QED and QCD with massless fermions in three dimensions

Nikhil Karthik and Rajamani Narayanan*

Florida International University

work supported in part by NSF grants 1205396 & 1515446

1705.11143 Submitted to Phys. Rev. D
1607.03905 Phys.Rev. D94 (2016) no.4, 045020
1606.04109 Phys.Rev. D94 (2016) no.6, 065026
1512.02993 Phys.Rev. D93 (2016) no.4, 045020
1505.01051 Phys.Rev. D92 (2015) no.2, 025003
Talk by N. Karthik, XQCD 2016

**Talk presented at*
Extreme QCD 2017 - The 15th international workshop on QCD in eXtreme conditions
26-28 Junt 2017, Pisa, Italy

Three dimensional QED and QCD with massless fermions

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Outline and Summary

Outline and Summary

- Lattice formalism and the phase of the fermion determinant. *1505.01051*

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Outline and Summary

- Lattice formalism and the phase of the fermion determinant. *1505.01051*
- Methodology to study the absence/presence of a bilinear condensate in theories with massless fermions. *1512.02993*

Outline and Summary

- Lattice formalism and the phase of the fermion determinant. *1505.01051*
- Methodology to study the absence/presence of a bilinear condensate in theories with massless fermions. *1512.02993*

Abelian Gauge Fields with even number(2N) of two component massless fermions preserving parity

Outline and Summary

- Lattice formalism and the phase of the fermion determinant. *1505.01051*
- Methodology to study the absence/presence of a bilinear condensate in theories with massless fermions. *1512.02993*

Abelian Gauge Fields with even number(2N) of two component massless fermions preserving parity

- N=1:

Outline and Summary

- Lattice formalism and the phase of the fermion determinant. *1505.01051*
- Methodology to study the absence/presence of a bilinear condensate in theories with massless fermions. *1512.02993*

Abelian Gauge Fields with even number(2N) of two component massless fermions preserving parity

- N=1:
  - Does not have a non-zero bilinear condensate. *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*

Outline and Summary

- Lattice formalism and the phase of the fermion determinant. *1505.01051*
- Methodology to study the absence/presence of a bilinear condensate in theories with massless fermions. *1512.02993*

Abelian Gauge Fields with even number(2N) of two component massless fermions preserving parity

- N=1:
  - Does not have a non-zero bilinear condensate. *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Low lying eigenvalues of the Dirac operator show critical behavior (neither ergodic, nor diffusive). *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*

Outline and Summary

- Lattice formalism and the phase of the fermion determinant. *1505.01051*
- Methodology to study the absence/presence of a bilinear condensate in theories with massless fermions. *1512.02993*

Abelian Gauge Fields with even number(2N) of two component massless fermions preserving parity

- N=1:
  - Does not have a non-zero bilinear condensate. *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Low lying eigenvalues of the Dirac operator show critical behavior (neither ergodic, nor diffusive). *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Vector and scalar correlators show three regions when computed on a ℓ^3 periodic torus: *1606.04109, **1705.11143**, Talk by N. Karthik in XQCD 2016*

Outline and Summary

- Lattice formalism and the phase of the fermion determinant. *1505.01051*
- Methodology to study the absence/presence of a bilinear condensate in theories with massless fermions. *1512.02993*

Abelian Gauge Fields with even number(2N) of two component massless fermions preserving parity

- N=1:
  - Does not have a non-zero bilinear condensate. *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Low lying eigenvalues of the Dirac operator show critical behavior (neither ergodic, nor diffusive). *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Vector and scalar correlators show three regions when computed on a ℓ^3 periodic torus: *1606.04109, **1705.11143**, Talk by N. Karthik in XQCD 2016*
    - A free field like behavior at short distances (x $\rightarrow$0). *Talk by N. Karthik in XQCD 2016*

Outline and Summary

- Lattice formalism and the phase of the fermion determinant. *1505.01051*
- Methodology to study the absence/presence of a bilinear condensate in theories with massless fermions. *1512.02993*

Abelian Gauge Fields with even number(2N) of two component massless fermions preserving parity

- N=1:
  - Does not have a non-zero bilinear condensate. *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Low lying eigenvalues of the Dirac operator show critical behavior (neither ergodic, nor diffusive). *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Vector and scalar correlators show three regions when computed on a ℓ^3 periodic torus: *1606.04109,* ***1705.11143****, Talk by N. Karthik in XQCD 2016*
    - A free field like behavior at short distances ($x \to 0$). *Talk by N. Karthik in XQCD 2016*
    - Anomalous (scalar) and non-anomalous (vector) power law behavior at large distances ($x \to \infty$; $x/\ell \to 0$) *Talk by N. Karthik in XQCD 2016*

Outline and Summary

- Lattice formalism and the phase of the fermion determinant. *1505.01051*
- Methodology to study the absence/presence of a bilinear condensate in theories with massless fermions. *1512.02993*

Abelian Gauge Fields with even number(2N) of two component massless fermions preserving parity

- N=1:
  - Does not have a non-zero bilinear condensate. *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Low lying eigenvalues of the Dirac operator show critical behavior (neither ergodic, nor diffusive). *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Vector and scalar correlators show three regions when computed on a ℓ^3 periodic torus: *1606.04109,* ***1705.11143****, Talk by N. Karthik in XQCD 2016*
    - A free field like behavior at short distances (x $\rightarrow$0). *Talk by N. Karthik in XQCD 2016*
    - Anomalous (scalar) and non-anomalous (vector) power law behavior at large distances (x $\rightarrow\infty$; x/$\ell \rightarrow$ 0) *Talk by N. Karthik in XQCD 2016*
    - Finite volume behavior (x $\rightarrow\infty$, x/ℓ fixed at a non-zero value) where we extract a mass proportional to 1/ℓ *Talk by N. Karthik in XQCD 2016*

Outline and Summary

- Lattice formalism and the phase of the fermion determinant. *1505.01051*
- Methodology to study the absence/presence of a bilinear condensate in theories with massless fermions. *1512.02993*

Abelian Gauge Fields with even number(2N) of two component massless fermions preserving parity

- N=1:
  - Does not have a non-zero bilinear condensate. *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Low lying eigenvalues of the Dirac operator show critical behavior (neither ergodic, nor diffusive). *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Vector and scalar correlators show three regions when computed on a ℓ^3 periodic torus: *1606.04109, **1705.11143**, Talk by N. Karthik in XQCD 2016*
    - A free field like behavior at short distances ($x \to 0$). *Talk by N. Karthik in XQCD 2016*
    - Anomalous (scalar) and non-anomalous (vector) power law behavior at large distances ($x \to \infty$; $x/\ell \to 0$) *Talk by N. Karthik in XQCD 2016*
    - Finite volume behavior ($x \to \infty$, x/ℓ fixed at a non-zero value) where we extract a mass proportional to $1/\ell$ *Talk by N. Karthik in XQCD 2016*
    - Flow of fermionic current central charge from ultra-violet to infra-red. ***1705.11143***

Outline and Summary

- Lattice formalism and the phase of the fermion determinant. *1505.01051*
- Methodology to study the absence/presence of a bilinear condensate in theories with massless fermions. *1512.02993*

Abelian Gauge Fields with even number(2N) of two component massless fermions preserving parity

- N=1:
  - Does not have a non-zero bilinear condensate. *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Low lying eigenvalues of the Dirac operator show critical behavior (neither ergodic, nor diffusive). *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Vector and scalar correlators show three regions when computed on a ℓ^3 periodic torus: *1606.04109, **1705.11143**, Talk by N. Karthik in XQCD 2016*
    - A free field like behavior at short distances ($x \to 0$). *Talk by N. Karthik in XQCD 2016*
    - Anomalous (scalar) and non-anomalous (vector) power law behavior at large distances ($x \to \infty$; $x/\ell \to 0$) *Talk by N. Karthik in XQCD 2016*
    - Finite volume behavior ($x \to \infty$, x/ℓ fixed at a non-zero value) where we extract a mass proportional to $1/\ell$ *Talk by N. Karthik in XQCD 2016*
    - Flow of fermionic current central charge from ultra-violet to infra-red. ***1705.11143***
    - Flavor and topological current correlators — Extended O(4) symmetry supporting strong duality. ***1705.11143***

Outline and Summary

- Lattice formalism and the phase of the fermion determinant. *1505.01051*
- Methodology to study the absence/presence of a bilinear condensate in theories with massless fermions. *1512.02993*

Abelian Gauge Fields with even number(2N) of two component massless fermions preserving parity

- N=1:
  - Does not have a non-zero bilinear condensate. *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Low lying eigenvalues of the Dirac operator show critical behavior (neither ergodic, nor diffusive). *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Vector and scalar correlators show three regions when computed on a ℓ^3 periodic torus: *1606.04109,* ***1705.11143****, Talk by N. Karthik in XQCD 2016*
    - A free field like behavior at short distances ($x \to 0$). *Talk by N. Karthik in XQCD 2016*
    - Anomalous (scalar) and non-anomalous (vector) power law behavior at large distances ($x \to \infty$; $x/\ell \to 0$) *Talk by N. Karthik in XQCD 2016*
    - Finite volume behavior ($x \to \infty$, x/ℓ fixed at a non-zero value) where we extract a mass proportional to $1/\ell$ *Talk by N. Karthik in XQCD 2016*
    - Flow of fermionic current central charge from ultra-violet to infra-red. ***1705.11143***
    - Flavor and topological current correlators — Extended O(4) symmetry supporting strong duality. ***1705.11143***

- N>1 theories show similar behavior with a reduction in the anomalous dimension. *1512.02993*

Outline and Summary

- Lattice formalism and the phase of the fermion determinant. *1505.01051*
- Methodology to study the absence/presence of a bilinear condensate in theories with massless fermions. *1512.02993*

Abelian Gauge Fields with even number(2N) of two component massless fermions preserving parity

- N=1:
  - Does not have a non-zero bilinear condensate. *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Low lying eigenvalues of the Dirac operator show critical behavior (neither ergodic, nor diffusive). *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Vector and scalar correlators show three regions when computed on a ℓ^3 periodic torus: *1606.04109, **1705.11143**, Talk by N. Karthik in XQCD 2016*
    - A free field like behavior at short distances ($x \to 0$). *Talk by N. Karthik in XQCD 2016*
    - Anomalous (scalar) and non-anomalous (vector) power law behavior at large distances ($x \to \infty$; $x/\ell \to 0$) *Talk by N. Karthik in XQCD 2016*
    - Finite volume behavior ($x \to \infty$, x/ℓ fixed at a non-zero value) where we extract a mass proportional to $1/\ell$ *Talk by N. Karthik in XQCD 2016*
    - Flow of fermionic current central charge from ultra-violet to infra-red. ***1705.11143***
    - Flavor and topological current correlators — Extended O(4) symmetry supporting strong duality. ***1705.11143***

- N>1 theories show similar behavior with a reduction in the anomalous dimension. *1512.02993*
- The quenched theory (N=0) has a non-zero bilinear condensate. ***1705.11143***

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Outline and Summary

- Lattice formalism and the phase of the fermion determinant. *1505.01051*
- Methodology to study the absence/presence of a bilinear condensate in theories with massless fermions. *1512.02993*

Abelian Gauge Fields with even number(2N) of two component massless fermions preserving parity

- N=1:
  - Does not have a non-zero bilinear condensate. *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Low lying eigenvalues of the Dirac operator show critical behavior (neither ergodic, nor diffusive). *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Vector and scalar correlators show three regions when computed on a ℓ^3 periodic torus: *1606.04109, **1705.11143**, Talk by N. Karthik in XQCD 2016*
    - A free field like behavior at short distances ($x \to 0$). *Talk by N. Karthik in XQCD 2016*
    - Anomalous (scalar) and non-anomalous (vector) power law behavior at large distances ($x \to \infty$; $x/\ell \to 0$) *Talk by N. Karthik in XQCD 2016*
    - Finite volume behavior ($x \to \infty$, x/ℓ fixed at a non-zero value) where we extract a mass proportional to $1/\ell$ *Talk by N. Karthik in XQCD 2016*
    - Flow of fermionic current central charge from ultra-violet to infra-red. ***1705.11143***
    - Flavor and topological current correlators — Extended O(4) symmetry supporting strong duality. ***1705.11143***
- N>1 theories show similar behavior with a reduction in the anomalous dimension. *1512.02993*
- The quenched theory (N=0) has a non-zero bilinear condensate. ***1705.11143***

Non-Abelian ($SU(N_c)$) Gauge Fields with even number($N_f = 2N$) of two component massless fermions preserving parity

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Outline and Summary

- Lattice formalism and the phase of the fermion determinant. *1505.01051*
- Methodology to study the absence/presence of a bilinear condensate in theories with massless fermions. *1512.02993*

Abelian Gauge Fields with even number(2N) of two component massless fermions preserving parity

- N=1:
  - Does not have a non-zero bilinear condensate. *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Low lying eigenvalues of the Dirac operator show critical behavior (neither ergodic, nor diffusive). *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Vector and scalar correlators show three regions when computed on a ℓ^3 periodic torus: *1606.04109, **1705.11143**, Talk by N. Karthik in XQCD 2016*
    - A free field like behavior at short distances ($x \to 0$). *Talk by N. Karthik in XQCD 2016*
    - Anomalous (scalar) and non-anomalous (vector) power law behavior at large distances ($x \to \infty$; $x/\ell \to 0$) *Talk by N. Karthik in XQCD 2016*
    - Finite volume behavior ($x \to \infty$, x/ℓ fixed at a non-zero value) where we extract a mass proportional to $1/\ell$ *Talk by N. Karthik in XQCD 2016*
    - Flow of fermionic current central charge from ultra-violet to infra-red. ***1705.11143***
    - Flavor and topological current correlators — Extended O(4) symmetry supporting strong duality. ***1705.11143***

- N>1 theories show similar behavior with a reduction in the anomalous dimension. *1512.02993*
- The quenched theory (N=0) has a non-zero bilinear condensate. ***1705.11143***

Non-Abelian ($SU(N_c)$) Gauge Fields with even number($N_f = 2N$) of two component massless fermions preserving parity

- 't Hooft limit ($N_c \to \infty$, N_f finite) has a non-zero bilinear condensate. *1607.03905, Talk by N. Karthik in XQCD 2016*

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Outline and Summary

- Lattice formalism and the phase of the fermion determinant. *1505.01051*
- Methodology to study the absence/presence of a bilinear condensate in theories with massless fermions. *1512.02993*

Abelian Gauge Fields with even number(2N) of two component massless fermions preserving parity

- N=1:
  - Does not have a non-zero bilinear condensate. *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Low lying eigenvalues of the Dirac operator show critical behavior (neither ergodic, nor diffusive). *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Vector and scalar correlators show three regions when computed on a ℓ^3 periodic torus: *1606.04109, **1705.11143**, Talk by N. Karthik in XQCD 2016*
    - A free field like behavior at short distances ($x \to 0$). *Talk by N. Karthik in XQCD 2016*
    - Anomalous (scalar) and non-anomalous (vector) power law behavior at large distances ($x \to \infty$; $x/\ell \to 0$) *Talk by N. Karthik in XQCD 2016*
    - Finite volume behavior ($x \to \infty$, x/ℓ fixed at a non-zero value) where we extract a mass proportional to $1/\ell$ *Talk by N. Karthik in XQCD 2016*
    - Flow of fermionic current central charge from ultra-violet to infra-red. ***1705.11143***
    - Flavor and topological current correlators — Extended O(4) symmetry supporting strong duality. ***1705.11143***

- N>1 theories show similar behavior with a reduction in the anomalous dimension. *1512.02993*
- The quenched theory (N=0) has a non-zero bilinear condensate. ***1705.11143***

