Semi-hard processes and BFKL

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Outline

1. **Introductory remarks**
   - **Gluon Reggeization in perturbative QCD**
   - **BFKL in the leading order**
   - **BFKL in the next-to-leading order**
   - **Other BFKL topics**
2. **Phenomenology**
   - **General scheme**
   - **Processes at e^+e^-, $e\gamma$ colliders**
   - **Processes at hadron colliders**
   - **Processes at ep colliders**
3. **Conclusions**

Semi-hard collision processes

Collision processes with the following scale hierarchy:

$$s \gg Q^2 \gg \Lambda^2_{\rm QCD}$$

Q is the hard scale of the process (*e.g.* photon virtuality, heavy quark mass, jet/hadron transverse momentum, t, *etc.*)

- **large $Q \implies \alpha_s(Q) \ll 1 \implies$ perturbative QCD**
- **large $s \implies$ large energy logs $\implies \alpha_s(Q) \log s \sim 1 \implies$ resummation**

The Balitsky-Fadin-Kuraev-Lipatov (BFKL) approach provides with the general framework for this resummation.

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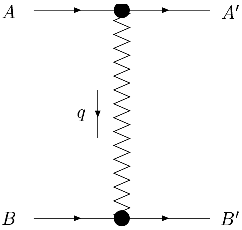
Gluon Reggeization in perturbative QCD

Elastic scattering process $A + B \longrightarrow A' + B'$

- **gluon quantum numbers in the t-channel: octet color representation, negative signature**
- **Regge limit:** $s \simeq -u \to \infty$, t **fixed (i.e. not growing with** s**)**
- **all-order resummation:**
  leading logarithmic approximation (LLA): $\alpha_s^n (\ln s)^n$
  next-to-leading logarithmic approximation (NLA): $\alpha_s^{n+1} (\ln s)^n$



$$\left(\mathcal{A}_8^-\right)_{AB}^{A'B'} = \Gamma_{A'A}^c \left[\left(\frac{-s}{-t} \right)^{j(t)} - \left(\frac{s}{-t} \right)^{j(t)} \right] \Gamma_{B'B}^c$$

$$j(t) = 1 + \omega(t) \; , \qquad j(0) = 1$$

$\omega(t)$ **– Reggeized gluon trajectory**

$$\Gamma_{A'A}^c = g \langle A' | T^c | A \rangle \Gamma_{A'A}$$

T^c **fundamental (quarks) or adjoint (gluons)**

Gluon Reggeization in perturbative QCD

Interlude: Sudakov decomposition

$$p = \beta p_1 + \alpha p_2 + p_\perp \ , \qquad p_\perp^2 = -\vec{p}^{\,2}$$

(p_1, p_2) **light-cone basis of the initial particle momenta plane**

$$p_A = p_1 + \frac{m_A^2}{s} p_2 \ , \qquad p_B = p_2 + \frac{m_B^2}{s} p_1 \ , \qquad 2\, p_1 \cdot p_2 = s \quad \blacksquare$$

The gluon Reggeization has been first verified in fixed order calculations, then rigorously proved

- **in the LLA** **[Ya.Ya. Balitsky, V.S. Fadin, L.N. Lipatov (1979)]**

$$\Gamma^{(0)}_{A'A} = \delta_{\lambda_{A'}\lambda_A} \ , \qquad \omega^{(1)}(t) = \frac{g^2 t}{(2\pi)^{D-1}} \frac{N}{2} \int \frac{d^{D-2}k_\perp}{k_\perp^2 (q-k)_\perp^2} = -g^2 \frac{N\Gamma(1-\epsilon)}{(4\pi)^{D/2}} \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} (\vec{q}^{\,2})^\epsilon$$

$$D = 4 + 2\epsilon \ , \qquad t = q^2 \simeq q_\perp^2$$

- **in the NLA** **[V.S. Fadin, R. Fiore, M.G. Kozlov, A.V. Reznichenko (2006)]**

$$\Gamma^{(1)}_{A'A} = \delta_{\lambda_{A'}\lambda_A} \Gamma^{(+)}_{AA} + \delta_{\lambda_{A'},-\lambda_A} \Gamma^{(-)}_{AA} \ , \qquad \omega^{(2)}(t)$$

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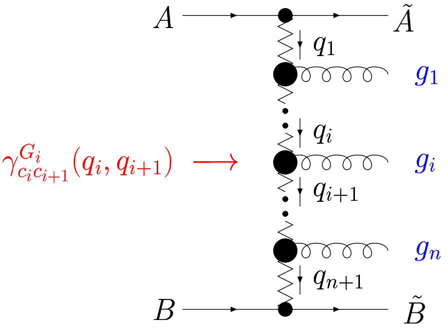
BFKL in the LLA

Inelastic scattering process $A + B \;\rightarrow\; \tilde{A} + \tilde{B} + n$ in the LLA



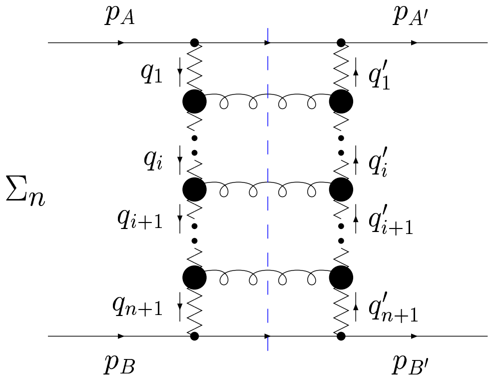
multi-Regge kinematics

$$\mathrm{Re}\mathcal{A}^{\tilde{A}\tilde{B}+n}_{AB} = 2s\,\Gamma^{c_1}_{\tilde{A}A}\left(\prod_{i=1}^{n}\gamma^{P_i}_{c_i c_{i+1}}(q_i, q_{i+1})\left(\frac{s_i}{s_R}\right)^{\omega(t_i)}\frac{1}{t_i}\right)\frac{1}{t_{n+1}}\left(\frac{s_{n+1}}{s_R}\right)^{\omega(t_{n+1})}\Gamma^{c_{n+1}}_{\tilde{B}B}$$

s_R **energy scale, irrelevant in the LLA**

BFKL in the LLA

Elastic amplitude $A + B \longrightarrow A' + B'$ in the LLA via s-channel unitarity

