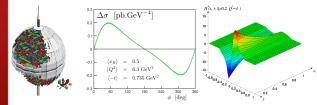
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Sketching the pion's valence-quark Generalized Parton Distribution



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Light Cone 2015 | Hervé MOUTARDE

Sept. 24th, 2015



Motivations 3D imaging of nucleon's partonic content but also...



Sketching the pion GPD

Correlation of the longitudinal momentum and the transverse position of a parton in the nucleon.

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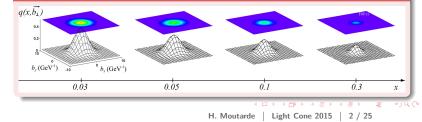
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Conclusions

- Insights on:
 - **Spin** structure.
 - **Energy-momentum** structure.
- Probabilistic interpretation of Fourier transform of $GPD(x, \xi = 0, t)$ in transverse plane.

Transverse plane density (Goloskokov and Kroll model)





Overview.

Development of a new GPD model in the Dyson-Schwinger and Bethe-Salpeter framework.



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Conclusions

 Important topic for several past, existing and future experiments: H1, ZEUS, HERMES, CLAS, CLAS12, JLab Hall A, COMPASS, EIC, ...

- GPD modeling / parameterizing is an essential ingredient for the interpretation of experimental data.
- Recent applications of the Dyson-Schwinger and Bethe-Salpeter framework to hadron structure studies.



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Development of a new GPD model in the Dyson-Schwinger and Bethe-Salpeter framework.



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- Important topic for several past, existing and future experiments: H1, ZEUS, HERMES, CLAS, CLAS12, JLab Hall A, COMPASS, EIC, ...
- GPD modeling / parameterizing is an essential ingredient for the interpretation of experimental data.
- Recent applications of the Dyson-Schwinger and Bethe-Salpeter framework to hadron structure studies.
- Here develop **pion GPD model** for simplicity.
- No planned experiment on pion GPDs but existing proposal of DVCS on a virtual pion.

Amrath et al., Eur. Phys. J. C58, 179 (2008)



Overview.

Development of a new GPD model in the Dyson-Schwinger and Bethe-Salpeter framework.



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- GPD modeling / parameterizing is an essential ingredient for the interpretation of experimental data.
- Recent applications of the Dyson-Schwinger and Bethe-Salpeter framework to hadron structure studies.
- GPDs: Theoretical Framework
- ② GPDs in the Dyson-Schwinger and Bethe-Salpeter Approach
 - Results: Theoretical Constraints and Phenomenology
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Extension: Implementing Positivity and Polynomiality

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GPDs: Theoretical Framework

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Pion Generalized Parton Distribution. Definition and symmetry relations.



Sketching the pion GPD

 $\frac{1}{2}\int \frac{\mathrm{d}z^{-}}{2\pi}$

with t =

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$$H_{\pi}^{q}(x,\xi,t) = \frac{1}{2} \int \frac{dz^{-}}{2\pi} e^{ixP^{+}z^{-}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q} \left(-\frac{z}{2} \right) \gamma^{+}q \left(\frac{z}{2} \right) \middle| \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^{+}=0\\z_{\perp}=0}}$$
with $t = \Delta^{2}$ and $\xi = -\Delta^{+}/(2P^{+})$.

References

Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
Ji, Phys. Rev. Lett. **78**, 610 (1997)
Radyushkin, Phys. Lett. **B380**, 417 (1996)

From isospin symmetry, all the information about pion GPD is encoded in $H^{u}_{\pi^+}$ and $H^{d}_{\pi^+}$. Further constraint from charge conjugation: $H^{u}_{\pi^{+}}(x,\xi,t) = -H^{d}_{\pi^{+}}(-x,\xi,t).$

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Properties. Generalization of form factors and Parton Distribution Functions.



Sketching the pion GPD

PDF forward limit

$$H^q(x,0,0) = q(x)$$

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Generalization of form factors and Parton Distribution Functions.



PDF forward limit

Form factor sum rule

$$\int_{-1}^{+1} dx \, H^q(x,\xi,t) = F_1^q(t)$$

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Generalization of form factors and Parton Distribution Functions.



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- PDF forward limit
- Form factor sum rule
- Polynomiality

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 $\int_{-1}^{+1} dx \, x^n H^q(x,\xi,t) = \text{polynomial in } \xi$

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- Form factor sum rule
- Polynomiality
- Positivity

$$H^{q}(x,\xi,t) \leq \sqrt{q\left(rac{x+\xi}{1+\xi}
ight)q\left(rac{x-\xi}{1-\xi}
ight)}$$

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- Positivity
- H^q is an **even function** of ξ from time-reversal invariance.



