D^0 mixing: experimental results from BaBar and Belle

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on behalf of the BaBar Collaboration

Flavor Physics and CP Violation 2010

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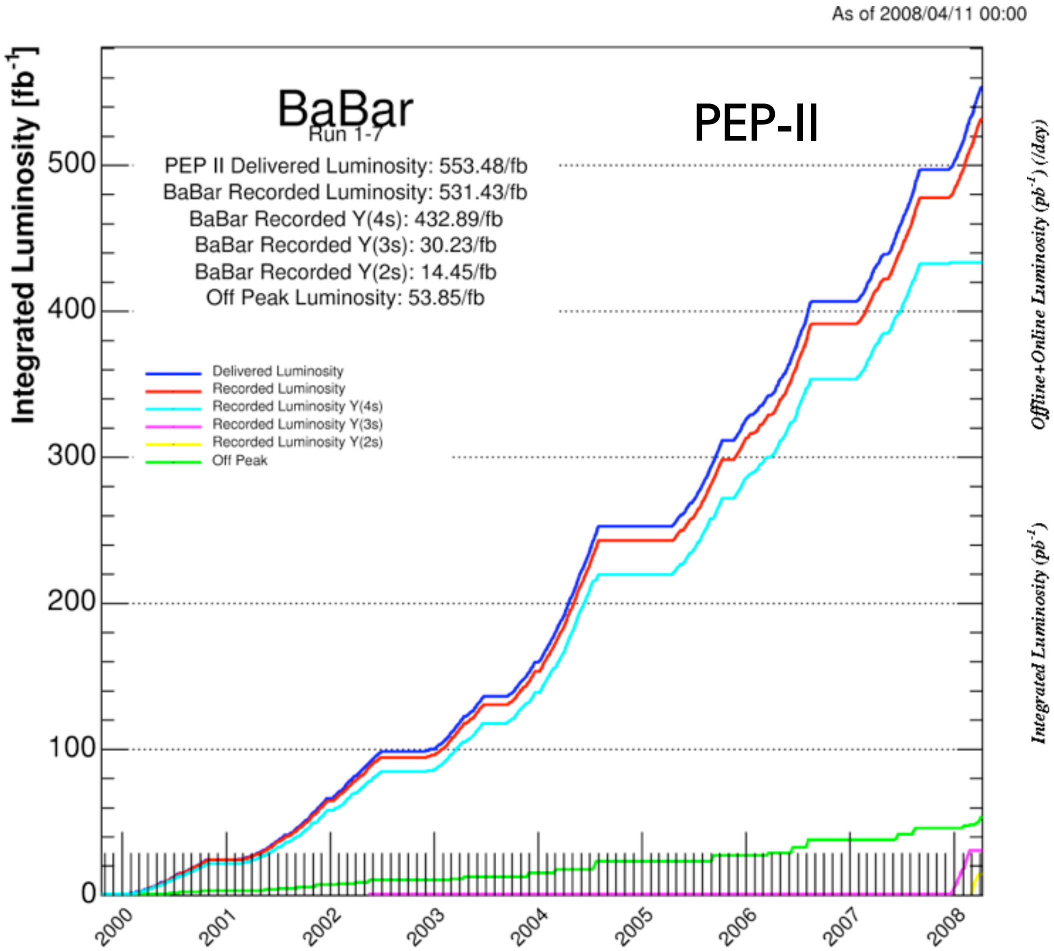
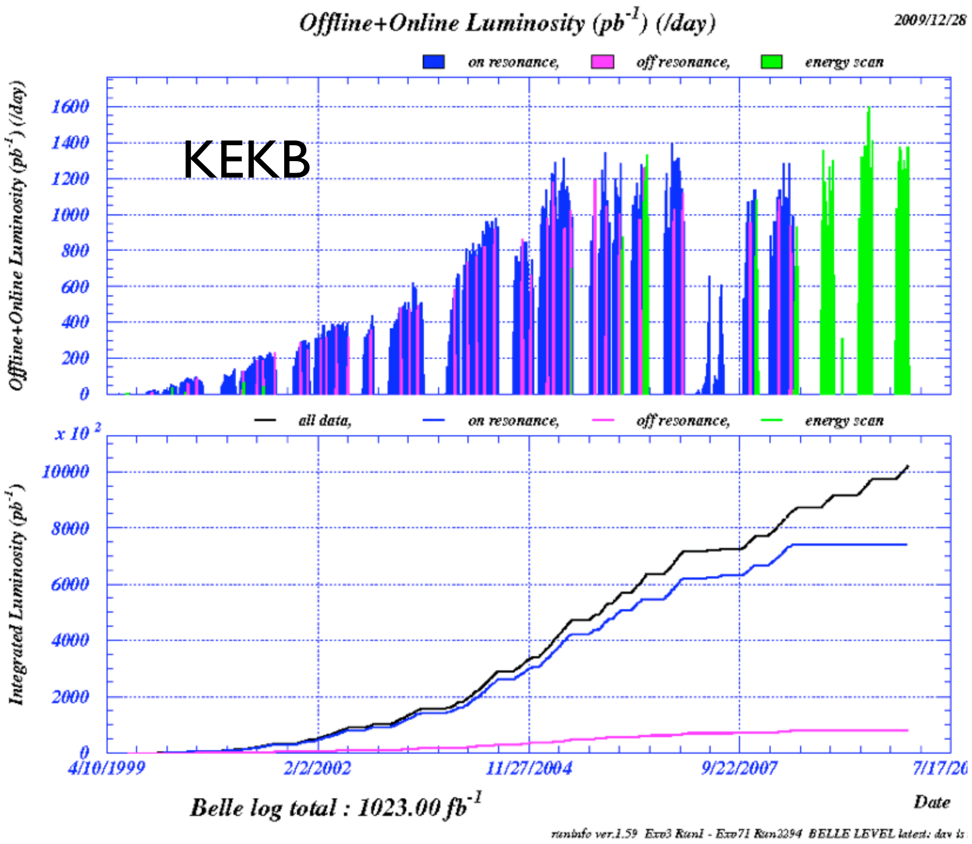
Outline

- B factories data samples
- D^0 mixing phenomenology
- Experimental status and recent results
- Summary

B Factories data samples





Recorded luminosity ~530 fb^{-1}
Peak luminosity ~$12 \times 10^{33} cm^{-2} s^{-1}$

Recorded luminosity = ~1.02 ab^{-1}
Peak luminosity ~$21 \times 10^{33} cm^{-2} s^{-1}$

D^0 mixing phenomenology

D^0 mixing notations

- Flavor mixing occurs when flavor eigenstates differ from mass eigenstates: well established phenomenon in neutral K, B_d, B_s systems.

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle \qquad |q|^2 + |p|^2 = 1$$

- Mixing parameters are expressed in terms of *x, y* functions of the mass and decay width differences:

$$x = \frac{m_1 - m_2}{\Gamma} \qquad y = \frac{\Gamma_1 - \Gamma_2}{2\Gamma} \qquad \text{where} \quad \Gamma = \frac{\Gamma_1 + \Gamma_2}{2}$$

- Three types of CP violation:

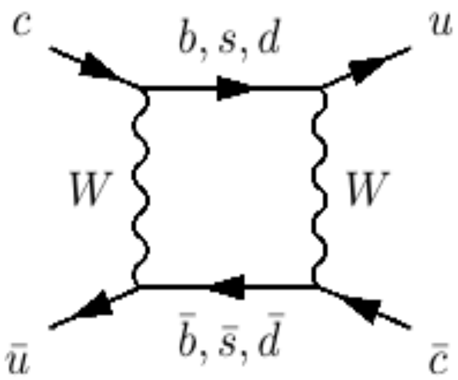
$$\langle f|H|D^0\rangle = A_f \qquad \langle f|H|\overline{D}^0\rangle = \overline{A}_f$$

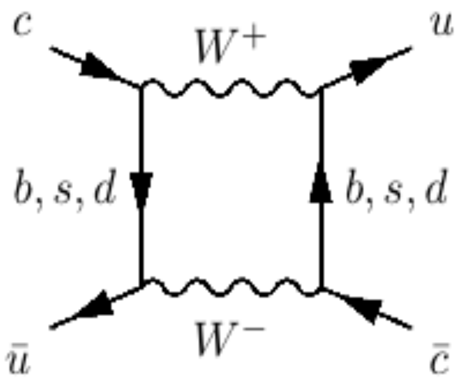
- in the decay (direct): $\left|\frac{\overline{A}_{\overline{f}}}{A_f}\right| \neq 1$
- in mixing (indirect): $r_m = \left|\frac{q}{p}\right| \neq 1$
- in the interference between mixing and decay: $\varphi_f \neq 0$

$$\lambda_f = \frac{q}{p}\frac{\overline{A}_f}{A_f} = r_m \left|\frac{\overline{A}_f}{A_f}\right| e^{i(\delta_f + \varphi_f)} \qquad \begin{array}{l} \varphi_f = \text{ weak phase} \\ \delta_f = \text{strong phase} \end{array}$$

Standard Model predictions

SM mixing loops has down type quarks in the loops:





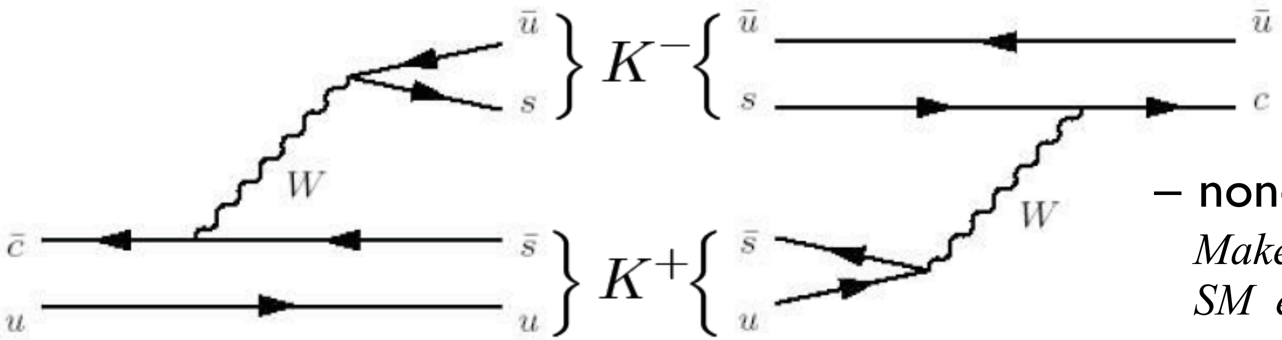
– *b* quark is CKM-suppressed

– *s, d* quark GIM suppressed

$$\Delta m \propto G_F^2 \frac{(m_s^2 - m_d^2)^2}{m_c^2}$$

$\rightarrow$ $\Delta m_{\rm box} \sim 10^{-5} \mathrm{ps}^{-1}$ Tiny!

Expect hadronic intermediate states to dominate:



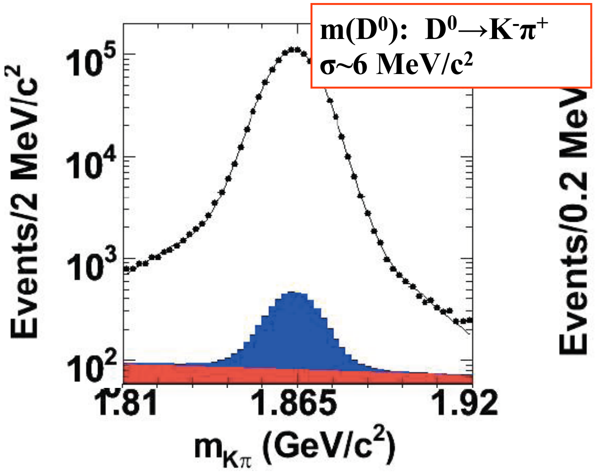
– non-perturbative contributions

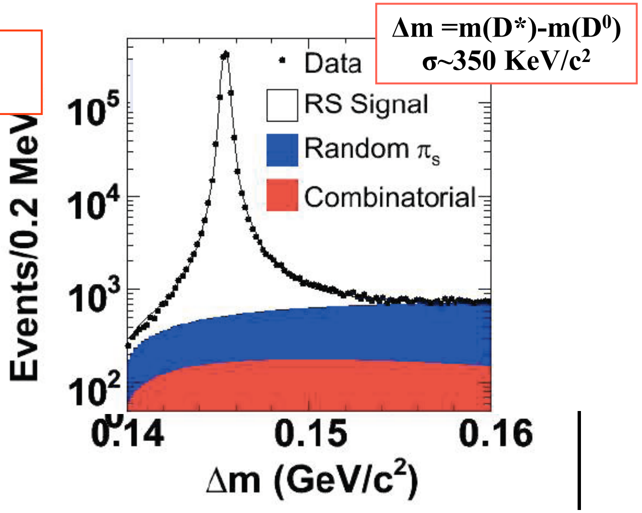
Makes it difficult to precisely predict SM expectations

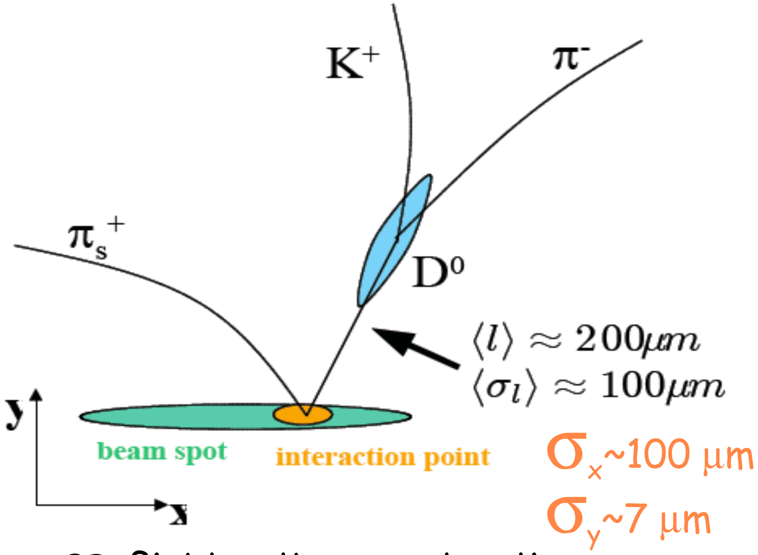
In SM expected |x|<10-2, |y|<10-2 and CP violation below the per mil level. New Physics contributions could enhance mixing rate and/or generate CP violation up to percent level.

