Theoretical uncertainties in the determination of the top mass

Gennaro Corcella

INFN - Laboratori Nazionali di Frascati

1. **Introduction**
2. **Top mass definitions**
3. **Top mass reconstruction and theory/Monte Carlo errors**
4. **Case study: bottom-fragmentation in top decays**
5. **Top mass from simulation of fictitious top hadrons**
6. **Perspectives for future LHC runs**
7. **Conclusions**

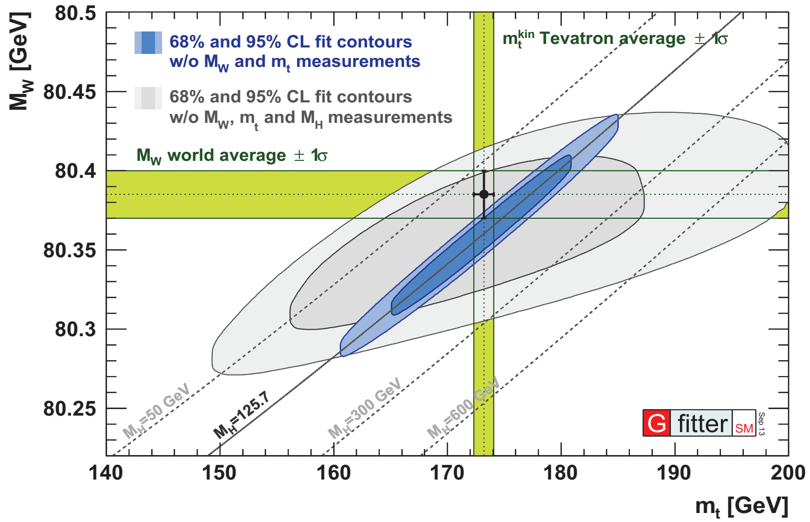
Based on work by G.C., F.Mescia, V.Drollinger, A.Mitov, M.Czakon, S.Frixione, A.Hoang, TeV+LHC top working groups

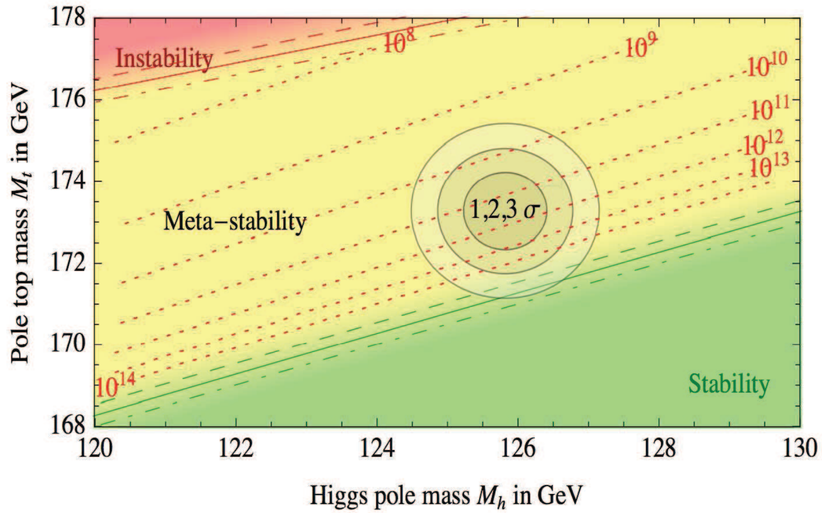
Work in progress with M.L. Mangano (t-flavoured hadrons)

Top physics among the main fields of investigation in theoretical and experimental particle physics

It is the heaviest elementary particle and its mass plays a crucial role in the electroweak symmetry breaking, constraining the Higgs mass even before its discovery

Stability of the SM vacuum depends on top and Higgs masses (Degrassi et al, JHEP'12)

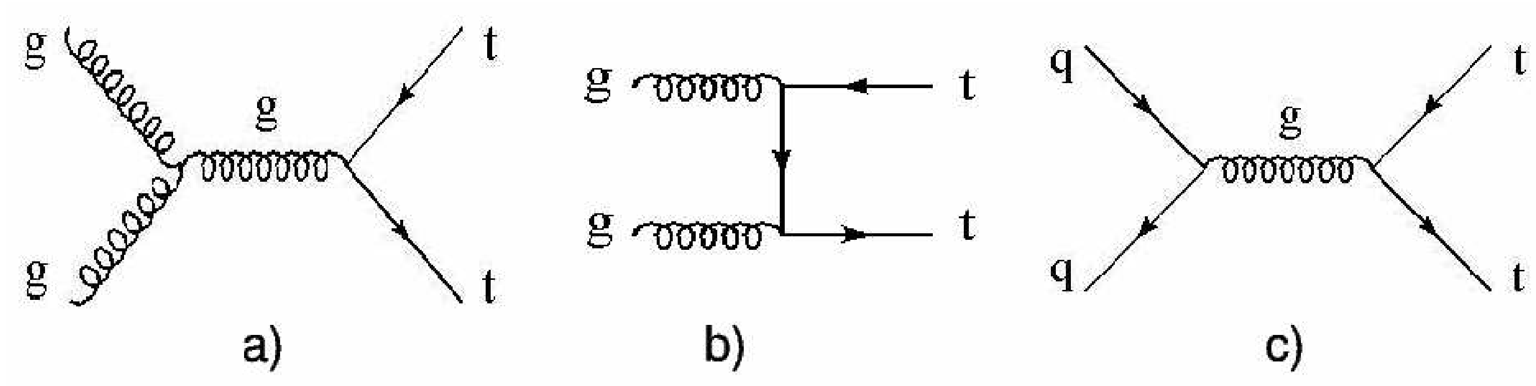




Latest world average (Tevatron and LHC):
$m_t^{\mathrm{TeV+LHC}} = [173.34 \pm 0.27(\mathrm{stat}) \pm 0.71(\mathrm{syst})]$ GeV (arXiv:1403.4427)

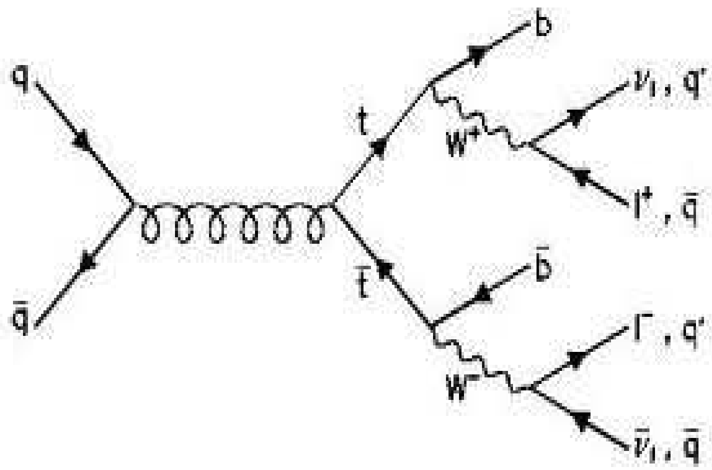
Issues to be discussed: How is the theory error evaluated? Which top mass is measured?

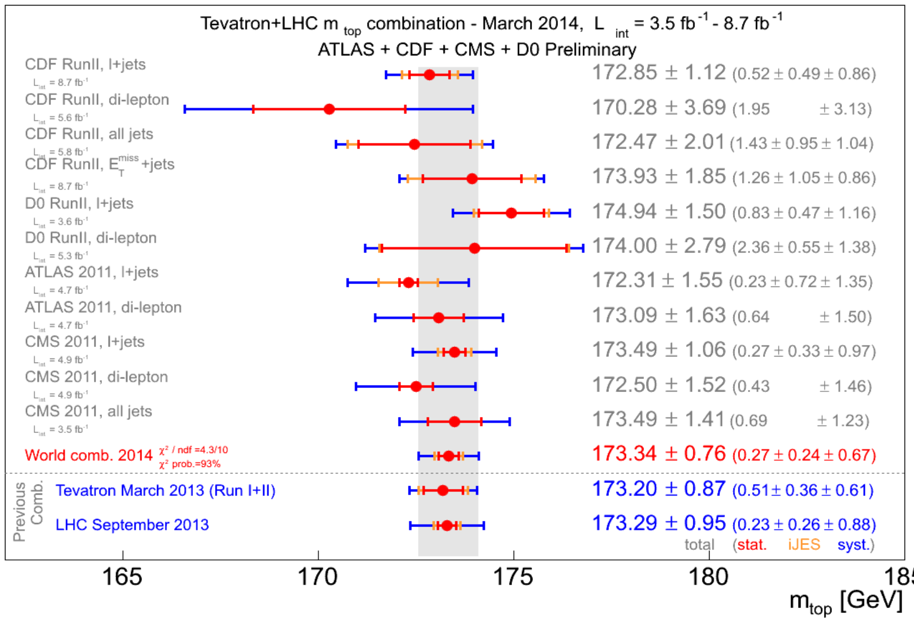
Top production and decay ar hadron colliders ($t\bar{t}$ pairs)



Top pairs produced through strong interactions (mostly $q\bar{q}$ at Tevatron, gg at LHC)

Top decays via $t \to bW$ with $\mathrm{BR} \simeq 1$





Final states as all-leptons, lepton+jets or all-jets according to W decays

Theoretical uncertainties on the top mass measurement

Monte Carlo systematics (MC); modelling QCD radiation effects (Rad); colour reconnection (CR); parton distribution functions (PDF)

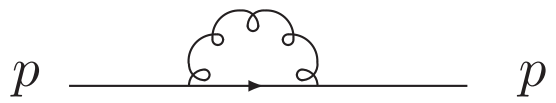
All values in GeV	CDF	D0	ATLAS	CMS	Tevatron	LHC	WA
m_{top}	173.19	174.85	172.65	173.58	173.58	173.28	173.34
Stat	0.52	0.78	0.31	0.29	0.44	0.22	0.27
iJES	0.44	0.48	0.41	0.28	0.36	0.26	0.24
stdJES	0.30	0.62	0.78	0.33	0.27	0.31	0.20
flavourJES	0.08	0.27	0.21	0.19	0.09	0.16	0.12
bJES	0.15	0.08	0.35	0.57	0.13	0.44	0.25
⟹ MC	0.56	0.62	0.48	0.19	0.57	0.25	0.38
Rad	0.09	0.26	0.42	0.28	0.13	0.32	0.21
CR	0.21	0.31	0.31	0.48	0.23	0.43	0.31
PDF	0.09	0.22	0.15	0.07	0.12	0.09	0.09
DetMod	<0.01	0.37	0.22	0.25	0.09	0.20	0.10
b-tag	0.04	0.09	0.66	0.11	0.04	0.22	0.11
LepPt	<0.01	0.20	0.07	<0.01	0.05	0.01	0.02
BGMC	0.10	0.16	0.06	0.11	0.11	0.08	0.10
BGData	0.15	0.19	0.06	0.03	0.12	0.04	0.07
Meth	0.07	0.15	0.08	0.07	0.06	0.06	0.05
MHI	0.08	0.05	0.02	0.06	0.06	0.05	0.04
Total Syst	0.85	1.25	1.40	0.99	0.82	0.92	0.71
Total	1.00	1.48	1.44	1.03	0.94	0.94	0.76
χ^2/ndf	1.09 / 3	0.13 / 1	0.34 / 1	1.15 / 2	2.45 / 5	1.81 / 4	4.33 / 10
χ^2 probability [%]	78	72	56	56	78	77	93

Overall theory error on world average mass: $\Delta m_t \simeq 0.54$ GeV

Mass central value and theory errors are driven by Monte Carlo event generators: can we connect them to a consistent mass definition?