Non-Abelian ($SU(N_c)$) Gauge Fields with even number($N_f = 2N$) of two component massless fermions preserving parity

- 't Hooft limit ($N_c \to \infty$, N_f finite) has a non-zero bilinear condensate. *1607.03905, Talk by N. Karthik in XQCD 2016*
- $N_c=2$, $N_f=1$ & 2 have a non-zero bilinear condensate. ***Work is complete***

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Outline and Summary

- Lattice formalism and the phase of the fermion determinant. *1505.01051*
- Methodology to study the absence/presence of a bilinear condensate in theories with massless fermions. *1512.02993*

Abelian Gauge Fields with even number(2N) of two component massless fermions preserving parity

- N=1:
  - Does not have a non-zero bilinear condensate. *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Low lying eigenvalues of the Dirac operator show critical behavior (neither ergodic, nor diffusive). *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Vector and scalar correlators show three regions when computed on a ℓ^3 periodic torus: *1606.04109, **1705.11143**, Talk by N. Karthik in XQCD 2016*
    - A free field like behavior at short distances ($x \to 0$). *Talk by N. Karthik in XQCD 2016*
    - Anomalous (scalar) and non-anomalous (vector) power law behavior at large distances ($x \to \infty$; $x/\ell \to 0$) *Talk by N. Karthik in XQCD 2016*
    - Finite volume behavior ($x \to \infty$, x/ℓ fixed at a non-zero value) where we extract a mass proportional to $1/\ell$ *Talk by N. Karthik in XQCD 2016*
    - Flow of fermionic current central charge from ultra-violet to infra-red. ***1705.11143***
    - Flavor and topological current correlators — Extended O(4) symmetry supporting strong duality. ***1705.11143***

- N>1 theories show similar behavior with a reduction in the anomalous dimension. *1512.02993*
- The quenched theory (N=0) has a non-zero bilinear condensate. ***1705.11143***

Non-Abelian ($SU(N_c)$) Gauge Fields with even number($N_f = 2N$) of two component massless fermions preserving parity

- 't Hooft limit ($N_c \to \infty$, N_f finite) has a non-zero bilinear condensate. *1607.03905, Talk by N. Karthik in XQCD 2016*
- N_c=2, N_f =1 & 2 have a non-zero bilinear condensate. ***Work is complete***
- The condensate for N_f = 2 is expected to come out lower than the one for N_f = 1. ***Work is essentially complete***

Outline and Summary

- Lattice formalism and the phase of the fermion determinant. *1505.01051*
- Methodology to study the absence/presence of a bilinear condensate in theories with massless fermions. *1512.02993*

Abelian Gauge Fields with even number(2N) of two component massless fermions preserving parity

- N=1:
  - Does not have a non-zero bilinear condensate. *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Low lying eigenvalues of the Dirac operator show critical behavior (neither ergodic, nor diffusive). *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Vector and scalar correlators show three regions when computed on a ℓ^3 periodic torus: *1606.04109, **1705.11143**, Talk by N. Karthik in XQCD 2016*
    - A free field like behavior at short distances ($x \to 0$). *Talk by N. Karthik in XQCD 2016*
    - Anomalous (scalar) and non-anomalous (vector) power law behavior at large distances ($x \to \infty$; $x/\ell \to 0$) *Talk by N. Karthik in XQCD 2016*
    - Finite volume behavior ($x \to \infty$, x/ℓ fixed at a non-zero value) where we extract a mass proportional to $1/\ell$ *Talk by N. Karthik in XQCD 2016*
    - Flow of fermionic current central charge from ultra-violet to infra-red. ***1705.11143***
    - Flavor and topological current correlators — Extended O(4) symmetry supporting strong duality. ***1705.11143***

- N>1 theories show similar behavior with a reduction in the anomalous dimension. *1512.02993*
- The quenched theory (N=0) has a non-zero bilinear condensate. ***1705.11143***

Non-Abelian ($SU(N_c)$) Gauge Fields with even number($N_f = 2N$) of two component massless fermions preserving parity

- 't Hooft limit ($N_c \to \infty$, N_f finite) has a non-zero bilinear condensate. *1607.03905, Talk by N. Karthik in XQCD 2016*
- $N_c=2$, $N_f=1$ & 2 have a non-zero bilinear condensate. ***Work is complete***
- The condensate for $N_f = 2$ is expected to come out lower than the one for $N_f = 1$. ***Work is essentially complete***

Abelian Gauge Fields with a single two component massless fermion

Outline and Summary

- Lattice formalism and the phase of the fermion determinant. *1505.01051*
- Methodology to study the absence/presence of a bilinear condensate in theories with massless fermions. *1512.02993*

Abelian Gauge Fields with even number(2N) of two component massless fermions preserving parity

- N=1:
  - Does not have a non-zero bilinear condensate. *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Low lying eigenvalues of the Dirac operator show critical behavior (neither ergodic, nor diffusive). *1512.02993, 1606.04109, Talk by N. Karthik in XQCD 2016*
  - Vector and scalar correlators show three regions when computed on a ℓ^3 periodic torus: *1606.04109, **1705.11143**, Talk by N. Karthik in XQCD 2016*
    - A free field like behavior at short distances ($x \to 0$). *Talk by N. Karthik in XQCD 2016*
    - Anomalous (scalar) and non-anomalous (vector) power law behavior at large distances ($x \to \infty$; $x/\ell \to 0$) *Talk by N. Karthik in XQCD 2016*
    - Finite volume behavior ($x \to \infty$, x/ℓ fixed at a non-zero value) where we extract a mass proportional to $1/\ell$ *Talk by N. Karthik in XQCD 2016*
    - Flow of fermionic current central charge from ultra-violet to infra-red. ***1705.11143***
    - Flavor and topological current correlators — Extended O(4) symmetry supporting strong duality. ***1705.11143***

- N>1 theories show similar behavior with a reduction in the anomalous dimension. *1512.02993*
- The quenched theory (N=0) has a non-zero bilinear condensate. ***1705.11143***

Non-Abelian (SU(N_c)) Gauge Fields with even number($N_f = 2N$) of two component massless fermions preserving parity

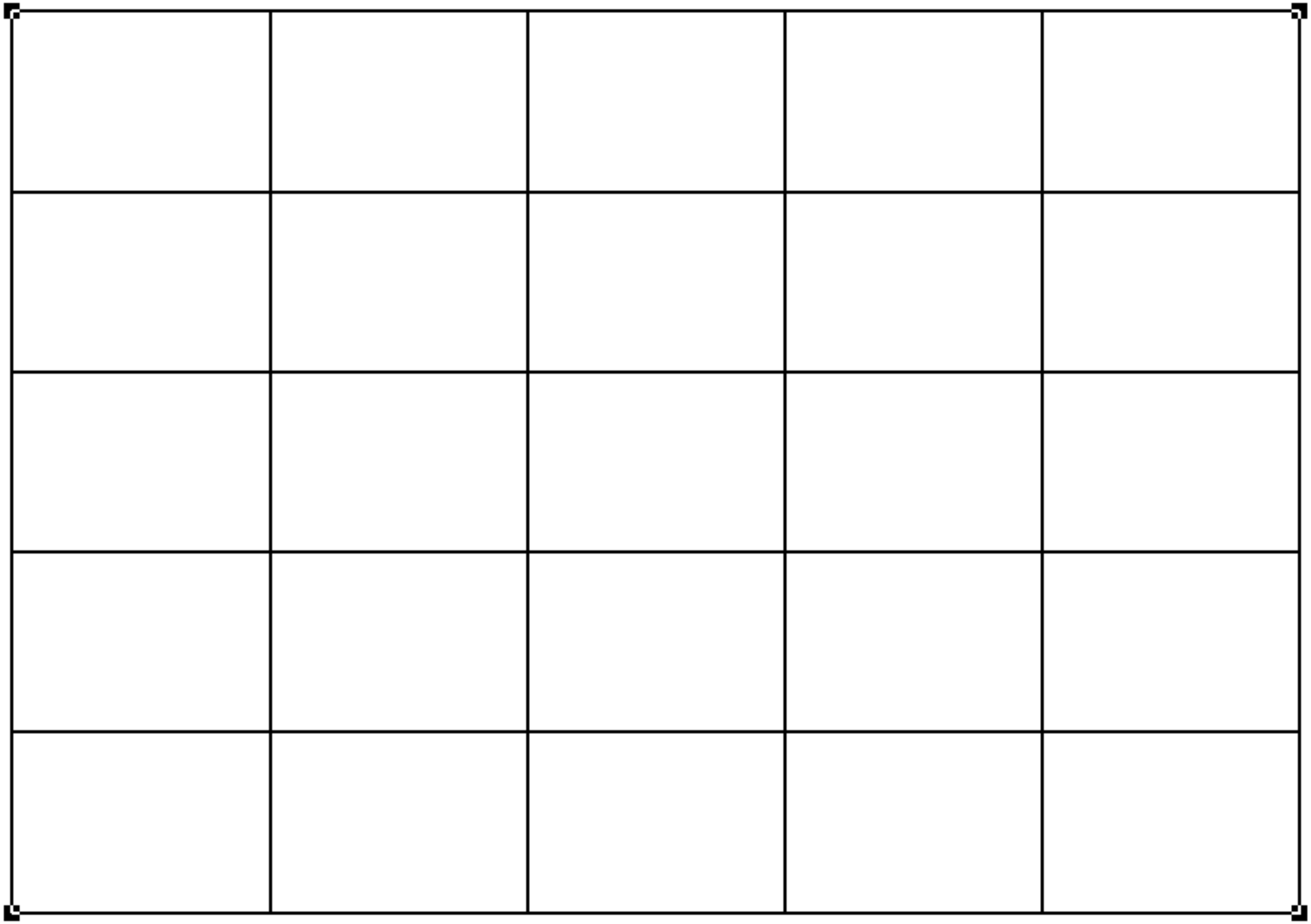
- 't Hooft limit ($N_c \to \infty$, N_f finite) has a non-zero bilinear condensate. *1607.03905, Talk by N. Karthik in XQCD 2016*
- $N_c=2$, $N_f=1$ & 2 have a non-zero bilinear condensate. ***Work is complete***
- The condensate for $N_f = 2$ is expected to come out lower than the one for $N_f = 1$. ***Work is essentially complete***

Abelian Gauge Fields with a single two component massless fermion

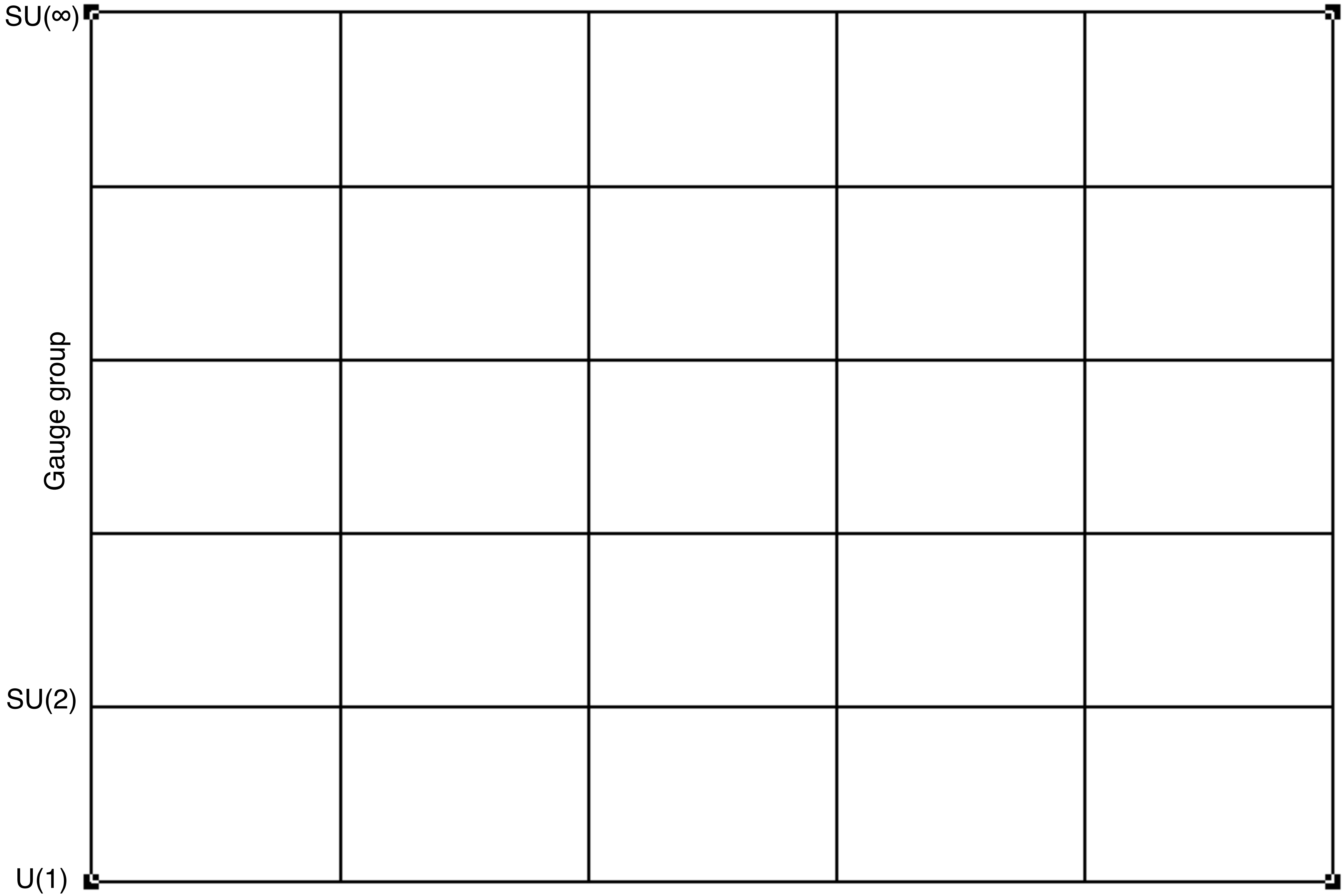
- A construction of a parity invariant formalism of a single two component massless charge 2 fermion coupled to a dynamical abelian gauge field — Son's model on the lattice. ***Work in progress***