Recent experimental results from B factories

Selection of D^0 mesons







Select D^0 mesons via $D^{*+} \rightarrow D^0\pi^+$ decay:

- charge of slow pion identifies the flavor of D^0 at production;
- exploit $m(D^0)$, D^0 reco invariant mass and $\Delta m = m(D^*) - m(D)$, D^{*+}-D^0 mass difference for bkg rejection;

Cut on D^0 momentum in center of mass frame, $p^*>2.5$-3.0 GeV/c rejects D^0 from B decays and combinatorial bkg.

3D flight path reconstruction

proper time $\quad t = \dfrac{\vec{L}\cdot\vec{p}}{p}\dfrac{m_{D^0}}{p}$

- D^0 vertex with beam spot (interaction region size) constraint applied. Determining decay time, t, and decay time error, σ_t , for each each event.

Typical resolution on proper-time: $\langle\sigma_t\rangle \simeq 0.5\tau_D = 0.2$ ps
thanks to the excellent performance of the Silicon Vertex Tracker.

Mixing analyses at the B factories

	Note:
$D^0 \rightarrow K^+\pi^-$	*study of the time dependence* See backup slides.
$D^0 \rightarrow K^+K^-, \pi^+\pi^-$	*lifetime ratio wrt* $D^0 \rightarrow K^-\pi^+$
$D^0 \rightarrow \phi K^0_S$	*lifetime difference between CP-even and CP-odd eigenstates*
$D^0 \rightarrow K^+\pi^-\pi^0$	*time-dependent Dalitz plot analysis*
$D^0 \rightarrow K^0_S\pi^+\pi^-$ New	*time-dependent Dalitz plot analysis*
$D^0 \rightarrow K^0_S K^+K^-$ New	*time-dependent Dalitz plot analysis*
$D^0 \rightarrow K^{(*)}l\nu$	*time-integrated analysis* Not covered in this talk

Legend: ★ = mixing evidence $> 3\sigma$

New = new result

At B factories events are selected from $e^+e^- \rightarrow c\bar{c}$ annihilations:
$\sigma\left(e^+e^- \rightarrow c\bar{c}\right) \simeq 1.3$ nb

D^0 to CP-even eigenstates K^+K^-, $\pi^+\pi^-$

- Mixing and CP violation alter decay time distribution of CP eigenstates to exponential with effective lifetimes $\tau_{hh}^{\pm}$:

$$\boxed{\begin{aligned} \tau_{hh}^{+} &= \tau(D^0 \to h^+h^-) \\ \tau_{hh}^{-} &= \tau(\overline{D}^0 \to h^+h^-) \\ \tau_{K\pi} &= \tau(D^0 \to K^-\pi^+) \end{aligned}} \longrightarrow$$

measured quantities

- Mixing and CP violation observables:

$$\boxed{\begin{aligned} \text{Mixing}: \quad y_{CP} &= \frac{\tau_{K\pi}}{\langle \tau_{hh} \rangle} - 1 \\ \text{CPV}: \quad \Delta Y &= \frac{\tau_{K\pi}}{\langle \tau_{hh} \rangle} A_\tau; \left(\Delta Y = -\frac{\tau_{K\pi}}{\langle \tau_{hh} \rangle} A_\Gamma \right) \end{aligned}}$$

Def: $\langle \tau_{hh} \rangle = \dfrac{\tau_{hh}^{+} + \tau_{hh}^{-}}{2}$

$$A_\tau = \frac{\tau_{hh}^{+} - \tau_{hh}^{-}}{\tau_{hh}^{+} + \tau_{hh}^{-}} = -A_\Gamma$$

$$\boxed{\text{if CP conserved} \Rightarrow y_{CP} \equiv y \quad \textit{and} \quad \Delta Y = 0 \quad (A_\Gamma = 0)}$$

Belle results for CP-even decays: y_{CP}, A_Γ

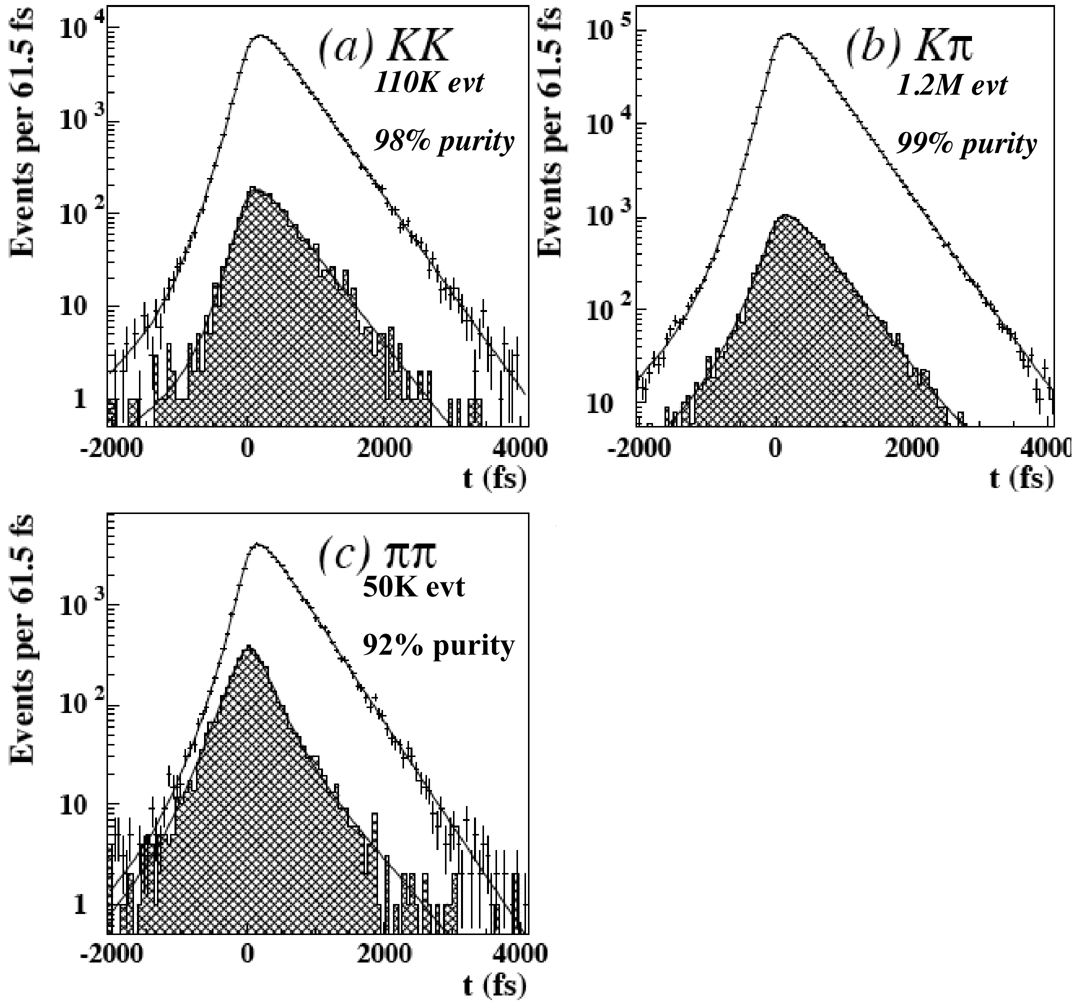
PRL 98:211803,2007

(540 fb^{-1})

- Use D* tagged events.
- Most of systematic errors cancel in the lifetime ratio.
- Bkg related systematics don't.

	y_{CP} (%)	A_Γ(%)
KK	1.25±0.39±0.28	0.15±0.34±0.16
ππ	1.44±0.57±0.42	-0.28±0.52±0.30
KK+ππ	1.31±0.32±0.25	0.01±0.30±0.15

Main syst. errors (σy_{CP},σA_Γ): acceptance (0.12%, 0.07%), res. function bias (0.14%, 0.08%), selection variations (0.11%, 0.05%)



Evidence of mixing at 3.2σ level.

No evidence for CP violation

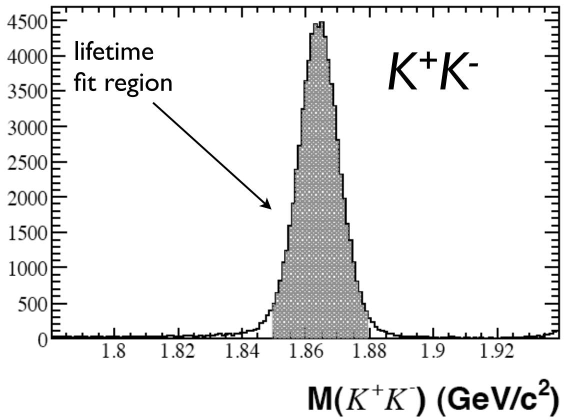
BaBar results for CP-even decays

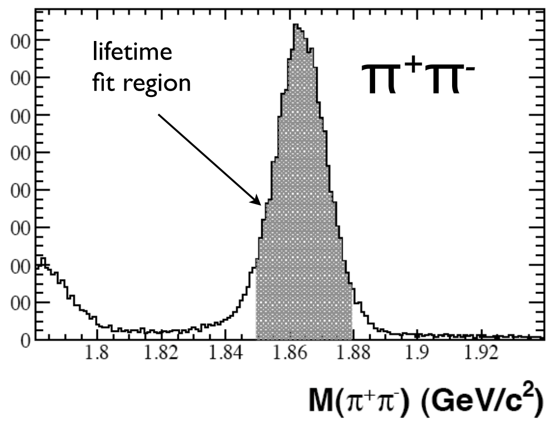
(384 fb^{-1})

- Using independent D* tagged and untagged events
- Untagged sample: 4x tagged sample but higher bkg; measure only y_{CP}

D* tagged events

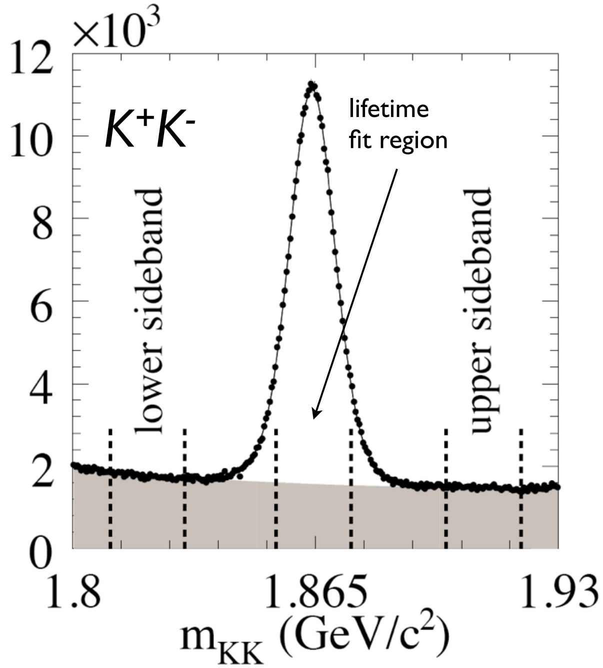
Sample	Size	Purity (%)
$K^-\pi^+$	730,880	99.9
K^-K^+	69,696	99.6
$\pi^-\pi^+$	30,679	98.0





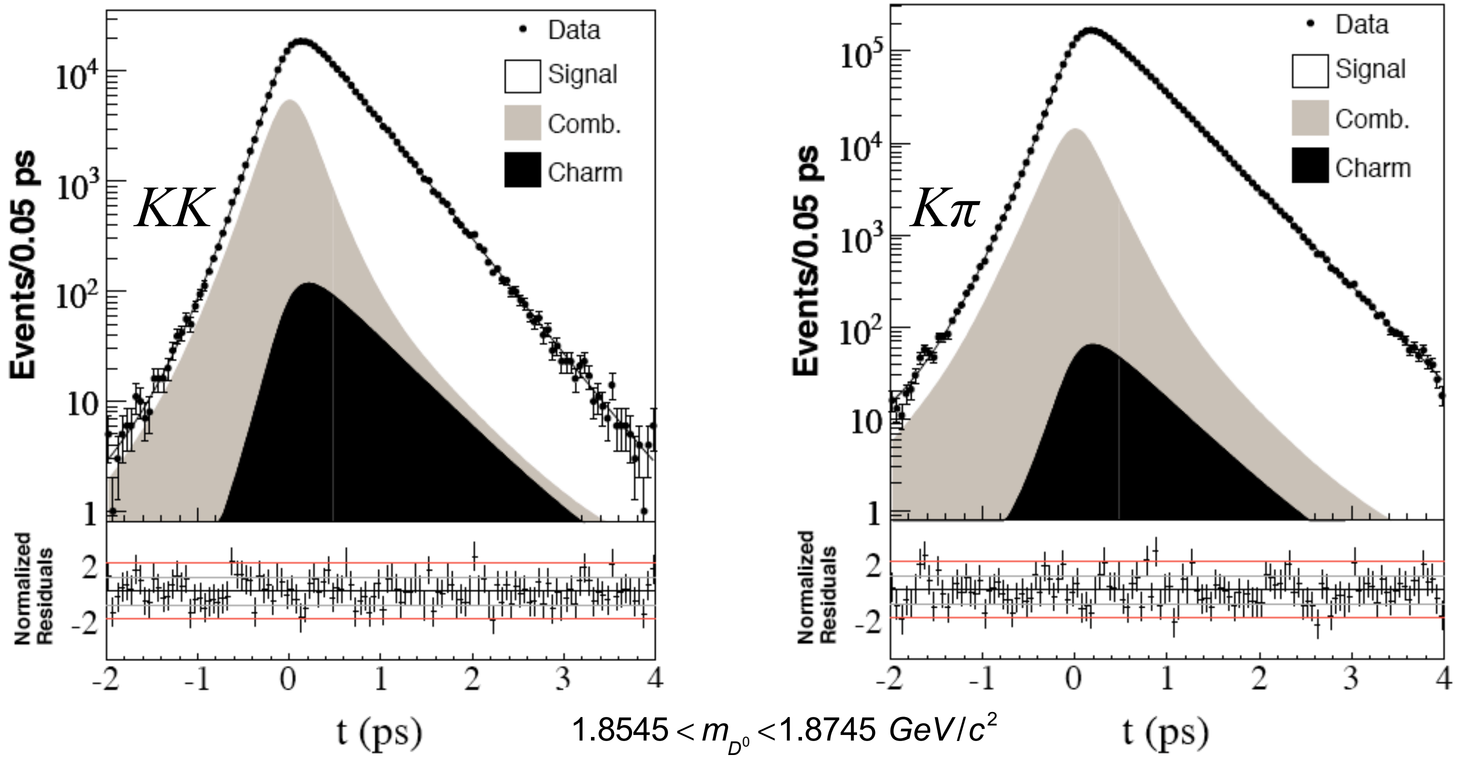
D* untagged events

Sample	Signal Yield (x 10^3)	Purity (%)
$K^-\pi^+$	2710.2 ± 3.4	94.2
K^+K^-	263.6 ± 1.0	80.9



y_{CP} from untagged $D^0 \to K^+K^-, K^-\pi^+$

PRD 80, 071103 (2009)



t_{KK} *(fs)* $= 405.85 \pm 1.00$ *(stat.)* $t_{K\pi}$ *(fs)* $= 410.39 \pm 0.38$ *(stat.)*

