AdS$_2$, SYK and wormholes

Juan Maldacena

Based on: - JM , Xiaoliang Qi .
- JM, Douglas Stanford and Zhenbin Yang.
- Ioanna Kourkoulou and JM
- JM, Douglas Stanford

+ many other papers by other authors

50th Anniversary of the Veneziano Model

Reflexions on the Veneziano Model

- It is important to have concrete and interesting formulas.
- Even if they are “wrong” for the phenomenon they were initially devised for.
- Maybe string theory will be useful for something else than what we imagine now!

AdS_2 , SYK and wormholes

Introduction

- Quantum mechanical systems with a finite but large number, N, of degrees of freedom (qubits).
- They have a 1/N expansion and are strongly coupled.
- Develop an approximate scale invariant behavior at relatively low energies.
- We will focus on universal features.
- We find these universal features in two systems:
- SYK model = relatively simple solvable model.
- Near extremal black holes = relatively simple gravitational system.

The first system

Sachdev, Ye, Kitaev model (SYK)

N Majorana fermions $\{\psi_i, \psi_j\} = \delta_{ij}$

Sachdev Ye Kitaev
Georges, Parcollet

$$H = \sum_{i_1, \cdots, i_4} J_{i_1 i_2 i_3 i_4} \psi_{i_1} \psi_{i_2} \psi_{i_3} \psi_{i_4}$$

Random couplings, gaussian distribution. $\langle J^2_{i_1 i_2 i_3 i_4} \rangle = J^2/N^3$

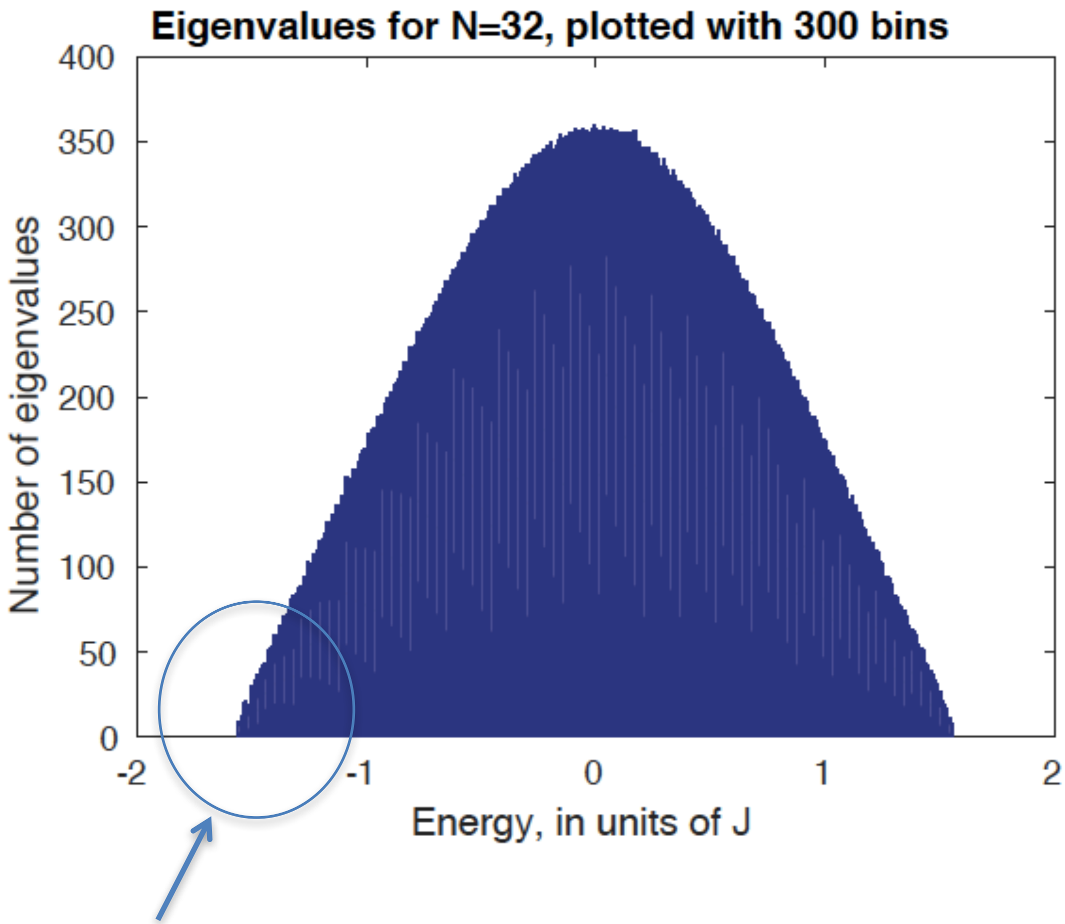
To leading order → treat J_{ijkl} as an additional "field" . (There are similar models with no disorder: tensor models Gurau, Witten, Klebanov et al…)

J = dimensionful coupling. We will be interested in the strong coupling region

$$1 \ll \beta J, \ \tau J \ll N$$

Spectrum

D. Stanford



$$\dim_H = 2^{\frac{N}{2}}$$

(specific J's drawn from the gaussian ensemble)

Exponentially large number of states contributes to the low energy region we consider

Large N effective action

Integrate out the fermions and the couplings to obtain an effective action for the singlets, the fermion bilinears.

$$S = \frac{N}{2}\left[\log\det(\partial_t - \Sigma) - \int d\tau d\tau' \Sigma(\tau,\tau')G(\tau,\tau') + \frac{J^2}{4}G(\tau,\tau')^4\right]$$

Outline of the derivation

$$Z = \int dj \int D\psi \exp\{\int dt \left[i \int \psi^i \dot{\psi}^i + j_{lkmr}\psi^l\psi^k\psi^m\psi^l\right] - j^2_{lkmr}N^3/J^2\}$$

Integrate over j_{lkmr}

$$Z = \int d\psi \exp\{i \int dt \psi^l \dot{\psi}^l + N \int dt dt' [\frac{1}{N}\psi^l(t)\psi^l(t')]^4\}$$

Insert a 1

$$1 = \int DG \delta(G - \frac{1}{N}\psi^i(t)\psi^i(t')) = \int DG D\Sigma e^{i\int dt dt' \Sigma(t,t)(NG(t,t') - \psi^i(t)\psi^i(t'))}$$

Integrate out fermions

$$Z = \int DG D\Sigma \exp\{N \left[Pf(\partial_t - \Sigma) + \int dt dt' \left(G(t,t')\Sigma(t,t') + J^2 G(t,t')^4\right)\right]\}$$

Large N effective action

$$S = \frac{N}{2}\left[\log\det(\partial_t - \Sigma) - \int d\tau d\tau' \Sigma(\tau,\tau')G(\tau,\tau') + \frac{J^2}{4}G(\tau,\tau')^4\right]$$

It is non-local in time. The bilocal terms come from the integral over the couplings.

This effective action is correct to leading orders, where we can ignore the replicas, $o(1/N^q)$

Similar actions were obtained for usual O(N) vector models.

Equations of motion from this action are relatively simple integral equations that can be solved numerically.

At low energies the solution is simple $\quad G_c(\tau,\tau') \propto \frac{1}{(\tau-\tau')^{2\Delta}} \quad \Delta = \frac{1}{4}$

It is scale invariant!

Scale vs conformal invariance

- Usually scale invariance → conformal invariance.
- In one dimensions: conformal invariance = full reparametrization symmetry.
- Is a symmetry of the low energy action

$$S = \frac{N}{2}\left[\log\det(\cancel{\partial_t} - \Sigma) - \int d\tau d\tau' \Sigma(\tau,\tau')G(\tau,\tau') + \frac{J^2}{4}G(\tau,\tau')^4\right]$$

If G is a solution, and we are given an arbitrary function f(τ), we can generate another solution:

$$G_c \longrightarrow G_{c,f}(\tau,\tau') = [f'(\tau)f'(\tau')]^{\Delta} G_c(f(\tau), f(\tau'))$$

Emergent reparametrization symmetry

Example: Go from zero the temperature to a finite temperature solution

$$G(\tau, \tau') \propto \frac{1}{(\tau - \tau')^{2\Delta}}$$

$$f(\tau) = \tan \frac{\pi \tau}{\beta}$$

$$G_f = \left[\frac{\pi}{\beta \sin \frac{\pi \tau}{\beta}} \right]^{2\Delta}$$

Zero modes of the action

Recall the conformal symmetry in the IR

$$G(\tau,\tau') \propto \frac{1}{(\tau-\tau')^{2\Delta}}$$

$$G \longrightarrow G_f(\tau,\tau') = [f'(\tau)f'(\tau')]^{\Delta} G(f(\tau), f(\tau'))$$

All these solutions have the same action in the strict IR limit.

Goldstone bosons → no action for f → would give a divergence if we do the path integral over f.

Solution: remember that the symmetry is also slightly broken.

Schwarzian action

Keep the leading term that breaks the symmetry and has the right properties

$$S = -\frac{N\alpha_s}{J}\int dt\, \mathrm{Sch}(f,t)\ , \qquad \mathrm{Sch}(f,t) = \left(\frac{f''}{f'}\right)' - \frac{1}{2}\frac{f''^2}{f'^2}$$

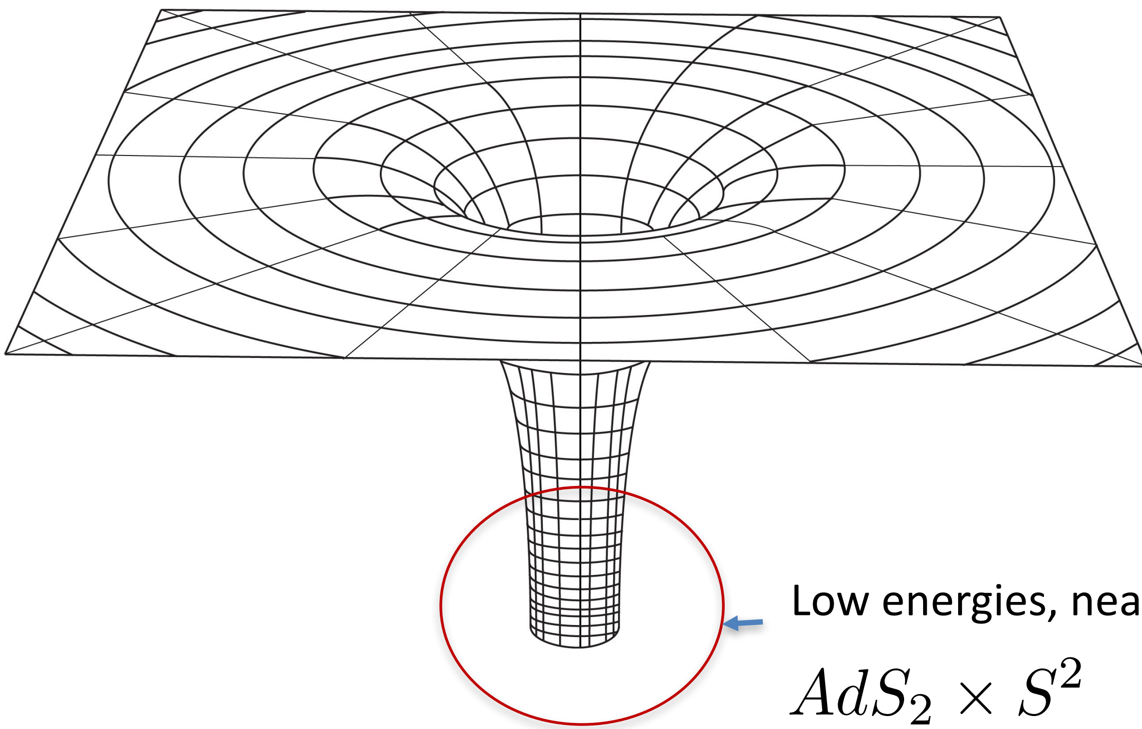
Numerical coefficient whose determination requires knowing the first deviation of the propagator from the IR conformal solution. Can be computed numerically.

This action governs several interesting aspects of the low energy dynamics.

It is coupled to another sector which (at this order) is exactly conformal: the non-zero modes of the effective action. They are organized in SL(2) representations.