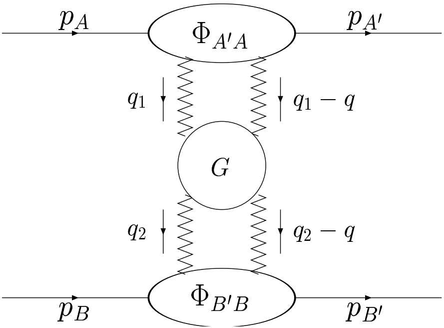


$$\mathcal{A}_{AB}^{A'B'} = \sum_{\mathcal{R}} (\mathcal{A}_{\mathcal{R}})_{AB}^{A'B'} \,, \qquad \mathcal{R} = \mathbf{1}\ \text{(singlet)},\ \mathbf{8}^-\ \text{(octet)}, \ldots$$

The 8^- color representation is important for the bootstrap, i.e. the consistency between the above amplitude and that with one Reggeized gluon exchange

BFKL in the LLA

Structure of the amplitude:



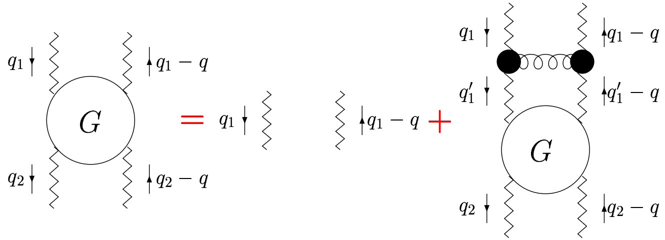
$$\mathrm{\textbf{Im}}_s(A_{\mathcal{R}})_{AB}^{A'B'} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}(\vec{q}_1 - \vec{q})^2} \int \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}(\vec{q}_2 - \vec{q})^2} \sum_{\nu} \Phi_{A'A}^{(\mathcal{R},\nu)}(\vec{q}_1; \vec{q})$$
$$\times \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^{\omega} G_{\omega}^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2, \vec{q}) \right] \Phi_{B'B}^{(\mathcal{R},\nu)}(-\vec{q}_2; -\vec{q})$$

BFKL in the LLA

- $G_\omega^{(\mathcal{R})}$ – **Mellin transform of the Green's functions for Reggeon-Reggeon scattering**

$$\omega G_\omega^{(\mathcal{R})}\left(\vec{q}_1, \vec{q}_2, \vec{q}\right) = \vec{q}_1^{\,2}(\vec{q}_1 - \vec{q})^{\,2}\delta^{(D-2)}\left(\vec{q}_1 - \vec{q}_2\right) + \int \frac{d^{D-2}q_r}{\vec{q}_r^{\,2}(\vec{q}_r - \vec{q})^{\,2}} \mathcal{K}^{(\mathcal{R})}\left(\vec{q}_1, \vec{q}_r; \vec{q}\right) G_\omega^{(\mathcal{R})}\left(\vec{q}_r, \vec{q}_2; \vec{q}\right)$$

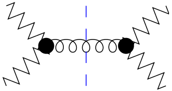
BFKL equation: $t = 0$ **and singlet color representation**
[Ya.Ya. Balitsky, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]



BFKL in the LLA

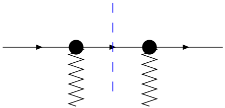
$$\mathcal{K}^{(\mathcal{R})}\left(\vec{q}_1, \vec{q}_2; \vec{q}\right) = \left[\omega\left(-\vec{q}_1^{\,2}\right) + \omega\left(-(\vec{q}_1 - \vec{q})^{\,2}\right)\right]\delta^{(D-2)}\left(\vec{q}_1 - \vec{q}_2\right) + \mathcal{K}_r^{(\mathcal{R})}\left(\vec{q}_1, \vec{q}_2; \vec{q}\right)$$

In the LLA: $\omega(t) = \omega^{(1)}(t)$, $\mathcal{K}_r = \mathcal{K}_{RRG}^{(B)}$



- $\Phi_{A'A}^{(\mathcal{R},\nu)}$ **– impact factors in the t-channel color state $(\mathcal{R},\nu)$**

$$\Phi_{A'A} = \sum_{\{f\}} \int \frac{d\kappa}{2\pi} d\rho_f \Gamma^{c}_{\{f\}A} (\Gamma^{c'}_{\{f\}A'})^*$$



constant in the LLA

BFKL Pomeron in the LLA

Pomeron channel: $t = 0$ **and singlet color representation in the** t**-channel**

Redefinition: $G_\omega(\vec{q}_1, \vec{q}_2) \equiv \frac{G_\omega^{(0)}(\vec{q}_1, \vec{q}_2, 0)}{\vec{q}_1^{\,2} \vec{q}_2^{\,2}}$, $\mathcal{K}(\vec{q}_1, \vec{q}_2) \equiv \frac{\mathcal{K}^{(0)}(\vec{q}_1, \vec{q}_2, 0)}{\vec{q}_1^{\,2} \vec{q}_2^{\,2}}$

$$\omega G_\omega\left(\vec{q}_1, \vec{q}_2\right) = \delta^{(D-2)}\left(\vec{q}_1 - \vec{q}_2\right) + \int d^{D-2} q_r \mathcal{K}\left(\vec{q}_1, \vec{q}_r\right) G_\omega\left(\vec{q}_r, \vec{q}_2\right)$$

$$\mathcal{K}\left(\vec{q}_1, \vec{q}_2\right) = 2\omega(-\vec{q}_1^{\,2})\delta^{(D-2)}\left(\vec{q}_1 - \vec{q}_2\right) + \mathcal{K}_r\left(\vec{q}_1, \vec{q}_2\right)$$