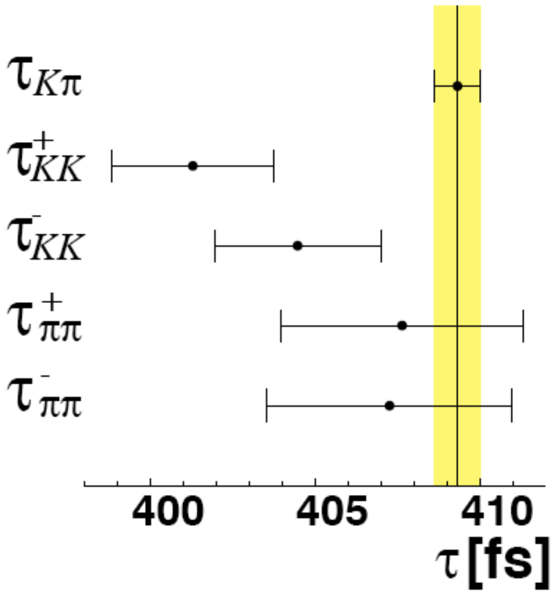
y_{CP} (untagged) = [1.12 ± 0.26 (stat) ± 0.22 (syst)]%

Evidence of mixing at 3.3 σ level

Main syst. errors: m_D window (0.110%), combinatorial lifetime PDF (0.115%), detector effects (0.093%)

D* tagged results: $y_{CP}, \Delta Y$

PRD 78 011105(R) (2008)



Sample	y_{CP}	ΔY
K^-K^+	$(1.60 \pm 0.46 \pm 0.17)\%$	$(-0.40 \pm 0.44 \pm 0.12)\%$
$\pi^-\pi^+$	$(0.46 \pm 0.65 \pm 0.25)\%$	$(\ \ 0.05 \pm 0.64 \pm 0.32)\%$
Combined	$(1.24 \pm 0.39 \pm 0.13)\%$	$(-0.26 \pm 0.36 \pm 0.08)\%$

Main syst. errors (σy_{CP}, $\sigma\Delta Y$): signal model (0.085%, 0.062%), detector model (0.064%, 0.054%), selection criteria (0.046%, 0.011%)

Evidence of mixing at 3σ level

No evidence of CP violation

Combined y_{CP} result: D* tagged+untagged

statistically uncorrelated samples, conservatively assuming 100% correlation in systematic errors

(384 fb^{-1})

PRD 80, 071103 (2009)

y_{CP} = [1.16 ± 0.22 (stat) ± 0.18 (syst)]%

Evidence of mixing at 4.1σ level

y_{CP} from untagged $D^0 \to K_S^0 K^+ K^-$

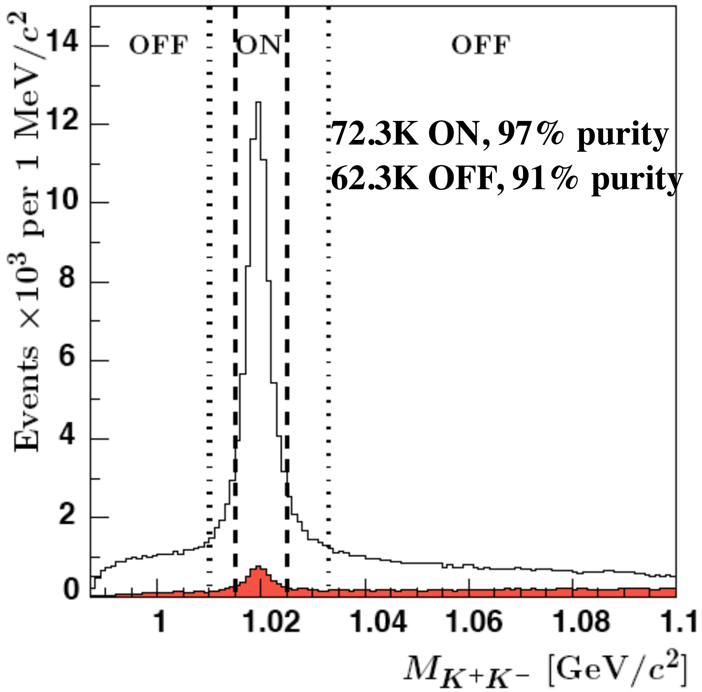
PRD 80, 052006 (2009)

(673 fb^{-1})

- Measurement of lifetime difference of CP-even and CP-odd eigenstates:

- ON region mainly CP-odd (φ(1020)K_S^0) and OFF region mainly CP-even (a_0(980)K_S^0)

- $\tau_{ON}(\tau_{OFF})$ effective lifetime and $f_{ON}(f_{OFF})$ CP-even fraction in ON (OFF) region. $f_{ON}(f_{OFF})$ is obtained from Dalitz Model taken from BaBar PRD 78, 034023 (2008).



$$y_{CP} = \frac{1}{f_{ON} - f_{OFF}} \frac{\tau_{OFF} - \tau_{ON}}{\tau_{OFF} + \tau_{ON}}$$

$\boxed{y_{CP} = [0.11 \pm 0.61 \text{ (stat.)} \pm 0.52 \text{ (syst.)}]\%}$

Main systematics: proper time resolution function offset ON-OFF (0.38%) and selection criteria (0.30%), Dalitz model error negligible (0.01%)

Time-dependent Dalitz plot analyses

Study event distribution as a function of Dalitz plot position and time

$$\frac{dN_f(s_{12},s_{13},t)}{ds_{12}ds_{13}dt} \propto e^{-\Gamma t}\left\{|A_f|^2 + \left[y\underbrace{\mathrm{Re}(A_f^*\bar{A}_f)} - x\underbrace{\mathrm{Im}(A_f^*\bar{A}_f)}\right](\Gamma t) + \frac{x^2+y^2}{4}(\Gamma t)^2|\bar{A}_f|^2\right\}$$

larger sensitivity in regions populated by Doubly Cabibbo Suppressed and CP eigenstates.

$$A_f = A(s_{12},s_{13}) \quad \bar{A}_f = \bar{A}(s_{12},s_{13}) \text{ and } (s_{12},s_{13}) = \text{Dalitz plot location}$$

- if f and $\bar{f}$ belong to the same Dalitz plot (e.g. $K^0_S\pi^+\pi^-$) by assuming CP conservation in decay $(\bar{A}_f = A_{\bar{f}})$ is possible to extract directly x, y mixing parameters, without $D^0 - \overline{D}^0$ relative strong phase uncertainty.

Method pioneered by CLEO Collaboration: D.Asner *et. al. Phys.Rev.D72:012001,2005*.

- if f and $\bar{f}$ do not belong to the same Dalitz plot (e.g $K^+\pi^-\pi^0$) the relative strong phase is not directly measurable at B Factories and we can extract effective mixing parameters, as for example:

$$x'' = x\cos\delta_{K\pi\pi^0} + y\sin\delta_{K\pi\pi^0} \qquad y'' = -x\sin\delta_{K\pi\pi^0} + y\cos\delta_{K\pi\pi^0}$$

$$\delta_{K\pi\pi^0} = \arg\left(\frac{A(D^0\to K^+\rho^-)}{A(\overline{D}^0\to K^+\rho^-)}\right)$$

Method pioneered by BaBar Collaboration: *Phys.Rev.Lett.103:211801,2009*

$D^0(t) \to K^+\pi^-\pi^0$ (WS) mixing fit (384 fb^{-1})

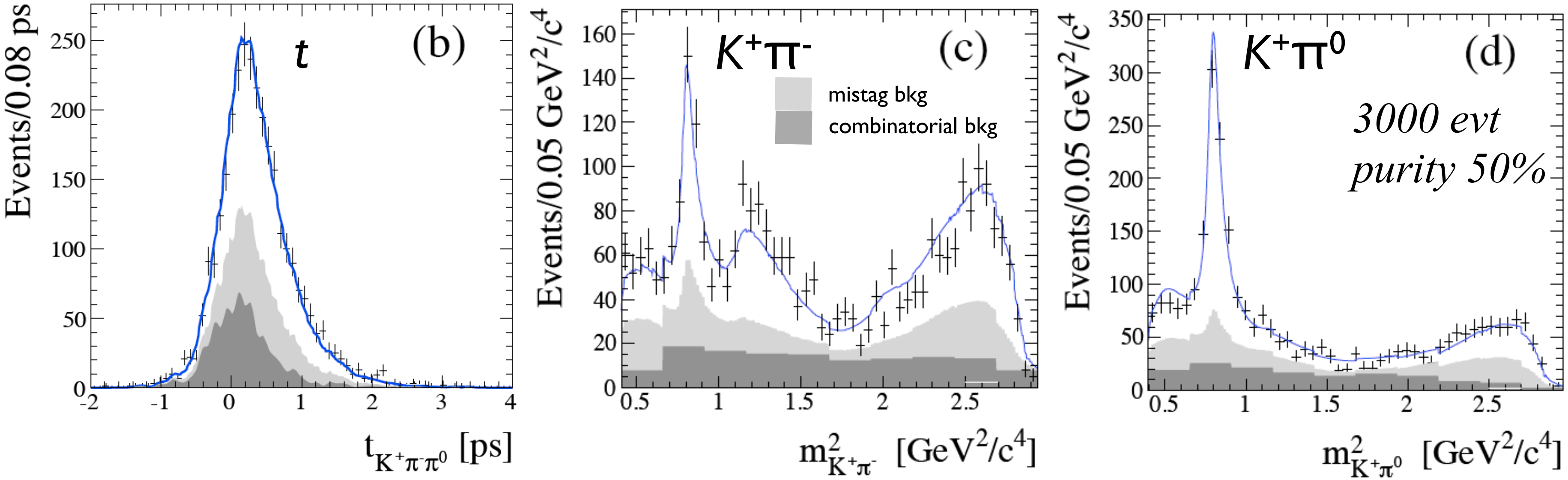
Phys.Rev.Lett.103:211801,2009

The WS time-evolution function contains both DCS (A_f) and CF amplitudes $(\bar{A}_f)$.
The CF amplitudes are determined in a time-independent Dalitz plot fit to the RS sample (~660K evt) and fixed in the WS time-dependent Dalitz plot mixing fit (~3000 evt).

DCS=Doubly Cabibbo Suppressed — CF = Cabibbo Favored
WS = Wrong Sign, $D^0 \to K^+\pi^-\pi^0$ — RS = Right Sign, $D^0 \to K^-\pi^+\pi^0$

Dalitz plot model:

Use Breit-Wigner functions for DCS and CF D^0 decay amplitudes.
$K\pi$ S-wave amplitude use BW together with effective range non-resonant component. ***D. Aston et al., Nucl. Phys. B 296, 493 (1988);***



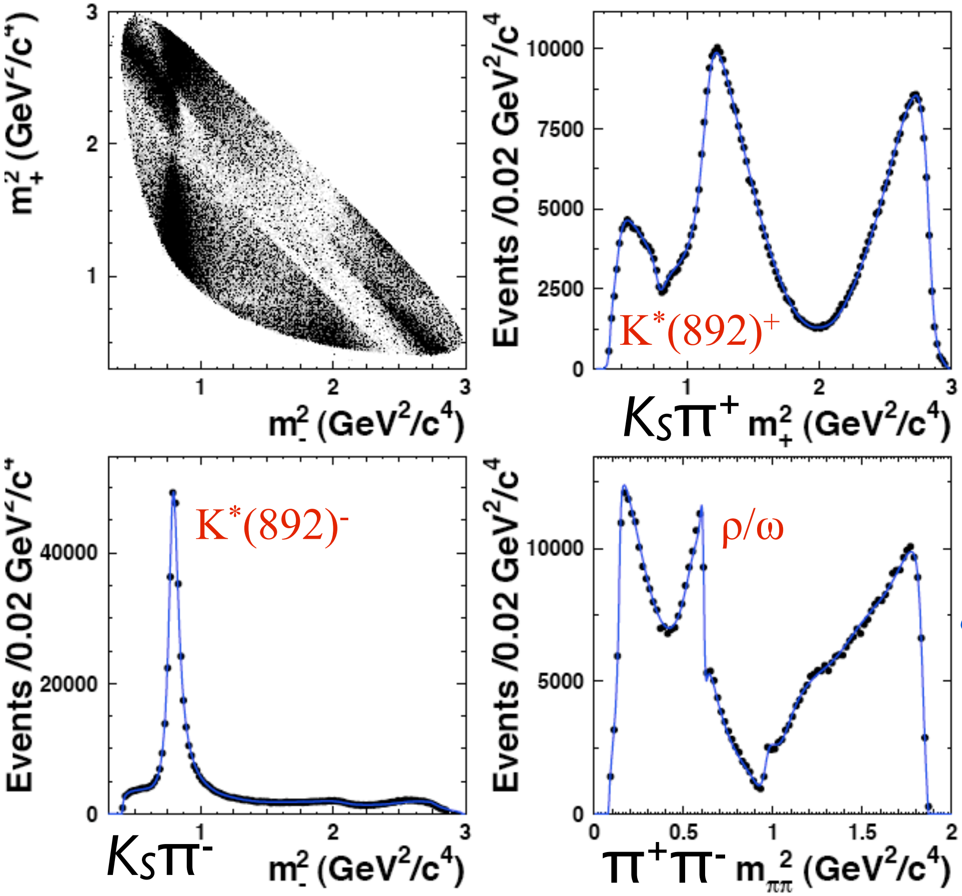
$$x'' = \left[2.61^{+0.57}_{-0.68}(\text{stat.}) \pm 0.39(\text{syst.})\right]\% \qquad y'' = \left[-0.06^{+0.55}_{-0.64}(\text{stat.}) \pm 0.34(\text{syst.})\right]\%$$

