Thermal Sommerfeld effect of P-wave quarkonium in lattice NRQCD

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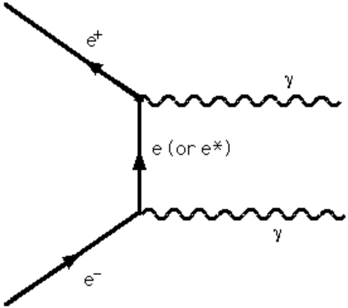
(work in progress)

Outline

1. Motivation
2. Method
3. Result/Discussion

Sommerfeld Effect

- annihilation cross-section

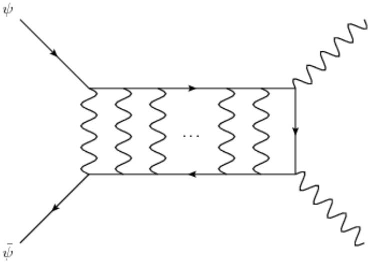


Sommerfeld Effect

- enhancement of annihilation for slowly moving particle :

$$\sim \frac{\alpha}{v}$$

per one rung in a ladder diagram, large for small v
and need to be resummed



Sommerfeld Effect

- Sommerfeld effect enhances the Born matrix elements

$$|\mathcal{M}_{\text{resummed}}|^2 = \mathcal{S}|\mathcal{M}_{\text{tree}}|^2$$

for the color singlet

$$\mathcal{S}_1 = \frac{X_1}{1-e^{-X_1}}, X_1 = C_F\frac{g^2}{4v}$$

and for the color octet

$$\mathcal{S}_8 = \frac{X_8}{e^{X_8}-1}, X_8 = (\frac{N_c}{2} - C_F)\frac{g^2}{4v}$$

- QCD is non-perturbative and decay of quarkonium state is described as (NRQCD factorization, G.T. Bodwin et al, PRD51 (1995) 1125)

$$\Gamma = \sum_i |\langle \mathcal{O}_i \rangle|^2 \Gamma^i_{\text{partonic}}$$

Thermal Sommerfeld Effect in QGP

• Lee-Weinberg equation (B.W. Lee and S. Weinberg, PRL39 (1977) 165)

$$\partial_t n = -\langle \sigma_{\rm eff} v \rangle (n^2 - n_{eq}^2)$$

• chemical equilibriation rate of heavy quark in Quark-Gluon Plasma (QGP)

• relic density of dark matter particle in early universe

• linearizing Lee-Weinberg equation (SK and M. Laine, JHEP 1607 (2016) 143)

$$\langle \sigma_{\rm eff} v \rangle \equiv \frac{\Gamma_{\rm chem}}{2n_{eq}}$$

How to calculate: Usual way

- consider Boltzmann equation*

$$\partial_t n \simeq -c(n^2 - n_{eq}^2) = \dot{n}_{\rm loss} + \dot{n}_{\rm gain}$$

with $\dot{n}_{\rm loss} = -cn^2$

- in equilibrium, $n(t) = n_{\rm eq}$ and $\delta\dot{n} \simeq -2cn\delta n$

$$\Gamma_{\rm chem} = \frac{\delta\dot{n}}{n}\Big|_{\rm eq} = -2\frac{\dot{n}_{\rm loss}}{n_{eq}}$$

- then perturbatively

$$\Gamma_{\rm chem} = \frac{2}{2N_c \int_{\mathbf{k}} f_F(E_k)} \int\int (2\pi)^4 \delta^4(P_1 + P_2 - K_1 - K_2) f_F(E_{k_1}) f_F(E_{k_2})$$

$$\left(\frac{1}{2}\sum |M_1|^2 [1 + f_B(\varepsilon_{p_1})][1 + f_B(\varepsilon_{p_2})] + N_f \sum |M_2|^2 [1 - f_F(\varepsilon_{p_1})][1 - f_F(\varepsilon_{p_2})]\right)$$

$\Gamma_{\rm chem}$ as a transport coefficient

• chemical equilibriation as a transport coefficient (D. Bödeker, M. Laine, JHEP07 (2012) 130, 01 (2013) 037)

