TMD factorization and evolution

Alexei Prokudin

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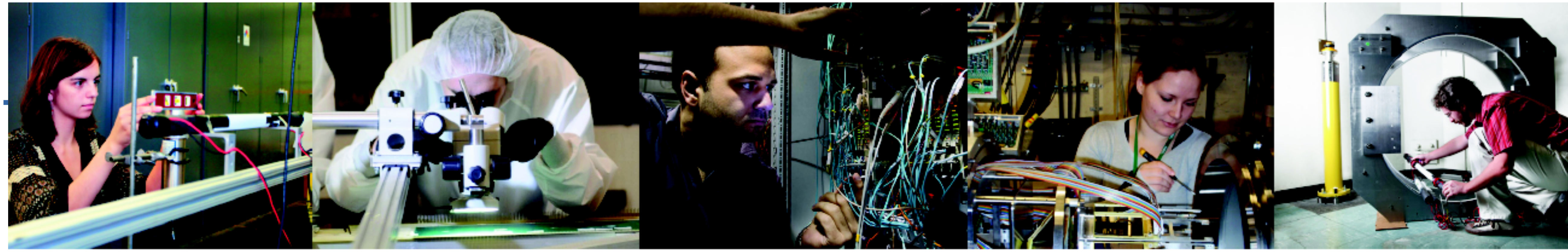


The 2015 LONG RANGE PLAN for NUCLEAR SCIENCE

RECOMMENDATION I

The progress achieved under the guidance of the 2007 Long Range Plan has reinforced U.S. world leadership in nuclear science. The highest priority in this 2015 Plan is to capitalize on the investments made.

- *With the imminent completion of the CEBAF 12-GeV Upgrade, its forefront program of using electrons to unfold the quark and gluon structure of hadrons and nuclei and to probe the Standard Model must be realized.*

…

- *The upgraded RHIC facility provides unique capabilities that must be utilized to explore the properties and phases of quark and gluon matter in the high temperatures of the early universe and to explore the spin structure of the proton.*

RECOMMENDATION III

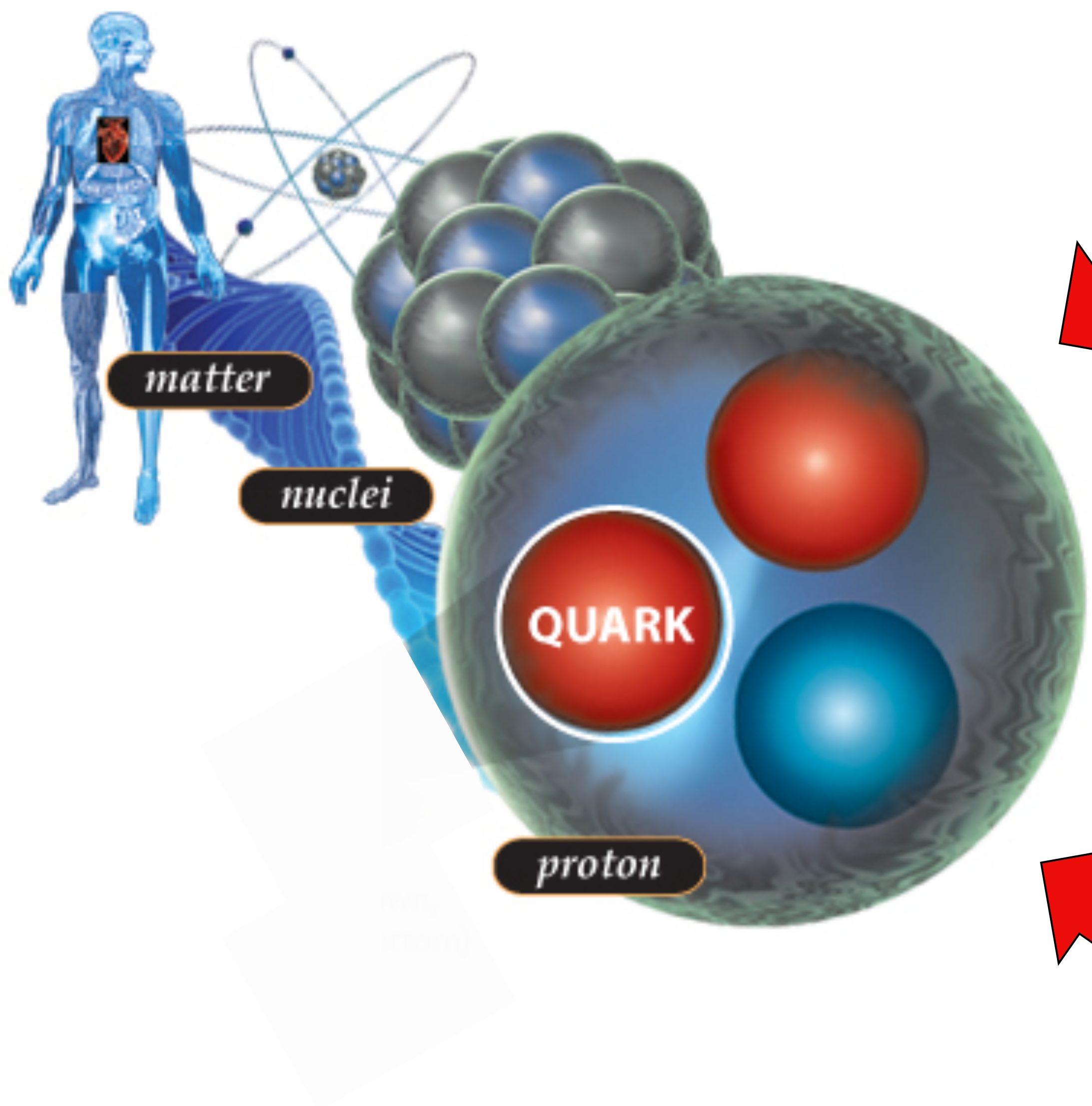
Gluons, the carriers of the strong force, bind the quarks together inside nucleons and nuclei and generate nearly all of the visible mass in the universe. Despite their importance, fundamental questions remain about the role of gluons in nucleons and nuclei. These questions can only be answered with a powerful new electron ion collider (EIC), providing unprecedented precision and versatility. The realization of this instrument is enabled by recent advances in accelerator technology.

We recommend a high-energy high-luminosity polarized EIC as the highest priority for new facility construction following the completion of FRIB.

The EIC will, for the first time, precisely image gluons in nucleons and nuclei. It will definitively reveal the origin of the nucleon spin and will explore a new quantum chromodynamics (QCD) frontier …

Why is it so important for theory? We are a data-driven science!

Exploring the nucleon: a fundamental quest



Know what we are made of !

Understand the strong force: "QCD"

Use protons as tool for discovery (e.g. LHC)

Xiangdong Ji at DIS08

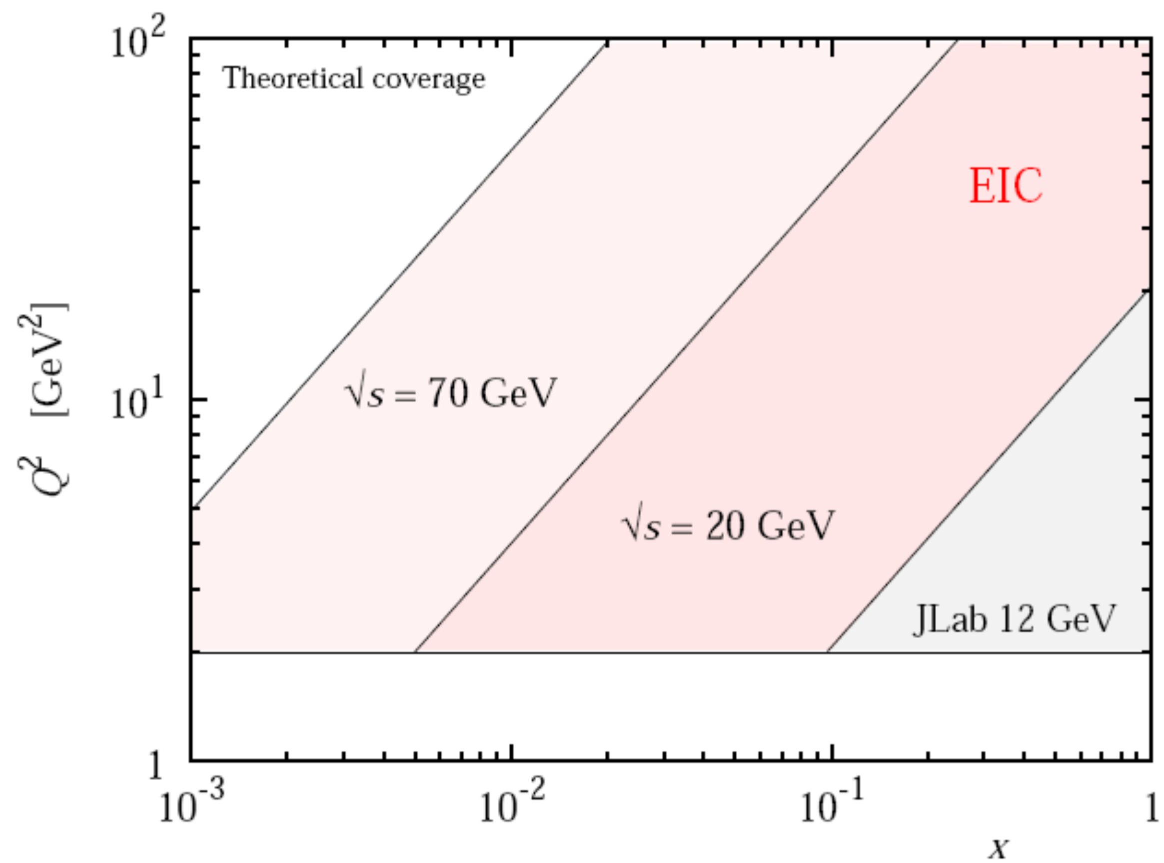
Spin is a fundamental quantum degree of freedom

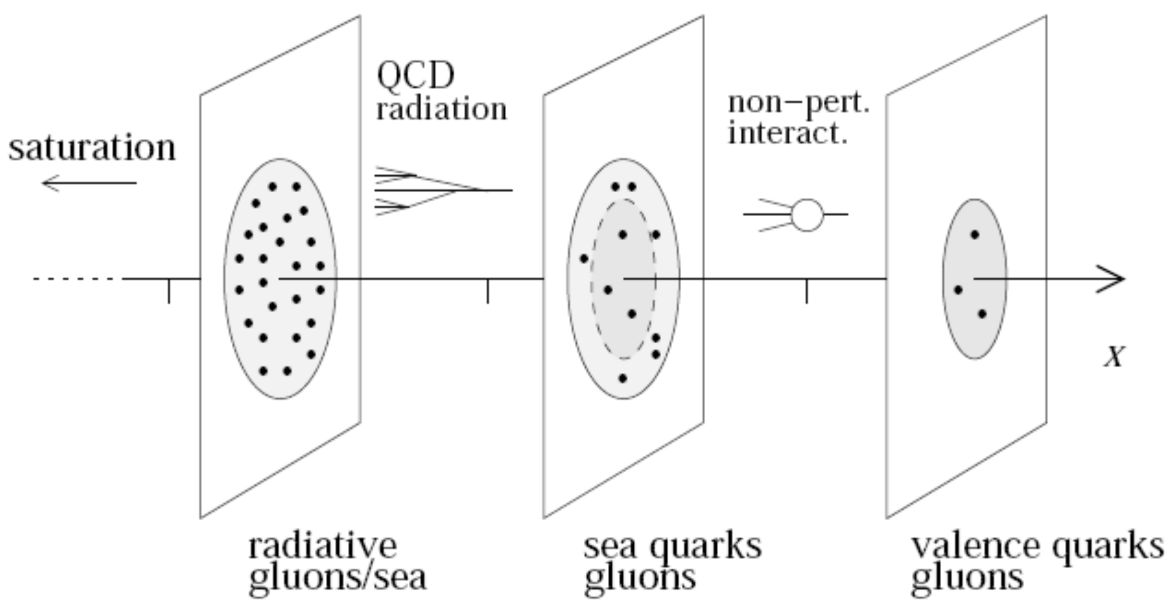
Spin plays a critical role in determining the basic structure of fundamental interactions

Test of a theory is not complete without a full test of spin-dependent decays and scattering

Spin provides a unique opportunity to probe the inner structure of a composite system (such as the proton)

Nucleon landscape





Nucleon is a many body dynamical system of quarks and gluons

Changing x we probe different aspects of nucleon wave function

How partons move and how they are distributed in space is one of the directions of development of nuclear physics

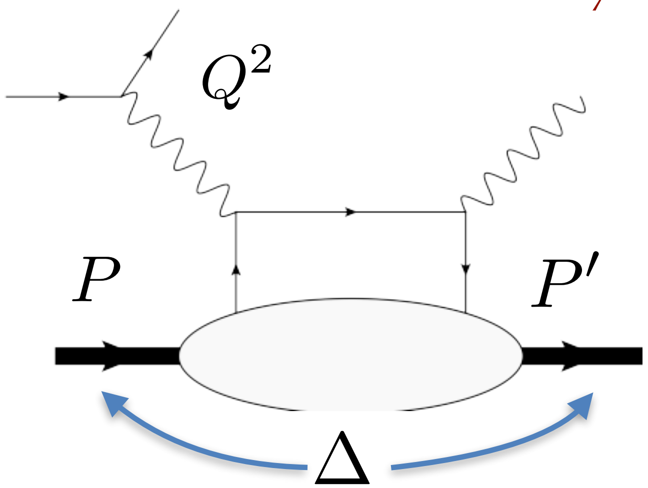
Technically such information is encoded into Generalised Parton Distributions (GPDs) and Transverse Momentum Dependent distributions (TMDs)

These distributions are also referred to as 3D (three-dimensional) distributions

GPDs TMDs

DVCS

Ji (1997)
Radyushkin (1997)



Q^2 ensures hard scale, pointlike interaction

$\Delta = P' - P$ momentum transfer can be varied independently

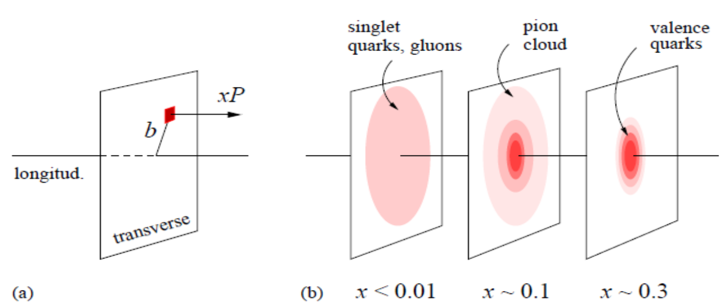
Connection to 3D structure

Burkardt (2000)
Burkardt (2003)

$$\rho(x, \vec{r}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{r}_\perp} H_q(x, \xi = 0, t = -\vec{\Delta}^2_\perp)$$

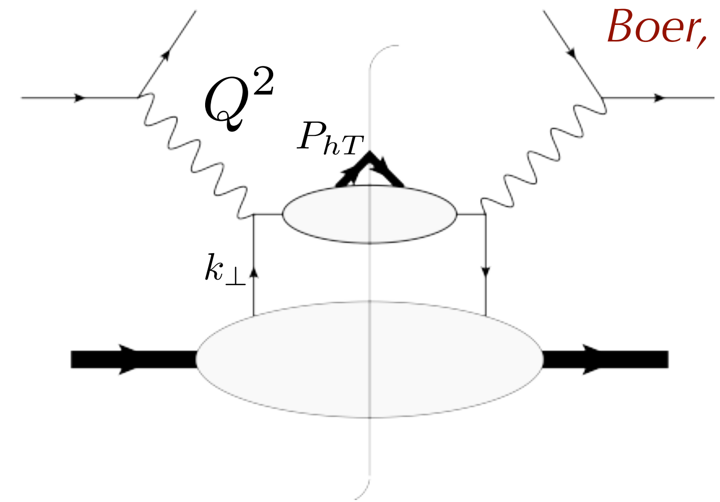
Drell-Yan frame $\Delta^+ = 0$

Weiss (2009)