Infrared divergences cancel in the singlet kernel

$\mathcal{K}\left(\vec{q}_1, \vec{q}_2\right)$ **is scale-invariant** $\longrightarrow$ **its eigenfunctions are powers of** $\vec{q}_2^{\,2}$**:**

$$\int d^{D-2} q_2 \mathcal{K}\left(\vec{q}_1, \vec{q}_2\right) (\vec{q}_2^{\,2})^{\gamma-1} = \frac{N\alpha_s}{\pi} \chi(\gamma) (\vec{q}_1^{\,2})^{\gamma-1}$$

$$\chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)\,, \quad \psi(\gamma) = \frac{\Gamma'(\gamma)}{\Gamma(\gamma)}$$

The set of functions $(\vec{q}_2^{\,2})^{\gamma-1}$**, with** $\gamma = 1/2 + i\nu$**,** $\nu \in (-\infty, +\infty)$ **is complete.**

BFKL Pomeron in the LLA

$$\sigma_{AB}(s) = \int_{-\infty}^{+\infty} \frac{d\nu}{2\pi^2} \int \frac{d^2\vec{q}_A}{2\pi} \int \frac{d^2\vec{q}_B}{2\pi} \left(\frac{s}{s_0}\right)^{\bar{\alpha}_s \chi(\nu)} \Phi_A(\vec{q}_A)(\vec{q}_A^{\,2})^{-i\nu-3/2} \Phi_B(-\vec{q}_B)(\vec{q}_B^{\,2})^{i\nu-3/2}$$

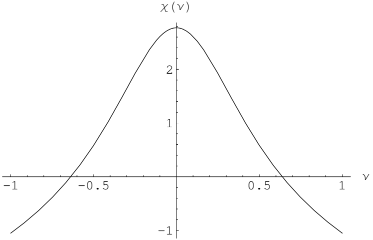
$$\bar{\alpha}_s \equiv \frac{N\alpha_s}{\pi}\,, \qquad \chi(\nu) \equiv \chi(1/2 + i\nu)$$

Saddle point approximation:

$$\chi(\nu) = 4\ln 2 + \psi''\left(\tfrac{1}{2}\right)\nu^2 + O(\nu^4)$$

$$\sigma_{AB}(s) \sim \frac{s^{4\bar{\alpha}_s \ln 2}}{\sqrt{\ln s}}$$

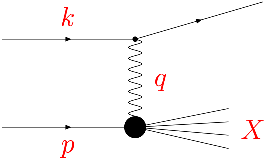
$\omega_P = 4\bar{\alpha}_s \ln 2 \simeq 0.40$ **for** $\alpha_s = 0.15$



- **unitarity is violated; BFKL cannot be applied at asymptotically high energies**
- **the scale of** s **and the argument of the running coupling constant are not fixed in the LLA** $\longrightarrow$ **NLA**

BFKL and DIS

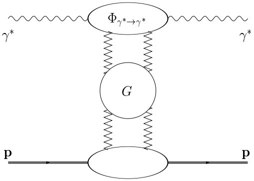
Deep inelastic electron-proton scattering: $e + p \to e + X$



$$Q^2 = -q^2 > 0$$
$$s = (p+k)^2$$
$$W^2 = (p+q)^2$$

$$x = \frac{Q^2}{2p \cdot q} \simeq \frac{Q^2}{Q^2 + W^2}\,, \qquad W^2 \simeq \frac{Q^2(1-x)}{x}$$

$$W^2 \gg Q^2 \gg M_P^2 \qquad \longrightarrow \qquad x \ll 1$$



$$F_2(x, Q^2) \sim \sigma_{\textbf{tot}}(\gamma^* p) \quad = \quad \frac{\mathcal{I}m\mathcal{A}(\gamma^* p \to \gamma^* p)|_{t=0}}{W^2}$$
$$\sim \quad x^{-4\bar{\alpha}_s \ln 2}$$

Outline

BFKL in the NLA

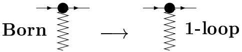
Production amplitudes keep the simple factorized form

$$\text{Re}\mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} = 2s\,\Gamma_{\tilde{A}A}^{c_1}\left(\prod_{i=1}^{n}\gamma_{c_i c_{i+1}}^{P_i}(q_i,q_{i+1})\left(\frac{s_i}{s_R}\right)^{\omega(t_i)}\frac{1}{t_i}\right)\frac{1}{t_{n+1}}\left(\frac{s_{n+1}}{s_R}\right)^{\omega(t_{n+1})}\Gamma_{\tilde{B}B}^{c_{n+1}}$$

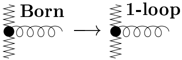
but, with respect to the LLA case, one replacement is allowed among the following:

multi-Regge kinematics (one α_s more)

- $\omega^{(1)} \longrightarrow \omega^{(2)}$
- $\Gamma_{P'P}^{c\,\text{(Born)}} \longrightarrow \Gamma_{P'P}^{c\,\text{(1-loop)}}$



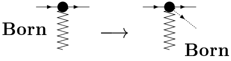
- $\gamma_{c_i c_{i+1}}^{G_i\text{(Born)}} \longrightarrow \gamma_{c_i c_{i+1}}^{G_i\text{(1-loop)}}$



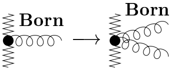
BFKL in the NLA

quasi-multi-Regge kinematics **(one** log s **less)**

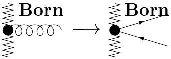
- $\Gamma^{c\,\textbf{(Born)}}_{P'P} \longrightarrow \Gamma^{c\,\textbf{(Born)}}_{\{f\}P}$



- $\gamma^{G_i\textbf{(Born)}}_{c_i c_{i+1}} \longrightarrow \gamma^{Q\overline{Q}\textbf{(Born)}}_{c_i c_{i+1}}$



- $\gamma^{G_i\textbf{(Born)}}_{c_i c_{i+1}} \longrightarrow \gamma^{GG\textbf{(Born)}}_{c_i c_{i+1}}$

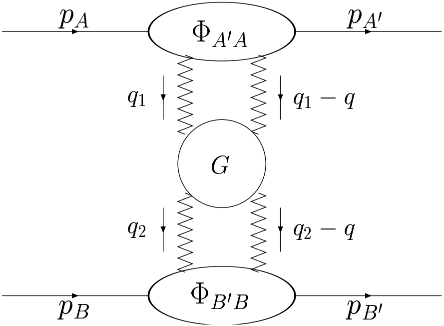


This is the program of calculation of radiative corrections to the LLA BFKL
[V.S. Fadin, L.N. Lipatov (1989)]

