PRECISION FORM FACTORS OF PIONS, KAONS & PROTONS
at the highest timelike momentum transfers
&
FIRST MEASUREMENTS OF HYPERONS

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Electromagnetic form factors provide the most transparent insight into the structure of hadrons. At large momentum transfers a photon interacts with the charges and spins of the constituent quarks in the hadron, and provides the deepest insight into the quark-gluon structure of the hadron.

At the most fundamental level the study of the structure of the lightest hadrons is most important, i.e.

**PROTON, PION & KAON**

I will talk about our latest precision measurements of timelike form factors of these.

And then I will talk about our first excursion into a different flavor, with the **first measurements** of hyperons:

**LAMBDA, SIGMA, CASCADE, and OMEGA.**
PRELIMINARIES

• Four momentum transfers defined as
  \[ Q(4 \text{ mom.})^2 = q(3 \text{ mom.})^2 \text{space} - (\text{energy})^2 \text{time} \]
  can be positive and spacelike, or negative and timelike.

• Form factors are analytic functions of momentum transfer, and therefore, a la Cauchy, for infinite momentum transfer
  \[ \text{FF}(\text{spacelike}, Q^2 = \infty) = \text{FF}(\text{timelike}, Q^2 = \infty) \]
  Because protons are available as targets, most of the early measurements were of spacelike form factors of protons via electron elastic scattering, \( e + p \rightarrow e' + p \)

• In 1960, the first proposals for electron-positron colliders were being considered at SLAC and Frascati. In anticipation of these, Cabibbo and Gatto wrote two classic papers (PRL 4,313(1960), PRD 124,1577 (1961)) pointing out that these colliders would provide the unique opportunity to measure timelike form factors of any hadrons, mesons and baryons.

• We are now realizing the full promise of the vision of Cabibbo and Gatto!
Timelike Momentum Transfers – Preliminary

• **For baryons**, there are two form factors, the Pauli and Dirac form factors, or more familiarly, the magnetic $G_M(s)$ and the electric $G_E(s)$ form factors.

• For $e^+ e^- \rightarrow p\bar{p}$ the differential cross section is

$$
\frac{d\sigma_0(s, \theta)}{d\Omega} = \frac{\alpha^2}{4s} \beta_B \left[ |G_B^M(s)|^2 (1 + \cos^2 \theta) + \tau/2 |G_B^E(s)|^2 \sin^2 \theta \right]
$$

• At large squared momentum transfers, $s$, the quantity $\tau = 4m_p^2/s$ becomes small, the contribution of $G_E^B$ becomes small, and it becomes difficult to determine it.

• According to the **dimensional counting rule of QCD**, the above cross section decreases as $s^{-5}$, making it extremely difficult, if not impossible, to measure baryon form factors for $|Q^2| \equiv s > 20$.

• **For pseudoscalar mesons, $\pi$ and $K$**, with zero spin, there is only one form factor, $F_m(s)$, and the differential cross section is

$$
\frac{d\sigma_0(s, \theta)}{d\Omega} = \frac{\alpha^2}{4s} \beta_m |F_m(s)|^2 \sin^2 \theta
$$

Further, the cross sections decrease only as $s^{-3}$, making life at large $|Q^2|$ easier!
**Timelike Form Factors of the Proton**

- **Spacelike form factors** of the proton have been measured since the 1980’s, and precision measurements have existed for $Q^2$ up to 31 GeV$^2$.

- Prior to 1993, measurements of the **timelike form factors** of the proton by the reaction $e^+e^- \rightarrow p\bar{p}$ were sparse, had large errors, and were confined to $|Q^2| < 5.7$ GeV$^2$.

- In 1993, at Fermilab we measured $G_M(|Q^2|)$ by $pp \rightarrow e^+e^-$ for $|Q^2| = 8.9$ to 13.11 GeV$^2$. While $Q^4G_M(|Q^2|)$ was found to vary as $\alpha^2$(strong) above 9 GeV$^2$, as predicted by QCD counting rules, a big surprise was discovered. It was found that $G_M(\text{timelike}) / G_M(\text{spacelike}) \approx 2$, in strong disagreement with the **pQCD expectation** of the two being equal at large momentum transfers.
Timelike Form Factors of the Proton

- Two possible explanations of the unexpected observation
  \( G_M(\text{timelike}) / G_M(\text{spacelike}) \approx 2, \)
  at \( |Q^2| = 8 - 13 \text{ GeV}^2 \) were offered.