QED and QCD with massless fermions in three dimensions

QED and QCD with massless fermions in three dimensions

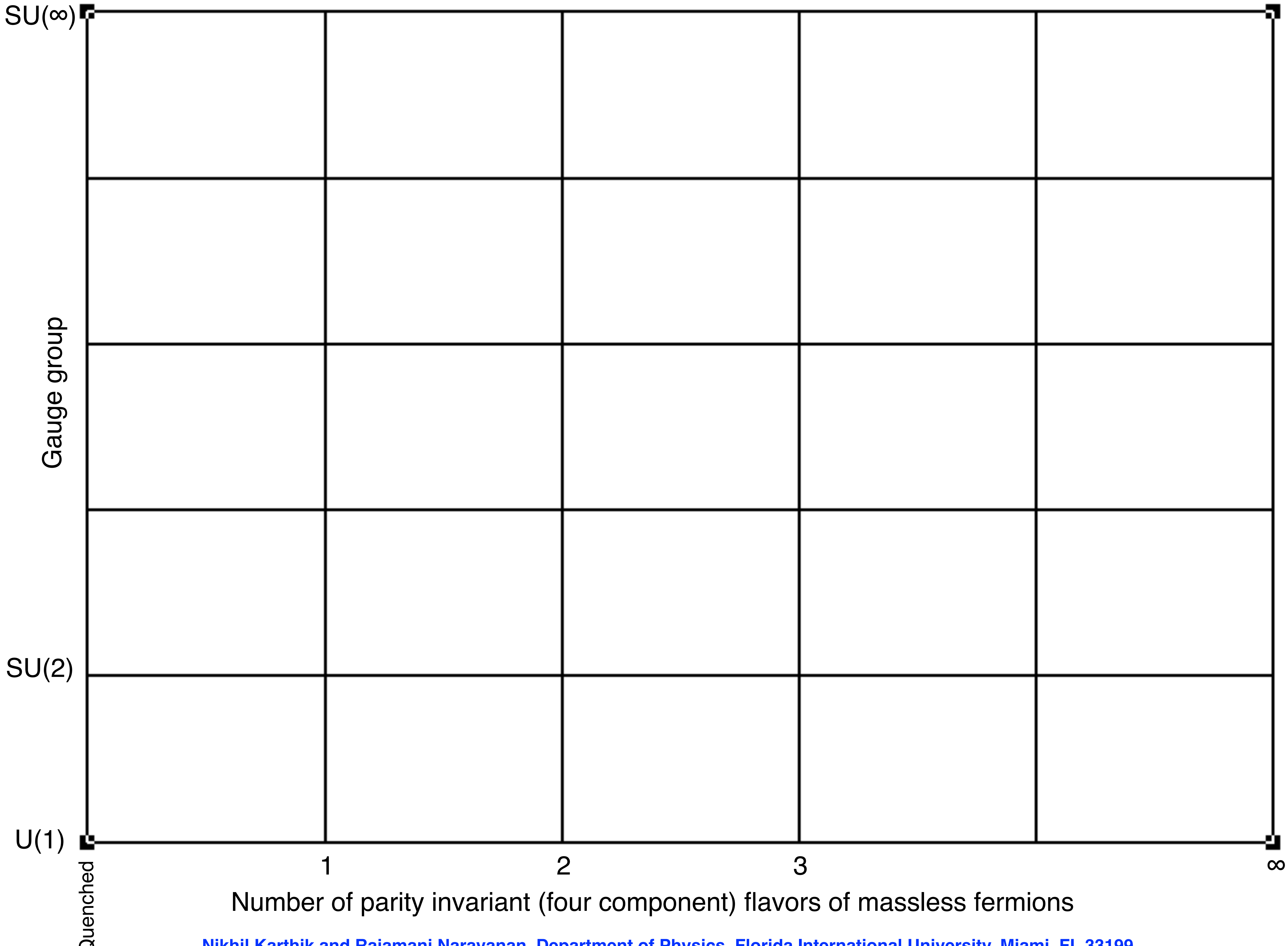


QED and QCD with massless fermions in three dimensions



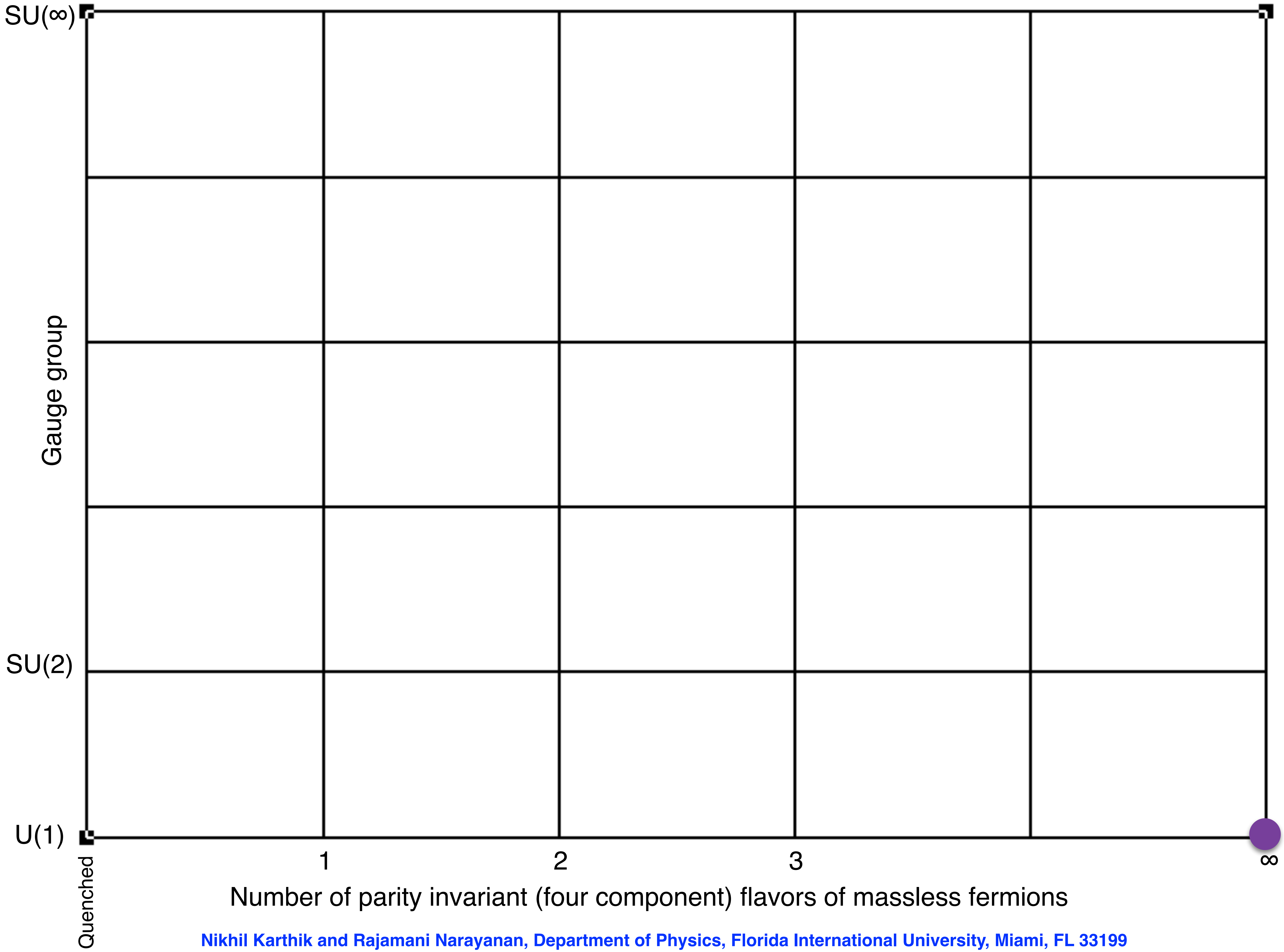
Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

QED and QCD with massless fermions in three dimensions



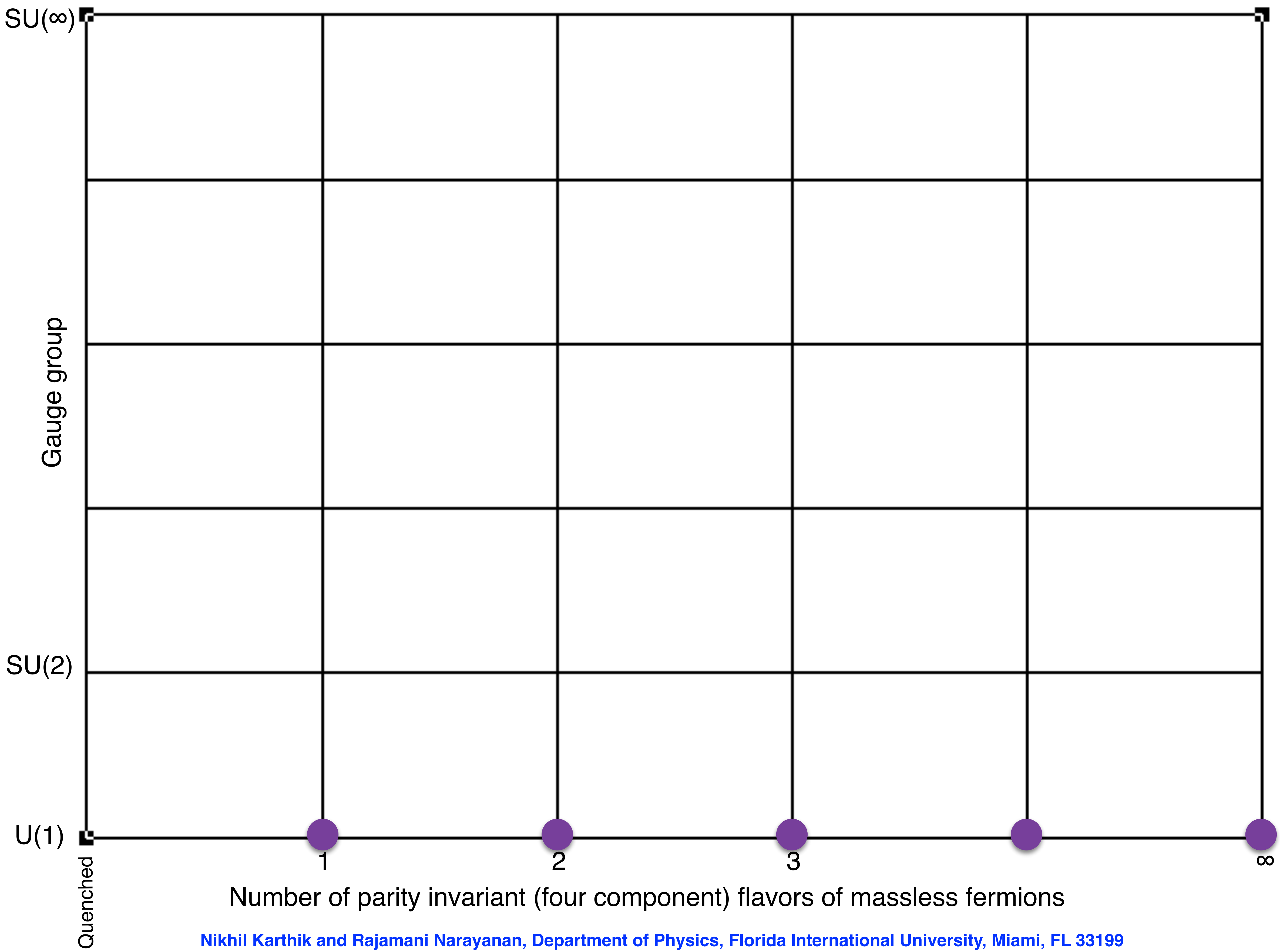
Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

QED and QCD with massless fermions in three dimensions



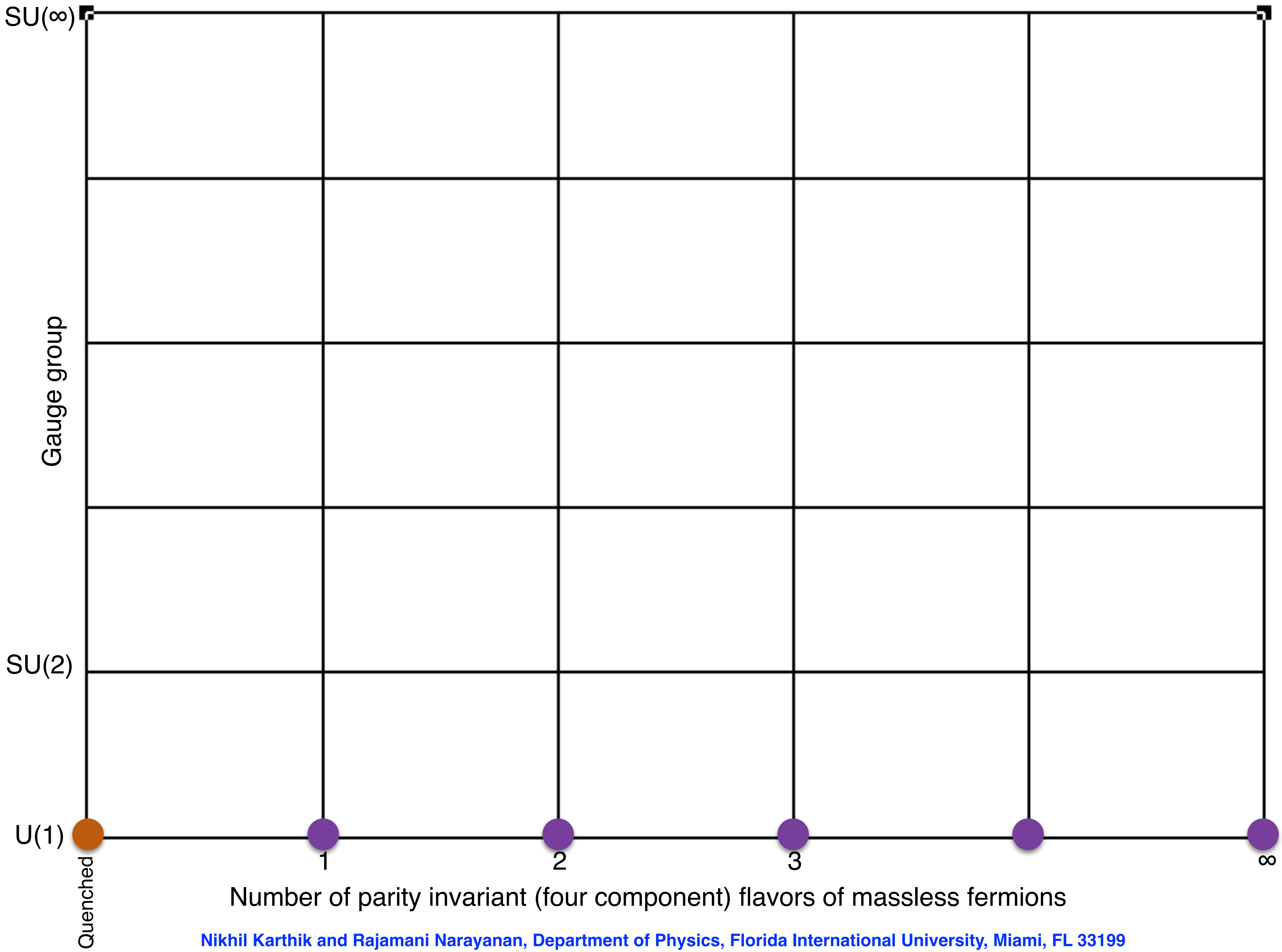
Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

