PATTERNS AND PARTNERS FOR CHIRAL SYMMETRY RESTORATION

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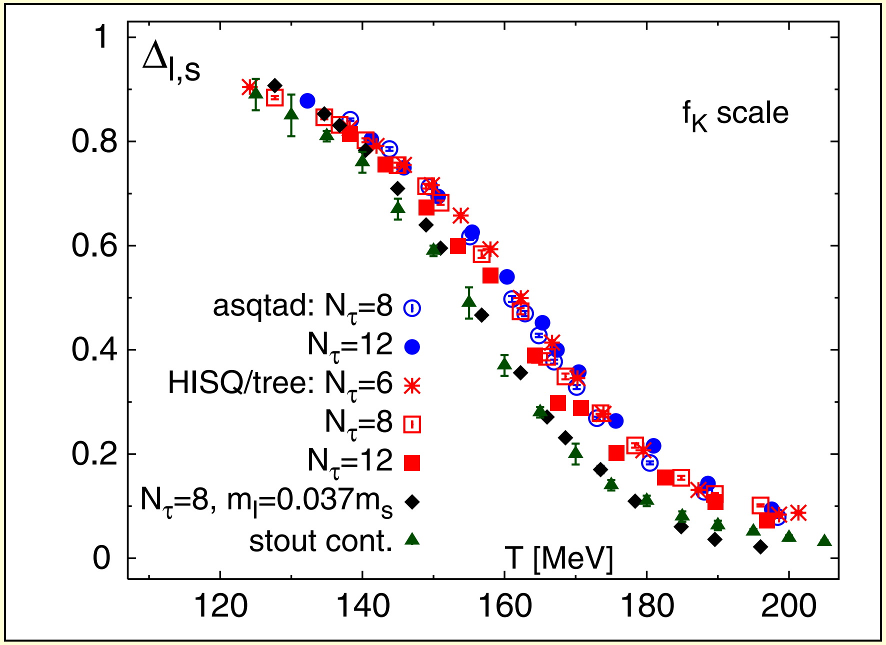
OUTLINE:

- $U(3)$ **Ward Identities:** $O(4)$ **vs** $O(4) \times U(1)_A$, **chiral partners**
- **WI and scaling of meson screening masses**
- **Role of thermal** $\sigma/f_0(500)$ **pole in chiral restoration**

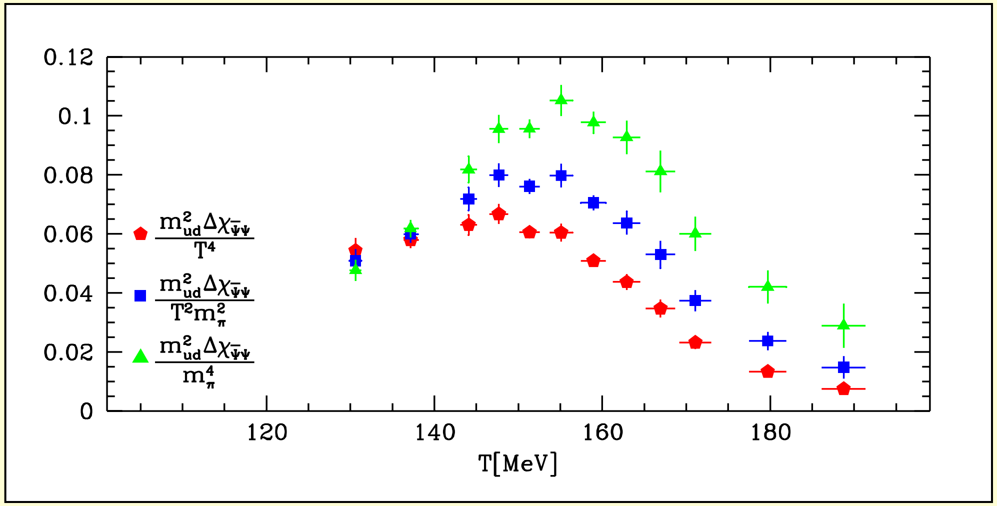
AGN, R.Torres Andrés, J.Ruiz de Elvira, PRD88, 076007 (2013)
AGN, J.Ruiz de Elvira, JHEP 1603 (2016) 186, arXiv:1704.05036

EXTREME QCD
PISA 26-28 JUNE 2017

Chiral Symmetry Restoration in QCD



$$\chi_S = -\frac{\partial}{\partial m_q}\langle \bar{q}q\rangle_T = \int_x \left[\langle \bar{q}q(x)\bar{q}q(0)\rangle_T - \langle \bar{q}q\rangle_T^2\right]$$



$$\Delta_{l,s} = \frac{\langle \bar{q}q\rangle_T - (2m_q/m_s)\,\langle \bar{s}s\rangle_T}{\langle \bar{q}q\rangle_0 - (2m_q/m_s)\,\langle \bar{s}s\rangle_0}$$

Y.Aoki, S. Borsanyi et al (Budapest-Wuppertal) 2009, 2010
A.Bazavov et al (Hot QCD), 2012, 2014

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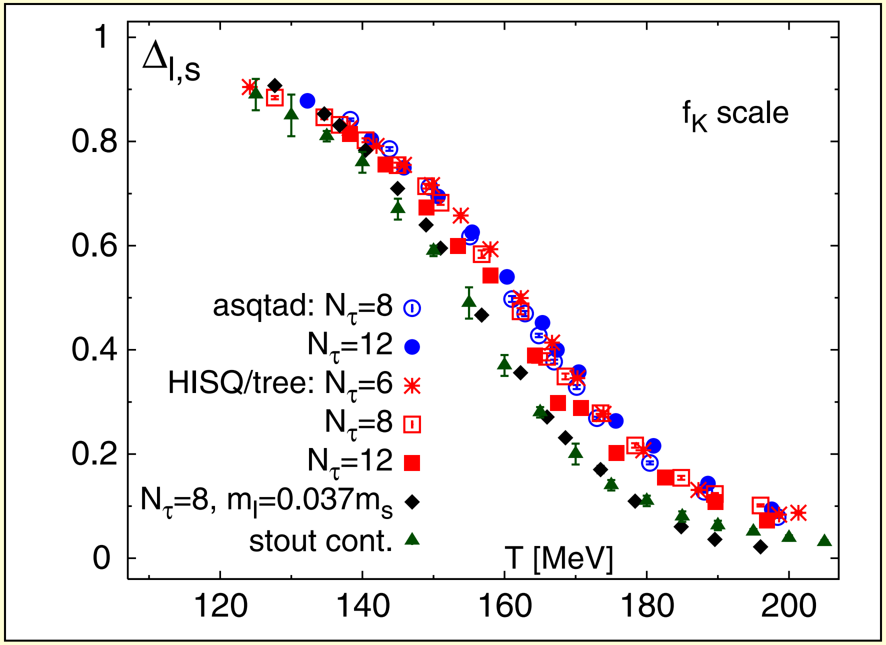

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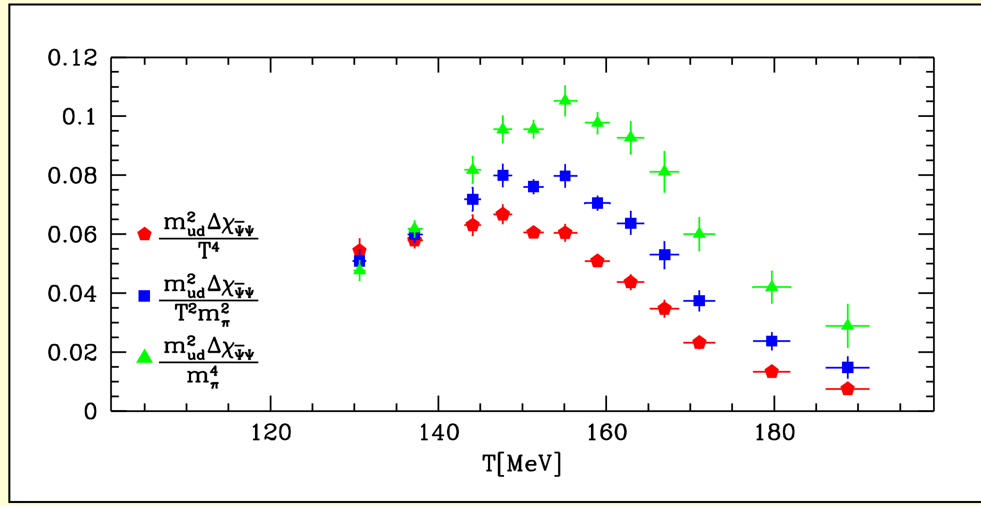

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CROSSOVER Transition @ $T_c \approx$ 155 MeV for N_f=2+1 and physical masses

Exact restoration → Phase transition for N_f=2 in chiral limit

Chiral Patterns and Partners

- $U(1)_A$ **asymptotic restoration could lead to** $O(4) \times U(1)_A$ **pattern instead of** $O(4)$

  Gross, Pisarski, Yaffe 1981

- **Affects the transition order, critical end point, etc**

  Pisarski, Wilczek, 1984. Pelissetto, Vicari 2013. Mitter, Schaefer 2014. Esser, Grahl, Rischke 2015

- **Observed** $M_{\eta'}$ **reduction points to** $U(1)_A$ **restoration.**
  Increase of η' **production would affect dileptons&diphotons**

  Kapusta, Kharzeev, McLerran 1996. Csorgo, Vertesi, Sziklai 2010

- **Chiral pattern still not settled in lattice in terms of chiral partner degeneration** $\rightarrow$

Chiral Patterns and Partners

Particle spectrum → degeneration of chiral partners:

$$\pi^a = \bar{\psi}_l \gamma_5 \tau^a \psi_l \quad \overset{SU_A(2)}{\longleftrightarrow} \quad \sigma = \bar{\psi}_l \psi_l$$

$$\updownarrow U_A(1) \qquad\qquad \updownarrow U_A(1)$$

$$\delta^a = \bar{\psi}_l \tau^a \psi_l \quad \overset{SU_A(2)}{\longleftrightarrow} \quad \eta_l = \bar{\psi}_l \gamma_5 \psi_l$$

$$\pi^a = i\bar{\psi}_l \gamma_5 \tau^a \psi, \quad \delta^a = \bar{\psi}_l \tau^a \psi_l \sim a_0(980)$$

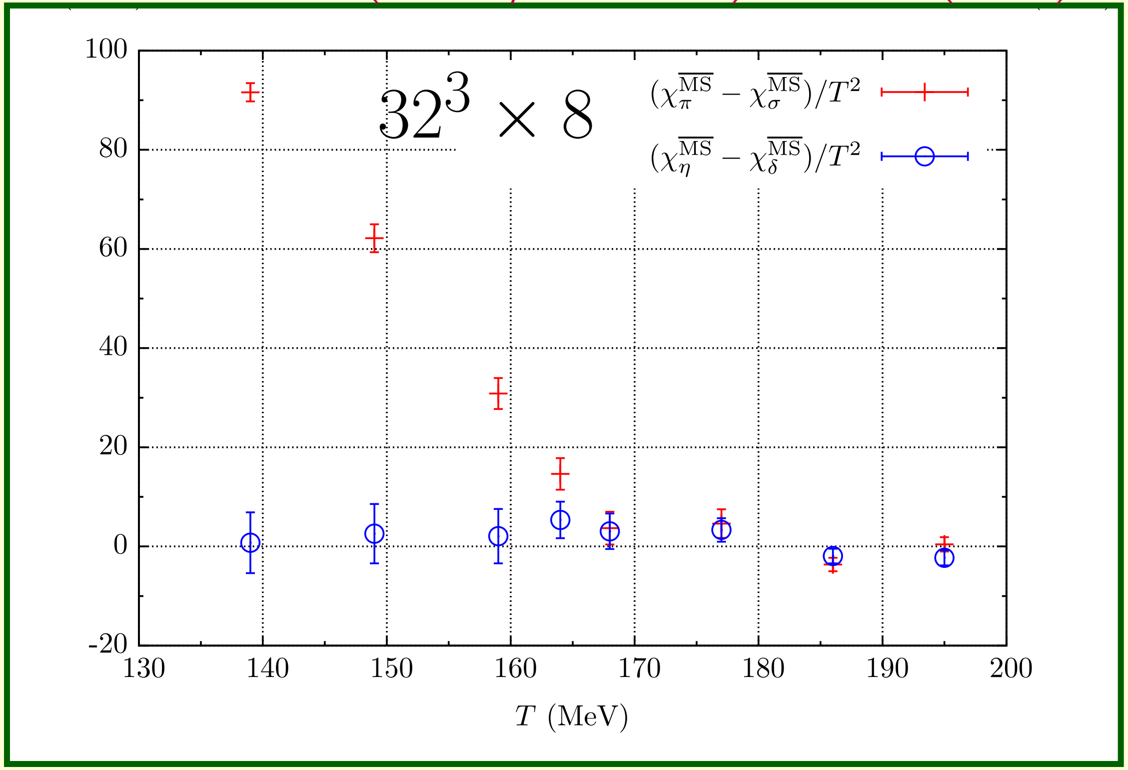
$$\sigma = \bar{\psi}_l \psi_l, \ \sigma_s = \bar{s}s \to f_0(500), f_0(980) \ \textbf{(mixed)}$$

$$\eta_l = i\bar{\psi}_l \gamma_5 \psi_l, \ \eta_s = i\bar{s}\gamma_5 s \to \eta, \eta' \ \textbf{(mixed)}$$