The second system

Near extremal black holes



Charge black holes.

$$M \geq Q$$

$$M \sim Q$$

Low energies, near horizon

$$\underline{AdS_2} \times S^2$$

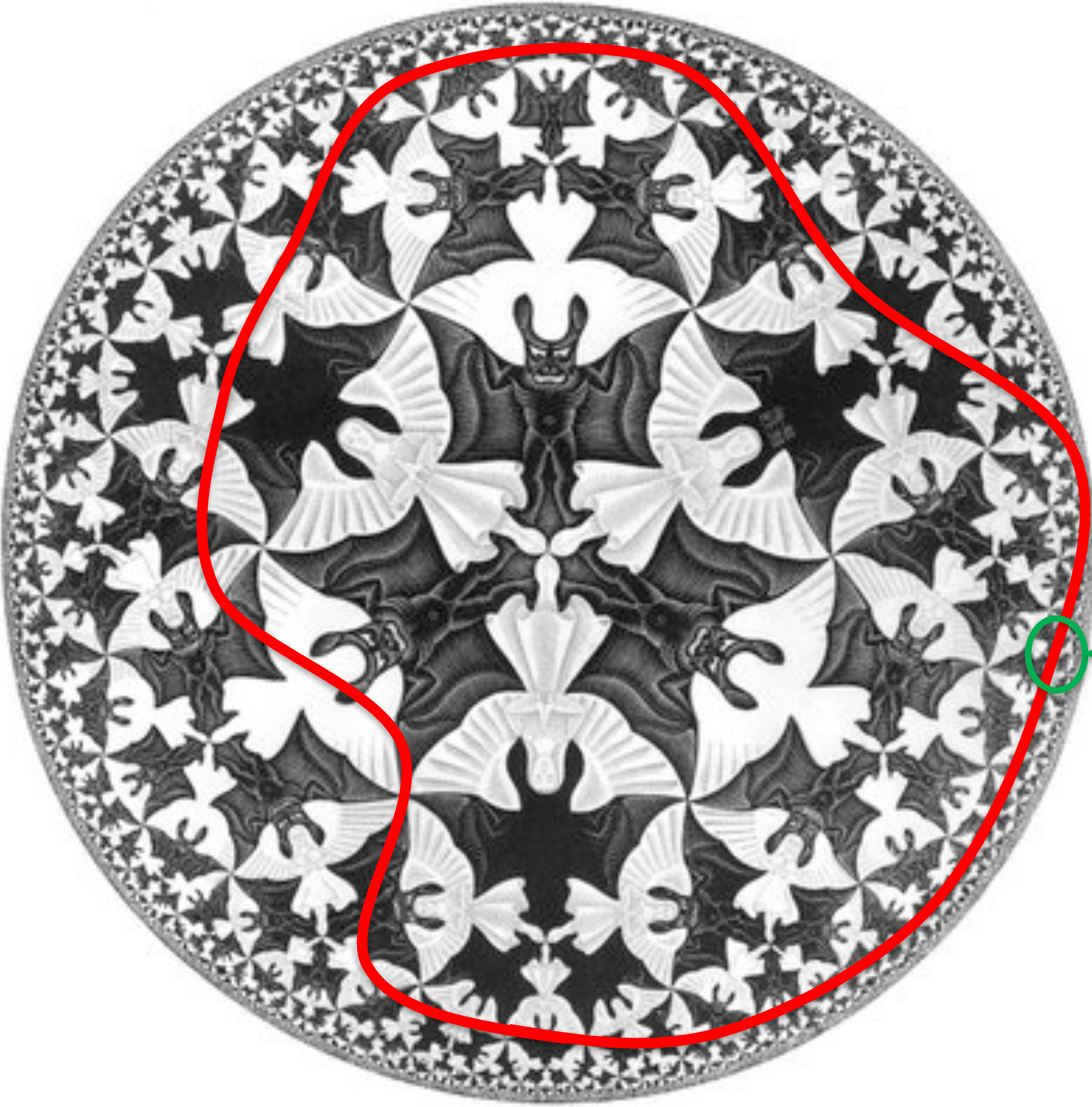
$$ds^2 = \frac{-dt^2 + dz^2}{z^2}$$

Scale invariant

Nearly AdS_2 gravity

Euclidean black hole

Region inside the red line



Infinite number of other configurations

Locally the same

Nearly AdS_2

Keep the leading effects that perturb away from AdS_2

Teitelboim Jackiw
Almheiri Polchinski

$$\int d^2x\sqrt{g}\phi(R+2) + \phi_0 \int d^2x\sqrt{g}R$$

Ground state entropy

Comes from the area of the additional dimensions, if we are getting this from 4 d gravity for a near extremal black hole.

Schwarzian action from Nearly AdS_2 gravity

No bulk propagating modes, only a boundary mode

$$S = \int d^2x\sqrt{g}\phi(R+2) - 2\frac{\phi_r}{\epsilon^2}\int duK \to$$

$$S = \frac{\phi_r\beta}{\epsilon^2} - \phi_r\int_0^\beta du Sch(t(u),u)$$

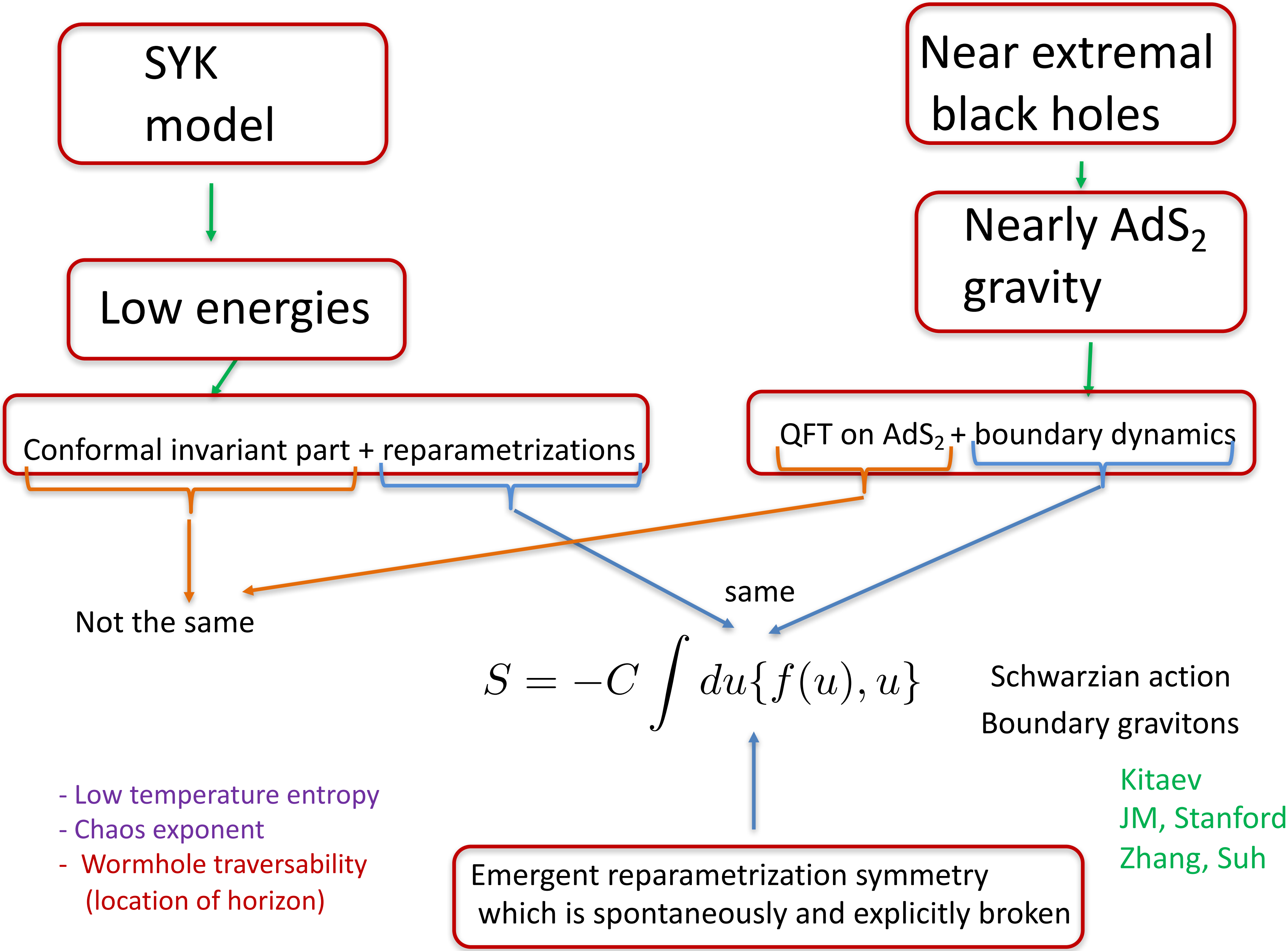
$$ds = \frac{du}{\epsilon} = \frac{t'du}{z} \to z = \epsilon t'(u)$$

Boundary time

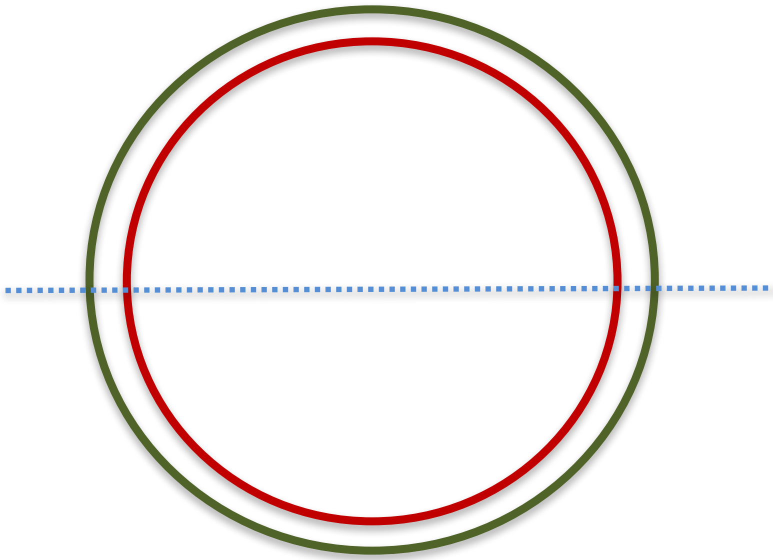
$$ds^2 = \frac{-dt^2+dz^2}{z^2}$$

Relation between the two

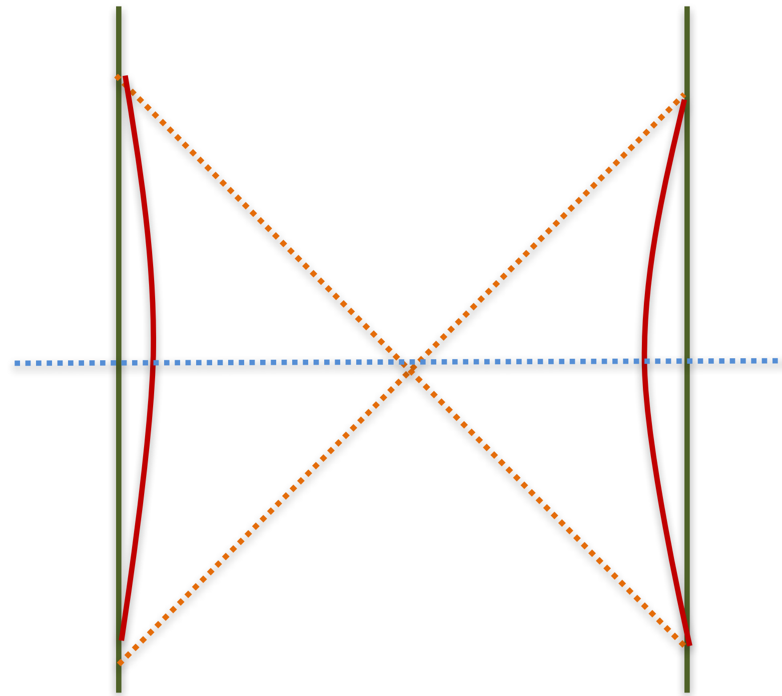
- Same general class.
- Analogy: like talking about the 3d Ising model and the 2nd order superfluid critical point.
- Both are conformal invariant.
- Both have a stress tensor
- But other operators are different.


SYK model
Low energies
Conformal invariant part + reparametrizations
Near extremal black holes
Nearly AdS_2 gravity
QFT on AdS_2 + boundary dynamics
Not the same
same
$$S = -C \int du \{f(u), u\}$$
Schwarzian action
Boundary gravitons
Kitaev
JM, Stanford
Zhang, Suh
- Low temperature entropy
- Chaos exponent
- Wormhole traversability (location of horizon)
Emergent reparametrization symmetry which is spontaneously and explicitly broken

Entangled states



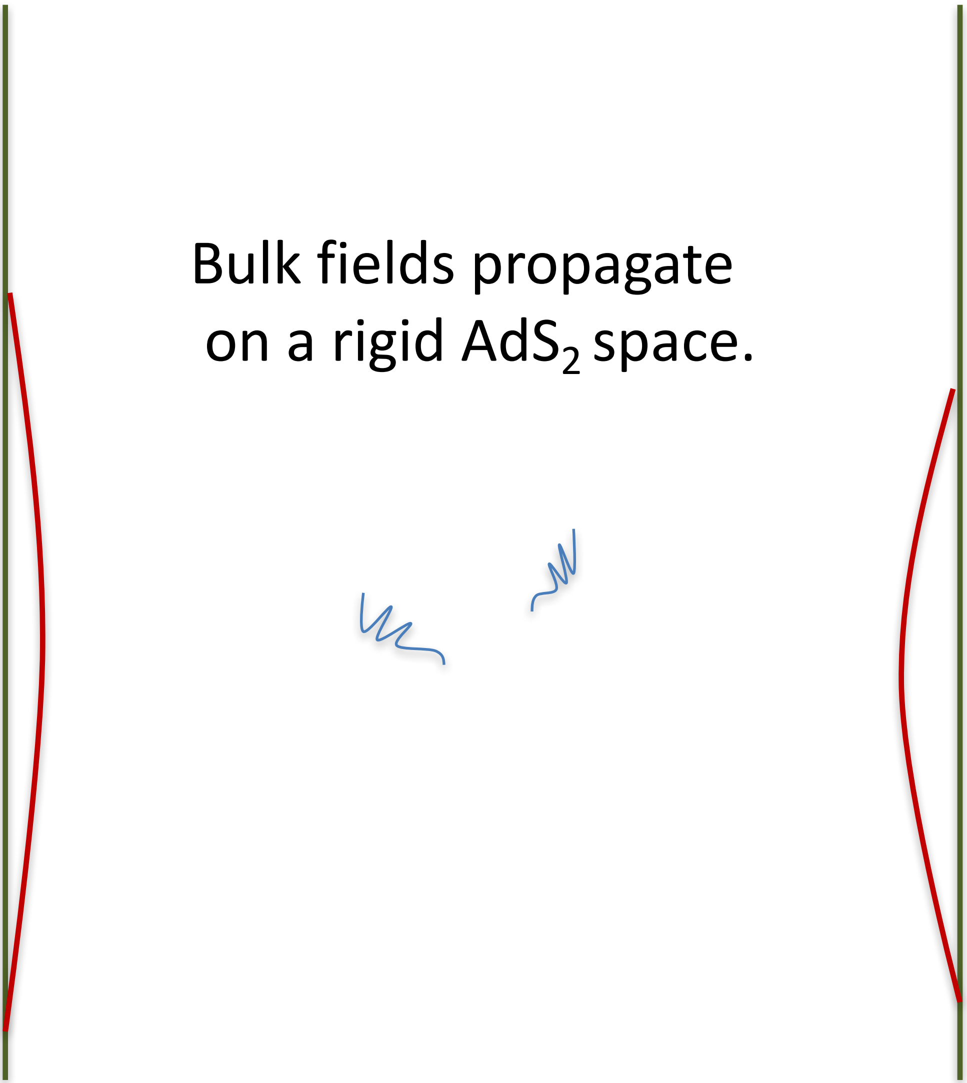
Euclidean black hole



Kruskal Schwarzschild AdS_2 wormhole

Thermofield double: $$|\Psi\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L \times |E_n\rangle_R$$