QED and QCD with massless fermions in three dimensions



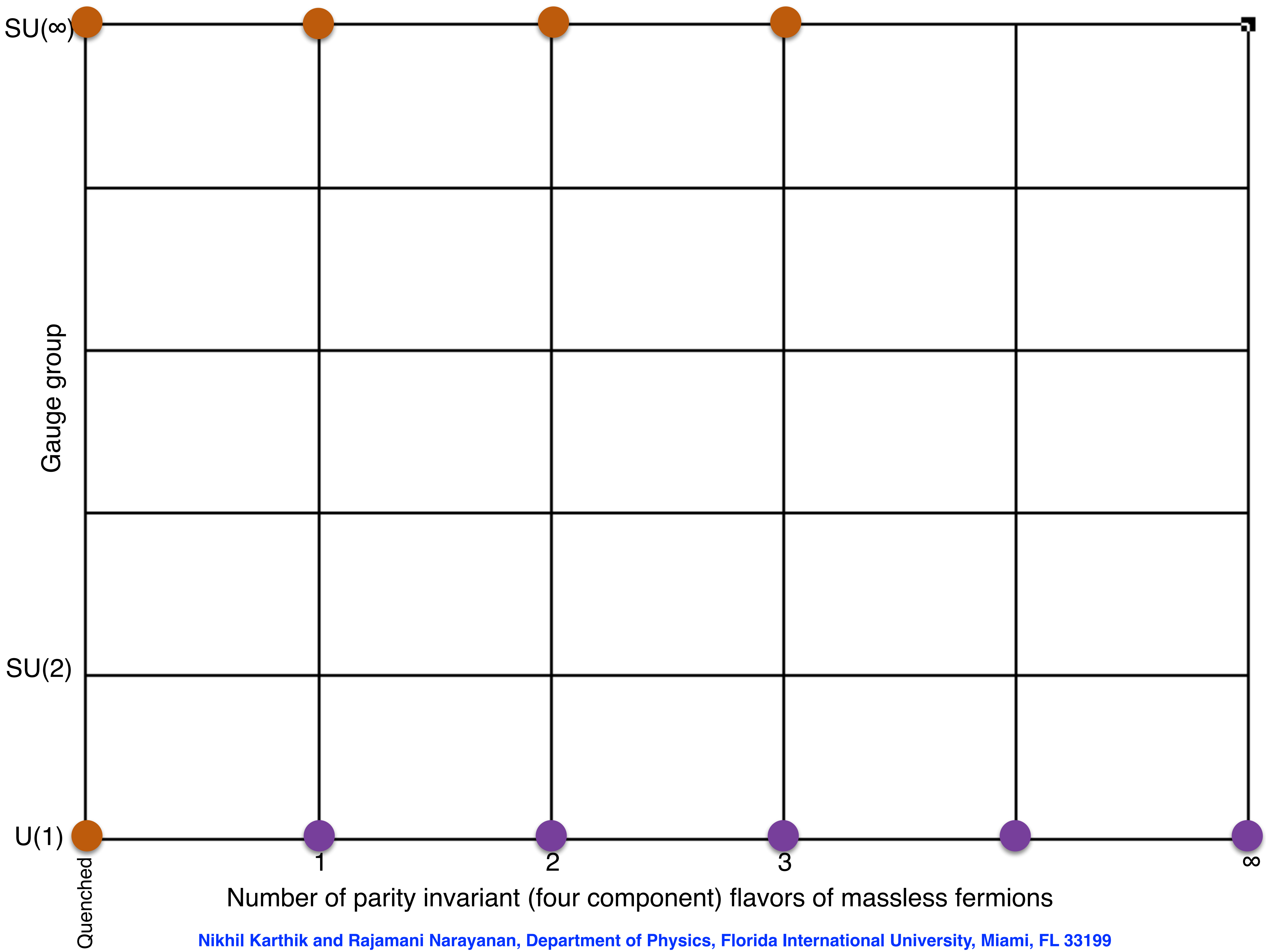
Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

QED and QCD with massless fermions in three dimensions



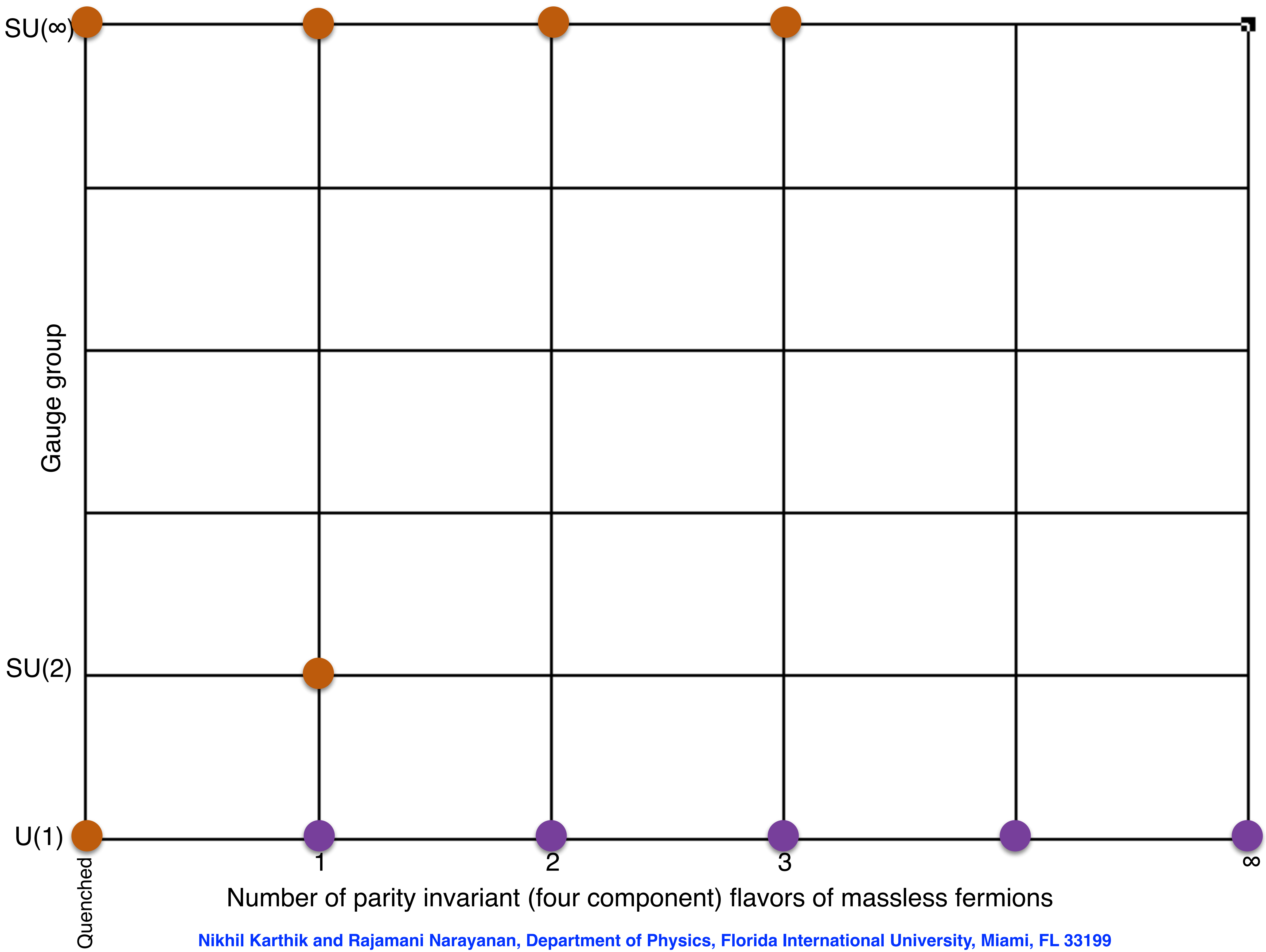
Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

QED and QCD with massless fermions in three dimensions



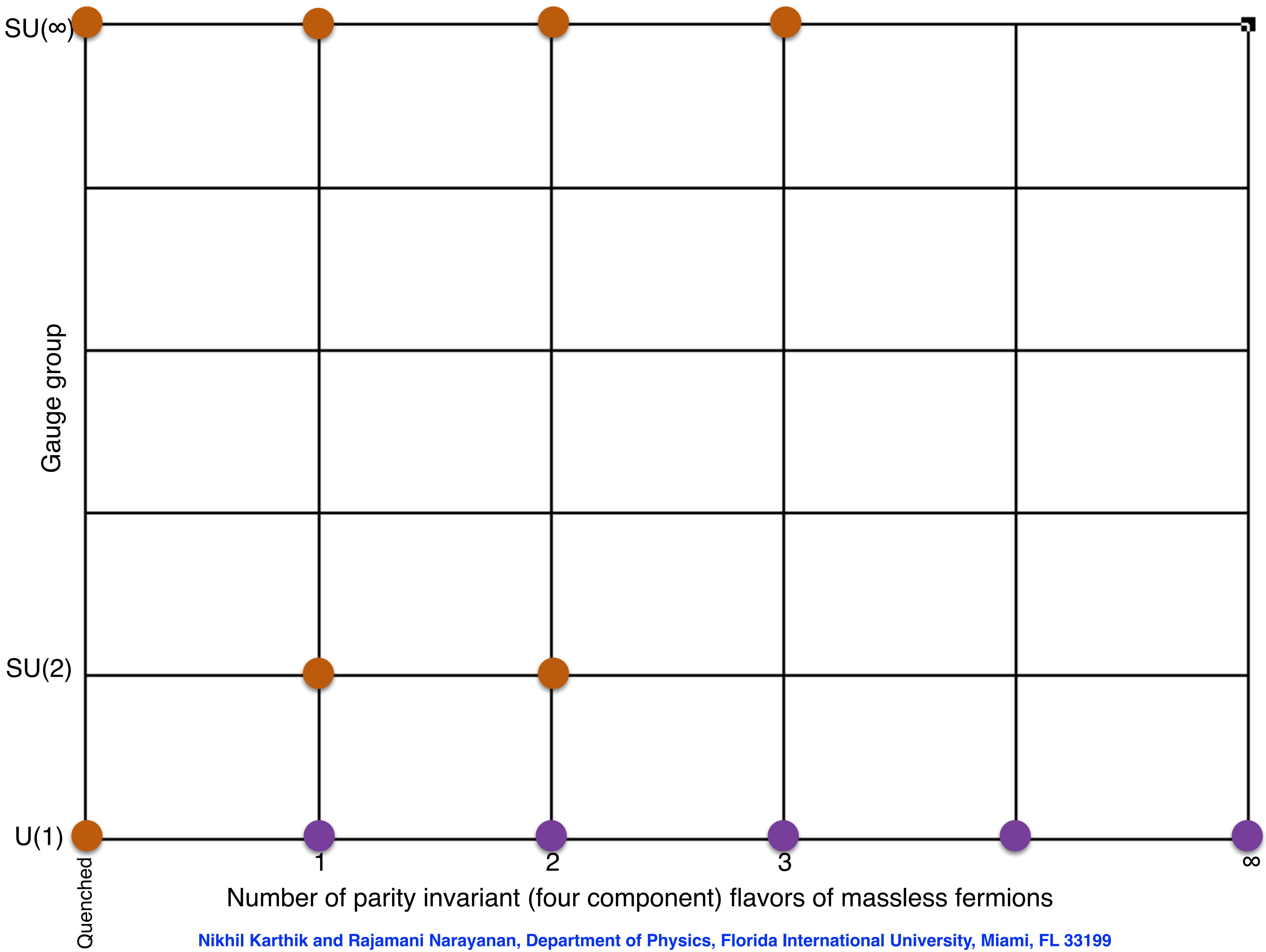
Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

QED and QCD with massless fermions in three dimensions



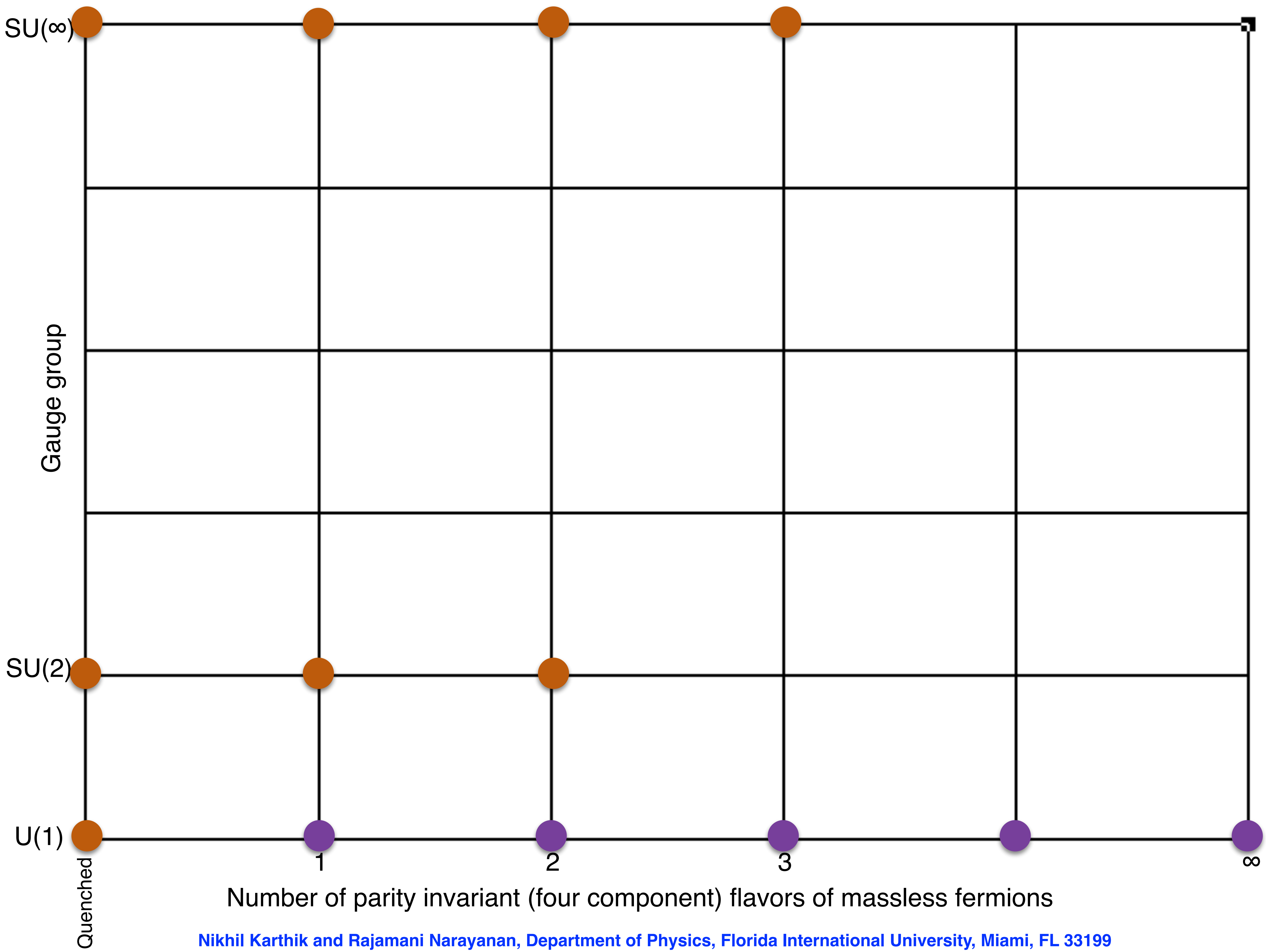
Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