Chiral Patterns and Partners

Lattice susceptibilities $SU_V(2) \times SU_A(2) \sim O(4)$ **pattern**

Buchoff et al (LLNL/RBC coll) PRD89 (2014)



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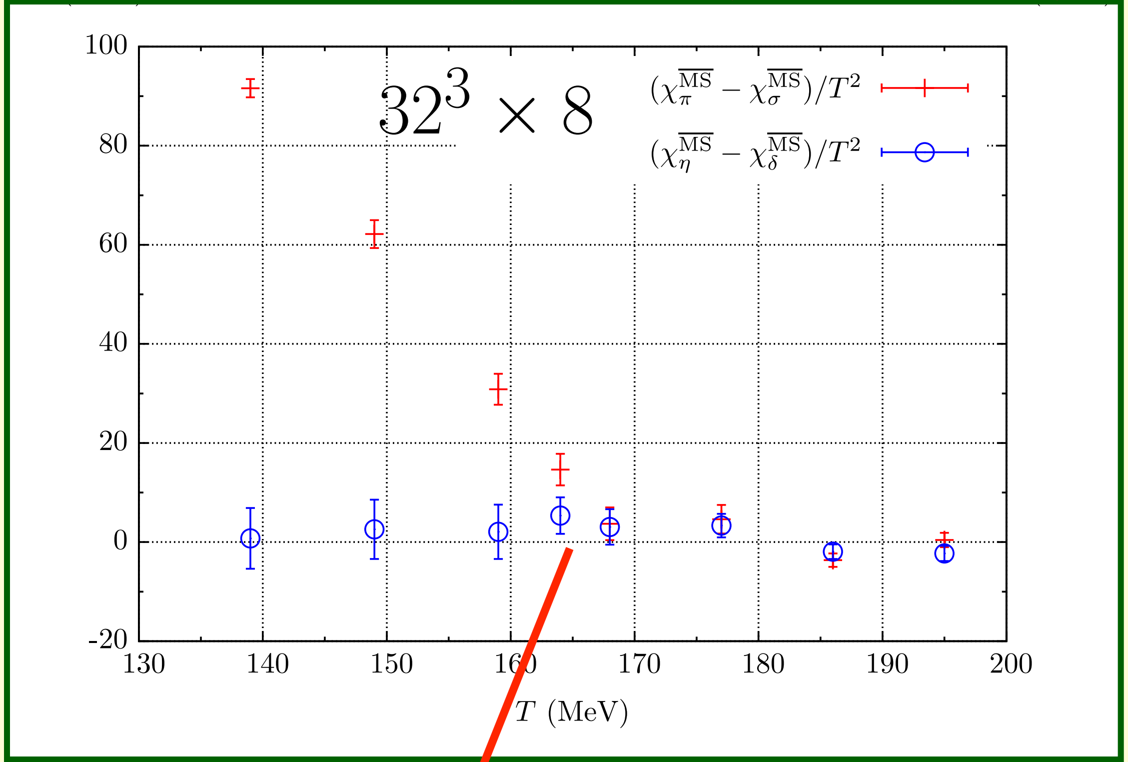

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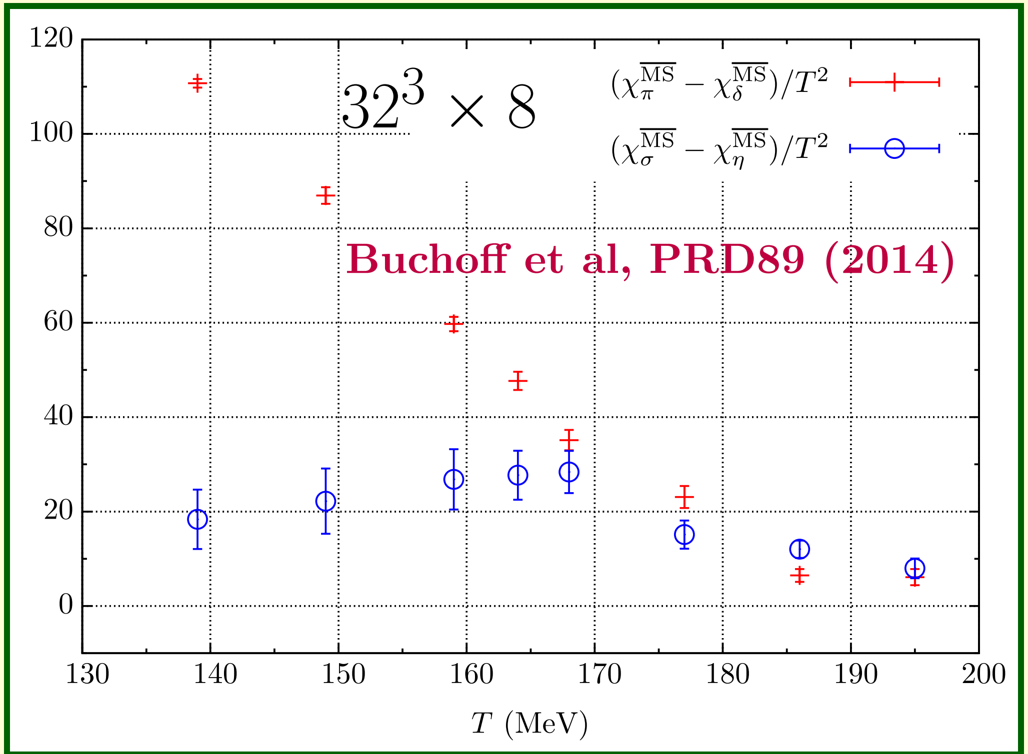
Chiral Patterns and Partners

$U_A(1)$ **restoration?** $\longrightarrow$ **degeneration of nonet partners of opposite parity e.g.** $\pi - a_0(980)(\delta)$



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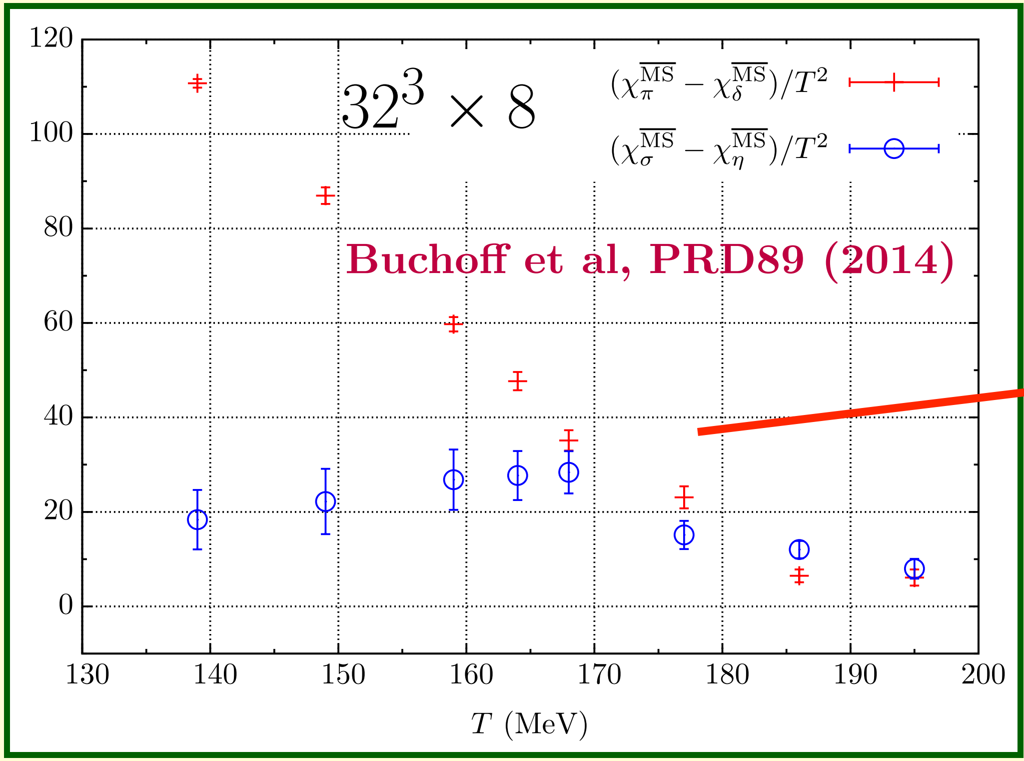

$U_A(1)$ **not restored at** T_c
$\Longrightarrow O(4)$ **pattern**

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$$\updownarrow_{U_A(1)} \qquad\qquad \updownarrow_{U_A(1)}$$

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Consistent with previous analysis of same group on meson screening masses
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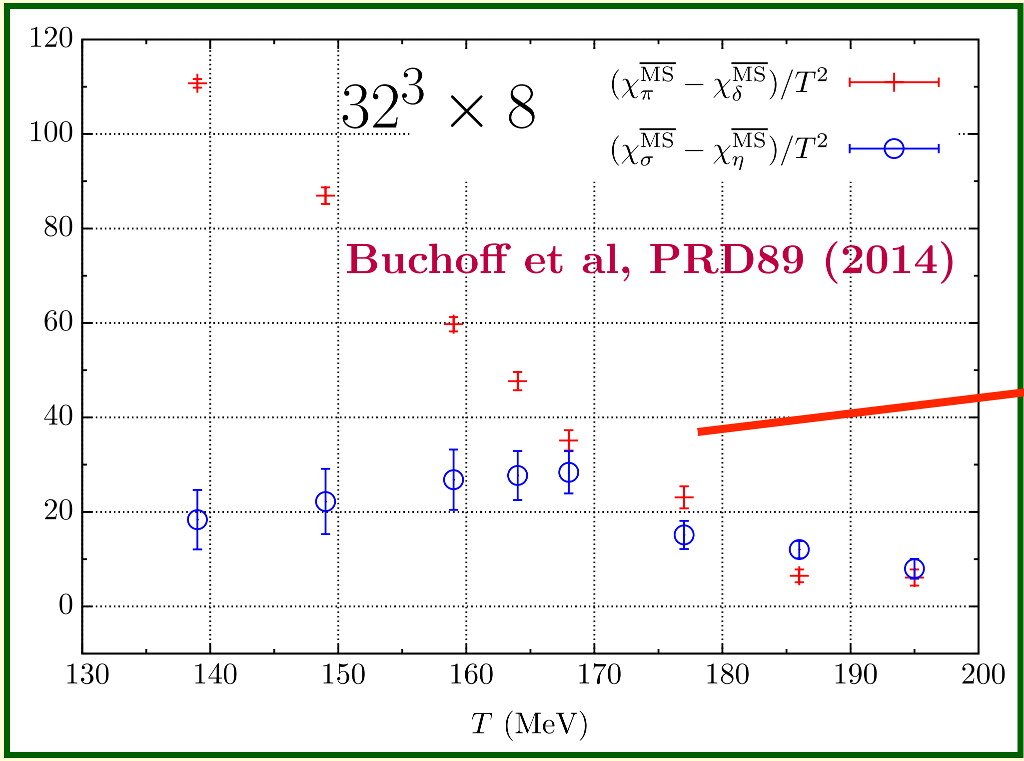