BFKL in the NLA

- $\omega^{(2)}(t)$ — **[V.S. Fadin (1995)] [V.S. Fadin, R. Fiore, M.I. Kotsky (1995)]**
  [V.S. Fadin, R. Fiore, A. Quartarolo (1996)]
  [V.S. Fadin, R. Fiore, M.I. Kotsky (1996)]
  [V.S. Fadin, M.I. Kotsky (1996)]
- $\gamma^{G_i}_{c_i c_{i+1}}$ **(1-loop)** — **[V.S. Fadin, L.N. Lipatov (1993)]**
  [V.S. Fadin, R. Fiore, A. Quartarolo (1994)]
  [V.S. Fadin, R. Fiore, M.I. Kotsky (1996)]
  [V.S. Fadin, R. Fiore, A. P. (2001)]
- $\Gamma^{c}_{P'P}$ **(1-loop)** — **[V.S. Fadin, R. Fiore (1992)] [V.S. Fadin, L.N. Lipatov (1993)]**
  [V.S. Fadin, R. Fiore, A. Quartarolo (1994)]
  [V.S. Fadin, R. Fiore, M.I. Kotsky (1995)]
- $\gamma^{Q\overline{Q}}_{c_i c_{i+1}}$ **(Born)** — **[V.S. Fadin, R. Fiore, A. Flachi, M.I. Kotsky (1997)]**
  [S. Catani, M. Ciafaloni, F. Hautmann (1990)]
  [G. Camici, M. Ciafaloni (1996)]
- $\gamma^{GG}_{c_i c_{i+1}}$**(Born)** — **[V.S. Fadin, L.N. Lipatov (1996)]**
  [V.S. Fadin, M.I. Kotsky, L.N. Lipatov (1997)]

Structure of the amplitude:



$$\mathbf{Im}_s(\mathcal{A}_\mathcal{R})_{AB}^{A'B'} = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}(\vec{q}_1-\vec{q})^2} \int \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}(\vec{q}_2-\vec{q})^2} \sum_\nu \Phi_{A'A}^{(\mathcal{R},\nu)}(\vec{q}_1;\vec{q};s_0)$$

$$\times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0} \right)^\omega G_\omega^{(\mathcal{R})}(\vec{q}_1,\vec{q}_2,\vec{q}) \right] \Phi_{B'B}^{(\mathcal{R},\nu)}(-\vec{q}_2;-\vec{q};s_0)$$

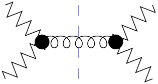
BFKL in the NLA

- $G_\omega^{(\mathcal{R})}$ – **Mellin transform of the Green's functions for Reggeon-Reggeon scattering**

$$\omega G_\omega^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_2,\vec{q}\right) = \vec{q}_1^{\,2}(\vec{q}_1-\vec{q}\,)^{\,2}\delta^{(D-2)}\left(\vec{q}_1-\vec{q}_2\right) + \int \frac{d^{D-2}q_r}{\vec{q}_r^{\,2}(\vec{q}_r-\vec{q}\,)^{\,2}}\mathcal{K}^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_r;\vec{q}\right) G_\omega^{(\mathcal{R})}\left(\vec{q}_r,\vec{q}_2;\vec{q}\right)$$

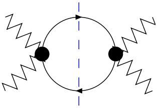
$$\mathcal{K}^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_2;\vec{q}\right) = \left[\omega\left(-\vec{q}_1^{\,2}\right)+\omega\left(-(\vec{q}_1-\vec{q}\,)^{\,2}\right)\right]\delta^{(D-2)}\left(\vec{q}_1-\vec{q}_2\right)+\mathcal{K}_r^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_2;\vec{q}\right)$$

In the NLA: $\omega(t)=\omega^{(1)}(t)+\omega^{(2)}(t)$, $\quad \mathcal{K}_r=\mathcal{K}_{RRG}^{(B)}+\mathcal{K}_{RRG}^{(1)}+\mathcal{K}_{RRQ\overline{Q}}^{(B)}+\mathcal{K}_{RRGG}^{(B)}$



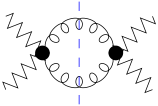
$\mathcal{K}_{RRG}^{(1)}$

$t=0$: [V.S. Fadin, L.N. Lipatov (1993)]
[V.S. Fadin, R. Fiore, A. Quartarolo (1994)]
[V.S. Fadin, R. Fiore, M.I. Kotsky (1996)]
$t\neq 0$: [V.S. Fadin, R. Fiore, A. P. (2001)]



$\mathcal{K}_{RRQ\overline{Q}}^{(B)}$

$t=0$: [V.S. Fadin, R. Fiore, A. Flachi, M.I. Kotsky (1997)]
$t\neq 0$: [V.S. Fadin, R. Fiore, A. P. (1999)]



$\mathcal{K}_{RRGG}^{(B)}$

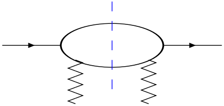
$t=0$: [V.S. Fadin, L.N. Lipatov, M.I. Kotsky (1997)]
$t\neq 0$: [V.S. Fadin, D.A. Gorbachev (2000)]
[V.S. Fadin, R. Fiore (2005)]

BFKL in the NLA

- $\Phi^{(\mathcal{R},\nu)}_{A'A}$ **– impact factors in the t-channel color state $(\mathcal{R},\nu)$**

$$\Phi_{A'A} = \sum_{\{f\}} \int \frac{d\kappa}{2\pi} d\rho_f \Gamma^{c}_{\{f\}A} (\Gamma^{c'}_{\{f\}A'})^{*}$$

$$\times \left(\frac{s_0}{\vec{q}_1^{\,2}} \right)^{\frac{\omega(-\vec{q}_1^{\,2})}{2}} \left(\frac{s_0}{(\vec{q}_1 - \vec{q})^2} \right)^{\frac{\omega(-(\vec{q}_1 - \vec{q})^2)}{2}}$$



