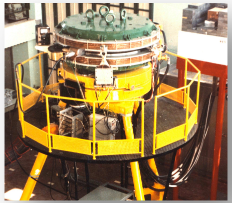

# Models for the total and inelastic pp cross-sections at LHC and beyond


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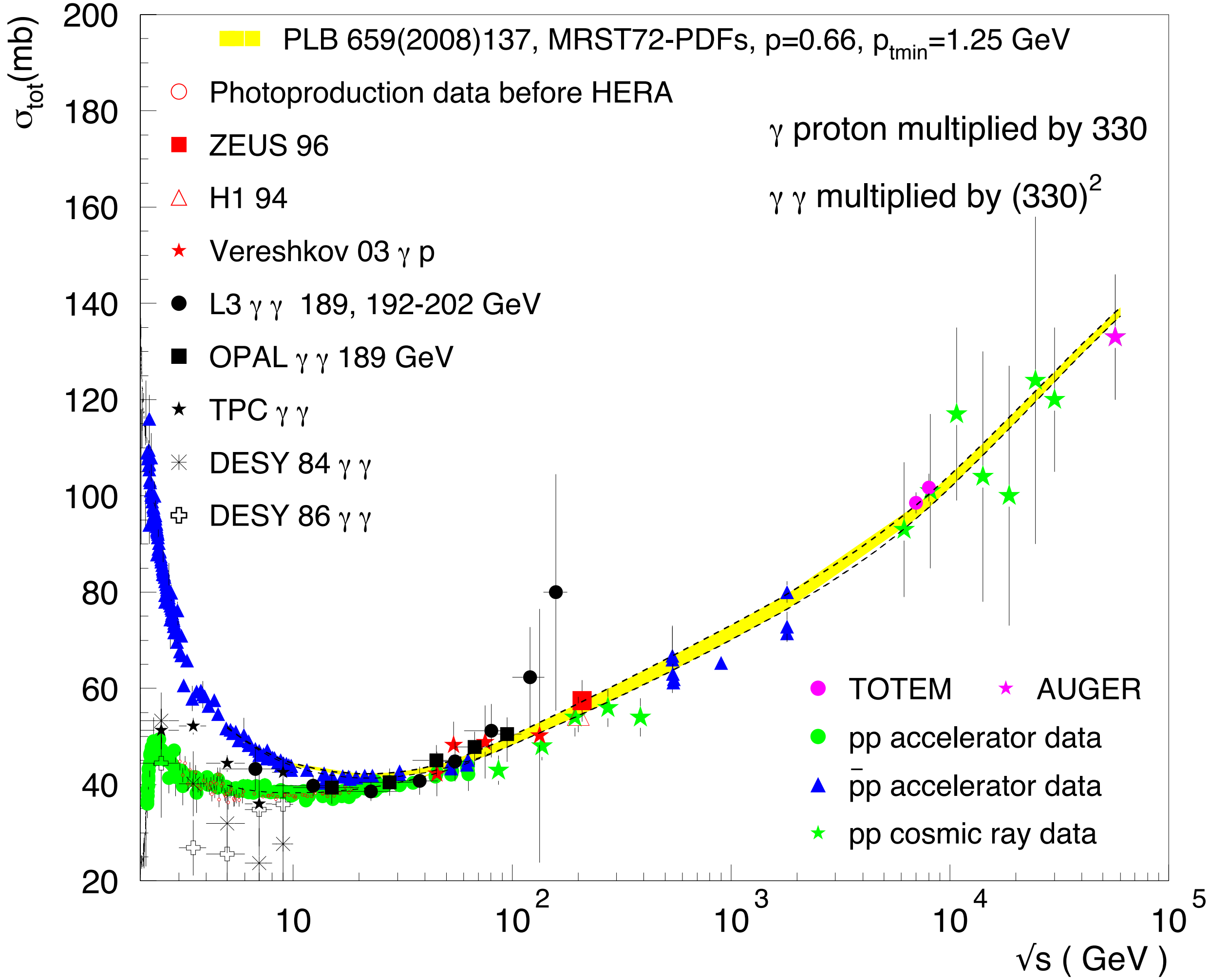
Cortona – NonPerturbative QCD, April 21st 2015

# All total cross-sections rise

Update of EPJC 2009, GP + R.M.Godbole, A. Grau, Y. Srivastava

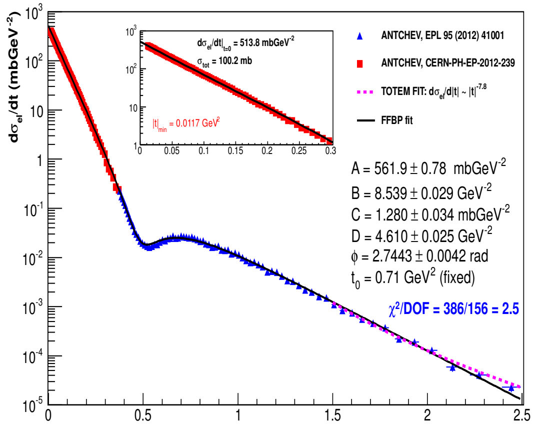


# The components of the total cross-section

- Total *pp-> everything*
- Inelastic *pp-> X (excluding pp elastic)*
  - Exp. def  complicated by cuts
  - Theor. calc. still unsolved

$$\sigma_{elastic} = \int_{-\infty}^{0} dt \frac{d\sigma_{el}}{dt}$$

- *Elastic  pp-> pp*
  - Differential elastic
    - Optical point
    - Forward peak
    - Dip and bump
    - Large |t| tail



# Basic fact: All total cross-sections rise… but not too much (Froissart dixit in 1961 + Martin 1962+Lukaszuk 1967)

$$\sigma_{total} \lesssim [\log s]^2$$

Asymptotically

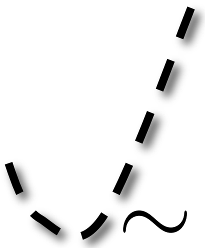
Where from?
A cut off in maximum angular momentum in partial wave expansion and
assumptions about large s behaviour of the pw amplitudes

=> b< b $_{max}$

$$\sigma_{tot} \lesssim \sum_{0,L} \simeq L^2_{max}$$

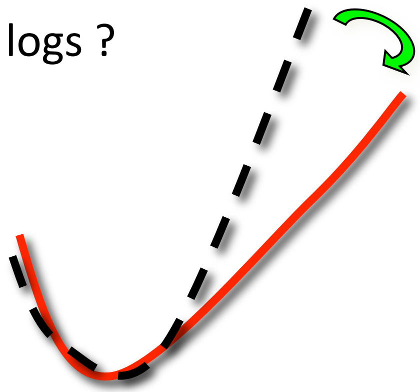
$$L_{max} = qb_{max} \sim \log s$$

What generates the rise, which is very fast at the start (ISR)?



$$\sim s^{0.3}$$

How to go from
power-law to logs ?



What tames the rise into to a Froissart-like behavior?

# Basic tension between Regge vs. eikonal: total x-section

- Regge + optical theorem:

t-space

$$\mathcal{A}(s,t) \simeq i\beta(t)s^{\alpha(t)-1}$$

$$\sigma_{tot} = 4\pi \Im m \mathcal{A}(s,t=0)$$

$$\simeq s^{\alpha(0)-1}$$

- Rise ~ $\alpha(0) = 1 + \epsilon > 1$

Donnachie Landshoff

$$\sigma_{total} = X s^{-\eta} + Y s^{\epsilon}$$

- NO Froissart bound

- Eikonal models:b-space

$$\mathcal{A}(s,t) = \int d^2\mathbf{b} e^{i\mathbf{q}\cdot\mathbf{b}}[1 - e^{i\chi(b,s)}]$$

Simplest : Black Disk Limit

$$i\chi(b,s) = -\theta(R(s) - b)$$

$$\sigma_{total} = \pi R^2(s)$$

Expanding radius ~ log s
Froissart bound OK because of cut-off in b-space

# One channel eikonal

$$\sigma_{total}(s) = 2\int d^2b[1 - \Re e^{i\chi(b,s)}]$$

$$\simeq \int d^2b[1 - e^{-\Im m\chi(b,s)}]$$

$$\sigma_{elastic}(s) = \int d^2b|1 - e^{i\chi(b,s)}|^2$$

$$\sigma_{inel} = \sigma_{total} - \sigma_{elastic} = \int d^2b[1 - e^{-2\Im m\chi(b,s)}]$$

Two channel eikonals : $|p >= c_1|p_1 > +c_2|p_2 >$

Or also 3 states, continuous distributions, etc.