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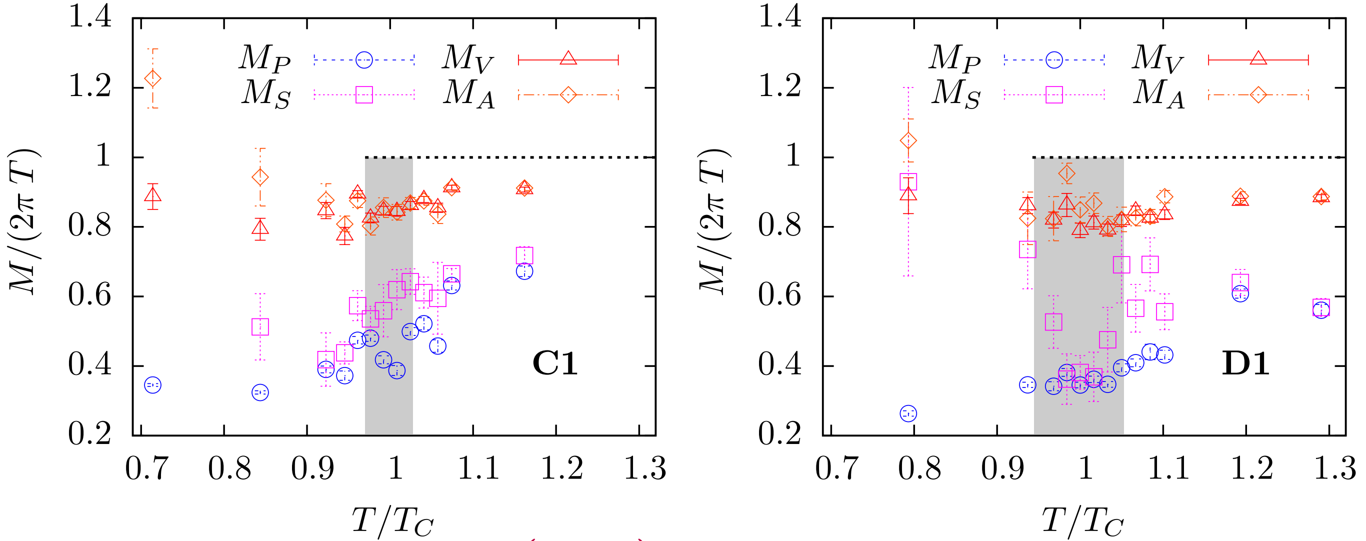
Not so clear for other lattice groups...

Chiral Patterns and Partners



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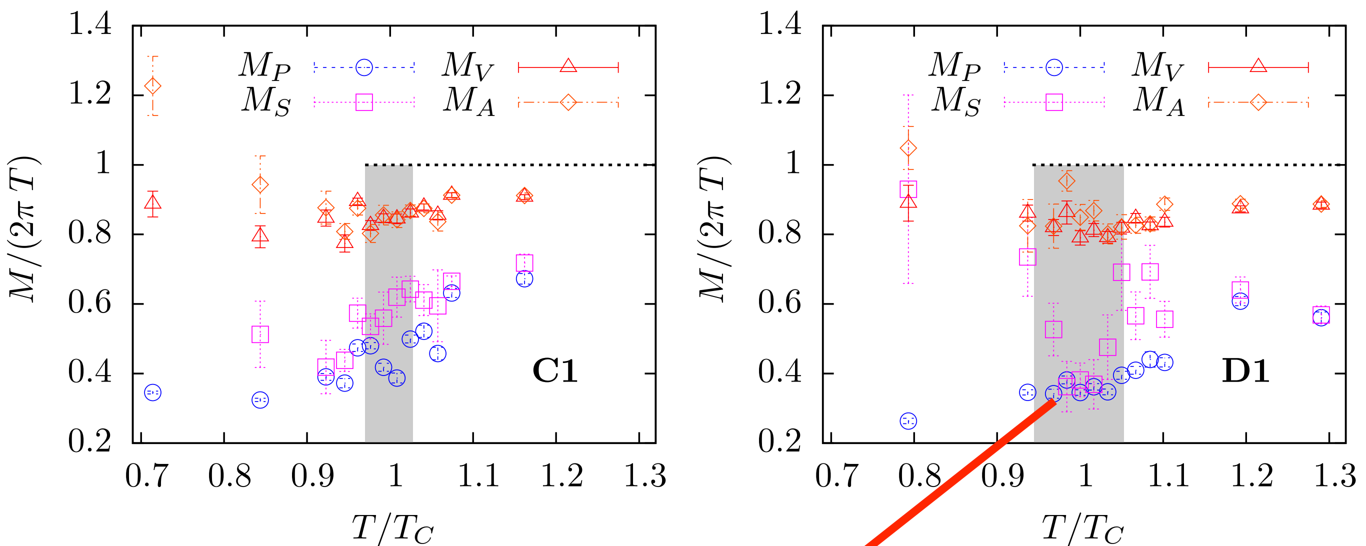
Chiral Patterns and Partners



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$U_A(1)$ **restored at** $T_c \Longrightarrow O(4) \times U_A(1)$ **pattern**

$$
\begin{array}{ccc}
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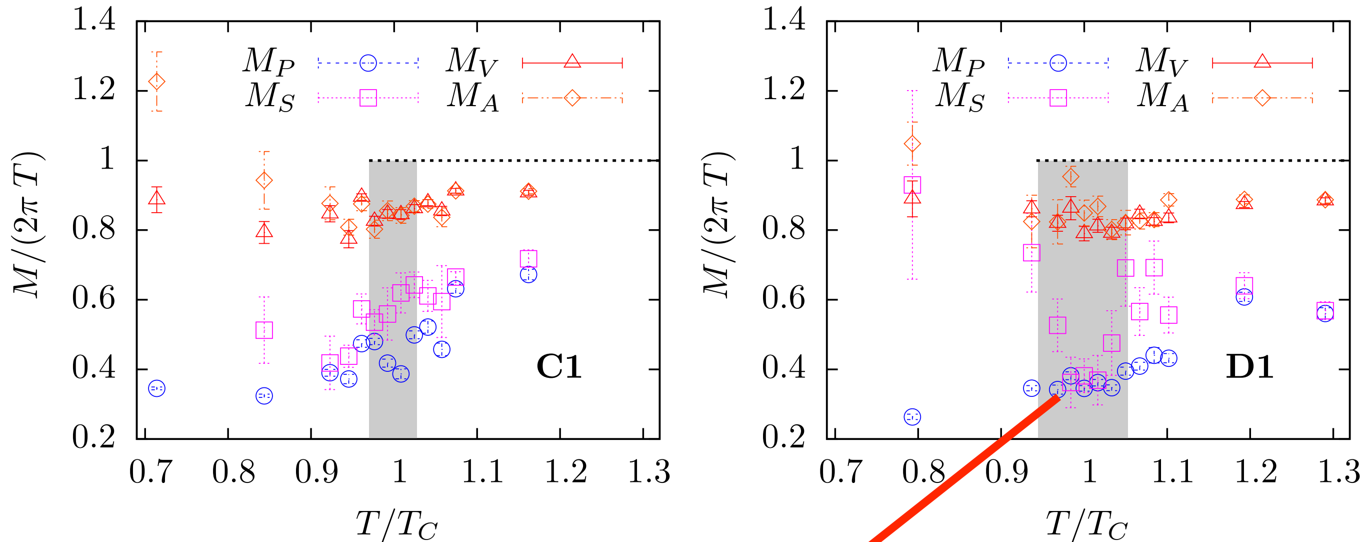
\end{array}
$$

Chiral Patterns and Partners



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$U_A(1)$ **restored at** $T_c \Longrightarrow O(4) \times U_A(1)$ **pattern**

In addition, $U_A(1)$ **restored for** $T \gtrsim T_c$ **close to chiral limit in**
Aoki et al, PRD86 (2012), Cossu et al, PRD87 (2013)

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Ward Identities

Formally from QCD through A/V transformations:

$$\left\langle \frac{\delta \mathcal{O}_P(y)}{\delta \alpha_A^a(x)} \right\rangle = -\left\langle \mathcal{O}_P(y) \bar{\psi}(x) \left\{ \frac{\lambda^a}{2}, \mathcal{M} \right\} \gamma_5 \psi(x) \right\rangle + i \frac{\delta_{a0}}{\sqrt{6}} \langle \mathcal{O}_P(y) A(x) \rangle$$

$$\left\langle \frac{\delta \mathcal{O}_S(y)}{\delta \alpha_V^a(x)} \right\rangle = \left\langle \mathcal{O}_S(y) \bar{\psi}(x) \left[\frac{\lambda^a}{2}, \mathcal{M} \right] \psi(x) \right\rangle$$

$$\lambda^0 = \sqrt{2/3}\, \mathbb{1}, \quad A(x) = \frac{3\alpha_s}{4\pi} Tr_c G_{\mu\nu} \tilde{G}^{\mu\nu}$$

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$\lambda^0 = \sqrt{2/3}\,\mathbb{1}$, $A(x) = \frac{3\alpha_s}{4\pi} Tr_c G_{\mu\nu}\tilde{G}^{\mu\nu}$

$\mathcal{O}_P^b = i\bar{\psi}\gamma_5\lambda^b\psi \equiv P^b \rightarrow$ **1p vs 2p fns** $\rightarrow \langle\bar{q}q\rangle$ **vs** χ_P

$\mathcal{O}_P^{bc} = P^b S^c \rightarrow$ **2p vs 3p** $\rightarrow$ **ch.partners vs meson vertices**
(e.g. $\chi^\sigma - \chi^\pi \sim \sigma\pi\pi, \ldots$**)**

$\mathcal{O}_S^b = \bar{\psi}\lambda^b\psi \equiv S^b \rightarrow \langle\bar{q}q\rangle$ **vs** χ_S **for** κ **sector** $b = 4,\ldots,7$

Ward Identities: quark condensates vs P suscept.

- π **SECTOR** $\rightarrow \langle \bar{q}q \rangle_l (T) = -\hat{m}\chi_P^\pi(T)$
- K **SECTOR** $\rightarrow \langle \bar{q}q \rangle_l (T) + 2 \langle \bar{s}s \rangle (T) = -(\hat{m}+m_s)\chi_P^K(T)$
- η **SECTOR** $\rightarrow \eta_0/\eta_8$ **mixing &** $U_A(1)$ **anomaly enter:**

$$\chi_P^{88} = -\frac{1}{3}\left(\frac{\langle \bar{q}q \rangle_l}{\hat{m}} + \frac{4\langle \bar{s}s \rangle}{m_s}\right) + \frac{\sqrt{3}}{9}\frac{\hat{m}-m_s}{\hat{m}m_s}\chi_P^{8A}$$

$$\chi_P^{80} = -\frac{\sqrt{2}}{3}\left(\frac{\langle \bar{q}q \rangle_l}{\hat{m}} - \frac{2\langle \bar{s}s \rangle}{m_s}\right) - \frac{\sqrt{6}}{18}\frac{\hat{m}+2m_s}{\hat{m}m_s}\chi_P^{8A}$$

$$\chi_P^{00} = -\frac{2}{3}\left(\frac{\langle \bar{q}q \rangle_l}{\hat{m}} + \frac{\langle \bar{s}s \rangle}{m_s}\right) - \frac{\sqrt{3}}{18}\frac{(\hat{m}+2m_s)^2}{\hat{m}m_s(m_s-\hat{m})}\chi_P^{8A}$$

$$\Rightarrow \chi_P^{\bar{s}s} = -\frac{\langle \bar{s}s \rangle}{m_s} + \frac{\hat{m}}{4\sqrt{3}m_s(\hat{m}-m_s)}\chi_P^{8A}$$

$\frac{\hat{m}}{m_s}$ **suppressed**

$$\chi_P^{ab} \equiv \int_T dx \langle P^a(x)P^b(0)\rangle, \; \chi_P^{8A} \equiv \int_T dx \langle P^8(x)A(0)\rangle, \; \langle \bar{q}q \rangle_l = \langle \bar{u}u + \bar{d}d \rangle, \; \hat{m} = m_u = m_d$$

Chiral Patterns and Partners from WI

Crossed ls correlator nonzero due to 08 mixing. From WI:

$$\chi_P^{ls}(T) = -2\frac{\hat{m}}{m_s}\chi_{5,disc}(T) = \frac{1}{2\sqrt{3}}\frac{1}{\hat{m}-m_s}\chi_P^{8A}(T)$$

where $\chi_{5,disc} = \frac{1}{4}\left(\chi_P^{\pi} - \chi_P^{\eta_l}\right)$ **measures** $O(4)\times U(1)_A$ **restoration**