Generalization of form factors and Parton Distribution Functions.



Sketching the pion GPD

- PDF forward limit
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- Positivity
- H^q is an **even function** of ξ from time-reversal invariance.
- H^q is **real** from hermiticity and time-reversal invariance.

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- H^q is an **even function** of ξ from time-reversal invariance.
- *H^q* is **real** from hermiticity and time-reversal invariance.

• H^q has support $x \in [-1, +1]$.



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- H^q is an **even function** of ξ from time-reversal invariance.
- *H^q* is **real** from hermiticity and time-reversal invariance.
 - H^q has support $x \in [-1, +1]$.
 - Soft pion theorem (pion target)

$$H^{q}(x,\xi=1,t=0) = \frac{1}{2}\phi_{\pi}^{q}\left(\frac{1+x}{2}\right)$$

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Generalization of form factors and Parton Distribution Functions.



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- H^q is an **even function** of ξ from time-reversal invariance.
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- H^q has support $x \in [-1, +1]$.
- **Soft pion theorem** (pion target)

Numerous theoretical constraints on GPDs.

- There is no known GPD parameterization relying only on first principles.
- Modeling becomes a key issue.



Double Distributions. Natural solution of the polynomiality problem.



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Conclusions

- A function satisfying a polynomiality property is the Radon transform of another function.
- Representation of GPD in terms of **Double Distributions**:

$$H^{q}(x,\xi,t) = \int_{\Omega} \mathrm{d}\beta \mathrm{d}\alpha \,\delta(x-\beta-\alpha\xi) \big(F^{q}(\beta,\alpha,t) + \xi G^{q}(\beta,\alpha,t)\big)$$

Müller *et al.*, Fortschr. Phys. **42**, 101 (1994) Radyushkin, Phys. Rev. **D59**, 014030 (1999) Radysuhkin, Phys. Lett. **B449**, 81 (1999)

- Support property: $x \in [-1, +1]$.
- Discrete symmetries: F^q is α -even and G^q is α -odd.

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Double Distributions. Identifying Double Distributions from GPD Mellin moments.



Sketching the pion GPD

Introduction

 Define Double Distributions F^q and G^q as matrix elements of twist-2 quark operators:

Theoretical framework Definition Double Distributions
$$\left\langle P + \frac{\Delta}{2} \middle| \bar{q}(0) \gamma^{\{\mu} i \stackrel{\leftrightarrow}{\mathbf{D}}{}^{\mu_1} \dots i \stackrel{\leftrightarrow}{\mathbf{D}}{}^{\mu_m\}} q(0) \middle| P - \frac{\Delta}{2} \right\rangle = \sum_{k=0}^m \binom{m}{k}$$

GPD modeling Diagrams $\left[F_{mk}^q(t) 2P^{\{\mu} - G_{mk}^q(t) \Delta^{\{\mu\}} \right] P^{\mu_1} \dots P^{\mu_{m-k}} \left(-\frac{\Delta}{2} \right)^{\mu_{m-k+1}} \dots \left(-\frac{\Delta}{2} \right)^{\mu_m}$

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with

$$\begin{split} F^{q}_{mk} &= \int_{\Omega} \mathrm{d}\beta \mathrm{d}\alpha \, \alpha^{k} \beta^{m-k} F^{q}(\beta, \alpha) \\ G^{q}_{mk} &= \int_{\Omega} \mathrm{d}\beta \mathrm{d}\alpha \, \alpha^{k} \beta^{m-k} G^{q}(\beta, \alpha) \end{split}$$

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GPDs in the Dyson-Schwinger and Bethe-Salpeter Approach

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GPDs in the rainbow ladder approximation. Evaluation of triangle diagrams.



Sketching the pion GPD

$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \left| \bar{q}(0) \gamma^+ (i\overleftrightarrow{D}^+)^m q(0) \right| \pi, P - \frac{\Delta}{2} \right\rangle$$

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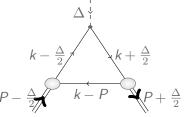
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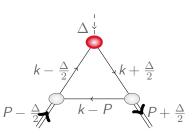
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- Compute **Mellin moments** of the pion GPD *H*.
- Triangle diagram approx.