QED and QCD with massless fermions in three dimensions



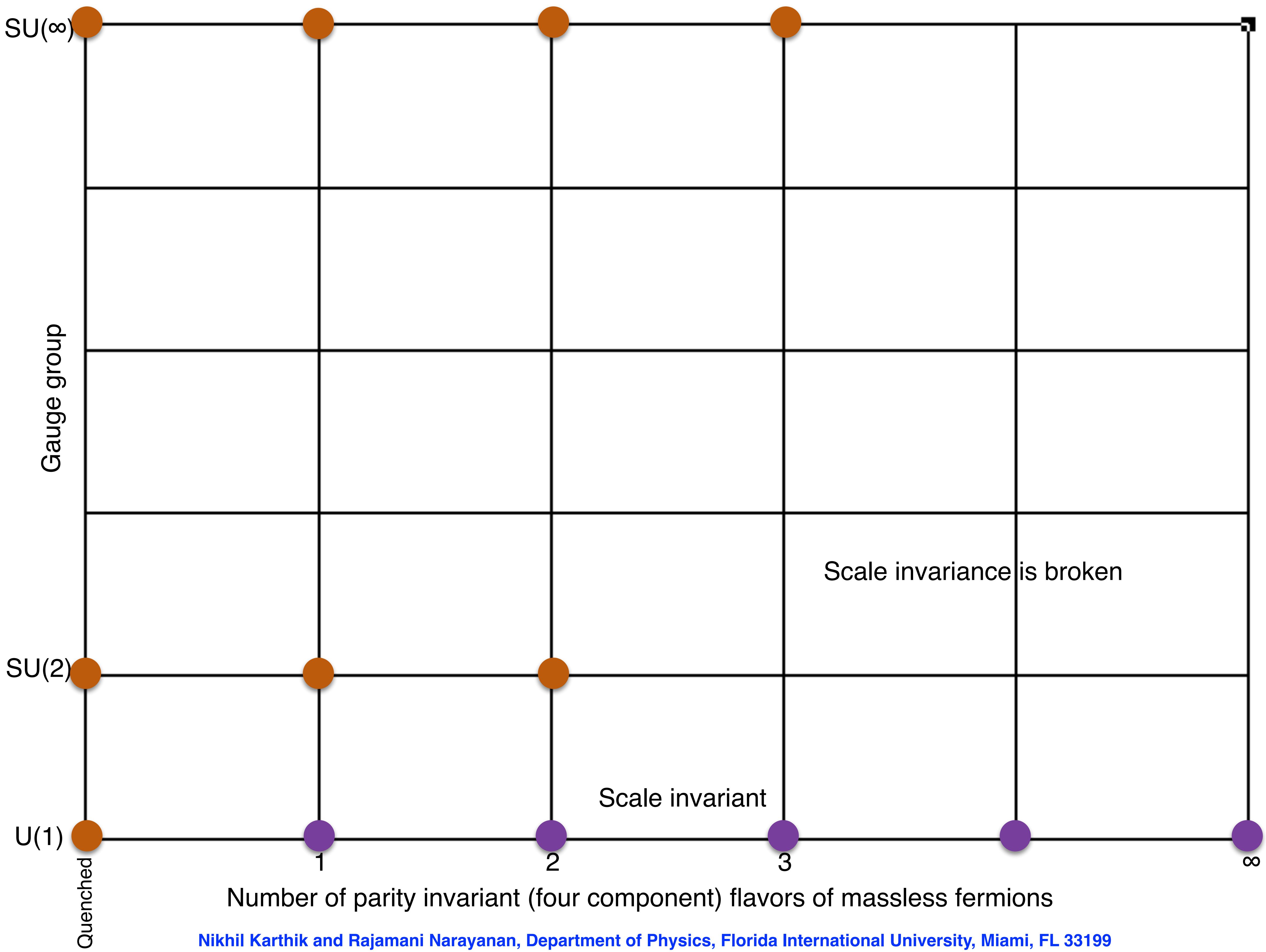
Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

QED and QCD with massless fermions in three dimensions



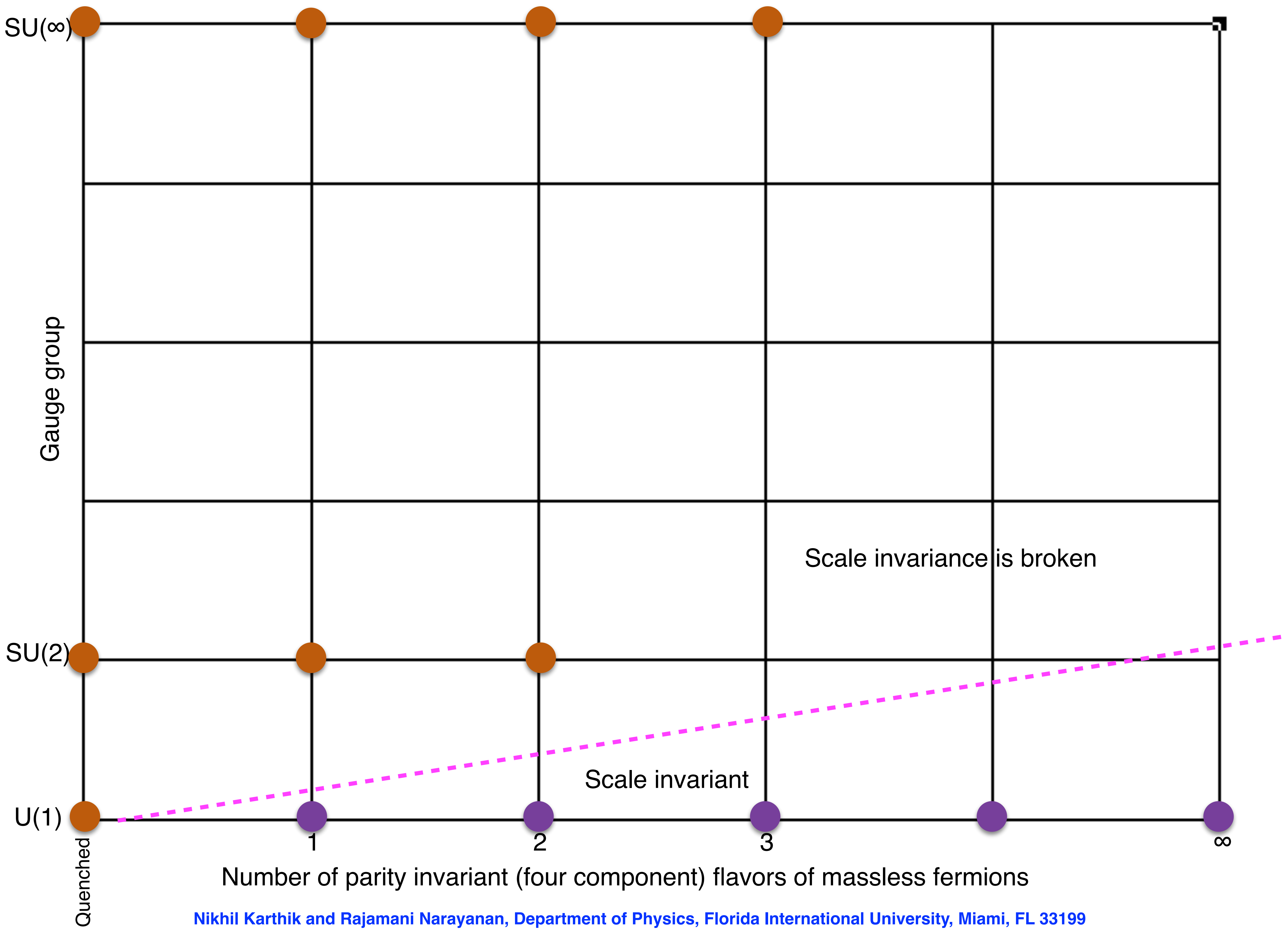
Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

QED and QCD with massless fermions in three dimensions



Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

QED and QCD with massless fermions in three dimensions



Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Lattice formalism - Overlap fermions

Lattice formalism - Overlap fermions

Background link variables on the lattice seen by the fermions are compact

Lattice formalism - Overlap fermions

Background link variables on the lattice seen by the fermions are compact

Fermionic partition function in a fixed gauge field background: $Z(\bar{\eta},\eta;m) = \det C_0(m) e^{\bar{\eta} G_0(m) \eta}$

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Lattice formalism - Overlap fermions

Background link variables on the lattice seen by the fermions are compact

Fermionic partition function in a fixed gauge field background: $Z(\bar{\eta}, \eta; m) = \det C_0(m) e^{\bar{\eta} G_0(m) \eta}$

Central character
A regularized
unitary operator, V,
that depends on the
background gauge fields

Lattice formalism - Overlap fermions

Background link variables on the lattice seen by the fermions are compact

Fermionic partition function in a fixed gauge field background: $Z(\bar{\eta},\eta;m) = \det C_0(m) e^{\bar{\eta} G_0(m)\eta}$

$$C_o(m) = \frac{1+V}{2} + m\frac{1-V}{2} \qquad m \in [-1,1]$$

Central character
A regularized
unitary operator, V,
that depends on the
background gauge fields

Lattice formalism - Overlap fermions

Background link variables on the lattice seen by the fermions are compact

Fermionic partition function in a fixed gauge field background: $Z(\bar{\eta},\eta;m) = \det C_0(m) e^{\bar{\eta} G_0(m) \eta}$

$$C_o(m) = \frac{1+V}{2} + m\frac{1-V}{2} \qquad m \in [-1,1]$$

$$C_o(1) = 1$$

Central character
A regularized
unitary operator, V,
that depends on the
background gauge fields

Lattice formalism - Overlap fermions

Background link variables on the lattice seen by the fermions are compact

Fermionic partition function in a fixed gauge field background: $Z(\bar{\eta}, \eta; m) = \det C_0(m) e^{\bar{\eta} G_0(m) \eta}$

$$C_o(m) = \frac{1+V}{2} + m\frac{1-V}{2} \qquad m \in [-1, 1]$$

$$C_o(1) = 1 \qquad\qquad C_o(-1) = V$$

Central character
A regularized
unitary operator, V,
that depends on the
background gauge fields

Lattice formalism - Overlap fermions

Background link variables on the lattice seen by the fermions are compact

Fermionic partition function in a fixed gauge field background: $Z(\bar{\eta}, \eta; m) = \det C_0(m) e^{\bar{\eta} G_0(m) \eta}$

$$C_o(m) = \frac{1+V}{2} + m\frac{1-V}{2} \qquad m \in [-1, 1]$$

$$C_o(1) = 1 \qquad C_o(0) = \frac{1+V}{2} \qquad C_o(-1) = V$$

Central character
A regularized
unitary operator, V,
that depends on the
background gauge fields

Lattice formalism - Overlap fermions

Background link variables on the lattice seen by the fermions are compact

Fermionic partition function in a fixed gauge field background: $Z(\bar{\eta},\eta;m) = \det C_0(m) e^{\bar{\eta}G_0(m)\eta}$

$$C_o(m) = \frac{1+V}{2} + m\frac{1-V}{2} \qquad m \in [-1,1] \qquad G_o(m) = \frac{1}{1-m}\left[C_o^{-1}(m) - 1\right] = \frac{A}{1+mA}$$

$$C_o(1) = 1 \qquad C_o(0) = \frac{1+V}{2} \qquad C_o(-1) = V$$

Central character
A regularized
unitary operator, V,
that depends on the
background gauge fields

Lattice formalism - Overlap fermions

Background link variables on the lattice seen by the fermions are compact

Fermionic partition function in a fixed gauge field background: $Z(\bar{\eta}, \eta; m) = \det C_0(m) e^{\bar{\eta} G_0(m) \eta}$

$$C_o(m) = \frac{1+V}{2} + m\frac{1-V}{2} \qquad m \in [-1, 1]$$

$$G_o(m) = \frac{1}{1-m}\left[C_o^{-1}(m) - 1\right] = \frac{A}{1+mA}$$

$$C_o(1) = 1 \qquad C_o(0) = \frac{1+V}{2} \qquad C_o(-1) = V$$

$$A = \frac{1-V}{1+V} \qquad A = -A^\dagger$$

Central character
A regularized
unitary operator, V,
that depends on the
background gauge fields

Lattice formalism - Overlap fermions

Background link variables on the lattice seen by the fermions are compact

Fermionic partition function in a fixed gauge field background: $Z(\bar{\eta}, \eta; m) = \det C_0(m) e^{\bar{\eta} G_0(m) \eta}$

$$C_o(m) = \frac{1+V}{2} + m\frac{1-V}{2} \qquad m \in [-1, 1] \qquad G_o(m) = \frac{1}{1-m}\left[C_o^{-1}(m) - 1\right] = \frac{A}{1+mA}$$

$$C_o(1) = 1 \qquad C_o(0) = \frac{1+V}{2} \qquad C_o(-1) = V \qquad A = \frac{1-V}{1+V} \qquad A = -A^\dagger$$

Central character
A regularized
unitary operator, V,
that depends on the
background gauge fields

Parallel transporter on the lattice: $T_k : [T_k \phi](n) = U_k(n) \phi\left(n + \hat{k}\right)$

Lattice formalism - Overlap fermions

Background link variables on the lattice seen by the fermions are compact

Fermionic partition function in a fixed gauge field background: $Z(\bar{\eta},\eta;m) = \det C_0(m) e^{\bar{\eta} G_0(m)\eta}$

$$C_o(m) = \frac{1+V}{2} + m\frac{1-V}{2} \qquad m \in [-1,1] \qquad G_o(m) = \frac{1}{1-m}\left[C_o^{-1}(m) - 1\right] = \frac{A}{1+mA}$$

$$C_o(1) = 1 \qquad C_o(0) = \frac{1+V}{2} \qquad C_o(-1) = V \qquad A = \frac{1-V}{1+V} \qquad A = -A^\dagger$$

Central character
A regularized
unitary operator, V,
that depends on the
background gauge fields

It is unitary: $T_k T_k^\dagger = 1$

Parallel transporter on the lattice: $T_k : [T_k\phi](n) = U_k(n)\phi\left(n+\hat{k}\right)$

Lattice formalism - Overlap fermions

Background link variables on the lattice seen by the fermions are compact

Fermionic partition function in a fixed gauge field background: $Z(\bar\eta,\eta;m) = \det C_0(m) e^{\bar\eta G_0(m)\eta}$

$$C_o(m) = \frac{1+V}{2} + m\frac{1-V}{2} \qquad m \in [-1,1]$$

$$G_o(m) = \frac{1}{1-m}\left[C_o^{-1}(m) - 1\right] = \frac{A}{1+mA}$$

$$C_o(1) = 1 \qquad C_o(0) = \frac{1+V}{2} \qquad C_o(-1) = V$$

$$A = \frac{1-V}{1+V} \qquad A = -A^\dagger$$

Central character
A regularized
unitary operator, V,
that depends on the
background gauge fields

Parallel transporter on the lattice: $T_k : [T_k\phi](n) = U_k(n)\phi\left(n+\hat{k}\right)$

It is unitary: $T_k T_k^\dagger = 1$

Under a gauge transformation:

$$T_k \to G T_k G^\dagger$$

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Lattice formalism - Overlap fermions

Background link variables on the lattice seen by the fermions are compact

Fermionic partition function in a fixed gauge field background: $Z(\bar\eta,\eta;m) = \det C_0(m) e^{\bar\eta G_0(m)\eta}$

$$C_o(m) = \frac{1+V}{2} + m\frac{1-V}{2} \qquad m\in[-1,1]$$

$$G_o(m) = \frac{1}{1-m}\left[C_o^{-1}(m) - 1\right] = \frac{A}{1+mA}$$

$$C_o(1) = 1 \qquad C_o(0) = \frac{1+V}{2} \qquad C_o(-1) = V$$

$$A = \frac{1-V}{1+V} \qquad A = -A^\dagger$$

Central character
A regularized
unitary operator, V,
that depends on the
background gauge fields

Parallel transporter on the lattice: $T_k : \left[T_k\phi\right](n) = U_k(n)\phi\left(n+\hat{k}\right)$

It is unitary: $T_k T_k^\dagger = 1$

Under a gauge transformation:

$$T_k \rightarrow G T_k G^\dagger$$

$$V = X\frac{1}{\sqrt{X^\dagger X}}$$

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Lattice formalism - Overlap fermions

Background link variables on the lattice seen by the fermions are compact

Fermionic partition function in a fixed gauge field background: $Z(\bar{\eta}, \eta; m) = \det C_0(m) e^{\bar{\eta} G_0(m) \eta}$

$$C_o(m) = \frac{1+V}{2} + m\frac{1-V}{2} \qquad m \in [-1, 1]$$

$$G_o(m) = \frac{1}{1-m}\left[C_o^{-1}(m) - 1\right] = \frac{A}{1+mA}$$

$$C_o(1) = 1 \qquad C_o(0) = \frac{1+V}{2} \qquad C_o(-1) = V$$

$$A = \frac{1-V}{1+V} \qquad A = -A^\dagger$$

Central character
A regularized
unitary operator, V,
that depends on the
background gauge fields

Parallel transporter on the lattice: $T_k : [T_k \phi](n) = U_k(n)\phi\left(n + \hat{k}\right)$