Top-quark mass definitions

Subtraction of the UV divergences in the self energy $\Sigma(p)$



Renormalized propagator: $S^{-1}(p) = -i[\not{p} - m_t^0 + \Sigma^R(p, m_t^0, \mu)]$

Mass is solution of equation $\not{p} - m_t + \Sigma^R(p, m_t, \mu) = 0$

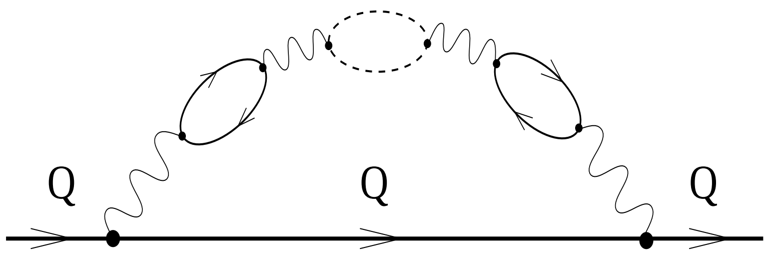
Pole mass:

$$\Sigma^R(p) = 0 \quad \text{and} \quad \frac{\partial \Sigma^R}{\partial \not{p}} = 0 \quad \text{for} \quad \not{p} = m$$

OK for electrons, but for quarks non-perturbative ambiguity: $\Delta m \sim \Lambda_{\mathrm{QCD}}$

Higher-order corrections lead to infrared renormalons:

$$\Sigma(m) \sim m \sum_n \alpha_S^{n+1} \, (2\beta_0)^n \; n!$$



$\overline{\rm MS}$ mass $\bar{m}_t(\mu)$– dimensional regularization $D = 4 - 2\epsilon$

$$\Sigma(p) = \frac{i\alpha_S C_F}{4\pi}\left\{\left[\frac{1}{\epsilon} - \gamma + \ln 4\pi + A(m_t^0, p, \mu)\right] \not{p} - \left[4\left(\frac{1}{\epsilon} - \gamma + \ln 4\pi\right) + B(m_t^0, p, \mu)\right] m_t^0\right\}$$

Counterterm to subtract $(1/\epsilon + \gamma_E - \ln 4\pi)$

Relation with the pole mass (coefficients c_i depending on $\ln[\mu^2/\bar{m}_t(\mu)^2]$)

$$m_t = \bar{m}_t(\mu)\left[1 + \alpha_S(\mu)c_1 + \alpha_S^2(\mu)c_2 + \ldots\right]$$

OK with off-shell quarks ($Z \to b\bar{b}$); at threshold contributions $\sim (\alpha_S/v^2)^k$

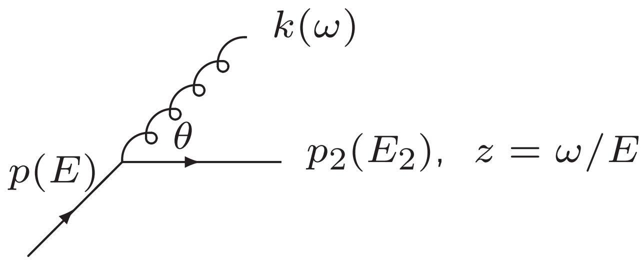
MSR masses in terms of an infrared scale R, e.g. μ_F, $\bar{m}(\mu)$, etc.

$$M_{\rm pole} = m(R,\mu) + \delta m(R,\mu) \quad ; \quad \delta m(R,\mu) = R\sum_{n=1}^{\infty}\sum_{k=0}^{n} a_{nk}\alpha_S(\mu)^n$$

$$\frac{dM_{\rm pole}}{d\ln\mu} = 0 \Rightarrow \frac{dm(R,\mu)}{d\ln\mu} = -R\gamma[\alpha_S(\mu)] \quad \text{(A. Hoang et al)}$$

To make a statement on the measured top mass one would need at least a NLO computation: how about Monte Carlo event generators?

Hard scattering through matrix-element generators at LO (ALPGEN, MadGraph, etc.) or NLO (POWHEG, MC@NLO), with HERWIG/PYTHIA for showers and hadronization



$$dP = \frac{\alpha_S}{2\pi}\, P(z)dz\, \frac{dQ^2}{Q^2}\, \Delta_S(Q^2_{\max}, Q^2)$$

Q^2: ordering variable

$\Delta_S(Q^2_{\max}, Q^2)$: no radiation in $[Q^2, Q^2_{\max}]$ (soft/collinear virtual corrections)

$$\Delta_S(Q^2_{\max}, Q^2) = \exp\left[-\frac{\alpha_S}{2\pi}\int_{Q^2}^{Q^2_{\max}} \frac{dQ'^2}{Q'^2}\int_{z_{\min}}^{z_{\max}} dz P(z)\right]$$

HERWIG : $Q^2 = E^2(1-\cos\theta) \simeq E^2\theta^2/2$ (angular ordering);

PYTHIA: $Q^2 = p^2$ or k_T^2

$\alpha_S(k_T^2)$ with two-loop evolution in HERWIG and one-loop in PYTHIA

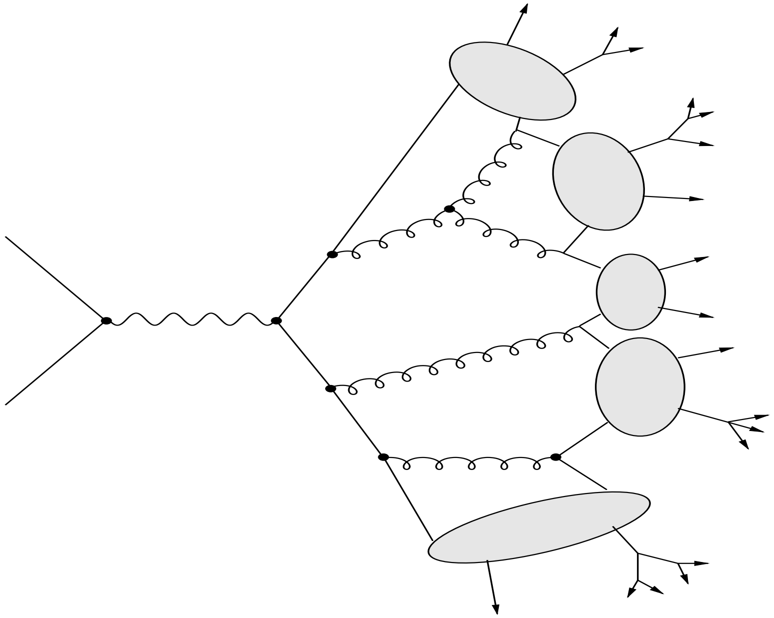
Total cross section LO thanks to unitarity $(1 = R + V)$

Distributions equivalent to threshold LL resummation, $+$ some NLLs

$\Lambda \to \Lambda_{\rm MC} = \Lambda \exp(4K\beta_0)$: NLL Sudakov form factor at large x (Catani, Marchesini and Webber)

NLO total cross section when hard-scattering via MC@NLO or POWHEG

Hadronization models



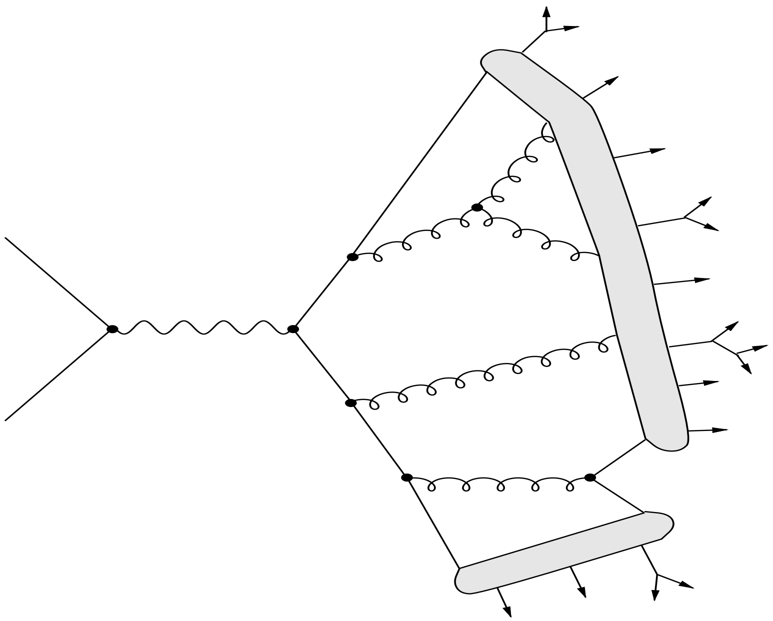
Cluster model (HERWIG)