# Eikonal + Regge: an example from Khoze, Martin, Ryskin(KMR)

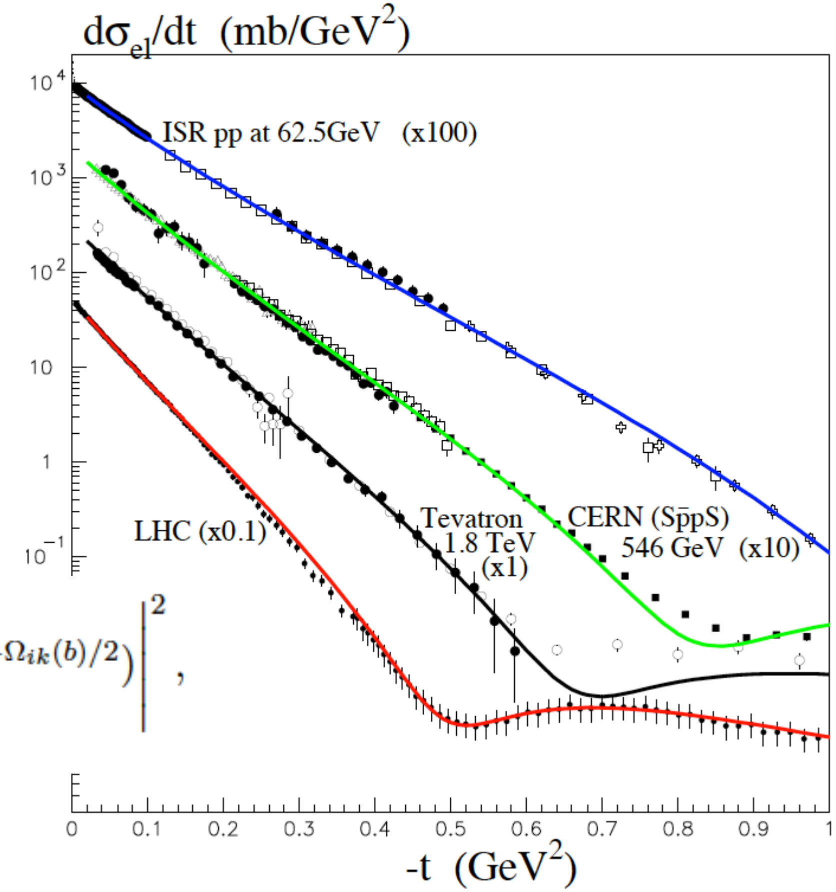
One channel

$$T_{el}(b,s) = i[1 - e^{-\Omega(b,s)/2}]$$

$$\Omega(b,s) = \int \frac{d^2 q_t}{4\pi^2} \sigma_0 F^2(q_t)[s/s_0]^{\alpha_P(t)-1}$$

Two or more channels, a la GW

$$\frac{d\sigma_{\rm el}}{dt} = \frac{1}{4\pi}\left|\int d^2b\, e^{iq_t\cdot b} \sum_{i,k} |a_i|^2|a_k|^2\, (1 - e^{-\Omega_{ik}(b)/2})\right|^2,$$




Eur.Phys.J. C74, 2756 (2014), arXiv:1312.3851

# Basic properties and models

- Donnachie and Landshoff based on Regge theory
  - Have decrease and rise

$$\boxed{\sigma_{AB}^{total} = X_{AB}s^{\epsilon} + Y_{AB}s^{-\eta_{AB}}}$$

  But -> violate Froissart bound

- Eikonal models satisfy unitarity

  But -> need impact parameter distribution
  - Single channel

$$\boxed{\mathcal{A}(s,t) = i\int d^2bJ_0(qb)[1 - e^{i\chi(b,s)}]}$$

  - Multichannel can also describe diffraction and $d\sigma_{elastic}/dt$
    But => need additional parameters
    - Two channel Khoze, Martin, Ryskin (etc.)
    - 3 channel Gostman, Levin, Maor (etc.)
    - Continuous distributions Lipari, Lusignoli (etc.)

# 1. How to generate a cut-off in b-space, asa confinement?

- Heisenberg 1952 shock wave model, cut off in b-space determined by the extension of the pion cloud

$$\boxed{\sigma_{total} \simeq \frac{\pi}{m_\pi^2}(\ln\frac{\sqrt{s}}{< E_0 >})^2}$$

$$< E_0 > \simeq constant \quad \sigma_{tot} \sim [\ln s]^2$$

$$< E_0 > \simeq \ln s \quad \sigma \simeq constant$$

- Form factors (most commonly used, early mini-jet models 1984-85), soft Pomeron models, etc.

- Resummation with singularity confinement in a eikonalized minijet model (GP et al. model)

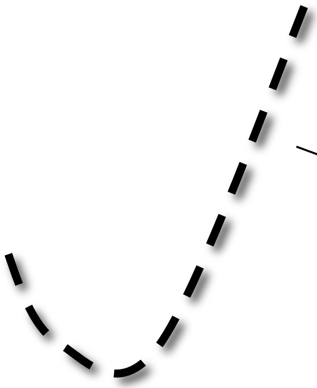
# Eikonal models

- Eikonal function built through Regge and Pomeron exchanges allows good overall phenomenological description of all, total, elastic, differential, but connection to pQCD not fully evident

- Mini-jet exchanges have clear pQCD origin and can drive the high energy behaviour of the eikonal : MC's or analytical calculations and use of parton standard PDFs, but….

# Mini-jet models : All total cross-sections rise… but not too much (Froissart dixit)

What generates the rise? Low-x parton collisions

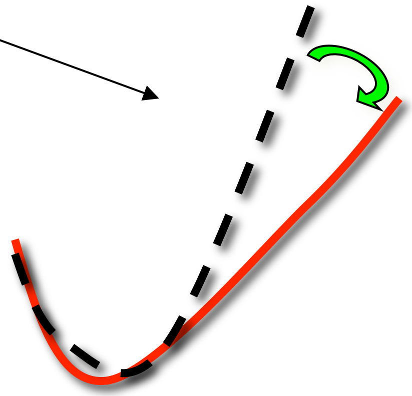


$s^{\epsilon} \quad \epsilon \sim 0.3$

Cline,Halzen &Luthe 1973
Gaisser, Halzen,Stanev 1985
G.P., Y.N. Srivastava 1986
Durand,Pi 1987
Sjostrand, van Zijl 1987
…

How to go from
hard to soft ?

What tames the rise into to a Froissart-like behavior?



A cut off obtained by [embedding into the eikonal]
the acollinearity induced by IR kt-emission
[our model, G.P. et al. **Phys.Lett.B382, 1996**]

# Minijets and the rise for √s ≈ 20 GeV

**pQCD**

$$\sigma^{AB}_{\rm jet} = \int_{ptmin}^{\sqrt{s}/2} dp_t \int_{4p_t^2/s}^{1} dx_1 \int_{4p_t^2/(x_1 s)}^{1} dx_2 \sum_{i,j,k,l} f_{i|A}(x_1,p_t^2) f_{j|B}(x_2,p_t^2) \frac{d\hat{\sigma}^{kl}_{ij}(\hat{s})}{dp_t}$$

- asymptotic freedom regime

$$\alpha_s(p_t) \to \alpha_{AF} = \frac{b_0}{\ln[p_t^2/\Lambda^2_{QCD}]}$$

$$p_t >> \Lambda_{QCD} \qquad p_t \simeq 1\ GeV$$

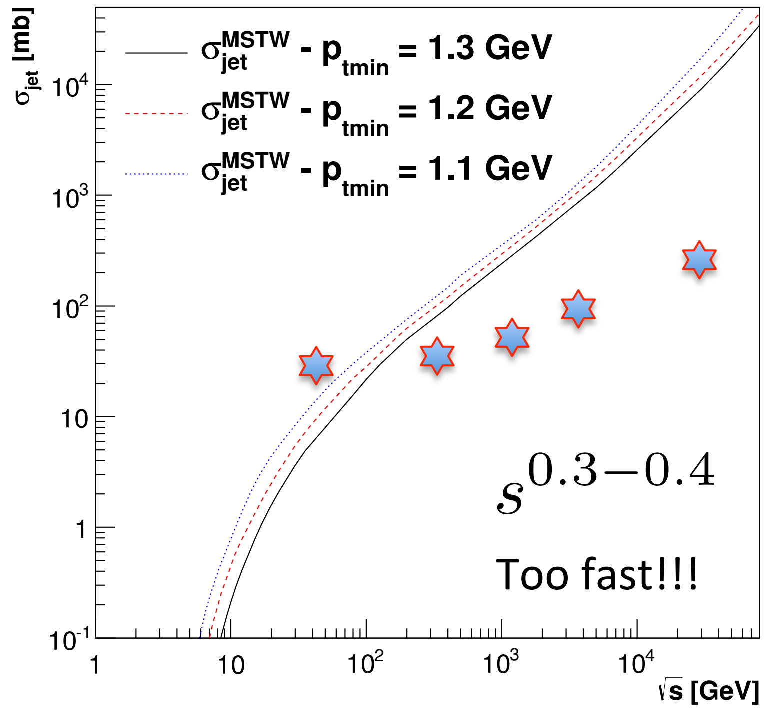
- parton-parton scattering with final parton $p_t \geq p_{tmin}$

for each initial parton

$f(x) \sim 1/x \qquad x \geq 2p_{tmin}/\sqrt{s}$

$x \leq 0.1 - 0.2 \quad and \quad \downarrow \quad \sigma_{mini-jet} \uparrow$

$$\boxed{\sqrt{s} \gtrsim 10 - 20\ GeV}$$




GP et al. , 1504.04890, to appear in PRD

# The mini-jet description is incomplete

- Hard scattering of color objects requires soft gluon emission (+ other finite corrections)
  - Soft ⇔ many undistinguishable soft gluons