1. The quark distribution in the proton is not like a Mercedes star, with the three quarks having identical distributions but diquark-quark, with a preferential pairing of the two identical u-quarks.

2. \( |Q^2| = 13 \text{ GeV}^2 \) is not large enough for pQCD to be valid.

- Although no alternate explanations have been offered, the diquark-quark model did not acquire acceptance. More about this later.

- To test the second possibility, the validity of pQCD at large \( |Q^2| \), we have made high precision measurements of \( G_M(p) \) to timelike \( |Q^2| = 14.2 \text{ and } 17.4 \text{ GeV}^2 \). using data taken at the \( e^+ e^- \) CESR collider at Cornell, and the detector CLEO-c.
PREJUDICES & OBSTACLES

- In trying to measure form factors at a collider like CESR at Cornell, one has to overcome two big obstacles.

1. The first is the prejudice that only weak interaction flavor physics is important, the rest has little priority. It is an uphill battle to get the required beam time allocated for form factor measurements.

2. The second obstacle is more generic. Everybody loves resonances, and they want to love to run on peaks of resonances.

- Unfortunately, hadron form factors are not weak interaction physics and you do not want to measure on the peaks of vector resonances which directly decay into $e^+e^-$. 

- Unless, of course, you can show that the resonances at which you want to run have negligibly small cross sections for decay into the hadron pairs of your interest, i.e., $\sigma(R) \not\rightarrow h^+ h^-$. Our measurements are based on just this fact being true for resonances above $D\bar{D}$ threshold at 3.73 GeV, so that we are able to use data taken at $\psi(3770)$ and $\psi(4160)$ to measure form factors.
An important pQCD prediction is that since both leptonic and hadronic decays of charmonium resonances depend on wave functions at the origin, the ratios of their branching fractions are identical,

$$\frac{B(\psi(n'))}{B(\psi(n))} \text{ to hadrons} = \frac{B(\psi(n'))}{B(\psi(n))} \text{ to leptons}$$

This simple prediction allows us to estimate branching fractions for a specific hadronic decay of a resonance $\psi(n')$ if that same decay has been measured at another resonance $\psi(n)$. Since, $B(\psi(3770), \psi(4170) \to e^+e^-)/B(\psi(3686) \to e^+e^-) = (0.36,1.04) \times 10^{-3}$, we conclude that the branching fractions for the hadronic decays of $\psi(3770)$ and $\psi(4160)$ are more than three orders of magnitude smaller than the corresponding measured decays of $\psi(3686)$.

With nearly 5 million $\psi(3772)$ and $\psi(4160)$ each, formed in the present measurements, and our detection efficiencies, we estimate resonance events

$$\begin{align*}
  \pi^+\pi^- & \quad K^+K^- & \quad p\bar{p} & \quad \Lambda\bar{\Lambda} & \quad \Sigma^+\bar{\Sigma}^+ & \quad \Sigma^0\bar{\Sigma}^0 & \quad \Sigma^-\bar{\Sigma}^- & \quad \Xi^0\bar{\Xi}^0 & \quad \Omega^-\bar{\Omega}^- \\
  \sim 0.04 & \quad 0.4 & \quad 1.3 & \quad 0.9 & \quad 0.2 & \quad 0.2 & \quad 0.2 & \quad 0.05 & \quad 0.03
\end{align*}$$

The observed counts for each decay turn out to be about 100 times larger than these resonance contributions. Therefore, all observed $e^+e^- \to \pi\pi, KK, p\bar{p}$, and hyperon yields we observe can be attributed to form factors.
The CLEO-c detector is a cylindrical general purpose detector. The detector components important for the present measurements are the CsI electromagnetic calorimeter, the drift chamber for charged particle detection, and the RICH detector, all of which are located in a 1 Tesla solenoidal magnetic field. The acceptance for photons and charged particles in the central detector is $|\cos \theta| < 0.8$.