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Sketching the pion GPD

$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \left| \bar{q}(0) \gamma^+ (i\overleftrightarrow{D}^+)^m q(0) \right| \pi, P - \frac{\Delta}{2} \right\rangle$$

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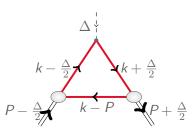
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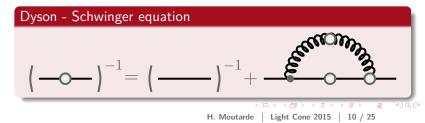
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- Compute **Mellin moments** of the pion GPD *H*.
- Triangle diagram approx.
- Resum infinitely many contributions.



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Sketching the pion GPD

$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \left| \bar{q}(0) \gamma^+ (i\overleftrightarrow{D}^+)^m q(0) \right| \pi, P - \frac{\Delta}{2} \right\rangle$$

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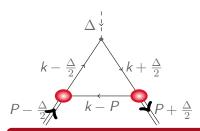
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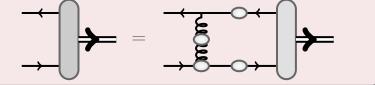
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- Compute **Mellin moments** of the pion GPD *H*.
- Triangle diagram approx.
- Resum infinitely many contributions.

Bethe - Salpeter equation



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Sketching the pion GPD

$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \left| \bar{q}(0) \gamma^+ (i\overleftrightarrow{D}^+)^m q(0) \right| \pi, P - \frac{\Delta}{2} \right\rangle$$

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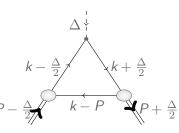
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- Compute **Mellin moments** of the pion GPD *H*.
- Triangle diagram approx.
- Resum infinitely many contributions.
- **Nonperturbative** modeling.

• Most GPD properties **satisfied by construction**.

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GPDs in the rainbow ladder approximation. Evaluation of triangle diagrams.



Sketching the pion GPD

$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \left| \bar{q}(0) \gamma^+ (i\overleftrightarrow{D}^+)^m q(0) \right| \pi, P - \frac{\Delta}{2} \right\rangle$$

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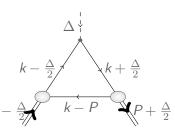
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- Compute **Mellin moments** of the pion GPD *H*.
- Triangle diagram approx.
- Resum infinitely many contributions.
- Nonperturbative modeling.

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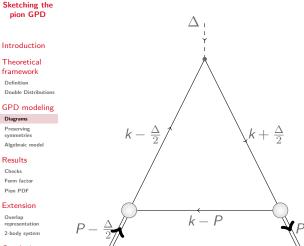
- Most GPD properties satisfied by construction.
- Also compute crossed triangle diagram.

Mezrag *et al.*, arXiv:1406.7425 [hep-ph] and Phys. Lett. **B741**, 190 (2015)

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GPDs in the rainbow ladder approximation. Physical content.





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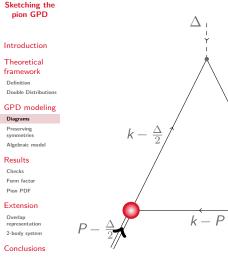
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GPDs in the rainbow ladder approximation. Physical content.



 Bethe-Salpeter vertex.



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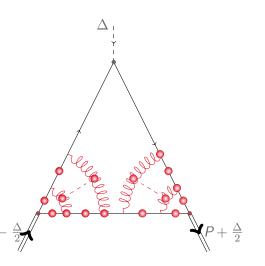
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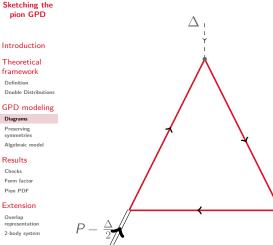
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GPDs in the rainbow ladder approximation. Physical content.





- Bethe-Salpeter vertex.
- Dressed quark propagator.

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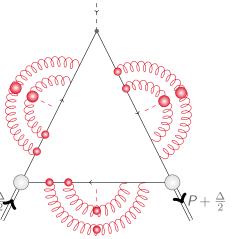
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- Bethe-Salpeter vertex.
 - Dressed quark propagator.
 - Much more than tree level perturbative diagram!

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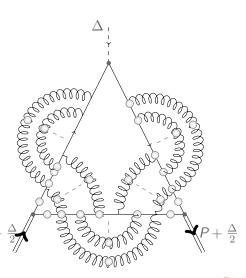
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- Bethe-Salpeter vertex.
- Dressed quark propagator.
- Much more than tree level perturbative diagram!
- Enable description of non perturbative phenomena.