Evidence for mixing at 3.2σ level

$D^0(t) \to K_S\pi^+\pi^-$ Dalitz-mixing fit

Phys.Rev.Lett.99:131803,2007

540 fb^{-1} data
N_{sig}= (534.4±0.8)x10^3
Purity= 95%



Isobar model fit results

$$\chi^2/ndof = 2.1 \;\; \text{with (3653-40) ndof}$$

Resonance	Amplitude	Phase (°)	Fit fraction
$K^*(892)^-$	1.629 ± 0.006	134.3 ± 0.3	0.6227
$K_0^*(1430)^-$	2.12 ± 0.02	-0.9 ± 0.8	0.0724
$K_2^*(1430)^-$	0.87 ± 0.02	-47.3 ± 1.2	0.0133
$K^*(1410)^-$	0.65 ± 0.03	111 ± 4	0.0048
$K^*(1680)^-$	0.60 ± 0.25	147 ± 29	0.0002
$K^*(892)^+$	0.152 ± 0.003	-37.5 ± 1.3	0.0054
$K_0^*(1430)^+$	0.541 ± 0.019	91.8 ± 2.1	0.0047
$K_2^*(1430)^+$	0.276 ± 0.013	-106 ± 3	0.0013
$K^*(1410)^+$	0.33 ± 0.02	-102 ± 4	0.0013
$K^*(1680)^+$	0.73 ± 0.16	103 ± 11	0.0004
$\rho(770)$	1 (fixed)	0 (fixed)	0.2111
$\omega(782)$	0.0380 ± 0.0007	115.1 ± 1.1	0.0063
$f_0(980)$	0.380 ± 0.004	-147.1 ± 1.1	0.0452
$f_0(1370)$	1.46 ± 0.05	98.6 ± 1.8	0.0162
$f_2(1270)$	1.43 ± 0.02	-13.6 ± 1.2	0.0180
$\rho(1450)$	0.72 ± 0.04	41 ± 7	0.0024
σ_1	1.39 ± 0.02	-146.6 ± 0.9	0.0914
σ_2	0.267 ± 0.013	-157 ± 3	0.0088
NR	2.36 ± 0.07	155 ± 2	0.0615

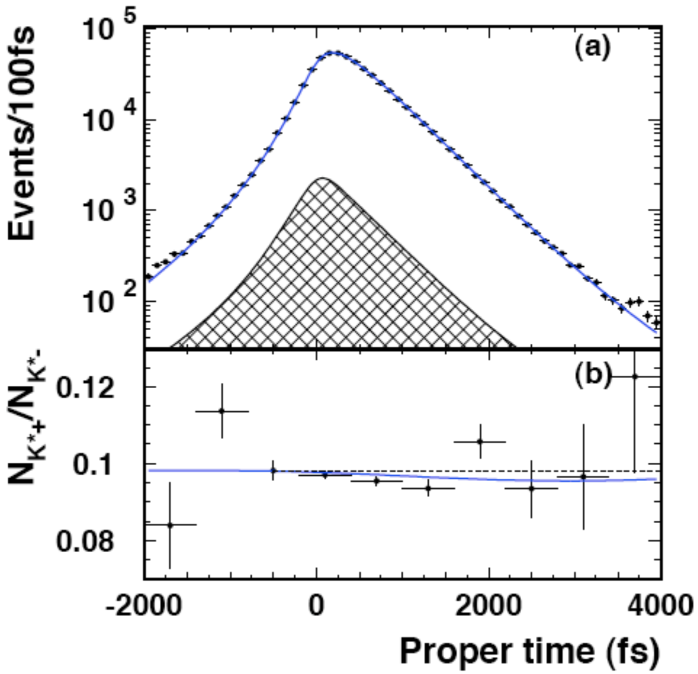
DCS states

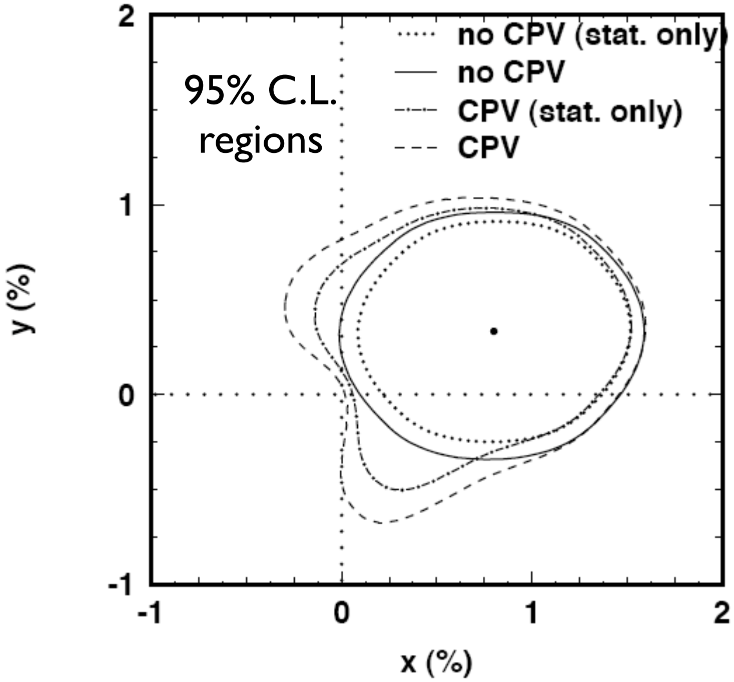
CP eigenstates

Mixing fit results

Phys.Rev.Lett.99:131803,2007

$\tau = 409.9 \pm 0.9$ fs consistent with PDG





$$x = \left[0.80 \pm 0.29(\text{stat.})^{+0.09}_{-0.07}(\text{syst.})^{+0.10}_{-0.14}(\text{model})\right]\ \%$$

$$y = \left[0.33 \pm 0.24(\text{stat.})^{+0.08}_{-0.12}(\text{syst.})^{+0.06}_{-0.08}(\text{model})\right]\ \%$$

$$|q/p| = 0.86^{+0.30}_{-0.29}(\text{stat.})^{+0.10}_{-0.09}(\text{syst.})$$

$$\phi = -0.24^{+0.28}_{-0.30}(\text{stat.}) \pm 0.09(\text{syst.})$$

No mixing disfavored at 2.2σ level

No evidence for CP violation

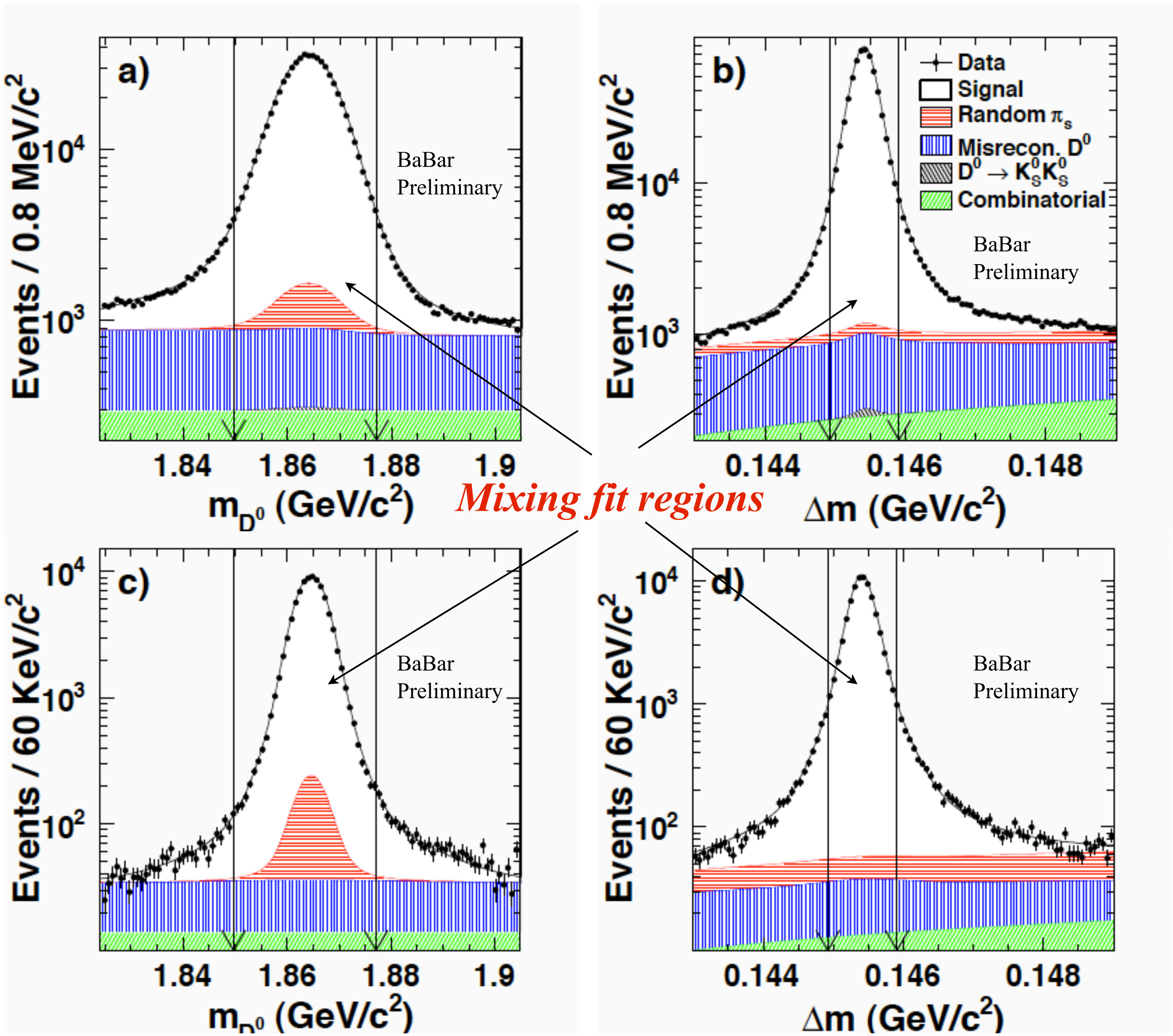
$D^0(t) \rightarrow K_S\pi^+\pi^-$ + $K_SK^+K^-$ analysis

e-Print: ***arXiv:1004.5053*** *[hep-ex]*

New

468.5 fb^{-1} data

- Select $D^{*+} \rightarrow D^0\pi^+$ events with high purity



$K_S\pi^+\pi^-$

N_{sig}= (540.8±0.8)x10^3
Purity= 98.5%

$K_SK^+K^-$

N_{sig}= (79.9±0.3)x10^3
Purity= 99.2%

Mixing time-dependent Dalitz plot fit

New

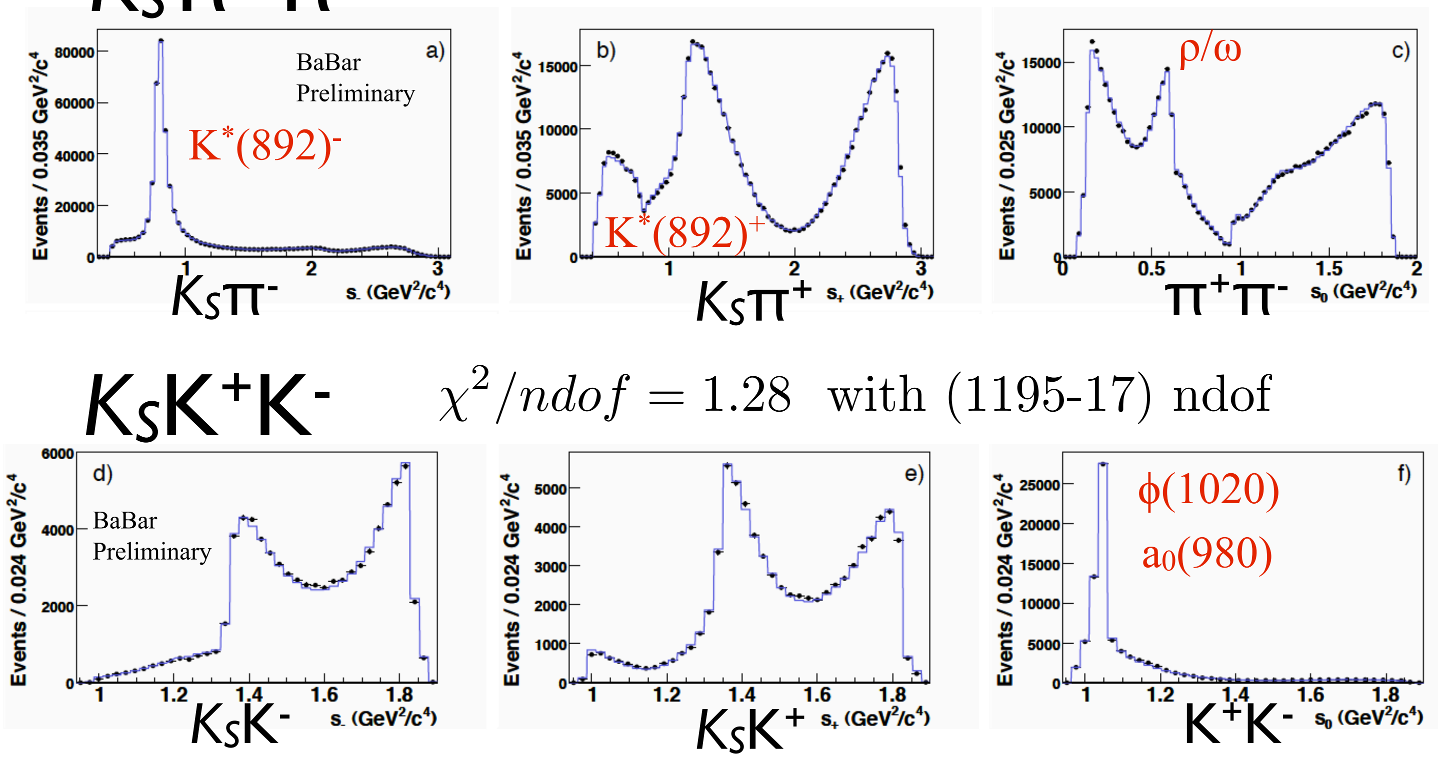
D^0 decay amplitude phenomenological parameterization:

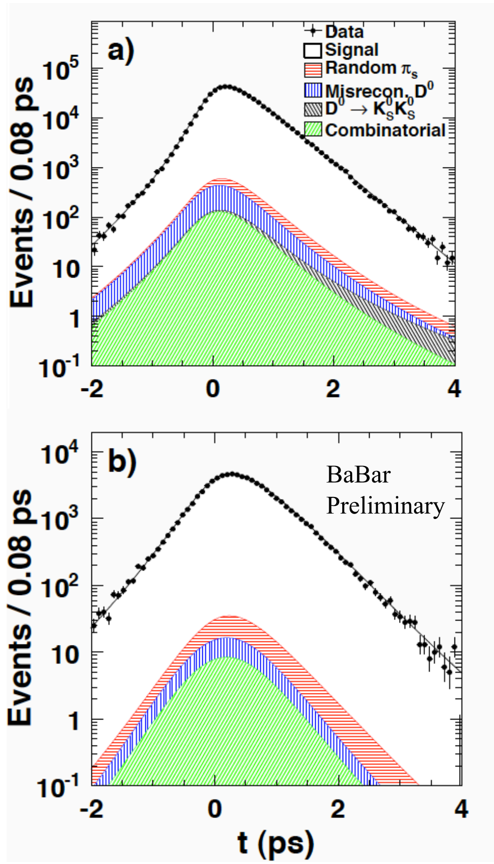
-for P- and D-wave amplitudes use Breit-Wigner (BW) model
-$\pi^+\pi^-$ S-wave dynamics use K-matrix formalism *V. V. Anisovich, A. V. Sarantsev, Eur. Phys. J. A 16 (2003) 229*
-$K\pi$ S-wave amplitude use BW with coherent non-resonant contribution *D. Aston et al., Nucl. Phys. B 296, 493 (1988); W. Dunwoodie, private communication.*
-K^+K^- S-wave use a coupled-channel BW for the $a_0(980)$ contribution.

$K_S\pi^+\pi^-$. DCS: $K^*(892)^+$, $K^*_0(1430)^+$, $K^*_2(1430)^+$.
CP eigenstates: $K_S\rho^0$ (CP=-1)
$K_SK^+K^-$. CP eigenstates: $K_S\phi$ (CP=-1), $K_Sa_0(980)$ (CP=+1).

$K_S\pi^+\pi^-$ $\chi^2/ndof = 1.21$ with (8626-41) ndof

$K_SK^+K^-$ $\chi^2/ndof = 1.28$ with (1195-17) ndof



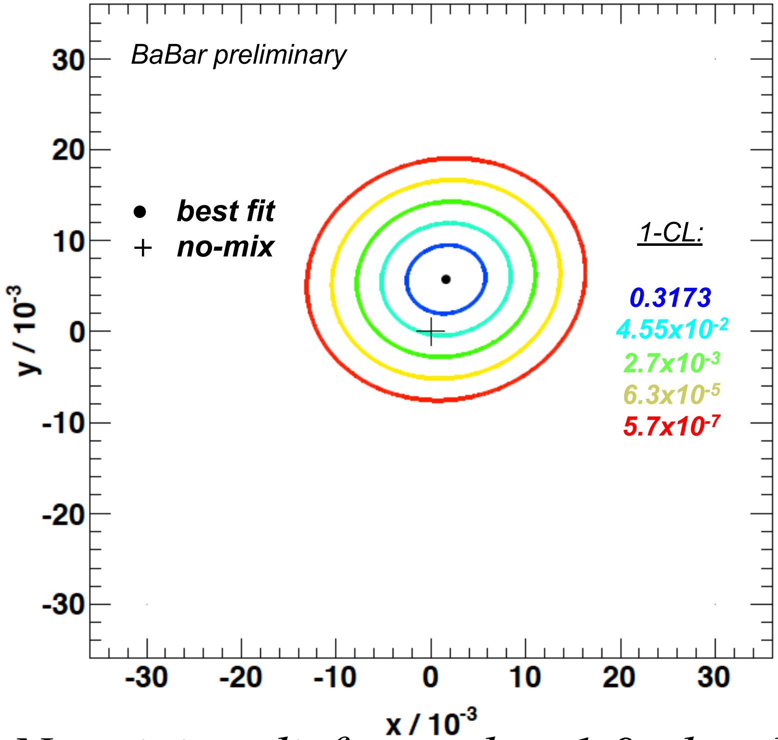


Mixing fit results

New

e-Print: ***arXiv:1004.5053*** *[hep-ex]*

Combined $K_S\pi^+\pi^- + K_SK^+K^-$ fit



No mixing disfavored at 1.9σ level

Experimental systematics

Source	$x[\%]$	$y[\%]$
SVT misalignment	0.0279	0.0826
Fit bias	0.0745	0.0662
Charge-flavor correlation (mistagging)	0.0487	0.0398
Event selection	0.0395	0.0508
Efficiency map	0.0367	0.0175
Background Dalitz-plot distribution	0.0331	0.0142
D^0 mass window	0.0250	0.0250
Proper lifetime PDF	0.0134	0.0128
Signal and background yields	0.0109	0.0069
Mixing in background	0.0103	0.0082
Dalitz-plot normalization	0.0106	0.0053
Proper lifetime error PDF	0.0058	0.0087
Experimental systematics	0.1177	0.1302

D^0 decay amplitude model systematics

Dominated by uncertainty on K*(892), K-matrix, $K\pi$ Lass parameters	0.0678	0.0532
Total	0.0830	0.0685

Combined $K_S\pi^+\pi^- + K_SK^+K^-$ fit results assuming CP conservation:

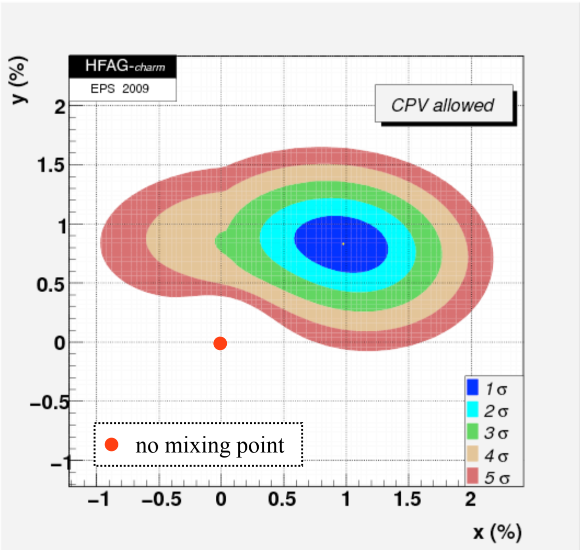
$$x = [0.16 \pm 0.23(\text{stat.}) \pm 0.12(\text{syst.}) \pm 0.08(\text{model})]\,\%$$

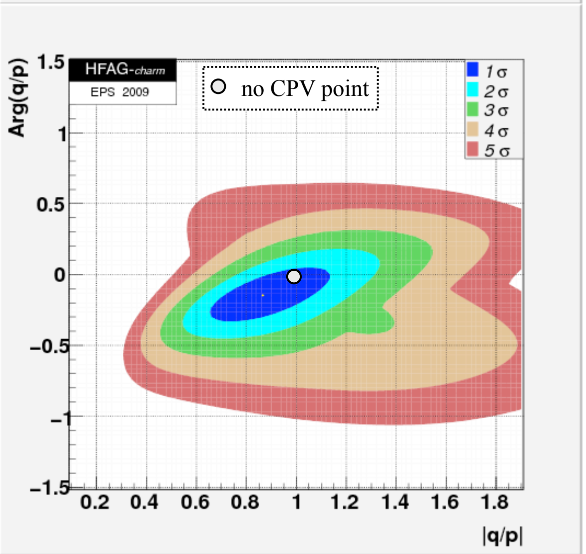
$$y = [0.57 \pm 0.20(\text{stat.}) \pm 0.13(\text{syst.}) \pm 0.07(\text{model})]\,\%$$

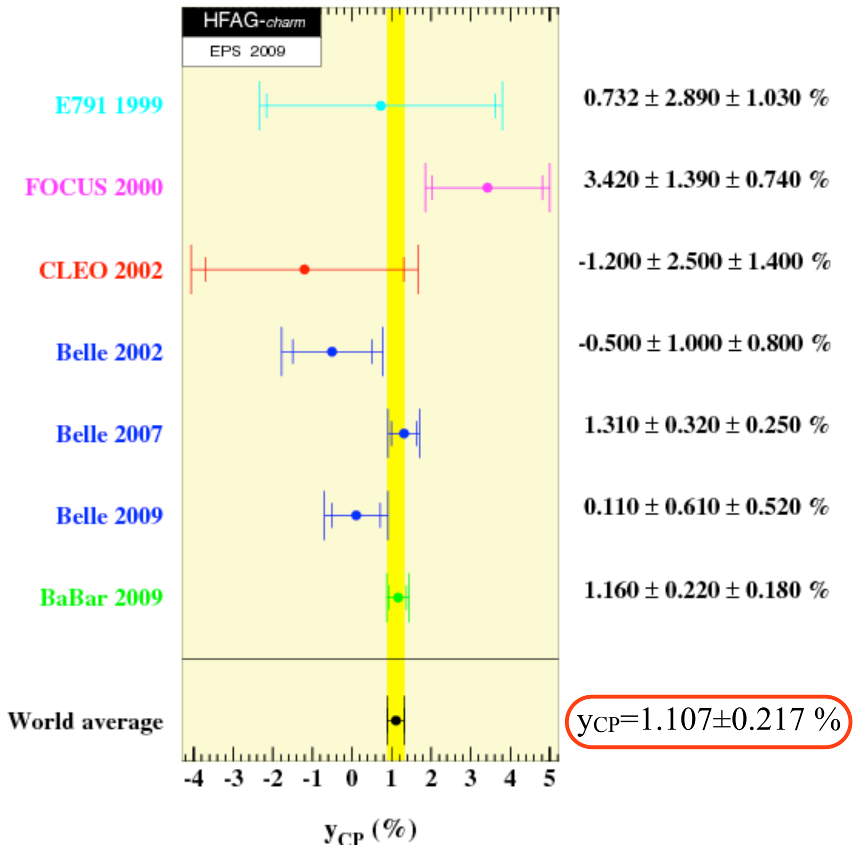
Best measurement of x parameter so far.

HFAG EPS 2009 results

http://www.slac.stanford.edu/xorg/hfag/charm/index.html







$x = (0.976 \pm 0.249)\%$ $|q/p| = 0.866 \pm 0.160$

$y = (0.833 \pm 0.160)\%$ $\varphi = -0.148 \pm 0.126$ rad

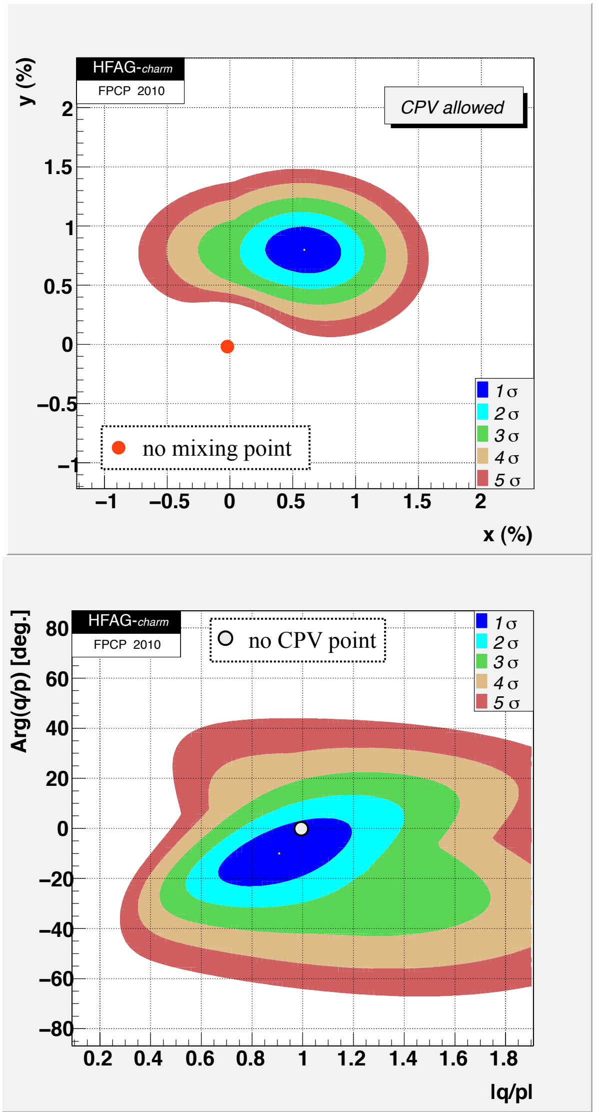
Mixing significance exceeds 10.2σ

No CPV point is within 1σ contour

New

HFAG preliminary FPCP2010 results

courtesy of Alan Schwartz on behalf of HFAG



note different vertical axis scale

HFAG averages including new BaBar $K_S\pi^+\pi^-$ + $K_SK^+K^-$ results:

- *sizable improvement in mixing contours*
- *noticeable effect on x parameter value*

EPS 2009	FPCP 2010
$x = (0.976 \pm 0.249)\%$	$x = (0.59 \pm 0.20)\%$
$y = (0.833 \pm 0.160)\%$	$y = (0.80 \pm 0.13)\%$
$\lvert q/p\rvert = 0.866 \pm 0.160$	$\lvert q/p\rvert = 0.91^{+0.19}_{-0.16}$
$\varphi = -0.148 \pm 0.126$ rad	$\varphi = -10\ ^{+9.3}_{-8.7}$ deg
	($\varphi = -0.175\ ^{+0.162}_{-0.152}$ rad)

Mixing significance still exceeding 10.2σ

No CPV point is within 1σ contour

Summary

- Evidence of $D^0 - \overline{D}^0$ mixing exceeds 10 σ combining all experimental results. No single measurement exceeds 5σ.