Perturbative evolution ends at $Q^2 = Q_0^2$

Angular ordering $\Rightarrow$ colour preconfinement

Forced gluon splitting $(g \to q\bar{q})$

Colour-singlet clusters decay
into the observed hadrons



String model (PYTHIA)

q and $\bar{q}$ move in opposite direction

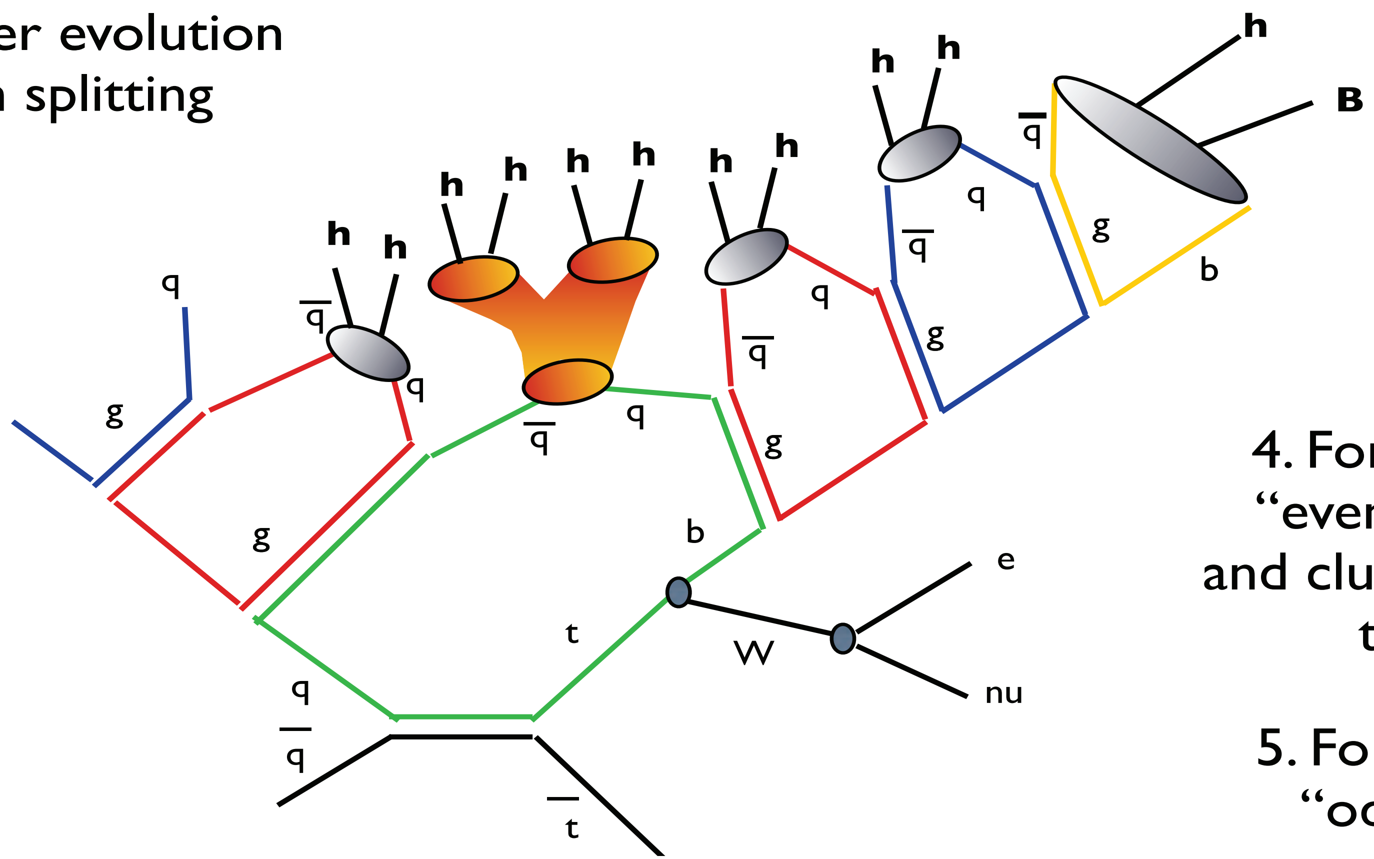
The colour field collapses into a string,
with uniform energy density

$q\bar{q}$ pairs are produced

The string breaks into the observed hadrons

Colour reconnection: hadrons can be formed via ‘even’ or ‘odd’ clusters: effects already studied at LEP for $e^+e^- \to W^+W^- \to 4$ jets M.L.Mangano, talk at TOP2013

1. Hard Process
2. Shower evolution
3. Gluon splitting



4. Formation of “even” clusters and cluster decay to hadrons

5. Formation of “odd” cluster

$m_t = (p_W + p_{b-\mathrm{jet}})^2$: the b-jet may come from either an even or an odd cluster

Colour reconnection can be investigated by varying relevant parameters in HERWIG and PYTHIA (so-called Perugia tunings) and affects the top mass about $\Delta m_t \leq 1$ GeV

Monte Carlo differences in parton showers and hadronization lead to the so-called Monte Carlo systematics

No unique way to estimate it: comparing two codes or one code and varying its parameters

CDF: comparing HERWIG and PYTHIA;

D0: ALPGEN+HERWIG vs. ALPGEN+PYTHIA;

Both CDF and D0 PYTHIA vs. MC@NLO to gauge the impact of NLO corrections

ATLAS: MC@NLO vs. POWHEG (NLO corrections), HERWIG vs. PYTHIA for hadronization

CMS: MadGraph vs. POWHEG

Radiation systematics: how much ISR and FSR affect the measurement of the top mass

CDF and D0: tuning PYTHIA $\Lambda_{\rm QCD}$ and initial Q^2 scale to Drell–Yan events ($q\bar{q}$ initial state) and propagate to $t\bar{t}$ events

ATLAS: playing around with ISR and FSR parameters of PYTHIA

CMS: varying factorization and renormalization scales of MadGraph, then matching scales for matching with parton showers

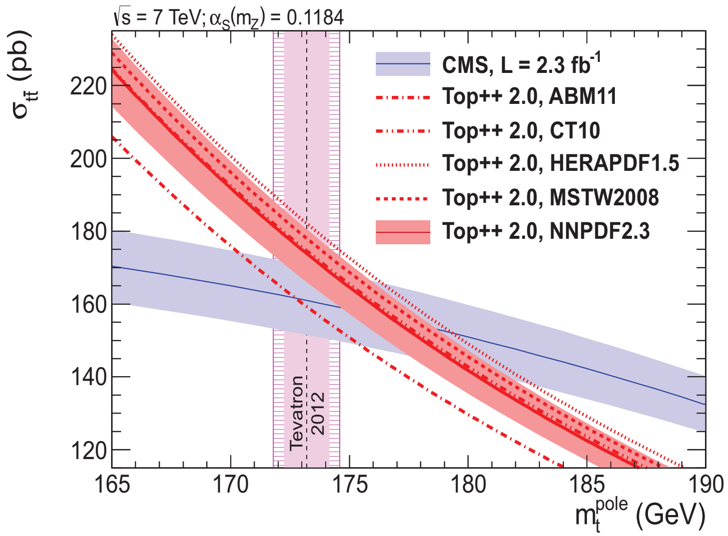
Top mass from total NNLO+NNLL $t\bar{t}$ cross section (Mitov, Fielder and Czakon, '13):

$$\sigma_{\rm tot} = \sum_{i,j} \int d\beta\ \Phi_{ij}(\beta, \mu_F^2)\ \hat{\sigma}_{ij} \quad , \quad \beta = \sqrt{1 - 4m^2/\hat{s}} \quad , \quad \Phi_{ij} = \frac{2\beta}{1-\beta^2}\, x\, (f_i \otimes f_j)$$

At NNLO, for $\mu = \mu_F = \mu_R$ and $L = \ln(m_t^2/\mu^2)$, using the pole mass:

$$\hat{\sigma} = \frac{\alpha_S^2}{m_t^2} \left\{ \sigma^{(0)} + \alpha_S \left[\sigma_{ij}^{(1)} + L \sigma_{ij}^{(1,1)} \right] + \alpha_S^2 \left[\sigma_{ij}^{(2)} + L \sigma_{ij}^{(2,1)} + L^2 \sigma^{(2,2)} \right] \right\}$$

Threshold logarithms $\alpha_S^n [\ln^m(1-z)/(1-z)]_+$ $z = m_t^2/(x_i x_j \hat{s})$, $m \leq 2n-1$



Scales: $\Delta\sigma \simeq 3\%$; pdfs: $\Delta\sigma \simeq 2.5\%$; α_S: $\Delta\sigma \simeq 1.5\%$, m_t: $\Delta\sigma \simeq 3\%$

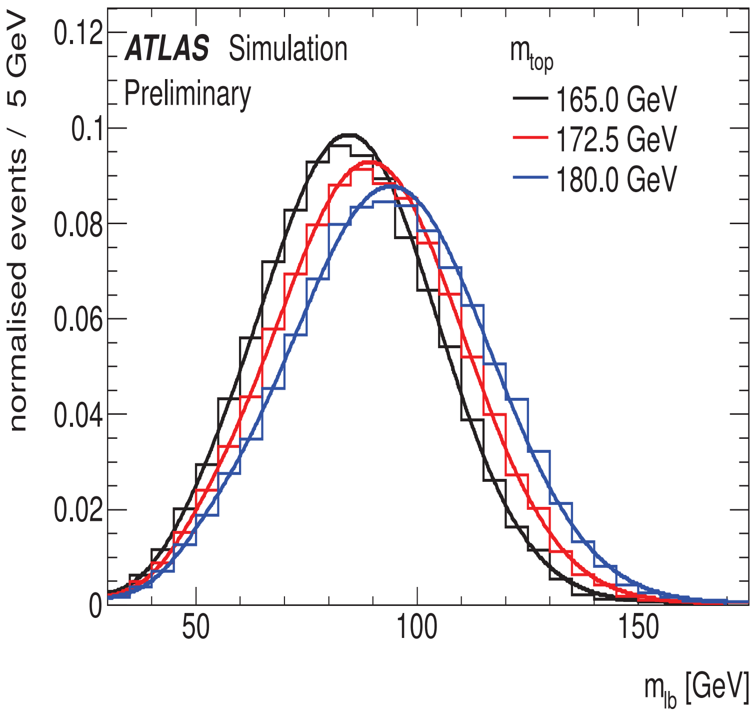
Extracted pole mass exhibits large errors: $m_t^{\rm pole} = \left(176^{+3.8}_{-3.4}\right)$ GeV

