Quantum Field Theory for Gravity

Can we have a QFT for quantum gravity?

What a weird
theoretical idea!

We DO have a QFT for quantum general relativity --- EFT

Diagnosis and obstacles for the UV

Inducing the Einstein action using Yang-Mills theories

The spin connection, confinement, inducement

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I. The problem of quantum gravity

If the Standard Model had been proposed in 1914, it would have developed a reputation as an impossible quantum theory

- SM grew together with QFT techniques
- each interaction required some new variation
- need renormalization, path integrals, FP ghosts, confinement, spontaneous and dynamical symmetry breaking, Higgs mechanism, dimensional regularization, anomalies, etc.

Quantum General Relativity needs Effective Field Theory also

<u>GR forms a good quantum effective field theory</u>

- predictive at low energy
- EFT points to demise at or below Planck energy
- still need a UV complete quantum theory

The Effective Field Theory of General Relativity

Can construct GR as a QFT and quantize it naturally

All divergences correspond to local terms in the action – renormalize
- but these are not interesting!

$$S_{grav} = \int d^4x \sqrt{-g} \left\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \dots \right\}$$

1/16πG

EFT tells us how to isolate low energy predictions
- parameters in L are not predictions of the EFT
- but non-local / non-analytic effects are

$$\sqrt{-q^2} \quad , \qquad \log(-q^2)$$

- real low energy propagation

Can make real unambiguous calculations at low energy

Power counting

Recall $R \sim \partial^2 g$

Expansion in the energy/curvature

- Loops create more powers of derivatives/curvatures

No loops ~ order $E^2 \sim R$

One loop ~ order $E^4 \sim R^2$

Matter loops also

Two loops ~ order $E^6 \sim R^3$

Dimensionful coupling constant:

$$\mathrm{Amp} \sim GE^2\left[1 + GE^2(a + b\log E^2) + G^2E^4 + ...\right]$$

Good for the EFT

Bad as property of a UV complete theory

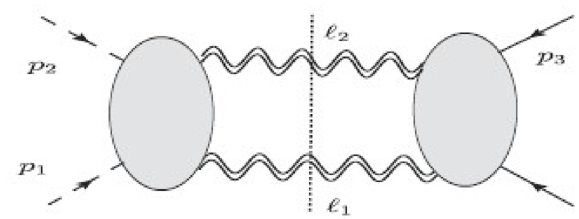
Some recent results:

Old result: Quantum correction to Newtonian potential

$$V(r) = -\frac{Gm_1m_2}{r}\left[1 + 3\frac{G(m_1+m_2)}{r} + \frac{41}{10\pi}\frac{G\hbar}{r^2}\right]$$

Holstein, Ross
JFD, Bjerrum-Bohr, Vanhove

Newer variant: One loop soft theorem
- unitarity techniques
- Compton amplitudes have soft theorems
- leading one loop amplitudes universal



Light bending at one loop: EP violation
- unitarity plus eikonal

$$\mathrm{bu}^{\varphi} = \frac{371}{120}, \qquad \mathrm{bu}^{\gamma} = \frac{113}{120}.$$

$$\theta_\eta = \frac{4GM}{b} + \frac{15}{4}\frac{G^2M^2\pi}{b^2} + \frac{8\mathrm{bu}^\eta - 47 + 64\log\frac{2r_0}{b}}{\pi}\frac{G^2\hbar M}{b^3}.$$

Bjerrum-Bohr, JFD, Holstein
Plante, Vanhove
Bai and Huang

-no longer moving on null geodesics

Non-local effective actions:

Barvinsky, Vilkovisky, Avrimidi

Perturbative running is contained in the R^2 terms

$$S_4 = \int d^4x \sqrt{g} \left[c_1(\mu)R^2 + c_2(\mu)R_{\mu\nu}R^{\mu\nu}\right]$$
$$+ \left[\bar{\alpha}R\log(\nabla^2/\mu^2)R + \bar{\beta}C_{\mu\nu\alpha\beta}\log(\nabla^2/\mu^2)C^{\mu\nu\alpha\beta}\right.$$
$$\left.+\bar{\gamma}\left(R_{\mu\nu\alpha\beta}\log(\nabla^2)R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}\log(\nabla^2)R^{\mu\nu} + R\log(\nabla^2)R\right)\right] + \mathcal{O}(R^3)$$

Hints of singularity avoidance:

JFD, El-Menoufi

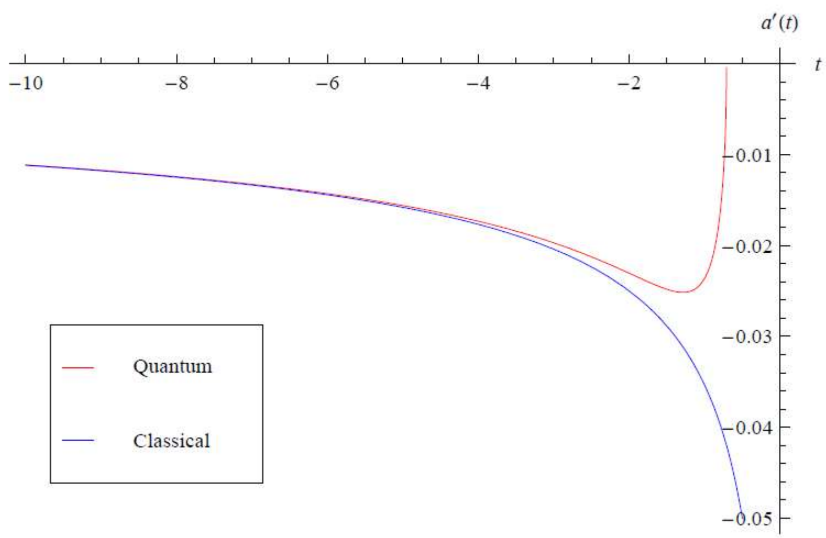


FIG. 12: Collapsing radiation-filled universe with gravitons only considered.