- No evidence of CPV in mixing and interference: present experimental sensitivity is at the level of 10^{-3}-10^{-2}.

- Mixing and CPV results in agreement with SM (within large theoretical uncertainties) provide useful constraints for Physics beyond SM.

- B Factories produced most precise measurements for D^0 mixing and CPV so far. Still room for improvements by exploiting the entire data sample and covering all the sensitive measurements.

- Experimental results are dominated by statistical error. Future experiments with very high luminosity will be able to test SM predictions with better precisions.

Backup

- Flavor mixing occurs when flavor eigenstates differ from mass eigenstates: well established phenomena in neutral K, B_d, B_s systems.

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle \qquad |q|^2 + |p|^2 = 1$$

- Mixing parameters are expressed in terms of *x*, *y* parameters, proportional to the mass and decay width differences of the mass eigenstates:

$$\boxed{x = \frac{m_1 - m_2}{\Gamma}; \quad y = \frac{\Gamma_1 - \Gamma_2}{2\Gamma}, \text{where } \Gamma = \frac{\Gamma_1 + \Gamma_2}{2}}$$

Three types of CP violation in D^0 meson system:

1.in the decay (direct):

$$a_{CP}^{f} = \frac{\Gamma(D^0 \to f) - \Gamma(\overline{D}^0 \to \overline{f})}{\Gamma(D^0 \to f) + \Gamma(\overline{D}^0 \to \overline{f})}$$

$$\langle f|H|D^0\rangle = A_f \qquad \langle f|H|\overline{D}^0\rangle = \overline{A}_f$$

$$a_{CP}^{f} \neq 0 \Longrightarrow \left|\frac{\overline{A}_{\overline{f}}}{A_f}\right| \neq 1 \quad \Longrightarrow \text{CPV}$$

2.in mixing (indirect):

$$r_m = \left|\frac{q}{p}\right| \neq 1 \quad \Longrightarrow \text{CPV}$$

3.in the interference between mixing and decay:

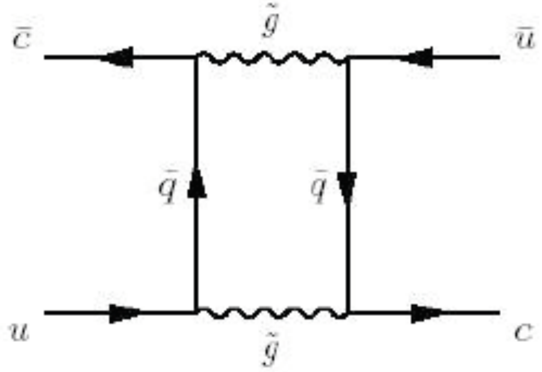
$$\lambda_f = \frac{q}{p}\frac{\overline{A}_f}{A_f} = r_m \left|\frac{\overline{A}_f}{A_f}\right| e^{i(\delta_f + \varphi_f)} \qquad \varphi_f \neq 0 \quad \Longrightarrow \quad \text{CPV}$$

strong phase weak phase

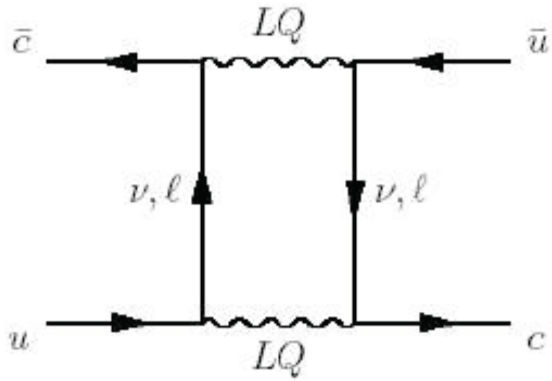
New physics in Charm Mixing

Charm mixing can be affected by possible new physics

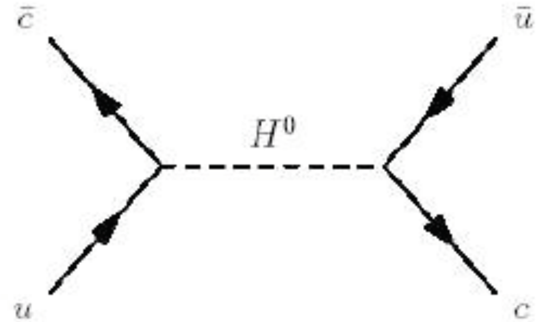
SUSY:



Leptoquarks:



Extended Higgs:



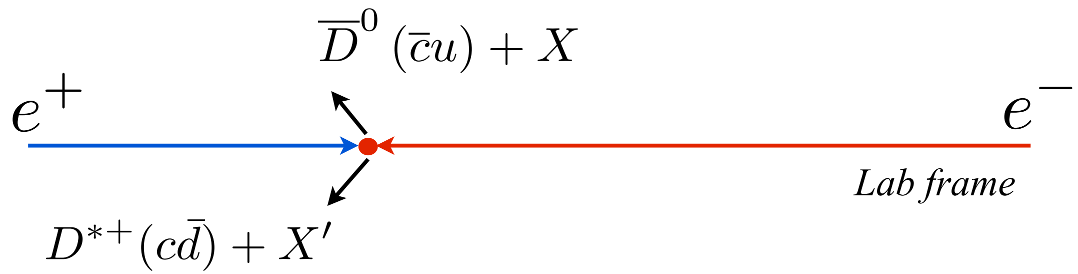
- new physics can increase *x* value, while *y* mostly unaffected: e.g. |x| >> |y| could be hint of New Physics;
- new physics contributions can generate CP violation up to few% level, more then one order of magnitude with respect to Standard Model expectations.

B Factories are also Charm Factories

At the $\Upsilon(4S)$ peak we have:

$$\sigma_{eff}\left(e^+e^- \to \Upsilon(4S) \to b\bar{b}\right) \simeq 1.1 \text{ nb} \qquad \sigma\left(e^+e^- \to c\bar{c}\right) \simeq 1.3 \text{ nb}$$

Example of $c\bar{c}$ event:



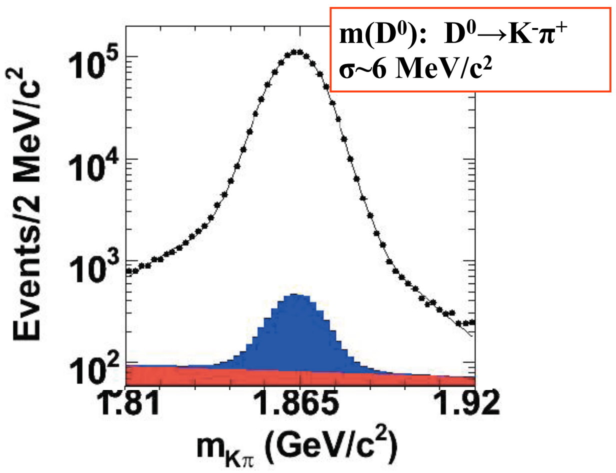
$D^{*+} \to D^0\pi^+$

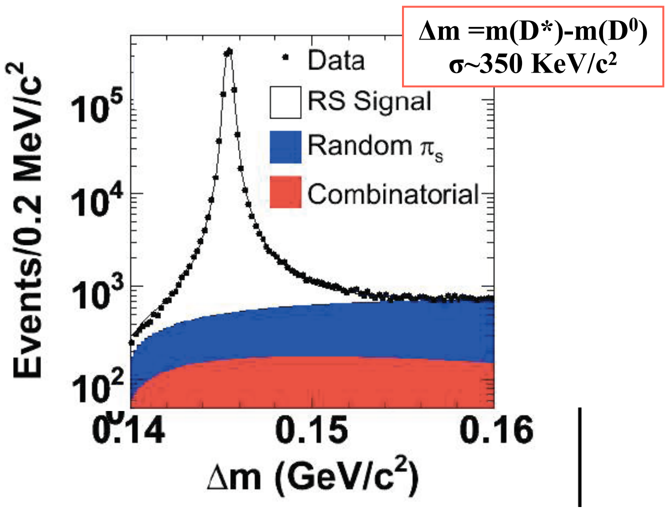
$D^0 \to K^0_S\pi^+\pi^-$

B Factories (BaBar + Belle) recorded about 1.5 ab^{-1} data sample,
i. e. about $2 \cdot 10^9$ $e^+e^- \to c\bar{c}$ events.
Average D^0 and D^{*+} multiplicity per e^+e^- annihilation at $\sqrt{s}$=10 GeV:
n(D^0) = 0.446±0.032 n(D^{*+}) = 0.177±0.022

Selection of D^0 mesons

- Select D^0 mesons via $D^{*+} \to D^0 \pi^+$ decay:
  - charge of slow pion identifies the flavor of D^0 at production;
  - exploit D^0 reco invariant mass, m_D, and D^{*+}-D^0 mass difference, $\Delta m = m(D^*) - m(D)$;
- Cut on D^0 momentum in center of mass frame, p^*, selects events from $e^+e^- \to c\bar{c}$ annihilations,
  - $\sigma\left(e^+e^- \to c\bar{c}\right) \simeq 1.3\ \mathrm{nb}$ larger than $\sigma_{eff}\left(e^+e^- \to \Upsilon(4S) \to b\bar{b}\right) \simeq 1.1\ \mathrm{nb}$
  - cut on p^*>2.5-3.0 GeV/c rejects D from B decays and combinatorial bkg.



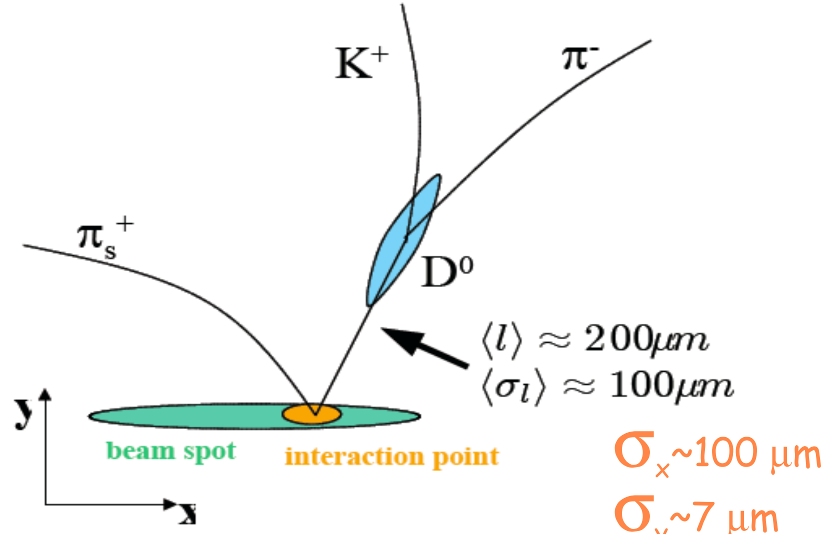


Selection of D^0 mesons

- identify the D^0 flavor at decay using the charge of the kaon;

- use PID, mainly for kaons, with typical efficiency of about 85% and pion misID of about 2%;

-D^0 vertex with beam spot (interaction region size) constraint applied. Determining decay time, t, and decay time error, σ_t , for each each event.



3D flight path reconstruction

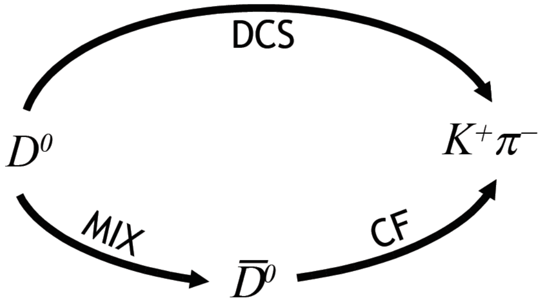
Typical resolution on proper-time: $\langle \sigma_t \rangle \simeq 0.5 \tau_D = 0.2$ ps
thanks to the excellent performance of the SVT.

Wrong sign $D^0 \to K^+\pi^-$ decays

- Wrong Sign (WS) final states from 2 sources: via double-Cabibbo-suppressed (DCS) decays or via mixing followed by Cabibbo-favored (CF) decays.