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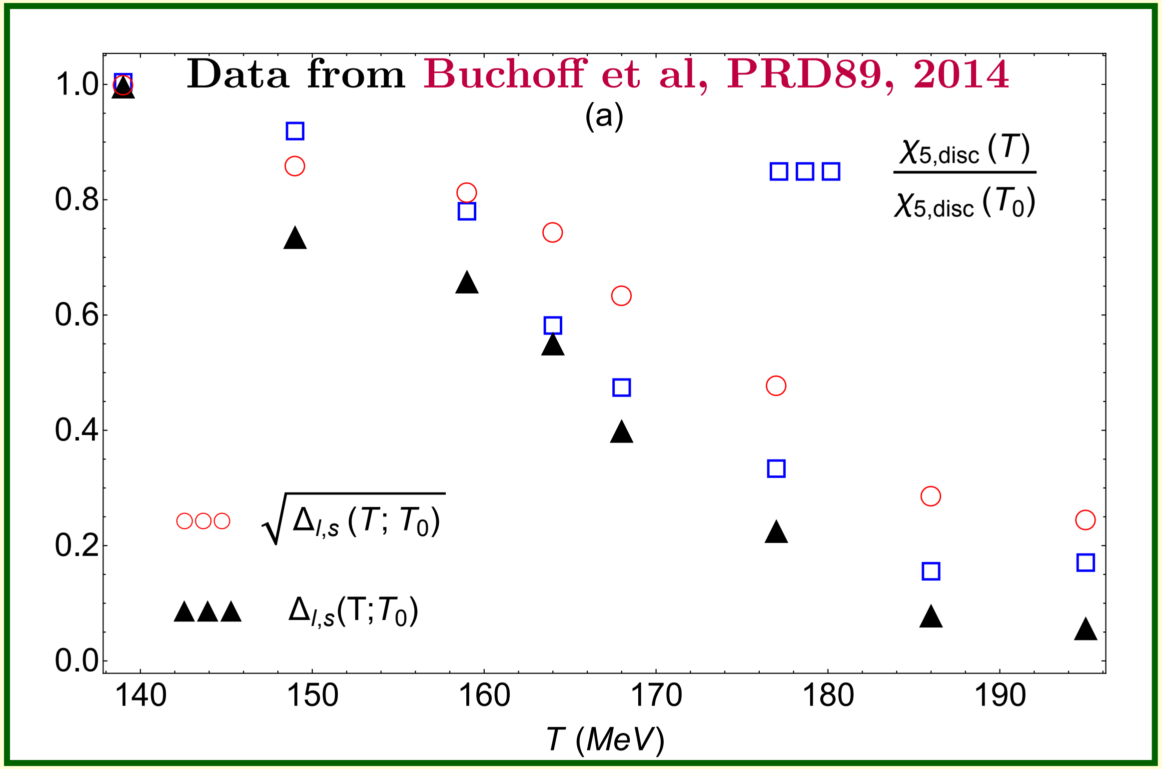
$\Rightarrow$ $SU(2)_A$ **transforms (**η_s **invariant)** $P_{ls} \to \langle \delta\eta_s \rangle = 0$ **by parity**

Hence at *exact* chiral restoration $\overset{O(4)}{\sim}$**:**
(e.g. two massless flavours @ $\langle \bar{q}q \rangle_l = 0$**)**

$$\chi_{5,disc} \overset{O(4)}{\sim} 0 \Rightarrow O(4)\times U(1)_A \text{ **pattern**}$$

Chiral Patterns and Partners from WI

Physical case, $O(4) \times U(1)_A$ restoration dictated by $\langle \bar{q}q \rangle_l$:



$32^3 \times 8$ lattice size.
$\hat{m}/m_s = 0.088$

WI connects with $a_0\eta\pi$ vertex: $P_{ls}(y) = \frac{1}{3}\hat{m}\int_T dx \, \langle \mathcal{T}\eta_s(y)\pi(x)\delta(0)\rangle$

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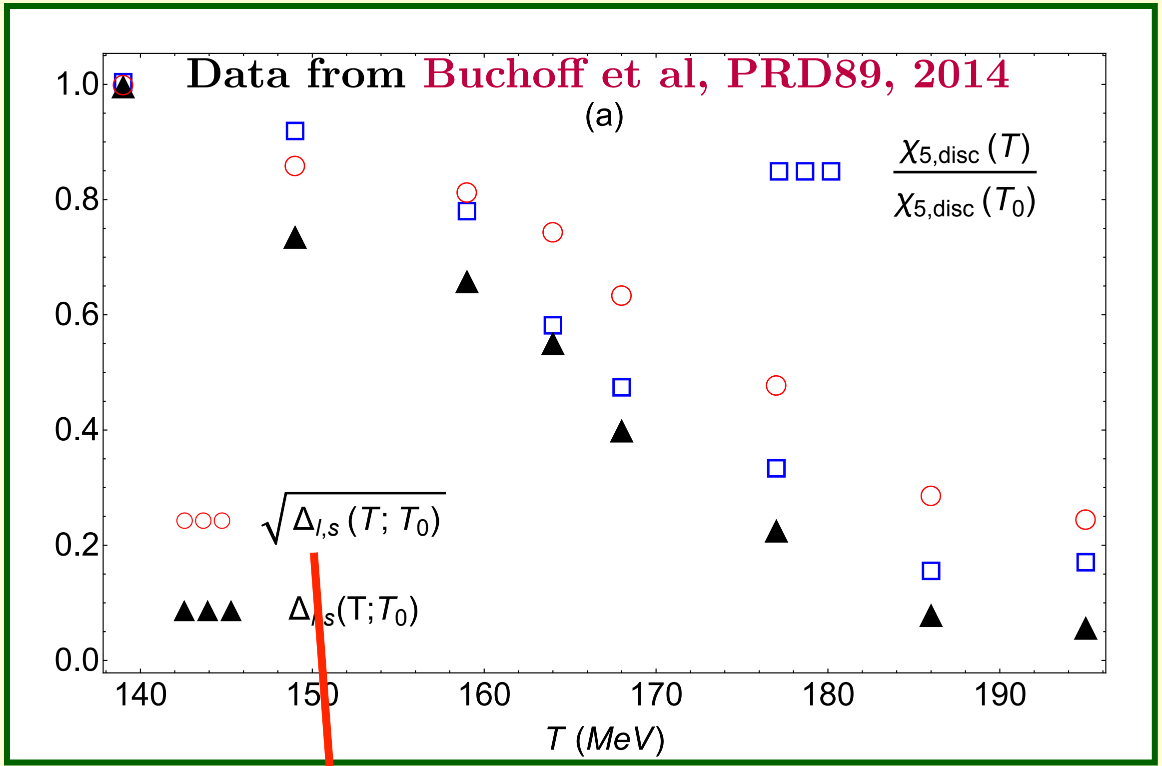

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Bilinear normalization $\pi\sim\sqrt{-\frac{\langle\bar{q}q\rangle_l}{G_\pi(0)}}$ **from** $\chi_P^\pi=-\frac{\langle\bar{q}q\rangle_l}{\hat{m}}$

Chiral Patterns and Partners from WI

$I=1/2$ SECTOR $(K-\kappa)$ DEGENERATION

$$\chi_S^\kappa(T)-\chi_P^K(T)=\frac{2}{m_s^2-\hat{m}^2}\left[m_s\left\langle\bar{q}q\right\rangle_l(T)-2\hat{m}\langle\bar{s}s\rangle(T)\right]$$

Chiral Patterns and Partners from WI

$I = 1/2$ **SECTOR** $(K - \kappa)$ **DEGENERATION**

$$\chi_S^\kappa(T) - \chi_P^K(T) = \frac{2}{m_s^2 - \hat{m}^2}\left[m_s \langle \bar{q}q \rangle_l (T) - 2\hat{m}\langle \bar{s}s\rangle(T)\right]$$

$\Rightarrow \chi_S^\kappa \stackrel{O(4)}{\sim} \chi_P^K$ **degeneration** $(\hat{m} = \langle \bar{q}q \rangle_l = 0)$

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$\Rightarrow$ $\chi_S^\kappa \overset{O(4)}{\sim} \chi_P^K$ **degeneration** $(\hat{m} = \langle \bar{q}q \rangle_l = 0)$

$\Rightarrow$ **Phys.case: dictated by (subtracted) condensate:**

$$\chi_S^\kappa - \chi_P^K = \frac{2m_s}{m_s^2 - \hat{m}^2}\Delta_{l,s}$$

measured in lattice

WI and Screening Masses

Assuming soft T behavior for residues and M_{sc}/M_{pole} of correlators $K_{P,S}$:

$\chi_{P,S} = K_{P,S}(p=0) \sim M_{pole}^{-2} \sim M_{sc}^{-2} \rightarrow$ **measured in lattice**

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$$\frac{M_\pi^{sc}(T)}{M_\pi^{sc}(0)} \sim \left[\frac{\chi_P^\pi(0)}{\chi_P^\pi(T)}\right]^{1/2} = \left[\frac{\langle \bar{q}q\rangle_l(0)}{\langle \bar{q}q\rangle_l(T)}\right]^{1/2}$$

$$\frac{M_K^{sc}(T)}{M_K^{sc}(0)} \sim \left[\frac{\chi_P^K(0)}{\chi_P^K(T)}\right]^{1/2} = \left[\frac{\langle \bar{q}q\rangle_l(0) + 2\langle \bar{s}s\rangle(0)}{\langle \bar{q}q\rangle_l(T) + 2\langle \bar{s}s\rangle(T)}\right]^{1/2}$$

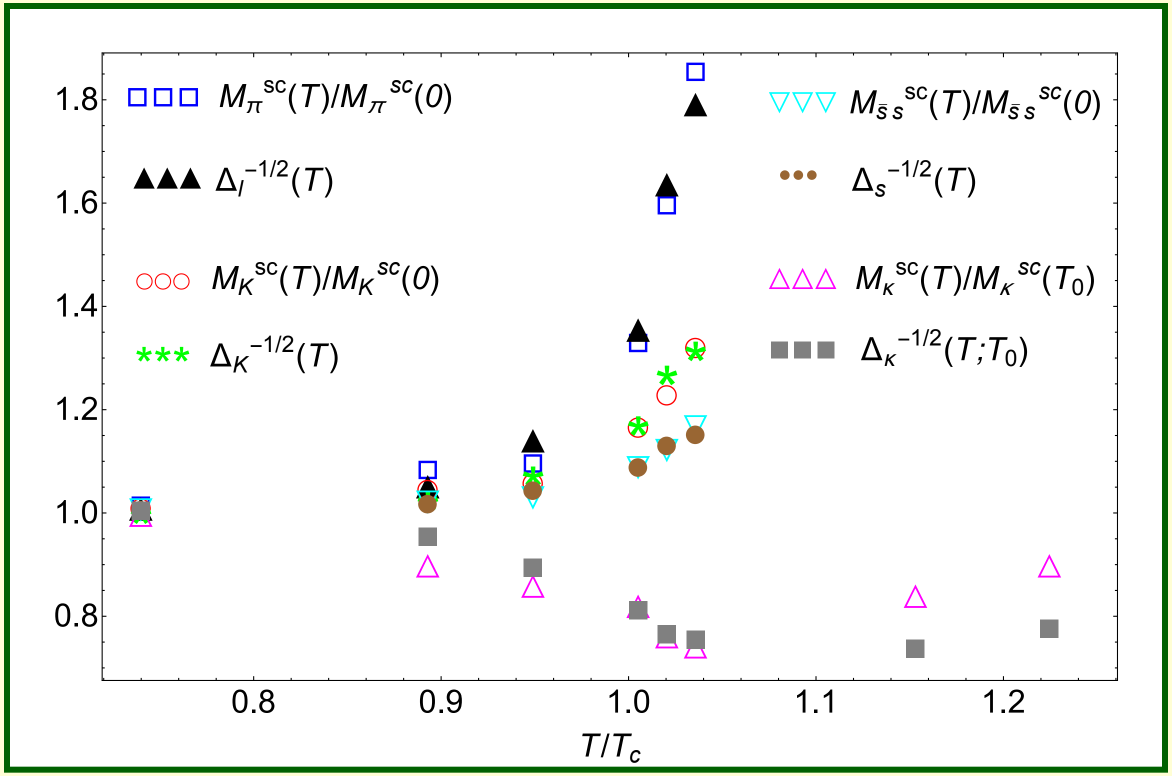
$$\frac{M_{\bar{s}s}^{sc}(T)}{M_{\bar{s}s}^{sc}(0)} \sim \left[\frac{\chi_P^{\bar{s}s}(0)}{\chi_P^{\bar{s}s}(T)}\right]^{1/2} \sim \left[\frac{\langle \bar{s}s\rangle(0)}{\langle \bar{s}s\rangle(T)}\right]^{1/2}$$

$$\frac{M_\kappa^{sc}(T)}{M_\kappa^{sc}(0)} \sim \left[\frac{\chi_S^\kappa(0)}{\chi_S^\kappa(T)}\right]^{1/2} = \left[\frac{\langle \bar{q}q\rangle_l(0) - 2\langle \bar{s}s\rangle(0)}{\langle \bar{q}q\rangle_l(T) - 2\langle \bar{s}s\rangle(T)}\right]^{1/2}$$