SIDIS

Kotzinian (1995),
Mulders,
Tangerman (1995),
Boer, Mulders (1998)



Q^2 ensures hard scale, pointlike interaction

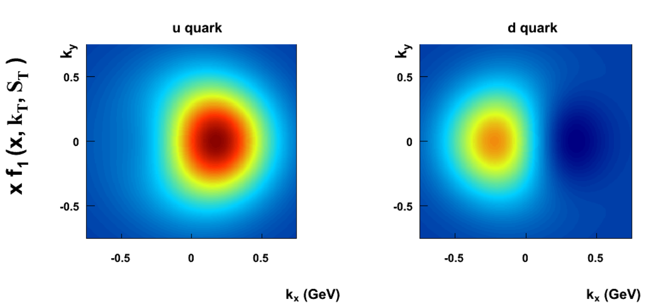
P_{hT} final hadron transverse momentum can be varied independently

Connection to 3D structure

Ji, Ma, Yuan (2004)
Collins (2011)

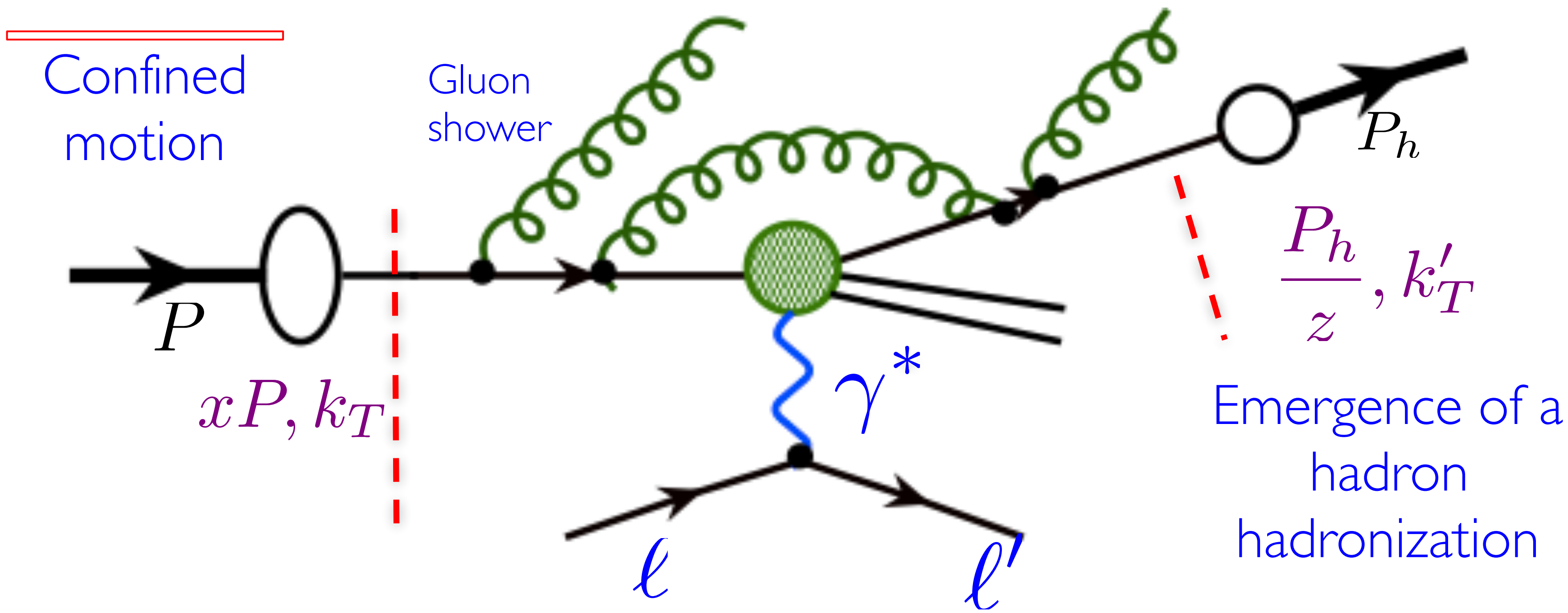
$$\tilde{f}(x, \vec{b}) = \int d^2 k_\perp e^{-i\vec{b}\cdot\vec{k}_\perp} f(x, \vec{k}_\perp)$$

$\vec{b}$ is the transverse separation of parton fields in configuration space



AP (2012)

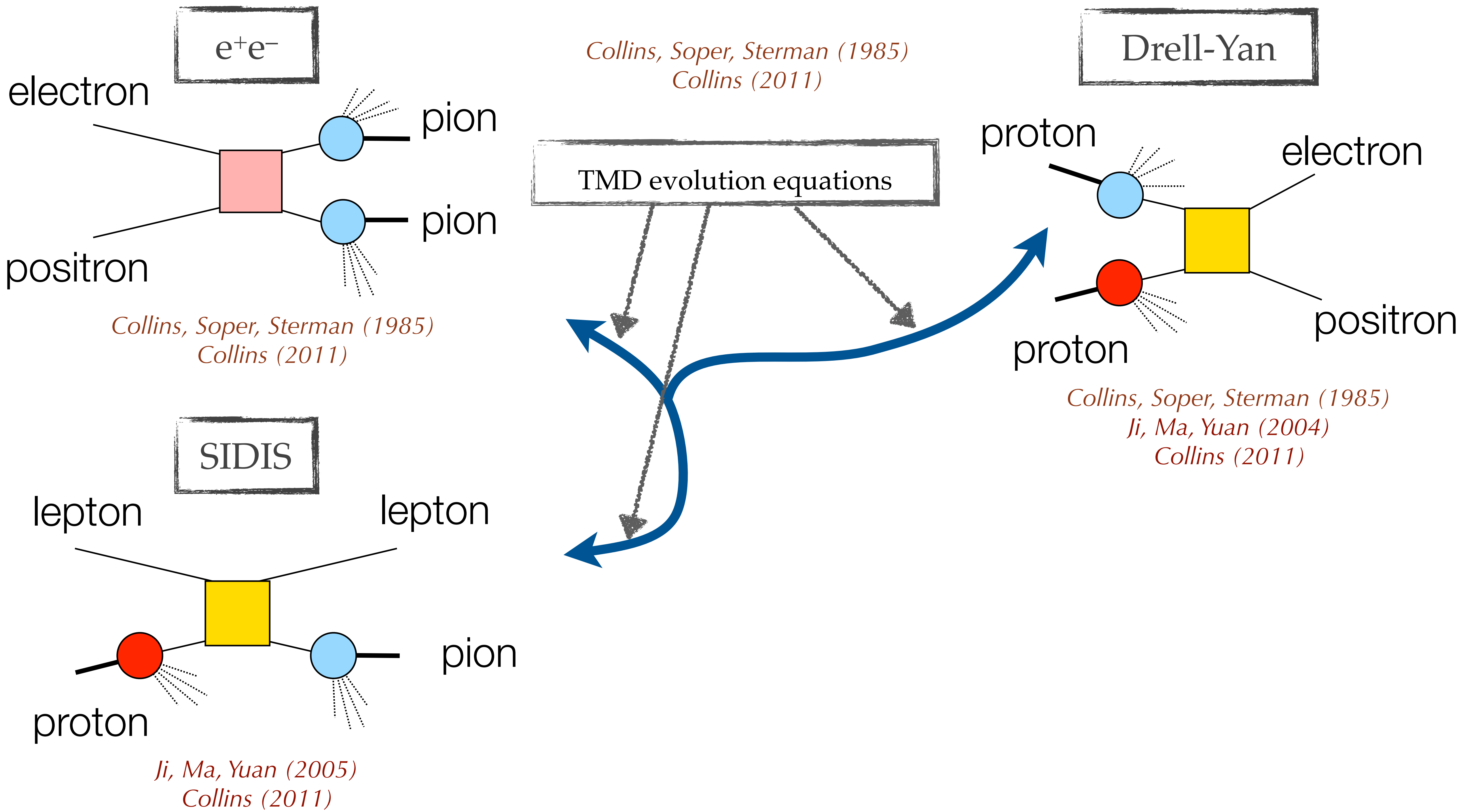
Study of evolution gives us insight on different aspects and origin of confined motion of partons, gluon radiation, parton fragmentation



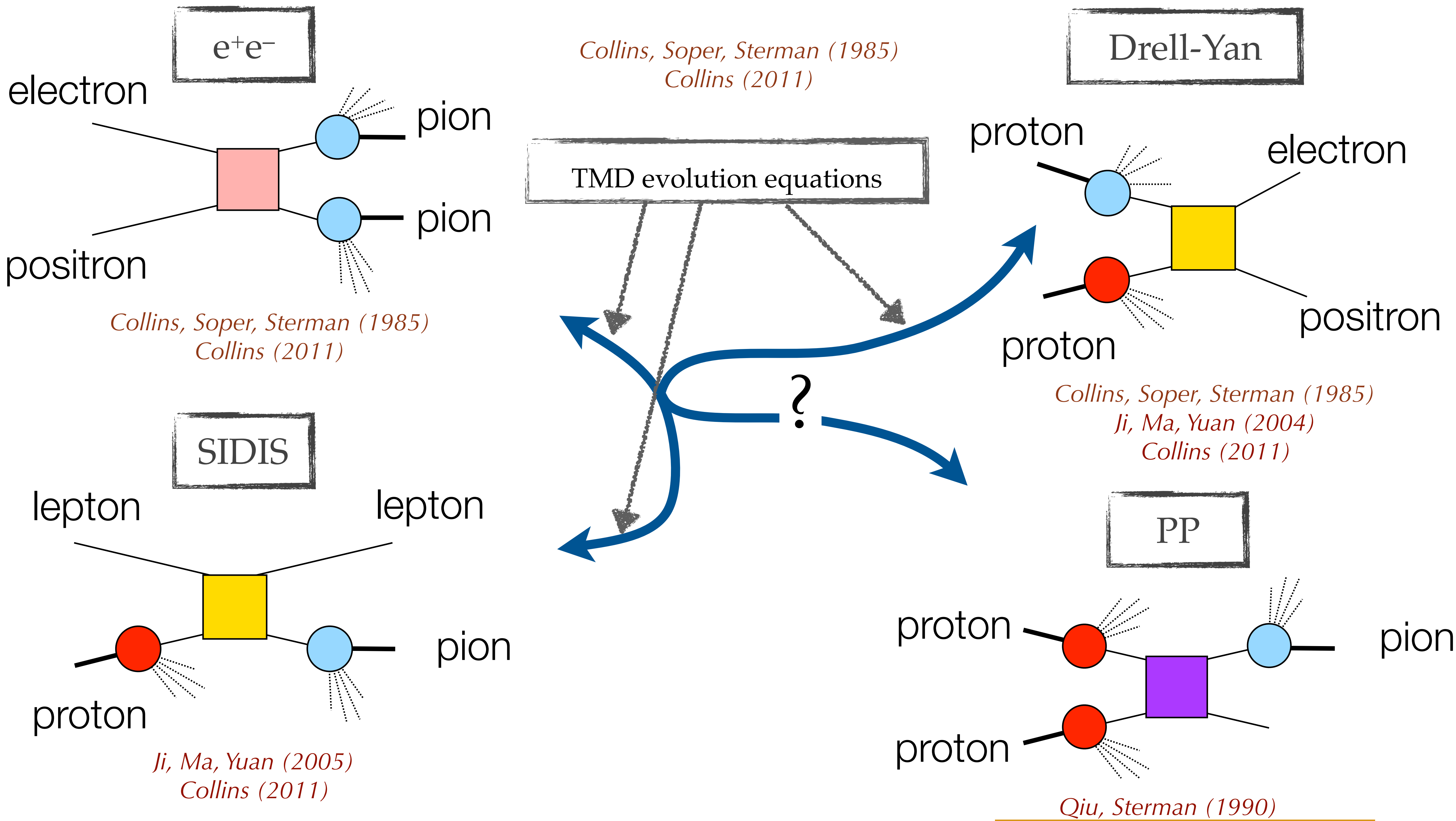
Evolution allows to connect measurements at very different scales.

TMD evolution has also a universal non-perturbative part. The result of evolution cannot be uniquely predicted using evolution equations until the non-perturbative part is reliably extracted from the data.

TMD factorization



TMD factorization



Only one scale is measured in PP
TMD factorization is not applicable?

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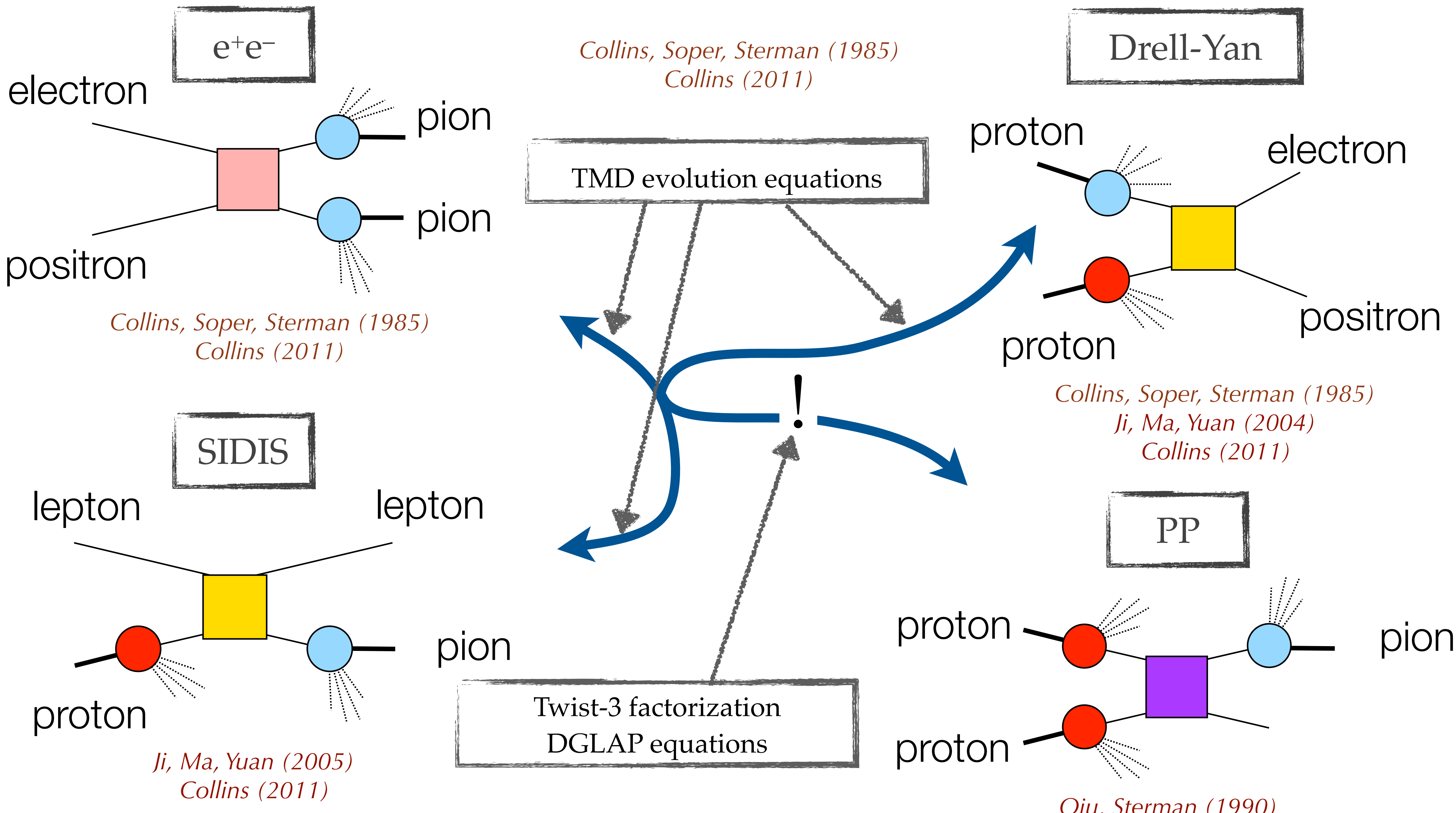
TMD factorization



- Twist-3 functions are related to TMD via OPE
- TMD and twist-3 factorizations are related in high QT region
- Global analysis of TMDs and twist-3 is possible:

All four processes can be used.