- No free parameters in this result

Modern view:

We have a quantum theory of gravity

It has the form of an effective field theory

We can make predictions at low energy

The effective theory points to the need of a UV completion

We will need to find a more complete theory eventually

This is clear progress!

What to do?

1) Nothing

- look for other problems that are testable

2) Try to form a non-perturbatively complete theory with GR

Asymptotic safety

3) Drastically change the theory

String theory
Loop quantum gravity
Dynamical triangulations
Gravity as quantum information
Causal sets
Entropic theories

4) Conventional QFT?

So far Nature has employed only gauge QFTs

Can we have a conventional QFT for gravity?
- with some variation

QFT nature explored in late 70's and abandoned
- partially for good physics reasons
- partially the lure of new ideas

But understanding of QFT advances

Lets revisit the topic with modern eyes

Diagnosis 1:

Matter loops renormalize R^2 terms

- at one loop – and at **all loop order**
- dimensionless coupling constant

R^2 terms must be in a fundamental QFT action

R^2 terms lead to quartic propagators

$$R^2 \sim \partial^2 g \partial^2 g \qquad\qquad D(q) = \frac{i}{q^4}$$

With quartic propagators, graviton loops also stop at R^2

- all orders
- dimensionless coupling constant
- R^2 theories renormalizeable (Stelle)

QFT "variation" needs quartic propagators

Quartic propagators and the physical spectrum

Traditionally we start an analysis of a theory with the free field limit
- form Hamiltonian
- define the asymptotic states

But we know theories for which the free field limit is meaningless concerning the asymptotic states

Confining theories
- Lagrangian fields do not appear in the physical spectrum at all

Stongly interacting theories can modify properties from the Lagarangian to the spectrum

Are some "forbidden" QFT aspects OK if they do not appear in the physical spectrum?

1) **Extra time derivatives and Ostrogradsky instability**

Most often start off analysis with the free Hamiltonian

Woodard
Scolarpedia

With $\mathcal{L} = \mathcal{L}(\phi, \dot{\phi}, \ddot{\phi}),$

Two coordinates ϕ and $\dot{\phi}$

Two momenta:

$$\pi_1 = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \ddot{\phi}}$$

$$\pi_2 = \frac{\partial \mathcal{L}}{\partial \ddot{\phi}} \quad .$$

Form Hamiltonian:

$$\mathcal{H} = \pi_1 \dot{\phi} + \pi_2 a(\phi, \dot{\phi}, \pi_2) - \mathcal{L}(\phi, \dot{\phi}, a(\phi, \dot{\phi}, \pi_2))$$

First term is signal of instability

But is free Hamiltonian always a reliable guide?

- what about confining theories?

Path Integrals sometimes tell a different story

Consider fourth order gauge theory:

$$\mathcal{L} = \frac{1}{2}(D^2\phi)^\dagger D^2\phi + ...$$

Doing PI yields:

$$\frac{1}{[\det(D^2D^2)]^{1/2}} = e^{-\frac{1}{2}Tr\log(D^2D^2)} = e^{-\frac{1}{2}\int d^4x <x|Tr\log(D^2D^2)|x>}$$

But we have:

$$\log(D^2D^2) = 2\log(D^2)$$

Up to constant, this is the usual gauge interaction

With PI, one can have a fine perturbation theory

Generating functional:

$$Z_0[J] = \int [d\phi] \exp i \int d^4x \frac{1}{2} \Box\phi\Box\phi - J\phi$$

$$Z_0[J] = Z_0[0] \exp -i \int d^4x d^4y \frac{1}{2} J(x)\Delta(x-y)J(y)$$

with propagator:

$$\Delta(x-y) = < x|\frac{1}{\Box^2}|y >= \int d^4q \; \frac{e^{-iq\cdot(x-y)}}{q^4}$$

Explicit calculations in literature without obvious sickness

Background field renormalization can be accomplished

- for suitably general Lagrangian

With $\mathcal{L}(U) = \mathcal{L}(\bar{U}) + \Delta^a \mathcal{O}^{ab} \Delta^b + \ldots$

$$\mathcal{O}^{ab} = \left[D^2 D^2 + A^{\alpha\beta\gamma} D_\alpha D_\beta D_\gamma + B^{\alpha\beta} D_\alpha D_\beta + C^\alpha D_\alpha + E\right]^{ab}$$

The divergences are captured in the heat-kernel coefficient:

$$Tr < x|\log \mathcal{D}|x > |_{div} = \frac{i}{(4\pi)^{d/2}} \Gamma(2 - \frac{d}{2})\ Tr\ a_2(x)$$