Dynamics

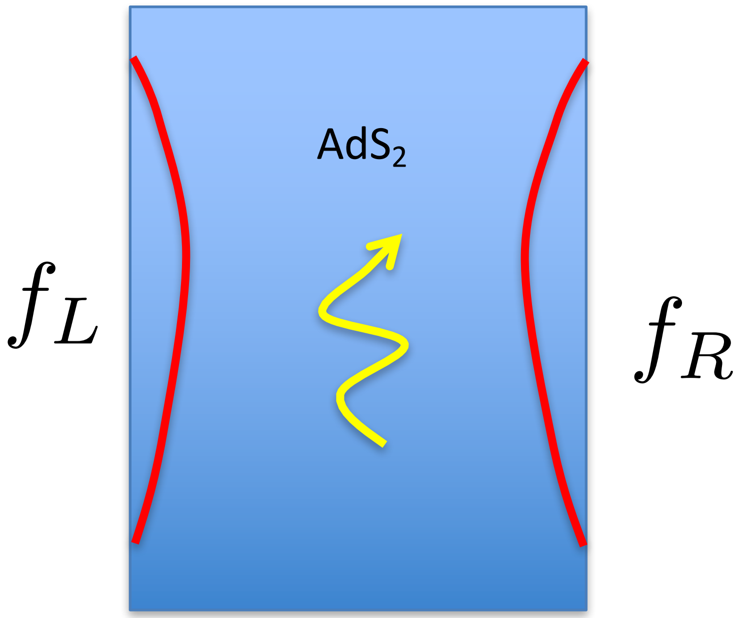


Boundaries also move in a rigid AdS_2 space, following local dynamical laws.

Schwarzian action describes this motion.

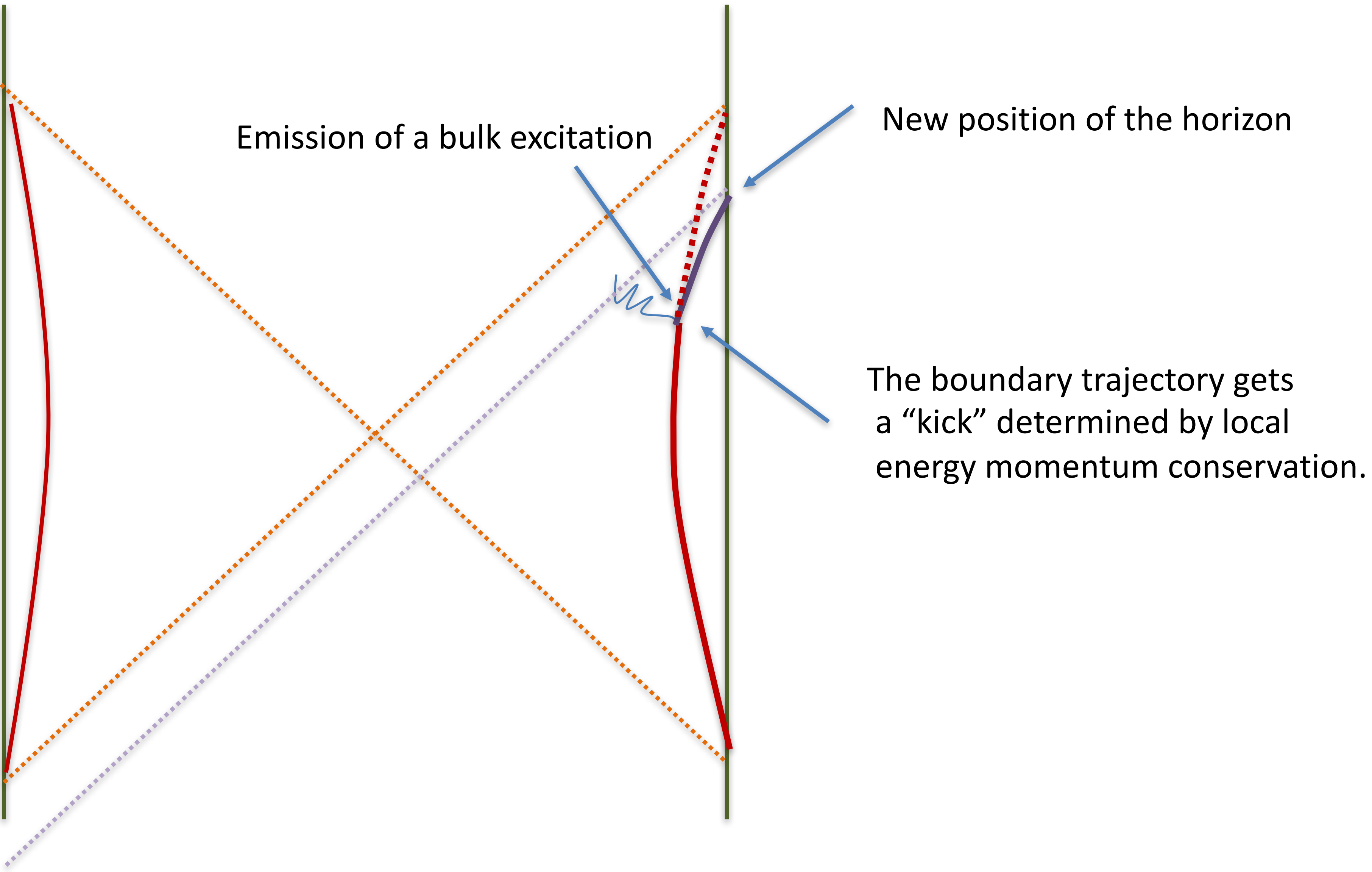
~ Mach principle

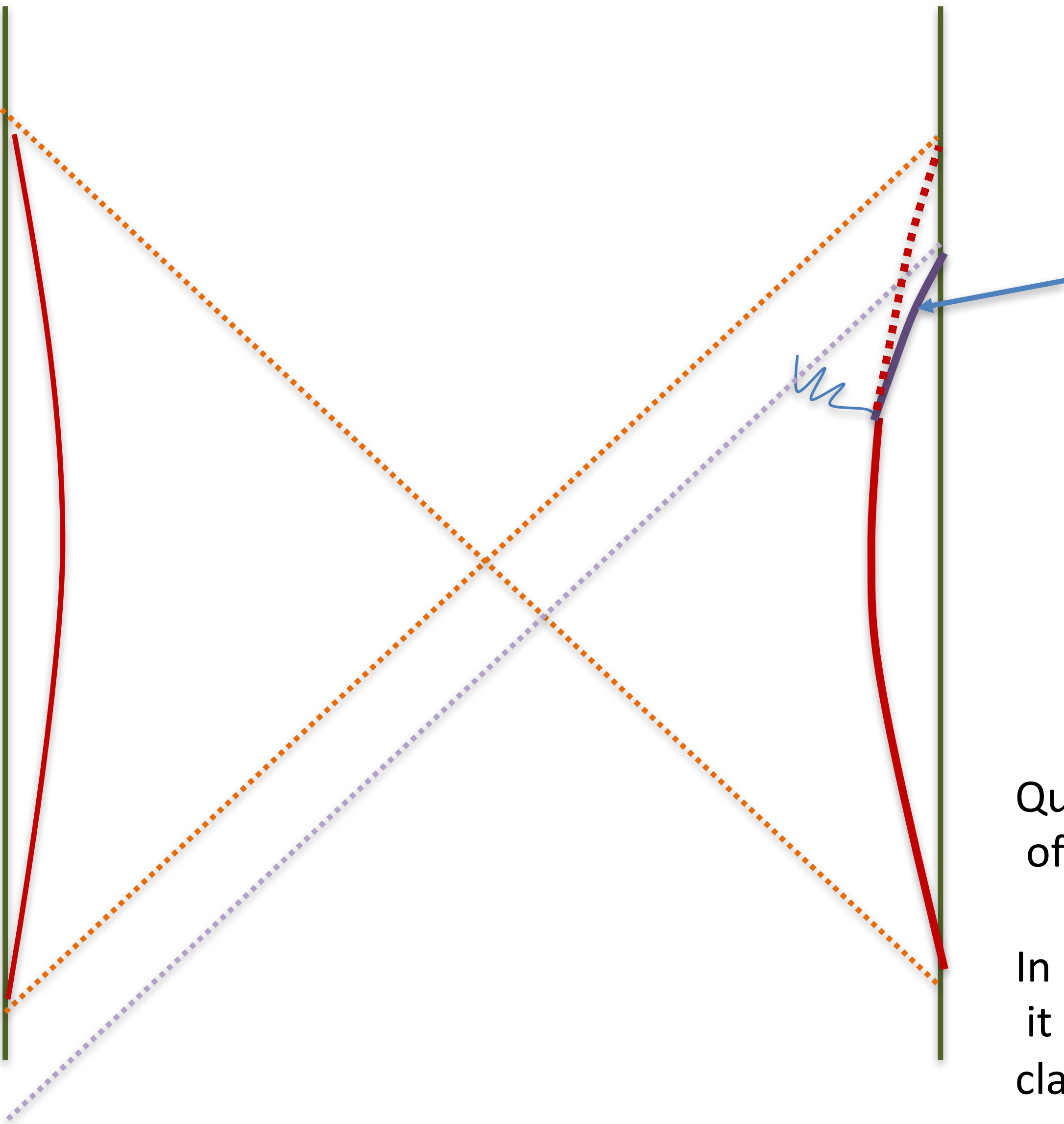
Gravitational dynamics



$$(H_{f_L} \times H_{\text{bulk}} \times H_{f_R})/SL(2,R)$$

Dynamics


Emission of a bulk excitation
New position of the horizon
The boundary trajectory gets
a "kick" determined by local
energy momentum conservation.



New trajectory diverges exponentially from the previous one

$$e^{\lambda t} = e^{2\pi T t}$$

This motion can be detected by OTOC and is directly related to the chaos exponent.

Quantum chaos = simple motion of a particle in AdS_2 , it is geometric.

In both the SYK model and gravity, it results from the motion of an essentially classical variable ! ~ motion in hyperbolic space.

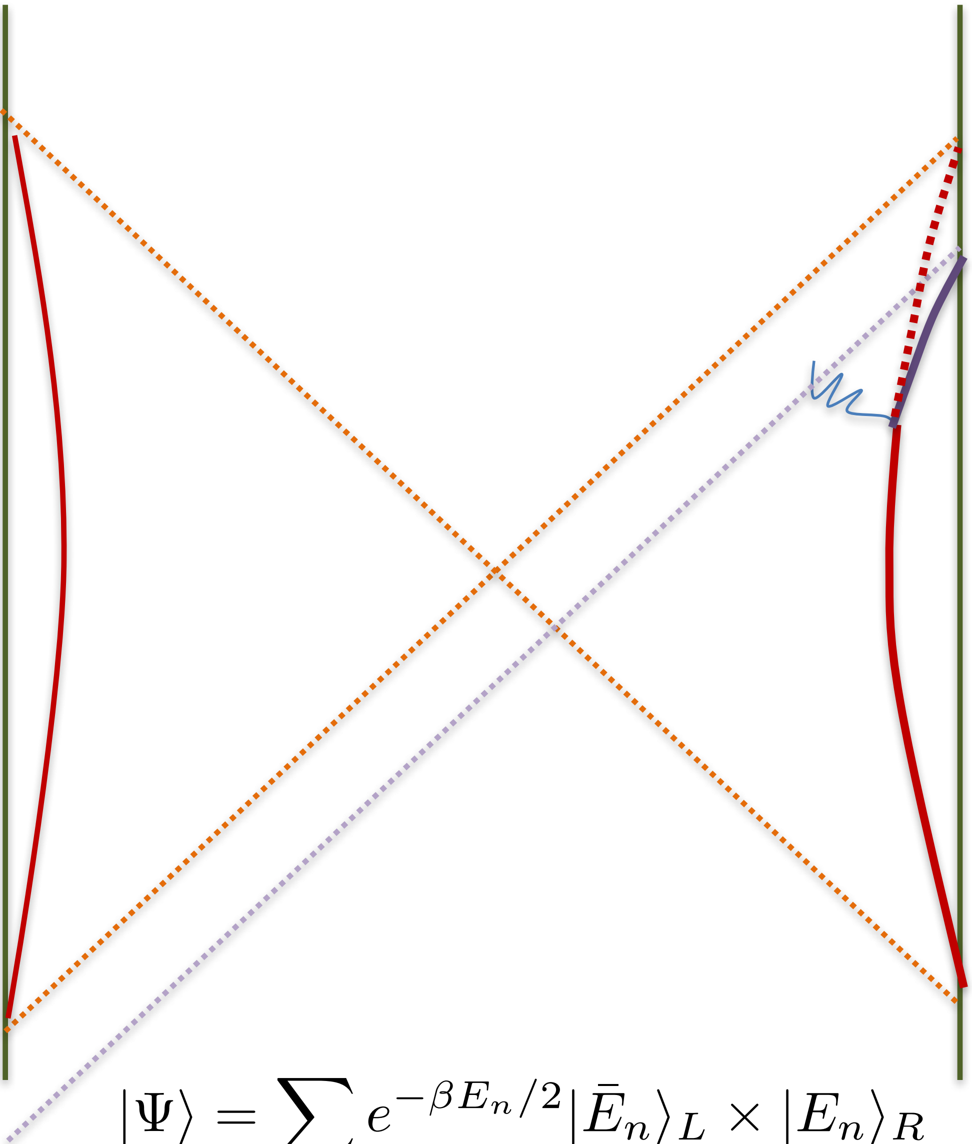
Quantum chaos from classical chaos

- The growth of out of time order correlators is related to the motion of a classical system.
- The one described by the Schwarzian action.
- Or the motion of the boundary particle.
- Roughly like motion in hyperbolic space : chaos from a geometric origin $\rightarrow$ structure of SL(2). Automatically maximal.