Charged particle resolution is

$$\sigma_p/p = 0.6\% @ 1\ GeV/c.$$  

Photon resolution is

$$\sigma_E/E = 2.2\% @ 1\ GeV,$$

and $5\% @ 100\ MeV$.

The data we use consists of

805 pb$^{-1}$ at $\psi(3770)$, $|Q^2| = 14.2\ GeV^2$, and

586 pb$^{-1}$ at $\psi(4170)$, $|Q^2| = 17.4\ GeV^2$. 
PROTON FORM FACTORS

\[ N_p \quad \sigma^p_B \text{ (pb)} \quad G^p_M \times 10^2 \quad Q^4 G^p_M / \mu_p \]

| \( \psi(3770), \ |Q^2| = 14.2 \text{ GeV}^2 \) | 213(15) | 0.46(4) | 0.88(4) | 0.64(3) |
| \( \psi(4170), \ |Q^2| = 17.4 \text{ GeV}^2 \) | 92(10) | 0.29(4) | 0.76(4) | 0.82(4) |

• With less than 5% errors, it is significant that \( Q^4 G^p_M / \mu_p \) at 14.2 GeV\(^2\) is (22 \pm 4)% smaller than at 17.4 GeV\(^2\).
The QCD counting rule prediction of $|Q^{-4}|$ variation of $G_M^p |Q^2|$ continues. However, there is an unexpected dip, with $G_M(14.2 \text{ GeV}^2)$ lower by $(22 \pm 4)\%$.

The $G_M(\text{timelike}) / G_M(\text{spacelike}) \approx 2$ persists, event at 17.4 GeV$^2$.

Despite $>300$ observed counts, we are not able to determine $G_E / G_M$. We obtain $G_E^p / G_M^p = 0.6^{+0.5}_{-1.6}$, with an upper limit of < 1.02 at 90% CL.
I now turn to the form factors of pions and kaons

1. The first thing to notice is that for spin zero pseudoscalars like $\pi^\pm$, $K^\pm$ there is no magnetic form factor*, and there is just one form factor.

$$\sigma(e^+e^- \rightarrow \pi^+\pi^-, K^+K^-) = \frac{\pi\alpha^2\beta}{3s} |F_{\pi,K}(s)|^2$$

Since $F_{\pi,K}(s)$ are expected to vary as $s^{-1}$, the cross sections decrease as $s^{-3} (\equiv |Q^{-6}|)$

2. Before 1990, almost no experimental data with any precision existed for pion and kaon spacelike or timelike form factors for $|Q^2| > 5$ GeV$^2$.

3. **Historical note:** Recall the famous Brodsky versus Isgur/Llwellyn-Smith debate (1984—1989) on when the momentum transfer is large enough for perturbative QCD to be valid. At that time, the discussion could only use the small amount of small $Q^2$ data for $F_\pi$ with larger errors which was available then.

*The quarks in pseudoscalar mesons have spins, and at large $|Q^2|$ one could, in principle, have magnetic form factors!
Form Factors of Pions and Kaons (pre-1990)

$F_{\pi,K}(\text{timelike})$

For $|Q^2| > 5 \text{ GeV}^2$

Up to $\pm 100\%$ errors

$F_{\pi,K}(\text{spacelike})$

Data limited to

$Q^2 < 2.5 \text{ GeV}^2$ for $\pi$

$Q^2 < 0.12 \text{ GeV}^2$ for $K$
Spacelike Form Factors of Pions and Kaons

- Spacelike form factors of mesons are very difficult to measure, because meson targets do not exist. Two different methods have been used.

1. \( F_\pi \) and \( F_K \) from Elastic Scattering of pions/kaons off atomic electrons, \( \pi(K)e^- \rightarrow \pi(K)e^- \). Unfortunately, in this approach the momentum transfer is very small. At CERN for 200 GeV pions, \( Q^2(\pi) \leq 0.25 \text{ GeV}^2 \) and \( Q^2(K) \leq 0.11 \text{ GeV}^2 \) were realized.