Barvinsky and Vilkovisky have worked this out

$$\begin{aligned}
a_2 \ =\ & \frac{1}{6} F_{\mu\nu} F^{\mu\nu} - \frac{1}{8} F_{\mu\nu} [D^\mu, A^\nu] + \frac{9}{80} F_{\mu\nu} A^{\mu\alpha\beta} A^{\nu}{}_{\alpha\beta} + \frac{9}{160} F_{\mu\nu} A^\mu A^\nu + \frac{1}{8} C_\mu A^\mu + \frac{1}{24} B_{\mu\nu} B^{\mu\nu} + \frac{1}{48} B^2 \\
- \ & \frac{1}{16} B D_\mu A^\mu + \frac{1}{8} B^{\mu\nu} D_\mu A_\nu - \frac{1}{8} B^{\mu\nu} D^\alpha A_{\mu\nu\alpha} + \frac{9}{80} A^{\mu\nu\alpha} D_\mu D_\nu A_\alpha - \frac{3}{80} A^\mu D_\mu D_\nu A^\nu - \frac{3}{160} A^\mu D^2 A_\mu \\
- \ & \frac{3}{40} A^{\mu\nu\alpha} D_\mu D_\beta A^{\beta}{}_{\nu\alpha} - \frac{1}{80} A^{\mu\nu\alpha} D^2 A_{\mu\nu\alpha} - \frac{1}{640} B(2A^{\mu\nu\alpha} A_{\mu\nu\alpha} + 3A^\mu A_\mu) \\
- \ & \frac{3}{320} B^{\mu\nu} (2A_{\mu\alpha\beta} A_\nu{}^{\alpha\beta} + A_\mu A_\nu + A_{\mu\nu\alpha} A^\alpha + A^\alpha A_{\mu\nu\alpha}) - \frac{1}{640} A^\beta D_\beta (2A^{\mu\alpha\beta} A_{\mu\alpha\beta} + 3A^\mu A_\mu) \\
- \ & \frac{1}{640} A^\beta [2(D_\beta A^{\mu\nu\alpha}) A_{\mu\nu\alpha} + 3(D_\beta A^\mu) A_\mu] - \frac{3}{160} A^{\mu\nu\alpha} D_\mu (A_\nu A_\alpha + 2A_{\nu\beta\gamma} A_\alpha^{\beta\gamma} + A_{\nu\alpha\beta} A^\beta + A^\beta A_{\nu\alpha\beta}) \\
+ \ & \frac{3}{320} A^{\mu\nu\alpha} (A_\nu D_\mu A_\alpha + 2A_{\nu\beta\gamma} D_\mu A_\alpha^{\beta\gamma} + A_{\nu\alpha\beta} D_\mu A^\beta + A^\beta D_\mu A_{\nu\alpha\beta}) \\
+ \ & \frac{1}{960 \times 32 \times 42} A^{\mu\nu\alpha} A^{\beta\gamma\delta} A^{\epsilon\sigma\rho} A^{\lambda\omega\eta} g_{\mu\nu\alpha\beta\gamma\epsilon\sigma\rho\lambda\omega\eta}
\end{aligned}$$

Higher derivatives in effective Lagrangians

For example, photons at low energy

$$\mathcal{L}_{eff} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\alpha}{60\pi m^2}\partial_\lambda F_{\mu\nu}\partial^\lambda F^{\mu\nu}$$

From vacuum polarization:

$$\begin{aligned} \Pi(q) &= \frac{e^2}{12\pi^2}\left[\frac{1}{\epsilon} + \log 4\pi - \gamma - 6\int_0^1 dx x(1-x)\log\left(\frac{m^2 - q^2 x(1-x)}{\mu^2}\right)\right] \\ &= \frac{e^2}{12\pi^2}\left[\frac{1}{\epsilon} + \log 4\pi - \gamma - \log\frac{m^2}{\mu^2} + \underset{**}{\frac{q^2}{5m^2}} + ...\right] \end{aligned}$$

But no instability of ground state

2) Expectation of ghosts/ negative norms

From:

$$\frac{-i}{q^4} \sim \frac{-i}{q^2(q^2-\mu^2)} = \frac{1}{\mu^2}\left(\frac{i}{q^2} - \frac{i}{q^2-\mu^2}\right)$$

But QED shows how this can be misleading:

$$\frac{1}{q^2 + \frac{\alpha}{15\pi m^2} q^4} = \frac{1}{q^2} - \frac{1}{q^2 + \frac{15\pi m^2}{\alpha}}$$

Propagator actually well defined at all energies
- no ghosts/negative norms

The Lee-Wick exception:

Example from Grinstein, O'Donnell Wise

Class of consistent higher derivative theories
- negative norm state is unstable – removed from spectrum

$$\mathcal{L}_{\rm hd} = \frac{1}{2}\partial_\mu\hat{\phi}\partial^\mu\hat{\phi} - \frac{1}{2M^2}(\partial^2\hat{\phi})^2 - \frac{1}{2}m^2\hat{\phi}^2 - \frac{1}{3!}g\hat{\phi}^3,$$

$$\hat{D}(p) = \frac{i}{p^2 - p^4/M^2 - m^2}.$$

Use auxiliary field to convert to quadratic propagators

$$-\tilde{\phi}\partial^2\hat{\phi} + \frac{1}{2}M^2\tilde{\phi}^2$$

Then straightforward manipulations

Second (ghost) state decays

Unitarity preserved in scattering amplitudes

Causality unclear on short time scales

Note: Loops do the same in the graviton propagator

Both matter and graviton loops
Effect of unitarity

Tomboulis 1977

$$iD^{\alpha\beta,\mu\nu}(q^2) = \frac{i\left[L^{\alpha\mu}L^{\beta\nu} + L^{\alpha\nu}L^{\beta\mu} - L^{\alpha\beta}L^{\mu\nu}\right]}{2q^2\left(1 - \frac{N_s G_N q^2}{120\pi}\log\left(-\frac{q^2}{\mu^2}\right)\right)} .$$

$$L^{\mu\nu}(q) = \eta^{\mu\nu} - q^\mu q^\nu / q^2 .$$

Decay of high mass ghost states could be a common feature

3) Kallen-Lehmann spectral representation:

For scalar field:

In order to derive a spectral representation for the propagator of a field $\Phi(x)$, one consider a complete set of states $\{|n\rangle\}$ so that, for the two-point function one can write

$$\langle 0|\Phi(x)\Phi^\dagger(y)|0\rangle = \sum_n \langle 0|\Phi(x)|n\rangle\langle n|\Phi^\dagger(y)|0\rangle.$$

We can now use Poincaré invariance of the vacuum to write down

$$\langle 0|\Phi(x)\Phi^\dagger(y)|0\rangle = \sum_n e^{-ip_n\cdot(x-y)}|\langle 0|\Phi(0)|n\rangle|^2.$$

Let us introduce the spectral density function

$$\rho(p^2)\theta(p_0)(2\pi)^{-3} = \sum_n \delta^4(p-p_n)|\langle 0|\Phi(0)|n\rangle|^2.$$

Leads to propagator:

$$\Delta(p) = \int_0^\infty d\mu^2 \rho(\mu^2)\frac{1}{p^2-\mu^2+i\epsilon},$$

Scalar propagator cannot fall faster than $1/p^2$ in UV

But this fails for Yang Mills:

Oehme
Zimmerman

Landau gauge:

$$\Delta(p^2) \sim \frac{1}{p^2}[\ln(\frac{p^2}{\mu^2})]^{-\alpha_0/2b}$$

$$\frac{\alpha_0}{2b} = \frac{13}{22}$$

Reasons discussed:

- confinement
- Unphysical d.o.f.
- Physical gauge d.o.f. are not Lorentz invariant
- Ghosts
- Related to Gribov problem?

JFD
PRD 2018

Aside: Test case

Triplet scalar in SU(2) with quartic propagator

SU(2) valued field

$U = e^{i\frac{\tau^a \phi^a}{f}}$ with $U \to V(x) U V^\dagger(x)$ $V(x)$ in SU(2).

with scale invariant Lagrangian

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{f^2}{4} Tr\left[(D^\mu D_\mu U)^\dagger (D^\nu D_\nu U)\right] \\
&+ d_1 \left(Tr\left[D^\mu U^\dagger D_\mu U\right]\right)^2 + d_2 Tr\left[D^\mu U^\dagger D^\nu U\right] Tr\left[D_\mu U^\dagger D_\mu U\right] \\
&+ d_3 Tr\left[D^\mu U^\dagger D^\nu U D_\mu U^\dagger D_\mu U\right] + d_4 Tr\left[U^\dagger D^2 U\right] Tr\left[D_\mu U^\dagger D^\mu U\right] \\
&+ d_5 Tr\left[U^\dagger D^2 U D_\mu U^\dagger D^\mu U\right] + d_6 Tr\left[U^\dagger D^2 U\right] Tr\left[U^\dagger D^2 U\right] \\
&+ d_7 F^i_{\mu\nu} Tr\left[\tau^i D^\mu U^\dagger D^\nu U\right] + d_8 F^i_{\mu\nu} F^{j\mu\nu} Tr\left[\tau^i U^\dagger \tau^j U\right]
\end{aligned}
$$

Scalar will be confined – usual discussions of g.s. not relevant.
Should be possible to simulate on lattice

Gauge renormalization is simple:

Expand to quadratic order

$$\mathcal{L} = \frac{1}{2}\phi^a D^2 D^2 \phi^a + ...$$

do the path integral and use $\log(D^2 D^2) = 2\log(D^2)$

The diverernces are just twice the usual ones:

$$S_{div} = \int d^4x \, \frac{1}{\epsilon}\frac{2a_2}{16\pi^2} = \int d^4x \, \frac{C_2}{48\pi^2\epsilon} F^i{\mu\nu} F^{i\mu\nu}$$

Result is asymptotically free

$$\beta(g) = -\left[\frac{22}{3} - \frac{1}{4}C_2\right]\frac{g^3}{16\pi^2}$$

Despite non-linearity, the scalar loops are renormalizeable also

Quartic propagators are the challenge for gravity QFT

Needed for renormalizability:
- matter loops generate divergences at R^2
- gravitational loops with quartic propagators become renomalizeable

But need pathway around the known pitfalls

Some disconnect between Lagrangian fields and physical spectrum
- starting with free field theory is not going to work
- need “normal” physical spectrum in the end
- perhaps inducing normal behavior on-shell
- perhaps confinement removes unphysical fields/behavior

Second aspect of diagnosis:

Despite quartic propagators, need usual ones at low energy
- well defined ground states
- experiment!

Low energy action is Einstein-Hilbert

Observation: Strongly interacting theories generate changes in Cosm. Const and in G

Can we use strong interactions to induce Einstein action?