Time evolution $(|x| \ll 1, |y| \ll 1)$:

$$\frac{dN_{WS}}{dt} \propto e^{-\Gamma t} \Bigg(\underbrace{R_D}_{\text{DCS}} + \underbrace{y' \sqrt{R_D} (\Gamma t)}_{\text{Interference}} + \underbrace{\frac{x'^2 + y'^2}{4} (\Gamma t)^2}_{\text{Mixing}} \Bigg)$$



$$R_D = \frac{\mathrm{B}(D^0 \to K^+\pi^-)}{\mathrm{B}(D^0 \to K^-\pi^+)} \simeq 3 \cdot 10^{-3}$$

phase between DCS and CF decays not directly measurable at B Factories

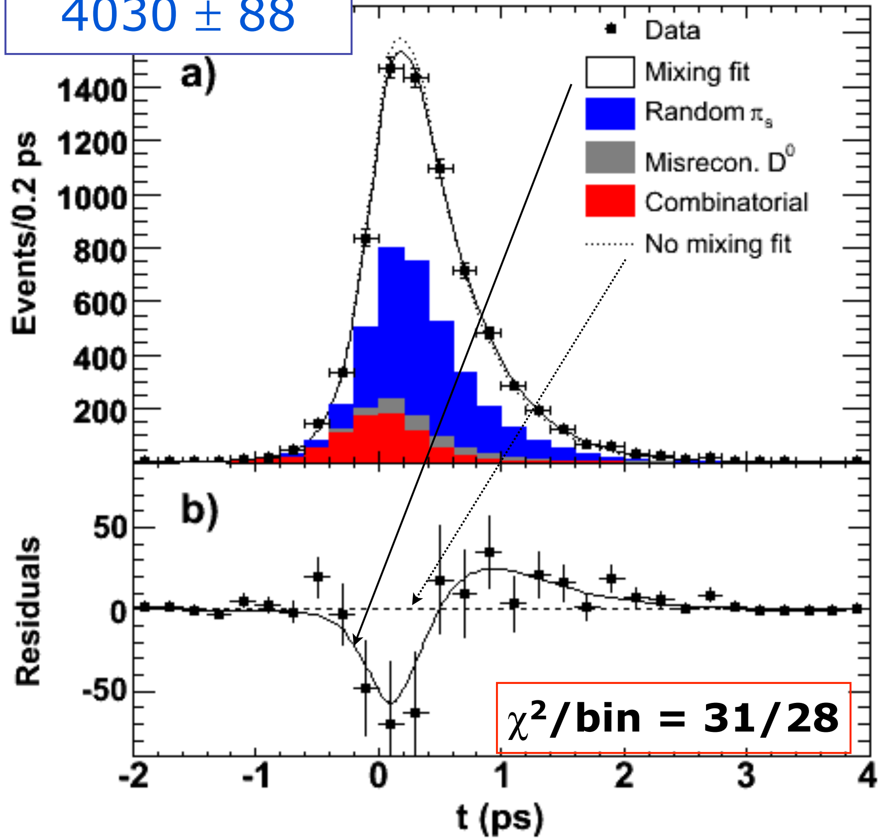
$$x' = x \cos \delta_{K\pi} + y \sin \delta_{K\pi} \qquad y' = -x \sin \delta_{K\pi} + y \cos \delta_{K\pi}$$

Analysis of the proper time distribution of WS events permits extraction of D^0 mixing parameters y', x'^{2}

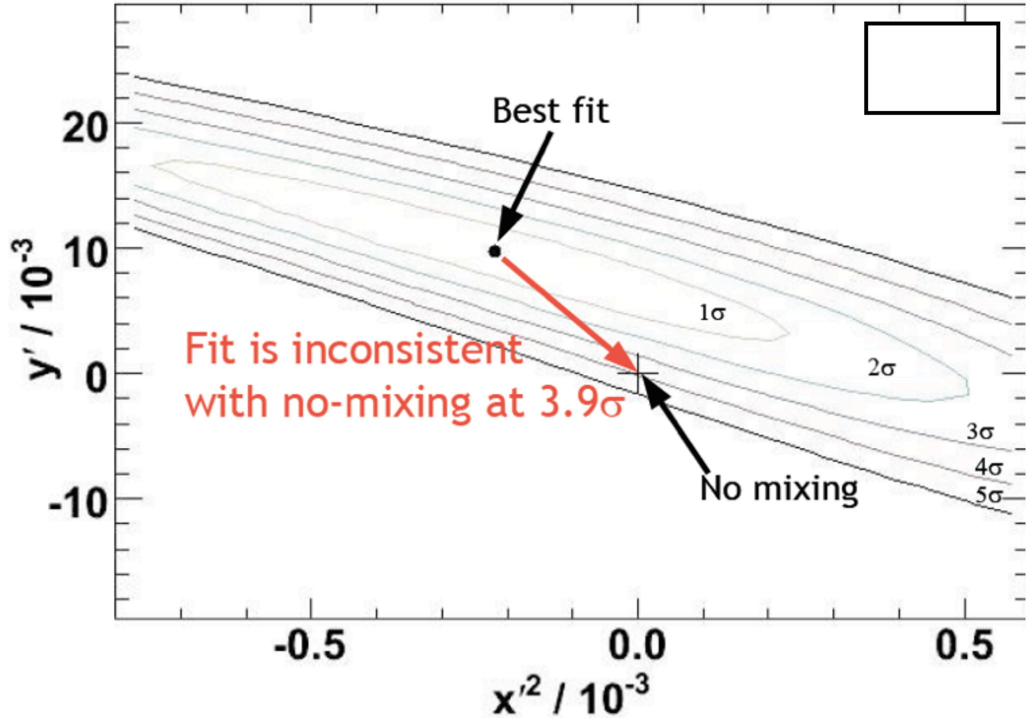
WS time fit: evidence of mixing at 3.9σ

PRL 98:211802,2007 (384 fb^{-1})

Fitted signal
4030 ± 88



WS *mixing* fit projection in signal region
1.843 GeV/c^2<m_D<1.883 GeV/c^2
0.1445 GeV/c^2<Δm< 0.1465 GeV/c^2



R_D: (3.03 ± 0.16 ± 0.10) x 10^{-3}

x'^2: (-0.22 ± 0.30 ± 0.21) x 10^{-3}

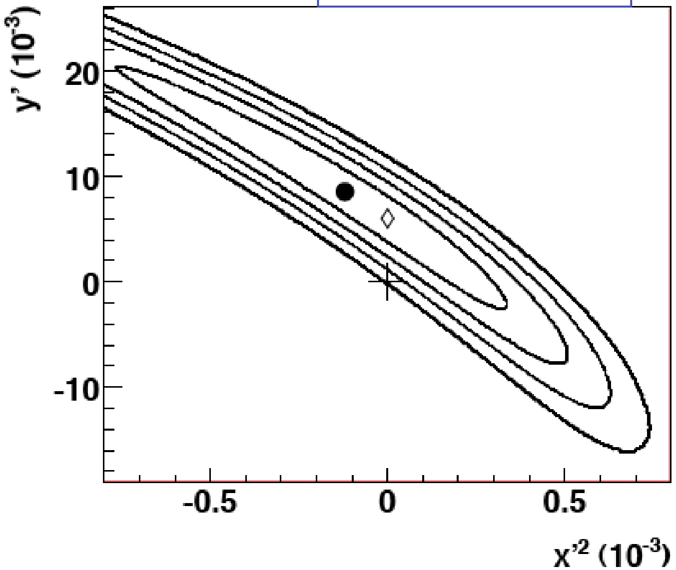
y': (9.7 ± 4.4 ± 3.1) x 10^{-3}

No evidence for CP violation fitting separately D^0 and $\overline{D}^0$

Belle & CDF measurements

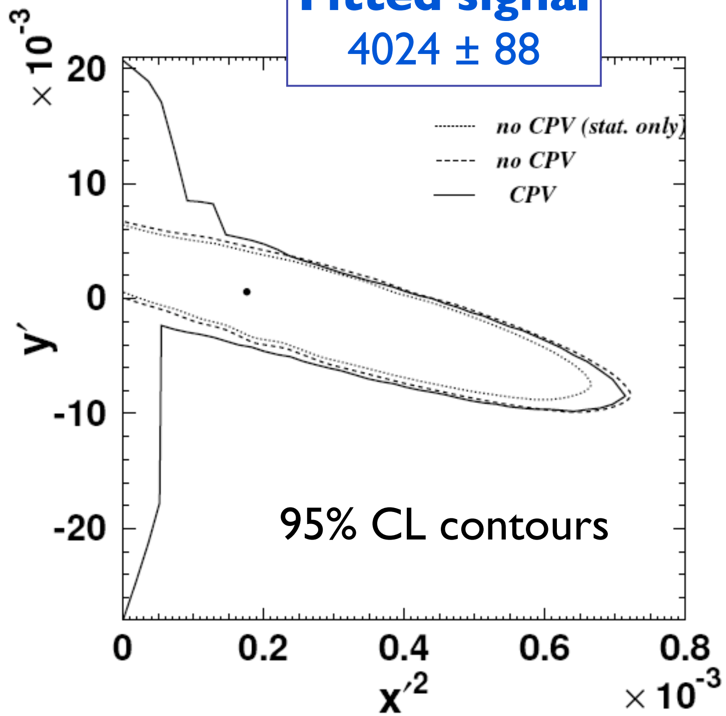
CDF: PRL 100:121802,2008 (1.5fb^{-1})

Fitted signal
(12.7 ± 0.3)K



Belle: PRL 96:151801,2006 (400 fb^{-1})

Fitted signal
4024 ± 88



Experiment	$R_D(10^{-3})$	$y'\,(10^{-3})$	$x'^2\,(10^{-3})$	Mixing Signif.
CDF	3.04 ± 0.55	8.5 ± 7.6	-0.12 ± 0.35	3.8
BABAR [8]	3.03 ± 0.19	9.7 ± 5.4	-0.22 ± 0.37	3.9
Belle [9]	3.64 ± 0.17	$0.6\,^{+4.0}_{-3.9}$	$0.18\,^{+0.21}_{-0.23}$	2.0

$$x'^2 = (0.18^{+0.21}_{-0.23}) \times 10^{-3}$$

$$y' = (0.6^{+4.0}_{-3.9}) \times 10^{-3}$$

Evidence of mixing at 3.8σ

No mixing point at 2σ

Systematic errors: $D^0 \rightarrow K^+\pi^-$

Two types of systematics considered:

- Variations in Fit Model
  - vary signal and background descriptions
    - Proper time resolution function, 3.6fs bias *(SVT mis-alignment)*
- Variations in selection criteria
  - most important: cuts on decay time and its error *t* and d*t*

Systematic source	R_D	y'	x'^2
Fit Model:	0.59σ	0.45σ	0.4σ
Selection Criteria:	0.24σ	0.55σ	0.57σ
Quadrature total:	0.63σ	0.71σ	0.7σ

Systematic errors: $D^0 \to K^+K^-$, $\pi^+\pi^-$

D* Tagged analysis

Systematic	$\sigma_{y_{CP}}$ (%) K^-K^+	$\pi^-\pi^+$	Av.	$\sigma_{\Delta Y}$ (%) K^-K^+	$\pi^-\pi^+$	Av.
Signal model	0.130	0.059	0.085	0.072	0.265	0.062
Charm bkg.	0.062	0.037	0.043	0.001	0.002	0.001
Combinatoric bkg.	0.019	0.142	0.045	0.001	0.005	0.002
Selection criteria	0.068	0.178	0.046	0.083	0.172	0.011
Detector model	0.064	0.080	0.064	0.054	0.040	0.054
Quadrature sum	0.172	0.251	0.132	0.122	0.318	0.083

D* Tagged analysis

Source	Δy_{CP} [%]	ΔA_Γ [%]
acceptance	0.12	0.07
equal t_0	0.14	0.08
M window position	0.04	< 0.01
signal/sideband background differences	0.09	0.06
opening angle distributions	0.02	- -
background distribution $B(t)$	0.07	0.07
(a)symmetric resolution function	0.01	0.01
selection variation	0.11	0.05
binning of t distribution	0.01	0.01
Total	0.25	0.15

Untagged analysis

Uncertainty Source	$\lvert\Delta y_{CP}\rvert$ (%)
Signal resolution model	0.016
Mass window	0.110
Misreconstructed Charm model	0.086
Combinatorial PDF	0.115
σ_t selection	0.069
Overlap candidate selection	0.017
Detector effects	0.093
Total	0.216

Systematic error on y_{CP}: $D^0 \to K_S^+ K^+ K^-$

Untagged analysis

Source	Systematic error (%)
Resolution function offset difference $t_0^{\mathrm{OFF}} - t_0^{\mathrm{ON}}$	± 0.38
Estimation of $\langle t \rangle_b$	± 0.10
$D^0 \to K^+ K^- \pi^+ \pi^-$ background	± 0.07
Selection of sideband	± 0.05
Variation of selection criteria	± 0.30
Fitting procedure	± 0.10
Proper decay time range and binning	± 0.07
Dalitz model	± 0.01
Total	± 0.52

TABLE II: Fraction difference $f_{\mathrm{ON}} - f_{\mathrm{OFF}}$ for the two Dalitz models. The nominal values are calculated from the data in Refs. [14, 21], and the fitted values from our fit results.

	$f_{\mathrm{ON}} - f_{\mathrm{OFF}}$	
Model	Nominal	Fitted
4 res. [21]	-0.730 ± 0.031	-0.732 ± 0.002
8 res. [14]	-0.753 ± 0.004	-0.769 ± 0.005

Systematic errors for $D^0 \to K^+\pi^-\pi^0$

Syst.	x''/r_0	y''/r_0
Dalitz model	0.338	0.472
t resolution function	0.259	0.0621
Background model	0.55	0.464
Signal and Background yields	0.168	0.0132
Dalitz plot efficiency	0.0876	0.0794
Selection	0.391	0.287
Total	0.858	0.745

Table 6: Systematic uncertainties on x''/r_0 and y''/r_0 (diagonal terms), in units of the statistical error σ.