    ⇔ resummation

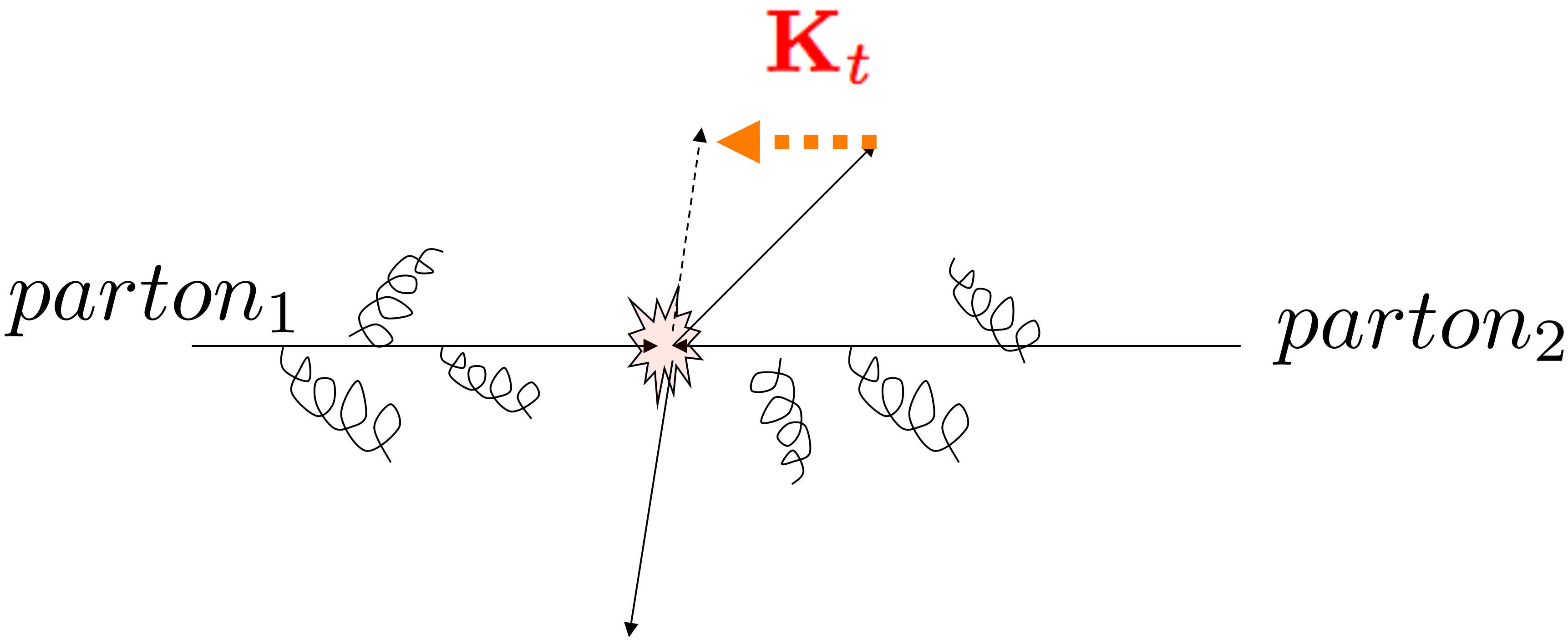
    exponentiation of regularized single spectrum

    obtained from integration in gluon momentum

⇒ Initial state gluon emission (and thus resummation) introduces an acollinearity => reduction of the x-section

- Embed into an eikonal formalism in b-space

# Soft gluon emission introduces acollinearity




Acollinearity reduces the collision cross-section as
partons do not scatter head-on any more, also explained as the gluon cloud
Becoming too thick for partons to see each other : gluon saturation

# Resummation: a semi-classical treatment

- photons= classical (Poisson distributions  constrained by overall Energy momentum conservation)+ QED (single photon emission spectrum)

$$d^2P(\vec{K}_t) = \sum P(n_k, \bar{n}_k)\delta^2(\vec{K}_t - \sum \vec{k}_t n_{k_t}) d^2\vec{K}_t$$
$$\to \frac{d^2\vec{K}_t}{(2\pi)^2}\int d^2\vec{b}\; e^{i\vec{K}_t\cdot\vec{b} - h_{QED}(b)}$$

- B. Touschek et al., Nuovo Cimento 1967

- Gluons=classical +QCD≠

Same as in QED

$$k_t \to 0$$

$$\alpha_{q\to q+\ gluon}$$

In full resummation it is not possible to use AF, i.e.

$$\alpha_{\overset{\text{NO!}}{\cancel{AF}}}(k_t) = \frac{b_0}{\ln[k_t^2/\Lambda^2_{QCD}]}$$

# Our proposal for running $\alpha_s(k_t)$ in the infrared region

$$V_{one\ gluon\ exchange} \sim r^{2p-1}$$

$$\boxed{\propto k_t^{-2p} \qquad k_t << \Lambda}$$

To reconcile with asymptotic Freedom

$$\propto \frac{1}{\log k_t^2/\Lambda^2} \qquad k_t >> \Lambda$$

A phenomenological interpolation

$$\alpha_{eff}(k_t) = \frac{12\pi}{11N_c - 2N_f} \frac{p}{\log[1 + p(k_t/\Lambda_{QCD})^{2p}]}$$

# Our QCD model for the total cross-section
R. Godbole, A. Grau, GP, YN Srivastava

$$\sigma_{total} \simeq 2 \int d^2\vec{b}[1 - e^{-\chi_I(b,s)}]$$

$$2\chi_I(b,s) = A_{FormFactor}(b,s)\sigma_{soft}(p_t < p_{tmin}) + \\ + A_{Resum}(b,s)\sigma_{mini-jet}(p_t > p_{tmin})$$

- Minijets to drive the rise
- Soft kt-resummation to tame the rise and introduce the cut-off in b-space needed to satisfy the Froissart bound

- Phenomenological singular but integrable soft gluon coupling to relate confinement with the rise
- Interpolation between soft and asymptotic freedom region

PLB1996-PRD2005

We model the impact parameter distribution as the Fourier-transform of ISR soft $k_t$ distribution and thus obtain a cut-off at large distances : Froissart bound?

$$A_{BN}(b,s) = N \int d^2\mathbf{K}_\perp \; e^{-i\mathbf{K}_\perp\cdot\mathbf{b}} \frac{d^2P(\mathbf{K}_\perp)}{d^2\mathbf{K}_\perp} = \frac{e^{-h(b,q_{max})}}{\int d^2\mathbf{b}\; e^{-h(b,q_{max})}}$$

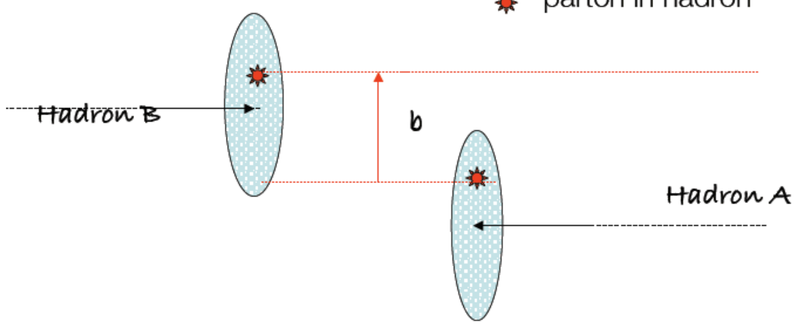
$$h(b,E) = \frac{16}{3\pi}\int_0^{q max} \frac{dk_t}{k_t}\alpha_{eff}(k_t)\ln\left(\frac{2q_{max}}{k_t}\right)[1 - J_0(bk_t)]$$