2. \( F_\pi \) from Electroproduction of pions, \( e^-p \rightarrow e^-\pi^+n \), has serious theoretical problems and uncertainties. The good precision data are confined to \( Q^2 < 2.45 \text{ GeV}^2 \) (JLab). \( F_K \) from Electroproduction of kaons — No data exist.

- As you will see, excellent timelike form factor data for \( \pi \) and \( K \) at large \( Q^2 \) now exist. It is a pity that the corresponding spacelike data do not exist to allow us to determine if the ratio \( F_{\pi,K}(\text{timelike})/F_{\pi,K}(\text{spacelike}) \approx 2 \), as it is for protons.
Timelike form factors of any hadron can be determined by measuring

\[ \sigma(e^+ e^- \rightarrow h^+ h^-), \quad h = \pi, K, p, \text{hyperons}, \]

but one has to reject 3 to 4 orders of magnitude larger background of QED–produced e^+e^- and \(\mu^+\mu^-\) pairs, and substantial tails of lighter hadrons.

- Cabibbo anecdote!
Measurements of Pion and Kaon Form Factors

- I will not bore you with the nitty-gritty of how using all the detector components of CLEO-c, the drift chambers, the central calorimeter, the RICH detector, and muon detector, we were able to identify $\pi$, $K$, and $p$ cleanly, in presence of the monstrous backgrounds of electrons and muons. Here is how clean!

$$X_h \equiv \frac{[E(h^+) + E(h^-)]}{\sqrt{s}}$$

3.77 GeV – MC

3.77 GeV – Data

4.17 GeV – Data
Pion and Kaon Form Factors

• The angular distributions for both pions and kaons at both $\sqrt{s} = 3772$ MeV and 4170 MeV fit very well the $\sin^2 \theta$ distribution for electric form factors. There is little evidence for $(1 + \cos^2 \theta)$ distribution contribution expected for a magnetic form factor.
# Pion and Kaon Form Factors – Results

(PRL 95, 261803 (2005), PRL 110, 022002 (2013))

![Graphs showing form factors for pions and kaons](image)

| $Q^2$ (GeV$^2$) | $N(\pi^+\pi^-)$ | $\sigma$(Born), pb. | $F_\pi(|Q^2|) \times 10$ | $Q^2 F_\pi(|Q^2|)$ (GeV$^2$) |
|-----------------|-----------------|---------------------|------------------------|-----------------------------|
| **CLEO(2005)**  | 13.5            | 26 ± 5              | 9.0 ± 1.8              | 0.75 ± 0.09                 | 1.02 ± 0.13                 |
| **NU(2013)**    | 14.2            | 661 ± 26            | 6.36 ± 0.25            | 0.65 ± 0.01                 | 0.92 ± 0.04                 |
| **NU(2013)**    | 17.4            | 213 ± 12            | 2.89 ± 0.16            | 0.48 ± 0.01                 | 0.84 ± 0.03                 |

| $Q^2$ (GeV$^2$) | $N(K^+K^-)$ | $\sigma$(Born), pb. | $F_K(|Q^2|) \times 10$ | $Q^2 F_K(|Q^2|)$ (GeV$^2$) |
|-----------------|-----------------|---------------------|------------------------|-----------------------------|
| **CLEO(2005)**  | 13.5            | 71 ± 9              | 5.7 ± 0.7              | 0.63 ± 0.04                 | 0.91 ± 0.14                 |
| **NU(2013)**    | 14.2            | 1564 ± 40           | 3.95 ± 0.10            | 0.54 ± 0.01                 | 0.76 ± 0.02                 |
| **NU(2013)**    | 17.4            | 644 ± 25            | 2.23 ± 0.09            | 0.44 ± 0.01                 | 0.77 ± 0.03                 |
The important experimental results are:

1. There is a remarkable agreement of the form factors for both pions and kaons with the dimensional counting rule prediction of QCD, that $|Q^2| F_{\pi,K}$ are nearly constant, varying with $|Q^2|$ only weakly as $\alpha_s(|Q^2|)$.

2. The existing theoretical predictions for pions underpredict the magnitude of $F_\pi(|Q^2|)$ at large $|Q^2|$ by large factors, ≥ 2.