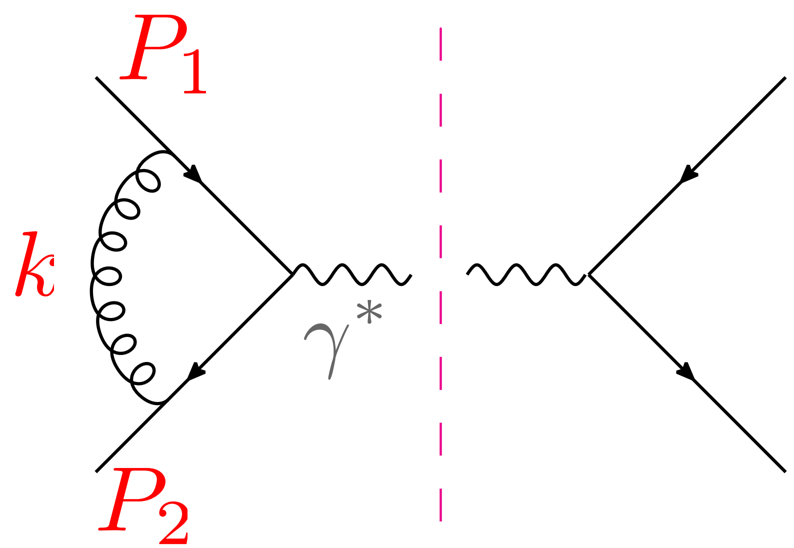
- Data are from HERMES, COMPASS, JLab,

BaBar, Belle, RHIC, LHC, Fermilab

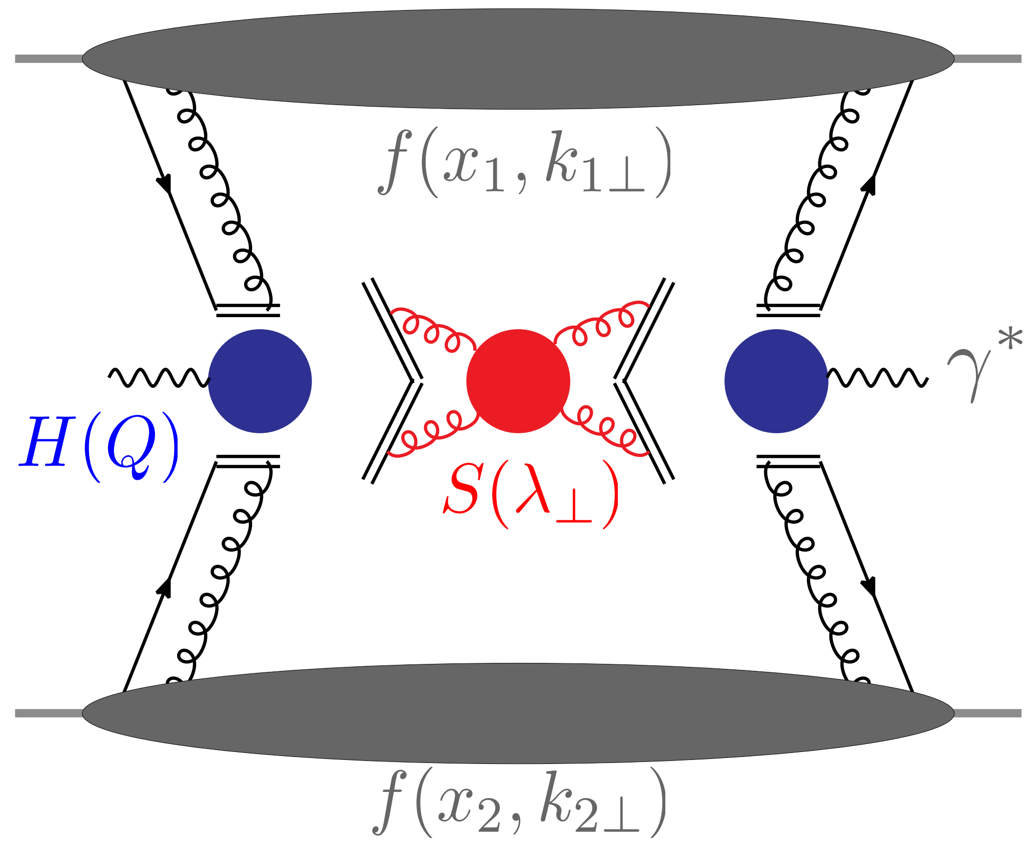
Global fit is needed.
Work in progress

TMD factorization in a nut-shell

- Drell-Yan: $p + p \to [\gamma^* \to \ell^+ \ell^-] + X$



Factorization of regions:
(1) k//P_1, (2) k//P_2, (3) k soft, (4) k hard



- Factorized form and mimicking "parton model"

$$\frac{d\sigma}{dQ^2 dy d^2 q_\perp} \propto \int d^2k_{1\perp}\, d^2k_{2\perp}\, d^2\lambda_\perp H(Q) f(x_1, k_{1\perp}) f(x_2, k_{2\perp}) S(\lambda_\perp) \delta^2(k_{1\perp} + k_{2\perp} + \lambda_\perp - q_\perp)$$

$$= \int \frac{d^2 b}{(2\pi)^2} e^{iq_\perp \cdot b} H(Q) f(x_1, b) f(x_2, b) S(b)$$

$$F(x, b) = f(x, b)\sqrt{S(b)}$$

$$\boxed{= \int \frac{d^2 b}{(2\pi)^2} e^{iq_\perp \cdot b} H(Q) F(x_1, b) F(x_2, b)}$$

mimic "parton model"

slide courtesy of Z. Kang

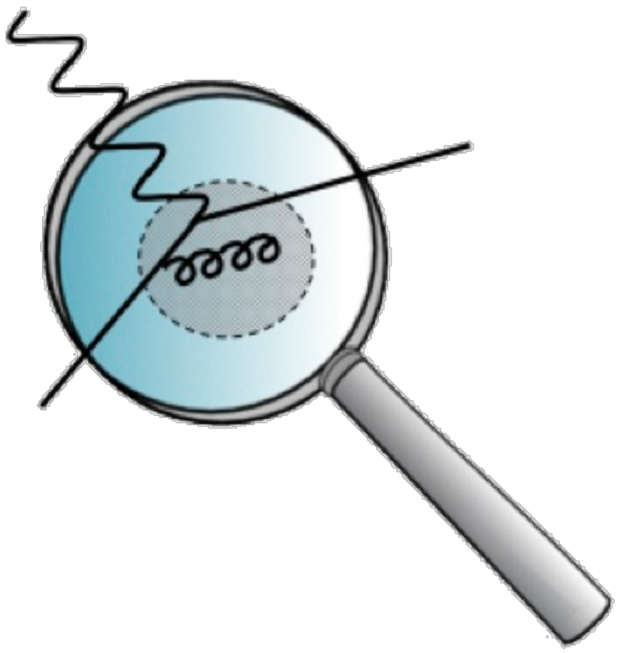
TMDs evolve

Just like collinear PDFs, TMDs also depend on the scale of the probe = evolution

Collinear PDFs

$F(x, Q)$

- ✓ DGLAP evolution
- ✓ Resum $\left[\alpha_s \ln(Q^2/\mu^2)\right]^n$
- ✓ Kernel: purely perturbative



TMDs

$F(x, k_\perp; Q)$

- ✓ Collins-Soper/rapidity evolution equation
- ✓ Resum $\left[\alpha_s \ln^2(Q^2/k_\perp^2)\right]^n$
- ✓ Kernel: can be non-perturbative when $k_\perp \sim \Lambda_{\rm QCD}$

$$F(x, Q_i)$$
$$\downarrow$$
$$R^{\rm coll}(x, Q_i, Q_f)$$
$$\downarrow$$
$$F(x, Q_f)$$

$$F(x, k_\perp, Q_i)$$
$$\downarrow$$
$$R^{\rm TMD}(x, k_\perp, Q_i, Q_f)$$
$$\downarrow$$
$$F(x, k_\perp, Q_f)$$

slide courtesy of Z. Kang

TMD evolution in b-space

$$F(x, k_\perp; Q)$$

- We have a TMD above measured at a scale Q. So far the evolution is written down in the Fourier transformed space (convolution → product)

$$F(x, b; Q) = \int d^2 k_\perp e^{-ik_\perp \cdot b} F(x, k_\perp; Q)$$

- In the small b region ($1/b >> \Lambda_{QCD}$), one can then compute the evolution to this TMD, which goes like

$$F(x, b; Q_f) = F(x, b; Q_i) \exp\left\{ -\int_{Q_i}^{Q_f} \frac{d\mu}{\mu} \left(A \ln \frac{Q_f^2}{\mu^2} + B \right) \right\} \left(\frac{Q_f^2}{Q_i^2} \right)^{-\int_{c/b}^{Q_i} \frac{d\mu}{\mu} A}$$

$$A = \sum_{n=1} A^{(n)} \left(\frac{\alpha_s}{\pi} \right)^n, \qquad B = \sum_{n=1} B^{(n)} \left(\frac{\alpha_s}{\pi} \right)^n$$

Only valid for small b

Collins-Sopoer-Sterman papers
Kang, Xiao, Yuan, PRL 11,
Aybat, Rogers, Collins, Qiu, 12,
Aybat, Prokudin, Rogers, 12,
Sun, Yuan, 13,
Echevarria, Idilbi, Schafer, Scimemi, 13,
Echevarria, Idilbi, Kang, Vitev, 14,
Kang, Prokudin, Sun, Yuan, 15, 16, …

TMD evolution and non-perturbative component

- Fourier transform back to the momentum space, one needs the whole b region (large b): need some non-perturbative extrapolation
  - Many different methods/proposals to model this non-perturbative part

$$F(x,k_\perp;Q)=\frac{1}{(2\pi)^2}\int d^2b e^{ik_\perp\cdot b}F(x,b;Q)=\frac{1}{2\pi}\int_0^\infty db\, bJ_0(k_\perp b)F(x,b;Q)$$

Collins, Soper, Sterman 85, ResBos, Qiu, Zhang 99, Echevarria, Idilbi, Kang, Vitev, 14, Aidala, Field, Gamberg, Rogers, 14, Sun, Yuan 14, D'Alesio, Echevarria, Melis, Scimemi, 14, Rogers, Collins, 15, …

- Eventually evolved TMDs in b-space

$$F(x,b;Q)\approx \boxed{C\otimes F(x,c/b^*)}\times \boxed{\exp\left\{-\int_{c/b^*}^{Q}\frac{d\mu}{\mu}\left(A\ln\frac{Q^2}{\mu^2}+B\right)\right\}}\times \boxed{\exp\left(-S_{\text{non-pert}}(b,Q)\right)}$$

longitudinal/collinear part

transverse part

✓ Non-perturbative: fitted from data

✓ The key ingredient – ln(Q) piece is spin-independent

Since the polarized scattering data is still limited kinematics, we can use unpolarized data to constrain/extract key ingredients for the non-perturbative part

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slide courtesy of Z. Kang

N \ q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

8 functions in total (at leading twist)

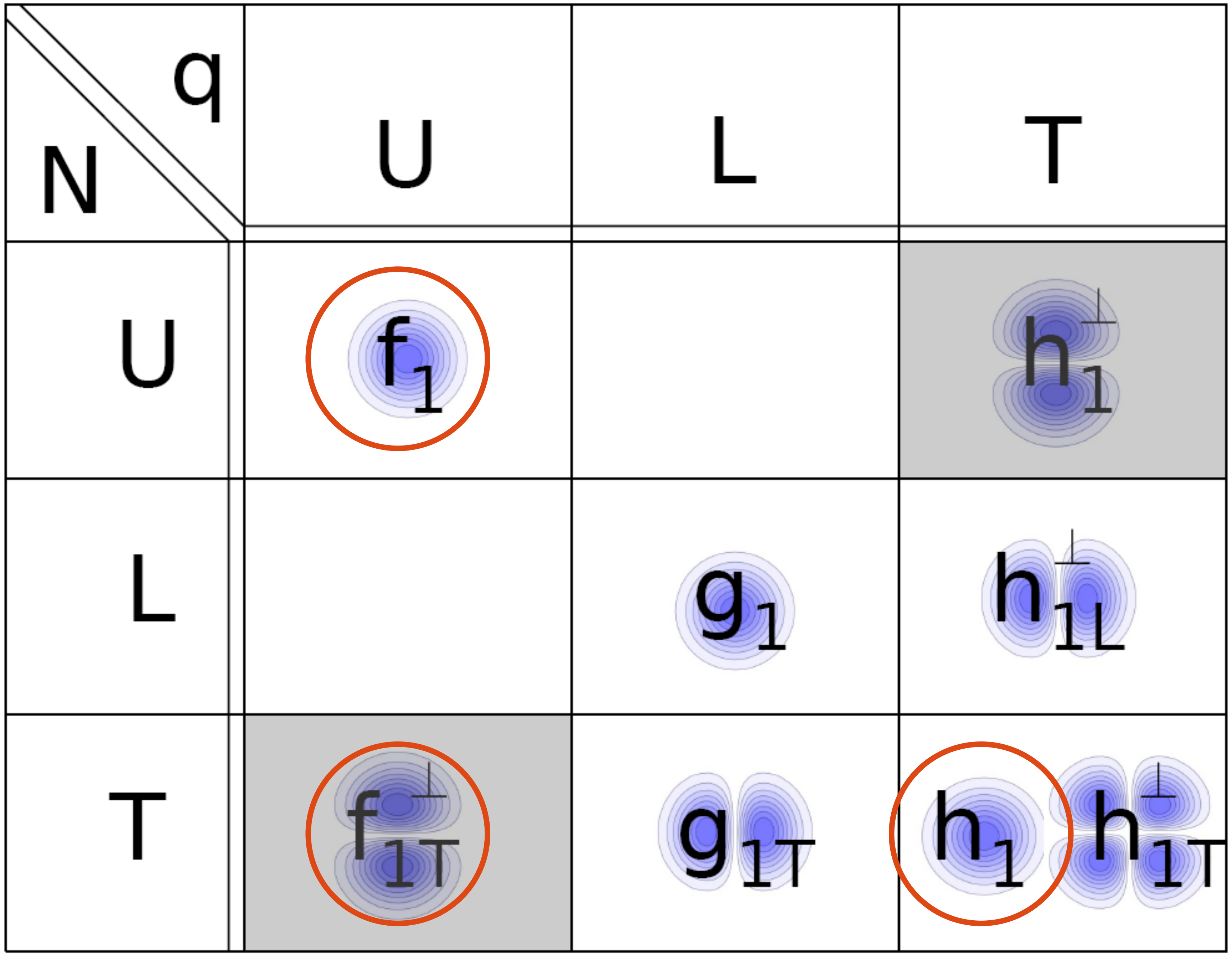
Each represents different aspects of partonic structure

Each depends on Bjorken-x, transverse momentum, the scale

Each function is to be studied

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

TMD distributions



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This talk

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

N \ q	U	L	T
U	D_1		$H_1^{\perp}$
L		G_{1L}	$H_{1L}^{\perp}$
T	$H_{1T}^{\perp}$	G_{1T}	H_1 $H_{1T}^{\perp}$