$-$ counterterm

non-trivial momentum and scale-dependence

BFKL Pomeron in the NLA

Pomeron channel: $t = 0$ and singlet color representation in the t-channel

$$\left(\mathcal{K}(\vec{q}_1, \vec{q}_2) \equiv \frac{\mathcal{K}^{(0)}(\vec{q}_1, \vec{q}_2, 0)}{\vec{q}_1^{\,2} \vec{q}_2^{\,2}} \; , \; \gamma = \frac{1}{2} + i\nu \right)$$

$$\int d^{D-2} q_2 \mathcal{K}\left(\vec{q}_1, \vec{q}_2\right) \left(\vec{q}_2^{\,2}\right)^{\gamma-1} = \frac{N\alpha_s(\vec{q}_1^2)}{\pi} \left(\chi(\gamma) + \frac{N\alpha_s(\vec{q}_1^2)}{\pi} \chi^{(1)}(\gamma) \right) \left(\vec{q}_1^{\,2}\right)^{\gamma-1}$$

- **broken scale invariance**
- **large corrections:** $-\left.\frac{\chi^{(1)}(\gamma)}{\chi(\gamma)}\right|_{\gamma=1/2} \simeq 6.46 + 0.05\frac{n_f}{N} + 0.96\frac{n_f}{N^3}$

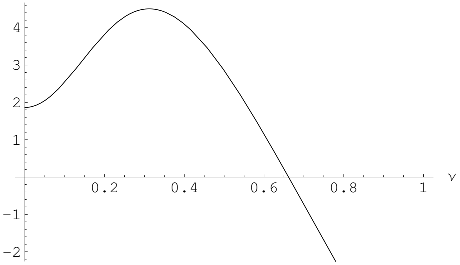
  [V.S. Fadin, L.N. Lipatov (1998)] [G. Camici, M. Ciafaloni (1998)]

BFKL Pomeron in the NLA

$\chi(\nu) + \bar{\alpha}_s(\vec{q}_1^{\,2})\chi^{(1)}(\nu)$ *vs* ν

$$\bar{\alpha}_s(\vec{q}_1^{\,2}) \equiv \frac{\alpha_s(\vec{q}_1^{\,2})N}{\pi} = 0.15$$

(omitted terms with the first derivative)



Double maxima $\longrightarrow$ oscillations in momentum space after ν-integration

Ways out:

- **rapidity veto** **[C.R. Schmidt (1999)]** **[J.R. Forshaw, D.A. Ross, A. Sabio Vera (1999)]**
- **collinear improvement** **[G. Salam (1998)] [M. Ciafaloni, D. Colferai (1999)]**
- **renormalization with a physical scheme** **[S.J. Brodsky, V.S. Fadin, V.T. Kim, L.N. Lipatov, G.B. Pivovarov (1999)]**
- **...**

Outline

Other BFKL topics

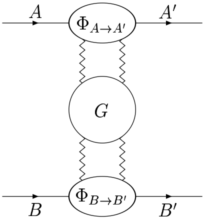
- Reggeization in other color channels [I.P. Ivanov (2005)]
- "diffusion" in the infrared
- unitarization problem
  - non-linear generalization of the BFKL equation, based on the idea of saturation of parton densities
    [L.V. Gribov, E.M. Levin, M.G. Ryskin (1983)]
    [I. Balitsky (1996)] [Yu. Kovchegov (1999)]
  - gauge-invariant effective theory for Reggeized gluon interactions
    [L.N. Lipatov (1995)]
- BFKL in coordinate representation and conformal invariance
  [V.S. Fadin, R. Fiore, A.P. (2007,2012)]
  [V.S. Fadin, R. Fiore, A.V. Grabovsky, A.P. (2007)]
  [V.S. Fadin, R. Fiore, A.V. Grabovsky, A.P. (2011)]
- BFKL in $N = 4$ SUSY [A.V. Kotikov, L.N. Lipatov (2000) and many others]

Outline

General scheme

Scattering $A+B \longrightarrow A'+B'$ in the Regge kinematical region $s\to\infty$, t fixed

BFKL approach: convolution of the Green's function of two interacting Reggeized gluons and of the impact factors of the colliding particles, valid both in the LLA and in the NLA.

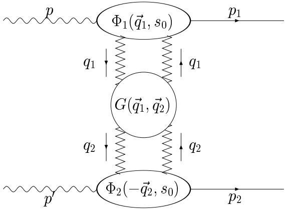


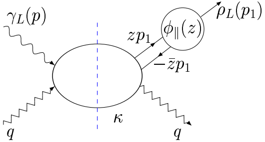
- **The BFKL Green's function is universal and takes care of the energy dependence**
- **Impact factors are process-dependent and depend on the hard scale, but not on the energy**

The list of processes which can be studied within NLA BFKL depends on the list of available NLO impact factors calculated so far.

Outline

$\gamma^*\gamma^* \to VV$, $V = \rho, \omega, \phi$





[D.Yu. Ivanov, M.I. Kotsky, A.P. (2004)]

Hard scales: photon virtualities, $Q_{1,2}$

- **Longitudinally polarized vector mesons are produced by longitudinally polarized photons; other helicity amplitudes power suppressed by $\sim m_\rho / Q_{1,2}$;**
- **forward scattering, i.e. zero transverse momenta of the produced mesons**

$$\frac{Q_1 Q_2}{D_1 D_2} \frac{\mathcal{I}m_s \mathcal{A}}{s} = \frac{1}{(2\pi)^2} \alpha_s(\mu_R)^2 \left[b_0 + \sum_{n=1}^{\infty} \bar{\alpha}_s(\mu_R)^n \, b_n \left(\ln\left(\frac{s}{s_0}\right)^n + d_n(s_0, \mu_R) \ln\left(\frac{s}{s_0}\right)^{n-1} \right) \right]$$

$$D_{1,2} = -\frac{4\pi e_q f_V}{N_c Q_{1,2}} \sqrt{N_c^2 - 1}\,, \qquad e_q \longrightarrow \frac{e}{\sqrt{2}}\,, \quad \frac{e}{3\sqrt{2}}\,, \quad -\frac{e}{3}$$

$$\rho^0\,, \qquad \omega\,, \qquad \phi$$

$Q_1 = Q_2 \equiv Q$ **"pure" BFKL regime** **[D.Yu. Ivanov, A.P. (2006)]**

LLA: b_n coefficients (Q-independent)

$$b_0 = 17.0664 \quad b_1 = 34.5920 \quad b_2 = 40.7609 \quad b_3 = 33.0618 \quad b_4 = 20.7467$$
$$b_5 = 10.5698 \quad b_6 = 4.54792 \quad b_7 = 1.69128 \quad b_8 = 0.554475$$