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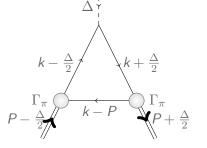
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Mellin moments.







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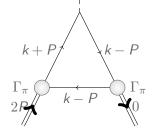
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- Mellin moments.
- Soft pion kinematics.



-2P



Symmetry-preserving truncation. Most of the GPD properties are obtained *a priori*.



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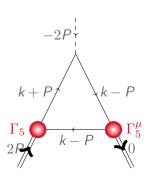
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- Mellin moments.
- Soft pion kinematics.
- Axial and axial vector vertices Γ₅, Γ^μ₅ in chiral limit.



Symmetry-preserving truncation. Most of the GPD properties are obtained *a priori*.



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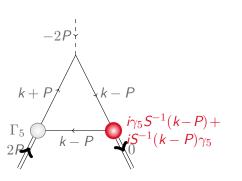
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- Mellin moments.
- Soft pion kinematics.
- Axial and axial vector vertices Γ₅, Γ₅^μ in chiral limit.
- Axial-vector Ward identity.



Symmetry-preserving truncation. Most of the GPD properties are obtained *a priori*.



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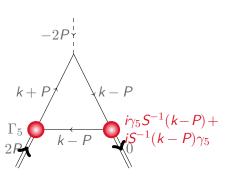
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- Mellin moments.
- Soft pion kinematics.
- Axial and axial vector vertices Γ₅, Γ₅^μ in chiral limit.
- Axial-vector Ward identity.
- Recover pion DA
 Mellin moments.

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Interaction strength and phenomenology. Constraints from the lattice and from spectroscopy.

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 k^2 [GeV²]



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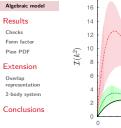
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0.5

- Gap equation kernel depends on interaction strength function $\mathcal{I}(k^2)$.
- Current model of $\mathcal{I}(k^2)$ yields ground and excited-state hadron masses with a **10-15 % accuracy** compared to experimental data.

Roberts et al., Few Body Syst. 51, 1 (2011)

Good agreement with independent evaluation from lattice data + Dyson-Schwinger equations.

> Binosi *et al.*, Phys. Lett. **B742**, 183 (2015)

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Towards an algebraic model. Dealing with the solutions of the gap and Bethe-Salpeter equations.



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- Numerical resolution of gap and Bethe-Salpeter equations in Euclidean space.
- Analytic continuation to Minkowskian space required.
- **III-posed** problem in the sense of Hadamard.
- Parameterize solutions and fit to numerical solution:

Gap Complex-conjugate pole representation:

$$S(k) = \sum_{i=0}^{N} \left[\frac{z_i}{i \not k + m_i} + \frac{z_i^*}{i \not k + m_i^*} \right]$$

Bethe-Salpeter Nakanishi representation of amplitude \mathcal{F}_{π} :

$$\mathcal{F}_{\pi}(q^2, q \cdot P) = \int_{-1}^{+1} \mathrm{d}\alpha \, \int_{0}^{\infty} \mathrm{d}\lambda \frac{\rho(\alpha, \lambda)}{(q^2 + \alpha q \cdot P + \lambda^2)^n}$$

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Intermediate step before using numerical solutions of Dyson-Schwinger and Bethe-Salpeter equations.



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$$\begin{split} S(p) &= \left[-i\gamma \cdot p + M \right] \Delta_M(p^2) \\ \Delta_M(s) &= \frac{1}{s + M^2} \\ \Gamma_\pi(k,p) &= i\gamma_5 \frac{M}{f_\pi} M^{2\nu} \int_{-1}^{+1} \mathrm{d}z \, \rho_\nu(z) \, \left[\Delta_M(k_{+z}^2) \right]^\nu \\ \rho_\nu(z) &= R_\nu (1 - z^2)^\nu \end{split}$$

with R_{ν} a normalization factor and $k_{+z} = k - p(1-z)/2$. Chang *et al.*, Phys. Rev. Lett. **110**, 132001 (2013) Only two parameters:

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 $S(p) = \left[-i\gamma \cdot p + \mathbf{M} \right] \Delta_{\mathbf{M}}(p^{2})$ $\Delta_{\mathbf{M}}(s) = \frac{1}{s + \mathbf{M}^{2}}$ $\Gamma_{\pi}(k, p) = i\gamma_{5} \frac{\mathbf{M}}{f_{\pi}} \mathbf{M}^{2\nu} \int_{-1}^{+1} dz \, \rho_{\nu}(z) \left[\Delta_{\mathbf{M}}(k_{+z}^{2}) \right]^{\nu}$ $\rho_{\nu}(z) = R_{\nu} (1 - z^{2})^{\nu}$

with R_{ν} a normalization factor and $k_{+z} = k - p(1-z)/2$. Chang *et al.*, Phys. Rev. Lett. **110**, 132001 (2013) Only two parameters:

Dimensionful parameter *M*.