$$\boxed{\alpha_{eff}(k_t \to 0) \sim k_t^{-2p}}$$

$$\boxed{A_{BN}(b,s) \sim e^{-(b\bar{\Lambda})^{2p}}}$$

$\sqrt{s}$= c.m. Energy hadrons AB

parton in hadron

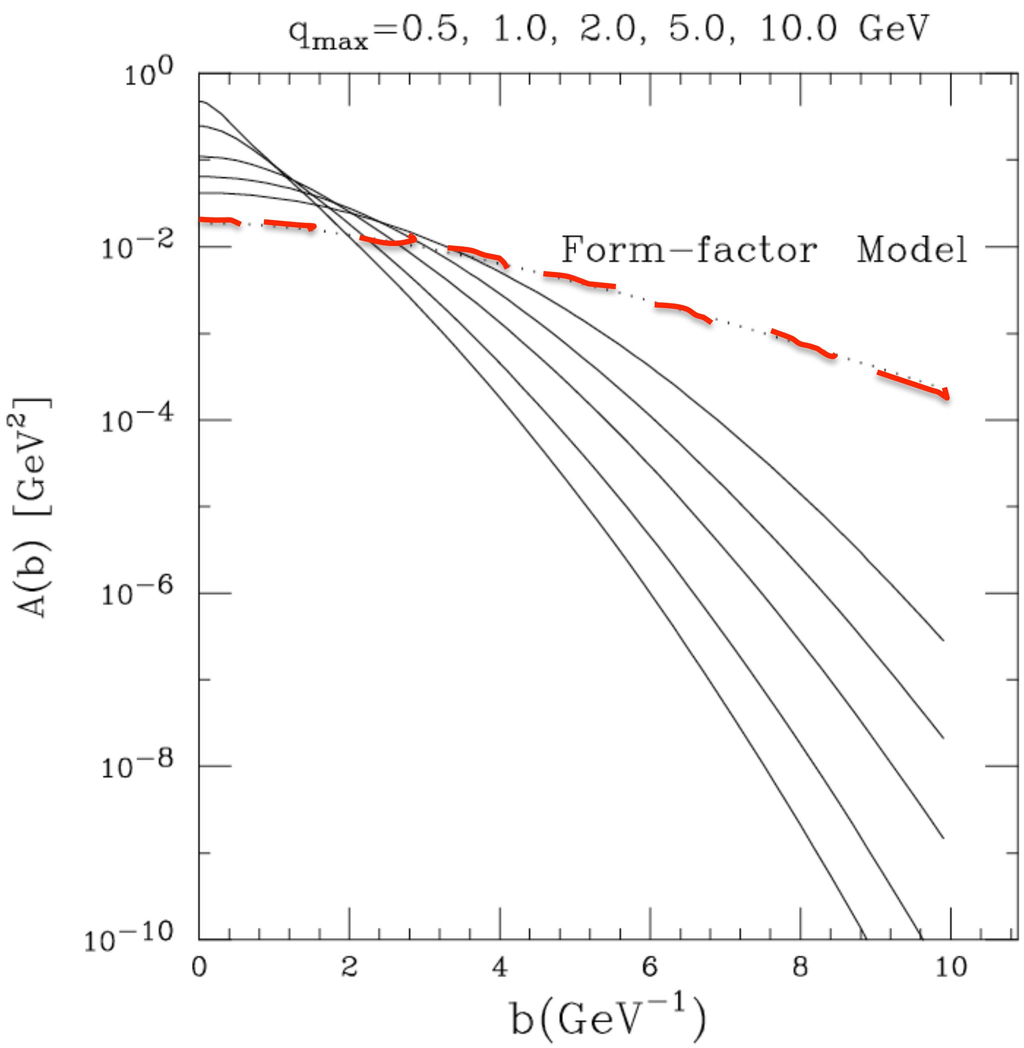


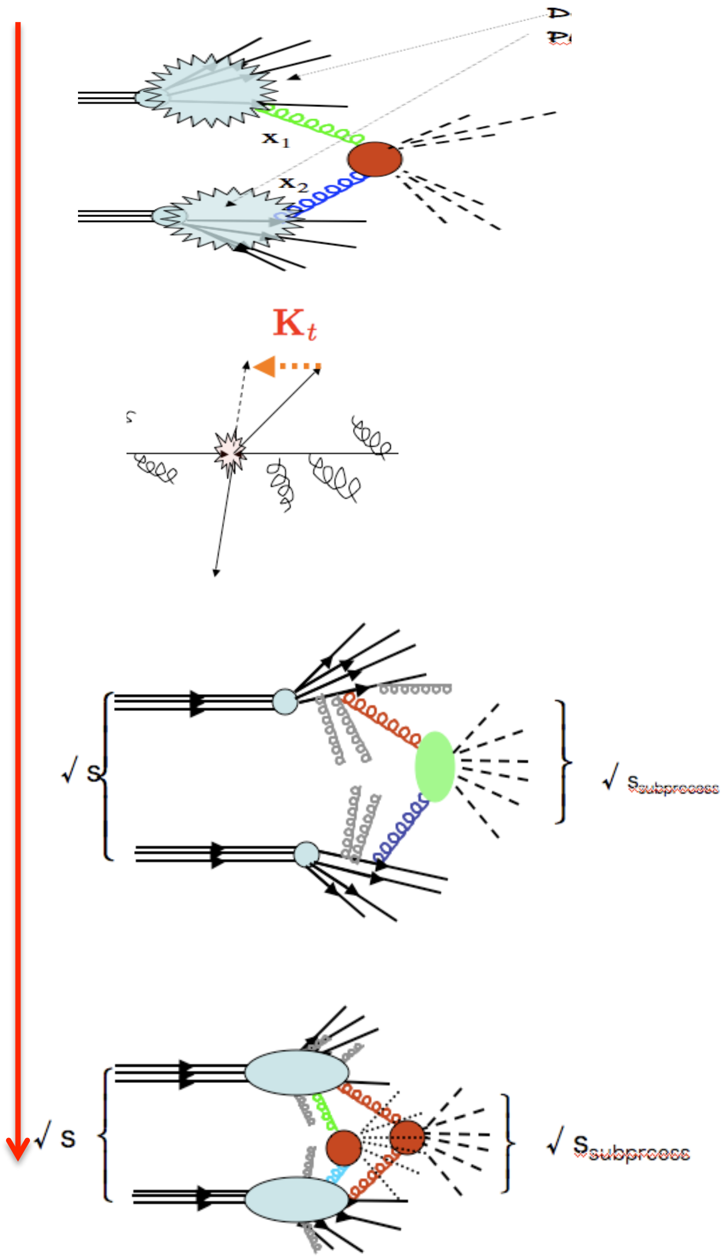

$\boxed{q_{tmax}}$ ?

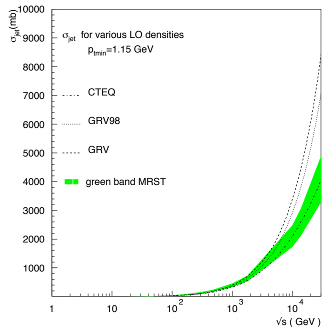
Fuzzy factorization (as in QED)
Fixed by single
gluon emission kinematics

**Parton b-distribution from form factor models vs resummation models**

Corsetti, Grau, Pancheri, Srivastava, PLB 1996



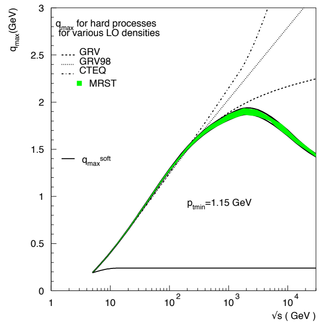



1. Calculate mini-jet cross-section Choosing densities and ptmin

$$\sigma_{mini-jet} \simeq s^{\epsilon}$$

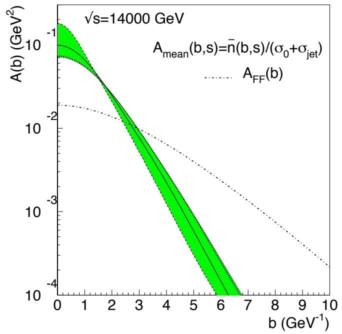
$$\epsilon \simeq 0.3 - 0.4$$




2. Calculate qmax: single soft gluon upper scale, for given PDF, ptmin

$$q_{max} \simeq p_{tmin}$$

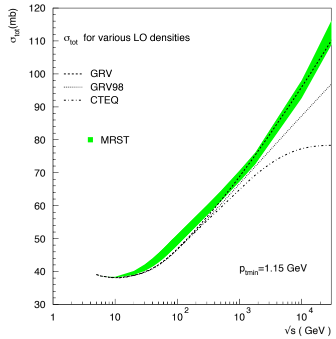
$$\lesssim 2 - 3\ GeV$$




3. Calculate impact parameter distribution for given qmax and given infrared parameter p

$$\chi(b,s) = \chi_{low\ energy} +$$

$$+ A(b, qmax)\sigma_{jet}$$




4. Eikonalize

$$\sigma_{total} = 2\int d^2\mathbf{b}[1 - e^{-\chi(b,s)}]$$

In our model, the emission of singular infrared gluons tames low-x gluon-gluon scattering ( mini-jets) and restores the Froissart bound