- The structure of the bulk is fixed and rigid. The boundary particle motion governs how this IR Hilbert space is embedded in the full exact Hilbert space. The same happens in SYK. The structure of the conformal solution is fixed and rigid, but the Schwarzian degreee of freedom governs its precise embedding.
- Like ants walking on a rotating sphere, but SU(2) $\rightarrow$ SL(2)
- Similar to hydrodynamics, where the fluid is locally the same but could be moving differently relative to the ambient space. Conservation of energy.

Entanglement and teleportation

No signals from one side to the other



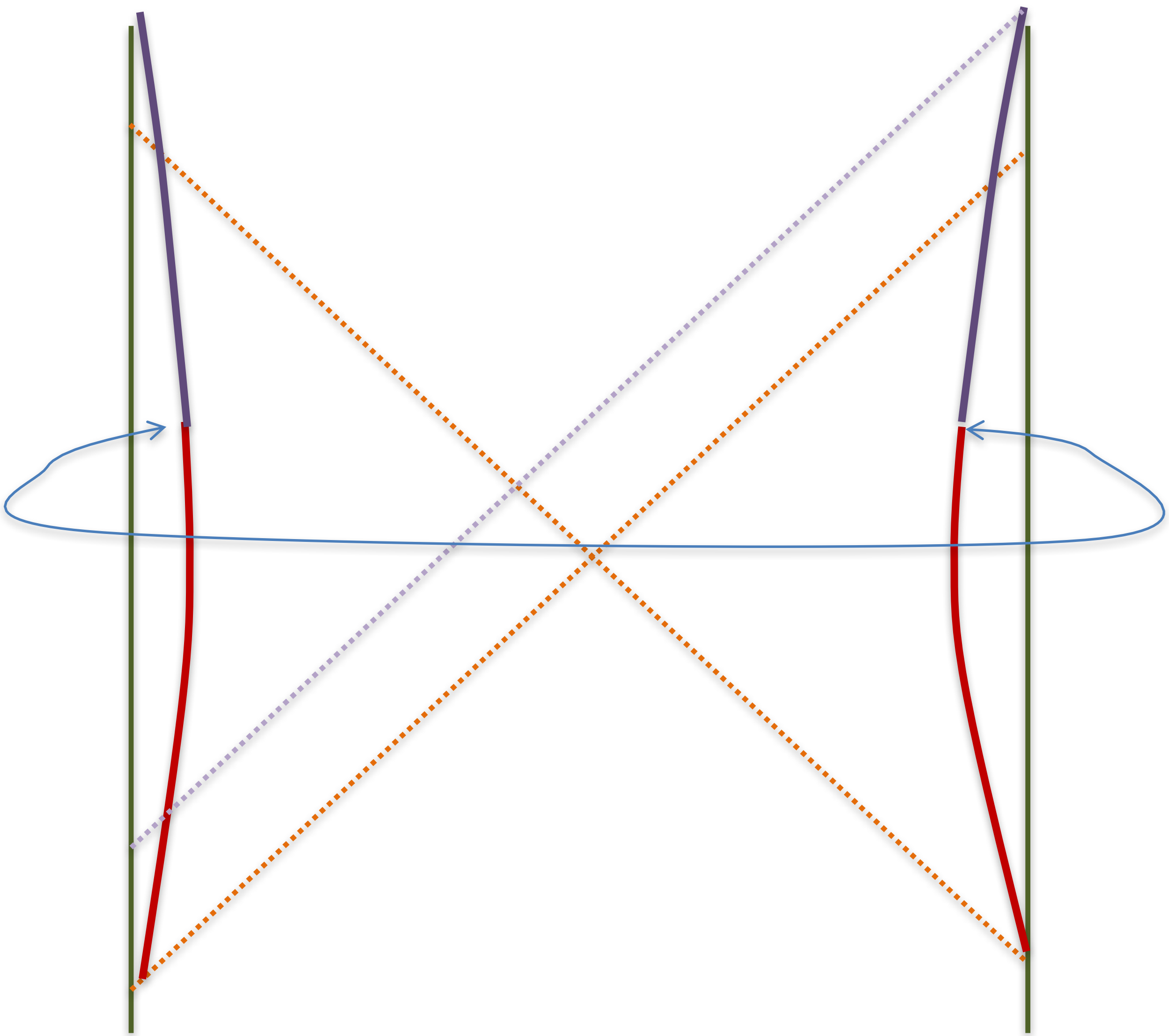
$$|\Psi\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L \times |E_n\rangle_R$$

Kicks are always "outwards" → no signal from one boundary to the other.

Consistent with entanglement.

Interaction between the two boundaries

Gao Jafferis Wall



Insert this in the path integral

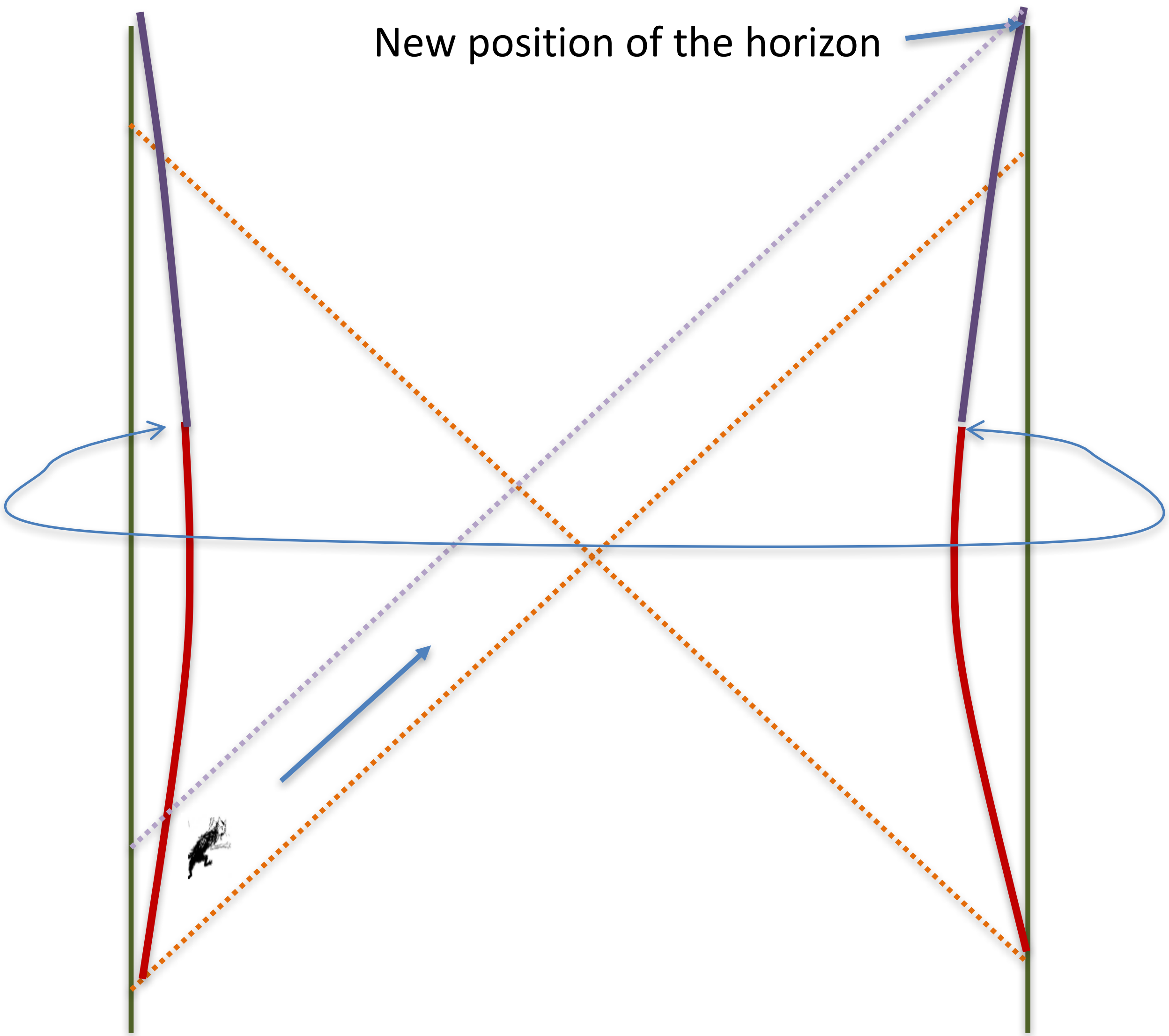
$$e^{ig\phi_L(t_L)\phi_R(t_R)}$$

approximate

$$e^{ig\langle\phi_L(t_L)\phi_R(t_R)\rangle}$$

Force between the two boundaries.
(Can be attractive for the right sign of g).
kicks the trajectories inwards

Interaction makes the wormhole traversable



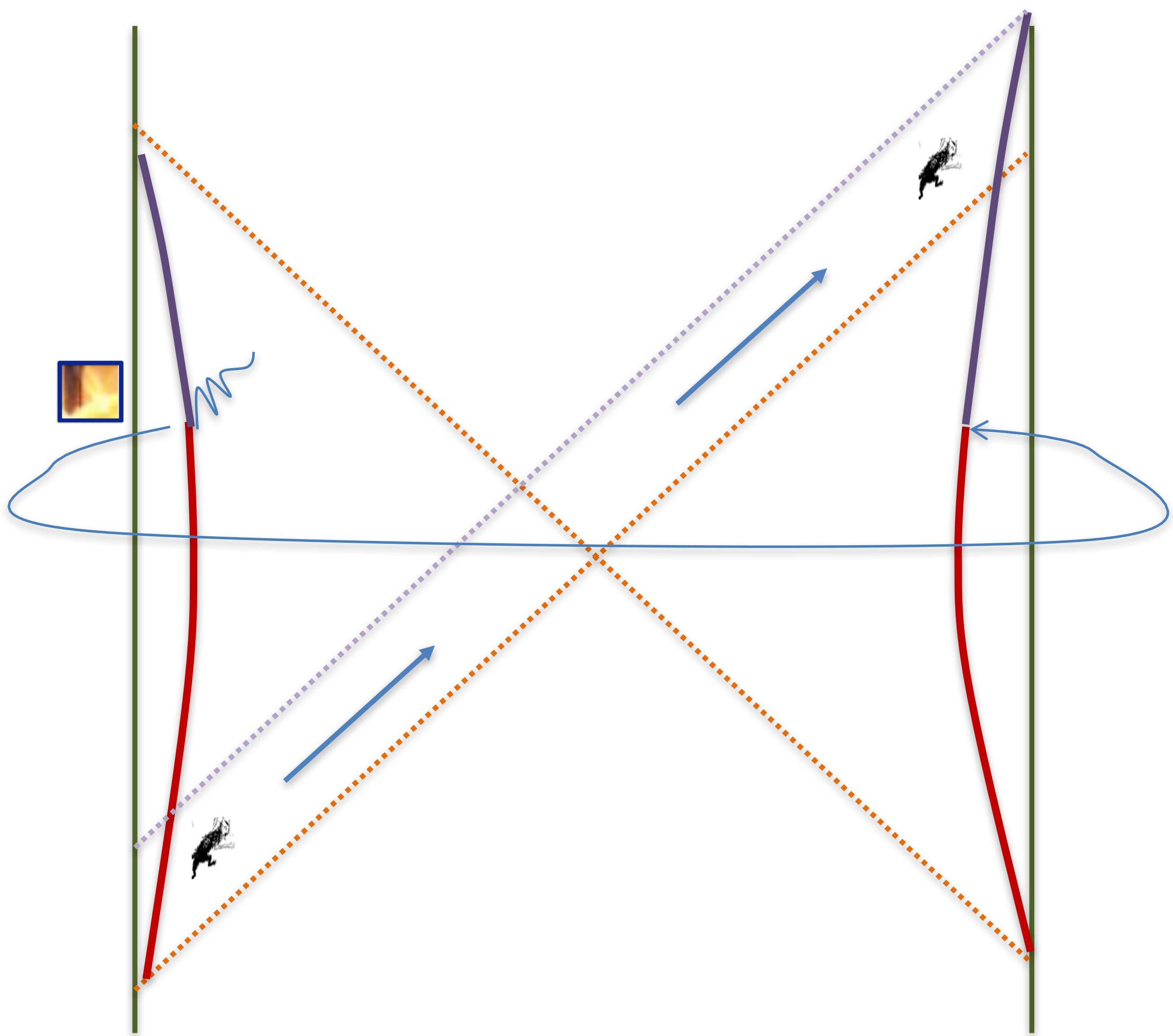
We can now send a signal
from the left to the right.

The wormhole has been
rendered traversable.

No contradiction because
we had a non-local interaction
between the two boundaries.

The point is not that it
we can send signals.
It is how signals get sent and
what they feel !

Quantum teleportation though the wormhole



$$e^{ig\phi_L(t_L)\phi_R(t_R)}$$

Measure $\phi_L \longrightarrow \sigma_L$

Act on the right with

$$e^{ig\sigma_L\phi_R(t_R)}$$

From the point of view of the right we get the same, whether we measure or not.