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- Dimensionful parameter M.
- Dimensionless parameter ν

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Intermediate step before using numerical solutions of Dyson-Schwinger and Bethe-Salpeter equations.



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- Dimensionful parameter *M*.
- Dimensionless parameter v. Fixed to 1 to recover asymptotic pion DA.

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Results: Theoretical Constraints and Phenomenology

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Sketching the pion GPD

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$$H_{x\geq\xi}^{u}(x,\xi,0) = \frac{48}{5} \left\{ \frac{3\left(-2(x-1)^{4}\left(2x^{2}-5\xi^{2}+3\right)\log(1-x)\right)}{20\left(\xi^{2}-1\right)^{3}} \right\}$$
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$$\frac{3\left(x^{3}(x(2(x-4)x+15)-30)-15(2x(x+5)+5)\xi^{4}\right)\log\left(x^{2}-\xi^{2}\right)}{20\left(\xi^{2}-1\right)^{3}} + \frac{3\left(x^{3}(x(2(x-4)x+15)-30)-15(2x(x+5)+5)\xi^{4}\right)\log\left(x^{2}-\xi^{2}\right)}{20\left(\xi^{2}-1\right)^{3}} + \frac{3\left(-5x(x(x(x+2)+36)+18)\xi^{2}-15\xi^{6}\right)\log\left(x^{2}-\xi^{2}\right)}{20\left(\xi^{2}-1\right)^{3}} + \frac{3\left(2(x-1)\left((23x+58)\xi^{4}+(x(x(x+67)+112)+6)\xi^{2}+x(x((5-2x)x+15)+3)\xi^{2}-15\xi^{6}\right)\log\left(1-\xi^{2}\right)\right)}{20\left(\xi^{2}-1\right)^{3}} + \frac{3\left(\left(15(2x(x+5)+5)\xi^{4}+10x(3x(x+5)+11)\xi^{2}\right)\log\left(1-\xi^{2}\right)\right)}{20\left(\xi^{2}-1\right)^{3}} + \frac{3\left(2x(5x(x+2)-6)+15\xi^{6}-5\xi^{2}+3\right)\log\left(1-\xi^{2}\right)\right)}{20\left(\xi^{2}-1\right)^{3}} \right\}$$

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Analytic expression in the DGLAP region. Sketching the pion GPD Similar expression in the ERBL region. Introduction Theoretical with correct powers of ξ . framework Also direct verification using Mellin moments of H. Definition Double Distributions GPD modeling

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Explicit check of **support property** and **polynomiality**

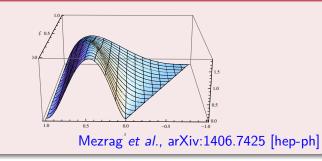




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- Analytic expression in the DGLAP region.
 - Similar expression in the ERBL region.
- **Explicit check** of **support property** and **polynomiality** with correct powers of *ξ*.
 - Also direct verification using Mellin moments of *H*.

Valence $H^u(x,\xi,t)$ as a function of x and ξ at vanishing t.



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Determination of the model dimensionful parameter M.



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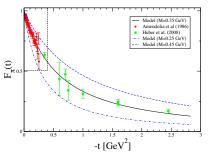
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Pion form factor obtained from isovector GPD:

$$\int_{-1}^{+1} \mathrm{d}x \, H^{l=1}(x,\xi,t) = 2F_{\pi}(t)$$

Single dimensionful parameter $M \simeq 350$ MeV.



Mezrag *et al.*, arXiv:1406.7425 [hep-ph]

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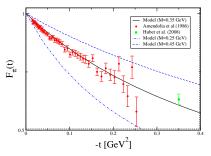
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$$\int_{-1}^{+1} \mathrm{d}x \, H^{l=1}(x,\xi,t) = 2F_{\pi}(t)$$

• Single dimensionful parameter $M \simeq 350$ MeV.