8 functions describing fragmentation of a quark into spin ½ hadron

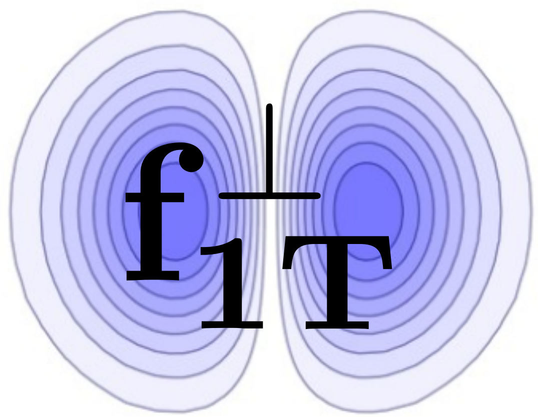
Mulders, Tangerman (1995), Meissner, Metz, Pitonyak (2010)

N \ q	U	L	T
U	D_1		$H_1^\perp$
L		G_{1L}	$H_{1L}^\perp$
T	$H_{1T}^\perp$	G_{1T}	H_1 $H_{1T}^\perp$

8 functions describing fragmentation of a quark into spin ½ hadron

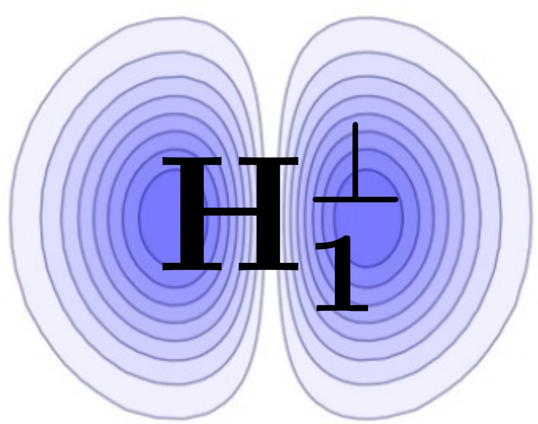
This talk

Mulders, Tangerman (1995), Meissner, Metz, Pitonyak (2010)



Sivers function

Non universal

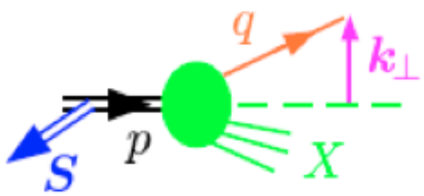


Collins function

Universal

Sivers function: unpolarized quark distribution inside a transversely polarized nucleon

Sivers 1989



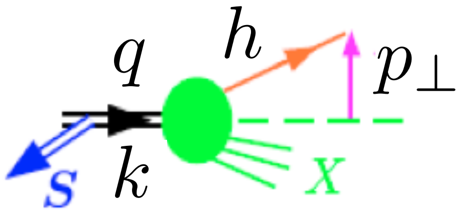
$$f_{q/h^\uparrow}(x,\vec{k}_\perp,\vec{S}) = f_{q/h}(x,k_\perp^2) - \frac{1}{M} f_{1T}^{\perp q}(x,k_\perp^2)\vec{S}\cdot(\hat{P}\times\vec{k}_\perp)$$

Spin independent

Spin dependent

Collins function: unpolarized hadron from a transversely polarized quark

Collins 1992



$$D_{q/h}(z,\vec{p}_\perp,\vec{s}_q) = D_{q/h}(z,p_\perp^2) + \frac{1}{zM_h} H_1^{\perp q}(z,p_\perp^2)\vec{s}_q\cdot(\hat{k}\times\vec{p}_\perp)$$

Definitions

Sivers function: $f_{1T}^{\perp q}$ describes strength of correlation $\vec{S} \cdot (\hat{P} \times \vec{k}_\perp)$

Sivers 1989

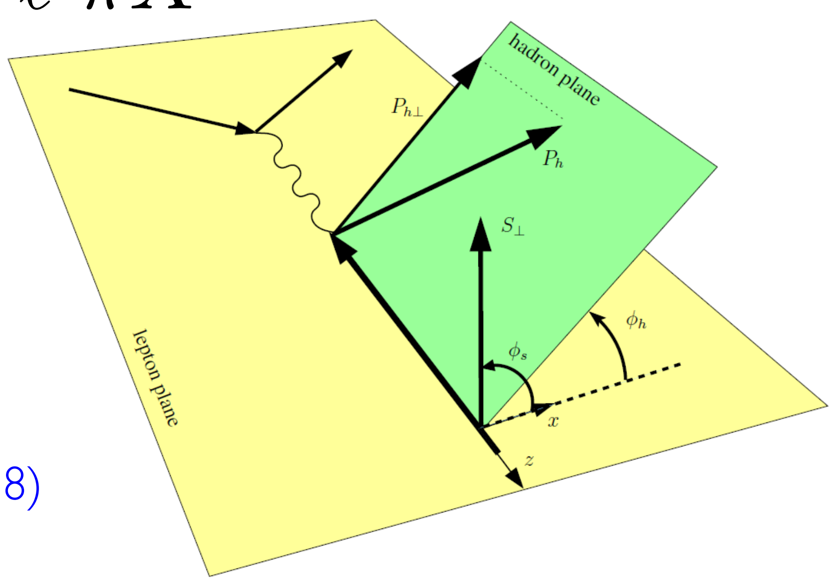
Collins function: $H_1^{\perp q}$ describes strength of correlation $\vec{s}_q \cdot (\hat{k} \times \vec{p}_\perp)$

Collins 1992

Both functions extensively studied experimentally, phenomenologically, theoretically

$$\ell P \to \ell' \pi X$$

Sivers function and Collins function can give rise to Single Spin Asymmetries in scattering processes. For instance in Semi Inclusive Deep Inelastic process



Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

$$d\sigma(S) \sim \sin(\phi_h + \phi_S) h_1 \otimes H_1^\perp + \sin(\phi_h - \phi_S) f_{1T}^\perp \otimes D_1 + ...$$

Sivers function

Large – N_c result $$f_{1T}^{\perp u} = -f_{1T}^{\perp d}$$ Pobylitsa 2003

→ Confirmed by phenomenological extractions

→ Confirmed by experimental measurements

Relation to GPDs (E) and anomalous magnetic moment Burkardt 2002

$$f_{1T}^{\perp q} \sim \kappa^q$$

→ Predicted correct sign of Sivers asymmetry in SIDIS

→ Shown to be model-dependent

Meissner, Metz, Goeke 2007

→ Used in phenomenological extractions

Bacchetta, Radici 2011

Sivers function

Sum rule

Burkardt 2004

→ Conservation of transverse momentum

→ Average transverse momentum shift of a quark inside a transversely polarized nucleon

$$\langle k_T^{i,q} \rangle = \varepsilon_T^{ij} S_T^j f_{1T}^{\perp(1)q}(x)$$

$$f_{1T}^{\perp(1)q}(x) = \int d^2 k_\perp \frac{k_\perp^2}{2M^2} f_{1T}^{\perp q}(x, k_\perp^2)$$

→ Sum rule

$$\sum_{a=q,g} \int_0^1 dx \langle k_T^{i,a} \rangle = 0 \qquad \sum_{a=q,g} \int_0^1 dx\, f_{1T}^{\perp(1)a}(x) = 0$$

Sivers function

Extractions

→ Many extractions without taking into account TMD evolution

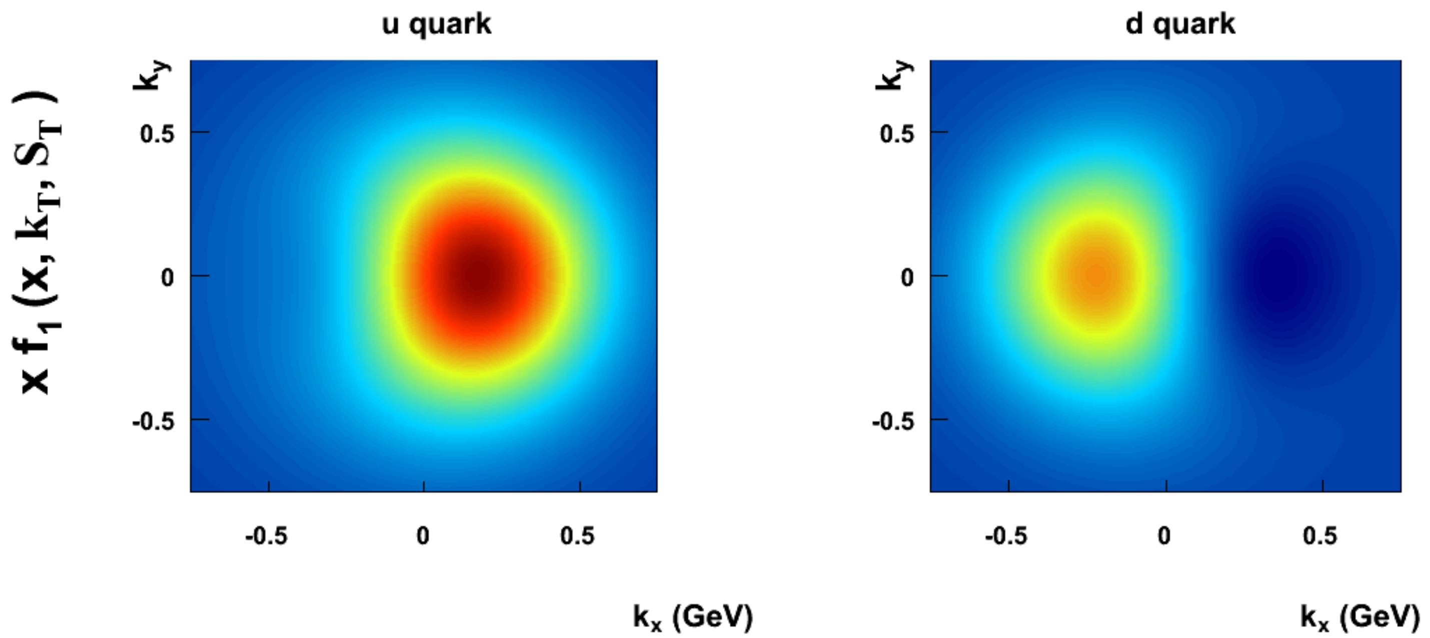
Efremov et al 2005, Vogelsang, Yuan 2005, Anselmino et al 2005, Collins et al 2006, Anselmino et al 2009, 2011, 2016, Bacchetta Radici 2011

→ Extractions with TMD evolution

Echevarria et al 2014, Sun Yuan 2013

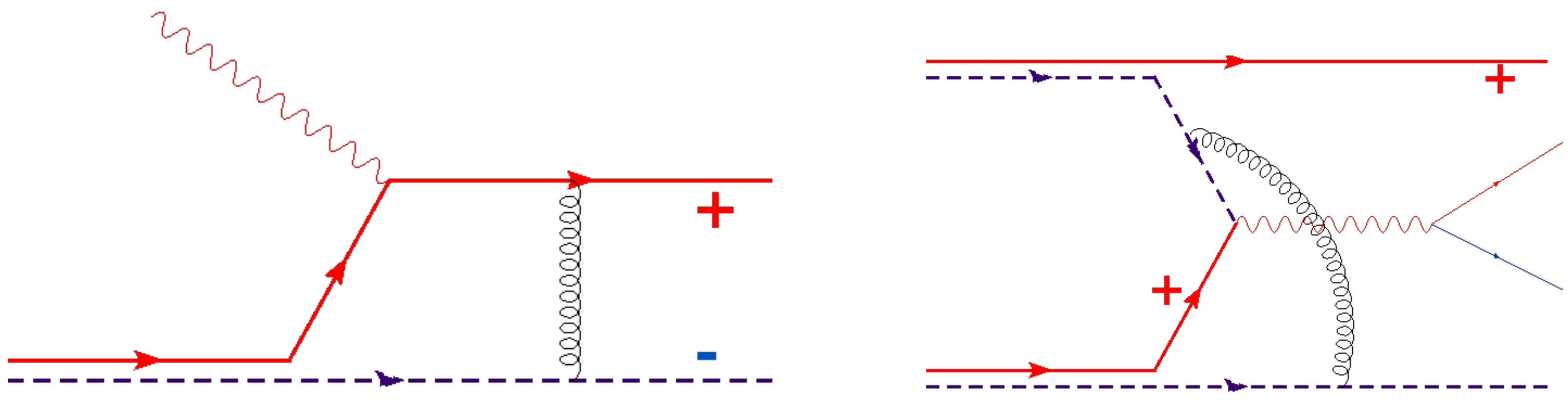
→ Relation to the tomography of the nucleon

Anselmino et al 2011



→ Agreement with the sum rule and large N_c prediction

Colored objects are surrounded by gluons, profound consequence of gauge invariance:
Sivers function has opposite sign when gluon couple after quark scatters (SIDIS) or before quark annihilates (Drell-Yan)



Brodsky,Hwang,Schmidt;
Belitsky,Ji,Yuan;
Collins;
Boer,Mulders,Pijlman;
Kang, Qiu;
Kovchegov, Sievert;
etc

$$f_{1T}^{\perp SIDIS} = -f_{1T}^{\perp DY}$$

Crucial test of TMD factorization and collinear twist-3 factorization
Several labs worldwide aim at measurement of Sivers effect in Drell-Yan
BNL, CERN, GSI, IHEP, JINR, FERMILAB etc
Barone et al., Anselmino et al., Yuan, Vogelsang, Schlegel et al., Kang, Qiu, Metz, Zhou etc
The verification of the sign change is an NSAC (DOE and NSF) milestone

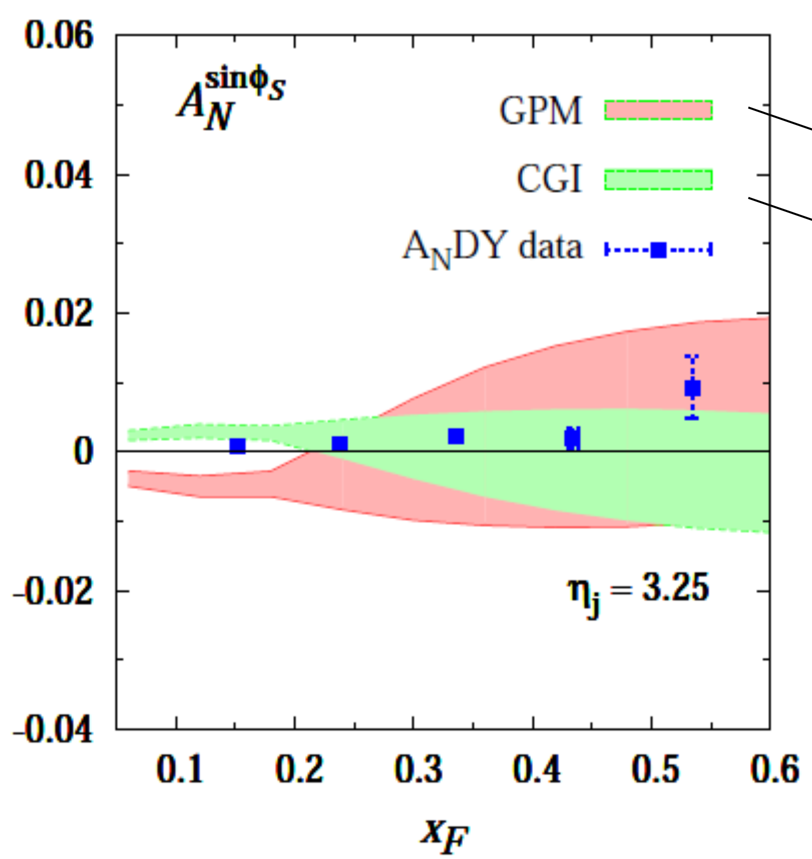
Process dependence of Sivers function

- Indication on process dependence of Sivers functions from analysis of A_N in $\ell N^{\uparrow} \to \ell X$

Metz et al 2012

- Indication on process dependence from AnDY data on A_N in $p^{\uparrow} p \to jet X$