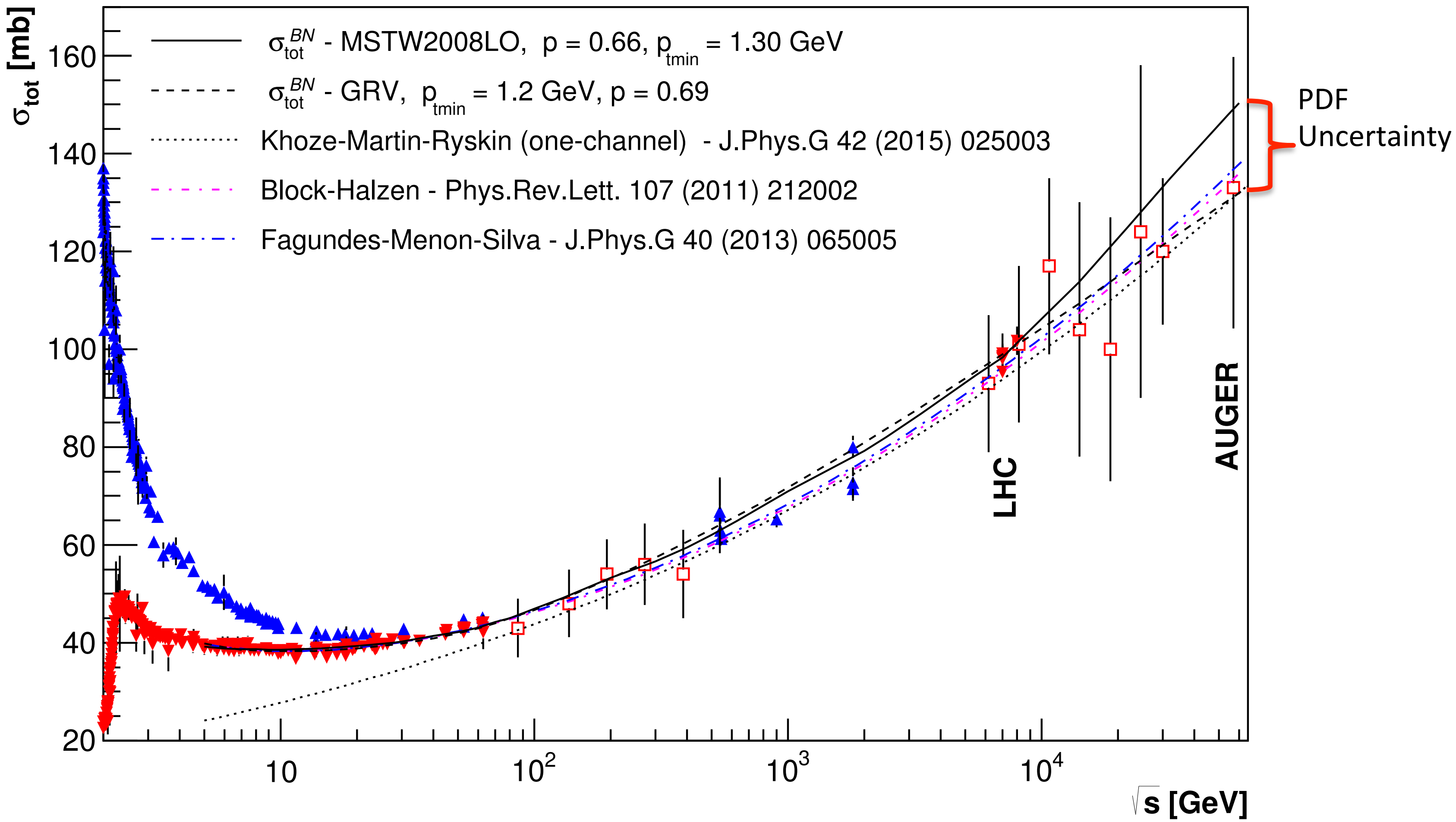
$s^{\varepsilon}$

$$\sigma_{tot}(s) \approx 2\pi \int_0^\infty db^2 [1 - e^{-C(s) e^{-(b\bar{\Lambda})^{2p}}}]$$

$$\sigma_{tot}(s) \rightarrow [\varepsilon \ln(s)]^{(1/p)} \qquad \frac{1}{2} < p < 1$$

Grau, Godbole,GP,Srivastava,PLB682 2009

# Some representative models: KMR, GP et al., Fagundes-Menon, Block-Halzen



# Total cross-section: model contents

- KMR model: one or two channel eikonal, soft Pomeron + hard Pomeron + triple Pomeron and form factors
  - Triple P to describe some regions of diffraction
  - Hard Pomeron for the rise
  - For elastic cross-section

- Similar models GLM, Ostapchenko

- Fagundes-Menon : minijets with a dynamical gluon and form factors

- Block Halzen : parametrization of an analytical model amplitude

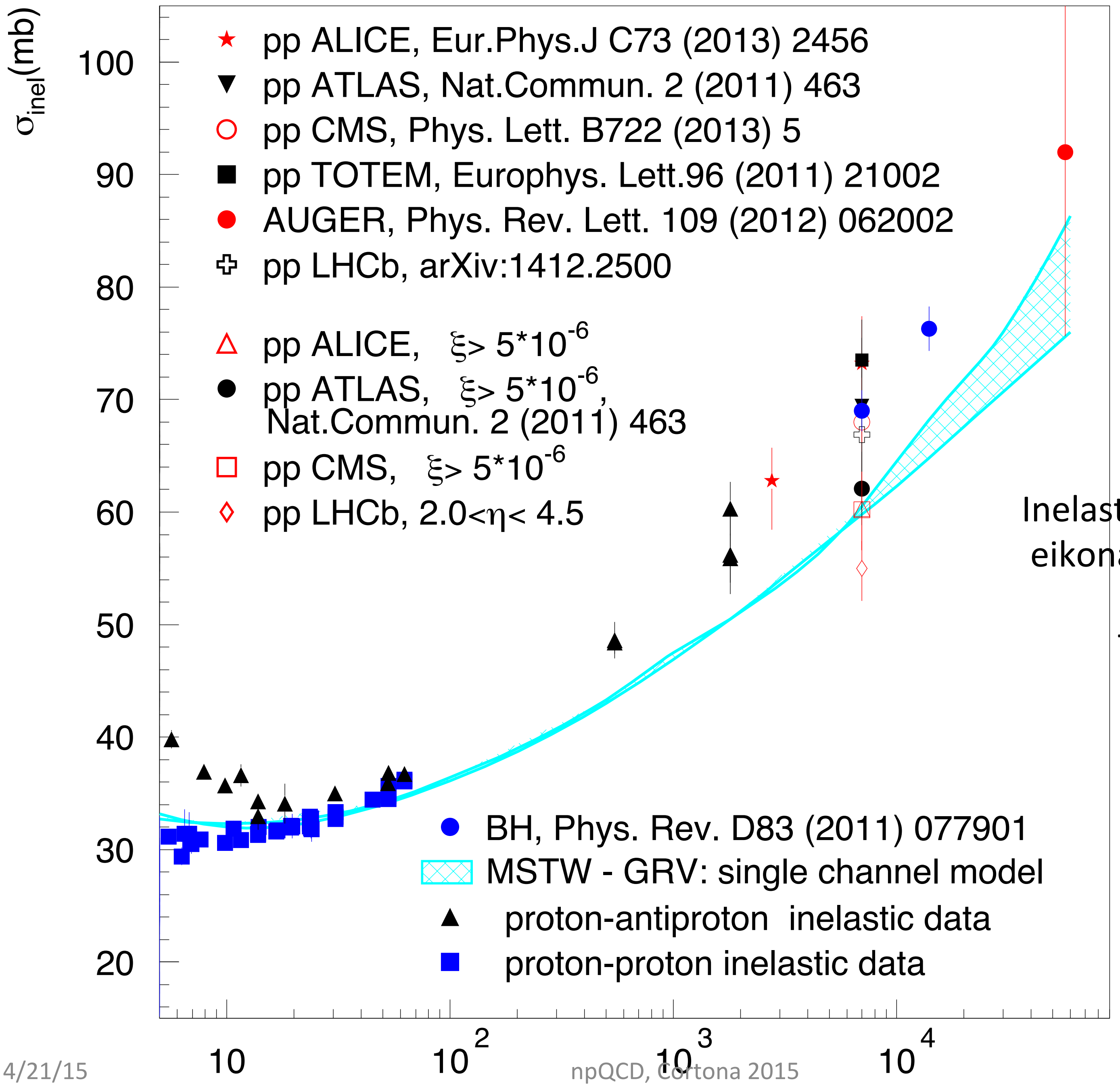
# The components of the total cross-section in one channel model

- Total pp-> everything  ok with one-channel eikonal minijet model

- Inelastic pp-> X (excluding pp elastic) low , something is missing

- Elastic  pp-> pp  too high
  - Differential elastic
    - Optical point
    - Forward peak
    - Dip and bump
    - Large |t| tail

$$\sigma_{elastic} = \int_{-\infty}^{0} dt \frac{d\sigma_{el}}{dt}$$




Inelastic in one channel eikonal with minijets

Too low

# The inelastic cross-section

- Basic definition

$$\sigma_{total} = \sigma_{elastic} + \underbrace{everything\ else}_{\sigma_{inelastic}}$$