One other variant of the same basic idea

JM and Xiaoliang Qi

Eternal traversable wormholes

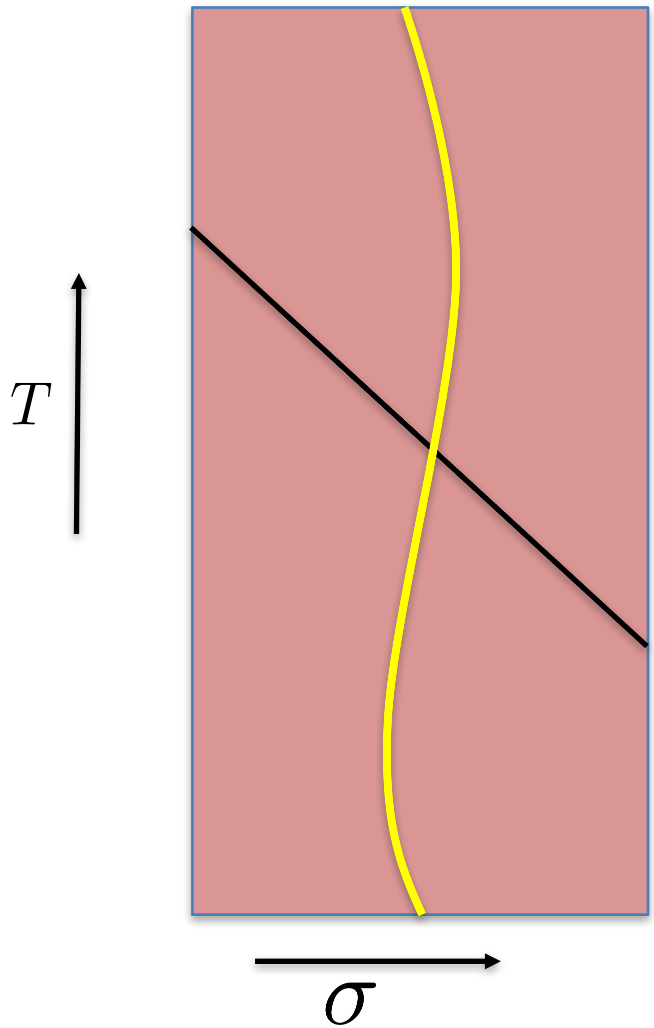
JM & Qi

$$H = H_L^{SYK} + H_R^{SYK} + \mu \sum_i \psi_L^i \psi_R^i$$

It looks like a relevant deformation.

It flows to a gapped system.

AdS_2 - Global coordinates

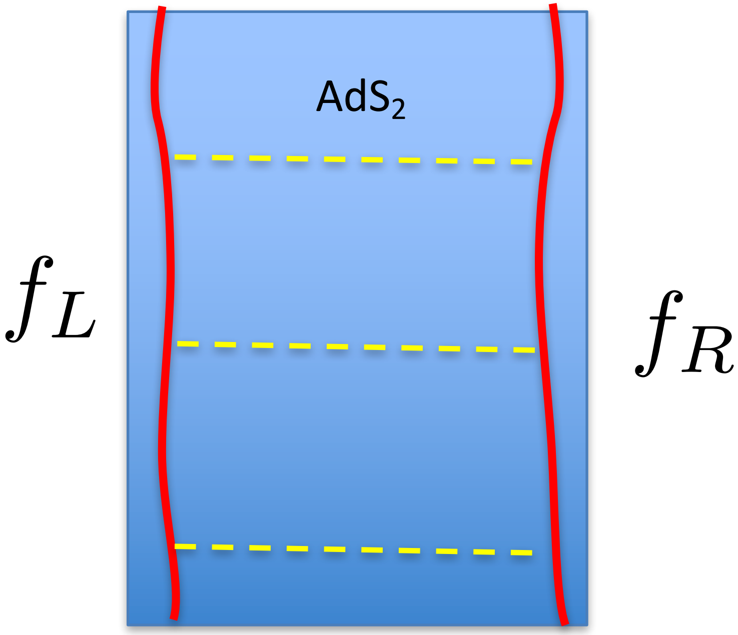


$$ds^2 = \frac{-dT^2 + d\sigma^2}{(\sin \sigma)^2}$$

- SL(2,R) isometries
- Two boundaries
- Causally connected
- Particle dynamics → oscillatory behavior → gapped spectrum
- Global coordinates

AdS$_2$ gravity +

Interaction



$$S = \frac{N\alpha_S}{J}\int du\{f_L(u),u\} + \{f_R(u),u\} + N\mu\int du\left[\frac{f_L'(u)f_R'(u)}{|f_L(u)-f_R(u)|^2}\right]^\Delta$$

+ Global SL(2,R) gauge symmetry → set total SL(2,R) charge to zero.

$$f(u) = \tan(T(u)/2)$$

Consequences of the symmetries

- Spectrum = Part determined by the SL(2) symmetry + part coming from the boundary degree of freedom.

$$E = w_0 \left[m\sqrt{2(1-\Delta)} + \sum_i (n_i + \Delta_i) \right] \ , \qquad m, \ n_i = \text{Integers}$$

Not determined by the symmetries, depends on μ

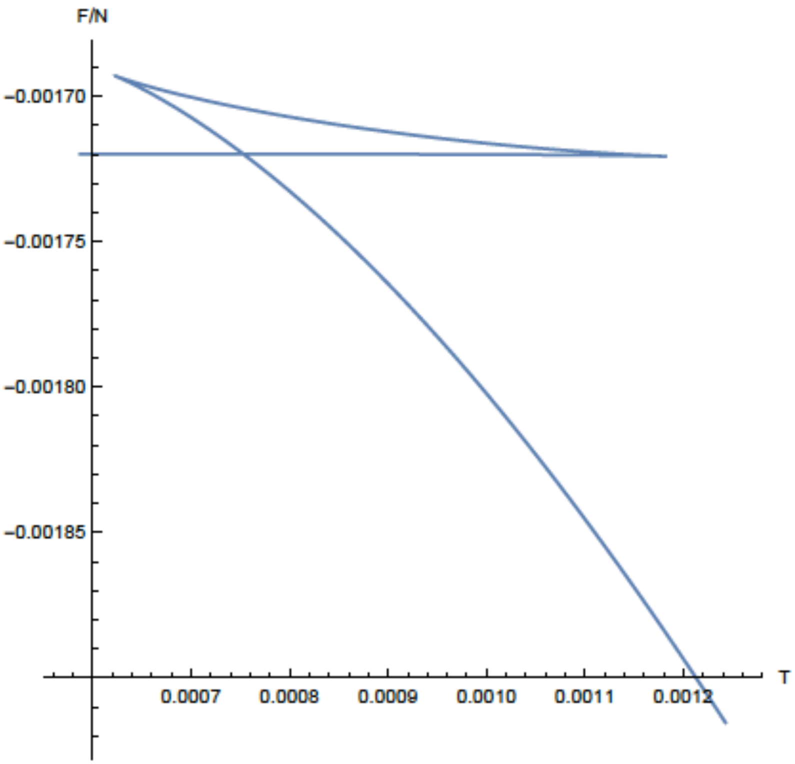
SL(2) representations. Bulk fields or conformal sector of the SYK model.

Motion of the boundary particles, of the Schwarzian action.

It is a bit like the Zeeman effect in atomic physics where an atom with non-zero spin, j, is put in a magnetic field. The spectrum is determined by the weakly broken rotational symmetry and it gives rise to $2j+1$ equally spaced levels.

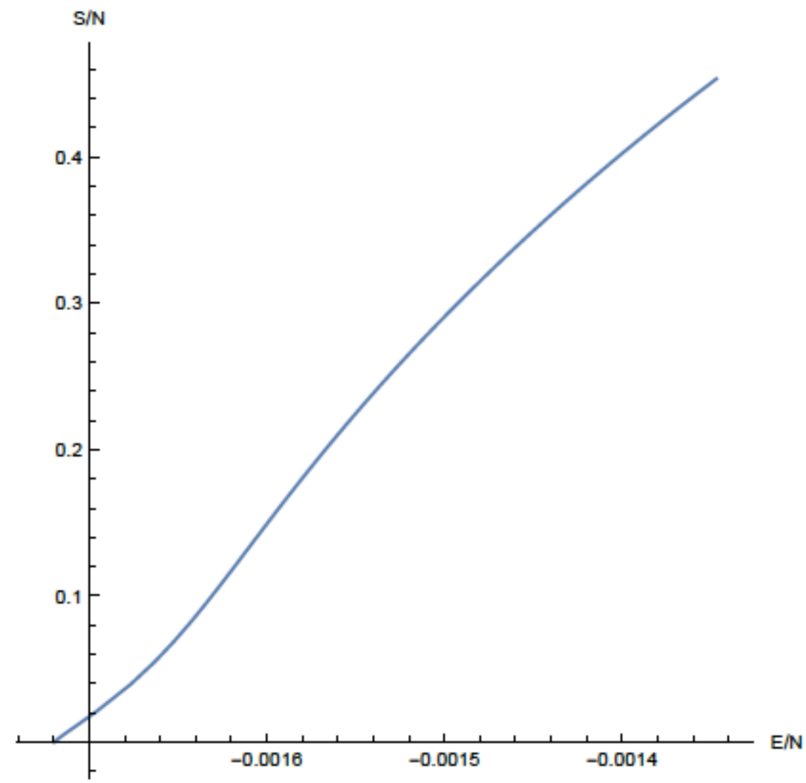
It is the analog of the operator $\rightarrow$ state mapping of higher dimensional CFTs.

Finite temperature SYK case



Free energy vs. Temperature

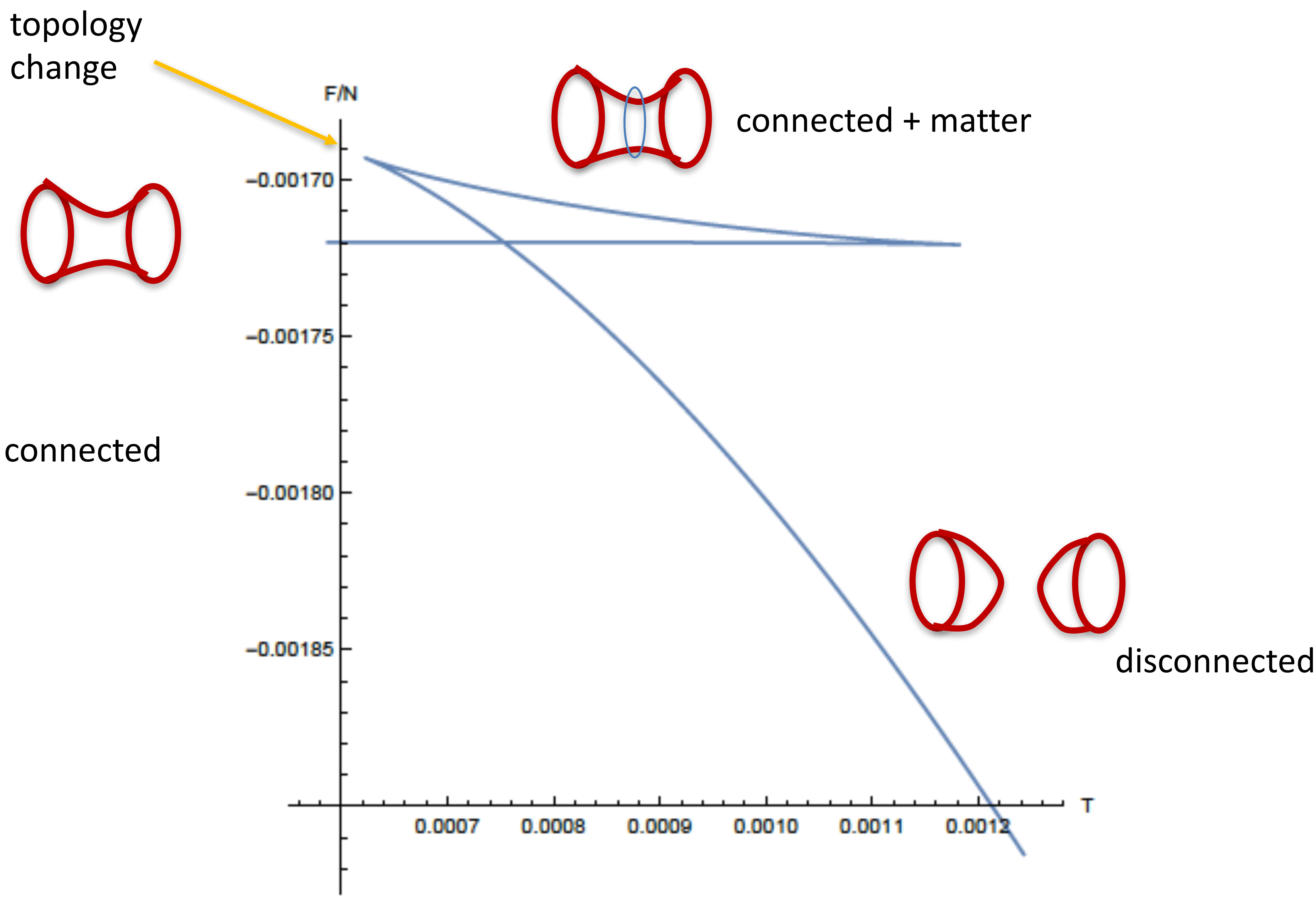
Canonical ensemble



Entropy vs Energy

Microcanonical ensemble

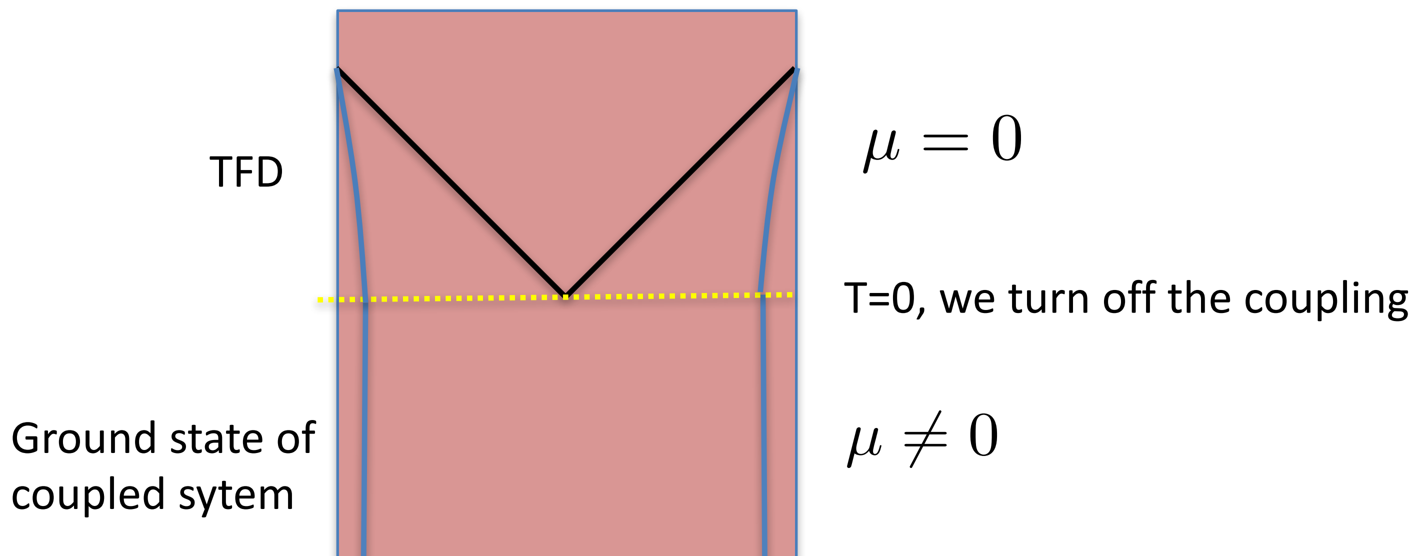
Finite temperature gravity



Making the TFD

- Create two SYK systems.
- Couple term. $\mu \neq 0$
- Couple them further to a heat sink and let them cool down to find its ground state.
- At t=0, turn off the left-right coupling. $\mu = 0$

- → Get a state that is close to the TFD.