3. The big surprise is that while pQCD predicts that $F_\pi/F_K=\left(f_\pi/f_K\right)^2=0.67\pm0.01$, we find: $F_\pi/F_K = 1.21 \pm 0.03$, at $|Q^2| = 14.2 \text{ GeV}^2$; $F_\pi/F_K = 1.09 \pm 0.04$, at $|Q^2| = 17.4 \text{ GeV}^2$. 
Theoretical Implications

- Lattice lives in Euclidean time, and is not capable of addressing timelike form factors. So we expect no lattice-based predictions for form factors, and have to live with predictions based on QCD–based models for timelike form factor predictions.

- The starting point of the existing calculations is factorization, with

\[ F(Q^2) = \psi_{\text{in}} \times T_H \times \psi_{\text{out}} \]

The meson wave functions \( \psi_{\text{in, out}} \) represent soft components, not calculable perturbatively. \( T_H \) represents the hard interaction, “hopefully calculable in perturbative QCD.”

- Since ab initio the quark wave functions are not known, various empirical wave functions have been used. Lepage and Brodsky used the asymptotic wave function

\[ \phi(x)_{\text{as}} \propto f_\pi x(1-x) \]

where the \( q \) and \( \bar{q} \) share momenta equally.

Chernyak and Zhitnitsky used the QCD sum-rule-inspired wave function

\[ \phi(x)_{\text{CZ}} \propto (1 - 2x^2) \phi(x)_{\text{as}} \]

which produces a two humped distribution.
Theoretical Implications (cont’d)

• The pQCD expression for form factors is (Lepage and Brodsky (1980))

\[ |Q^2| F_{\pi,K}(|Q^2|) = 16\pi \alpha_s |Q^2| \times f_{\pi,K}^2 \]

With the known decay constants \( f_{\pi} = 130.41 \text{ MeV} \), and \( f_{K} = 156.1 \text{ MeV} \),
this leads, for \(|Q^2| = 17.4 \text{ GeV}^2\), to

\[ |Q^2| F_{\pi}(|Q^2|) = 0.21 \text{ GeV}^2 \], factor 4 smaller than what we measure,
\[ |Q^2| F_{K}(|Q^2|) = 0.31 \text{ GeV}^2 \], factor 2.6 smaller than what we measure,

• This leads to the serious problem that even the ratio, which is supposed to
remove the dependence on the assumed identity of pion and kaon wave
functions, is predicted to be

\[ F_{\pi} / F_{K} = f_{\pi}^2 / f_{K}^2 = 0.67 \pm 0.01 \]

This is (36±1)% smaller than our measurement of 1.09±0.04 for \(|Q^2| = 17.4 \text{ GeV}^2\).

• With the precision of our measurements, it is quite obvious that something is
very wrong.

Could it be the assumed identity of the \(\pi\) and \(K\) wave functions ?
Could it be that pQCD is not valid even for \(|Q^2| = 17.4 \text{ GeV}^2\) ?
Theoretical Implications (cont’d)

• Since the relation $F_\pi(|Q^2|) / F_\pi(|Q^2|) = f_\pi^2 / f_K^2$ is based on assuming identical wave functions for pions and kaons, Lepage and Brodsky (1980) conjectured that because the s-quark in the kaon is $\sim 27$ times heavier than the $\langle u, d \rangle$ quarks in the pion, and the SU(3) flavor symmetry is broken, the kaon wave function may differ from the pion wave function by acquiring an asymmetric component, and account for the observed violation of the above relation.

• While large differences in pion and kaon wave functions were proposed by Chernyak & Zhitnitsky (1984), recent quenched lattice calculations (Braun et al., PRD 74, 074501 (2006)), and AdS/CFT light–front QCD model calculations (Brodsky and de Teremond, arXiv:0802.0514[hep-ph] (2008)), predict a much smaller asymmetric component in the kaon wave function, and a much smaller effect of SU(3)-breaking than CZ proposed.