Anomalous contrib. $\frac{\hat{m}}{m_s}$ **suppressed**

WI and Screening Masses

Same lattice setup for masses (Cheng et al EPJC'11) and condensates (PRD'08)



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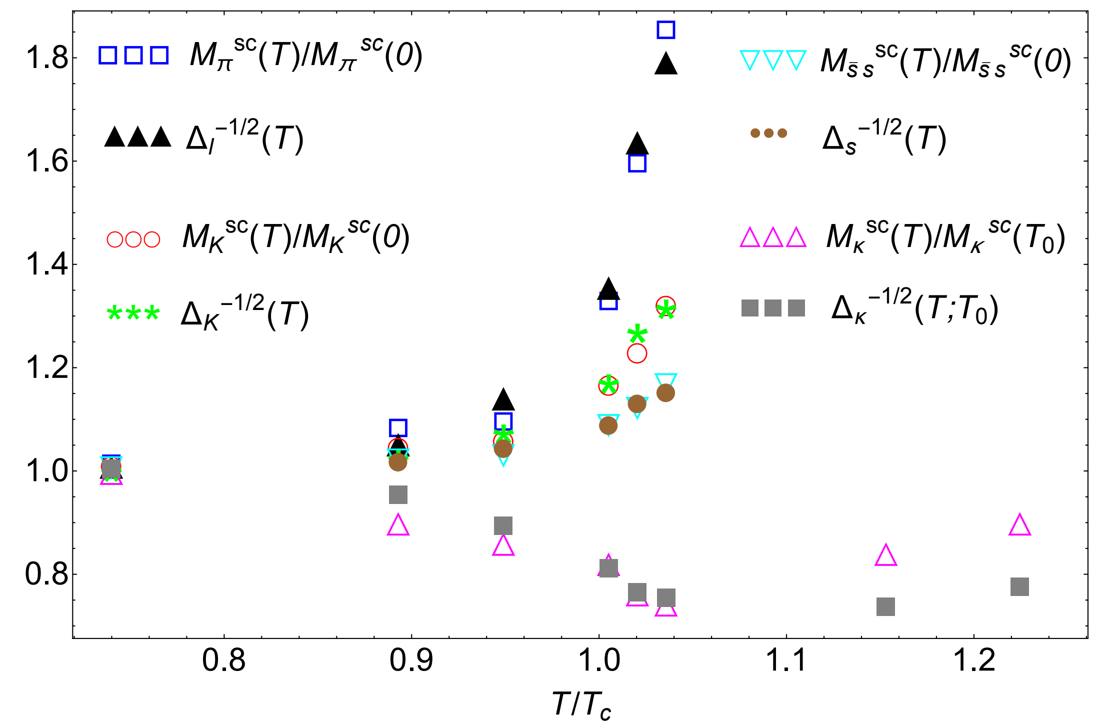

- **$< 5\%$ deviations** below T_c from predicted WI scaling
- **Δ_i subtracted condensates** with two fit parameters to eliminate $T = 0$ lattice divergences $\langle \bar{q}_i q_i \rangle \sim m_i/a^2 + \ldots$
- **Rapid T_c increase** due to $M_\pi^{sc} \sim \langle \bar{q} q \rangle_l^{-1/2}$. **Softer** $M_K^{sc} \sim (\langle \bar{q} q \rangle_l + 2\langle \bar{s} s \rangle)^{-1/2}$ (soft T-dep $\langle \bar{s} s \rangle$). **Even softer** $M_{\bar{s}s}^{sc} \sim \langle \bar{s} s \rangle^{-1/2}$ (no light contrib.)
- **κ minimum** from condensate diff. (last two points not fitted)

Low-energy realization: effective meson theories

- **WI defined only formally in QCD, up to renormalization.**
- **Effective Theories needed below the transition to verify WI and study partner degeneration.**
- **ChPT model-independent framework for π, K, η, η'.**
- **HRG approach to include (free) heavier states (T_c reduction)**
- **$\pi\pi$ scattering dominant interaction process.**
- **Unitarized ChPT (scattering) generates (thermal) resonances**

Gasser, Gerber, Leutwyler, 1987, 1989

Karsch, Tawfik, Redlich 2003, Tawfik-Toublan 2005, Jankowski, Blaschke, Spalinski 2013

Cabrera, Dobado, Fernández-Fraile, AGN, Llanes-Estrada, Peláez, Ruiz de Elvira, Torres-Andrés 2002-

Low-energy realization: effective meson theories

$\Rightarrow$ **WI verified in $U(3)$ ChPT* to NNLO in $\delta \sim 1/N_c \sim m_q \sim T^2$**

* **to account consistently for $U_A(1)$ anomaly and η'**

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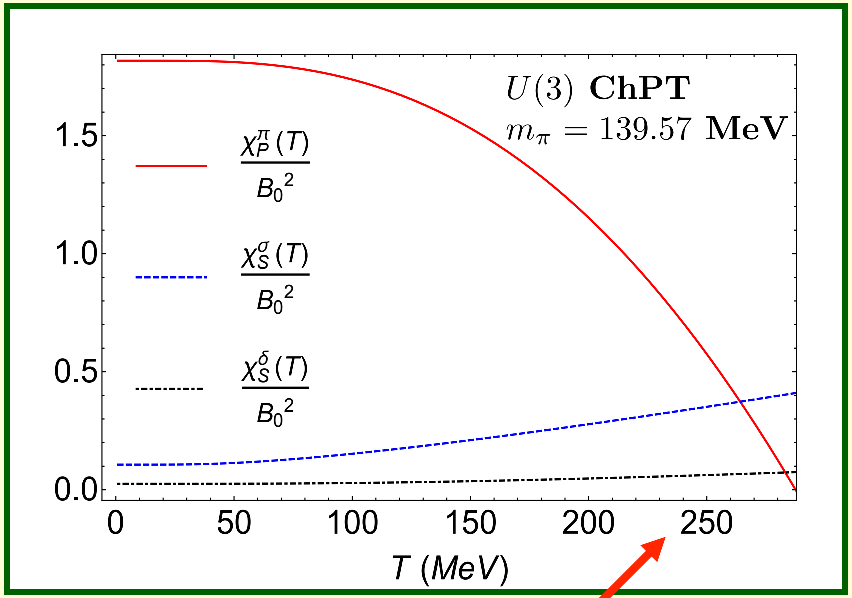
*** to account consistently for $U_A(1)$ anomaly and η'**



Differences within ChPT uncertainty in massive case.

Low-energy realization: effective meson theories

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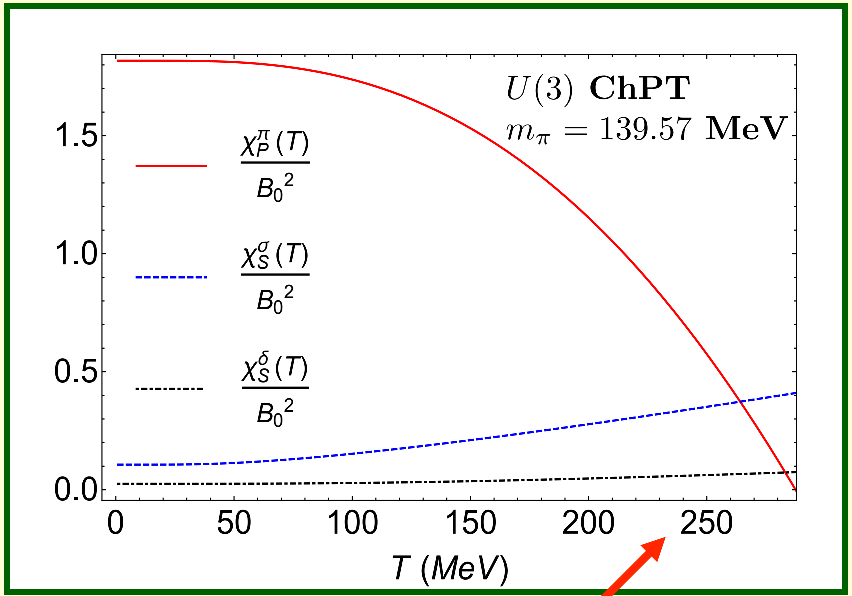
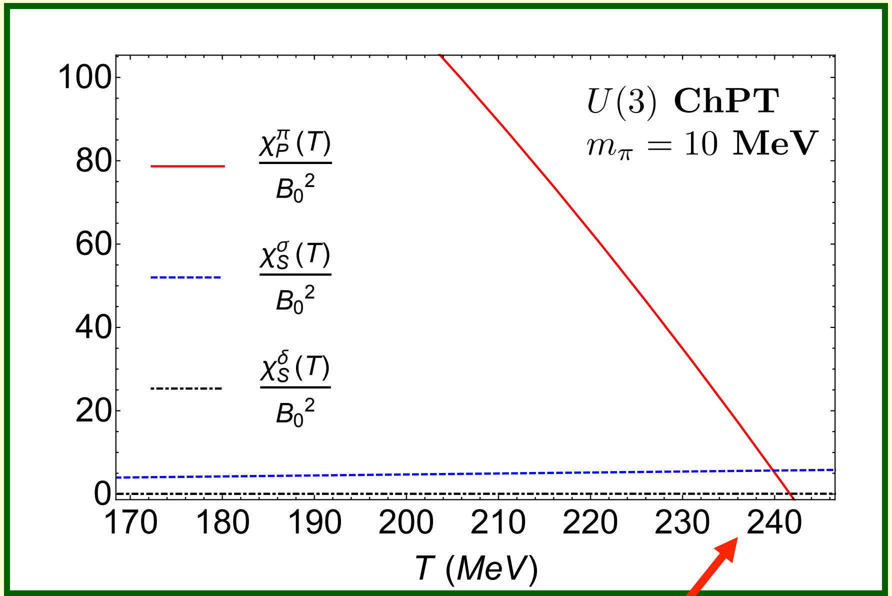




Differences within ChPT uncertainty in massive case.

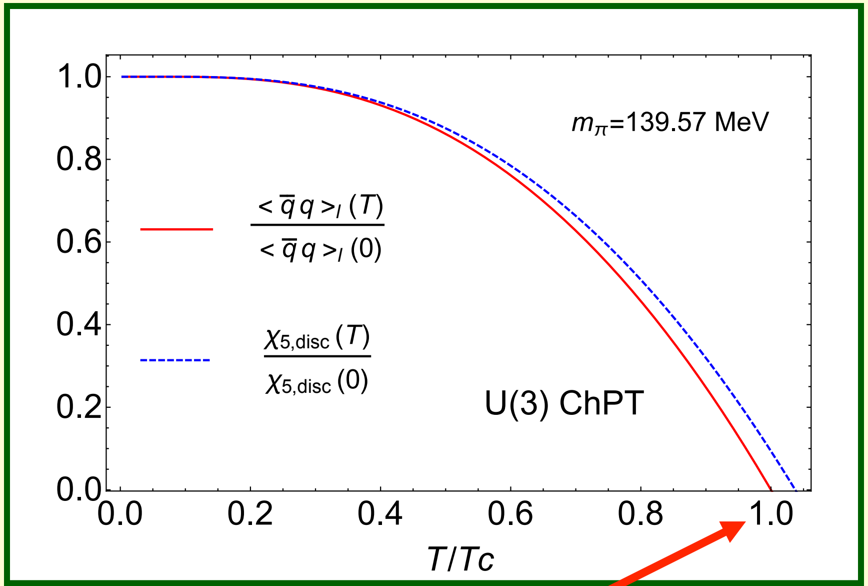
$\rightarrow O(4) \times U_A(1)$ **in chiral limit**

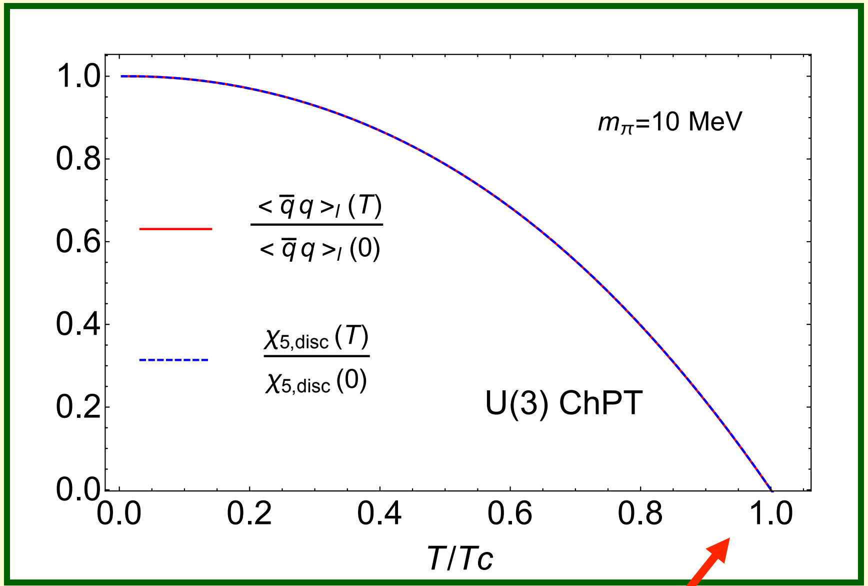
with $\frac{\chi_{5,disc}(T)}{\chi_{5,disc}(0)} \rightarrow \frac{\langle \bar{q}q \rangle_l (T)}{\langle \bar{q}q \rangle_l (0)}$

Low-energy realization: effective meson theories

$\Rightarrow$ **WI verified in $U(3)$ ChPT* to NNLO in $\delta \sim 1/N_c \sim m_q \sim T^2$**

* **to account consistently for $U_A(1)$ anomaly and η'**





Differences within ChPT uncertainty in massive case.