<u>Whenever you move in some direction in GR, there are always others ahead of you</u>

Important work done by:

Stelle
Fradkin and Tseytlin
Adler
Smilga
Zee
Tomboulis
Hasslacher and Mottola
Mannheim
Holdom and Ren
Einhorn and Jones
Salvio and Strumia
Lu , Perkins, Pope, Stelle

And many others

<u>Analogy – Two flavor massless QCD</u>

Lagrangian is <u>scale invariant</u>

QCD is weakly coupled in <u>both</u> UV and IR

UV story is well known – asymptotic freedom

As we come down in energy – strong coupling region 2 GeV to 0.5 GeV

But at low energy, the chiral symmetry requires massless degrees of freedom
 - organized as an effective field theory

$$\mathcal{L} = \frac{F^2}{4}\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) \qquad \text{with} \qquad U = exp\left[\frac{i\tau \cdot \pi}{F}\right]$$

This is weakly coupled in the IR
 - explicitly depends on QCD scale
 - going up in energy enters the strong coupling region

If we had uncovered pionic theory first, we would think that there was an impassable barrier at 1 GeV.

Dimensional transmutation at work:

Low energy actions are not scale-invariant

Not just logarithmic corrections in running

Scale of QCD features in all low energy physics

Purely perturbative corrections are logarithmic:

$$V_{\text{eff}} = \frac{3e^4}{64\pi^2}\,\phi_c^4\left(\ln\frac{\phi_c^2}{\langle\phi\rangle^2} - \frac{1}{2}\right)$$

Non-perturbative corrections carry power dependence on scale of the theory

$$\Lambda \sim M e^{-\frac{8\pi^2}{g^2(M)}}$$

Dimensional transmutation with gravity?

Start with **scale/conformal** invariant action

Couplings will run

– asymptotically free but strong coupling at low energy

Running coupling defines the Planck scale

Low energy theory is EFT for the metric

- using dimensional transmutation for the scale

$$S = \int d^4x \sqrt{-g} \left[-\Lambda - \frac{2}{\kappa^2} R + ... \right]$$

Weakly coupled in both the UV and IR

Scale invariant gravity - generalities

Einstein Hilbert action is not scale invariant:

$$\frac{2}{\kappa^2}R \sim M_P^2 \partial g \partial g + \ldots$$

Curvature squared terms are scale invariant

$$c_i R^2 \sim \partial^2 g \partial^2 g + \ldots$$

R+R^2 theories have ghosts in general

- but disagreements over importance and inevitability

Pure R^2 theories have quartic propagators

- infrared issues

R^2 theories are perturbatively renomalizeable (Stelle)
and asymptotically free (Fradkin, Tseytlyn)

<u>Power-counting modifications</u>

1) **Difference in power-counting of gauge fields and gravity**
   - at one loop both are of order R^2
   - but gravity loops keep increasing powers R^n
   - further gauge loops stay are order R^2

2) **Pure quadratic gravity also stops at order R^2**
   - no scale in theory
   - no generation of higher dimension operators
   - with usual expansion, a bit more subtle

$$\text{propagator} \sim \frac{M_P^2}{k^4} \qquad\qquad \text{vacuum polarization} \sim \kappa k^2 \; (\frac{M_P^2}{k^4})^2 \; \kappa k^2 \;\; \sim \frac{M_P^2}{k^4}$$

QCD plus scale invariant gravity

Could induce Einstein Hilbert action via dimensional transmutation

Divergences from QCD loops proportional to Weyl-squared

But finite induced effects proportional to QCD scale

$$S_{induced} \sim \int d^4x \ \left[\Lambda_{QCD}^4 \ + \ \Lambda_{QCD}^2 R + ...\right]$$

Can we repeat this at the Planck scale?

Adler, Smilga, Zee, Hasslacher + Mottola1982

SU(N) + gravity

Take the YM theory with the highest energy scale

- coupled to onformal gravity

YM should induce largest contribution to M_P

$$S = \int d^4x \sqrt{-g}\left[-\frac{1}{4g_N^2} g^{\mu\alpha} g^{\nu\beta} F^a_{\mu\nu} F^a_{\alpha\beta} - \frac{1}{2\xi^2} C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta}\right]$$

Both couplings are asymptotically free. At one loop,

$$\begin{aligned} \beta(g) &= -\frac{11N}{3\pi^2} g^3 \\ \beta(\xi) &= -\frac{199}{480\pi^2}\xi^3 - \frac{N^2-1}{160\pi^2}\xi^3 \end{aligned}$$

Induced Einstein-Hilbert action makes physical spectrum better

- matches on to effective field theory
- avoiding Ostrogradsky instability

Perhaps this is sufficient!?

Calculating the induced G for QCD

Adler-Zee formula:

$$\frac{1}{16\pi G} = \frac{i}{96} \int d^4x \; x^2 < 0|T \; T(x)T(0)|0 >$$

where:

$$T(x) = \eta_{\mu\nu} T^{\mu\nu}(x)$$

Quick derivation:

Weak field limit:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Gravitaional coupling:

$$\mathcal{L} = -\frac{1}{2} h^{\mu\nu} T_{\mu\nu}$$

To second order in the gravitational field:

$$i \int d^4x \mathcal{L}_{eff} = \frac{1}{2}\left(\frac{-i}{2}\right)^2 \int d^4x\ d^4y\ h_{\mu\nu}(x) h_{\alpha\beta}(y) < 0|T\ T^{\mu\nu}(x) T^{\alpha\beta}(y)|0 >$$

For convenience, consider special case (see also Brown, Zee)

$$h_{\mu\nu}(x) = \frac{1}{4}\eta_{\mu\nu} h(x)$$

Slowly varying fields (on QCD scale):

$$h(y) = h(x) + (y-x)^\mu \partial_\mu h(x) + \frac{1}{2}(y-x)^\mu (y-x)^\nu \partial_\mu \partial_\nu h(x) + ...$$