It is unitary: $T_k T_k^\dagger = 1$

Under a gauge transformation:

$$T_k \rightarrow G T_k G^\dagger$$

$$V = X\frac{1}{\sqrt{X^\dagger X}}$$

$$X = B + D$$

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Lattice formalism - Overlap fermions

Background link variables on the lattice seen by the fermions are compact

Fermionic partition function in a fixed gauge field background: $Z(\bar\eta, \eta; m) = \det C_0(m) e^{\bar\eta G_0(m)\eta}$

$$C_o(m) = \frac{1+V}{2} + m\frac{1-V}{2} \qquad m \in [-1,1] \qquad G_o(m) = \frac{1}{1-m}\left[C_o^{-1}(m) - 1\right] = \frac{A}{1+mA}$$

$$C_o(1) = 1 \qquad C_o(0) = \frac{1+V}{2} \qquad C_o(-1) = V \qquad A = \frac{1-V}{1+V} \qquad A = -A^\dagger$$

Central character
A regularized
unitary operator, V,
that depends on the
background gauge fields

Parallel transporter on the lattice: $T_k : [T_k\phi](n) = U_k(n)\phi\left(n+\hat{k}\right)$

It is unitary: $T_k T_k^\dagger = 1$

Under a gauge transformation:

$$T_k \to G T_k G^\dagger$$

$$V = X\frac{1}{\sqrt{X^\dagger X}}$$

$$X = B + D$$

$$D = \frac{1}{2}\sum_{k=1}^{3}\sigma_k\left[T_k - T_k^\dagger\right] \qquad B = \frac{1}{2}\sum_{k=1}^{3}\left[2 - T_k - T_k^\dagger\right] - m_w \qquad 0 < m_w < 2$$

$$D^\dagger = -D \qquad B^\dagger = B$$

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Lattice formalism - Overlap fermions

Background link variables on the lattice seen by the fermions are compact

Fermionic partition function in a fixed gauge field background: $Z(\bar\eta,\eta;m) = \det C_0(m) e^{\bar\eta G_0(m)\eta}$

$$C_o(m) = \frac{1+V}{2} + m\frac{1-V}{2} \qquad m \in [-1,1]$$

$$G_o(m) = \frac{1}{1-m}\left[C_o^{-1}(m) - 1\right] = \frac{A}{1+mA}$$

$$C_o(1) = 1 \qquad C_o(0) = \frac{1+V}{2} \qquad C_o(-1) = V$$

$$A = \frac{1-V}{1+V} \qquad A = -A^\dagger$$

Central character
A regularized
unitary operator, V,
that depends on the
background gauge fields

Parallel transporter on the lattice: $T_k : [T_k\phi](n) = U_k(n)\phi\left(n+\hat{k}\right)$

It is unitary: $T_k T_k^\dagger = 1$

Under a gauge transformation:

$$T_k \to G T_k G^\dagger$$

Under parity

$$T_k \to T_k^\dagger$$

$$V = X\frac{1}{\sqrt{X^\dagger X}}$$

$$X = B + D$$

$$D = \frac{1}{2}\sum_{k=1}^{3}\sigma_k\left[T_k - T_k^\dagger\right] \qquad B = \frac{1}{2}\sum_{k=1}^{3}\left[2 - T_k - T_k^\dagger\right] - m_w \qquad 0 < m_w < 2$$

$$D^\dagger = -D \qquad B^\dagger = B$$

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Lattice formalism - Overlap fermions

Background link variables on the lattice seen by the fermions are compact

Fermionic partition function in a fixed gauge field background: $Z(\bar{\eta},\eta;m) = \det C_0(m) e^{\bar{\eta} G_0(m)\eta}$

$$C_o(m) = \frac{1+V}{2} + m\frac{1-V}{2} \qquad m \in [-1,1]$$

$$G_o(m) = \frac{1}{1-m}\left[C_o^{-1}(m) - 1\right] = \frac{A}{1+mA}$$

$$C_o(1) = 1 \qquad C_o(0) = \frac{1+V}{2} \qquad C_o(-1) = V$$

$$A = \frac{1-V}{1+V} \qquad A = -A^\dagger$$

Central character
A regularized
unitary operator, V,
that depends on the
background gauge fields

Parallel transporter on the lattice: $T_k : [T_k\phi](n) = U_k(n)\phi\left(n+\hat{k}\right)$

It is unitary: $T_k T_k^\dagger = 1$

Under a gauge transformation:

$$T_k \to G T_k G^\dagger$$

Under parity

$$T_k \to T_k^\dagger$$

Under parity

$$V \to V^\dagger$$

$$V = X\frac{1}{\sqrt{X^\dagger X}}$$

$$X = B + D$$

$$D = \frac{1}{2}\sum_{k=1}^{3}\sigma_k\left[T_k - T_k^\dagger\right] \qquad B = \frac{1}{2}\sum_{k=1}^{3}\left[2 - T_k - T_k^\dagger\right] - m_w \qquad 0 < m_w < 2$$

$$D^\dagger = -D \qquad B^\dagger = B$$

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Lattice formalism - Overlap fermions

Background link variables on the lattice seen by the fermions are compact

Fermionic partition function in a fixed gauge field background: $Z(\bar\eta,\eta;m) = \det C_0(m) e^{\bar\eta G_0(m)\eta}$

$$C_o(m) = \frac{1+V}{2} + m\frac{1-V}{2} \qquad m \in [-1,1]$$

$$G_o(m) = \frac{1}{1-m}\left[C_o^{-1}(m) - 1\right] = \frac{A}{1+mA}$$

$$C_o(1) = 1 \qquad C_o(0) = \frac{1+V}{2} \qquad C_o(-1) = V$$

$$A = \frac{1-V}{1+V} \qquad A = -A^\dagger$$

Central character
A regularized
unitary operator, V,
that depends on the
background gauge fields

Parity

$$G_o^\dagger(m) = -G_o(-m)$$

It is unitary: $T_k T_k^\dagger = 1$

Parallel transporter on the lattice: $T_k : [T_k\phi](n) = U_k(n)\phi\left(n+\hat{k}\right)$

Under a gauge transformation:

$$T_k \to G T_k G^\dagger$$

Under parity

$$V \to V^\dagger$$

$$V = X\frac{1}{\sqrt{X^\dagger X}}$$

$$X = B + D$$

Under parity

$$T_k \to T_k^\dagger$$

$$D = \frac{1}{2}\sum_{k=1}^{3}\sigma_k\left[T_k - T_k^\dagger\right] \qquad B = \frac{1}{2}\sum_{k=1}^{3}\left[2 - T_k - T_k^\dagger\right] - m_w \qquad 0 < m_w < 2$$

$$D^\dagger = -D \qquad B^\dagger = B$$

Lattice formalism - Overlap fermions

Background link variables on the lattice seen by the fermions are compact

Fermionic partition function in a fixed gauge field background: $Z(\bar\eta,\eta;m) = \det C_0(m) e^{\bar\eta G_0(m)\eta}$

$$C_o(m) = \frac{1+V}{2} + m\frac{1-V}{2} \qquad m \in [-1,1]$$

$$G_o(m) = \frac{1}{1-m}\left[C_o^{-1}(m) - 1\right] = \frac{A}{1+mA}$$

$$C_o(1) = 1 \qquad C_o(0) = \frac{1+V}{2} \qquad C_o(-1) = V$$

$$A = \frac{1-V}{1+V} \qquad A = -A^\dagger$$

Parity

$$C_o^\dagger(m) = \frac{C_o(-m)}{V}$$

Central character
A regularized
unitary operator, V,
that depends on the
background gauge fields

Parity

$$G_o^\dagger(m) = -G_o(-m)$$

It is unitary: $T_k T_k^\dagger = 1$

Parallel transporter on the lattice: $T_k : [T_k\phi](n) = U_k(n)\phi\left(n+\hat{k}\right)$

Under a gauge transformation:

$$T_k \to G T_k G^\dagger$$

Under parity

$$V \to V^\dagger$$

$$V = X\frac{1}{\sqrt{X^\dagger X}}$$

$$X = B + D$$

Under parity

$$T_k \to T_k^\dagger$$

$$D = \frac{1}{2}\sum_{k=1}^{3}\sigma_k\left[T_k - T_k^\dagger\right] \qquad B = \frac{1}{2}\sum_{k=1}^{3}\left[2 - T_k - T_k^\dagger\right] - m_w \qquad 0 < m_w < 2$$

$$D^\dagger = -D \qquad B^\dagger = B$$

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Three dimensional QED and QCD with massless fermions

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Phase of the fermion determinant with infinite mass

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Phase of the fermion determinant with infinite mass

$$\det C_o(m) = \det\left[\frac{1+V}{2} + m\frac{1-V}{2}\right]$$

Phase of the fermion determinant with infinite mass

$$\det C_o(m) = \det\left[\frac{1+V}{2} + m\frac{1-V}{2}\right] \qquad \det C_o(-1) = \det V$$

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Phase of the fermion determinant with infinite mass

$$\det C_o(m) = \det\left[\frac{1+V}{2} + m\frac{1-V}{2}\right] \qquad \det C_o(-1) = \det V$$

Perturbative gauge fields: $$\det V = \exp\left[i\frac{1}{4\pi}\int d^3x\ \epsilon_{\mu\nu\rho}A_\mu\partial_\nu A_\rho\right]$$

Phase of the fermion determinant with infinite mass

$$\det C_o(m) = \det\left[\frac{1+V}{2} + m\frac{1-V}{2}\right] \qquad \det C_o(-1) = \det V$$

Perturbative gauge fields: $$\det V = \exp\left[i\frac{1}{4\pi}\int d^3x\ \epsilon_{\mu\nu\rho}A_\mu\partial_\nu A_\rho\right]$$

A standard non-perturbative gauge field:

Phase of the fermion determinant with infinite mass

$$\det C_o(m) = \det\left[\frac{1+V}{2} + m\frac{1-V}{2}\right] \qquad \det C_o(-1) = \det V$$

Perturbative gauge fields: $\det V = \exp\left[i\frac{1}{4\pi}\int d^3x\ \epsilon_{\mu\nu\rho}A_\mu\partial_\nu A_\rho\right]$

A standard non-perturbative gauge field: $F_{12} = \frac{2\pi}{L^2} \quad A_3 = \frac{2\pi h}{L} \quad h \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Phase of the fermion determinant with infinite mass

$$\det C_o(m) = \det\left[\frac{1+V}{2} + m\frac{1-V}{2}\right] \qquad \det C_o(-1) = \det V$$

Perturbative gauge fields: $\det V = \exp\left[i\frac{1}{4\pi}\int d^3x\ \epsilon_{\mu\nu\rho}A_\mu\partial_\nu A_\rho\right]$

A standard non-perturbative gauge field: $F_{12} = \frac{2\pi}{L^2} \quad A_3 = \frac{2\pi h}{L} \quad h \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

$\det V = \exp[i2\pi h]$ is gauge invariant

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Phase of the fermion determinant with infinite mass

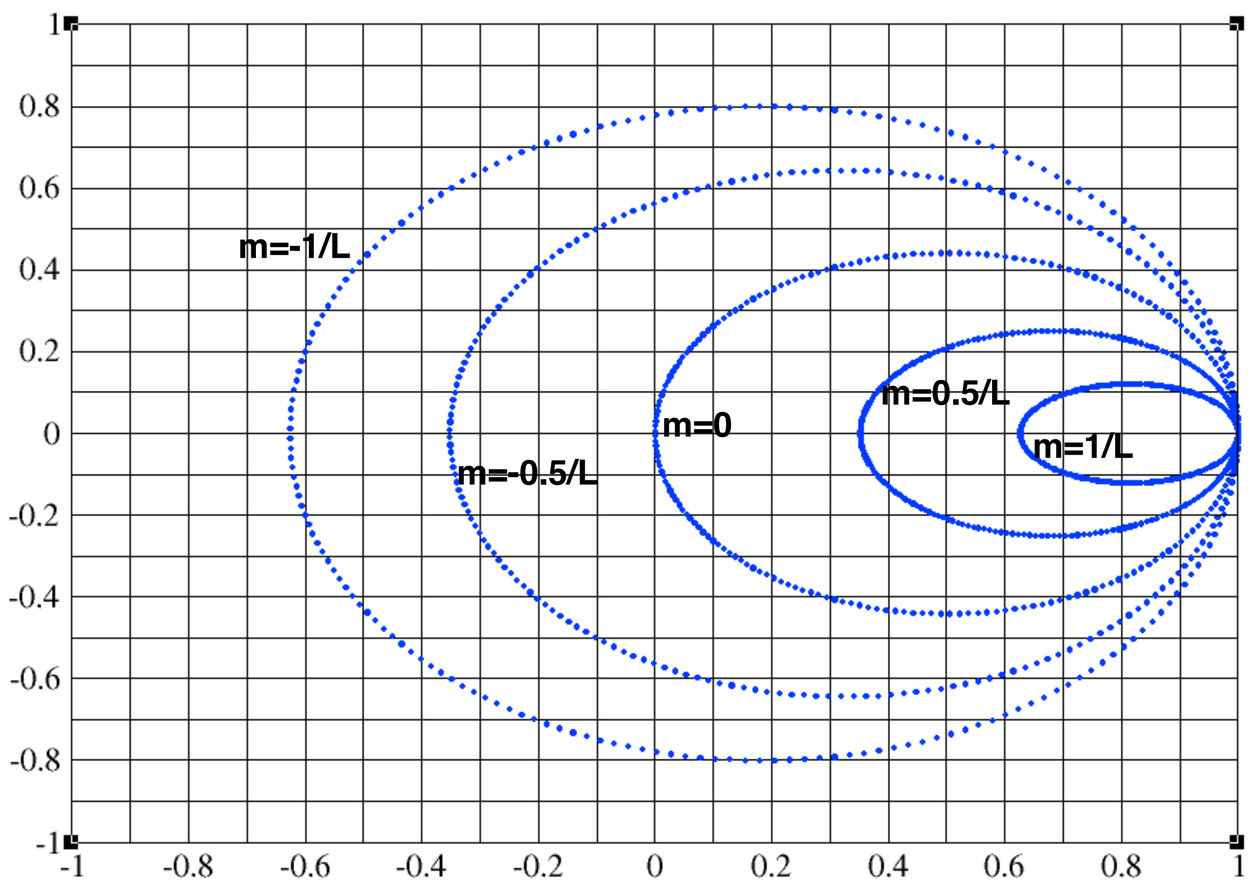
$$\det C_o(m) = \det\left[\frac{1+V}{2} + m\frac{1-V}{2}\right] \qquad \det C_o(-1) = \det V$$

Perturbative gauge fields: $\det V = \exp\left[i\frac{1}{4\pi}\int d^3x\ \epsilon_{\mu\nu\rho}A_\mu\partial_\nu A_\rho\right]$

A standard non-perturbative gauge field: $F_{12} = \frac{2\pi}{L^2} \quad A_3 = \frac{2\pi h}{L} \quad h \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

$\det V = \exp[i2\pi h]$ is gauge invariant

Orbit of the determinant as h is varied over a full period



Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Phase of the fermion determinant with infinite mass