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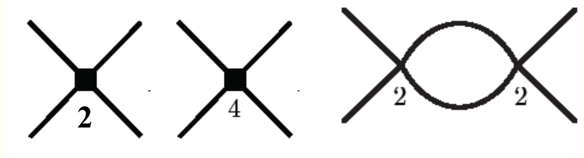
Unitarizing scattering: resonances



ChPT Partial waves $t^{IJ} = t_2^{IJ} + t_4^{IJ} + \ldots$

Unitarity $\rightarrow$ **Im** $t(s) = \sigma(s)|t(s)|^2 \;\; (s \geq 4M^2) \Rightarrow$ **Im** $t^{-1} = -\sigma$

$$\sigma(s) = \sqrt{1 - \frac{4M^2}{s}} = \frac{p_{CM}}{\sqrt{s}}$$ **two-particle phase space**

$$t^U(s;T) = \frac{[t_2(s)]^2}{t_2(s) - t_4(s;T)}$$

(IAM)

Exactly proven for large NGB and chiral limits:

S.Cortés, AGN, J.Morales '16

FINITE TEMPERATURE:

$$t_4(s) \rightarrow t_4(s;T)$$

$$\sigma \rightarrow \sigma\left[1 + 2n_B(\sqrt{s}/2)\right] \equiv \sigma_T$$

Thermal phase Space.

Bose net enhancement $(1+n)^2 - n^2$

A.Dobado, D.Fernández-Fraile, AGN, F.J.Llanes-Estrada, J.R.Peláez,E.Tomás-Herruzo, '02 '05 '07

The σ/f0(500) and chiral symmetry restoration

★ **Saturate the scalar correlator with the $f_0(500)$ thermal state:**

(assuming $p=0$ pole not very diff. from s_p)

$$\chi_S^U(T) = \frac{\chi_S^{ChPT}(0) M_S^2(0)}{M_S^2(T)}$$

$$s_p = (M_p - i\Gamma_p/2)^2 \rightarrow M_S^2 = M_p^2 - \Gamma_p^2/4$$

$$\chi_S \propto G_S(p=0) \sim \frac{1}{\mathbf{Re}(\Sigma_S)} \sim \frac{1}{M_S^2}$$

Normalization to match $T=0$ ChPT.
Compensates pole diff.

★ **Unitarized condensate from χ^U requires additional scaling assumptions (holding in ChPT):**

$$\delta f(T) = f(T) - f(0)$$

$$\delta \langle \bar{q} q \rangle^U (T, M) = B_0 T^2 g(T/M)$$

$x_0 \ll 1$ matching point

$$\delta \chi_S^U = B_0^2 h(T/M)$$

$$g(x) = g(x_0) + \int_{x_0}^{x} \frac{h(y)}{y^3} dy \quad (x > x_0)$$

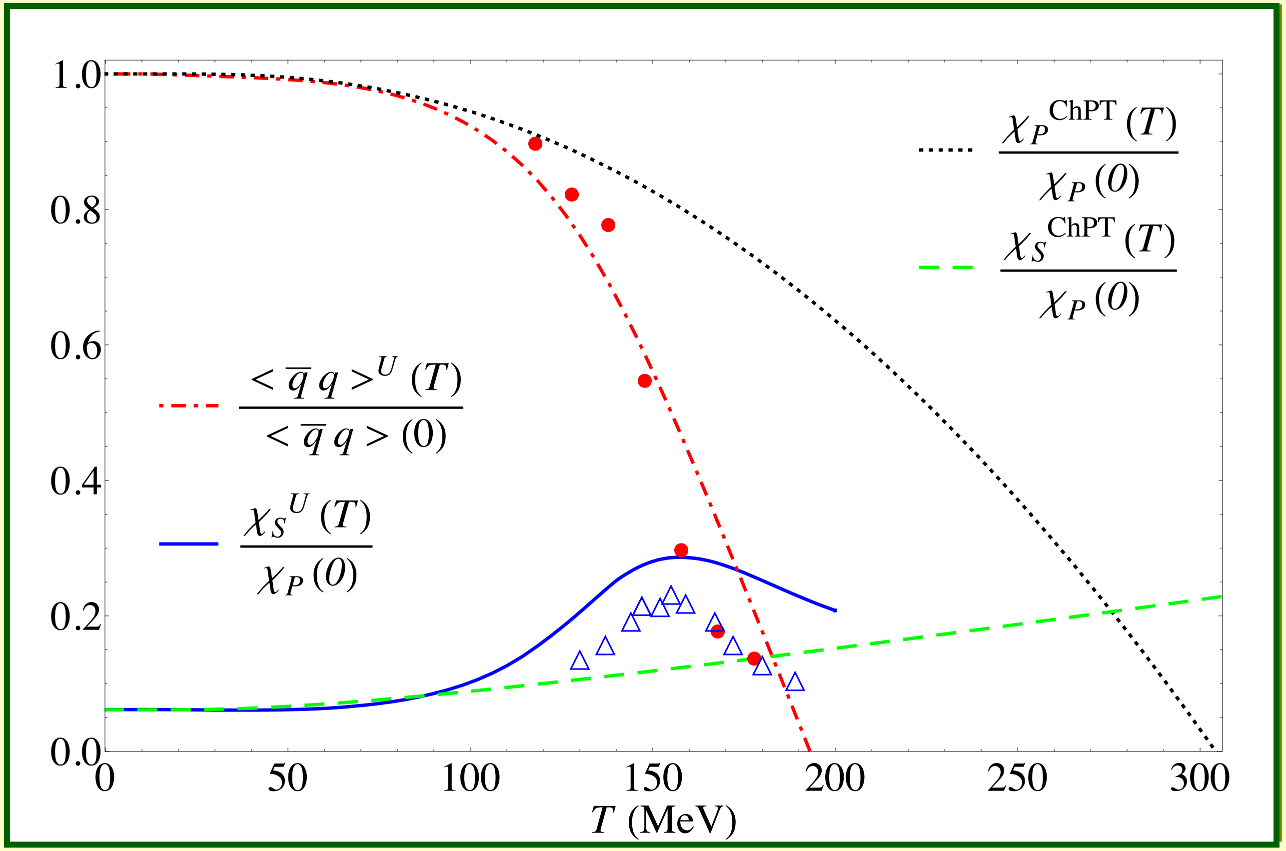
$$g(x) = g_{ChPT}(x) \quad (x \leq x_0)$$

The σ/f0(500) and chiral symmetry restoration

Data from
Y.Aoki et al JHEP 09

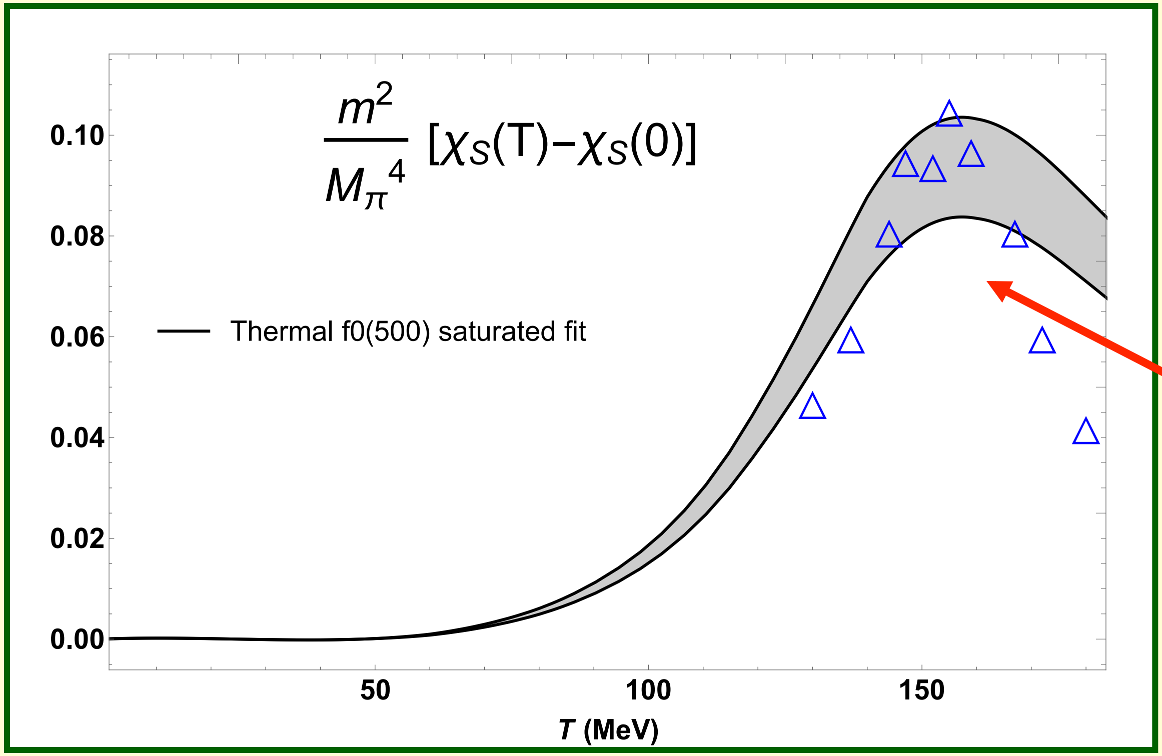
Not a fit!
↓
LEC fixed at $T = 0$:
$M_p = 441$ **MeV**
$\Gamma_p =$ **466 MeV**

Robust under unit.method and LEC uncert.