This results in the effective Lagrangian:

$$\begin{aligned} i\mathcal{L}_{eff}(x) &= -\frac{1}{128}h^2(x)\int d^4z \;\; < 0|T\;T(z)T(0)|0 > \\ &+ \frac{1}{1024}(\partial_\mu h(x))^2\int d^4z\; z^2 \;\; < 0|T\;T(z)T(0)|0 > \end{aligned}$$

The first term is part of the cosmological constant, the second is the Einstein action. Identify via:

$$\sqrt{-g}R = -\frac{3}{32}(\partial_\mu h(x))^2$$

This gives the Adler-Zee formula:

$$\frac{1}{16\pi G} = \frac{i}{96}\int d^4x\; x^2 < 0|T\;T(x)T(0)|0 >$$

Example: QCD

For QCD (with no quarks or massless quarks)

$$T^{\mu}_{\ \mu} = \frac{\beta(g)}{2g} F^{a}_{\mu\nu} F^{a\mu\nu}$$

From trace anomaly

F^2 creates a scalar glueball from the vacuum

The scalar correlator

$$< 0|F^2(x)\ F^2(0)|0 >$$

has been well studied in QCD.

Induced G – in SU(N) theories

JFD, Menezes
Dec. 2017 arXiv

Construct the sum rule for QCD – lattice data, OPE and pert. theory

Separate long and short distance techniques at $x=x_0$

$$\psi(x) = <0|T\, T(x)T(0)|0> = [\psi_{pert}(x) + \psi_{OPE}(x)]\, \Theta(x_0 - x) + \psi_{lattice}(x)\Theta(x - x_0)$$

Perturbative:

$$\psi_{pert} = \frac{C_\psi}{x^8(\log(1/\Lambda^2 x^2))^2} \ , \qquad C_\psi = \frac{96}{\pi^4}$$

(Adler)

OPE:

$$\psi_{OPE} = \left(\frac{b}{8\pi}\right)^2 \left[\frac{\alpha_s^2 b}{\pi^3 x^4} < \alpha_s G^2 > + \frac{2\alpha_s^2}{\pi^2 x^2} < gG^3 > + \frac{29\alpha_s^3 \log(\mu^2 x^2)}{2\pi^2 x^2} < gG^3 > \right]$$

(NSVZ, Bagan, Steele)

Lattice: $\psi_{lattice} = \frac{t_0^2 M_g}{4\pi^2 x} K_1(M_g x)$ (Wightman function)

$t_0 = 1.1 \pm 0.22 \ \ \mathrm{GeV}^3$ $\qquad M_g = 1.71 \pm 0.05 \pm 0.08 \ \ \mathrm{GeV}$

(Chen, et al.2006)

There are subtle features

$$v = -\log(\Lambda^2 x^2)$$

The perturbative part needs to be regularized:
- Adler contour (dim. reg.)
- We have a second regularization (agrees)

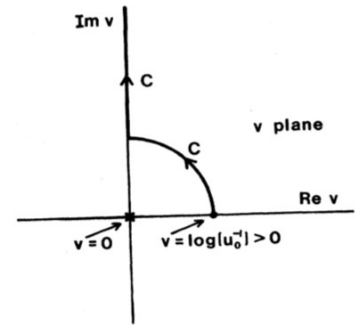


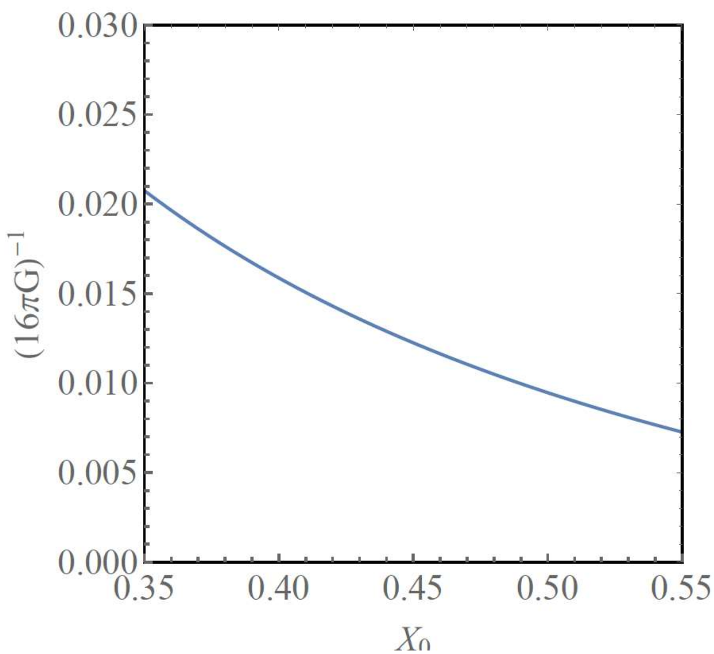
FIG. 5. Contour of integration C to be used in evaluating Eq. (5.56). The contour begins at $v = \log u_0^{-1} = \log(\mathscr{M}^2 t_0)^{-1}$ and must avoid the singularity at $v = 0$.