Dalitz Plot fit results for DCS amplitudes: $D^0 \to K^+\pi^-\pi^0$

Resonance	Amplitude	Phase (degrees)	Fit Fraction (%)
$\rho(770)$	1 (fixed)	0 (fixed)	39.8 ± 6.5
$K_2^{*0}(1430)$	0.088 ± 0.017	-17.2 ± 12.9	2.0 ± 0.7
$K_0^{*+}(1430)$	6.78 ± 1.00	69.1 ± 10.9	13.1 ± 3.3
$K^{*+}(892)$	0.899 ± 0.005	-171.0 ± 5.9	35.6 ± 5.5
$K_0^{*0}(1430)$	1.65 ± 0.59	-44.4 ± 18.5	2.8 ± 1.5
$K^{*0}(892)$	0.398 ± 0.038	24.1 ± 9.8	6.5 ± 1.4
$\rho(1700)$	5.4 ± 1.6	157.4 ± 20.3	2.0 ± 1.1
$\chi^2/ndof = 188/215 = 0.876$			
Total fit fraction = 102%			

Dalitz Plot fit results for CF amplitudes: $D^0 \to K^-\pi^+\pi^0$

Resonance	Amplitude	Phase (degrees)	Fit Fraction (%)
$\rho(770)$	1 (fixed)	0 (fixed)	63.6 ± 5.2
$K^{*-}(1680)$	4.46 ± 0.04	141.4 ± 0.7	4.0 ± 0.3
$K_2^{*-}(1430)$	0.023 ± 0.001	-147.9 ± 2.6	0.12 ± 0.01
$K_2^{*0}(1430)$	0.0408 ± 0.0008	-8.4 ± 1.1	0.51 ± 0.04
$K^{*-}(1410)$	0.16 ± 0.01	43.1 ± 4.4	0.09 ± 0.01
$K_0^{*-}(1430)$	2.28 ± 0.04	170.9 ± 0.9	2.2 ± 0.2
$K^{*-}(892)$	0.380 ± 0.001	162.1 ± 0.2	9.2 ± 0.7
$K^{*0}(1410)$	0.19 ± 0.01	-281.5 ± 2.6	0.15 ± 0.02
$K_0^{*0}(1430)$	2.67 ± 0.01	82.8 ± 0.4	7.8 ± 0.6
$K^{*0}(1680)$	5.07 ± 0.04	-40.4 ± 0.6	6.0 ± 0.5
$K^{*0}(892)$	0.399 ± 0.001	0.5 ± 0.3	9.5 ± 0.8
$\rho(1700)$	4.06 ± 0.07	152.9 ± 0.9	1.9 ± 0.2
Total fit fraction = 105%			

Systematic errors for $D^0 \to K_S^+ \pi^+ \pi^-$

Systematics of $D^0 \to K_s^0 \pi^+ \pi^-$ Dalitz

Experimental

Source	Δx (%)	Δy (%)
Event selection	$^{+0.076}_{-0.001}$	$^{+0.018}_{-0.078}$
Dalitz dep. effi.	$+0.004$	-0.009
Background	$^{+0.041}_{-0.068}$	$^{+0.077}_{-0.086}$
Total	$^{+0.09}_{-0.07}$	$^{+0.08}_{-0.12}$

Model dependence

Source	Δx (%)	Δy (%)
M&Γ errors	± 0.020	± 0.010
$F_r = F_D = 1$	-0.031	$+0.006$
$\Gamma(q^2) = \text{const.}$	-0.051	-0.041
K-Matrix	± 0.073	± 0.058
No NR	-0.015	$+0.003$
No $K^*(1680)^+$	-0.003	-0.008
No $\rho(1450)$	-0.005	-0.006
$K_0^*(1430)$ DCS/CF	-0.103	$+0.001$
$K_2^*(1430)$ DCS/CF	$+0.069$	-0.025
$K^*(1410)$ DCS/CF	-0.016	$+0.009$
Total	$^{+0.10}_{-0.14}$	$^{+0.06}_{-0.08}$

$D^0 \rightarrow K_S \pi^+ \pi^-$ + $D^0 \rightarrow K_S K^+ K^-$ fit results for different subsamples

TABLE I: Results from the mixing fits. The first uncertainty is statistical, the second systematic and the third systematic from the amplitude model. For the nominal fit, the corresponding correlation coefficients between x and y are 3.5%, 16.0% and −2.7%, respectively.

Fit type	$x/10^{-3}$	$y/10^{-3}$
Nominal	$1.6 \pm 2.3 \pm 1.2 \pm 0.8$	$5.7 \pm 2.0 \pm 1.3 \pm 0.7$
$K^0_S \pi^+ \pi^-$	2.6 ± 2.4	6.0 ± 2.1
$K^0_S K^+ K^-$	-13.6 ± 9.2	4.4 ± 5.7
D^0	0.0 ± 3.3	5.5 ± 2.8
$\overline{D}^0$	3.3 ± 3.3	5.9 ± 2.8

$D^0 \to K_S\pi^+\pi^-$ Dalitz plot fit results

TABLE I: $D^0 \to K_S^0\pi^+\pi^-$ complex amplitudes, $\pi\pi$ P-vector and $K\pi$ S-wave parameters, and fit fractions, as obtained from the mixing fit. The $\pi\pi$ S-wave parameters β_5, f_{14}^{prod}, and f_{15}^{prod} are fixed to zero due to the lack of sensitivity. We also report the mass and the width of the $K^*(892)^{\mp}$ resonance. Errors are statistical only. The fit fraction is defined as the integral over the entire DP of a single component divided by the coherent sum of all components. The sum of fit fractions is 103.3%. A detailed description of the parameters can be found elsewhere [14]. Equations (14) and (15) in [14] have been corrected as follows, $\mathcal{A}_{K\pi\ L=0}(s) = T_{K\pi\ L=0}(s)/\rho(s)$, where $\rho(s) = q/\sqrt{s}$ is the phase-space factor and $T_{K\pi\ L=0}(s) = F\sin(\delta_F + \phi_F)e^{i(\delta_F+\phi_F)} + R\sin\delta_R e^{i(\delta_R+\phi_R)}e^{i2(\delta_F+\phi_F)}$, with $\tan\delta_R = M_{K_0^*(1430)}\Gamma_{K_0^*(1430)}(s)/(M^2_{K_0^*(1430)} - s)$, $\cot\delta_F = 1/(aq) + rq/2$, s the invariant mass squared of the $K\pi$ system, and q the momentum of the spectator particle in the $K\pi$ rest frame [21]. The symbol † indicates the parameters fixed in the mixing fit to the values extracted from a time-integrated DP fit to the same data. The results from this time-integrated DP fit for the amplitude model parameters agree within statistical errors with the results reported here.

	Component	Amplitude	Phase (rad)	Fit fraction (%)
	$K^*(892)^-$	1.735 ± 0.005	2.331 ± 0.004	57.0
CP	$\rho(770)^0$	1	0	21.1
	$K_0^*(1430)^-$	2.650 ± 0.015	1.497 ± 0.007	6.1
	$K_2^*(1430)^-$	1.303 ± 0.013	2.498 ± 0.012	1.9
	$\omega(782)$	0.0420 ± 0.0006	2.046 ± 0.014	0.6
DCS	$K^*(892)^+$	0.164 ± 0.003	-0.768 ± 0.019	0.6
	$K^*(1680)^-$	0.90 ± 0.03	-2.97 ± 0.04	0.3
	$f_2(1270)$	0.410 ± 0.013	2.88 ± 0.03	0.3
	$K_0^*(1430)^+$	0.145 ± 0.014	1.78 ± 0.10	< 0.1
DCS	$K_2^*(1430)^+$	0.115 ± 0.013	2.69 ± 0.11	< 0.1
	$\lceil$ $\pi\pi$ S-wave			15.4
	β_1	5.54 ± 0.06	-0.054 ± 0.007	
	β_2	15.64 ± 0.06	-3.125 ± 0.005	
	β_3	44.6 ± 1.2	2.731 ± 0.015	
	β_4	9.3 ± 0.2	2.30 ± 0.02	
	f_{11}^{prod}	$11.43 \pm 0.11^\dagger$	$-0.005 \pm 0.009^\dagger$	
	f_{12}^{prod}	$15.5 \pm 0.4^\dagger$	$-1.13 \pm 0.02^\dagger$	
	f_{13}^{prod}	$7.0 \pm 0.7^\dagger$	$0.99 \pm 0.11^\dagger$	
	$\lfloor$ s_0^{prod}	Parameter value		
	s_0^{prod}	-3.92637		
	$\lceil$ $K\pi$ S-wave parameters			
	$M_{K_0^*(1430)}$ (MeV/c^2)	$1421.5 \pm 1.6^\dagger$		
	$\Gamma_{K_0^*(1430)}$ (MeV/c^2)	$247 \pm 3^\dagger$		
	F	$0.62 \pm 0.04^\dagger$		
	ϕ_F (rad)	$-0.100 \pm 0.010^\dagger$		
	R	1		
	ϕ_R (rad)	$1.10 \pm 0.02^\dagger$		
	a (GeV/c^{-1})	$0.224 \pm 0.003^\dagger$		
	$\lfloor$ r (GeV/c^{-1})	$-15.01 \pm 0.13^\dagger$		
	$\lceil$ $K^*(892)$ parameters			
	$M_{K^*(892)}$ (MeV/c^2)	$893.70 \pm 0.07^\dagger$		
	$\lfloor$ $\Gamma_{K^*(892)}$ (MeV/c^2)	$46.74 \pm 0.15^\dagger$		

$D^0 \rightarrow K_S K^+ K^-$ Dalitz plot fit results

TABLE II: $D^0 \to K^0_S K^+ K^-$ complex amplitudes and fit fractions, as obtained from the mixing fit. We also report the mass and the width of the $\phi(1020)$ resonance, and the $a_0(980)$ coupling constant to $K\overline{K}$ as determined from the fit. Errors are statistical only. The fit fraction is defined as the integral over the entire DP of a single component divided by the coherent sum of all components. The sum of fit fractions is 163.4%. A detailed description of the parameters can be found elsewhere [14]. The symbol † indicates the parameters fixed in the mixing fit to the values extracted from a time-integrated DP fit to the same data. The results from this time-integrated DP fit for the amplitude model parameters agree within statistical errors with the results reported here.

	Component	Amplitude	Phase (rad)	Fit fraction (%)
	$a_0(980)^0$	1	0	51.8
CP	$\phi(1020)$	0.2313 ± 0.0011	-0.977 ± 0.008	44.1
	$a_0(1450)^+$	$0.93 \pm 0.03^\dagger$	$1.66 \pm 0.07^\dagger$	25.6
	$a_0(980)^+$	0.635 ± 0.006	-2.91 ± 0.02	19.5
	$a_0(1450)^0$	$0.83 \pm 0.10^\dagger$	$-1.93 \pm 0.12^\dagger$	19.3
CP	$f_0(1370)$	$0.16 \pm 0.05^\dagger$	$0.2 \pm 0.2^\dagger$	1.7
	$f_2(1270)$	0.385 ± 0.015	-3.08 ± 0.04	0.7
DCS	$a_0(980)^-$	0.125 ± 0.008	2.47 ± 0.04	0.7

$\phi(1020)$ and $a_0(980)$ parameters	Value
$M_{\phi(1020)}$ (MeV/c^2)	$1019.55 \pm 0.02^\dagger$
$\Gamma_{\phi(1020)}$ (MeV/c^2)	$4.60 \pm 0.04^\dagger$
$g_{K\overline{K}}$ (MeV/c^2)	$537 \pm 9^\dagger$

$D^0 \to K_S\pi^+\pi^-$ + $D^0 \to K_S K^+K^-$ Dalitz model systematics

We measure model-dependent systematics using 10 toy Monte Carlo samples generated with the nominal model. Each toy sample is fitted with the nominal model, and with each of the alternative models, and the mean change in fit result is taken as the uncertainty due to an alternative model.

Systematic source	$x(\%)$	$y(\%)$
$\pi\pi$ $S-$wave: K-matrix solution-I	0.0121 $\pm$ 0.0116	-0.0077 $\pm$ 0.0077
$\pi\pi$ $S-$wave: K-matrix solution-IIa	-0.0033 $\pm$ 0.0020	0.0020 $\pm$ 0.0012
$\pi\pi$ $S-$wave: Alternative NR term production vector	-0.0040 $\pm$ 0.0032	-0.0174 $\pm$ 0.0052
$\pi\pi$ $P-$wave: $\rho(770)$ and $\omega(782)$ float mass and width	0.0279 $\pm$ 0.0284	-0.0080 $\pm$ 0.0227
$\pi\pi$ $P-$wave: $\rho(770)$ BW line shape	-0.0010 $\pm$ 0.0063	0.0052 $\pm$ 0.0052
$K\pi$ $P-$wave: $K^*(1680)$ mass variation	-0.0125 $\pm$ 0.0023	0.0020 $\pm$ 0.0031
$K\pi$ $P-$wave: $K^*(1680)$ width variation	-0.0033 $\pm$ 0.0017	0.0025 $\pm$ 0.0015
$K\pi$ $P-$wave: $K^*(1680)$ mass and width from PDG	-0.0172 $\pm$ 0.0042	0.0037 $\pm$ 0.0046
$K\pi$ $D-$wave: $K_2^*(1430)$ mass variation	0.0013 $\pm$ 0.0014	-0.0007 $\pm$ 0.0014
$K\pi$ $D-$wave: $K_2^*(1430)$ width variation	-0.0005 $\pm$ 0.0013	0.0012 $\pm$ 0.0009
More $K_S^0\pi^+\pi^-$ resonances: $K^*(1410)$ and $\rho(1450)$	-0.0001 $\pm$ 0.0036	-0.0010 $\pm$ 0.0025
K-matrix, LASS, $K^*(892)$, $\phi(1020)$, and $g_{K\bar{K}}$ parameters	0.0678	0.0532
KK $S-$wave: $a_0(980)$ mass variation	0.0001 $\pm$ 0.0004	0.0010 $\pm$ 0.0002
KK $S-$wave: $g_{\eta\pi}$ variation	0.0003 $\pm$ 0.0009	0.0032 $\pm$ 0.0006
KK $S-$wave: $f_0(1370)$ mass variation	-0.0003 $\pm$ 0.0004	-0.0012 $\pm$ 0.0006
KK $S-$wave: $f_0(1370)$ width variation	-0.0001 $\pm$ 0.0002	-0.0005 $\pm$ 0.0004
KK $S-$wave: $f_0(1370)$ from E791	-0.0004 $\pm$ 0.0004	-0.0009 $\pm$ 0.0007
KK $S-$wave: $a_0(1450)$ mass variation	-0.0002 $\pm$ 0.0004	0.0007 $\pm$ 0.0003
KK $S-$wave: $a_0(1450)$ width variation	0.0001 $\pm$ 0.0003	0.0003 $\pm$ 0.0002
More $K_S^0K^+K^-$ resonances: $a_0(1450)$ DCS and $f_0(980)$	-0.0007 $\pm$ 0.0013	-0.0003 $\pm$ 0.0026
Fewer $K_S^0K^+K^-$ resonances: $f_0(1370)$ and $f_2(1270)$	-0.0165 $\pm$ 0.0109	0.0226 $\pm$ 0.0091
$\pi\pi - KK$ $D-$waves: $f_2(1270)$ mass variation	-0.0007 $\pm$ 0.0009	-0.0008 $\pm$ 0.0008
$\pi\pi - KK$ $D-$waves: $f_2(1270)$ width variation	0.0006 $\pm$ 0.0012	0.0006 $\pm$ 0.0010
$\pi\pi - KK$ $P-, D-$waves: Blatt-Weisskopf factors	0.0025 $\pm$ 0.0058	0.0026 $\pm$ 0.0077
$\pi\pi - KK$ $P-, D-$waves: resonance frame	-0.0244 $\pm$ 0.0248	-0.0233 $\pm$ 0.0173
$\pi\pi - KK$ $P-, D-$waves: Helicity formalism	0.0005 $\pm$ 0.0245	-0.0172 $\pm$ 0.0170
Total Dalitz model systematics	0.0830	0.0685