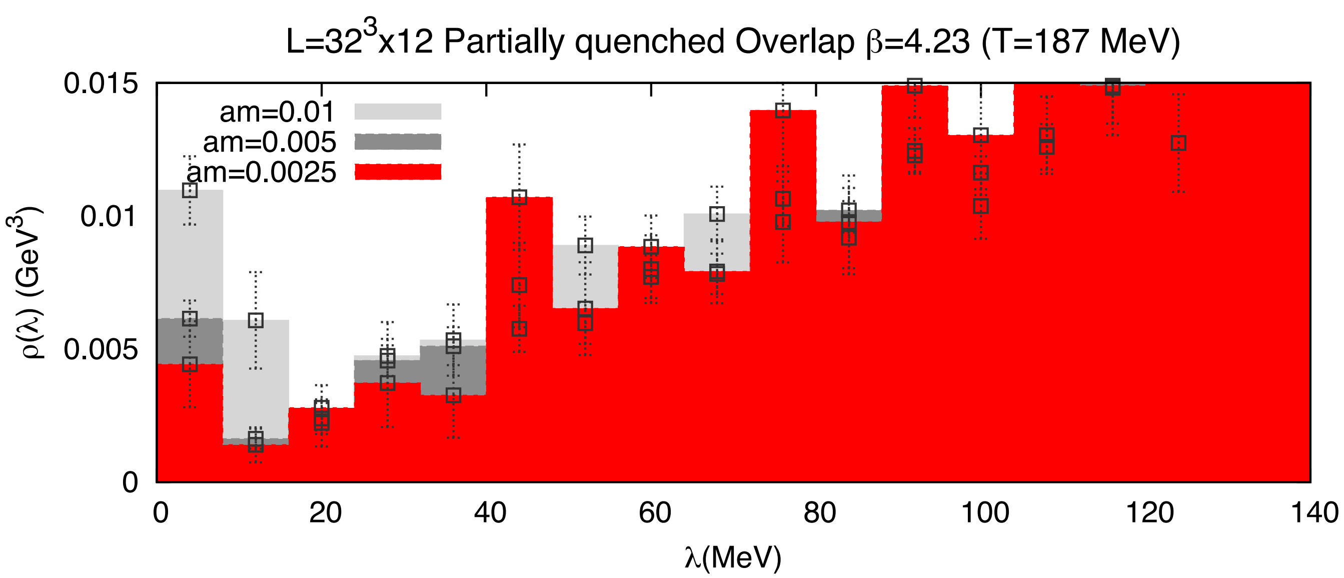


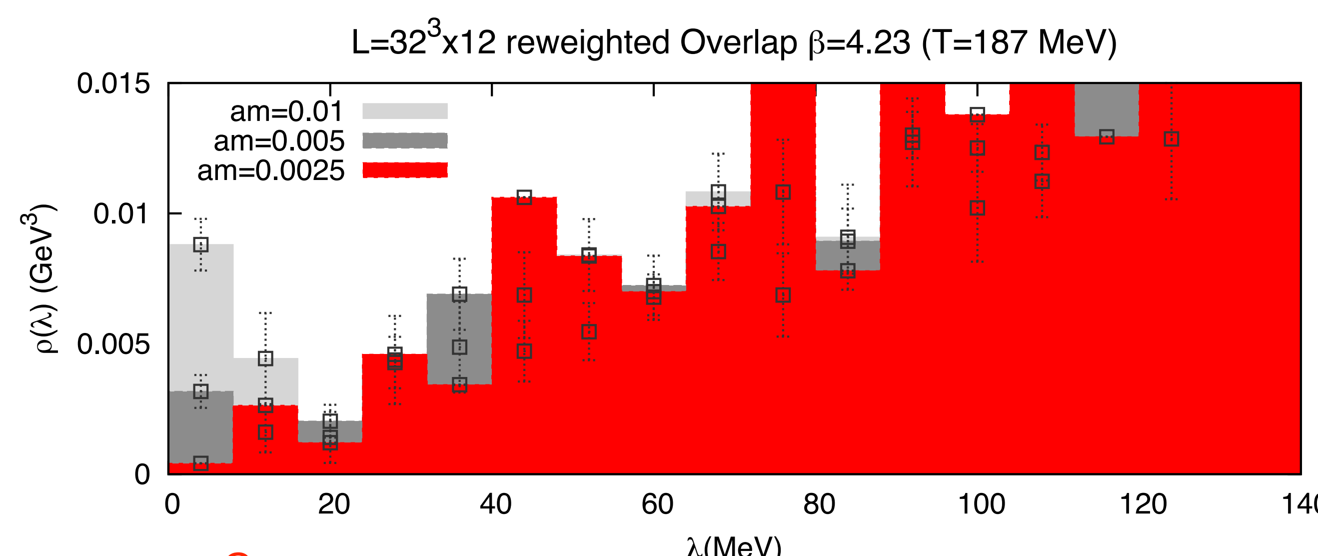


Preliminary

smaller lattice spacing

$$a(N_t = 12)/a(N_t = 8) = 8/12 = 2/3$$





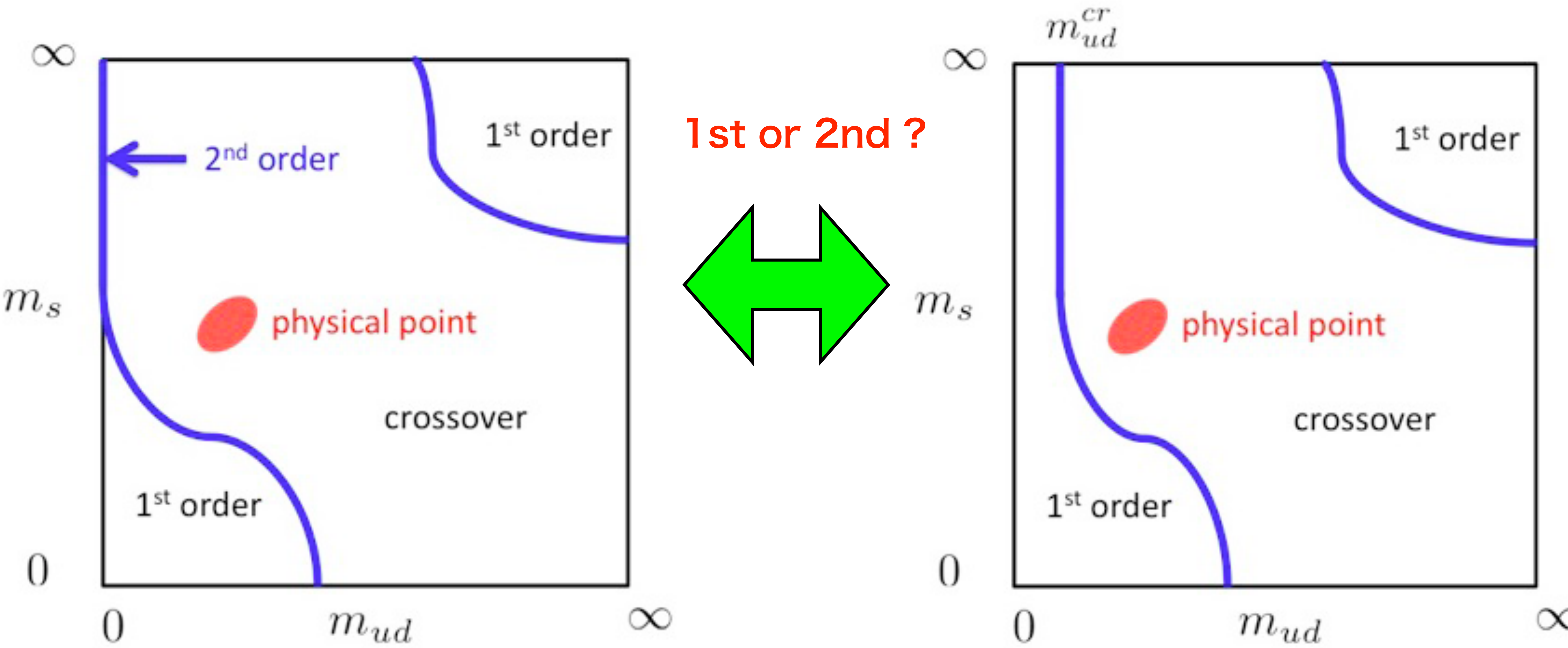
gap ?

Existence of the gap is unclear.

Further inverstigation is necessary. (Chiral zero modes must be removed.)

3. Conclusion

Order of phase transition in 2-flavor QCD



gapless $SU(2)_L \otimes SU(2)_R \otimes Z_4$

gap $SU(2) \otimes SU(2) \otimes U(1)$

Conformal bootstrap method may help.

Even if the phase transition is of 2nd order, its universality class might be different from O(4).