NLA: $d_n(s_0, \mu_R)$ coefficients ($s_0 = Q^2 = \mu_R^2$, $n_f = 5$)

$$d_1 = -3.71087 \quad d_2 = -11.3057 \quad d_3 = -23.3879 \quad d_4 = -39.1123$$
$$d_5 = -59.207 \quad d_6 = -83.0365 \quad d_7 = -111.151 \quad d_8 = -143.06$$

NLA: $d_n^{\rm imp}(s_0, \mu_R)$ coefficients ($s_0 = Q^2 = \mu_R^2$, impact factor contribution)

$$d_1^{\rm imp} = -3.71087 \quad d_2^{\rm imp} = -8.4361 \quad d_3^{\rm imp} = -13.1984 \quad d_4^{\rm imp} = -18.0971$$
$$d_5^{\rm imp} = -23.0235 \quad d_6^{\rm imp} = -27.9877 \quad d_7^{\rm imp} = -32.9676 \quad d_8^{\rm imp} = -37.9618$$

Large NLA corrections!

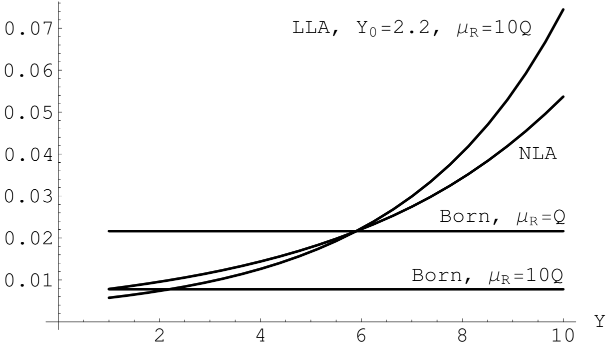
d_n coefficients negative and increasingly large in absolute value.
The contribution from the kernel dominates only for $n \geq 4$.

Optimization of the perturbative expansion needed!
Principle of minimal sensitivity (PMS) [P.M. Stevenson (1981)]: require the minimal sensitivity to the change of both s_0 and μ_R.

$\frac{Q^2}{D_1 D_2} \frac{\mathcal{I}m_s \mathcal{A}}{s}$ vs Y

$Y \equiv \log s/Q$
$Y_0 \equiv \log s_0/Q$

Q^2**=24 GeV**^{2}**,** $n_f = 5$



Lessons

- **The Born approximation does not give necessarily the estimate from below.**
- **The optimal values for** μ_R **are “unnaturally” larger than** Q **(new scale or nature of the BFKL series?).**

$\longrightarrow$ **Since NLA corrections are large and since the exact amplitude should be renorm- and energy scale invariant, the NNLA terms should be large and of the opposite sign with respect to the NLA.**
If the NNLA corrections were known and we would apply PMS to the NNLA amplitude, we would obtain more natural values of μ_R**.**

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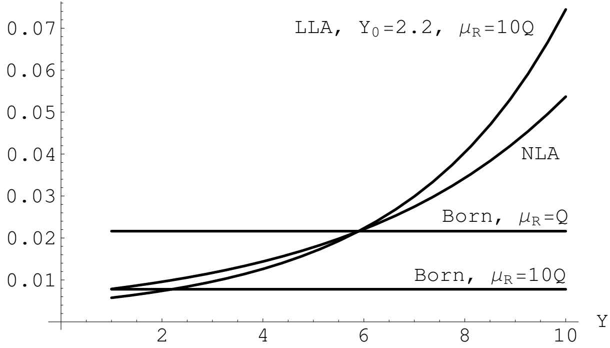

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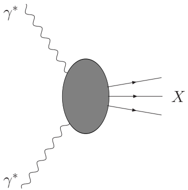
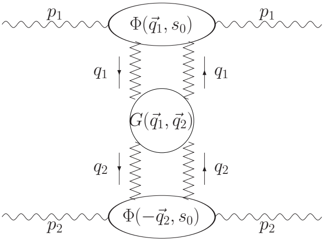
$\gamma^*\gamma^* \to$ hadrons





Hard scales: photon virtualities, $Q_{1,2}$

The NLO impact factor, a long saga ...
[J. Bartels, S. Gieseke, C.F. Qiao (2001)]
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[V.S. Fadin, D.Yu. Ivanov, M.I. Kotsky (2003)]
[J. Bartels, A. Kyrieleis (2004)]

... till the recent breakthrough
[I. Balitsky, G.A. Chirilli (2013)]
[G.A. Chirilli, Yu.V. Kovchegov (2014)]

Compatibility between the different approaches not yet established.