Gamberg, Kang, AP 2013
D'Alesio et al 2013



No sign change

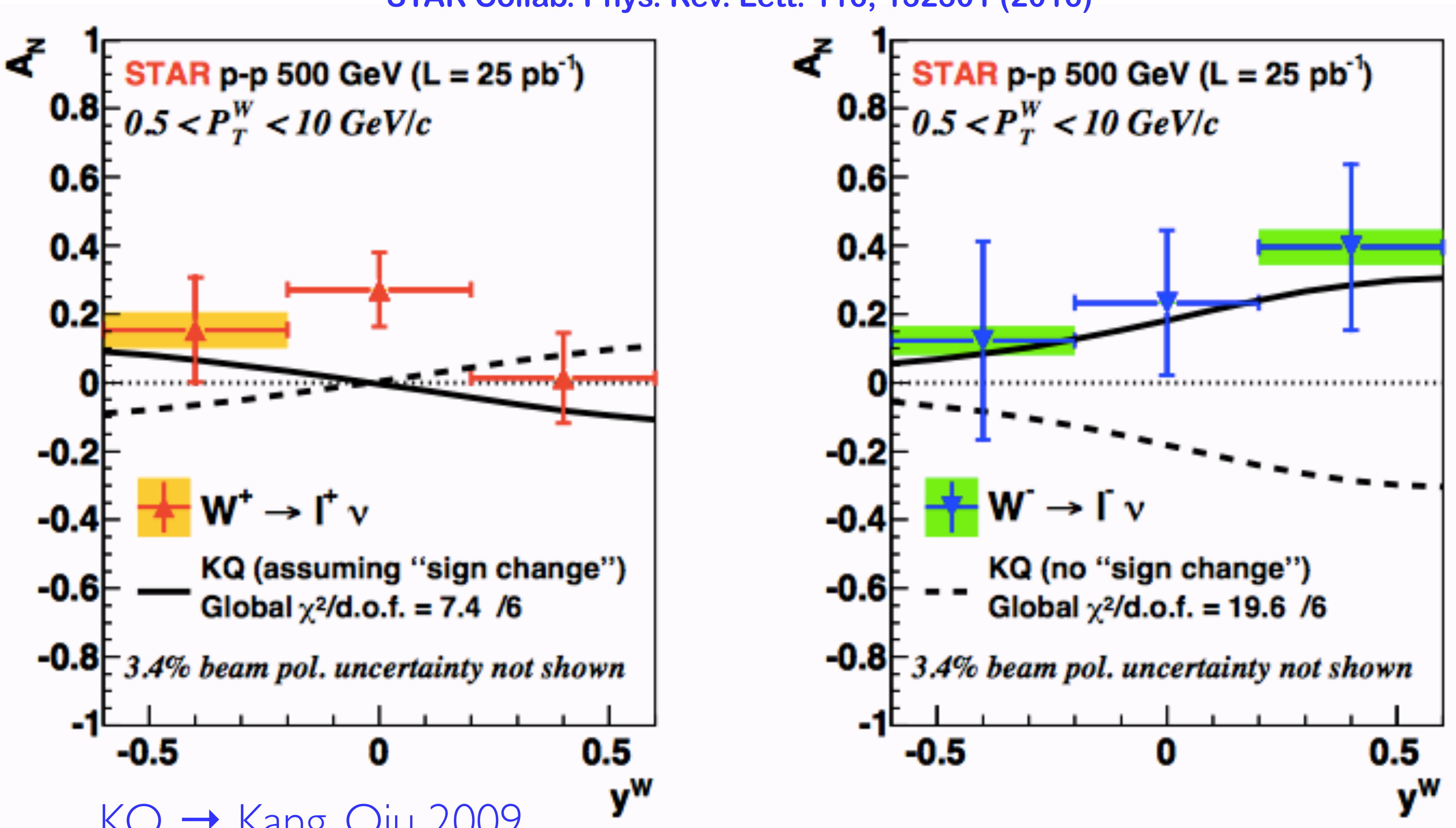
Sign change

Process dependence of Sivers function

STAR 2015

→ First experimental hint on the sign change: A_N in W and Z production

STAR Collab. Phys. Rev. Lett. 116, 132301 (2016)



KQ → Kang, Qiu 2009

$$p^\uparrow p \to W^\pm X$$
$$p^\uparrow p \to Z^0 X$$

→ No sign change

$$\chi^2/d.o.f \sim 1.2$$

$$\chi^2/d.o.f \sim 3.2$$

Process dependence of Sivers function

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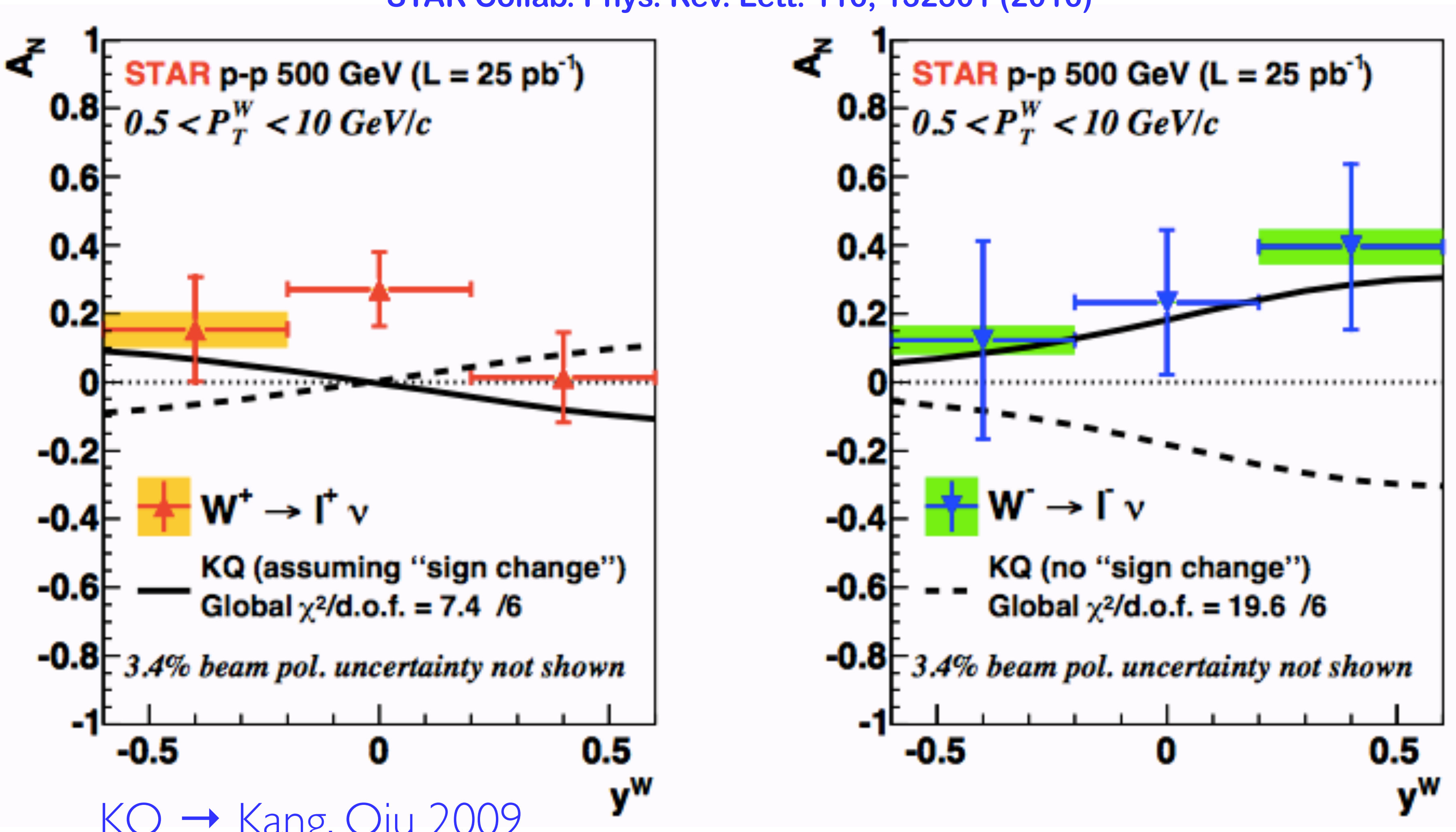

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- No sign change

$$\chi^2/d.o.f \sim 1.2$$

$$\chi^2/d.o.f \sim 3.2$$

- Large uncertainties of predictions
- No antiquark Sivers functions

Process dependence of Sivers function

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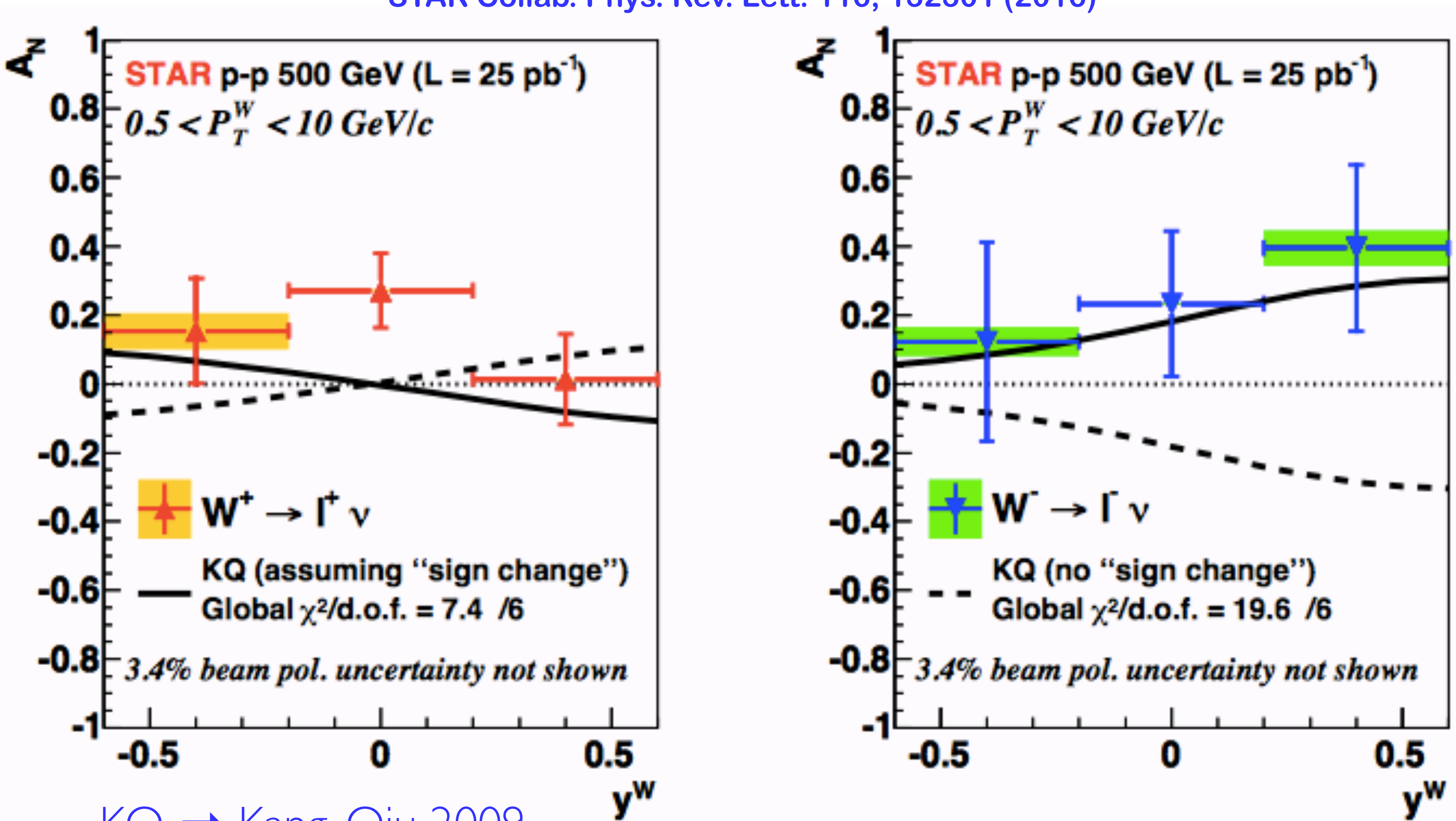

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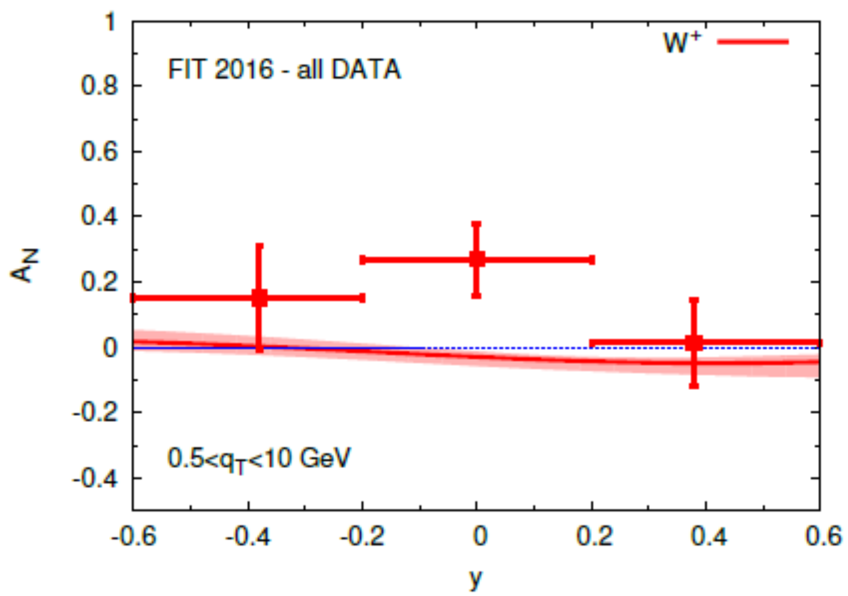
- Large uncertainties of predictions
- ~~No antiquark Sivers functions~~

Anselmino et al 2016

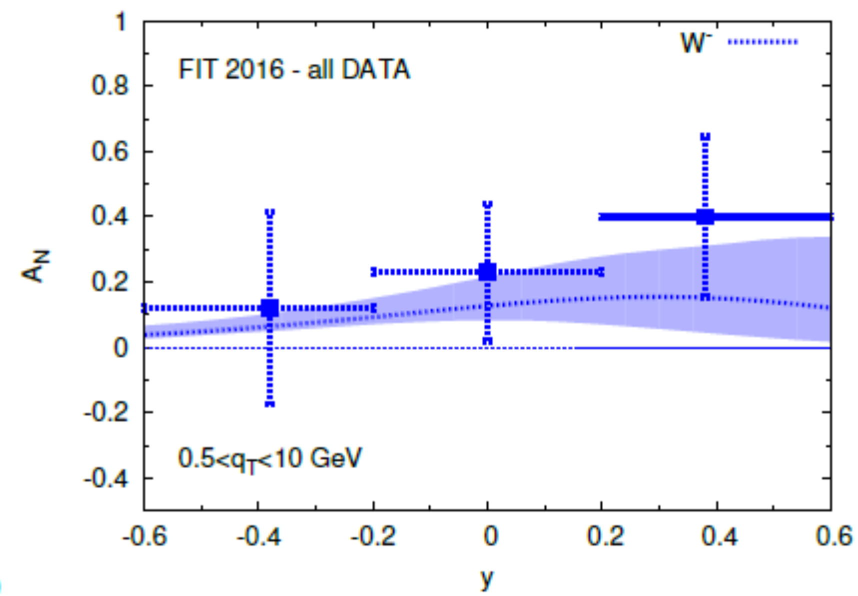
Process dependence of Sivers function

Anselmino et al 2016

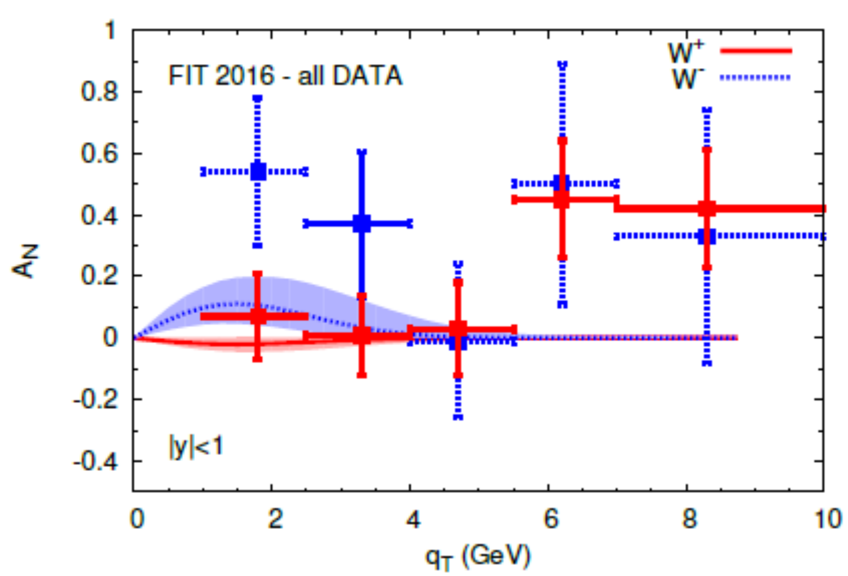
→ First experimental hint on the sign change: A_N in W and Z production