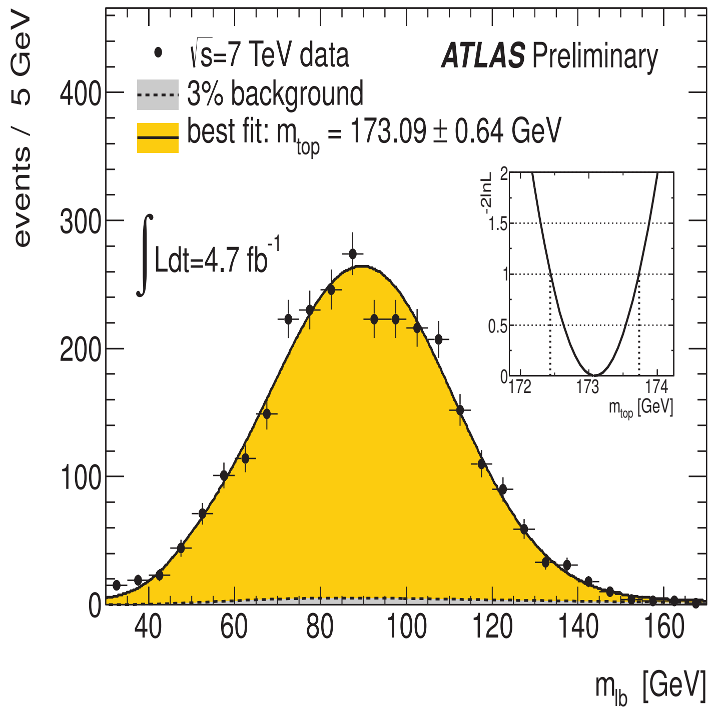
Note: CMS cross section still relies on m_t value in the MC generator

Standard top mass reconstruction relies on top decays: template method

Distribution of observables sensitive to m_t and reconstruction of under the assumption that the final state is $WbWb$ and the W mass is fixed

Data confronted with Monte Carlo templates and m_t is the value minimizing the χ^2





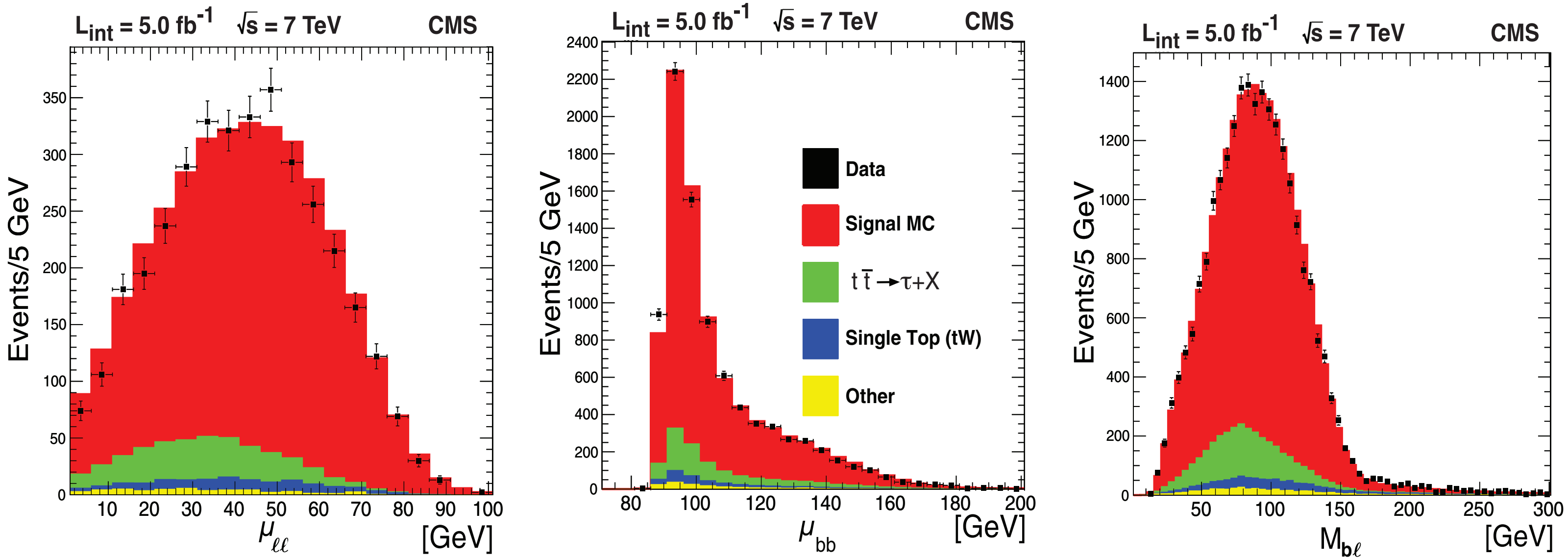
ATLAS: b-jet+lepton invariant mass in dilepton channel

m_t is the parameter in HERWIG and PYTHIA, often called Monte Carlo mass

Endpoint method (CMS): in dilepton channels, the endpoint of kinematic distributions depends on m_t, m_W and m_ν

Constraining m_W and m_ν, one can get the top mass

Endpoints of $\ell+b$-jet, '$\ell\ell$' or 'bb' invariant mass distributions are sensitive to m_t



$$\mu_{bb}^{\max} = \frac{m_t}{2}\left(1 - \frac{m_W^2}{m_t^2}\right) + \sqrt{\frac{m_t^2}{4}\left(1 - \frac{m_W^2}{m_t^2}\right) + m_W^2}\ , \quad \mu_{\ell\ell}^{\max} = \frac{m_W}{2}\left(1 - \frac{m_\nu^2}{m_t^2}\right) + \sqrt{\frac{m_t^2}{4}\left(1 - \frac{m_\nu^2}{m_t^2}\right) + m_\nu^2}$$

Final result is $m_t = 173.9\pm$ 0.9 (stat) $^{+1.7}_{-2.1}$ (syst) ; $\delta m_t(\mathrm{th}) \simeq \pm 0.6$ GeV

Theory error mostly driven by colour reconnection effects, with minor contributions due to renormalization/factorization scales, PDFs and parton/hadron matching scale

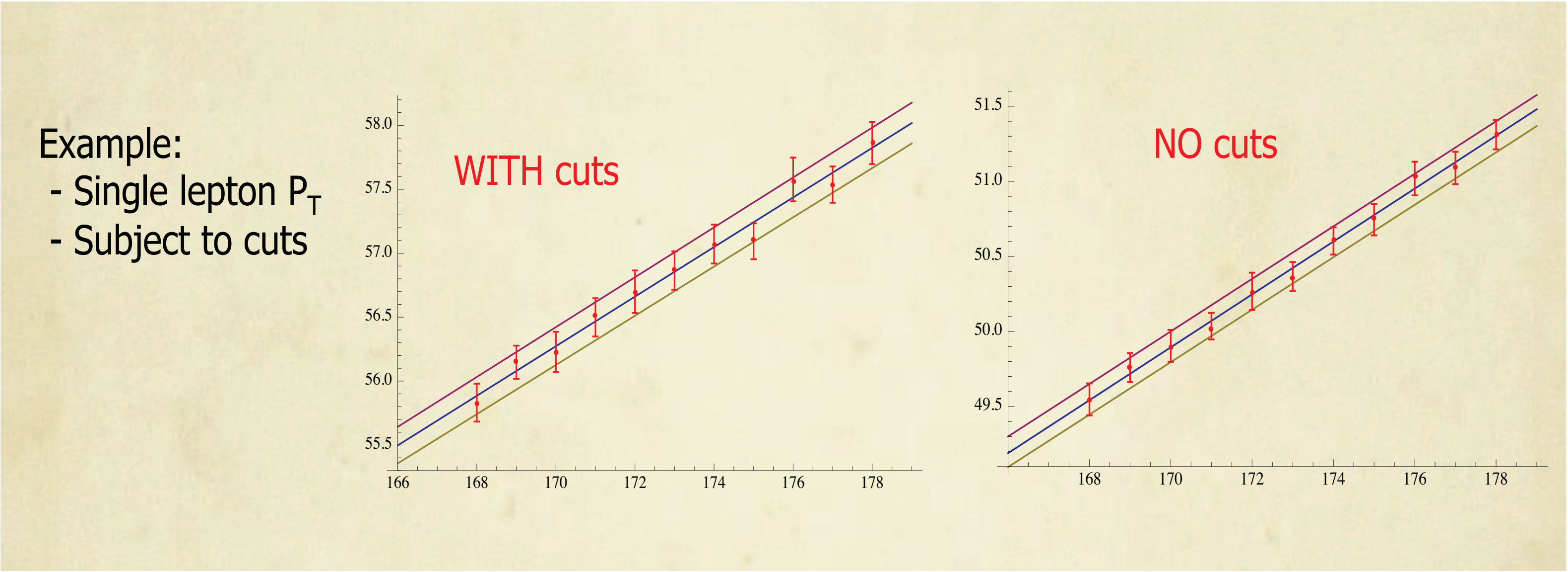
Newer ideas (A.Mitov and S.Frixione, JHEP'14): dilepton observables in moment space:

$$p_T(\ell)\ ,p_T(\ell^+\ell^-)\ ,\ m(\ell^+\ell^-)\ ,\ E(\ell^+)+E(\ell^-)\ ,\ p_T(\ell^+)+p_T(\ell^-)\ ,\ \mu_{O,N}=\frac{1}{\sigma}\int d\sigma O^N$$

σ and $d\sigma$ total and differential $t\bar{t}$ cross sections, possibly including cuts

$\langle p_T\rangle$ without and with cuts: $|\eta_\ell|\leq 2.4$, $|\eta_b|\leq 2.4$, $p_{T,\ell}\geq 20$ GeV, $p_{T,b}\geq 30$ GeV

Linear fits: $$\mu_{O,N}=\alpha_{O,i}(173\ \mathrm{GeV})^i+\beta_{O,i}m_t^i$$



Observables are completely inclusive with respect to strong physics and exhibit very little dependence on hadronization corrections and no need to reconstruct the top quarks

Analysis carried out at $\sqrt{s}=8$ TeV using NLO hard scattering (aMC@NLO, with pole mass), HERWIG parton showers and MadSpin for spin correlations: $\Delta m_t\simeq 0.8$ GeV