Conclusions

- The SYK is a nice solvable model.
- It has many features in common with near extremal black holes.
- In both cases we have a low energy almost conformal symmetry
- It is maximally chaotic.
- Chaos is described by a simple classical variable (scramblon).
- Connection to wormholes.
- Traversability and teleportation.

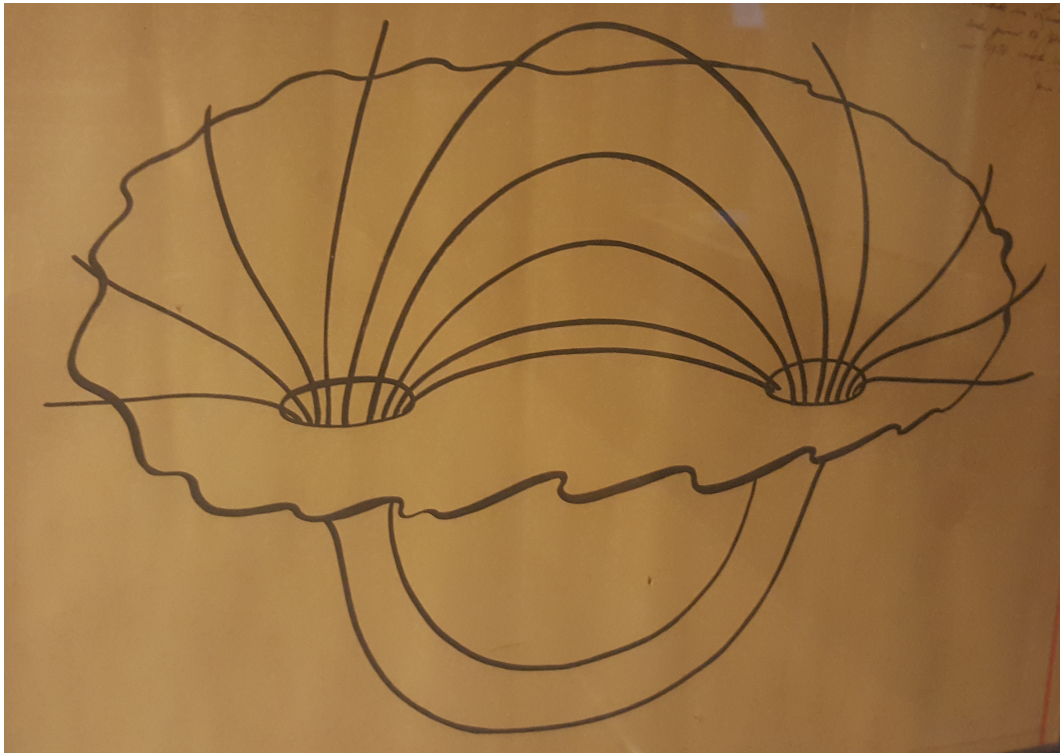
One application:
New Wormhole Solutions

Based on work in progress with:

Alexey Milekhin

Fedor Popov

Drawing by John Wheeler, 1966



Charge without charge.

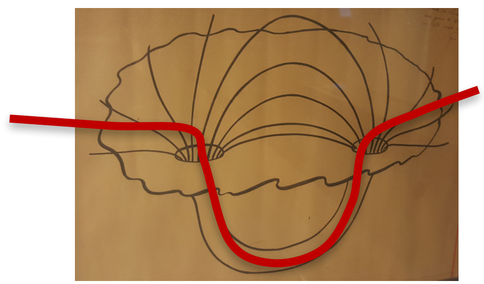
Spatial geometry. Traversable wormhole

There are no science fiction wormholes!

- No wormhole allows you to travel faster than the speed of light in the ambient space.
- Forbidden by:
- I) The Achronal Average Null Energy Condition

Not yet proven in a general spacetime, but believed to hold in QFT

$$\int dx^- T_{--} \geq 0$$



- II) Einstein equations.

Achronal = fastest line

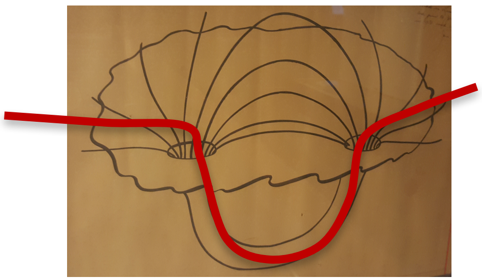
Friedman Schleich, Witt, Galloway, Woolgar
Gao Wald

Longer wormholes

- What if it takes longer to go through the wormhole ?
- Not possible in classical physics due to the Null Energy Condition.

Topological censorship: Friedman Schleich, Witt, Galloway, Woolgar

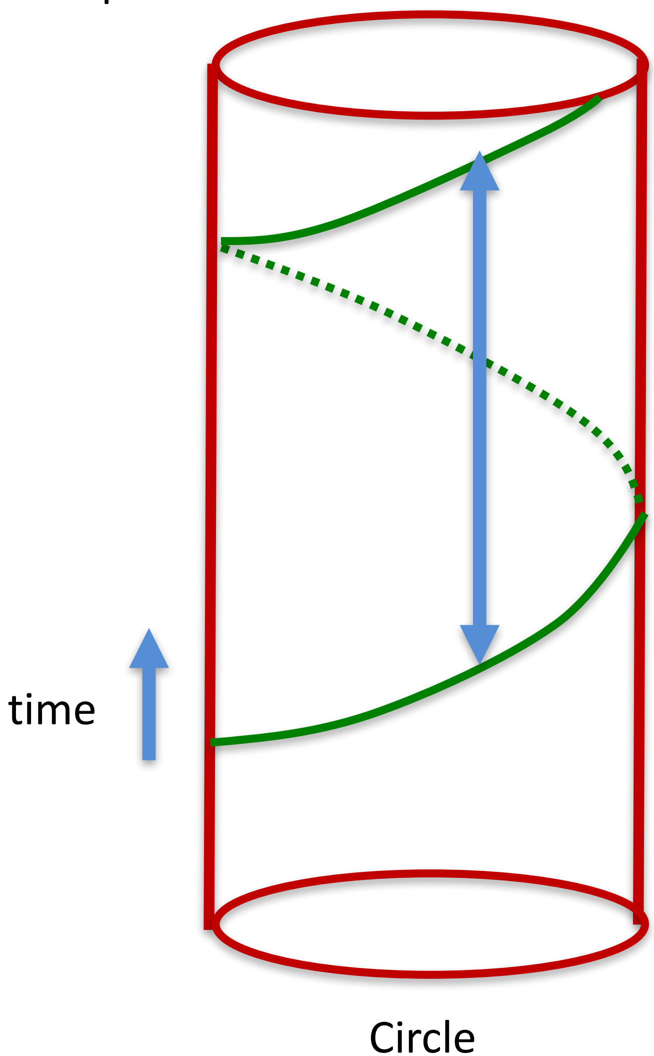
- → We need quantum effects to find a solution. Casimir-like energy.



- Can we do it in a controllable way ?

Negative energy from quantum mechanics

Eg. Two spacetime dimensions



$$T_{++} < 0$$

$$E \propto -\frac{1}{L}$$

Negative Casimir energy

Quantum effect

The null energy condition does not hold for null lines that are not the fastest

Some necessary elements

- We need something looking like a circle to have negative Casimir energy.
- Large number of bulk fields to enhance the size of quantum effects.

- We will show how to assemble these elements in a few steps.

The theory

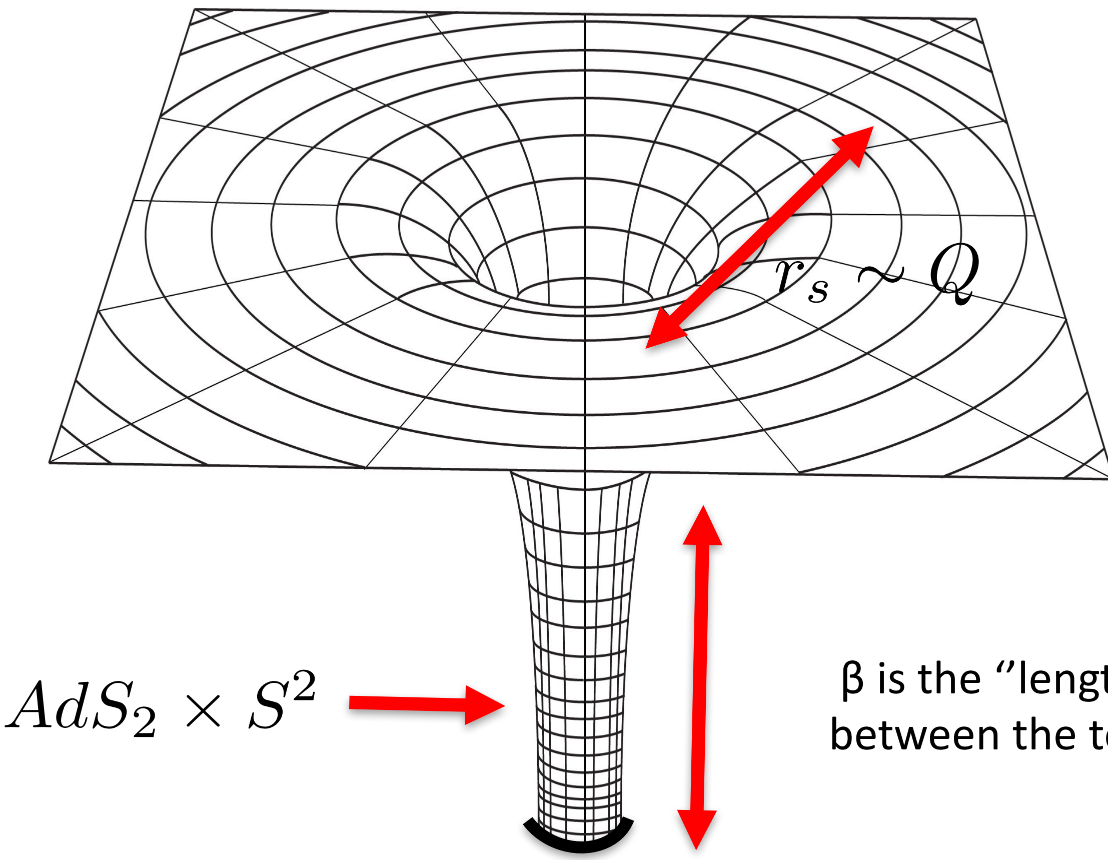
$$S = \int d^4x \sqrt{g}\left[R + F^2 + \bar{\psi}\ \not{D}\psi\right]$$

Einstein + U(1) gauge field + massless charged fermion

Could be the Standard Model at very small distances, smaller than the electroweak scale where the fermions are effectively massless and the U(1) would be hypercharge. SU(3) x SU(2) x U(1).

The first solution

Extremal, or near extremal, magnetically charged black hole, magnetic charge Q.