(a)



$$p^{\uparrow}p \to W^{\pm}X$$



→ Results with sign change

→ No TMD evolution

→ Antiquark Sivers functions included

Process dependence of Sivers function

Anselmino et al 2016

STAR 2015

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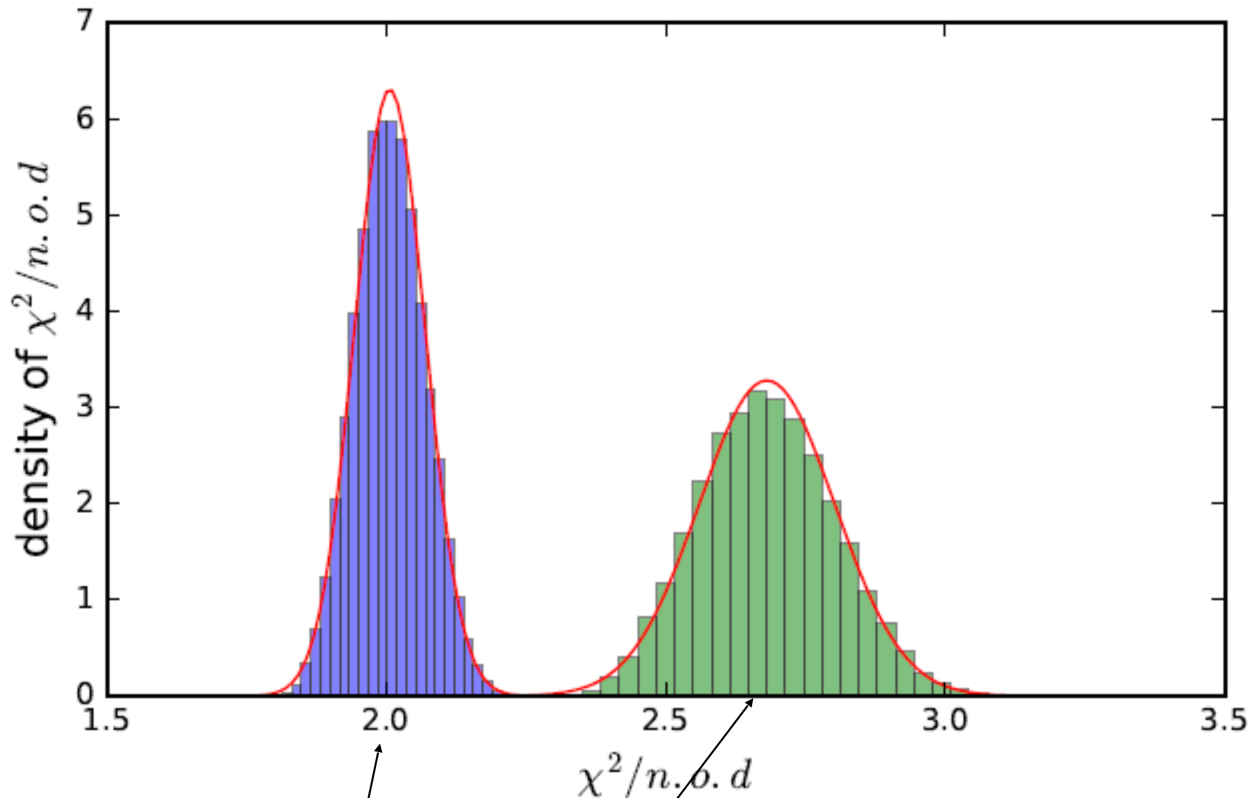


- Sign change
- No sign change

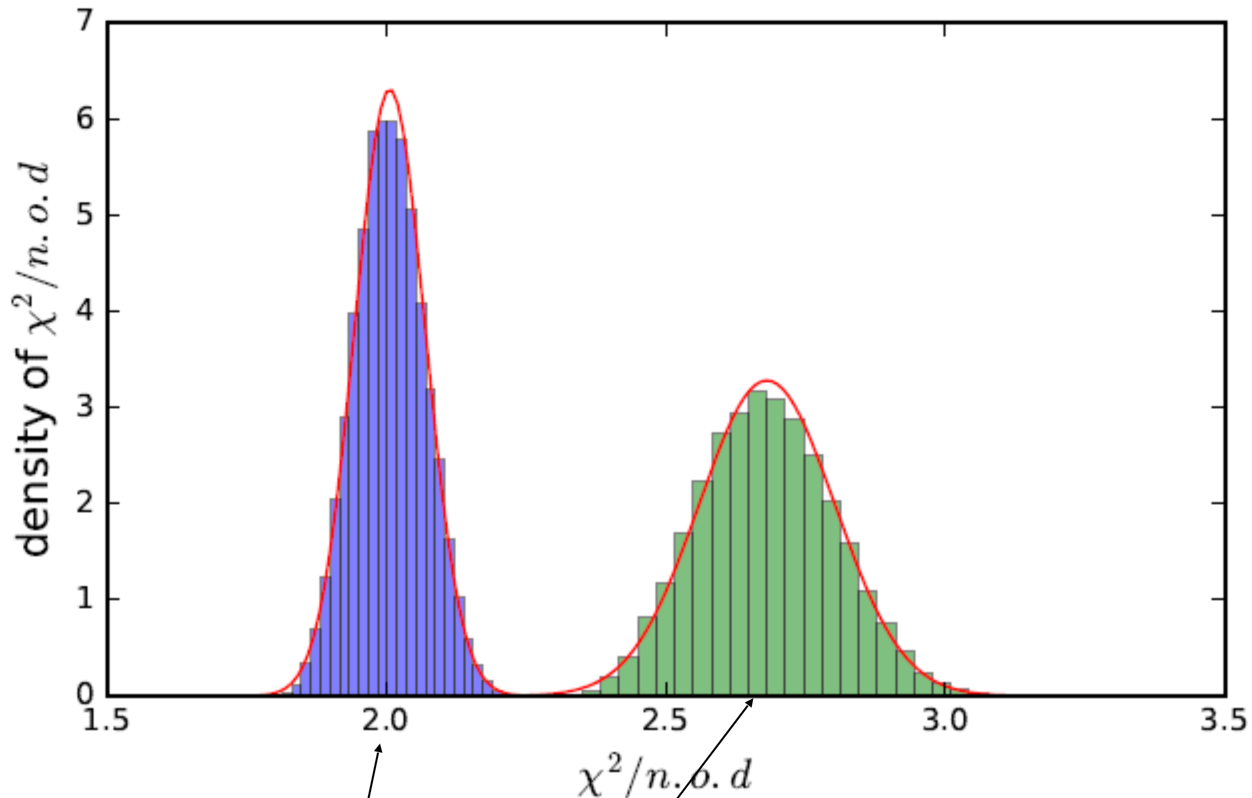
$\chi^2/d.o.f \sim 2$

$\chi^2/d.o.f \sim 2.68$

Process dependence of Sivers function

Anselmino et al 2016

- First experimental hint on the sign change: A_N in W and Z production



- Sign change
- No sign change

$\chi^2/d.o.f \sim 2$

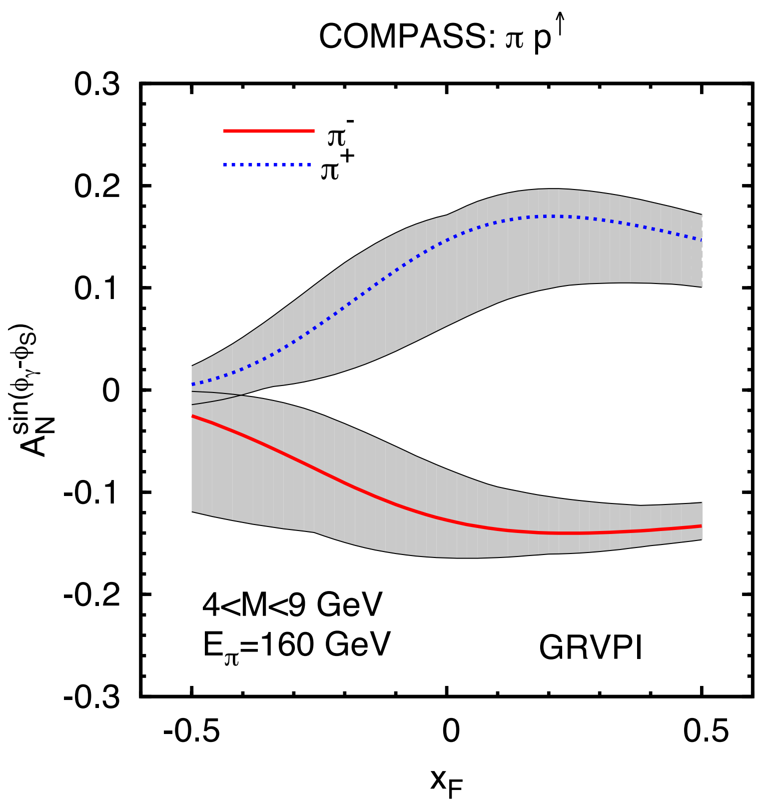
$\chi^2/d.o.f \sim 2.68$

- STAR results hint on sign change
- More precise data is needed
- Drell-Yan measurements are needed

Predictions for Sivers asymmetry in Drell-Yan

As the issue of the sign of the Sivers asymmetry in Drell-Yan processes is so important, let us discuss in details the choices adopted here and their motivation. We define our kinematical configuration with hadron $A^\uparrow$ moving along the positive z-axis, and hadron B opposite to it, in the $A-B$ center of mass frame. We choose the "up" ($\uparrow$) polarization direction as the positive y-axis ($\phi_S = \pi/2$). The transverse momenta have azimuthal angles

$$\boldsymbol{q}_T = q_T(\cos\phi_\gamma,\, \sin\phi_\gamma,\, 0) \qquad \boldsymbol{k}_{\perp i} = k_{\perp i}(\sin\varphi_i,\, \cos\varphi_i,\, 0) \qquad (i=1,2)\,, \tag{12}$$



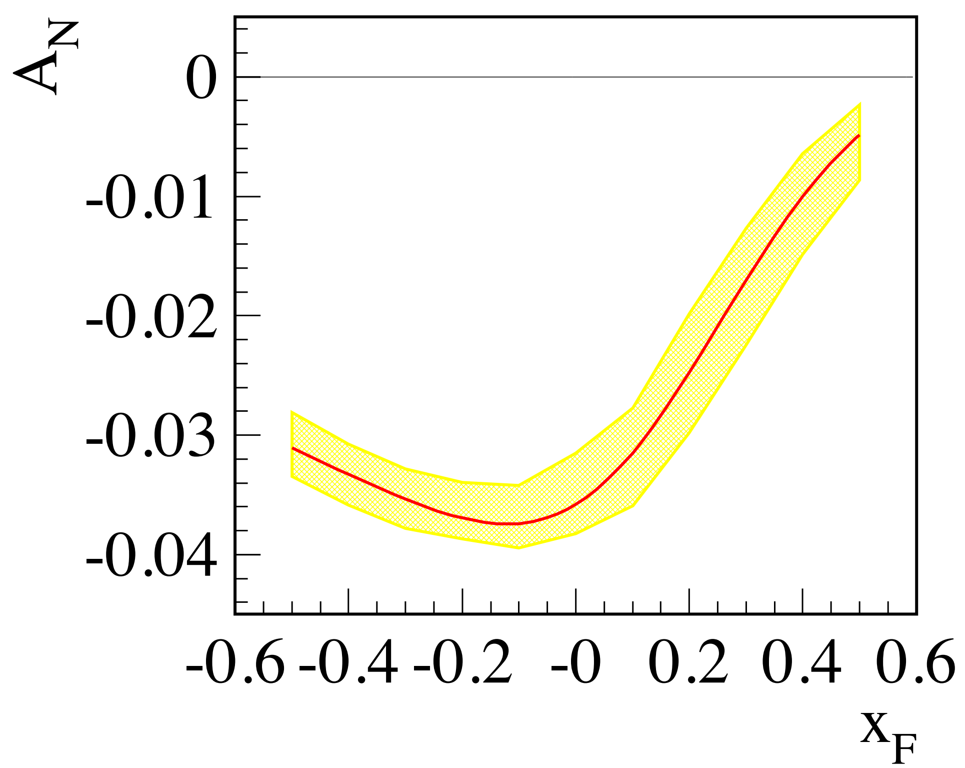
Anselmino et al 2009

$$A_N^{\sin(\phi_\gamma-\phi_S)}(x_F, M, q_T) = \frac{\int d\phi_\gamma\, [N(x_F, M, q_T, \phi_\gamma)]\, \sin(\phi_\gamma - \phi_S)}{\int d\phi_\gamma\, [D(x_F, M, q_T)]}$$