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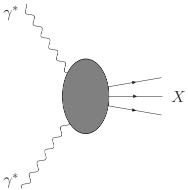

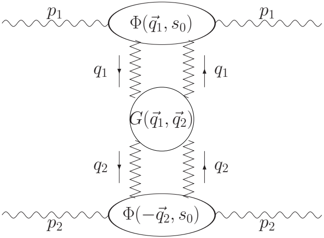

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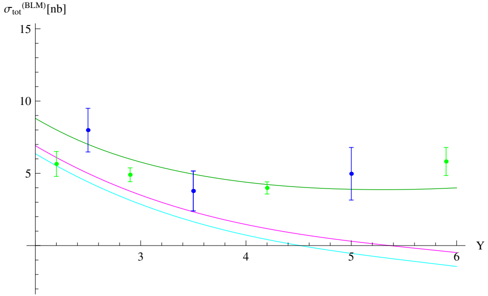
$Q_1^2 = Q_2^2 \equiv Q^2$=17 GeV^2

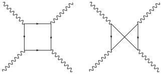
$Y \equiv \log s/Q$

- **OPAL** (Q^2=18 GeV^2)
- **L3** (Q^2=16 GeV^2)



Green: NLA Green's function + LO impact factors (PMS scale setting)
[F. Caporale, D.Yu. Ivanov, A.P. (2008)]

Cyan/Magenta: full NLA (BLM scale) **[D.Yu. Ivanov, B. Murdaca, A.P. (2014)]**

- **quark box included** 
- **[S.J. Brodsky, G.P. LePage, P.B. MacKenzie (1983)] (BLM) scale setting: μ_R set at the value that makes the β_0-dependent terms in the amplitude vanish.**

$\longrightarrow$ **High-energy linear colliders needed!**

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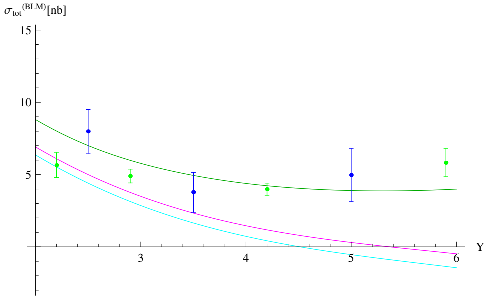

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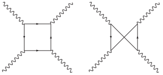
- **quark box included** 
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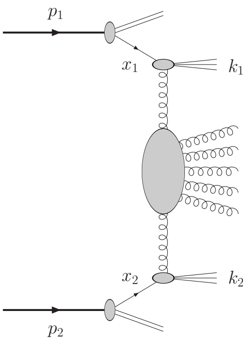
$\longrightarrow$ **High-energy linear colliders needed!**

Outline

Mueller-Navelet jets

$$\text{proton}(p_1) + \text{proton}(p_2) \to \text{jet}_1(k_1) + \text{jet}_2(k_2) + X$$

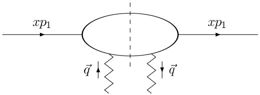


- **large jet transverse momenta (hard scales):** $\vec{k}_1^{\,2} \sim \vec{k}_2^{\,2} \gg \Lambda^2_{\rm QCD}$
- **large rapidity gap between jets,** $\Delta y \equiv Y = y_{J_1} - y_{J_2}$**,**
  which requires large c.m. energy of the proton collisions, $s = 2p_1 \cdot p_2 \gg \vec{k}_{1,2}^{\,2}$

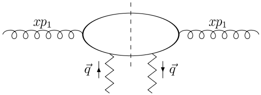
[A.H. Mueller, H. Navelet (1987)]

- **Step 0**: take the impact factors for colliding partons

[V.S. Fadin, R. Fiore, M.I. Kotsky, A.P. (2000)]
[M. Ciafaloni and G. Rodrigo (2000)]

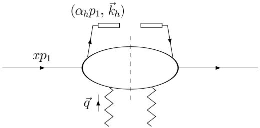


quark impact factor

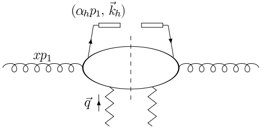


gluon impact factor

- **Step 1**: "open" one of the integrations over the phase space of the intermediate state to "allow" one parton to generate the jet



quark jet vertex

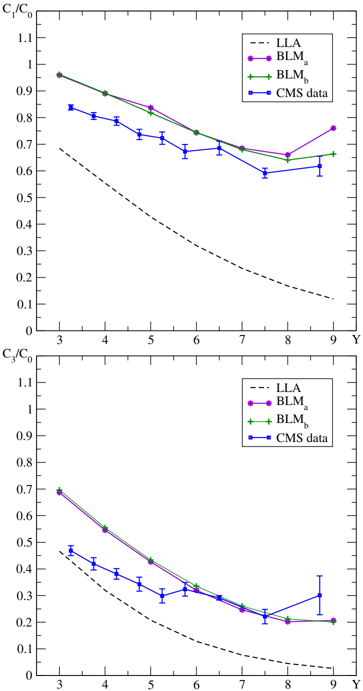


gluon jet vertex

- **Step 2**: take the convolution with leading-twist PDFs

$$\sum_{a=q,\bar{q}} f_a \otimes (\text{quark jet vertex}) \quad + \quad f_g \otimes (\text{gluon jet vertex})$$

[J. Bartels, D. Colferai, G.P. Vacca (2003)]
[F. Caporale, D.Yu. Ivanov, B. Murdaca, A.P., A. Perri (2011)]
[D.Yu. Ivanov, A.P. (2012)] (small-cone approximation)

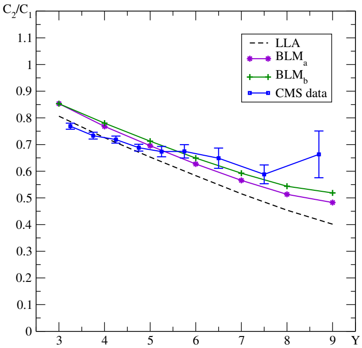


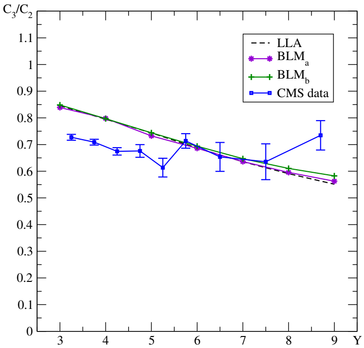
$$C_n/C_0 = \langle \cos[n(\phi_{J_1} - \phi_{J_2} - \pi)] \rangle$$
$$vs\ Y = y_{J_1} - y_{J_2}$$

small-cone approximation
BLM scale setting

□ **CMS (7 TeV)**

[F. Caporale, D.Yu. Ivanov, B. Murdaca, A.P. (2014)]





$$C_n/C_m = \frac{\langle \cos(n\phi)\rangle}{\langle \cos(m\phi)\rangle} \; vs \; Y = y_{J_1} - y_{J_2}$$