• treat the approach to the equilibrium as a Langevin process

$$\delta\dot{n}(t) = -\Gamma_{\rm chem}\delta n(t) + \xi(t)$$

$$\langle\langle\xi(t)\xi(t')\rangle\rangle = \Omega_{\rm chem}\delta(t-t'), \qquad \langle\langle\xi(t)\rangle\rangle = 0$$

where $\delta n(t)$ is the deviation from the equilibrium and $\xi(t)$ is a stochastic noise

Γ_{chem} as a transport coefficient

- to access Ω_{chem}, consider correlators (for heavy quark, or non-relativistic QCD, the heavy quark number is replaced by hamiltonian) in imaginary time

$$\Omega(\tau) = \frac{1}{V}\langle \partial_t H(\tau) \partial_t H(0) \rangle_{\text{qm}}$$

or

$$\Delta(\tau) = \frac{1}{V}\langle H(\tau) H(0) \rangle_{\text{qm}}$$

then

$$\Omega_{\text{chem}} = \lim_{\Gamma_{\text{chem}} \ll \omega \ll \omega_{UV}} 2T \frac{\rho_\Omega(\omega)}{\omega} \quad \text{or} \quad = \lim_{\omega \ll T} 2T \omega \rho_\Delta(\omega)$$

and

$$\Gamma_{\text{chem}} = \frac{\lim_{\omega \to 0^+} 2T \frac{\rho_\Omega(\omega)}{\omega}}{2\chi_f M^2} \quad \text{or} \quad = \frac{\lim_{\omega \ll T} 2T \omega \rho_\Delta(\omega)}{2\chi_f M^2}$$

$\Gamma_{\rm chem}$ as a transport coefficient

- consider

$$
\begin{aligned}
&\int_{\vec{x},\vec{y}} \langle H(\tau,\vec{x})\, H(0,\vec{y})\, (\psi^\dagger\chi)(\tau_1,\vec{0})\, (\chi^\dagger\psi)(\tau_2,\vec{0})\rangle\Big|_{\tau_1>\tau_2} \left[\theta(\tau-\tau_1)+\theta(\tau_2-\tau)\right] \\
= \;& \frac{1}{\mathcal{Z}} \mathrm{Tr}\Big[e^{-\beta\hat{\mathcal{H}}} (\hat\psi^\dagger\hat\chi)(\tau_1,\vec{0})\, (\hat\chi^\dagger\hat\psi)(\tau_2,\vec{0}) \int_{\vec{x},\vec{y}} \hat{H}(0,\vec{x})\hat{H}(0,\vec{y})\Big] \\
= \;& \frac{1}{\mathcal{Z}} \sum_{m,n} \langle q\bar{q},m| e^{-\beta\hat{\mathcal{H}}} (\hat\psi^\dagger\hat\chi)(\tau_1,\vec{0})\,|n\rangle\, \langle n|(\hat\chi^\dagger\hat\psi)(\tau_2,\vec{0}) \int_{\vec{x},\vec{y}} \hat{H}(0,\vec{x})\hat{H}(0,\vec{y})|
\\
= \;& \frac{4M^2}{\mathcal{Z}} \sum_{m,n} e^{-\beta E_m} e^{(\tau_1-\tau_2)(E_m-\varepsilon_n)}\, \langle q\bar{q},m|\hat\psi^\dagger\hat\chi|n\rangle \langle n|\hat\chi^\dagger\hat\psi|q\bar{q},m\rangle
\end{aligned}
$$

E_m are the states with heavy quarks, ε_n are states without heavy quarks

$\Gamma_{\rm chem}$ as a transport coefficient

- with $\mathcal{C}_{mn} = \frac{4M^2}{\mathcal{Z}} e^{-\beta E_m} \langle q\bar{q}, m|\hat{\psi}^\dagger \hat{\chi}|n\rangle \langle n|\hat{\chi}^\dagger \hat{\psi}|q\bar{q}, m\rangle$

$$\varepsilon(\tau) = \sum_{m,n} \mathcal{C}_{mn} \int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \frac{e^{(\tau_1-\tau_2)(E_m-\varepsilon_m)}}{\tau_1-\tau_2} \left[\theta(\tau-\tau_1)+\theta(\tau_2-\tau)\right]$$