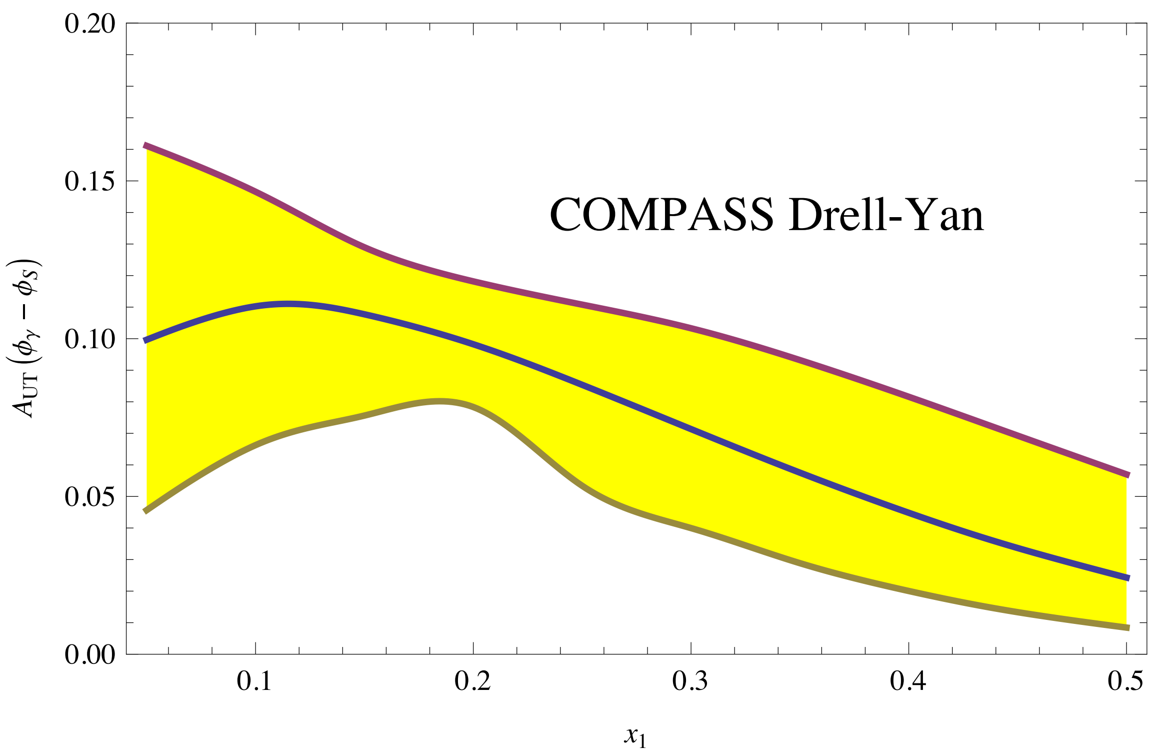
$$\begin{aligned} N(x_F, M, q_T, \phi_\gamma) &\equiv \frac{d^4\sigma^\uparrow}{dx_F\, dM^2\, d^2\boldsymbol{q}_T} - \frac{d^4\sigma^\downarrow}{dx_F\, dM^2\, d^2\boldsymbol{q}_T} \\ &= \frac{4\pi\alpha^2}{9M^2 s}\sum_q \frac{e_q^2}{x_1+x_2}\, \Delta^N f_{q/A^\uparrow}(x_1)\, f_{\bar q/B}(x_2)\, \sqrt{2e}\, \frac{q_T}{M_1}\, \frac{\langle k_S^2\rangle^2 \exp\left[-q_T^2/\left(\langle k_S^2\rangle + \langle k_{\perp 2}^2\rangle\right)\right]}{\pi\left[\langle k_S^2\rangle + \langle k_{\perp 2}^2\rangle\right]^2 \langle k_{\perp 2}^2\rangle}\, \sin(\phi_S - \phi_\gamma) \end{aligned} \tag{20}$$

$$\begin{aligned} D(x_F, M, q_T) &\equiv \frac{1}{2}\left[\frac{d^4\sigma^\uparrow}{dx_F\, dM^2\, d^2\boldsymbol{q}_T} + \frac{d^4\sigma^\downarrow}{dx_F\, dM^2\, d^2\boldsymbol{q}_T}\right] = \frac{d^4\sigma^{unp}}{dx_F\, dM^2\, d^2\boldsymbol{q}_T} \\ &= \frac{4\pi\alpha^2}{9M^2 s}\sum_q \frac{e_q^2}{x_1+x_2} f_{q/A}(x_1)\, f_{\bar q/B}(x_2)\, \frac{\exp\left[-q_T^2/\left(\langle k_{\perp 1}^2\rangle + \langle k_{\perp 2}^2\rangle\right)\right]}{\pi\left[\langle k_{\perp 1}^2\rangle + \langle k_{\perp 2}^2\rangle\right]}\,. \end{aligned} \tag{21}$$

- No TMD evolution
- TMD PDF width $\langle k_\perp^2 \rangle = 0.25\ GeV^2$
- Pion width the same as proton



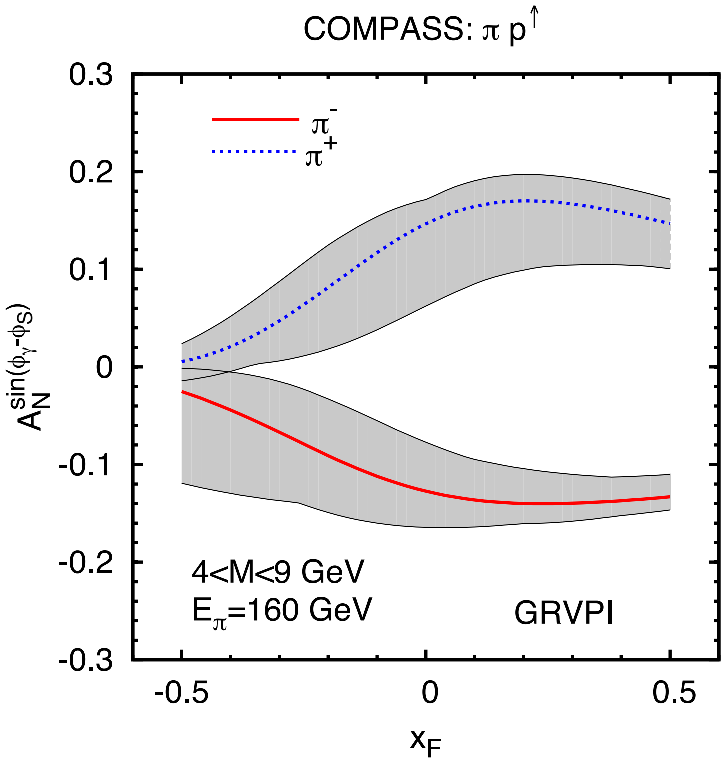
Echevarria et al 2014



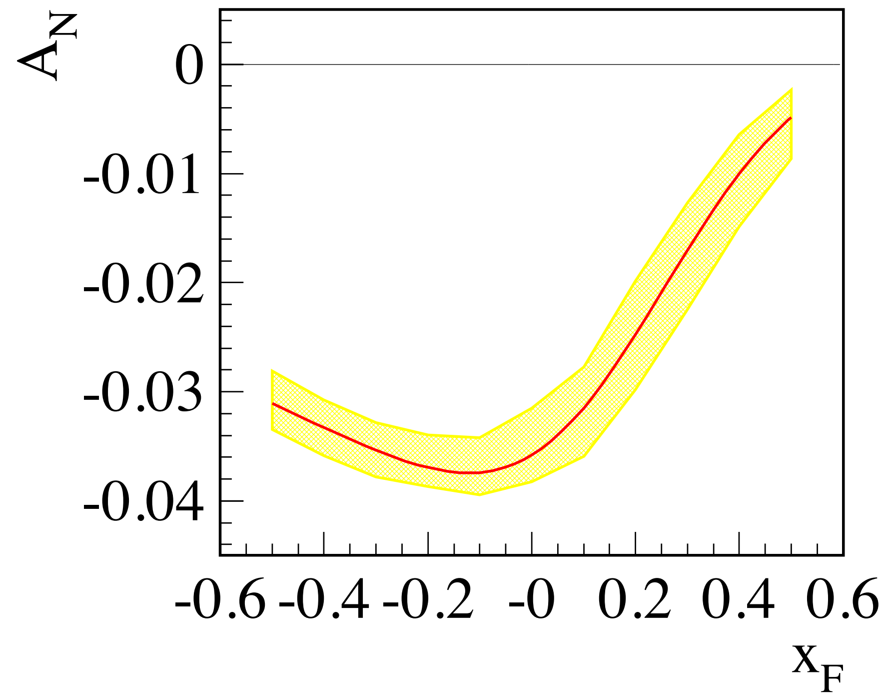
Sun, Yuan 2013

It is important to realize that the A_N defined above is opposite to the so-called weighted asymmetry $A_N^{\sin(\phi_\gamma - \phi_s)}$ defined in the literature, see, e.g., Refs. [63, 83].

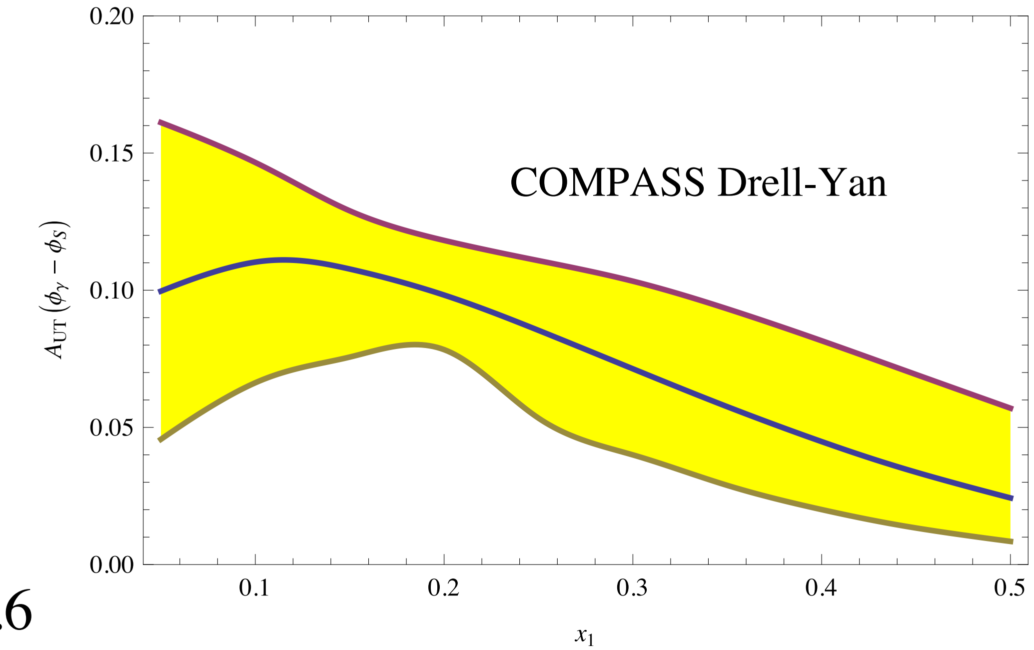
- Why predictions with TMD evolution are different?
- They use different forms of non perturbative evolution kernel
- COMPASS will be sensitive to this choice



Anselmino et al 2009

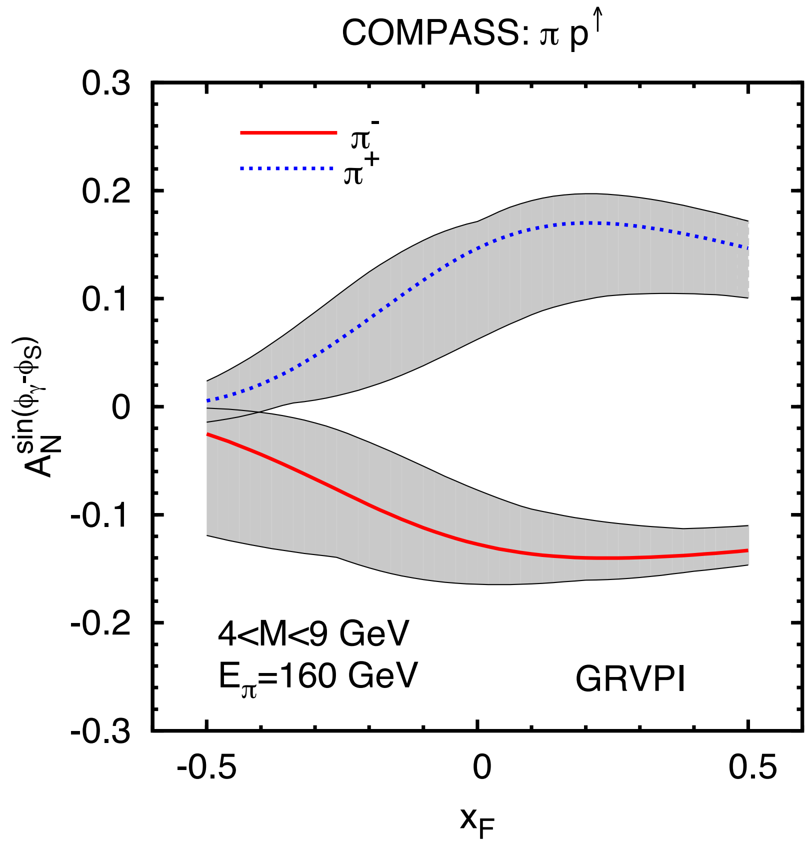


Echevarria et al 2014

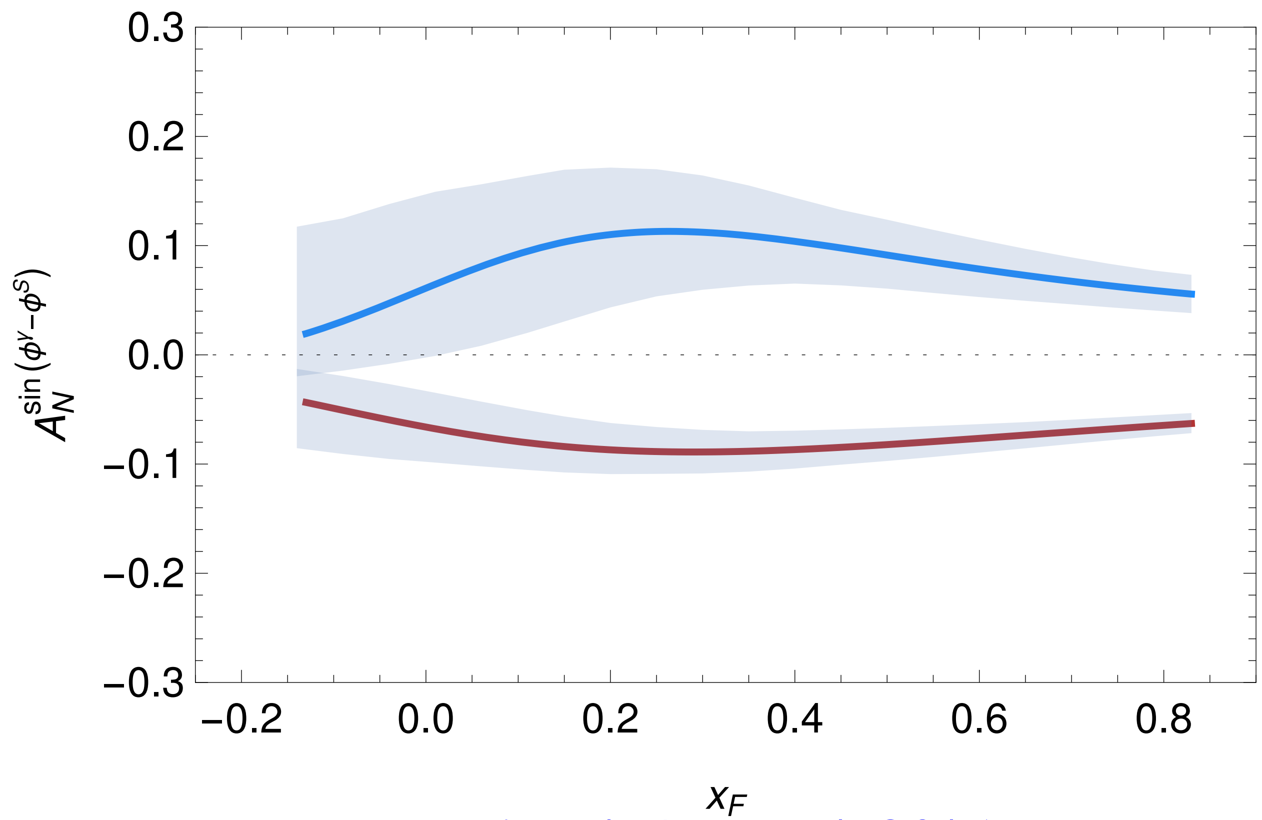


Sun, Yuan 2013

- ➔ Why predictions with TMD evolution and no TMD evolution are different?
- ➔ TMD evolution typically results in **suppression** due to soft gluon radiation
- ➔ Thus results with TMD evolution are lower compared to no evolution
- ➔ **End of the story?**



Anselmino et al 2009



Anselmino et al 2016

- However also no TMD evolution prerdictions are sensitive to the choice of widths!
- Asymmetry is **suppressed**, but not due to TMD evolution

- TMD PDF width $\langle k_\perp^2 \rangle = 0.25\ GeV^2$

  Values in accordance with EMC data

- TMD PDF width $\langle k_\perp^2 \rangle = 0.57\ GeV^2$

  Values in accordance with HERMES and COMPASS unpolarized SIDIS data

- Unpolarized Drell-Yan cross section is needed!
- The width of pion TMD PDF is to be extracted from the data

Collins function

Schafer-Teryaev sum rule

Schafer Teryaev 1999
Meissner, Metz, Pitonyak 2010

→ Conservation of transverse momentum

$$\langle P_T^i(z)\rangle \sim H_1^{\perp(1)}(z) \qquad H_1^{\perp(1)}(z) = \int d^2p_\perp \frac{p_\perp^2}{2z^2M_h^2} H_1^\perp(z,p_\perp^2)$$