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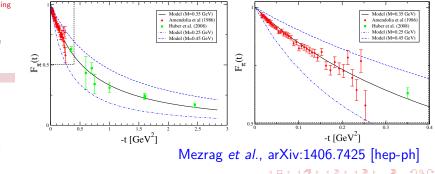
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Overlap representation 2-body system Pion form factor obtained from isovector GPD:

$$\int_{-1}^{+1} \mathrm{d}x \, H^{l=1}(x,\xi,t) = 2F_{\pi}(t)$$

• Single dimensionful parameter $M \simeq 350$ MeV.



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Pion Parton Distribution Function. Determination of the model initial scale.



Sketching the pion GPD Pion PDF obtained from forward limit of GPD:

 $q(x) = H^q(x, 0, 0)$

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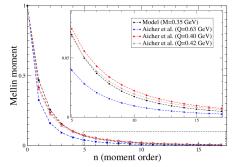
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Conclusions

 Use LO DGLAP equation and compare to PDF extraction. Aicher *et al.*, Phys. Rev. Lett. **105**, 252003 (2010)



Mezrag et al., arXiv:1406.7425 [hep-ph]

Extension: Implementing Positivity and Polynomiality

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Overlap representation. A first-principle connection with Light Front Wave Functions.



Sketching the pion GPD

Decompose an hadronic state $|H; P, \lambda\rangle$ in a Fock basis:

Introduction
$$|H; P, \lambda\rangle = \sum_{N,\beta} \int [\mathrm{d}x \mathrm{d}\mathbf{k}_{\perp}]_N \psi_N^{(\beta,\lambda)}(x_1, \mathbf{k}_{\perp 1}, \dots, x_N, \mathbf{k}_{\perp N}) |\beta, k_1, \dots, k_N\rangle$$

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GPD modeling

Diagrams

Preserving symmetries

Algebraic model

Results

Chocks

Form factor

Pion PDF

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2-body system

Conclusions

Derive an expression for the pion GPD in the DGLAP region $\xi < x < 1$:

$$H^{q}(x,\xi,t) \propto \sum_{\beta,j} \int [d\bar{x} d\bar{\mathbf{k}}_{\perp}]_{N} \delta_{j,q} \delta(x-\bar{x}_{j}) \psi_{N}^{(\beta,\lambda)*}(\hat{x}',\hat{\mathbf{k}}_{\perp}') \psi_{N}^{(\beta,\lambda)}(\tilde{x},\tilde{\mathbf{k}}_{\perp})$$

with $\tilde{x}, \tilde{\mathbf{k}}_{\perp}$ (resp. $\hat{x}', \hat{\mathbf{k}}'_{\perp}$) generically denoting incoming (resp. outgoing) parton kinematics.

Diehl et al., Nucl. Phys. **B596**, 33 (2001)

Similar expression in the ERBL region $-\xi \le x \le \xi$, but with overlap of *N*- and (N+2)-body LFWF.

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Pion Parton Distribution Function. Algebraic model in the overlap representation.

Overlap — Triangle diagram



Sketching the pion GPD

Evaluate LFWF in algebraic model:
$$\psi(\textbf{x},\textbf{k}_{\perp}) \propto \frac{\textbf{x}(1-\textbf{x})}{[(\textbf{k}_{\perp}-\textbf{x}\textbf{P}_{\perp})^2+\textit{M}^2]^2}$$

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a(x)

1.5

1.0

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Overlap representation 2-body system



$$\psi(x, \mathbf{k}_{\perp}) \propto \frac{\chi(1-\chi)}{[(\mathbf{k}_{\perp} - x\mathbf{P}_{\perp})^2 + \mathbf{E}_{\perp}]^2}$$

Expression for the GPD at $t = 0$:

$$H(x,\xi,0) \propto \frac{(1-x)^2(x^2-\xi^2)}{(1-\xi^2)^2}$$

Manifest 2-body symmetry.

• Expression for the PDF:

 $q(x) = 30x^2(1-x)^2$

• Off-forward case: in progress.

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Conclusions

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Conclusions and prospects. Symmetry-preserving GPD modeling.



Sketching the pion GPD

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- Preserving symmetries
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- Overlap representation 2-body system
- Conclusions

- Computation of GPDs, DDs, PDFs, LFWFs and form factors in the nonperturbative framework of Dyson-Schwinger and Bethe-Salpeter equations.
- **Explicit check** of several theoretical constraints, including polynomiality, support property and soft pion theorem.
- Simple algebraic model exhibits most features of the numerical solutions of the Dyson-Schwinger and Bethe-Salpeter equations.
- Very good agreement with existing pion form factor and PDF data.
- In progress: a priori implementation of polynomiality and positivity.

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