Result is forced to be real

Matching at $x=x_0$ is not perfect
- scalar glueball sum rule analysis historically difficult

Results

Pure QCD – **Induced G is positive**



Matching at $X_0^{-1} = 2$ GeV:

$$\frac{1}{16\pi G_{\rm ind}} = 0.095 \pm 0.030 \ \mathrm{GeV}^2$$

Issues:

Still need to include dynamical gravity:

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{4g_N^2} g^{\mu\alpha} g^{\nu\beta} F^a_{\mu\nu} F^a_{\alpha\beta} - \frac{1}{2\xi^2} C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta} \right]$$

This result corresponds to dynamical gravity being still weakly coupled when SU(N) gets strong

Cosmological constant also induced

$$\Lambda_{cc} = \frac{-11}{64\pi^2} < g^2 G^2 >$$

Perhaps can play off gravity vs SU(N)
Or Unimodular…
Or Spin Connection

Also should study effect in curved background…

Note: Maybe can use metric itself to do dimensional transmutation

Becomes strongly coupled

Different limits of YM plus R^2

- I am thinking of scale of YM larger that running gravity scale
  YM determines Planck scale

- But if YM scale is smaller, maybe running gravity scale does it

Holdom Ren
Einhorn Jones
Salvio Strumia

Intermediate summary #1

**Dimensional transmutation may allow a
renormalizeable model to lead to Einstein-Hilbert action**

Could scale a QCD-like theory up to the Planck scale

Still work to do to be convinced that this is a good pathway

JFD
PRD 2018

Spin connection and confinement?

Setting: In construction of GR with fermions, naturally have two fields
-vierbein (tetrad) e_μ^a and spin connection ω_μ^{ab}
- ω_μ^{ab} appears naturally as a gauge field

Recover GR only by extra assumption – metricity for vierbein

$$\nabla_\mu e_\nu^a = 0 = \partial_\mu e_\nu^a + \omega^a{}_{b\mu} e_\nu^b - \Gamma_{\mu\nu}{}^\lambda e_\lambda^a$$

Removes ω_μ^{ab} as independent field

$$\omega_\mu^{ab}(x) = e^{a\nu}(\partial_\mu e_\nu^b - \Gamma_{\mu\nu}{}^\lambda e_\lambda^b)$$

What if we do not assume metricity?

Explorations:

1) With usual gauge action, spin connection is asymptotically free

2) Is the spin connection confined (or condensed, gapped)
   - would yield metric theory without extra assumption

3) In scale invariant theory for ω_μ^{ab} , dimensional transmutation
   will give Einstein-Hilbert action

4) With conformally invariant theory for ω_μ^{ab}, richer set of invariants
   $\rightarrow$ conformal model for gravitons

Quick review: Vierbein and spin connection

From Equivalence Principle one can write the metric in terms of vierbein variables

$$g_{\mu\nu}(x) = \eta_{ab}\ e^a_\mu(x)\ e^b_\nu(x)$$

In addition to general covariance

$$e'^a_\mu = \frac{\partial x^\nu}{\partial x'^\mu} e^a_\nu$$

there is an extra **local Lorentz** symmetry

$$e'^a(x) = \Lambda^a{}_c(x)\ e^c(x) \quad \text{with} \quad \eta_{ab}\ \Lambda^a{}_c(x)\ \Lambda^b{}_d(x) = \eta_{cd}$$

For scalars, this feature is irrelevant. But for fermions, it is important

$$\mathcal{L} = \bar{\psi}[i\gamma^a e^\mu_a(x)\partial_\mu + ...]\psi$$

To include the local Lorentz symmetry

$$\psi \to \psi'(x') = S(x)\psi(x)$$

where

$$S(x) = \exp\left(\frac{-i}{2} J_{ab}\alpha^{ab}(x)\right) \quad , \quad J_{ab} = \frac{\sigma_{ab}}{2} \quad \text{with} \quad \sigma_{ab} = \frac{i}{2}[\gamma_a, \gamma_b] \quad .$$

To include this, need spin connection and gauge covariant derivative

$$\mathcal{L} = \bar{\psi}[i\gamma^a e_a^\mu(x) D_\mu]\psi$$

$$D_\mu = \partial_\mu - ig\frac{J_{ab}}{2}\omega_\mu^{ab} \equiv \partial_\mu - ig\boldsymbol{\omega}_\mu$$

with gauge transformation

$$\boldsymbol{\omega}'_\mu \; = \; S\boldsymbol{\omega}_\mu S^{-1} - \frac{2i}{g}(\partial_\mu S)S^{-1}$$

$$S^{-1}(x)\gamma^a S(x)\Lambda_a^b(x) = \gamma^b$$

$$e_a^{\mu'} \; = \; \Lambda_a^b(x) e_b^\mu$$

<u>Relation to GR:</u>

- at this stage we have two fields

- field strength tensor

$$[D_\mu, D_\nu] = -ig\frac{J_{ab}}{2}R^{ab}_{\mu\nu}$$

$$R^{ab}_{\mu\nu} = \partial_\mu \omega^{ab}_\nu - \partial_\nu \omega^{ab}_\mu + g(\omega^{ac}_\mu \ \omega_{\nu c}{}^{b} - \omega^{ac}_\nu \ \omega_{\mu c}{}^{b})$$

Impose metricity (or first order formalism) (g absorbed here)

$$\nabla_\mu e^a_\nu = 0 = \partial_\mu e^a_\nu + \omega^a{}_{b\mu} e^b_\nu - \Gamma_{\mu\nu}{}^{\lambda} e^a_\lambda$$

Obtain GR with Riemann tensor

$$R_{\mu\nu\alpha\beta} = e_{a\alpha} e_{b\beta} R^{ab}_{\mu\nu}$$

Asymptotic Freedom:

Consider usual gauge Lagrangian

$$\mathcal{L} = -\frac{1}{4} R^{ab}_{\mu\nu} R^{\mu\nu}_{ab}$$

This has SO(3,1) gauge symmetry (**non-compact**)

$$[J_{ab}, J_{cd}] = i \left(\eta_{ad} J_{bc} + \eta_{bc} J_{ad} - \eta_{ac} J_{bd} - \eta_{bd} J_{ac}\right)$$

which can be repackaged in more usual gauge notation

$$[J_{ab}, J_{cd}] = 2i f_{[ab][cd][ef]} J^{ef}$$

$$\begin{aligned} f_{[ab][cd][ef]} &= -\frac{1}{4} [\eta_{bc}\eta_{de}\eta_{fa} - \eta_{bd}\eta_{ce}\eta_{fa} - \eta_{bc}\eta_{df}\eta_{ea} + \eta_{bd}\eta_{cf}\eta_{ea} \\ &\qquad - \eta_{bca}\eta_{de}\eta_{fb} + \eta_{ad}\eta_{ce}\eta_{fb} + \eta_{ac}\eta_{df}\eta_{eb} - \eta_{ad}\eta_{cf}\eta_{eb}] \\ &\equiv 2\eta_{b][c}\eta_{d][e}\eta_{f][a} \end{aligned}$$

and

$$R^{[ab]}_{\mu\nu} = \partial_\mu \omega^{[ab]}_\nu - \partial_\nu \omega^{[ab]}_\mu + g f^{[ab]}{}_{[cd][ef]} \omega^{[cd]}_\mu \omega^{[ef]}_\nu$$

Gauge loops then proceed in the usual way, with substitution

$$f_{imn} f_{jmn} = C_2 \delta_{ij} \qquad \rightarrow \qquad f_{[ab][cd][ef]} f^{[gh][cd][ef]} = C_2 \delta_{[ab]}^{[gh]}$$

with

$$C_2 = 2$$

This then yields the beta function

$$\beta(g) \;=\; -\frac{11 C_2}{3} \frac{g^3}{16\pi^2} \;=\; -\frac{22}{3} \frac{g^3}{16\pi^2}$$

Note: Fermion loops do not contribute to this coupling. **Return to this later**

Confined, condensed, gapped?

Spin connection weakly coupled in UV

Strongly coupled in IR

Running defines a scale – perhaps M_P

Analogies would suggest confinement, but non-compact group?

Singlet channel is attractive, then perhaps condensation

Assume spin connection is not propagating at low energy
- then symmetry must be realized with metric only
- **explains metric theory without need to assume metricity of vierbein**

Should be able to be answered by lattice work

Note: Smilga and Holdom + Ren have suggested confinement for the metric field

What happens at low energy?

Euclidean gravity

For lattice studies transform to Euclidean space

Symmetry:

Lorentzian SO(3,1) goes to Euclidean O(4) (compact)
Beta function is the same
O(4) Yang Mills is confining

But is Euclidean gravity valid?
- most numerical studies assume this

If Euclidean gravity makes sense, the spin connection will be confined with usual action.

A scale invariant model:

Include Weyl squared term:

$$S_{s.i.} = \int d^4x\sqrt{-g}\left[-\frac{1}{4g^2}R_{\mu\nu}^{ab}R_{ab}^{\mu\nu} - \frac{1}{2\xi^2}C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta}\right]$$

This is peturbatively complete – all perturbative divergences controlled

It is also doubly asymptotically free

$$\begin{aligned}\beta(g) &= -\frac{22}{3\pi^2}g^3 \\ \beta(\xi) &= -\frac{199}{480\pi^2}\xi^3 - \frac{3}{80\pi^2}\xi^3\end{aligned}$$

Intermediate summary #2

Spin connection provides possible model for dimensional transmutation program

Treated as independent field, spin connection can have strong dynamics

It is already part of the gravitational theory

Will it be confined/condensed?

Can this make a useful model of gravity?

Pathway:

Start with fundamental C^2 action for gravity and extra "helper" gauge theory (renom. and AF)

- weakly coupled at high energy

Use path integral to define the theory

Induce the regular Einstein action at low energy

- weakly coupled at low energy
- vacuum stability

Graviton propagator transitions from standard to quartic

- imaginary parts from loops
- ghosts unstable
- causality fuzziness at Planck scale

Speculations:

Still need to prove this is a viable pathway, but…

Dark energy:

- interplay between gravitational and SU(N) induced effects?
- does spin connection generate $\Lambda = 0$?

$$4\Lambda =< 0|T^{\mu}_{\ \mu}|0 >=< 0|\frac{\beta}{2g} R^{ab}_{\mu\nu} R^{\mu\nu}_{ab}|0 >$$

Dark matter:

- any role for bound states?

Singularity resolution in black holes

Comments:

The Planck scale may not be the ultimate barrier

- certainly EFT indicates strong coupling
- but can emerge as weak coupling in the UV

If gravity can be a conventional field theory, it probably should look like this

- scale/conformal invariant actions are most promising
- extra conformal symmetry attractive for fundamental gravity

Dimensional transmutation can yield Einstein action

- explicit calculation for QCD-like theories

The spin connection can live as an independent field

- most natural as a gauge field

The spin connection (with usual gauge interaction) is asymptotically free

- Confined or condensed?
- weak coupling beyond Planck scale