- then with $\omega, \varepsilon_m << E_m$

$$\rho_\Delta(\omega) = \frac{\mathrm{Im} f_1({}^1S_0)}{M^2} \sum_{m,n} \mathcal{C}_{mn} \frac{e^{\beta\omega} - e^{-\beta\omega}}{\omega^2}$$

$\Gamma_{\rm chem}$ as a transport coefficient

- finally,

$$
\begin{aligned}
\Omega_{\rm chem} &= \frac{4\mathrm{Im}f_1({}^1S_0)}{M^2}\sum_{m,n}\mathcal{C}_{mn} \\
&= 16\,\mathrm{Im}f_1({}^1S_0)\,\frac{1}{\mathcal{Z}}\sum_{m,n}e^{-\beta E_m}\,\langle q\bar{q},m|\hat{\psi}^\dagger\hat{\chi}|n\rangle\langle n|\hat{\chi}^\dagger\hat{\psi}|q\bar{q},m\rangle \\
&= 16\,\mathrm{Im}f_1({}^1S_0)\,\frac{1}{\mathcal{Z}}\,\mathrm{Tr}\left[e^{-\beta\hat{\mathcal{H}}}(\hat{\psi}^\dagger\hat{\chi})(0^+,\vec{0})\,(\hat{\chi}^\dagger\hat{\psi})(0,\vec{0})\right] \\
&= 16\,\mathrm{Im}f_1({}^1S_0)\,\big\langle(\psi^\dagger\chi)(0^+,\vec{0})\,(\chi^\dagger\psi)(0,\vec{0})\big\rangle
\end{aligned}
$$

$\Gamma_{\rm chem}$ as a transport coefficient

$$
\begin{aligned}
P_1 &\equiv \frac{1}{2N_c}\mathrm{Re}\left\langle G^{\theta}_{\alpha\alpha;ii}(\beta,\vec{0};0,\vec{0})\right\rangle ,\\
P_2 &\equiv \frac{1}{2N_c}\left\langle G^{\theta}_{\alpha\gamma;ij}(\beta,\vec{0};0,\vec{0})G^{\theta\dagger}_{\gamma\alpha;ji}(\beta,\vec{0};0,\vec{0})\right\rangle ,\\
P_3 &\equiv \frac{1}{2N_c^2}\left\langle G^{\theta}_{\alpha\alpha;ij}(\beta,\vec{0};0,\vec{0})G^{\theta\dagger}_{\gamma\gamma;ji}(\beta,\vec{0};0,\vec{0})\right\rangle .
\end{aligned}
$$

- singlet Sommerefeld factor

$$
\bar{S}_1 = \frac{P_2}{P_1^2} .
$$

- octet Sommerefeld factor

$$
\bar{S}_8 = \frac{N_c^2 P_3 - P_2}{(N_c^2-1)P_1^2} .
$$

Γ_{chem} as a transport coefficient

- P-wave Sommerfeld factor

$$\bar{S}_p = \frac{P_p}{M^2 P_1^2}$$

with

$$p_p = \mathrm{Tr}\langle \Delta_i G_V(\beta,\vec{0};0,\vec{0};i) G^\dagger(\beta,\vec{0};0,\vec{0})\rangle - \mathrm{Tr}\langle G_V(\beta,\vec{0};0,\vec{0};i) \Delta_i G^\dagger(\beta,\vec{0};0,\vec{0})\rangle$$

- P-wave state may have a better signal-to-noise ratio in an experimental situation or a better detection strategy

- bound states are important in thermal Sommerfeld effect and P-wave has lower melting temperature

Lattice setup

• anisotropic Euclidean lattices (i.e., the time direction lattice spacing is different from the space direction lattice spacing, $a_s/a_t = 3.5$), $a_s = 0.1227(8)$ fm

• $N_f = 2+1$ light quark flavors ($M_\pi \simeq 400$ MeV, $M_K \simeq 500$ MeV)