A case study to estimate the theory uncertainty: the J/ψ method

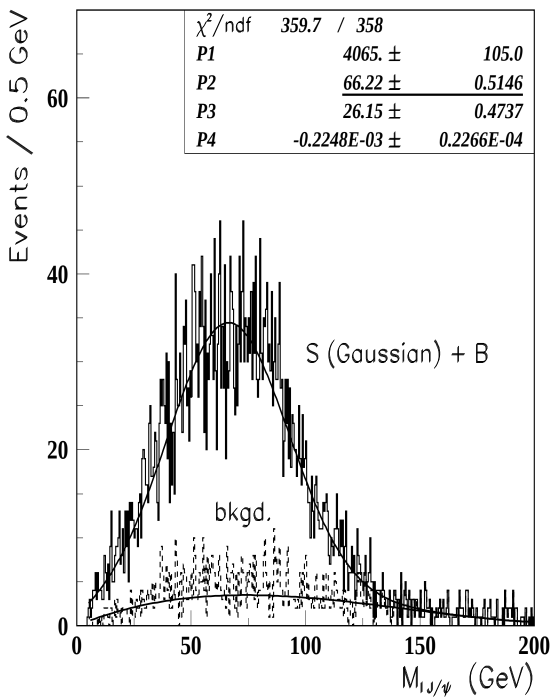
Final states with leptons and J/ψ, i.e. $W \to \ell\nu$ and $B \to J/\psi X$, $J/\psi \to \mu^+\mu^-$

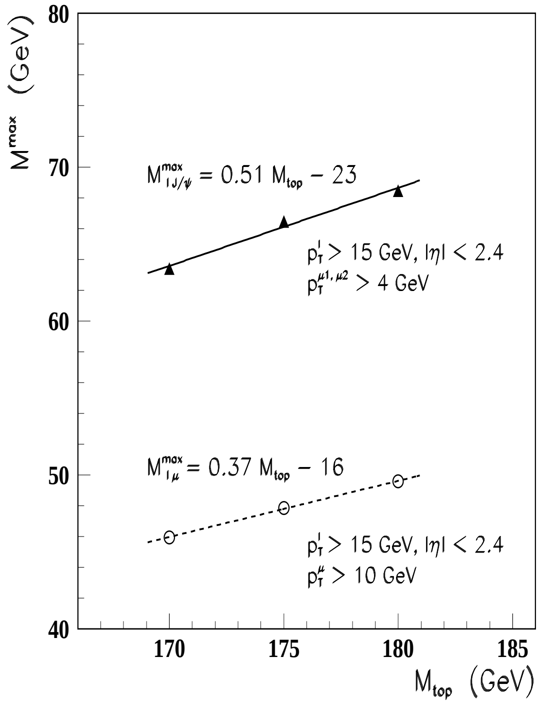
A. Kharchilava, PLB 476 (2000) 73, R. Chierici and A. Dierlamm, CMS Note 2006/058

Linear fits of peak values of $m_{\ell J/\psi}$ and $m_{\ell\mu}$

Systematics (theo + exp): $\Delta m_t(\mathrm{syst}) \simeq 1.47$ GeV

b-fragmentation (PYTHIA+Peterson model): $\Delta m_t(\mathrm{frag}) \simeq 0.51$ GeV



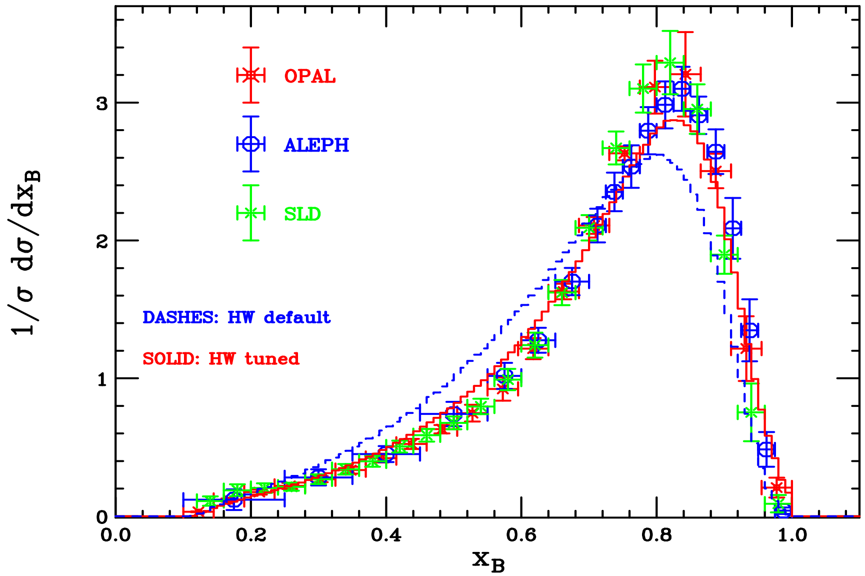


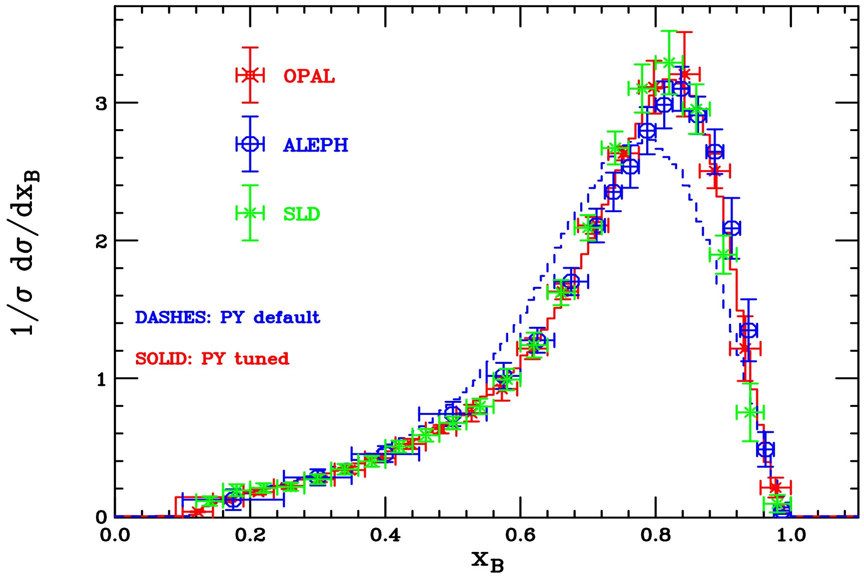
Monte Carlo tuning: $e^+e^- \to Z^0 \to b\bar{b} \to BX_{\bar{b}} \quad x_B = 2E_B/m_Z$

(G. C. and V. Drollinger, NPB 730 (2005) 82)

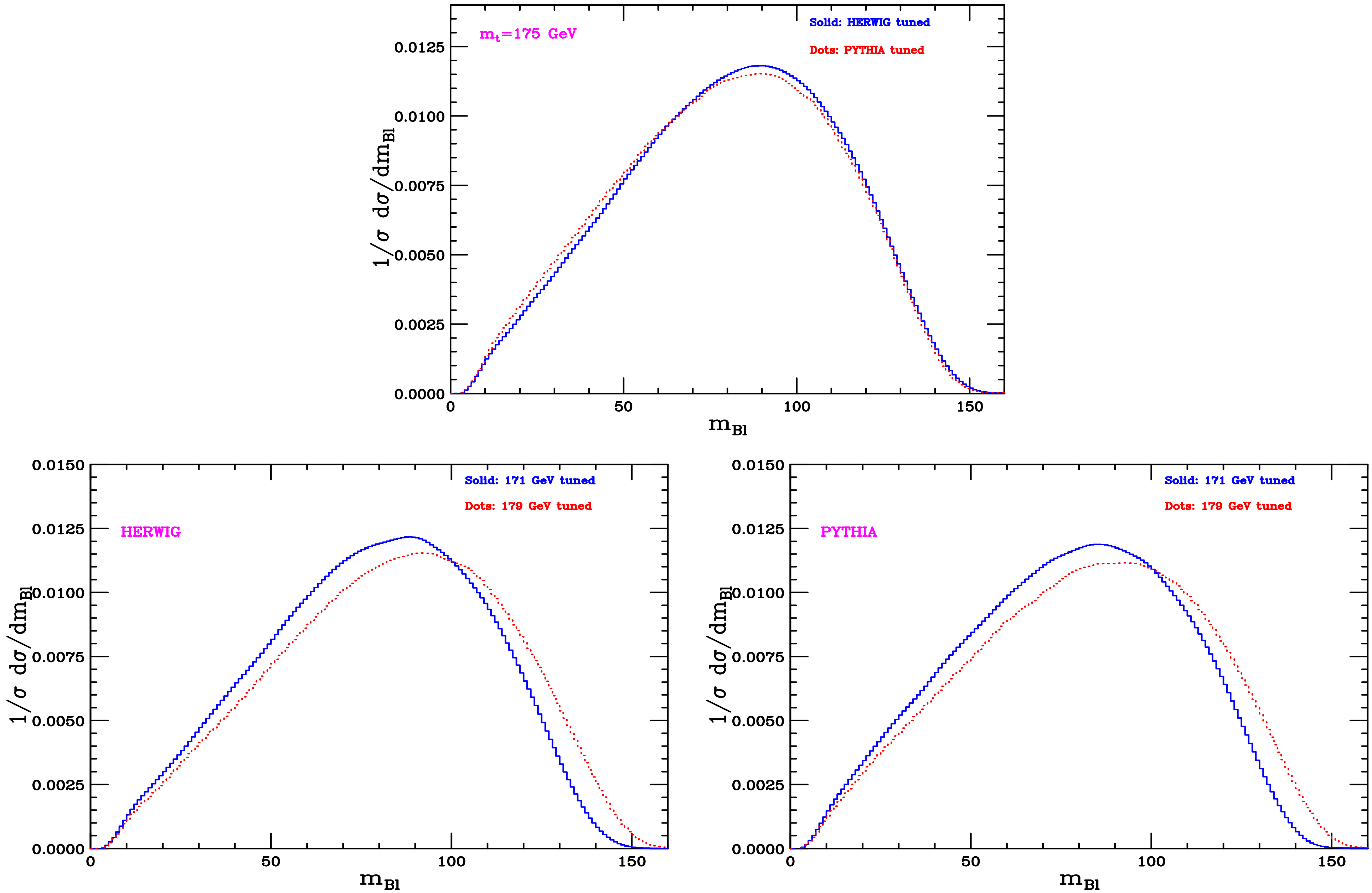
HERWIG	PYTHIA
CLSMR(1) $=$ 0.4 (0.0)	
CLSMR(2) $=$ 0.3 (0.0)	PARJ(41) $=$ 0.85 (0.30)
DECWT $=$ 0.7 (1.0)	PARJ(42) $=$ 1.03 (0.58)
CLPOW $=$ 2.1 (2.0)	PARJ(46) $=$ 0.85 (1.00)
PSPLT(2) $=$ 0.33 (1.00)	
χ^2/dof $=$ 222.4/61 (739.4/61)	χ^2/dof $=$ 45.7/61 (467.9/61)