- Experimentally : since forward (-t≈0), one needs to make cuts => measurement depends on cuts and extrapolations from central regions

- Theoretically : consensus on there being different regimes in central region and "diffraction "region, but separation is ill defined

# A strategy for decomposing the inelastic cross-section into diffractive and mini-jets driven

Strategy:

- Obtain the $\boxed{\chi_I(b,s)}$ from model for total cross-section
- Input $\boxed{\chi_I(b,s)}$ to obtain

$$\sigma_{inelastic} = \sigma_{inel}^{correlated} + \sigma_{inel}^{uncorrelated \simeq Poisson}$$

- Use a convenient parametrization for diffractive processes
- Compare with data

# 1. The uncorrelated part of the inelastic [Achilli et al., G, … PRD 2011]

**Uncorrelated independent events**
**Poisson distributed in b-space**

$$\bar{n}(b,s) \rightarrow \chi_I(b,s)/2$$

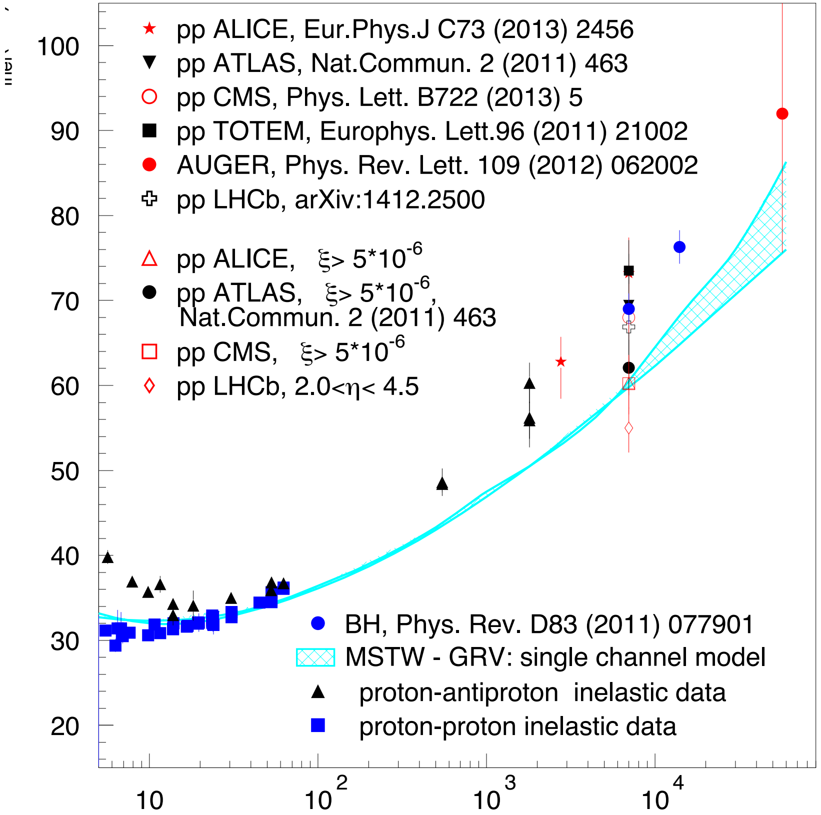
$$\Pi_n(\bar{n}) = \frac{\bar{n}^n e^{-\bar{n}}}{n!}$$