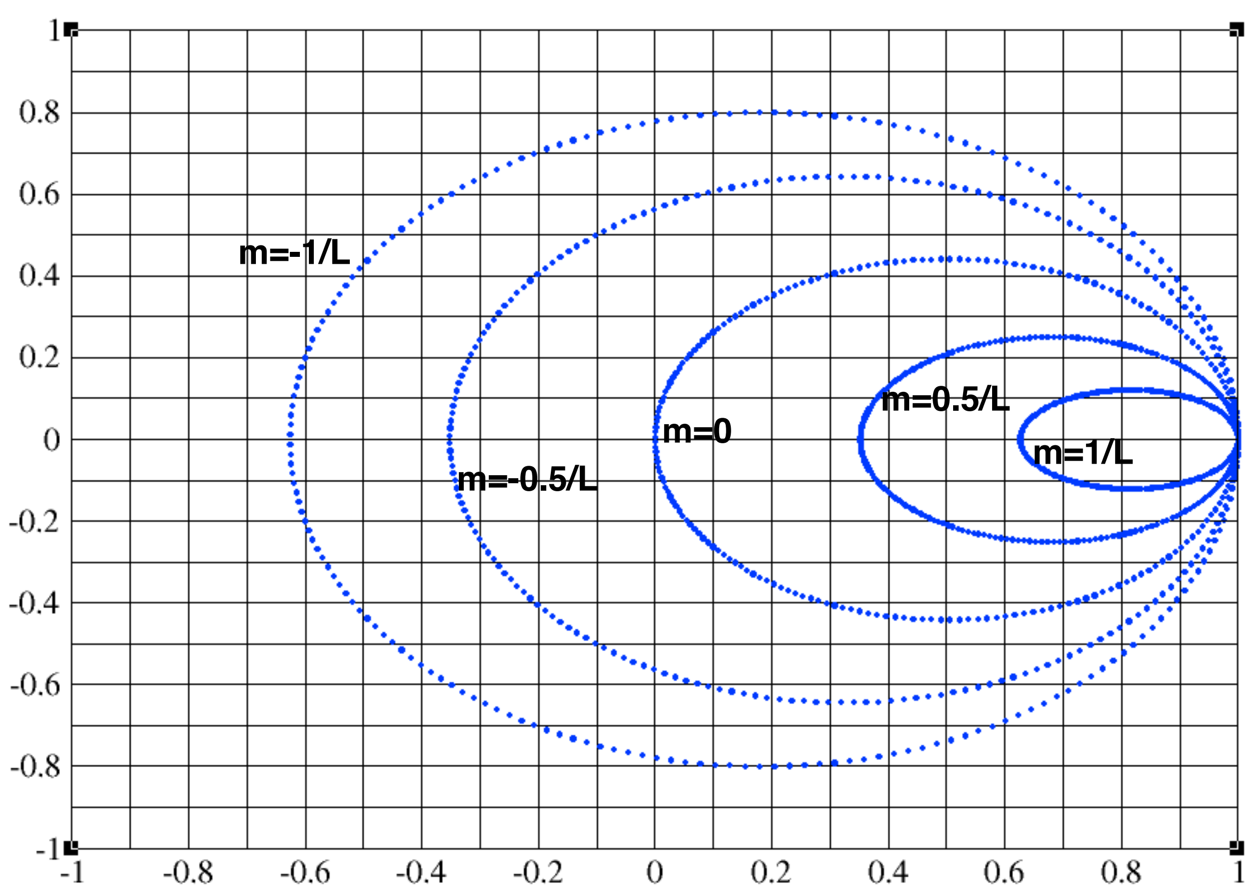
$$\det C_o(m) = \det\left[\frac{1+V}{2} + m\frac{1-V}{2}\right] \qquad \det C_o(-1) = \det V$$

Perturbative gauge fields: $\det V = \exp\left[i\frac{1}{4\pi}\int d^3x\ \epsilon_{\mu\nu\rho}A_\mu\partial_\nu A_\rho\right]$

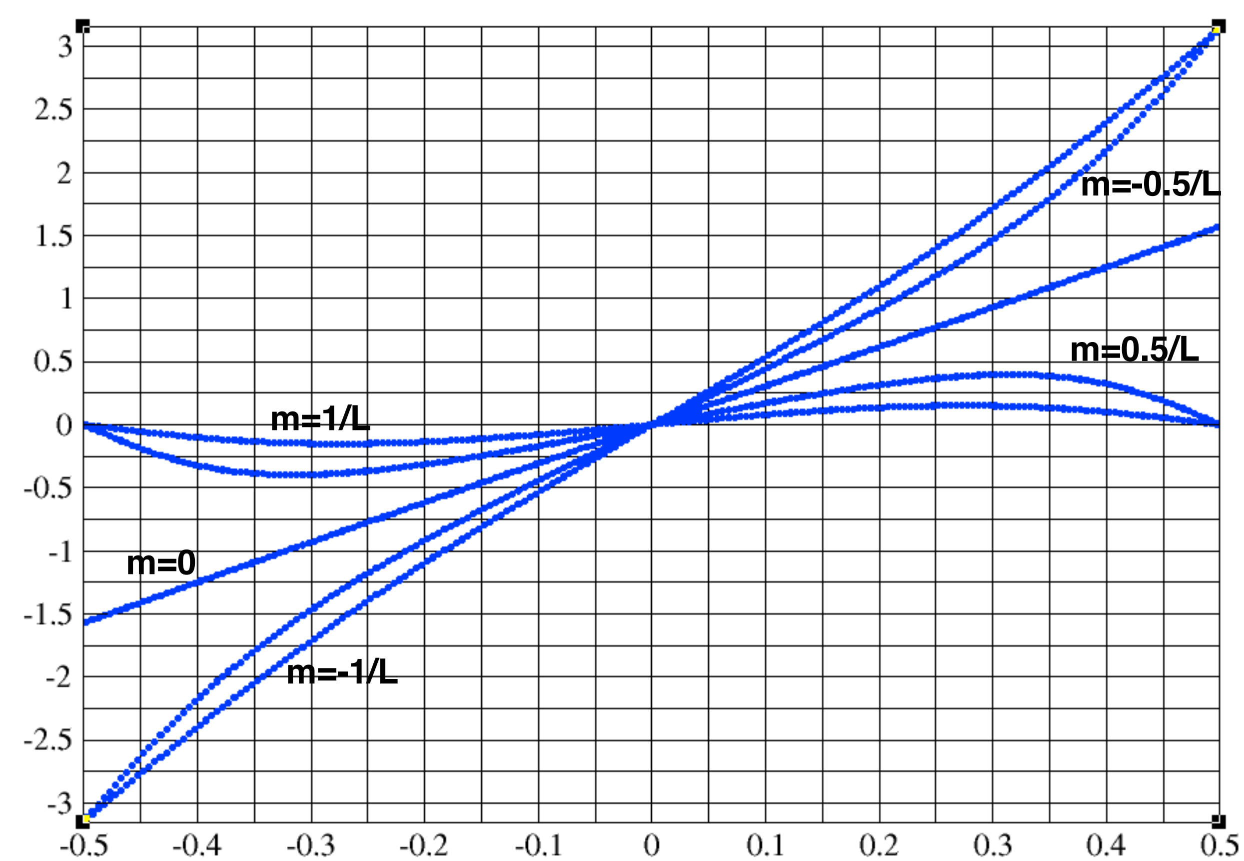
A standard non-perturbative gauge field: $F_{12} = \frac{2\pi}{L^2} \quad A_3 = \frac{2\pi h}{L} \quad h \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

$\det V = \exp[i2\pi h]$ is gauge invariant

Orbit of the determinant as h is varied over a full period



Phase of the determinant as a function of h



Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Parity invariant theory with massless fermions on the lattice

Parity invariant theory with massless fermions on the lattice

One pair of two component fermions, $N_f = 1$

Parity invariant theory with massless fermions on the lattice

One pair of two component fermions, $N_f = 1$

$$Z(\bar{\eta}_+, \bar{\eta}_-, \eta_+, \eta_-) = \det \frac{1+V}{2} \det \frac{1+V^\dagger}{2} e^{\bar{\eta}_+ A \eta_+ - \bar{\eta}_- A \eta_+} \qquad A = \frac{1-V}{1+V}$$

Parity invariant theory with massless fermions on the lattice

One pair of two component fermions, $N_f = 1$

$$Z(\bar\eta_+, \bar\eta_-, \eta_+, \eta_-) = \det \frac{1+V}{2} \det \frac{1+V^\dagger}{2} e^{\bar\eta_+ A \eta_+ - \bar\eta_- A \eta_+} \qquad A = \frac{1-V}{1+V}$$

Continuum physics from lattice

Parity invariant theory with massless fermions on the lattice

One pair of two component fermions, $N_f = 1$

$$Z(\bar{\eta}_+, \bar{\eta}_-, \eta_+, \eta_-) = \det \frac{1+V}{2} \det \frac{1+V^\dagger}{2} e^{\bar{\eta}_+ A \eta_+ - \bar{\eta}_- A \eta_+} \qquad A = \frac{1-V}{1+V}$$

Continuum physics from lattice

- We used a non-compact single plaquette action for the U(1) gauge fields.

Parity invariant theory with massless fermions on the lattice

One pair of two component fermions, $N_f = 1$

$$Z(\bar{\eta}_+, \bar{\eta}_-, \eta_+, \eta_-) = \det \frac{1+V}{2} \det \frac{1+V^\dagger}{2} e^{\bar{\eta}_+ A \eta_+ - \bar{\eta}_- A \eta_+} \qquad A = \frac{1-V}{1+V}$$

Continuum physics from lattice

- We used a non-compact single plaquette action for the U(1) gauge fields.
- We used the standard Wilson plaquette action for $SU(N_c)$ gauge fields.

Parity invariant theory with massless fermions on the lattice

One pair of two component fermions, $N_f = 1$

$$Z(\bar\eta_+, \bar\eta_-, \eta_+, \eta_-) = \det \frac{1+V}{2} \det \frac{1+V^\dagger}{2} e^{\bar\eta_+ A \eta_+ - \bar\eta_- A \eta_+} \qquad A = \frac{1-V}{1+V}$$

Continuum physics from lattice

- We used a non-compact single plaquette action for the U(1) gauge fields.
- We used the standard Wilson plaquette action for $SU(N_c)$ gauge fields.
- We worked on a periodic lattice with L points in each direction.

Parity invariant theory with massless fermions on the lattice

One pair of two component fermions, $N_f = 1$

$$Z(\bar\eta_+,\bar\eta_-,\eta_+,\eta_-) = \det\frac{1+V}{2}\det\frac{1+V^\dagger}{2}e^{\bar\eta_+ A\eta_+ - \bar\eta_- A\eta_+} \qquad A = \frac{1-V}{1+V}$$

Continuum physics from lattice

- We used a non-compact single plaquette action for the U(1) gauge fields.
- We used the standard Wilson plaquette action for $SU(N_c)$ gauge fields.
- We worked on a periodic lattice with L points in each direction.
- The standard lattice gauge coupling, β, was set to $2L/\ell$.

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Parity invariant theory with massless fermions on the lattice

One pair of two component fermions, $N_f = 1$

$$Z(\bar{\eta}_+, \bar{\eta}_-, \eta_+, \eta_-) = \det \frac{1+V}{2} \det \frac{1+V^\dagger}{2} e^{\bar{\eta}_+ A \eta_+ - \bar{\eta}_- A \eta_+} \qquad A = \frac{1-V}{1+V}$$

Continuum physics from lattice

- We used a non-compact single plaquette action for the U(1) gauge fields.
- We used the standard Wilson plaquette action for $SU(N_c)$ gauge fields.
- We worked on a periodic lattice with L points in each direction.
- The standard lattice gauge coupling, β, was set to $2L/\ell$.
- We worked at a fixed ℓ and first took the continuum limit by extracting the results at several values of L.

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Parity invariant theory with massless fermions on the lattice

One pair of two component fermions, $N_f = 1$

$$Z(\bar\eta_+, \bar\eta_-, \eta_+, \eta_-) = \det \frac{1+V}{2} \det \frac{1+V^\dagger}{2} e^{\bar\eta_+ A \eta_+ - \bar\eta_- A \eta_+} \qquad A = \frac{1-V}{1+V}$$

Continuum physics from lattice

- We used a non-compact single plaquette action for the U(1) gauge fields.
- We used the standard Wilson plaquette action for $SU(N_c)$ gauge fields.
- We worked on a periodic lattice with L points in each direction.
- The standard lattice gauge coupling, β, was set to $2L/\ell$.
- We worked at a fixed ℓ and first took the continuum limit by extracting the results at several values of L.
- We then varied ℓ to study the behavior as a function of the dimensionless volume.

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Quenched QED ($N = 0$) has a non-zero condensate.

Quenched QED (N = 0) has a non-zero condensate.

Random Matrix Model

Quenched QED (N = 0) has a non-zero condensate.

Random Matrix Model

$$K_N(x,y) = \frac{\pi\sqrt{xy}}{2(x-y)}\left[J_{N+\frac{1}{2}}(\pi x)J_{N-\frac{1}{2}}(\pi y) - J_{N-\frac{1}{2}}(\pi x)J_{N+\frac{1}{2}}(\pi y)\right]$$

Quenched QED (N = 0) has a non-zero condensate.

Random Matrix Model

$$K_N(x,y) = \frac{\pi\sqrt{xy}}{2(x-y)}\left[J_{N+\frac{1}{2}}(\pi x)J_{N-\frac{1}{2}}(\pi y) - J_{N-\frac{1}{2}}(\pi x)J_{N+\frac{1}{2}}(\pi y)\right]$$

- N=0

Quenched QED (N = 0) has a non-zero condensate.

Random Matrix Model

$$K_N(x,y) = \frac{\pi\sqrt{xy}}{2(x-y)}\left[J_{N+\frac{1}{2}}(\pi x)J_{N-\frac{1}{2}}(\pi y) - J_{N-\frac{1}{2}}(\pi x)J_{N+\frac{1}{2}}(\pi y)\right]$$

- N=0
- Distributions of low lying eigenvalues can be obtained from Fredholm determinants.

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

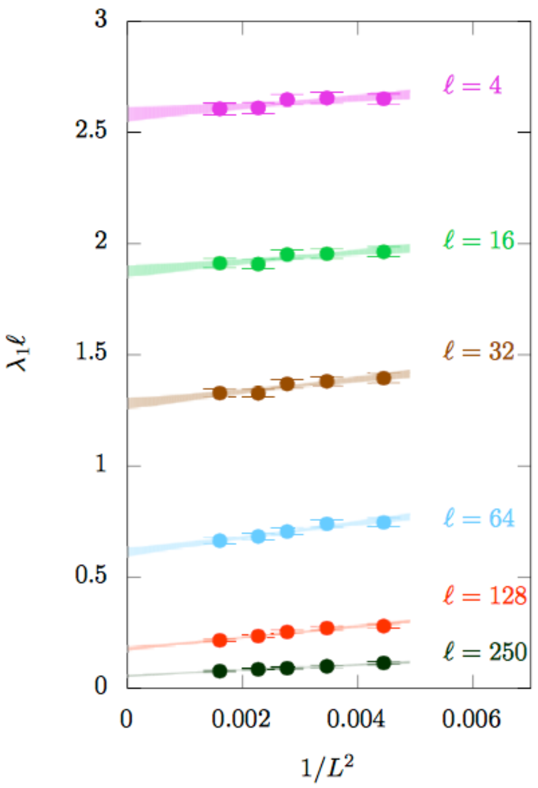
Quenched QED (N = 0) has a non-zero condensate.

Random Matrix Model

$$K_N(x,y) = \frac{\pi\sqrt{xy}}{2(x-y)}\left[J_{N+\frac{1}{2}}(\pi x)J_{N-\frac{1}{2}}(\pi y) - J_{N-\frac{1}{2}}(\pi x)J_{N+\frac{1}{2}}(\pi y)\right]$$

- N=0
- Distributions of low lying eigenvalues can be obtained from Fredholm determinants.
- Numerical evaluation of the eigenvalues of the kernel and the subsequent computation of determinants and traces of resolvents is the most efficient method to obtain the distributions.