• $24^3 \times N_t$ lattices

• T_c = 185 MeV, $a_s M = 2.92$ (bottom) and 1.5 ($\sim$ charm)

• lattices used for bottomonium at $T \neq 0$ study (G. Aarts et al, JHEP07 (2014) 097) and electric conductivity of QGP (G. Aarts et al, JHEP-2 (2015) 186)

Lattice setup

$$\begin{aligned}
G^{\theta}(0,\mathbf{x};\cdot) &= \frac{\delta_{\mathbf{x},\vec{0}}}{a_s^3}\,, \\
G^{\theta}(a_t,\mathbf{x};\cdot) &= \left(1-\frac{a_t\mathcal{H}_0}{2n}\right)^n U_0^{\dagger}(0,\mathbf{x})\left(1-\frac{a_t\mathcal{H}_0}{2n}\right)^n G^{\theta}(0,\mathbf{x};\cdot)\,, \\
G^{\theta}(\tau+a_t,\mathbf{x};\cdot) &= \left(1-\frac{a_t\mathcal{H}_0}{2n}\right)^n U_0^{\dagger}(\tau,\mathbf{x})\left(1-\frac{a_t\mathcal{H}_0}{2n}\right)^n \left(1-a_t\,\delta\mathcal{H}\right) G^{\theta}(\tau,\mathbf{x}
\end{aligned}$$

where U_0 is a time-direction gauge link. The lowest-order Hamiltonian reads

$$\mathcal{H}_0 = -\frac{\Delta^{(2)}}{2M}\,,$$

where $\Delta^{(2)}$ is a discretized gauge Laplacian.

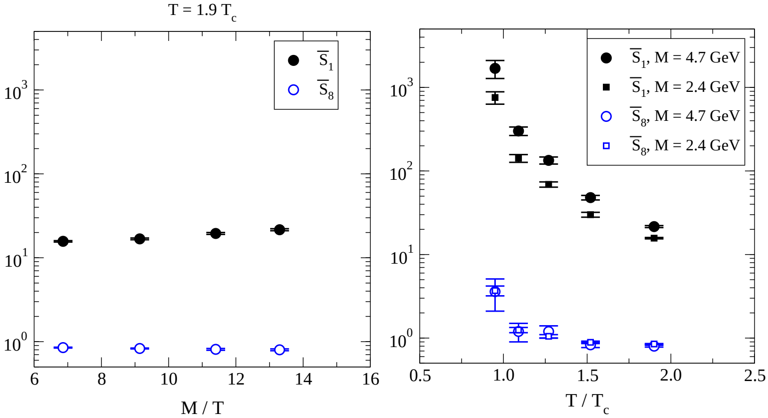
Lattice setup

The higher order correction is

$$\begin{aligned}\delta\mathcal{H} &= -\frac{(\Delta^{(2)})^2}{8M^3} + \frac{ig_0\,(\nabla\cdot\mathbf{E}-\mathbf{E}\cdot\nabla)}{8M^2} - \frac{g_0\,\sigma\cdot(\nabla\times\mathbf{E}-\mathbf{E}\times\nabla)}{8M^2} \\ &\quad -\frac{g_0\,\sigma\cdot\mathbf{B}}{2M} + \frac{a_s^2\Delta^{(4)}}{24M} - \frac{a_t(\Delta^{(2)})^2}{16nM^2}\,,\end{aligned}$$

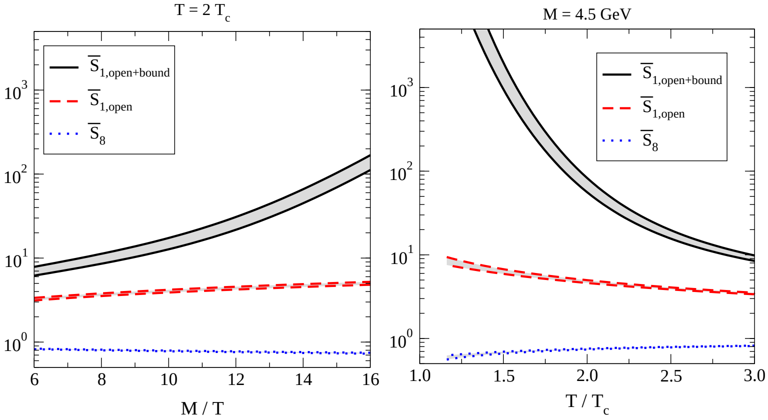
Lattice result of thermal Sommerfeld factor (S-Wave)