Lund/Bowler fragmentation function : $f_B(z) \propto \dfrac{1}{z^{1+brm_b^2}}(1-z)^a \exp(-bm_T^2/z)$





B-lepton invariant mass according to tuned HERWIG and PYTHIA (G.C. and F. Mescia)



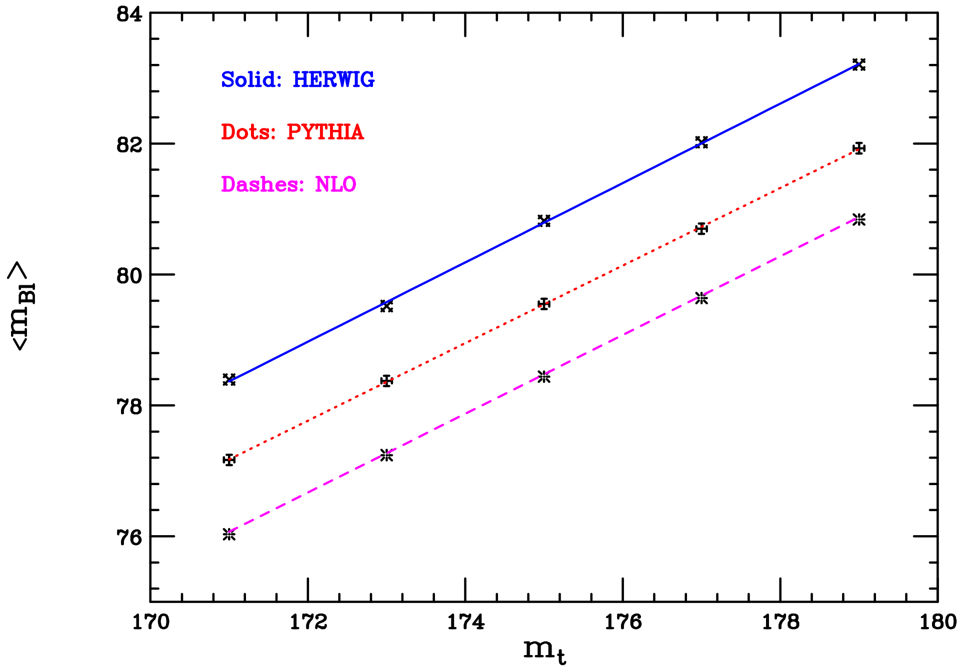
Linear fits to extract m_t from $m_{B\ell}$

HERWIG: $\langle m_{B\ell}\rangle_{\rm H} \simeq -25.31\ {\rm GeV} + 0.61\ m_t$; $\delta = 0.043\ {\rm GeV}$

PYTHIA: $\langle m_{B\ell}\rangle_{\rm P} \simeq -24.11\ {\rm GeV} + 0.59\ m_t$; $\delta = 0.022\ {\rm GeV}$

NLO: $\langle m_{B\ell}\rangle_{\rm NLO} \simeq -26.7\ {\rm GeV} + 0.60\ m_t$; $\delta = 0.004\ {\rm GeV}$

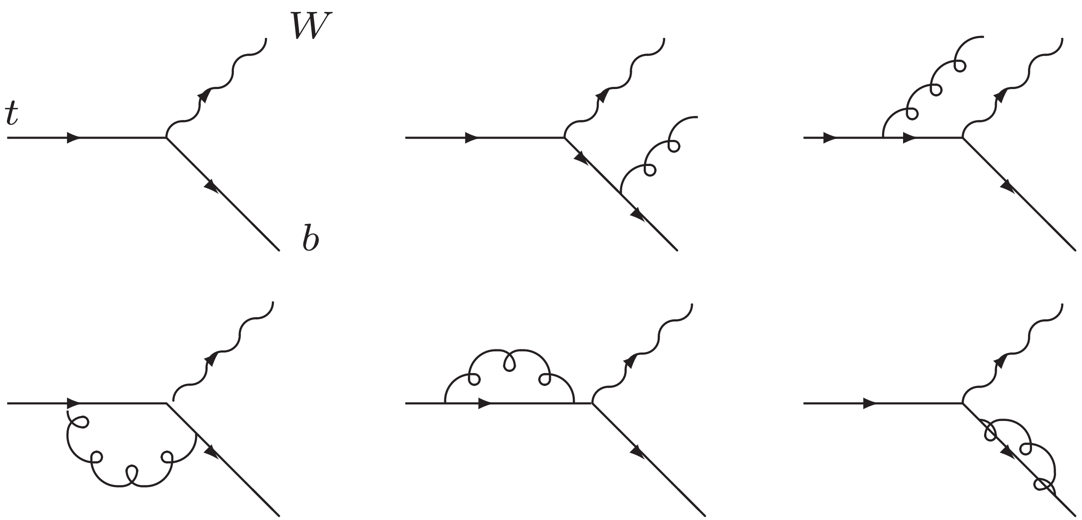
S.Biswas, K.Melnikov and M.Schulze, JHEP 1008 (2010) 048: $m_{B\ell}$ at NLO



$\Delta\langle m_{B\ell}\rangle_{\rm H,P} \simeq 1.2$ GeV ; $\Delta\langle m_{B\ell}\rangle_{\rm H,NLO} \simeq 2.2$ GeV ; $\Delta\langle m_{B\ell}\rangle_{\rm P,NLO} \simeq 1.1$ GeV

m_t is pole mass in NLO, Monte Carlo mass in HERWIG/PYTHIA

NLO top decays would be necessary to have a consistent top mass definition



$t(q) \to b(p_b)W(p_W)\ (g(p_g))$

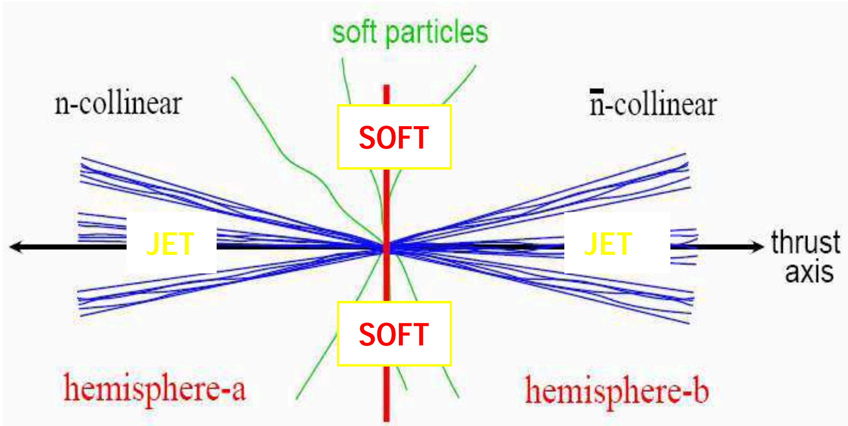
Total width up to NNLO; b spectrum at NLO+NLL threshold resummation available the PFF formalism and using the top pole mass

Parton showers matched to tree-level $\Gamma(t \to bWg)$ (hard/large-angle radiation)

The top mass in top decays in HERWIG/PYTHIA should be related to the pole mass (on-shell) and in fact the world average (relying on MC's) agrees with the pole mass extracted from the NNLO cross section

Open questions: width ambiguity? Missing higher orders in the top self-energy? Hadronization corrections?

Attempt to address the MC mass using the SCET formalism $Q \gg m_t \gg \Gamma_t \gg \Lambda_{\rm QCD}$



Top-decay invariant masses: $M_t^2 = \left(\sum_{i\in a} p_i^\mu\right)^2,\ M_{\bar t}^2 = \left(\sum_{i\in b} p_i^\mu\right)^2$

Factorization theorem: (Hoang et al.)

$$\frac{d\sigma}{dM_t^2 dM_{\bar t}^2} \sim H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \int d\ell^+ d\ell^- B_+\left(\hat s_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_+\left(\hat s_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S(\ell^+, \ell, \mu$$

H_Q, H_m: hard scattering coefficient functions at scales Q and m_t

$B_\pm$: heavy-quark jet functions, describing top evolution into jets

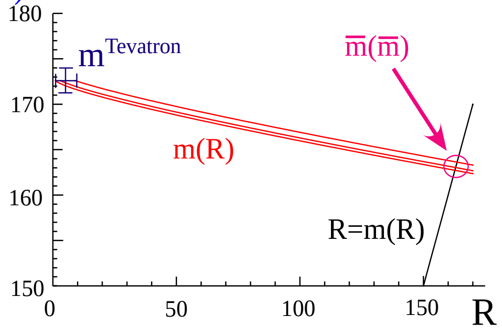
$S(\ell^+, \ell^-, \mu)$: non-perturbative fragmentation function, depending on soft emissions

Large logarithms $\ln(Q/m_t)$, $\ln(m_t/\Gamma_t)$, etc.