- ★ **Improving of critical behaviour** → χ_S^U **peak at** $T_c =$ **157 MeV**
  $T_c \downarrow$ **and more abrupt** χ_S^U **near chiral limit** ⇒ **Thermal** $f_0(500)$ **crucial!**
- ★ **low-T χ_S^U and $\langle \bar q q \rangle^U$ OK with ChPT**
- ★ **S/P intersection near χ_S^U peak**

Unitarized susceptibility fits & HRG

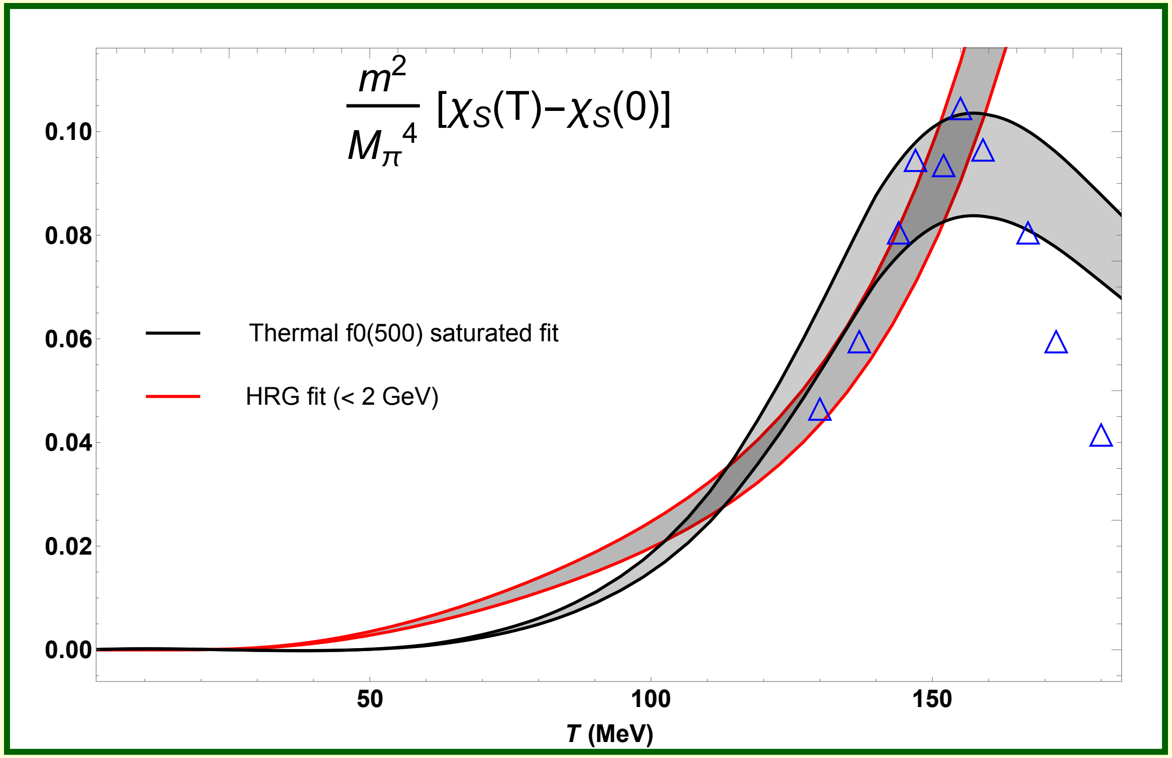


data above T_c not fitted

$$\chi_S^U(T) = A\frac{M_\pi^4}{4m_q^2}\frac{M_S^2(0)}{M_S^2(T)}$$

$A = 0.1 \pm 0.01$ **fit param.**

Unitarized susceptibility fits & HRG



data above T_c not fitted

- HRG fit based on Jankowski et al (2013) HRG quark mass dependence
- f0(500) saturated model accounts better for data around T_c

CONCLUSIONS

- ★ **WI useful for chiral pattern and related partner degeneration**

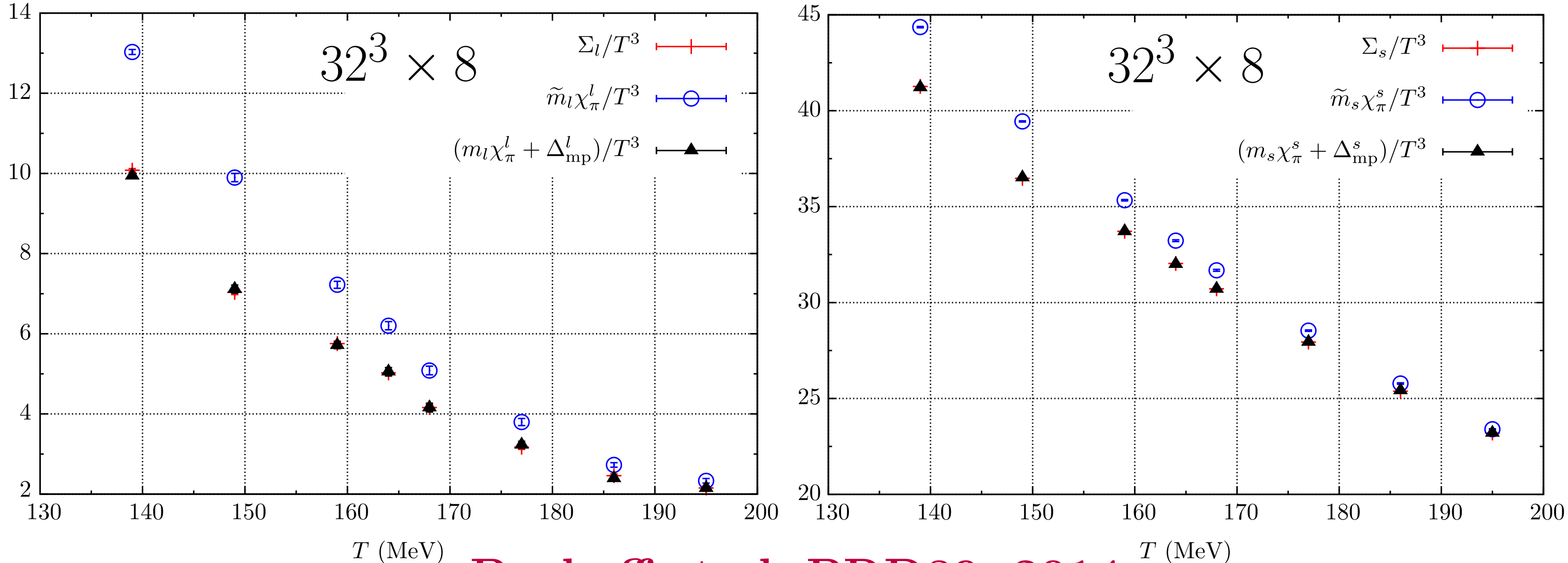
- ★ **Consistent with $O(4)\times U(1)_A$ for exact restoration.**
  $\chi_{5,disc}$ **scaling governed by $\langle \bar{q}q \rangle_l$ in phys.lim.**

- ★ **WI scaling of meson screening masses consistent with lattice**

- ★ **Thermal $f_0(500)$ relevant $\rightarrow$ saturated χ_S^U OK with lattice data**

BACKUP SLIDES

Check of WI in lattice



Buchoff et al, PRD89, 2014

- ⋆ **Both π and $\bar{s}s$ channel need compensating lattice current to reduce finite-size effects**
- ⋆ **Small deviations in $\bar{s}s$ channel compatible with anomaly suppression**
- ⋆ **No results for K channel (so far) which would test $\langle \bar{q}q \rangle_l + 2\langle \bar{s}s \rangle$ combination**

Chiral Patterns and Partners from WI

2p vs 3p WI FOR CHIRAL PARTNERS

$O(4)$ **partners**

$$P_{\pi\pi}(y) - S_{ll}(y) = \hat{m}\int_T dx\,\langle \mathcal{T}\sigma_l(y)\pi(x)\pi(0)\rangle$$

$$P_{ll}(y) - S_{\delta\delta}(y) = \hat{m}\int_T dx\,\langle \mathcal{T}\delta(y)\pi(x)\eta_l(0)\rangle$$

$$P_{ls}(y) = \frac{1}{3}\hat{m}\int_T dx\,\langle \mathcal{T}\eta_s(y)\pi(x)\delta(0)\rangle$$

$$S_{ls}(y) = -\frac{1}{3}\hat{m}\int_T dx\,\langle \mathcal{T}\sigma_s(y)\pi(x)\pi(0)\rangle$$

$$d^{abc}\left[P_{KK}(y) - S_{\kappa\kappa}(y)\right] = \hat{m}\int_T dx\,\left\langle \mathcal{T}K^b(y)\kappa^c(x)\pi^a(0)\right\rangle$$

Chiral Patterns and Partners from WI

2p vs 3p WI FOR CHIRAL PARTNERS

$O(4)$ **partners**

$\sigma\pi\pi$ **vertex**

$\rightarrow$ $\pi\pi$ **scattering** $I = J = 0$

$$P_{\pi\pi}(y) - S_{ll}(y) = \hat{m}\int_T dx\,\langle\mathcal{T}\sigma_l(y)\pi(x)\pi(0)\rangle$$

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Chiral Patterns and Partners from WI

2p vs 3p WI FOR CHIRAL PARTNERS

$U(1)_A$ **partners**

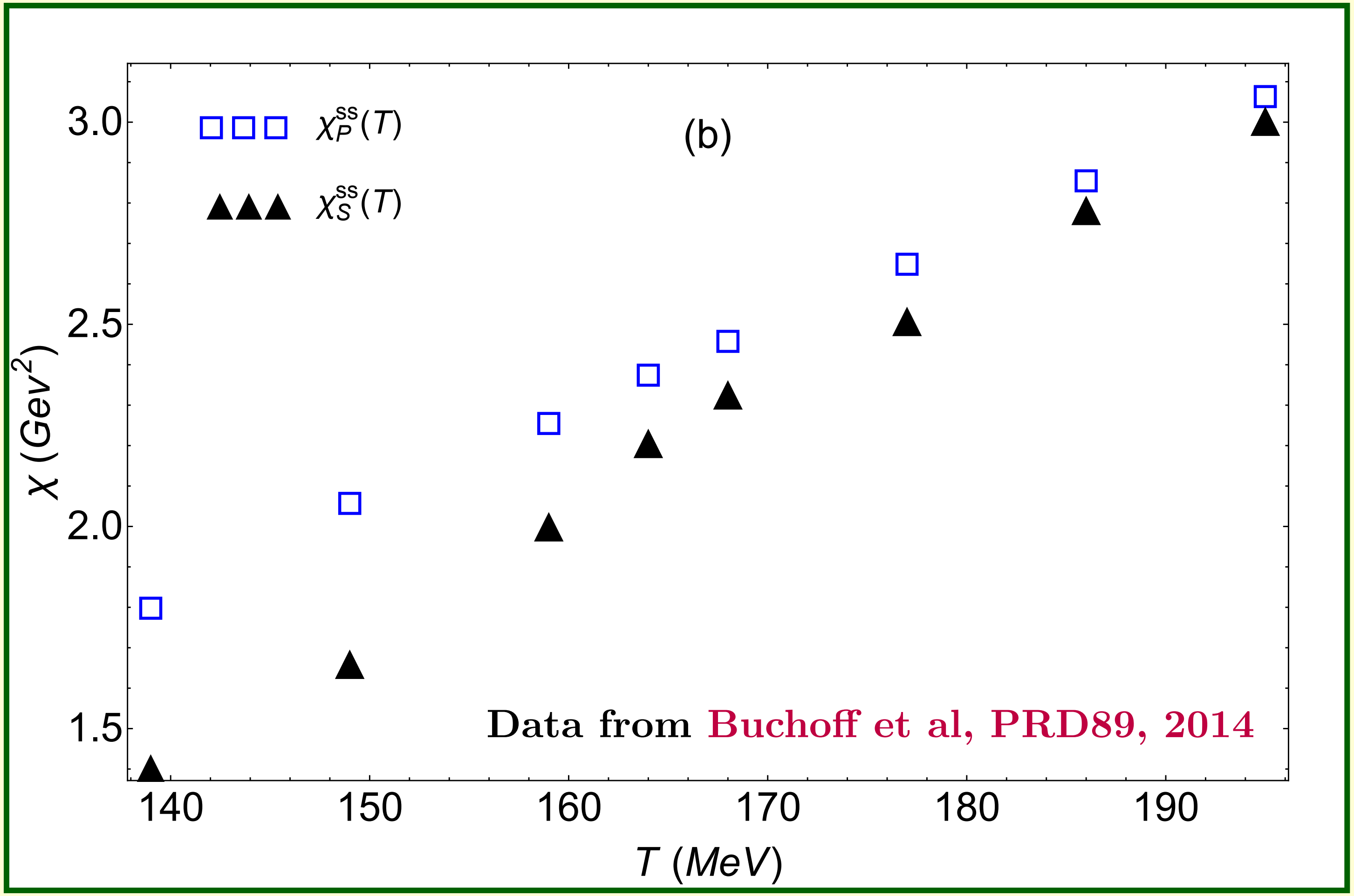
$$
\begin{aligned}
P_{\pi\pi}(y) - S_{\delta\delta}(y) &= \int_T dx\, \langle \mathcal{T} \pi(y) \delta(0) \tilde{\eta}(x) \rangle \\
P_{ll}(y) - S_{ll}(y) &= \int_T dx\, \langle \mathcal{T} \eta_l(y) \sigma_l(0) \tilde{\eta}(x) \rangle \\
P_{ls}(y) - S_{ls}(y) &= \int_T dx\, \langle \mathcal{T} \eta_l(y) \sigma_s(0) \tilde{\eta}(x) \rangle \\
P_{ss}(y) - S_{ss}(y) &= \int_T dx\, \langle \mathcal{T} \eta_s(y) \sigma_s(0) \tilde{\eta}(x) \rangle \\
P_{KK}(y) - S_{\kappa\kappa}(y) &= \int_T dx\, \langle \mathcal{T} K(y) \kappa(0) \tilde{\eta}(x) \rangle
\end{aligned}
$$

$\tilde{\eta}(x) = \hat{m}\eta_l(x) + m_s \eta_s(x) + \frac{1}{2} A(x)$ **three sources of** $U(1)_A$ **breaking**

Chiral Patterns and Partners from WI

Under $U(1)_A$ rotations $P_{ss} \overset{U(1)_A}{\sim} S_{ss}$.