- SK and M. Laine, JHEP1607 (2016) 143

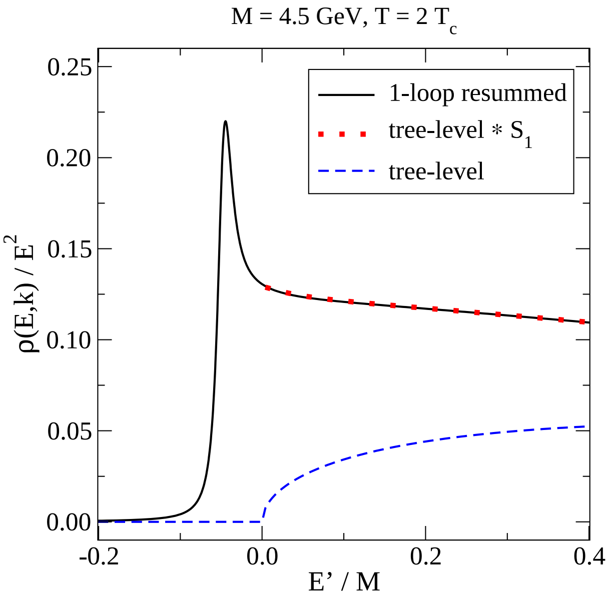


Analytic estimate of thermal Sommerfeld factor (S-wave)

- SK and M. Laine, JHEP1607 (2016) 143



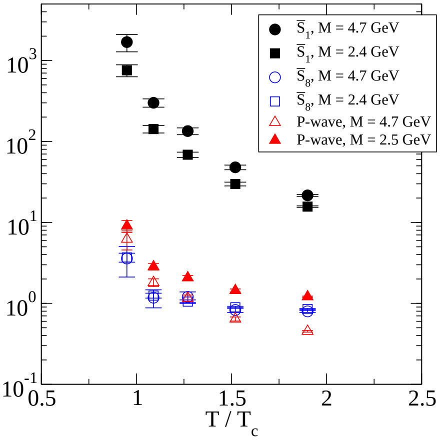
Analytic estimate of thermal Sommerfeld factor (S-wave)



Lattice result of thermal Sommerfeld factor (S & P-Wave)

N_t	T/T_c	$a_s M$	M/T	$\bar{S}_1$	$\bar{S}_P$
32	0.95	2.92	26.7	1690(410)	6.2(1.6)
28	1.09	2.92	23.4	301(35)	1.8(2)
24	1.27	2.92	20.0	134(13)	1.2(1)
20	1.52	2.92	16.7	48(3)	0.64(4)
16	1.90	2.92	13.3	21.6(6)	0.45(1)
32	0.95	1.50	13.7	758(128)	9.0(1.5)
28	1.09	1.50	12.0	142(15)	2.8(3)
24	1.27	1.50	10.3	69(5)	2.1(1)
20	1.52	1.50	8.57	30(2)	1.44(6)
16	1.90	1.50	6.86	15.7(3)	1.20(2)

Lattice result of thermal Sommerfeld factor (S & P-Wave)



Discussion

• a real time quantity, chemical equilibriation rate, is calculated non-perturbatively using Euclidean lattice without analytic continuation for the first time.

• in the current relativistic heavy ion collision experiments, charm quark and bottom quarks do not chemically equilibriate.

• thermal Sommerfeld effect for S- & P-wave bottomonium annihilation is calculated using lattice NRQCD.

• bound state is the cause of large enhancement for S-wave thermal Sommerfeld factor. for P-wave, we expect quarkonium melting within the investigated temperature range