$$\sum_{1,\infty} \Pi_n = 1 - e^{-\bar{n}(b,s)}$$

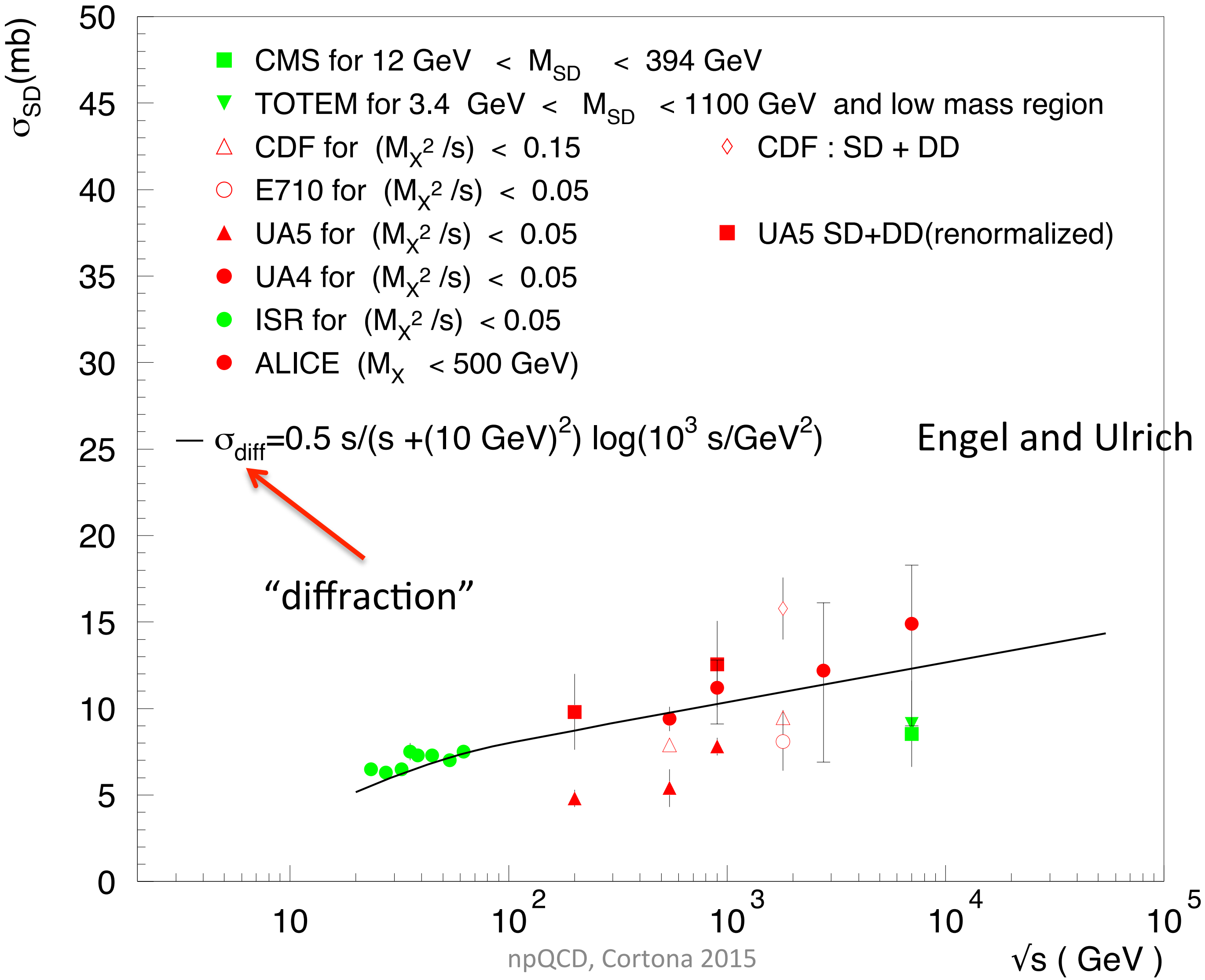
$$\sigma_{inel}^{uncorrelated}(s) =$$

$$\int d^2b\, \Pi_n(\bar{n}) =$$

$$\int d^2b[1 - e^{-\bar{n}(b,s)}]$$

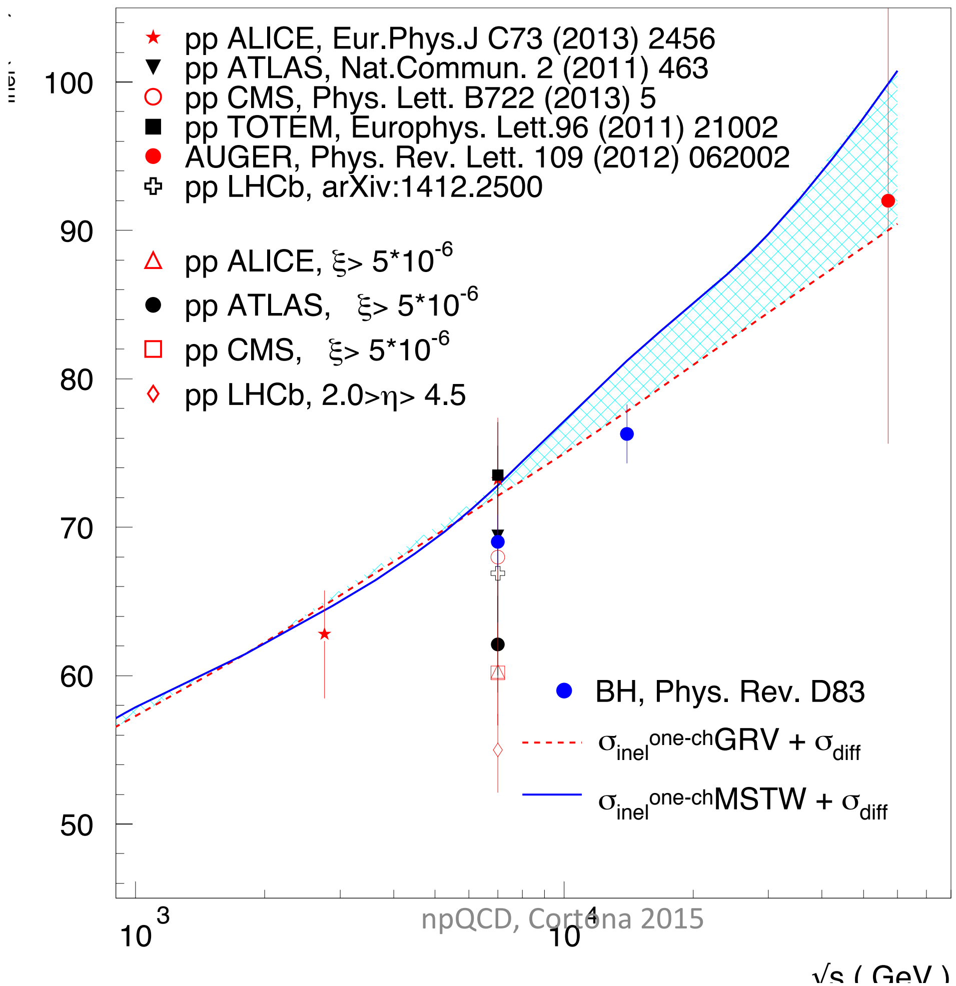


2. Parametrize diffraction at high energy from
R. Engel and R. Ulrich, Internal Pierre Auger Note GAP-2012 (March, 2012)



# Inelastic cross-section at high energy= uncorrelated + "diffraction"

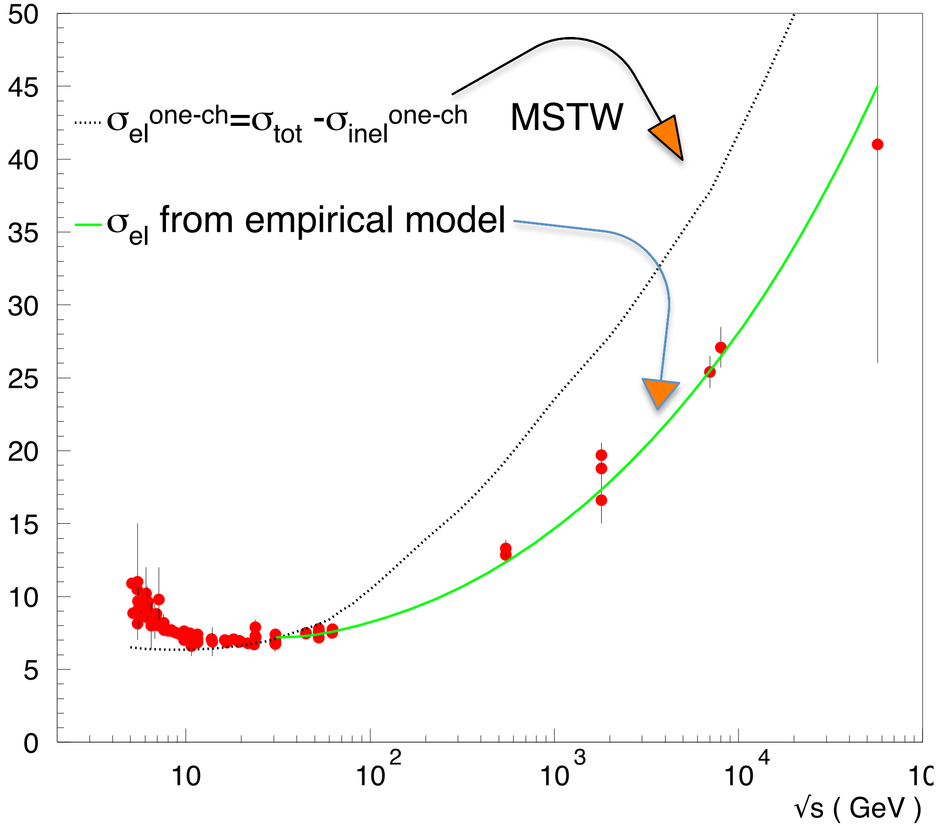
- The high energy part

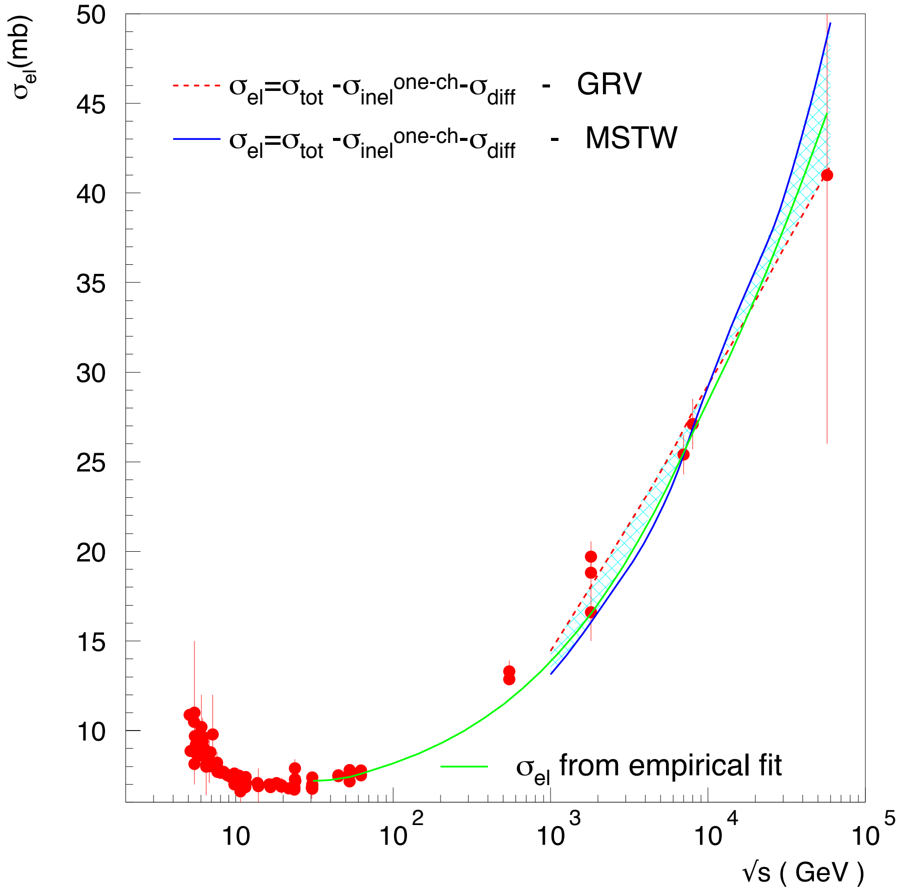


# Correspondingly, for the elastic

**Subtract "diffraction" from one-channel at high energy**

**One-channel elastic**



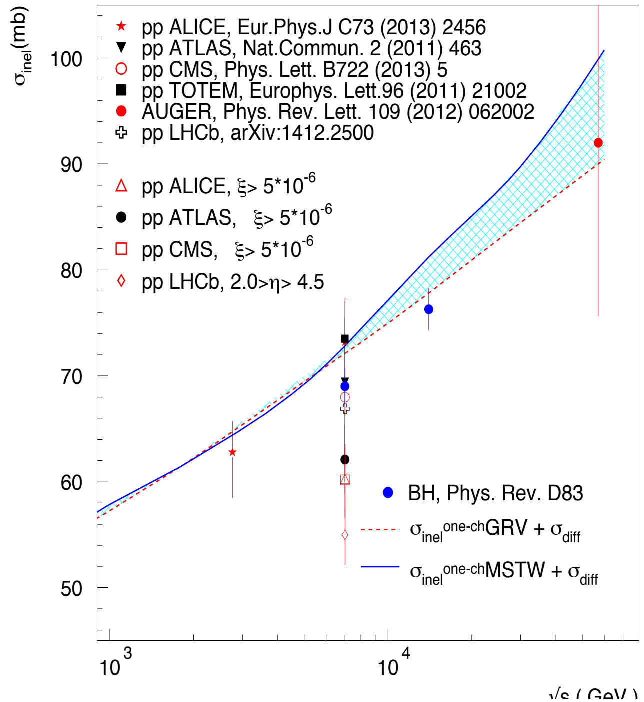




Empirical : D. A. Fagundes, A. Grau, S. Pacetti, G. Pancheri, and Y. N. Srivastava, Phys.Rev. D88, 094019 (2013)

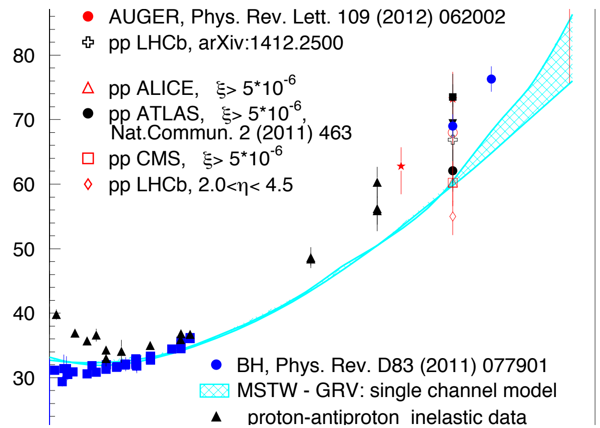
# What are learning from this exercise?