• It has been suggested that an experimental determination of the effect of SU(3)-breaking can be made by measuring the form factor of the neutral kaon, and we are making such a measurement
Estimating SU(3) Breaking in the Kaon

- **Lepage and Brodsky (1980)** suggested that a large violation of the \( \frac{F_\pi}{F_K} = \frac{f_\pi^2}{f_K^2} \) identity can arise if there is a substantial SU(3) breaking effect in the kaon wave function. They predicted that a large SU(3) breaking effect would lead to a large form factor for the neutral kaon, and \( \frac{F_{K_S K_L}}{F_{K^+ K^-}} \) of the order one, and suggested that \( F_{K_S K_L} \) should be measured.

- Following this suggestion, we have made the first ever measurement of the form factor \( F_{K_S K_L}(|Q|^2) \) at \(|Q|^2| = 17.4 \text{ GeV}^2\).

- Since the cross section for \( e^+ e^- \rightarrow K_S K_L \) is expected to be small, and we do not attempt to detect \( K_{L=0} \), careful criteria for event identification had to be developed, and their efficacy tested. We have done so by measuring \( \psi(2S) \rightarrow K_S K_L \) for \(|Q|^2| = 13.6 \text{ GeV}^2\) using the same event selection criteria as for \(|Q|^2| = 17.4 \text{ GeV}^2\), and confirmed that we obtain \( \mathcal{B}(\psi(2S) \rightarrow K_S K_L) \) in agreement with its known value.
• For $e^+ e^- \rightarrow K_S K_L$ at $\sqrt{s} = 4.17$ GeV, we obtain 4 events in the signal region, and a Monte Carlo background estimate of 2 counts. This leads for $|Q^2| = 17.4$ GeV$^2$ to:

$$F_{K_SK_L}(|Q^2|) = 3.9 \times 10^{-3}, \quad 90\% \text{ CL of } (0 - 7.0) \times 10^{-3}$$

$$F_{K_SK_L}(|Q^2|) / F_{K^+K^-}(|Q^2|) = 0.09, \quad 90\% \text{ CL of } 0 - 0.16.$$

• In other words, the SU(3) breaking effect on the ratio is found to be small, certainly much less than of “the order of one”.

• To come back to the original problem of $F_{\pi}/F_K(\text{expt.}) \neq f_\pi^2 / f_K^2$, it is now apparent that it can not be attributed to SU(3) breaking alone. The problem remains unresolved.

• Here is a challenge worthy of the best theoretical attempts.
Form Factors of Hyperons

• I already told you that in 1960, before quarks were even proposed, but strangeness and strange baryons, the hyperons, were known, Cabibbo and Gatto wrote the classic papers on the measurement of timelike form factors by $e^+e^- \rightarrow \text{hadron–antihadron}$. They discussed the proton and neutron, and pion and kaon, and went on to say that it would be very interesting to measure hyperon form factors. But they noted that the cross sections are likely to be very small, and despaired whether they could be measured.

• And now we have measured hyperon form factors for the first time* with good precision at the large momentum transfer of $|Q^2| = 14.2 \text{ GeV}^2$.

• We identify the hyperons by their dominant decays. These (and their branching fractions) are:

\[
\begin{align*}
\Lambda^0 &\rightarrow p\pi^- \text{ (64%) } \\
\Sigma^+ &\rightarrow p\pi^0 \text{ (52%) } \\
\Sigma^0 &\rightarrow \Lambda\gamma \text{ (100%) } \\
\Xi^- &\rightarrow \Lambda\pi^- \text{ (100%) } \\
\Xi^0 &\rightarrow \Lambda\pi^0 \text{ (100%) } \\
\Omega^- &\rightarrow \Lambda K^- \text{ (68%) }
\end{align*}
\]

*Recently BaBar [PRD 76, 092006 (2007)] reported form factor measurements for $\Lambda\bar{\Lambda}$ and $\Sigma^0\bar{\Sigma}^0$ using the ISR method. While they have good statistical precision near threshold, the number of observed events decreases rapidly, and for $|Q^2| > 9 \text{ GeV}^2$ they are only able to obtain upper limits.
What Do We Expect to Learn from Hyperon Form Factors?

• As we go from protons to hyperons, serially replacing one, two, or three up/down quarks with strange quarks, what do we expect to learn at $|Q^2| = 14.2$ GeV$^2$?