→ Sum rule

$$\sum_h \int_0^1 dz \langle P_T^i(z)\rangle = 0$$

→ If only pions are considered $H_1^{\perp fav}(z) \sim -H_1^{\perp unf}(z)$

Universality of TMD fragmentation functions

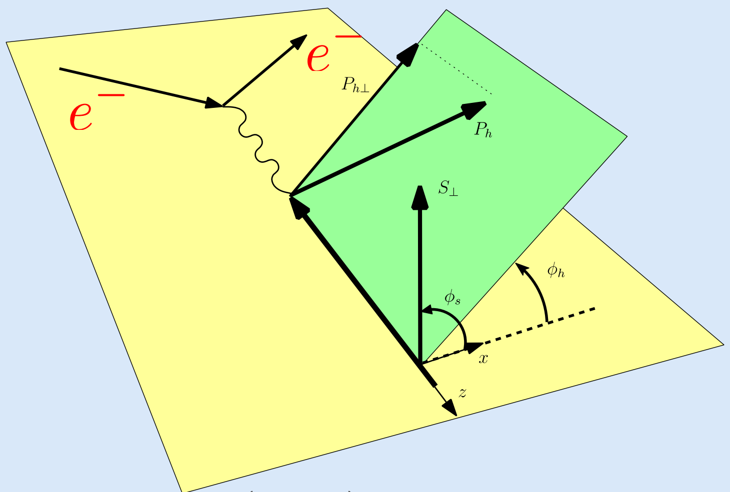
Metz 2002, Metz, Collins 2004, Yuan 2008
Gamberg, Mukherjee, Mulders 2011
Boer, Kang, Vogelsang, Yuan 2010

$$H_1^\perp(z)|_{SIDIS} = H_1^\perp(z)|_{e^+e^-} = H_1^\perp(z)|_{pp}$$

→ Very non trivial results

→ Agrees with phenomenology, allows global fits

- SIDIS and e+e-: combined global analysis

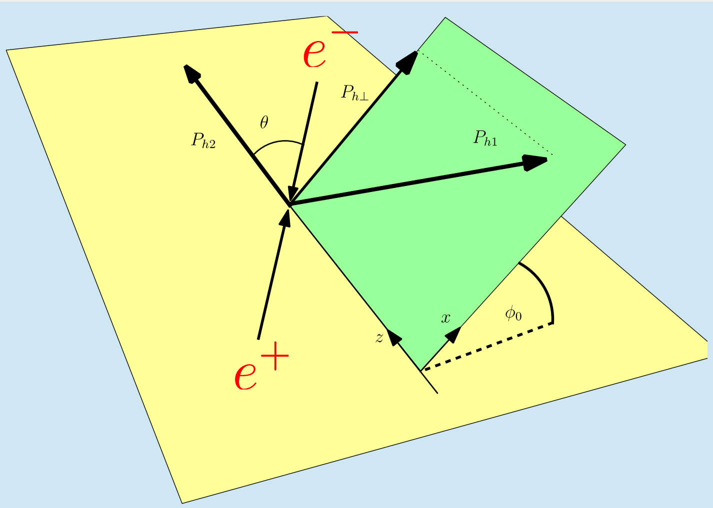


$$F_{UT}^{\sin(\phi_h+\phi_s)} \sim h_1(x_B, k_\perp) H_1^\perp(z_h, p_\perp)$$

transversity

Collins function

$$\frac{d\sigma(S_\perp)}{dx_B\, dy\, dz_h\, d^2 P_{h\perp}} = \sigma_0(x_B, y, Q^2)\left[F_{UU} + \sin(\phi_h+\phi_s)\,\frac{2(1-y)}{1+(1-y)^2}\,F_{UT}^{\sin(\phi_h+\phi_s)} + \ldots\right]$$



$$Z_{\text{collins}}^{h_1 h_2} \sim H_1^\perp(z_1, p_{1\perp}) H_1^\perp(z_2, p_{2\perp})$$

Collins function

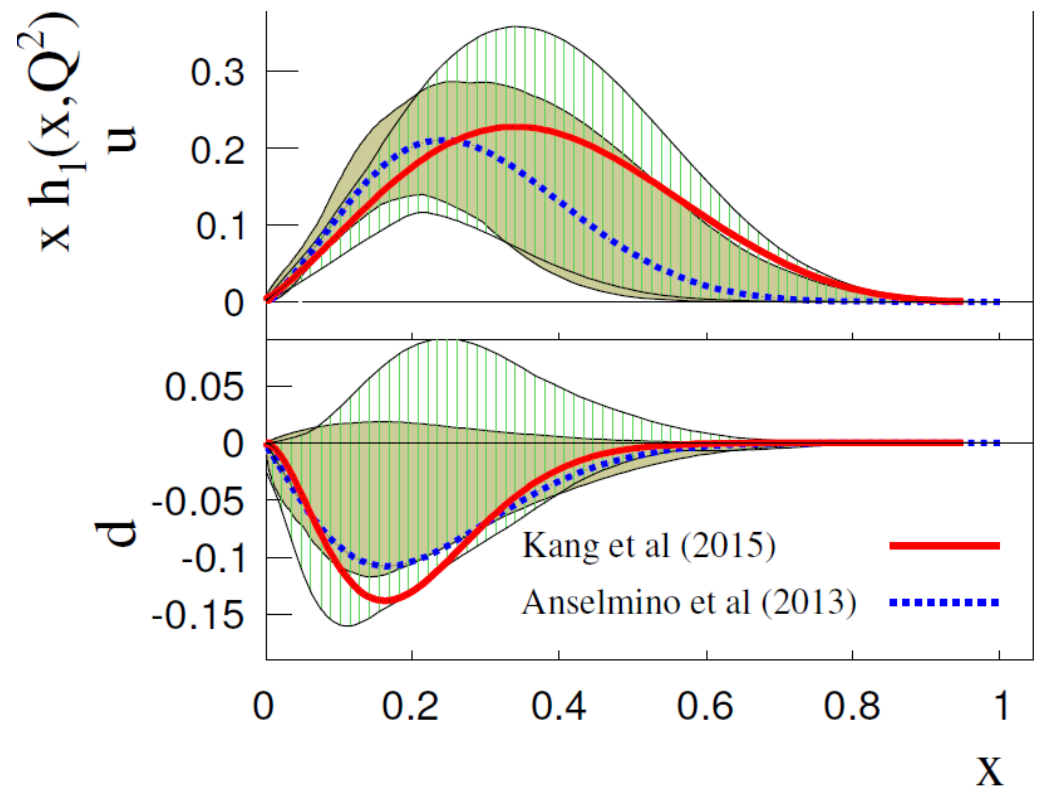
Collins function

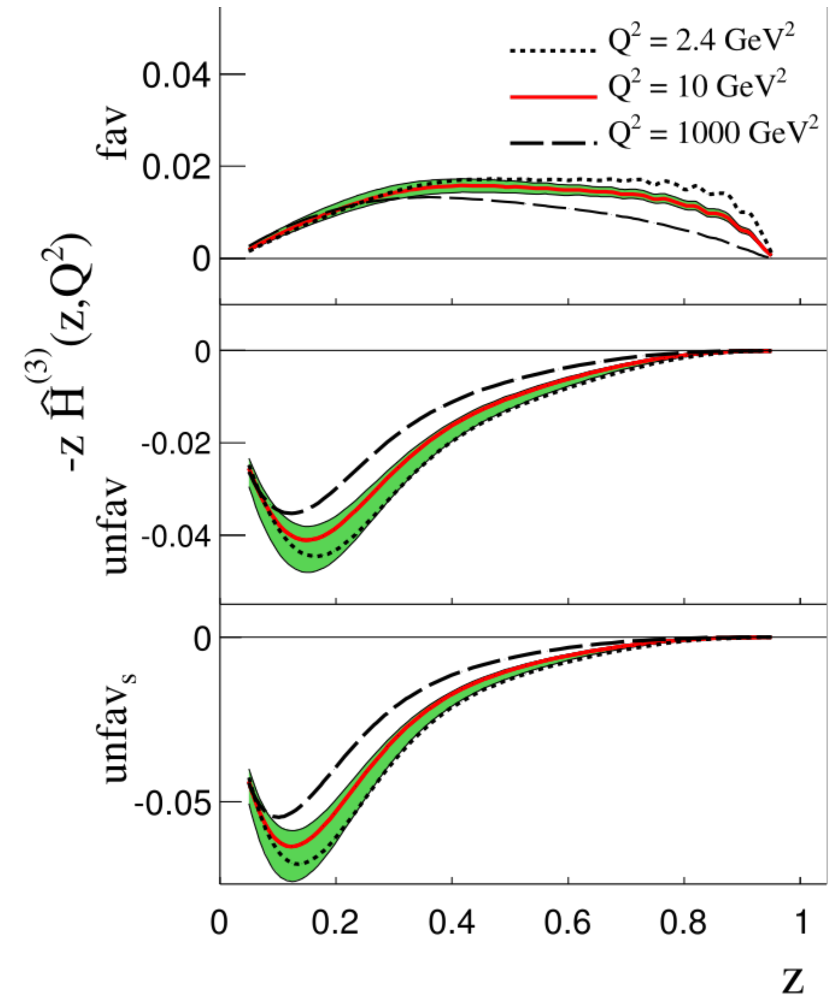
$$\frac{d\sigma^{e^+e^-\to h_1 h_2 + X}}{dz_{h1}\, dz_{h2}\, d^2 P_{h\perp}\, d\cos\theta} = \frac{N_c \pi \alpha_{\text{em}}^2}{2Q^2}\left[\left(1+\cos^2\theta\right) Z_{uu}^{h_1 h_2} + \sin^2\theta \cos(2\phi_0) Z_{\text{collins}}^{h_1 h_2}\right]$$

Transversity and Collins FF

Kang-Prokudin-Sun-Yuan 2015
Anselmino et al 2015

- Fitted quark transversity and Collins function: x (z) -dependence



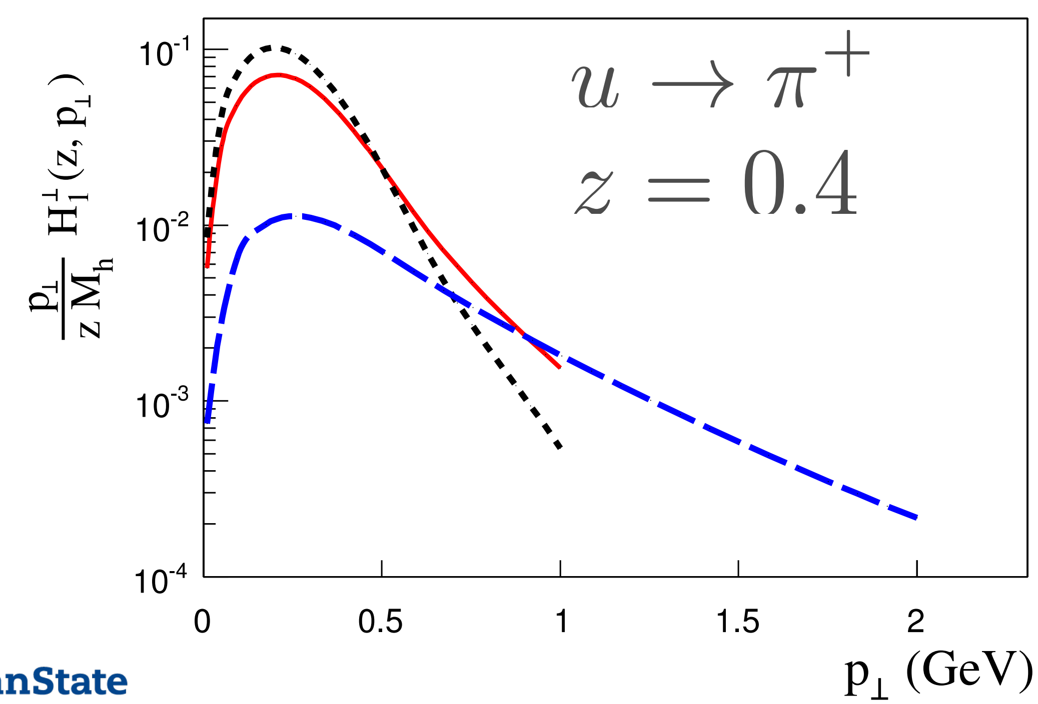


$\text{fav}: \quad u \to \pi^+$

$\text{unfav}: \quad d \to \pi^+$

$\text{unfav}_s: \quad s \to \pi$

- Collins function: pt-dependence



Compatible with LO extraction
Anselmino et al 2009, 2013, 2015

Precision of extraction depends on precision of calculations

Leading Log (LL):	$A^{(1)}$		
Next-to Leading Log (NLL):	$A^{(1,2)}$	$B^{(1)}$	$C^{(1)}$
Next-to-Next-to Leading Log (NNLL):	$A^{(1,2,3)}$	$B^{(1,2)}$	$C^{(1)}$

Kang, AP, Sun, Yuan 2015
Echevarria, Scimemi, Vladimirov 2016

Precision is important!

$C^{(1)}$ means that one should use NLO collinear distributions

Is the phenomenology complete at this point?

Presently *or soon* available fits

	Framework	HERMES	COMPASS	DY	Z production	N of points
KN 2006 hep-ph/0506225	NLL	✘	✘	✔	✔	98
Pavia 2013 (+Amsterdam,Bilbao) arXiv:1309.3507	No evo	✔	✘	✘	✘	1538
Torino 2014 (+JLab) arXiv:1312.6261	No evo	✔ (separately)	✔ (separately)	✘	✘	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NNLL	✘	✘	✔	✔	223
EIKV 2014 arXiv:1401.5078	NLL	1 (x,Q^2) bin	1 (x,Q^2) bin	✔	✔	500 (?)
Pavia 2016	NLL	✔	✔	✔	✔	8156

From Alessandro Bacchetta's talk at QCD Evolution 2016

No good understanding of asymmetries is possible without unpolarized cross-section description

- Phenomenology/theory is not yet complete
- Relation to collinear treatment should be refined
- Phenomenology with transition to collinear treatment (Y term) is to be performed
- Target mass corrections are not yet included in TMD formalism
- Better understanding of factorization and process mechanisms is needed

- TMD related studies have been extremely active in the past few years, lots of progress have been made
- We look forward to the future experimental results from COMPASS, RHIC, Jefferson Lab, LHC, Fermilab, future Electron Ion Collider
- Many TMD related groups are created throughout the world:

Italy, Netherlands, Belgium, Germany, Japan, China, Russia, and the USA

DOE funded Topical Collaboration for theory

TMD Collaboration

5 years of funding of **$2,160,000**
18 institutions
Theory, phenomenology, lattice QCD
Several postdoc positions.
2 tenure track positions: Temple, NSU
Support of undergraduates.

The TMD Collaboration
Spokespersons: William Detmold (MIT) and Jianwei Qiu (BNL)

Co-Investigators - (in alphabetical order of institutions):
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Thomas Mehen (Duke University)
Ted Rogers (Jefferson Laboratory and Old Dominion University)
Alexei Prokudin (Jefferson Laboratory and Penn State University at Berks)
Feng Yuan (Lawrence Berkeley National Laboratory)
Christopher Lee and Ivan Vitev (Los Alamos National Laboratory)
William Detmold, John Negele and Iain Stewart (MIT)
Matthias Burkardt and Michael Engelhardt (New Mexico State University)
Leonard Gamberg (Penn State University at Berks)
Andreas Metz (Temple University)
Sean Fleming (University of Arizona)
Keh-Fei Liu (University of Kentucky)
Xiangdong Ji (University of Maryland)
Simonetta Liuti (University of Virginia)

- 5 years of funding
- 18 institutions
- Theory, phenomenology, lattice QCD
- Several postdoc and tenure track positions to be created
- "To address the challenges of extracting novel quantitative information about the nucleon's internal landscape"
- "To provide compelling research, training, and career opportunities for young nuclear theorists"