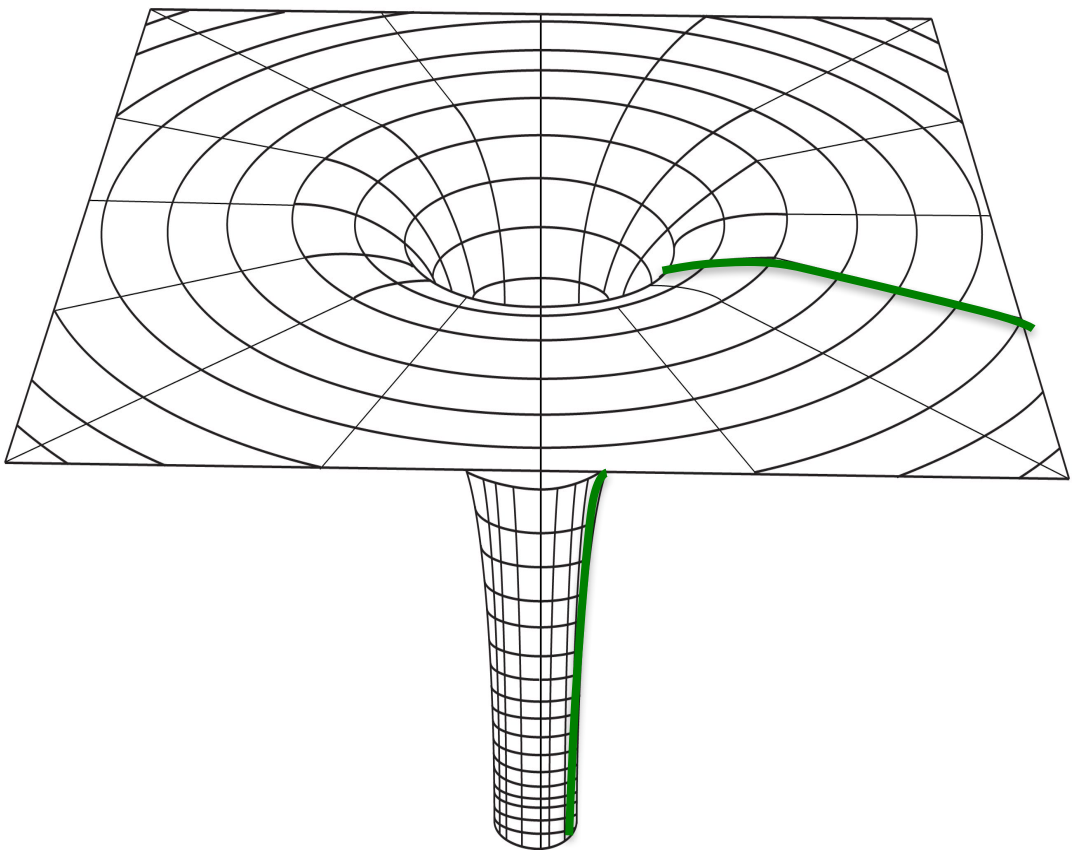
$$M = Q + Q^3 T^2 = Q + \underbrace{\frac{Q^3}{\beta^2}}_{\text{Very small}}$$

$$Q \gg 1$$

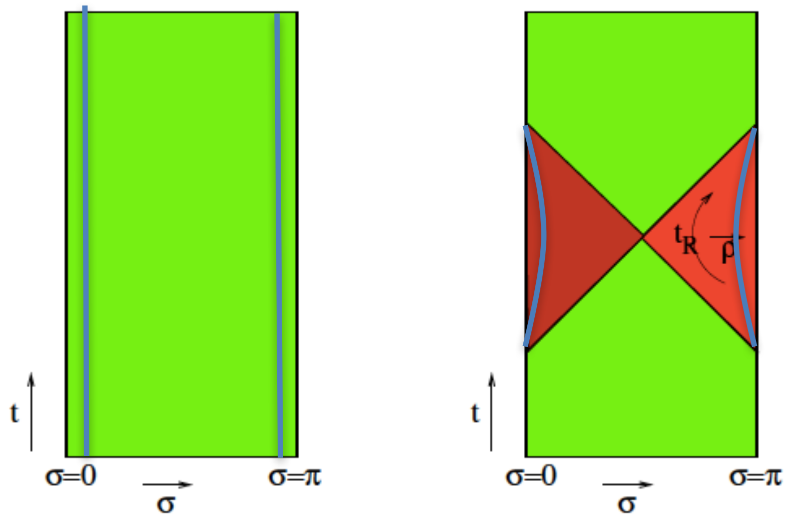
β is the "length" of the throat. Redshift factor between the top and the bottom

Motion of charged fermions

- Magnetic field on the sphere.
- There is a Landau level with precisely zero energy.
- Orbital and magnetic dipole energies precisely cancel.
- Degeneracy Q = flux of the magnetic field on the sphere
- We effectively get Q massless two dimensional fermions along the time and radial direction.
- We can think of each of them as following a magnetic field line.



AdS$_2$



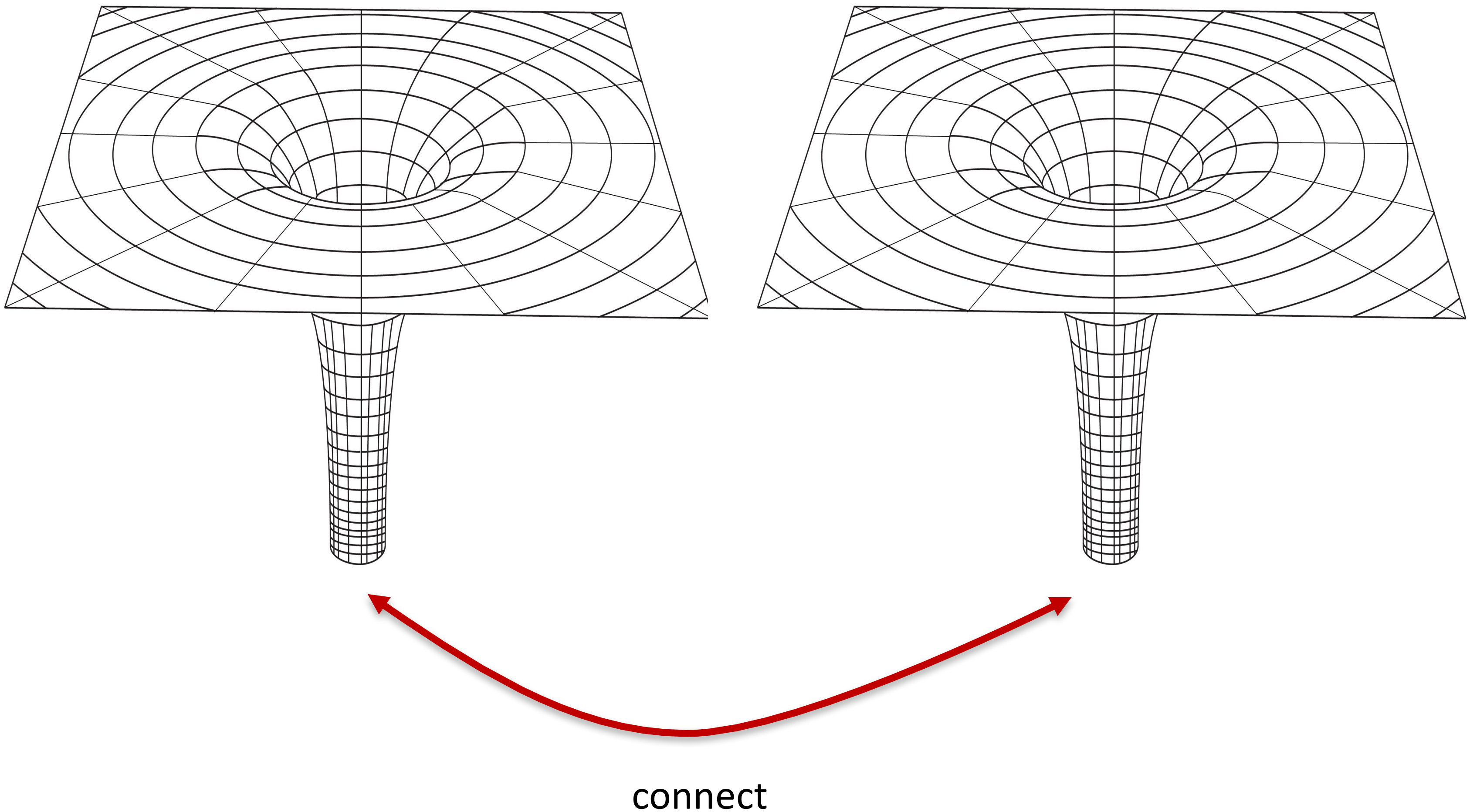
$$ds^2 = \frac{-dt^2 + d\sigma^2}{\sin^2 \sigma} \qquad\qquad ds^2 = -(r^2 - 1)dt^2 + \frac{dr^2}{(r^2 - 1)}$$

Two black holes connected in various ways. All equally valid solutions in the exact extremal limit (infinite length throat).

They acquire non-zero energy when the throat has finite length

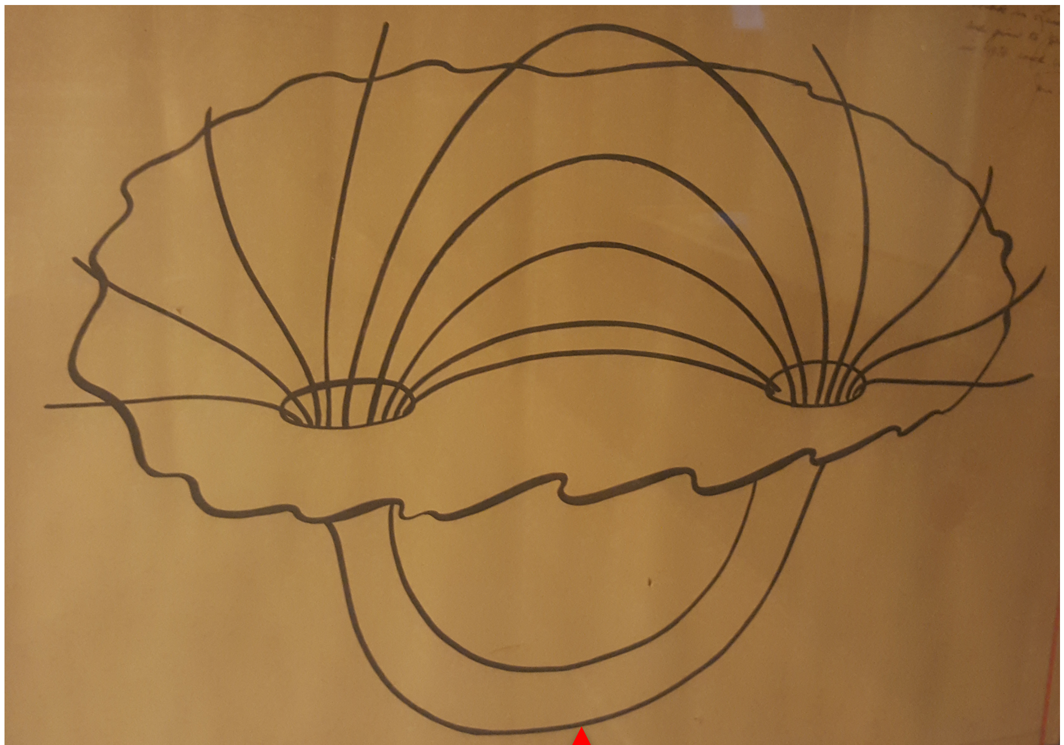
$$M = Q + Q^3 T^2 = Q + \frac{Q^3}{\beta^2}$$

Connect a pair black holes



Connect a pair black holes

Positive magnetic charge



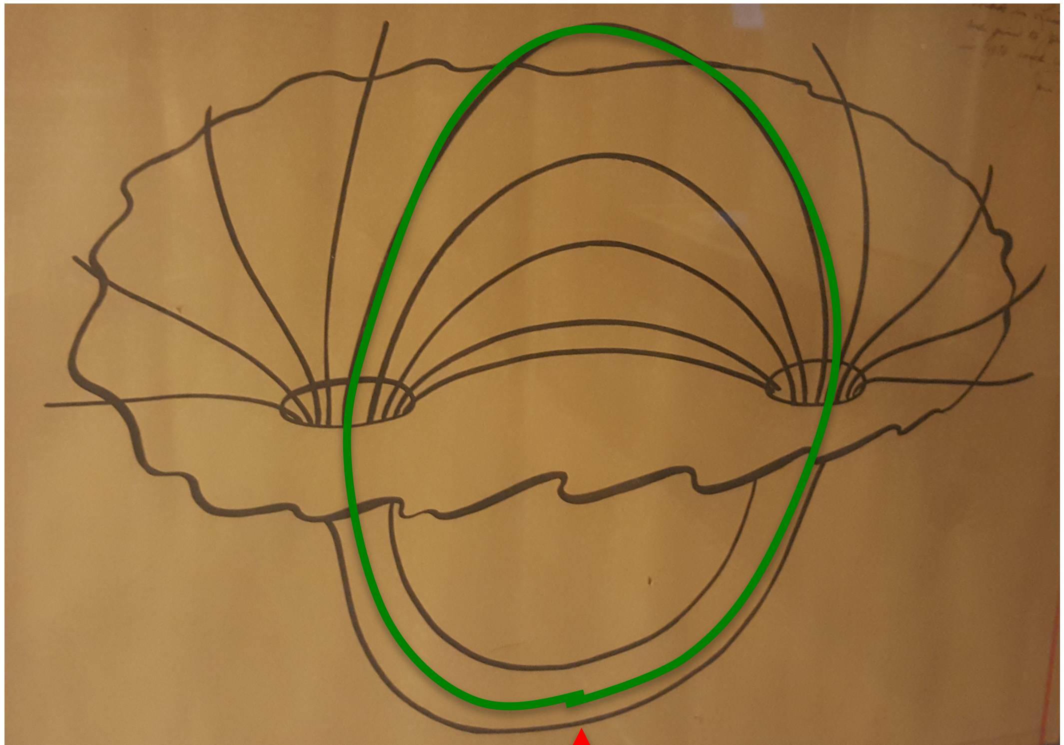
Negative magnetic charge

Not a solution yet.
Not a black hole.