• Do we see SU(3) breaking effects?  
  Do we see diquark correlation effects?  
  Are $G_M(B)$ for hyperons proportional to $\mu_B$, as for nucleons?  
  Do neutral hyperons have finite $G_E(Q^2)$ as the neutron?

• One strange quark ($\Lambda^0 (uds), \Sigma^0 (uds), \Sigma^+ (uus), \Sigma^- (dds)$), $J = 1/2$
  – Is there evidence for a SU(3)-breaking effects?  
    Are there diquark effects related to isospin differences: $\Lambda^0$ (I=0) and $\Sigma^0$ (I=1)?

• Two strange quarks ($\Xi^- (dss), \Xi^0 (uss)$), $J = 1/2, I = 1/2$
  – Do the Cascades show any large differences from Sigmas?  
    Is there evidence for diquark effects with two strange quarks in Cascades?  
    Do the Cascades (I=1/2) resemble Nucleons (I=1/2)?

• Three strange quarks ($\Omega^- (sss)$) $J = 3/2, I = 0$
  – How does $\Omega^-$ with 3 s-quarks differ from proton with three u/d-quarks?

Obviously not all these questions can be answered by the first measurements of hyperon form factors we report here, but they indicate the physics potential of such measurements.
Form Factors of Hyperons

- We have developed very successful event selection criteria to measure branching fractions for $\psi(2S) \rightarrow \text{hyperons}$. The figure shows the extremely clean hyperon event distributions as a function of $X = (E(h) + E(\bar{h}))/\sqrt{s}$.

- Using the same event selections we have analyzed our data of $L = 805 \text{ pb}^{-1}$ at $\sqrt{s} = 3.772 \text{ GeV}$, or $|Q^2| = 14.2 \text{ GeV}^2$ for form factor decays of hyperons.
Form Factors of Hyperons – Results

- The numbers of events $N(B\bar{B})$ in the signal region leads to the Born cross section $\sigma = N(B\bar{B})/(\epsilon L C)$, where $\epsilon$ is the MC-determined efficiency, $L = 802$ pb$^{-1}$ is the $e^+e^-$ luminosity, and $C = 0.76 – 0.78$ is the correction factor for initial state radiation. The cross section is related to the form factors as

$$\sigma(s) = (4\pi\alpha^2\beta_B /3s)[|G_M(s)|^2 + \tau /2|G_E(s)|^2].$$

- In the table we quote our results for $G_M(14.2 \text{ GeV}^2)$ assuming $G_E(s) = G_M(s)$ for both charged and neutral hyperons, because at $s \equiv |Q^2| = 14.2 \text{ GeV}^2$, finite values of $G_E(s)$ are possible even for the neutral hyperons.

<table>
<thead>
<tr>
<th></th>
<th>$N(B\bar{B})$</th>
<th>$\sigma(B\bar{B}) \text{ pb}$</th>
<th>$G_M(B\bar{B}) \times 10^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$ (uud)</td>
<td>213(15)</td>
<td>0.46( 3)</td>
<td>0.88( 3)</td>
</tr>
<tr>
<td>$\Lambda^0$ (uds)</td>
<td>105(10)</td>
<td>0.80( 8)</td>
<td>1.18( 6)</td>
</tr>
<tr>
<td>$\Sigma^0$ (uds)</td>
<td>15( 4)</td>
<td>0.28( 7)</td>
<td>0.71( 9)</td>
</tr>
<tr>
<td>$\Sigma^+$ (uus)</td>
<td>29( 5)</td>
<td>0.99(18)</td>
<td>1.32(12)</td>
</tr>
<tr>
<td>$\Xi^-$ (dss)</td>
<td>38( 5)</td>
<td>0.70(11)</td>
<td>1.14( 9)</td>
</tr>
<tr>
<td>$\Xi^0$ (uss)</td>
<td>5$^+_{-2}$</td>
<td>0.35$^{+0.20}_{-0.16}$</td>
<td>0.80$^{+0.20}_{-0.21}$</td>
</tr>
<tr>
<td>$\Omega^-$ (sss)</td>
<td>3$^+_{-1}$</td>
<td>0.14$^{+0.11}_{-0.09}$</td>
<td>0.59$^{+0.20}_{-0.23}$</td>
</tr>
</tbody>
</table>