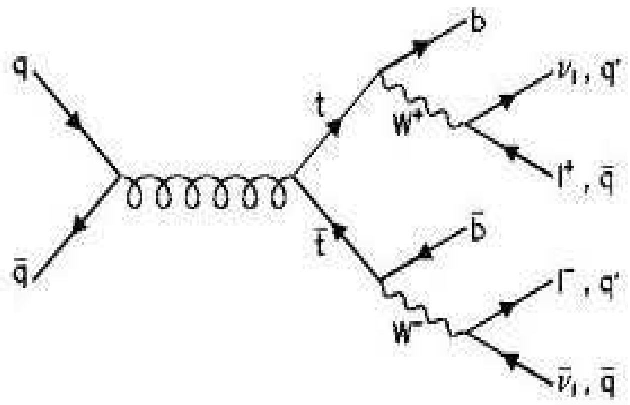
Jet mass: short-distance (resonance) mass with $R \sim \Gamma_t$

$$m_J(\mu) \sim \left[\frac{d \ln \tilde{B}(y,\mu)}{dy}\right]_{y=-ie^{-\gamma_E}/R} \quad \Rightarrow \quad m_{\rm pole} = m_J(\mu) + e^{\gamma_E}\Gamma_t \frac{\alpha_S(\mu) C_F}{\pi}\left(\ln\frac{\mu}{\Gamma_t} + \frac{1}{2}\right) + \mathcal{O}(\alpha_S^2)$$

MC mass is interpreted as $m_J(Q_0) \simeq 173$ GeV and then R-evolved to $\bar{m}(\bar{m}) \simeq 163$ GeV



MC mass is close to the pole mass: top decay kinematics is driven by the pole mass



$$\frac{1}{[(p_W+p_b)^2 - m^2]^2 + m^2\Gamma^2} \sim \frac{1}{\pi}\,\delta[(p_W+p_b)^2 - m^2] \text{ for } \Gamma \ll m$$

On-shell final state (factorization): $(p_W+p_b)^2 \sim m^2$ with $m = m_{\rm pole}$

Using the $\overline{\rm MS}$ mass is possible, but $m = m_{\overline{\rm MS}}[1+\mathcal{O}(\alpha_S)]$ recovers the pole mass

World average (CDF, D0, ATLAS, CMS): $m_t = 173.34 \pm 0.27\ (\mathrm{stat})\ \pm 0.71\ (\mathrm{syst})$ GeV relies on Monte Carlo generators

Reconstructed mass $m_t^2 = (p_{b-\mathrm{jet}} + p_\nu + p_\ell)^2$ (with cuts on jets and leptons) with on-shell tops should be close to the top mass, up to widths and higher-order corrections

Attempts based on SCET have in fact shown that $m_t \simeq m_t^{\mathrm{pole}} + \mathcal{O}(\alpha_S \Gamma)$

A possible way out: HERWIG with fictitious top-hadron states (G.C. and M.L.Mangano)

Top quarks hadronize ($T^{\pm,0}$) and decay, e.g., through the spectator model

From a given observable R extract the Monte Carlo mass m_T^{MC}

Study the same observable R with standard top samples, get m_t^{MC} and compare the extracted masses $m_T^{\mathrm{MC}} = m_t^{\mathrm{MC}} + \Delta m$

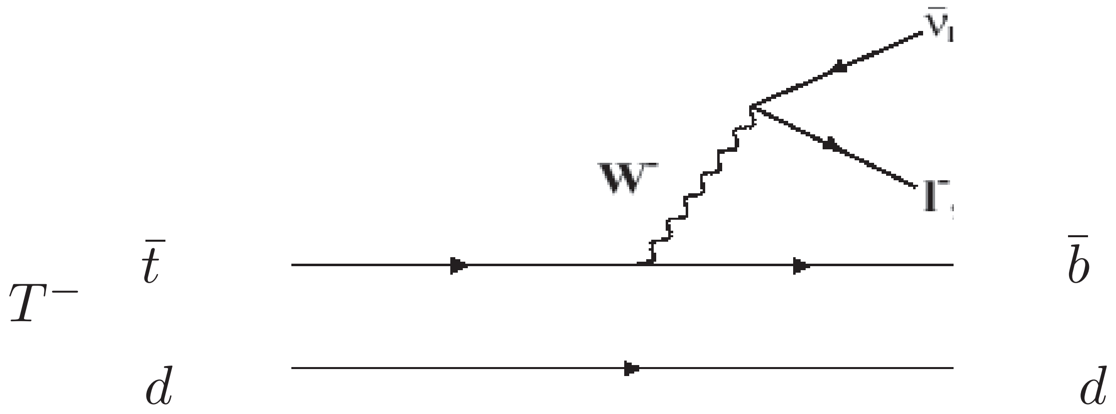
In the hadronized samples, the Monte Carlo mass can be related to the T-meson mass M_T and ultimately to the pole or $\overline{\mathrm{MS}}$ top-quark masses by using lattice, potential models, HQET, NRQCD, etc.

Connection between pole/$\overline{\mathrm{MS}}$ mass and the Monte Carlo mass

Investigate the dependence of the results on the specific analysis/observable and contributions to Δm (colour flow, gluon radiation, hadron decay models)

HERWIG for $e^+e^- \to t\bar{t}$ at $\sqrt{s} = 1$ TeV with top quarks hadronizing before decaying

t-flavoured mesons in the dilepton channel, i.e. $T^+ = (t\bar{d})$, $T^0 = (t\bar{u})$, $T^- = (\bar{t}d)$, etc.
Spectator model decays: $T^- \to (\bar{b}d)\ell^-\bar{\nu}_\ell + X \ldots \quad p_T^2 = (p_{\bar{b}} + p_W + p_q + p_X)^2$

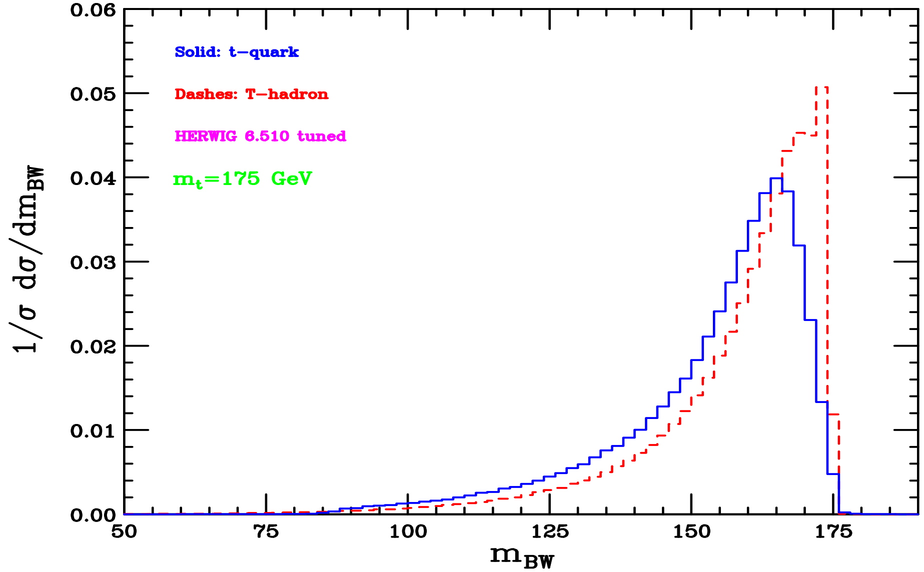


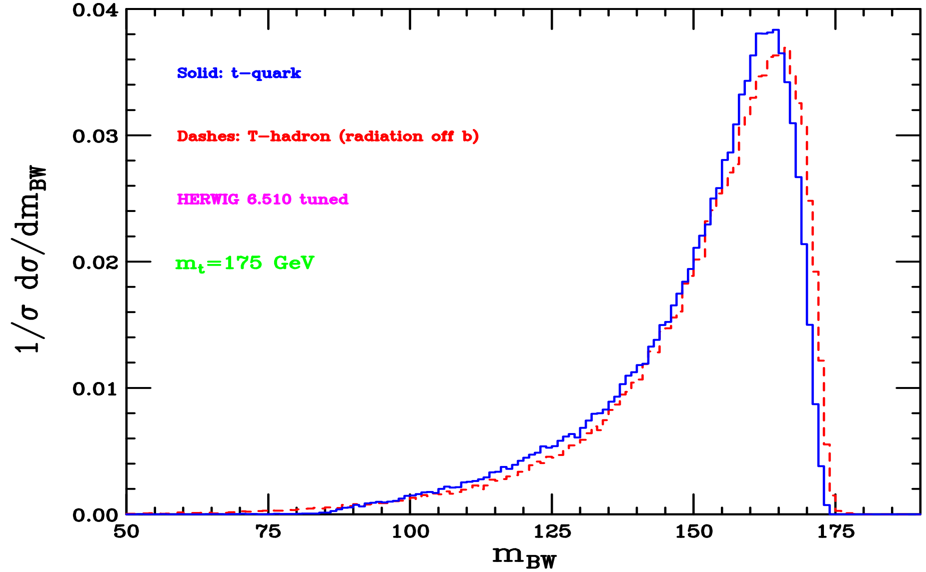
In a fraction of events, proportional to $\Delta_S(Q_b^2, Q_0^2)$, the b quarks in T decays do not radiate gluons: the $(\bar{b}q)$ cluster yields a B meson plus a soft hadron, e.g. pions

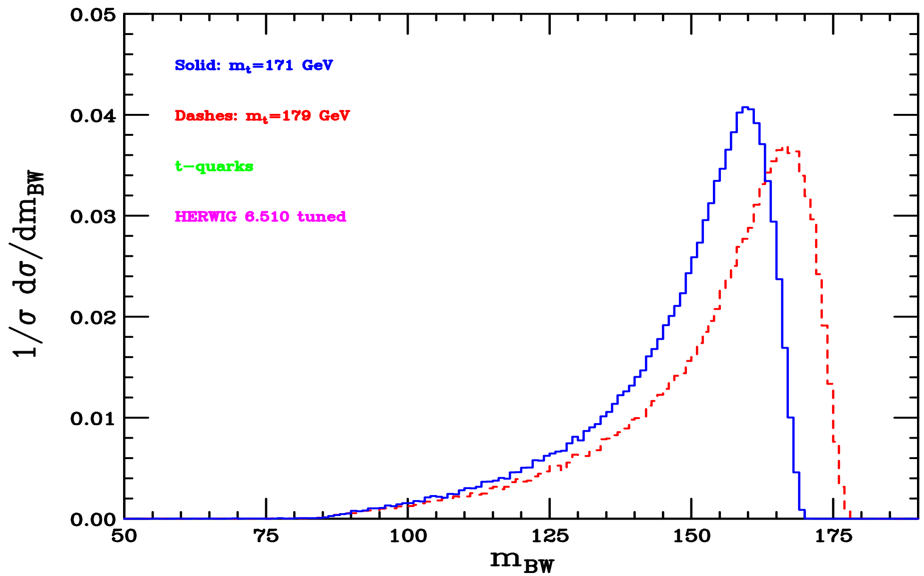
Spectator quarks likely do not radiate

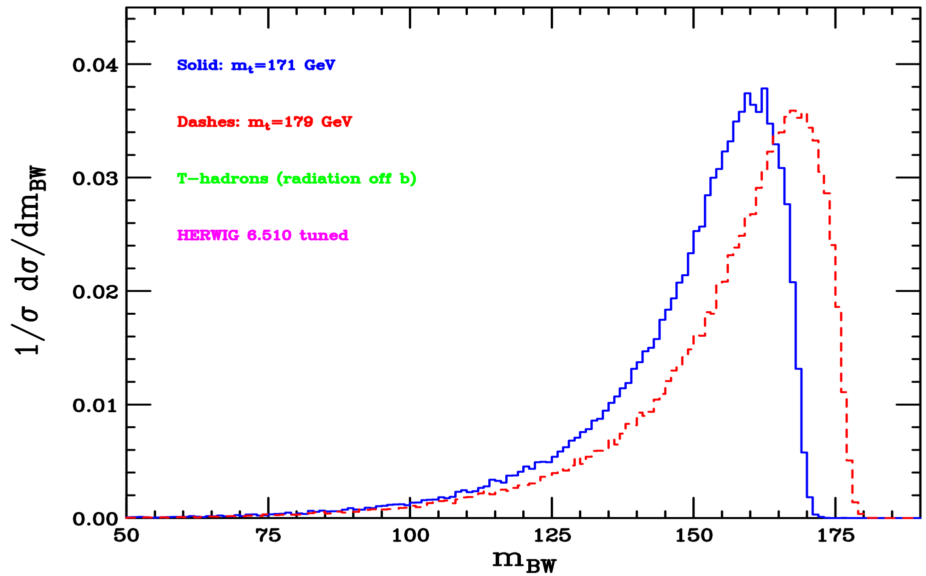
In usual top decays before hadronization, the b-quark manages to form hard clusters decaying into B's and more energetic hadrons