Parametrizing diffraction (as shown)

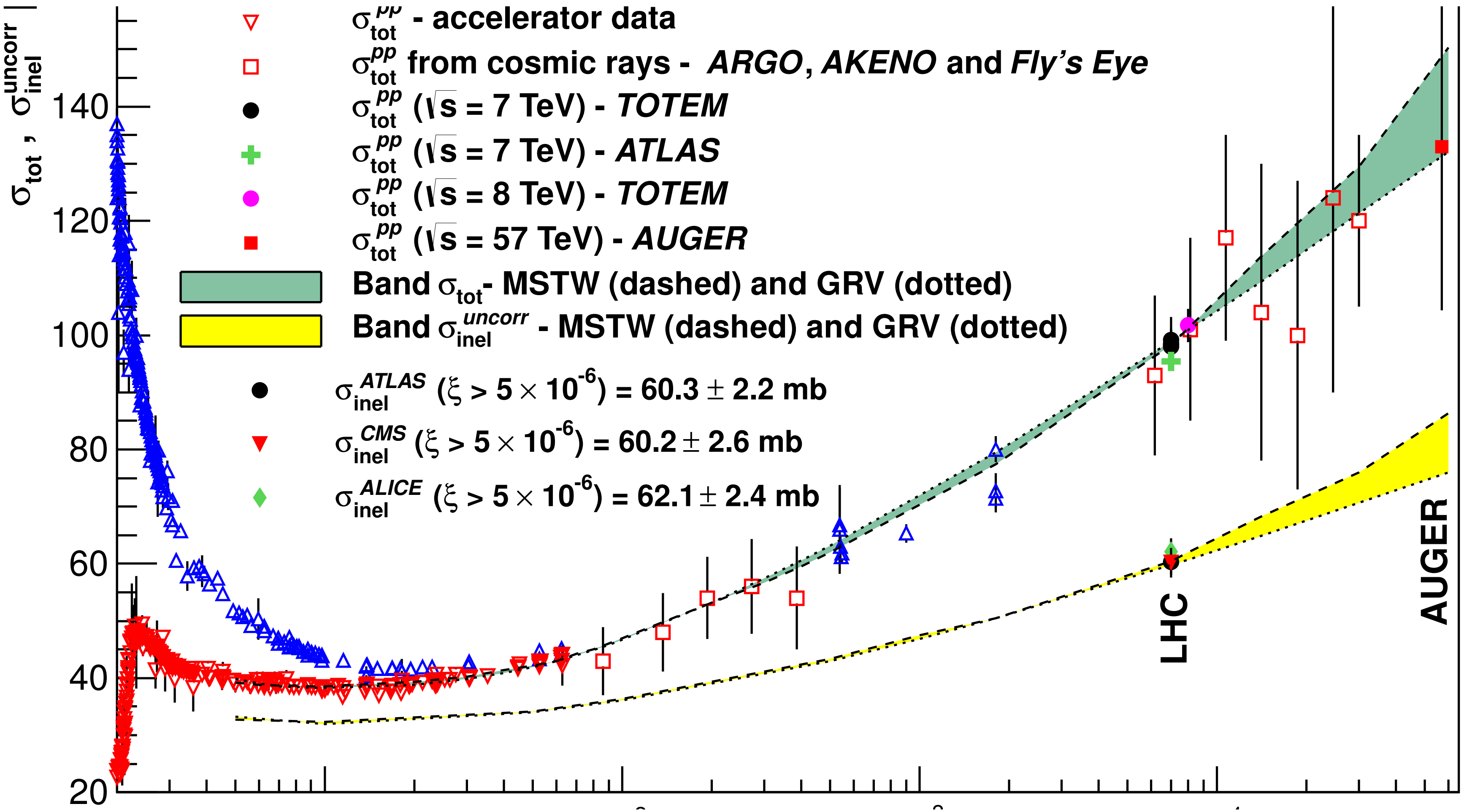
+ one-channel for uncorrelated Poisson distributed collisions




=> we can identify the non-diffractive high energy component in inelastic cross-section



# Total and inelastic uncorrelated x-sections: bands ⇔ PDFs uncertainty



# Conclusion

- Our QCD minijet model with infrared soft gluon resummation (PLB 1996-PRD 2005) has been applied to LHC cross-sections (PRD 2011- PRD 2015 to be published)

- Uncertainties from different PDF sets have been highlighted

- A decomposition of the inelastic cross-section into uncorrelated and diffractive collisions (parametrized) indicates good agreement with LHC data for $M_X^2/s < 5\ 10^{-6}$

- Work is in progress to extend the model
  - to describe diffraction and
  - to cosmic ray production p-air cross-section [D. Fagundes, A. Grau, G. Pancheri, Y. Srivastava, and O. Shekhovtsova, (2014), arXiv:1408.2921]

# Our model: A. Grau, R. Godbole, GP, Y.N. Srivastava, 1999 etc.

- Standard Eikonal with one channel (for now) purely imaginary at large b

- Minijets with standard LO PDFs (GRV, MSTW2008) driving the rise: well known calculation, except that we use up-to-date PDFs and a constant ptmin ~ 1 GeV

- Soft gluon resummation to tame the rise:
  connection with a confining potential and the Froissart bound

# Different impact parameter distribution for diffraction or minijets in needed, at low energy no-minijets, it does not matter

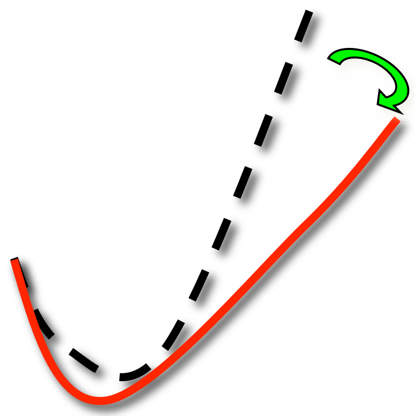
- For central collisions, pQCD described [minijets] impact parameter distribution must be energy dependent, partons on one side see partons on the other proton, the b-distribution is only defined when partons collide and start moving across the hadronic matter -> our 1996 proposal

- For diffractive production - what our decomposition defined as *correlated*-

b-distribution may be different -> work in progress

# Facts about total cross-section

- About $\sigma_{total}$
  - Decrease up to ~ 5-10 GeV
  - Increase after 10-20 GeV
- Froissart bound limits the rise
- Fast rise at the beginning s



# Facts about total and elastic

- About $\sigma_{total}$
  - Decrease up to ~ 5-10 GeV
  - Increase after 10-20 GeV
- Froissart bound limits the rise
- About $\frac{d\sigma_{el}}{dt}$
  - Very fast decrease –t~ 0.2 GeV^2
  - A dip at –t =√(sigtot sigel)
  - Bump and then decrease -t^8
- Optical theorem anchors it at t=0

- # Regge-Pomeron exchanges

$$A(s,t) = \sum_{i=1,n} \beta_i(t) s^{\alpha_i(t)}$$

$$\alpha_i(t) = \alpha_i(t=0) + \alpha_i' t$$

$$\left\{\begin{array}{l} \alpha_i(0) \leq 0 \quad Regge\ trajectories \quad \alpha_i' \simeq 1\ GeV^{-2} \\ \alpha_i(0) \geq 0 \quad Pomeron\ pole\ \alpha_i'\ small\ or\ zero \end{array}\right.$$

Donnachie and Landshoff phenomenological proposal for all total cross-sections, 1996

$$\sigma_{AB}^{total} = X_{AB} s^{\epsilon} + Y_{AB} s^{-\eta_{AB}}$$

$$\boxed{\begin{array}{r} \epsilon \simeq 0.08 \div 0.1\ universal \\ \eta \simeq 0.5[\rho\ trajectory] \end{array}}$$

# Optical theorem

$$ImA(s,t=0) = \int dt|A(s,t)|^2 + G_{inelastic}$$

3 simple relations in one-channel eikonal models

$$\sigma_{total}(s) = \int d^2b[1-e^{i\chi(b,s)}] = \int d^2b[1-e^{-\Im m\chi(b,s)}] \simeq \int d^2b[1-e^{-\chi_I(b,s)}]$$

$$\sigma_{elastic}(s) = \int d^2b|1-e^{i\chi(b,s)}|^2$$

# The inelastic cross-section

**From ISR to AUGER**

**The high energy region**