[A. Sabio Vera (2006)]
[A. Sabio Vera, F. Schwennsen (2007)]

small-cone approximation
BLM scale setting

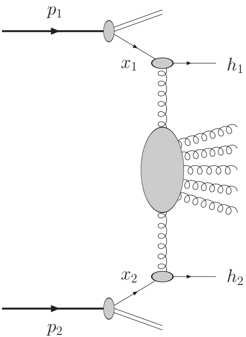
□ **CMS (7 TeV)**

Similar results obtained with the exact jet vertices
[B. Ducloué, L. Szymanowski, S. Wallon (2013,2014)]

Discrimination between BFKL and fixed-order (DGLAP) in progress
[F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A.P. (in progress)]

Di-hadrons with "Mueller-Navelet" kinematics

$$\text{proton}(p_1) + \text{proton}(p_2) \rightarrow \text{hadron}_1(k_1) + \text{hadron}_2(k_2) + X$$



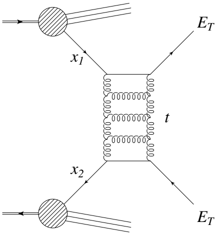
- **large hadron transverse momenta (hard scales):** $\vec{k}_1^2 \sim \vec{k}_2^2 \gg \Lambda^2_{\rm QCD}$
- **large rapidity gap between the identified hadrons,** $\Delta y \equiv Y = y_{h_1} - y_{h_2}$
- **The relevant NLO impact factor is known** **[D.Yu. Ivanov, A.P. (2012)]**
- **Same observables as for Mueller-Navelet jets**
  [F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A.P. (in progress)]

Other processes

- **Mueller-Tang jets**

$$\text{proton} + \text{proton} \to \text{jet}_1 + \text{jet}_2 + \text{gap}$$

  - **Test of BFKL at $t \neq 0$**
  - **The relevant NLO impact factor is known [M. Hentschinski, J.D. Madrigal Martinez, B. Murdaca, A. Sabio Vera (2014)]**
  - **Non-perturbative gap survival probability to be accounted for**



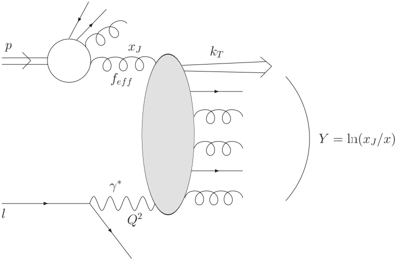
[from S. Wallon, arXiv:0710.0833]

- **Inclusive central + forward jet production, Mueller-Navelet + central jet production,** ***etc.***

Outline

Forward jets in DIS

$$\gamma^* + \text{proton} \to \text{jet} + \text{X}$$



[from O. Kepka *et al.*, EPJ C55 (2008) 259]

- **large photon virtuality and jet transverse momentum (hard scales)**
- **large** $Y \equiv \log\left(\frac{x_J}{x_B}\right)$
- **Past analyses based on LO forward jet vertex and LO photon impact factor agree with HERA data**
  [O. Kepka, C. Royon, C. Marquet, R.B. Peschanski (2008)]
- **Inclusion of NLO corrections in the forward jet impact factor in progress**
  [F. Caporale, F.G. Celiberto, D.Yu. Ivanov, B. Murdaca, A.P. (in progress)]
- **Similar process: identified hadron instead of the jet**

Conclusions

- **The BFKL approach gives a common basis for the description of semi-hard processes; it is based on a remarkable property of perturbative QCD, the gluon Reggeization.**
- **Physical amplitudes in NLA are written in terms of a universal Green's function and of process-dependent impact factors of the colliding particles.**
- **Several impact factors have been calculated with NLO accuracy, thus allowing for tests of the NLA BFKL approach in several processes.**
- **The advent of LHC has allowed for some successful tests of NLA BFKL in the jet sector; previously, also HERA had shown compatibility with partially-NLA BFKL approaches. High-energy linear colliders could consolidate these results and open the way to further tests.**

BACKUP

NLO BFKL kernel "eigenvalues"

Pomeron channel: $t = 0$ **and singlet color representation in the** t**-channel**

$$\left(\mathcal{K}(\vec{q}_1, \vec{q}_2) \equiv \frac{\mathcal{K}^{(0)}(\vec{q}_1, \vec{q}_2, 0)}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}} \;,\; \gamma = \frac{1}{2} + i\nu\right)$$

$$\int d^{D-2}q_2 \mathcal{K}\left(\vec{q}_1, \vec{q}_2\right)\left(\vec{q}_2^{\,2}\right)^{\gamma-1} = \frac{N\alpha_s(\vec{q}_1^2)}{\pi}\left(\chi(\gamma) + \frac{N\alpha_s(\vec{q}_1^2)}{\pi}\chi^{(1)}(\gamma)\right)\left(\vec{q}_1^{\,2}\right)^{\gamma-1}$$

$$\chi^{(1)}(\nu) = -\frac{\beta_0}{8\,N_c}\left(\chi^2(\nu) - \frac{10}{3}\chi(\nu) - i\chi'(\nu)\right) + \bar{\chi}(\nu)\;, \qquad \beta_0 = \frac{11\,N_c}{3} - \frac{2n_f}{3}$$

$$\begin{aligned}\bar{\chi}(\nu) \quad &= \quad -\frac{1}{4}\left[\frac{\pi^2-4}{3}\chi(\nu) - 6\zeta(3) - \chi''(\nu) - \frac{\pi^3}{\cosh(\pi\nu)}\right.\\ &+ \quad \left.\frac{\pi^2\sinh(\pi\nu)}{2\nu\cosh^2(\pi\nu)}\left(3 + \left(1 + \frac{n_f}{N_c^3}\right)\frac{11+12\nu^2}{16(1+\nu^2)}\right) + 4\phi(\nu)\right]\end{aligned}$$

$$\phi(\nu) = 2\int_0^1 dx\, \frac{\cos(\nu\ln(x))}{(1+x)\sqrt{x}}\left[\frac{\pi^2}{6} - \mathrm{Li}_2(x)\right]$$