Results with hadronized top quarks for BW invariant mass for fixed m_t^{MC} with and possibly without gluon radiation off the b (top plots) and varying m_t^{MC} (bottom)









Mellin moments - m_{BW} spectrum, allowing gluon emissions off the b quarks

T-hadrons:

m_t (GeV)	$\langle m_{BW} \rangle$ (GeV)	$\langle m_{BW}^2 \rangle$ (GeV^2)	$\langle m_{BW}^3 \rangle$ (GeV^3)	$\langle m_{BW}^4 \rangle$ (GeV^4)
171	148.76	2.24×10^4	3.41×10^6	5.24×10^8
173	150.44	2.29×10^4	3.53×10^6	5.48×10^8
175	152.18	2.35×10^4	3.66×10^6	5.74×10^8
177	153.80	2.40×10^4	3.77×10^6	5.99×10^8
179	155.61	2.45×10^4	3.91×10^6	6.28×10^8

t-quarks:

m_t (GeV)	$\langle m_{BW} \rangle$ (GeV)	$\langle m_{BW}^2 \rangle$ (GeV^2)	$\langle m_{BW}^3 \rangle$ (GeV^3)	$\langle m_{BW}^4 \rangle$ (GeV^4)
171	148.08	2.21×10^4	3.35×10^6	5.11×10^8
173	149.56	2.26×10^4	3.46×10^6	5.32×10^8
175	151.00	2.30×10^4	3.56×10^6	5.54×10^8
177	152.60	2.36×10^4	3.67×10^6	5.78×10^8
179	153.97	2.40×10^3	3.78×10^6	6.00×10^8

Perspectives on top mass measurement (Snowmass report, $\Delta m_t^{\rm th} \simeq 3/4 \Delta m_t^{\rm syst}$)

Conventional methods (extrapolation of CMS lepton+jets analysis, assuming future tuning of soft-QCD parameters to LHC data)

	Ref.[70]	Projections				
CM Energy	7 TeV	14 TeV				
Cross Section	167 pb	951 pb				
Luminosity	$5 fb^{-1}$	$100 fb^{-1}$		$300 fb^{-1}$		$3000 fb^{-1}$
Pileup	9.3	19	30	19	30	95
Syst. (GeV)	0.95	0.7	0.7	0.6	0.6	0.6
Stat. (GeV)	0.43	0.04	0.04	0.03	0.03	0.01
Total	**1.04**	**0.7**	**0.7**	**0.6**	**0.6**	**0.6**
Total (%)	0.6	0.4	0.4	0.3	0.3	0.3

End-point method (CMS)

	Ref.[76]	Projections		
CM Energy	7 TeV	14 TeV		
Cross Section	167 pb	951 pb		
Luminosity	$5 fb^{-1}$	$100 fb^{-1}$	$300 fb^{-1}$	$3000 fb^{-1}$
Syst. (GeV)	1.8	1.0	0.7	0.5
Stat. (GeV)	0.90	0.10	0.05	0.02
Total	**2.0**	**1.0**	**0.7**	**0.5**
Total (%)	1.2	0.6	0.4	0.3

Perspectives using the J/ψ method

Theory errors: current estimation is based on NLO calculation of $\langle m_{B\ell}\rangle$ and mostly driven by scale and b-quark fragmentation

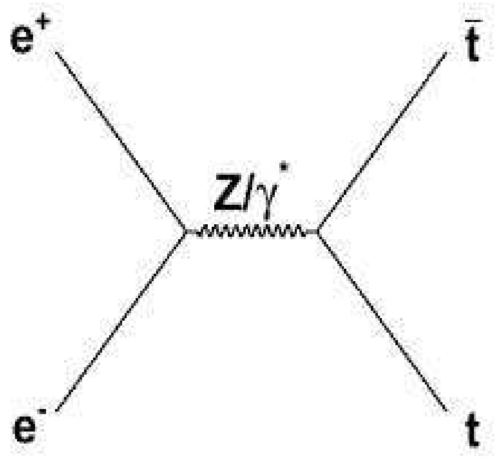
Results are obtained assuming that NNLO $\langle m_{B\ell}\rangle$ can be available in the near future and would reduce scale variation by a factor 2.5

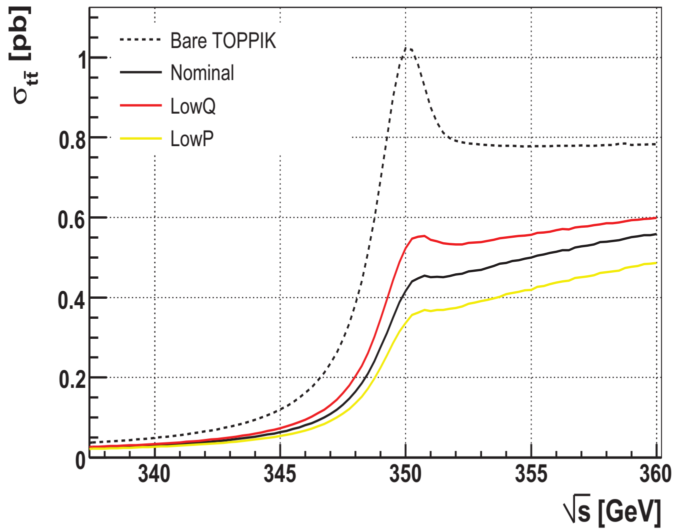
	LHC 13 TeV				LHC 33 TeV			
	$\delta_{\rm scale}[\%]$		$\delta_{\rm pdf}[\%]$		$\delta_{\rm scale}[\%]$		$\delta_{\rm pdf}[\%]$	
	MSTW	NNPDF	MSTW	NNPDF	MSTW	NNPDF	MSTW	NNPDF
NLO	$^{+12.1}_{-12.1}$	$^{+11.8}_{-11.9}$	$^{+1.9}_{-2.3}$	$^{+1.8}_{-1.8}$	$^{+11.5}_{-10.3}$	$^{+11.2}_{-10.0}$	$^{+1.2}_{-1.5}$	$^{+1.0}_{-1.0}$
NNLO	$^{+3.4}_{-5.6}$	$^{+3.5}_{-5.7}$	$^{+1.8}_{-2.0}$	$^{+1.8}_{-1.8}$	$^{+3.1}_{-4.7}$	$^{+3.1}_{-4.7}$	$^{+1.0}_{-1.4}$	$^{+1.0}_{-1.0}$

Extrapolation with CMS foreseen statistics and assuming that lepton colliders may reduce b-fragmentation uncertainty:

	Ref. analysis	Projections				
CM Energy	8 TeV	14 TeV			33 TeV	100 TeV
Cross Section	240 pb	951 pb			5522 pb	25562 pb
Luminosity	$20fb^{-1}$	$100fb^{-1}$	$300fb^{-1}$	$3000fb^{-1}$	$3000fb^{-1}$	$3000fb^{-1}$
Theory (GeV)	-	1.5	1.5	1.0	1.0	0.6
Stat. (GeV)	7.00	1.8	1.0	0.3	0.1	0.1
Total	**-**	**2.3**	**1.8**	**1.1**	**1.0**	**0.6**
Total (%)	**-**	**1.3**	**1.0**	**0.6**	**0.6**	**0.4**

Perspectives for top mass measurement at lepton colliders (ILC, CLIC, TLEP)





Top mass from threshold scan: $t\bar{t}$ bound states (strong interactions), albeit smeared by the large width (weak interactions)

Cross section calculated in NRQCD peaked at $\sqrt{s} \simeq 2m_t$ and strongly dependent on mass, width and coupling constant: $\sigma_{\rm res} \simeq \alpha_S^3/(m_t\Gamma_t)$

Calculations known up to NNLO, with NNNLO being a challenge in pQCD

Experimental simulations: 9-step scan between 346 and 354 GeV at $\mathcal{L} = 300$ fb $^{-1}$

Theoretical uncertainty (higher orders, uncertainties on m_t, Γ_t and α_S) about 3%

Total uncertainty about 100 MeV, after summing in quadrature statistics (30 MeV), luminosity (50 MeV), beam energy (35 MeV) and errors on $f(\sqrt{s_{\rm res}}, m_t)$ (80 MeV)

Conclusions and outlook

Theory error on the top mass in traditional methods mostly driven by Monte Carlo generators

Extraction of the pole mass from NNLO+NNLL cross section, but large errors

Theory errors about 3/4 of total systematic errors and dominate especially in the measurement using $J/\psi + \ell$ final states (1 GeV)

Bottom fragmentation in top decays as a case study to determine the systematic uncertainty on m_t in the J/ψ method

Relating the extracted mass to the theoretical definitions is an open issue, although it must be close to the pole mass (decay final states)

Simulations of top-flavoured hadrons may eventually become a useful benchmark

Perpectives at high energy and luminosity: $\Delta m_t \simeq 0.6$ GeV (standard methods), 0.5-0.7 GeV (endpoint), 1.1 GeV ($J/\psi + \ell$), 100 MeV (lepton colliders)

Small and understood theory errors and a clear relation between MC and pole masses should be the ultimate goal of these top quark studies