Note: $G_M(\Lambda^0) = 1.66(23)G_M(\Sigma^0)$

Kamal K. Seth, 10/28/2013
What do the Hyperon Form Factors Tell Us

- No modern predictions of hyperon form factors for $|Q^2| > 1 \text{ GeV}^2$ exist. The old (1977) VDM-based predictions of $\sigma(e^+e^- \rightarrow B\bar{B})$ by Körner & Kuroda are factor $\sim 10$ smaller than what we measure. We can therefore only discuss qualitative features and patterns in our data.

1. No evidence is seen for $G_M(B)$ being proportional to $\mu_B$. Actually, none is expected for timelike form factors.
2. $G_M(B)$ vary from $0.6 \times 10^{-2}$ to $1.3 \times 10^{-2}$ relatively smoothly, except for $\Sigma^0$.
3. $G_M(\Sigma^+) = 1.3(1) \approx G_M(\Xi^-) = 1.1(1)$. Is their near equality due to spin correlations? In both $\Sigma^+(uus)$ and $\Xi^-(dss)$ the two like-quarks are coupled to spin-singlet, $S = 0$.

Closed circles : $G_E = G_M$,
Open circles: $G_E = 0$
$G_M(\Lambda^0)$ and $G_M(\Sigma^0)$ and Diquark Correlations

The Dramatic Difference Between $\Lambda^0$ and $\Sigma^0$:

- While both $G_M(\Lambda^0)$ and $G_M(\Sigma^0)$ have a $|uds\rangle$ construct, $G_M(\Lambda^0)$ is $\sim 70\%$ larger than $G_M(\Sigma^0)$. Why?

  We note that the isospins of $\Lambda^0$ and $\Sigma^0$ are different: $I(\Lambda^0) = 0, I(\Sigma^0) = 1$. Since only the $u$ and $d$ quarks carry isospin, it is extremely suggestive that the observed difference in $G_M$ arises due to differences in the configurations of the $u$ and $d$ quarks in $\Lambda^0$ and $\Sigma^0$.

- The spins in isoscalar $\Lambda^0$ are coupled to $u \uparrow + d \downarrow$, or $s = 0$, and the spins in isovector $\Sigma^0$ are coupled to $s = 1$. This leads to much stronger spatial correlation between the $u$ and $d$ quarks in $\Lambda^0$ compared to $\Sigma^0$. With large $|Q^2| = 14.2$ GeV$^2$ our measurements are sensitive to it. We suggest that this gives rise to $G_M(\Lambda^0)$ being much larger than $G_M(\Sigma^0)$.

**Question:** How reasonable is this explanation based on the diquark correlations between the two light quarks, $u$ and $d$?

**Answer:** Quite reasonable!
Diquark Correlations

Two-body correlations are known to play an important role in many aspects of physics, ranging from Cooper pairs in superconductivity, to pairing interactions in nuclear physics. Diquark-quark models of nucleons have been proposed to explain many observations in hadron physics, particularly the observed \( G_M(\text{timelike}) / G_M(\text{spacelike}) \approx 2 \) for the proton.

• Recently, Wilczek and colleagues have drawn attention to the fact that “it is plausible that several of the most profound aspects of low-energy QCD dynamics are connected to diquark correlations.” Wilczek goes on to actually state that
  
  – “The \( \Lambda \) is isosinglet, so it features the good diquark [ud], while \( \Sigma \), being isotriplet, features the bad diquark (ud).”
  
  – “the good diquark would be significantly more likely to be produced than the bad diquark”, and that “this would reflect in a large \( \Lambda / \Sigma \) ratio.”

• We claim that this is exactly what we are observing in the difference between \( G_M(\Lambda^0) \) and \( G_M(\Sigma^0) \) and \( \sigma(\Lambda^0) / \sigma(\Sigma^0) \approx 3 \).

• Our observations of hyperon form factors thus constitute one of the best evidence for diquark correlations.