Also testable in lattice



WI and Lattice Screening Masses

Subtracted Condensates have the right critical behavior in lattice, avoiding $T=0$ finite-size divergences $\langle \bar{q}_i q_i \rangle \sim m_i/a^2 + \ldots$:

$$\Delta_l(T) = \frac{\langle \bar{q}q \rangle_l (T) - \langle \bar{q}q \rangle_l (0) + \langle \bar{q}q \rangle_l^{ref}}{\langle \bar{q}q \rangle_l^{ref}}$$

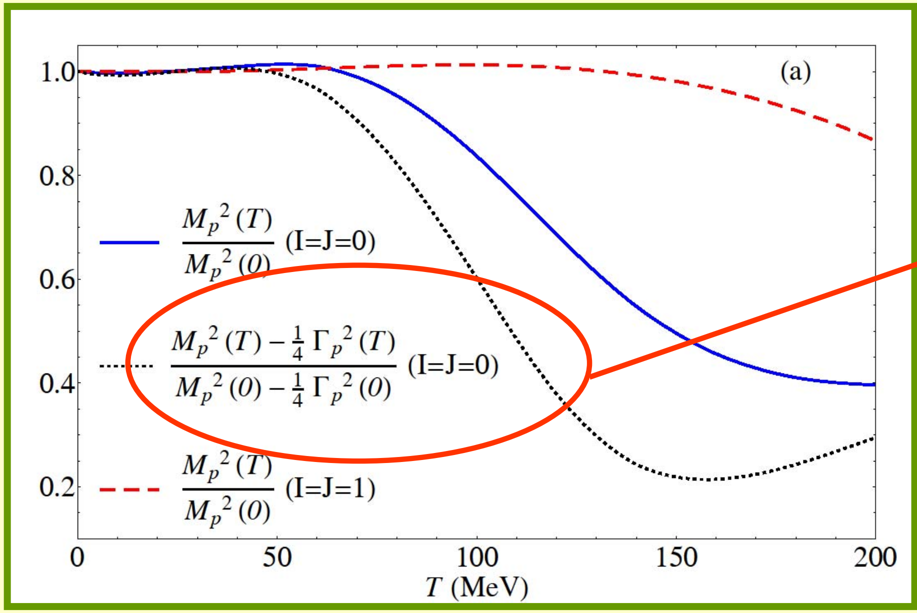
$$\Delta_K(T) = \frac{\langle \bar{q}q \rangle_l (T) - \langle \bar{q}q \rangle_l (0) + 2\left[\langle \bar{s}s \rangle (T) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{q}q \rangle_l^{ref} + \langle \bar{s}s \rangle^{ref}}{\langle \bar{q}q \rangle_l^{ref} + \langle \bar{s}s \rangle^{ref}}$$

$$\Delta_s(T) = \frac{2\left[\langle \bar{s}s \rangle (T) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{s}s \rangle^{ref}}{\langle \bar{s}s \rangle^{ref}}$$

$$\Delta_\kappa(T;T_0) = \frac{\langle \bar{q}q \rangle_l (T) - \langle \bar{q}q \rangle_l (0) - 2\left[\langle \bar{s}s \rangle (T) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{q}q \rangle_l^{ref} - \langle \bar{s}s \rangle^{ref}}{\langle \bar{q}q \rangle_l (T_0) - \langle \bar{q}q \rangle_l (0) - 2\left[\langle \bar{s}s \rangle (T_0) - \langle \bar{s}s \rangle (0)\right] + \langle \bar{q}q \rangle_l^{ref} - \langle \bar{s}s \rangle^{ref}}$$

$$\begin{aligned} r_1^3 \langle \bar{q}q \rangle_l^{ref} &= 0.750 \\ r_1^3 \langle \bar{s}s \rangle^{ref} &= 1.061 \\ r_1 &\simeq 0.31 \text{ fm} \end{aligned}$$

The σ/f0(500) and chiral symmetry restoration



$$M_S^2(T) = M_p^2(T) - \Gamma_p^2(T)/4$$

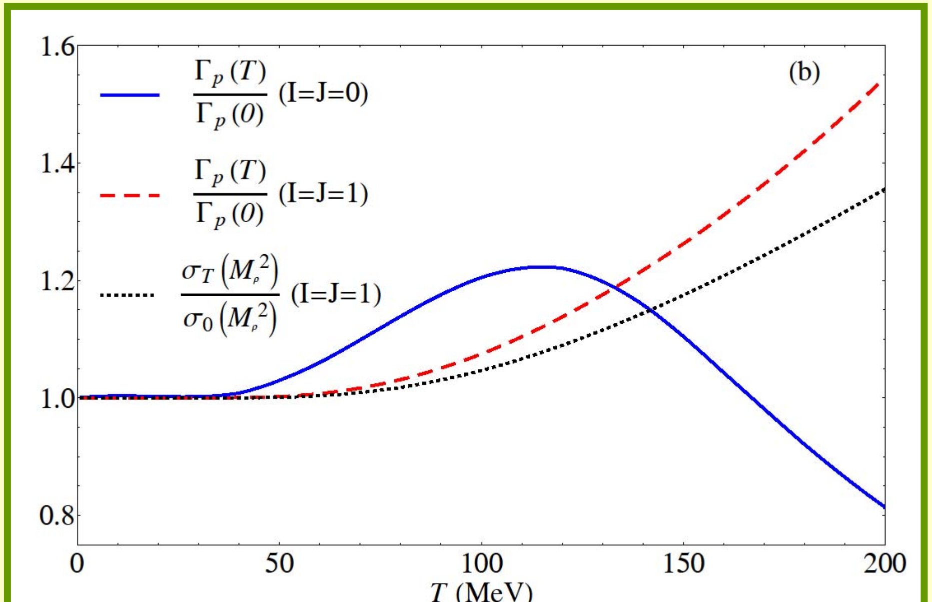
scalar pole mass

Chiral restoring behaviour !

LEC fixed at $T=0$ comp.with PDG:

$f_0(500) : M_p^{00} = 441$ MeV; $\Gamma_p^{00} = 466$ MeV

$\rho(770 : M_p^{11} = 756$ MeV; $\Gamma_p^{11} = 151$ MeV



Pole position:

$$s_p(T) = [M_p(T) - i\Gamma_p(T)/2]^2$$

(2nd Riemann sheet)

S/P susceptibilities at low energies

$$\chi_P(T)\delta^{ab} = \int_0^\beta \int d^3\vec{x}\, \langle \mathcal{T}\left(\bar{q}\gamma_5\tau^a q\right)(x)\left(\bar{q}\gamma_5\tau^b q\right)(0)\rangle$$

$$\chi_S(T) = -\frac{\partial}{\partial m}\langle\bar{q}q\rangle_T = \int_0^\beta d\tau \int d^3\vec{x}\left[\langle\mathcal{T}(\bar{q}q)(x)(\bar{q}q)(0)\rangle_T - \langle\bar{q}q\rangle_T^2\right]$$

Expected to be saturated by π and σ-like poles:

$$\chi_P = 4B_0^2F_\pi^2 G_\pi(p^2=0) \sim 4B_0^2\frac{F_\pi^2}{M_\pi^2} = -\frac{\langle\bar{q}q\rangle}{m_q}$$

from PCAC+GOR $(T=0)$ **or LO ChPT**

$$B_0 = M_\pi^2/2m_q$$

$$\chi_S = 4B_0^2F_\pi^2 G_\sigma(p^2=0) \sim \frac{4B_0^2F_\pi^2}{M_\sigma^2}$$

from $\mathcal{L}_{SB} = 2B_0F_\pi s(x)\sigma(x)$

But no need to deal with a particle-like σ state.
$\Rightarrow$ suitable for ChPT (model independent) and UChPT

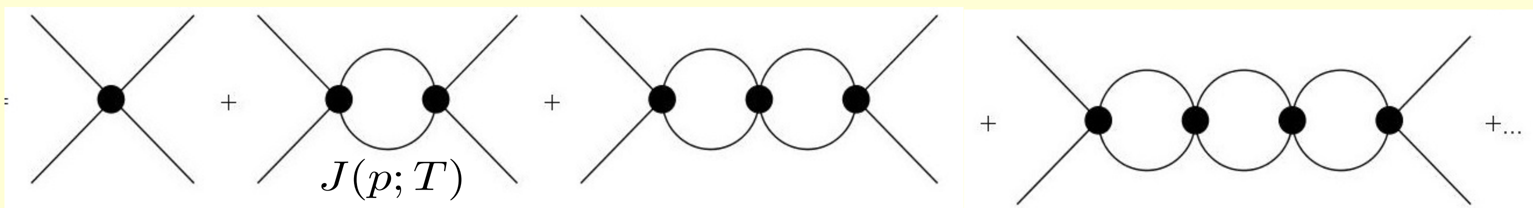
Large-N_{GB} NLSM at finite temperature (chiral limit)

S.Cortés, AGN, J.Morales, PRD93 (2016) 036001

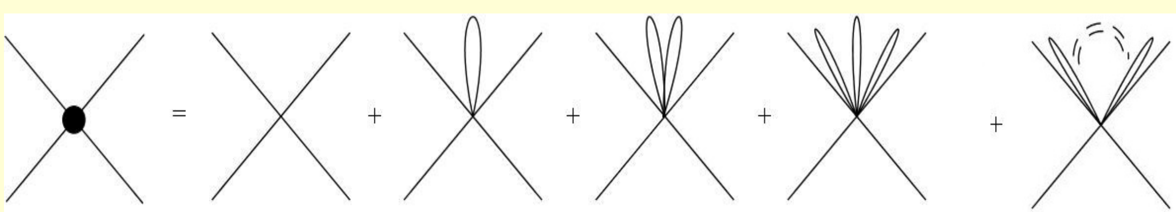
- $S^N = \dfrac{O(N+1)}{O(N)}$ **formulation:**

$$\mathcal{L}_{NLSM} = \frac{1}{2} g_{ab}(\pi)\partial_\mu \pi^a \partial^\mu \pi^b; \qquad g_{ab}(\pi) = \delta_{ab} + \frac{1}{NF^2}\frac{\pi_a \pi_b}{1-\pi^2/NF^2}$$

- **Leading order scattering at finite T:**

$A(p;T) =$ 

$$= \boxed{\frac{s}{NF^2}\frac{f(T)}{1-\frac{s}{2F^2}f(T)J(p;T)}} \qquad f(T) = \frac{1}{1-\frac{T^2}{12F^2}}$$

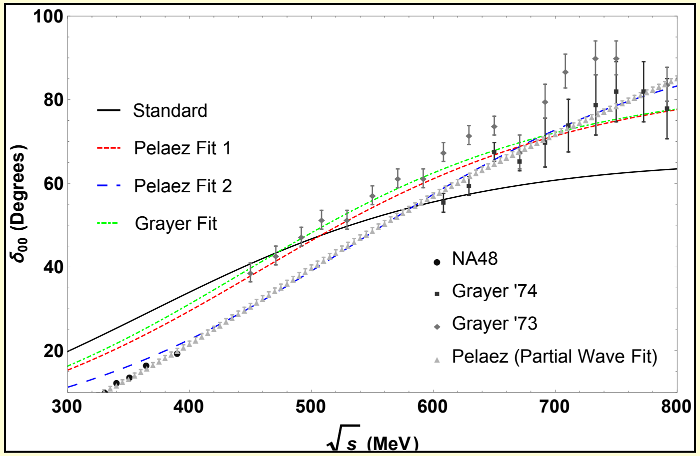


$$= \frac{s}{NF^2} f(T)$$

Large-N_{GB} NLSM at finite temperature (chiral limit)

S.Cortés, AGN, J.Morales, PRD93 (2016) 036001

- **Thermal Unitarity exact:** $\text{Im}\, t_{IJ}(s;T) = \sigma_T |t_{IJ}(s;T)|^2$
- **Renormalizable with $T=0$ scheme $\rightarrow$ two free parameters F, μ**
- **$I=J=0$ phase shift and $f_0(500)$ thermal pole consistent with data and 2nd order chiral symmetry restoration (chiral limit)**



Parameter set	T_c (MeV)
Grayer	92.33
Peláez 1	96.00
Peláez 2	129.07
IAM	118.23