Nearly $AdS_2 \times S^2$ wormhole of finite length

Fermion trajectories

Positive magnetic charge

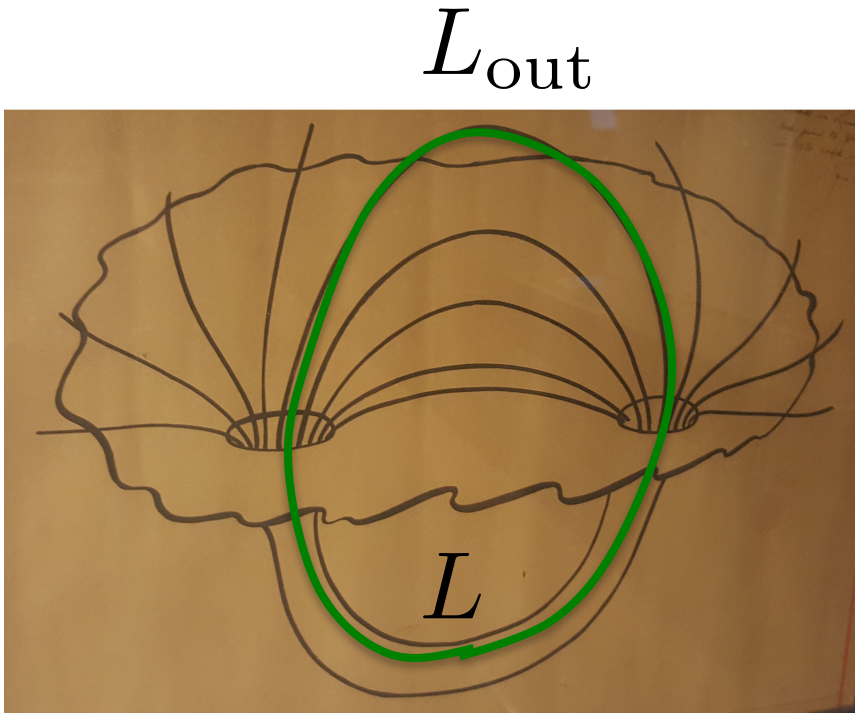


Negative magnetic charge

Charged fermion moves along this closed circle.

Casimir energy

Assume: “Length of the throat” is larger than the distance.



Casimir energy is of the order of

$$E \propto -\frac{Q}{L + L_{\mathrm{out}}} \sim -\frac{Q}{L}$$

Finding the solution

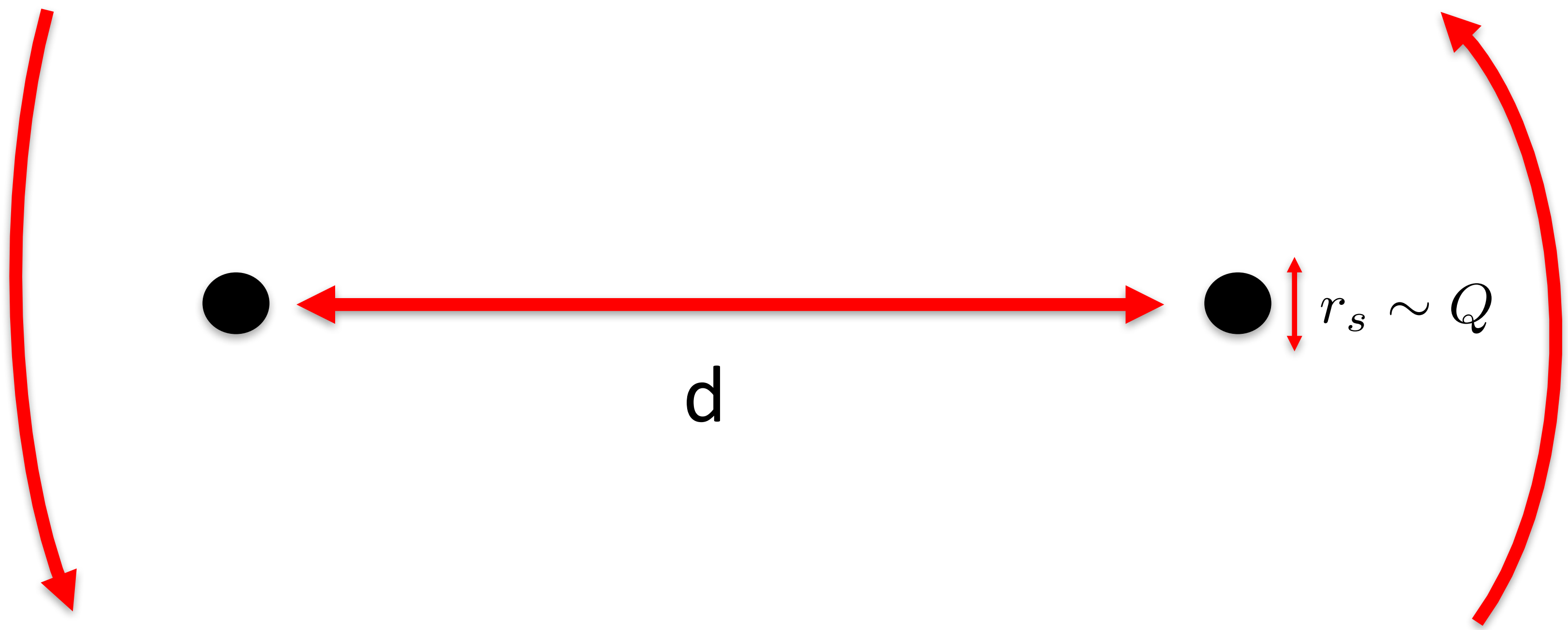
Balance the classical curvature + gauge field energy vs the Casimir energy.

$$E = Q + \frac{Q^3}{L^2} - \frac{Q}{L}, \qquad \frac{\partial E}{\partial L} = 0 \longrightarrow L \sim Q^2 \ , \quad E_{\rm min} - Q \sim -\frac{1}{Q} \sim -\frac{1}{r_s}$$

Now the throat is stabilized. Negative binding energy.

This is not yet a solution: The two objects attract and would fall on to each other

Adding rotation



Some necessary inequalities

$$L \sim Q^2$$ From stabilized throat solution

$$d \ll L \longrightarrow d \ll Q^2$$ Black holes close enough to that Casmir energy computation was correct.

$$\sqrt{\frac{Q}{d^3}} = \Omega \ll \frac{1}{L} \sim \frac{1}{Q^2} \longrightarrow Q^{\frac{5}{3}} \ll d$$ Black holes far enough so that they rotate slowly compared to the energy gap.

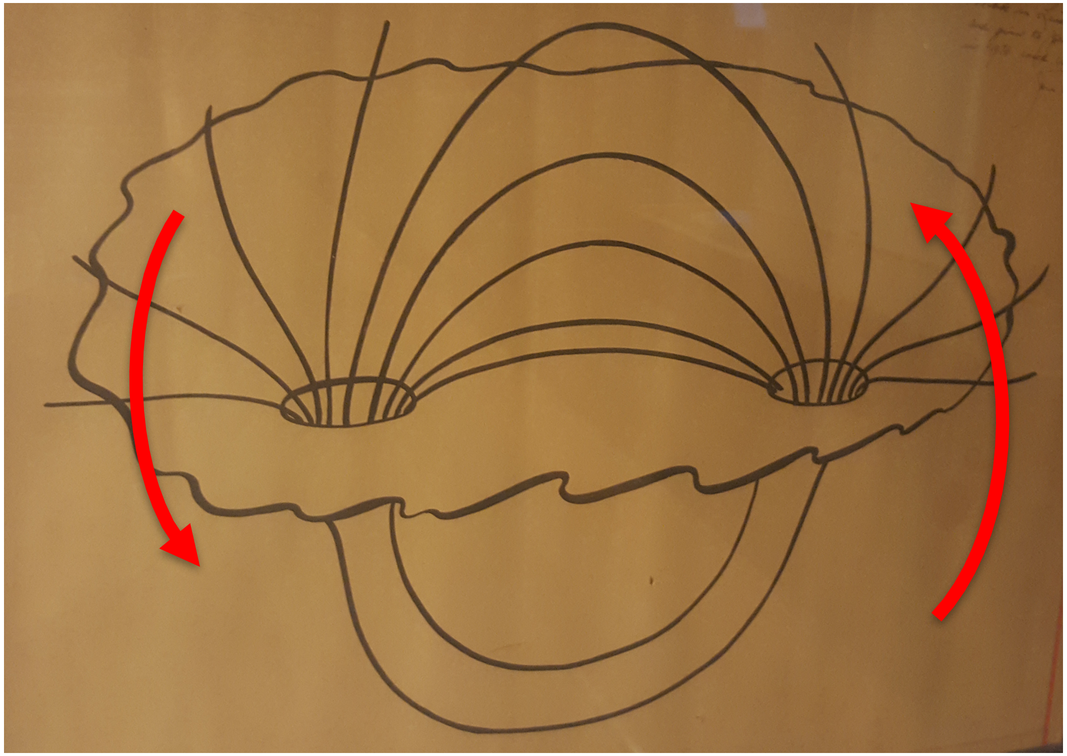
Kepler
rotation frequency

Unruh-like temperature less than energy gap

They are compatible $$Q^{\frac{5}{3}} \ll d \ll Q^2$$

Other effects we could think off are also small :
can allow small eccentricity, add electromagnetic and gravitational radiation, etc.

Final solution



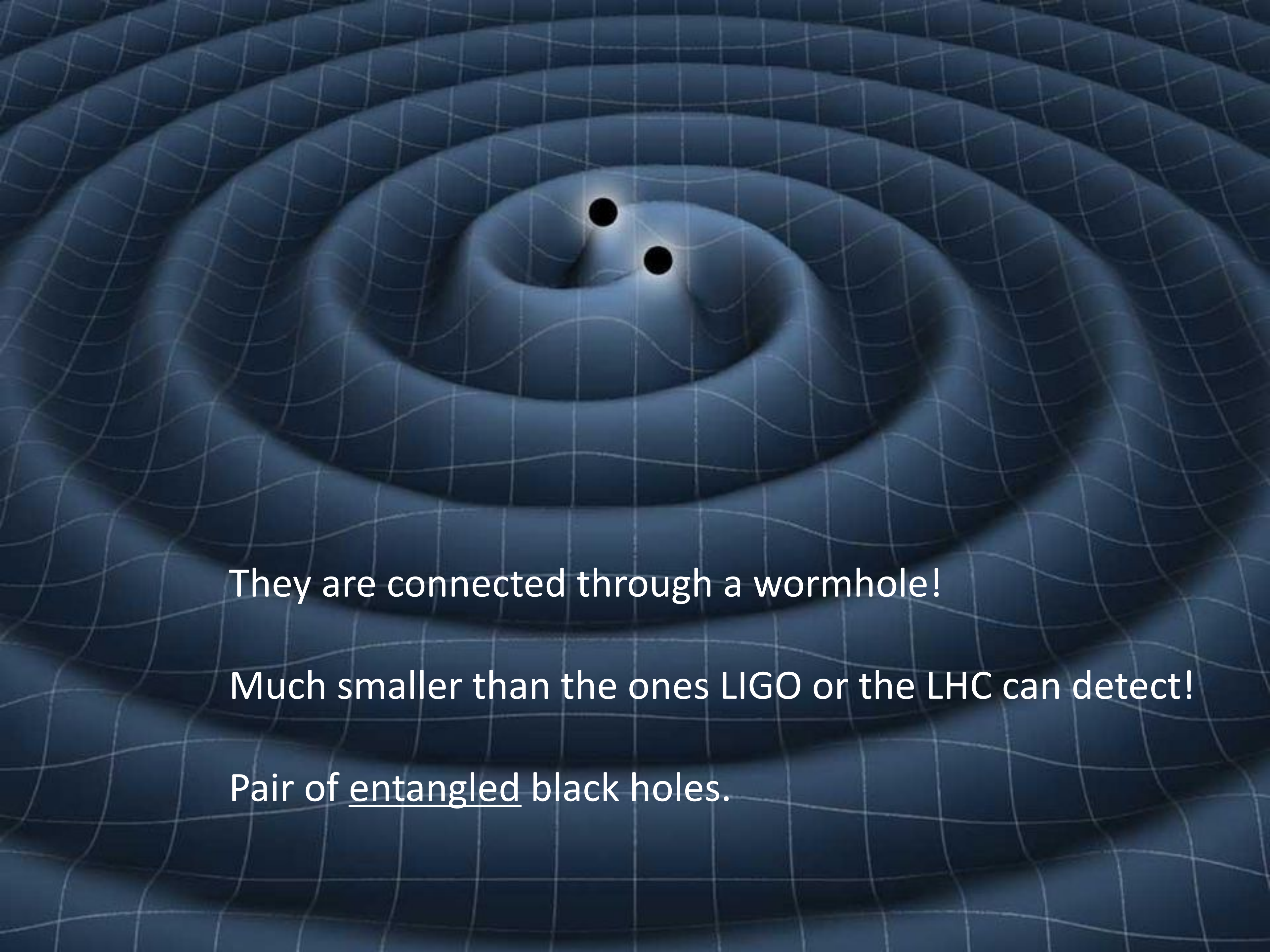
Looks like two near extremal black holes if you do not get to the middle of wormhole
But there is no horizon !. Zero entropy solution.
It has a small binding energy.

It could exist if nature is described by the Standard Model at short distances and d is smaller than the electroweak scale,

$$1 \ll Q \ll 10^8$$

If the standard model is not valid → it is possible that similar ingredients are present in the true theory.

That it <u>can</u> exist, does not mean that it is easily produced by some natural or artificial process.



They are connected through a wormhole!

Much smaller than the ones LIGO or the LHC can detect!

Pair of <u>entangled</u> black holes.

Conclusions

- We displayed a solution of an Einstein Maxwell theory with charged fermions.
- It is a traversable wormhole in four dimensions and with no exotic matter.
- It balances classical and quantum effects.
- It has a non-trivial spacetime topology, which is forbidden in the classical theory.
- It does <u>not</u> violate causality.
- It has no horizon and no entropy.
- Can be viewed as two entangled black holes.

Thank you, Gabriele,
for your wonderful gift !

Precise formula for the 2pt function

$$C = \langle e^{-igV} \chi_R(t) e^{igV} \chi_L(-t) \rangle \ , \qquad V = g\phi_L(0)\phi_R(0)$$

$$C \sim \int dp (p)^{2\Delta-1} e^{-ip} e^{-ig} e^{i\frac{g}{(1+pe^{t})^{2\Delta}}}$$

Amount of information we can send is roughly g

$$\langle V \rangle = 1 \ , \quad \langle \phi_L^2(0) \rangle \sim 1$$