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

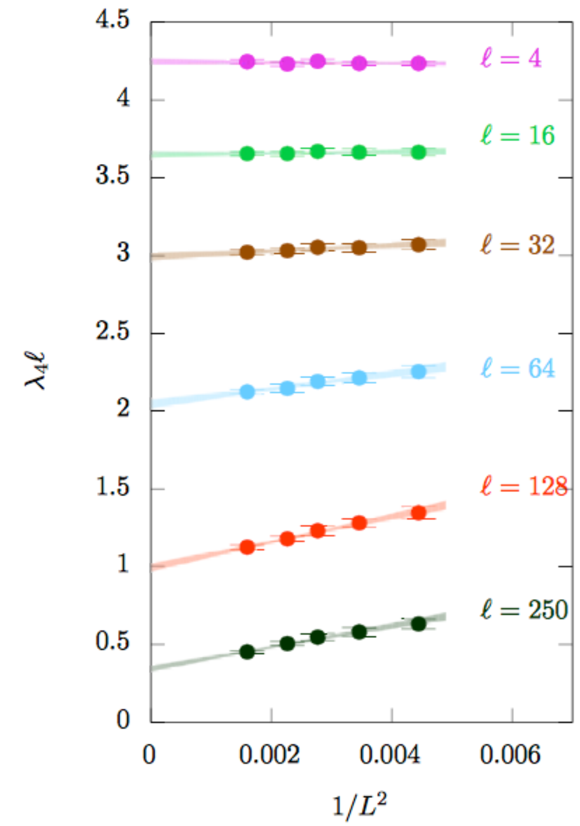


Quenched QED (N = 0) has a non-zero condensate.

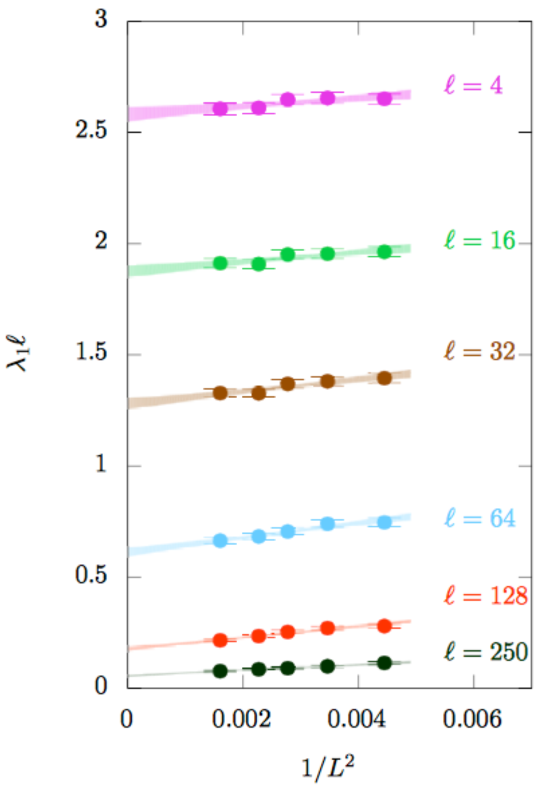
Random Matrix Model

$$K_N(x,y) = \frac{\pi\sqrt{xy}}{2(x-y)}\left[J_{N+\frac{1}{2}}(\pi x)J_{N-\frac{1}{2}}(\pi y) - J_{N-\frac{1}{2}}(\pi x)J_{N+\frac{1}{2}}(\pi y)\right]$$

- N=0
- Distributions of low lying eigenvalues can be obtained from Fredholm determinants.
- Numerical evaluation of the eigenvalues of the kernel and the subsequent computation of determinants and traces of resolvents is the most efficient method to obtain the distributions.



Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

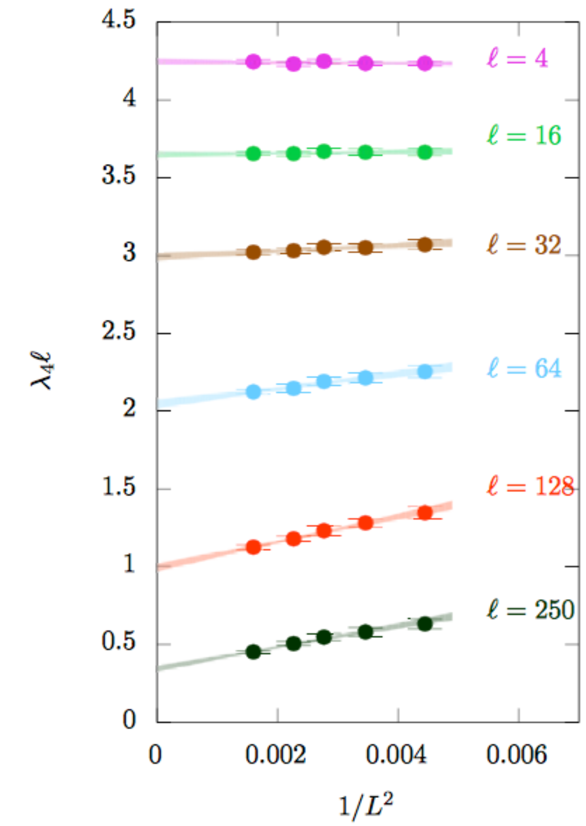


Quenched QED (N = 0) has a non-zero condensate.

Random Matrix Model

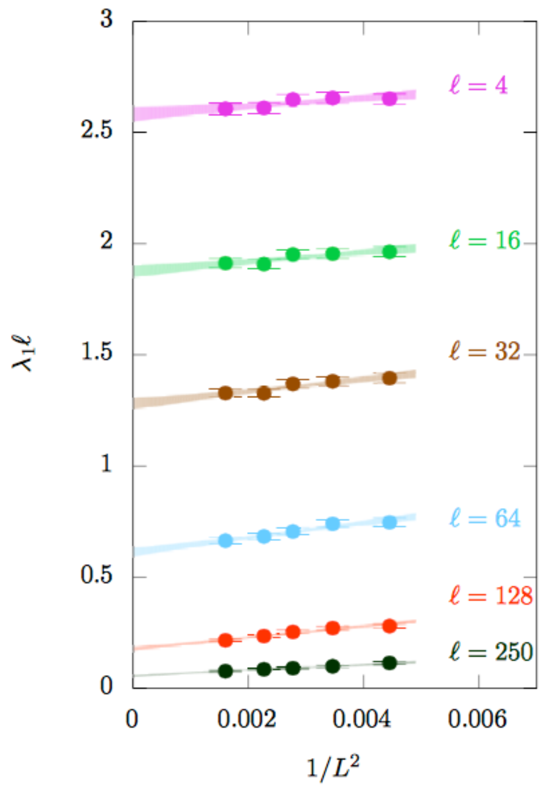
$$K_N(x,y) = \frac{\pi\sqrt{xy}}{2(x-y)}\left[J_{N+\frac{1}{2}}(\pi x)J_{N-\frac{1}{2}}(\pi y) - J_{N-\frac{1}{2}}(\pi x)J_{N+\frac{1}{2}}(\pi y)\right]$$

- N=0
- Distributions of low lying eigenvalues can be obtained from Fredholm determinants.
- Numerical evaluation of the eigenvalues of the kernel and the subsequent computation of determinants and traces of resolvents is the most efficient method to obtain the distributions.



$$K(x,y) = \frac{\sin(\pi(x-y))}{\pi(x-y)}$$

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

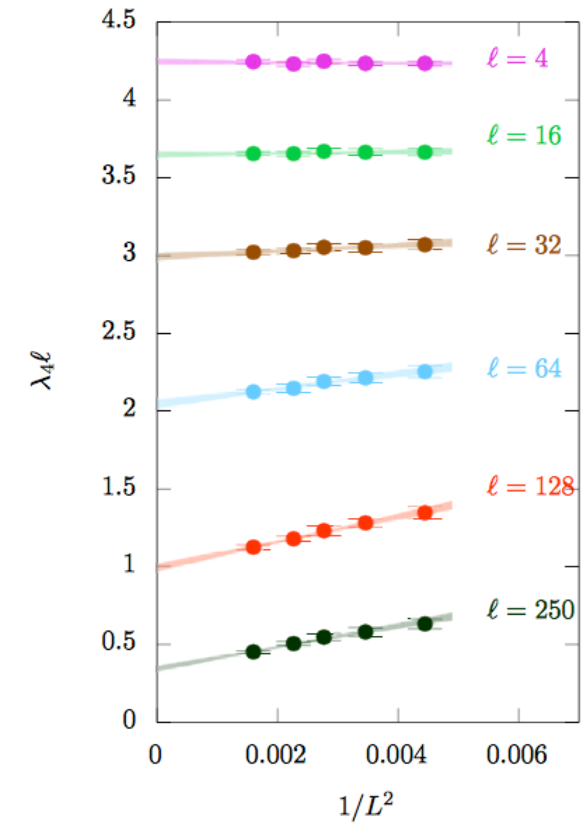


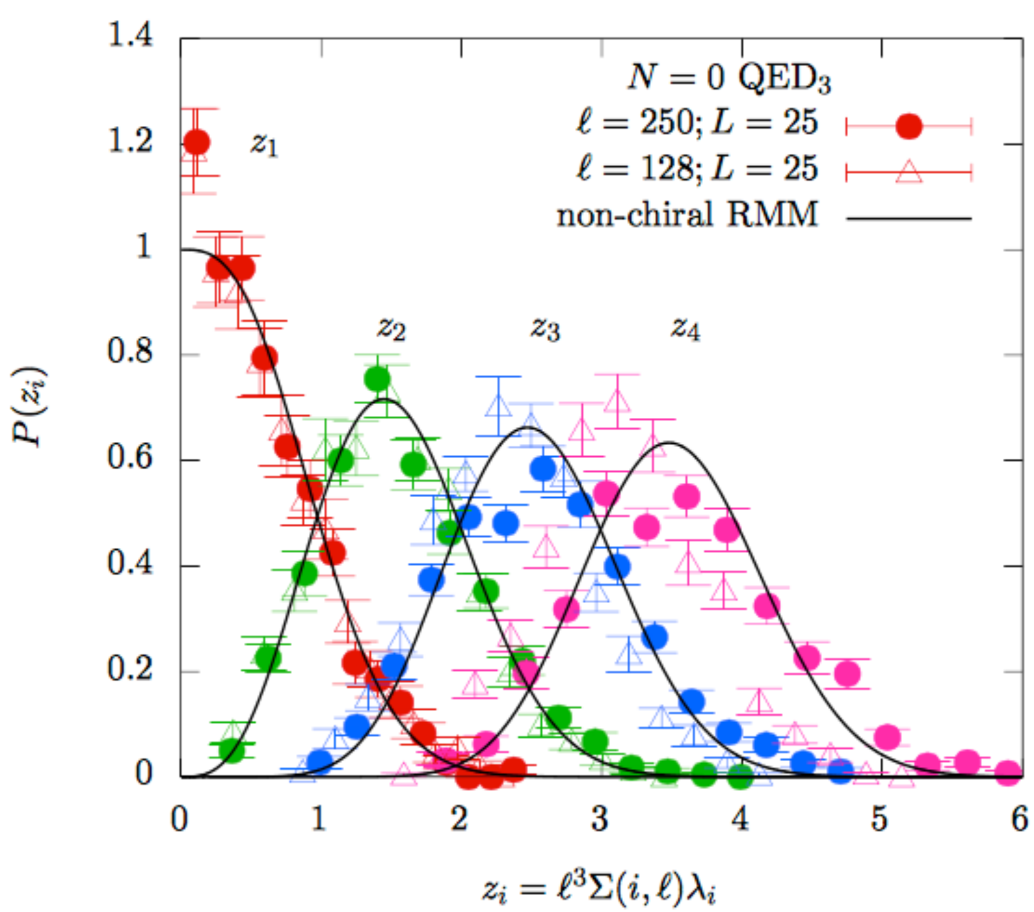
Quenched QED (N = 0) has a non-zero condensate.

Random Matrix Model

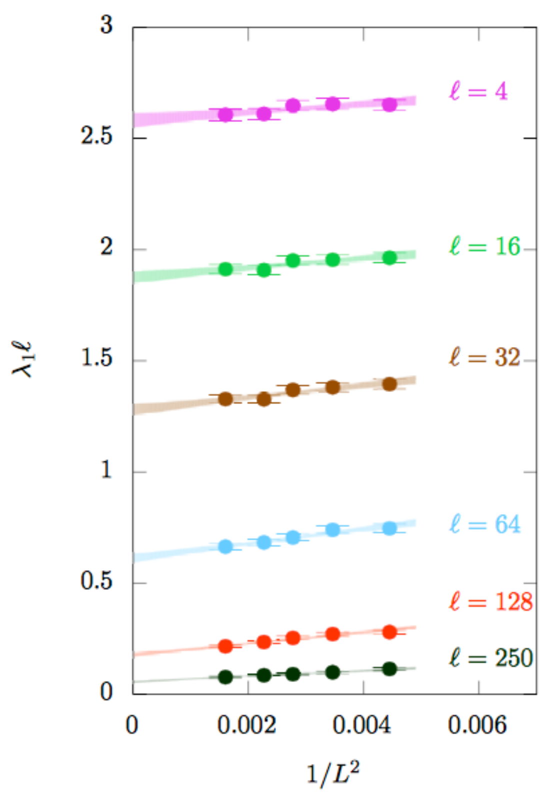
$$K_N(x,y) = \frac{\pi\sqrt{xy}}{2(x-y)}\left[J_{N+\frac{1}{2}}(\pi x)J_{N-\frac{1}{2}}(\pi y) - J_{N-\frac{1}{2}}(\pi x)J_{N+\frac{1}{2}}(\pi y)\right]$$

- N=0
- Distributions of low lying eigenvalues can be obtained from Fredholm determinants.
- Numerical evaluation of the eigenvalues of the kernel and the subsequent computation of determinants and traces of resolvents is the most efficient method to obtain the distributions.





$$K(x,y) = \frac{\sin(\pi(x-y))}{\pi(x-y)}$$

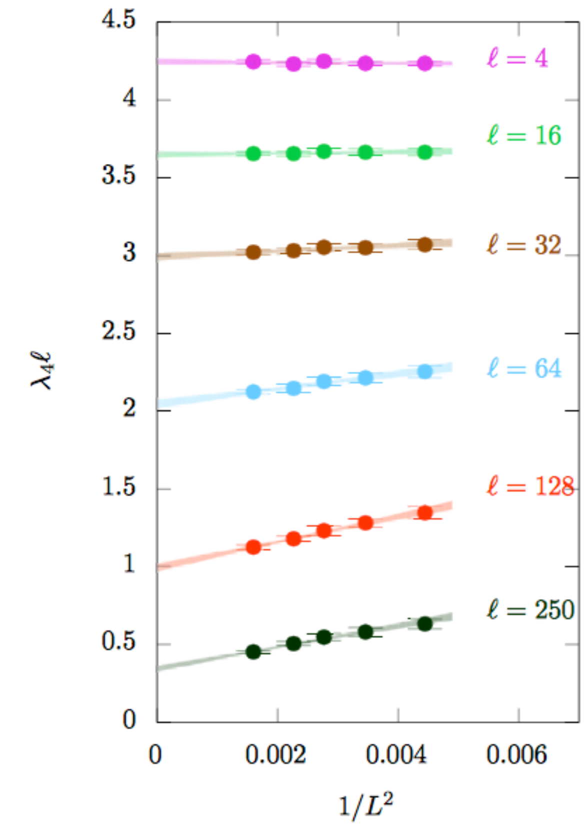


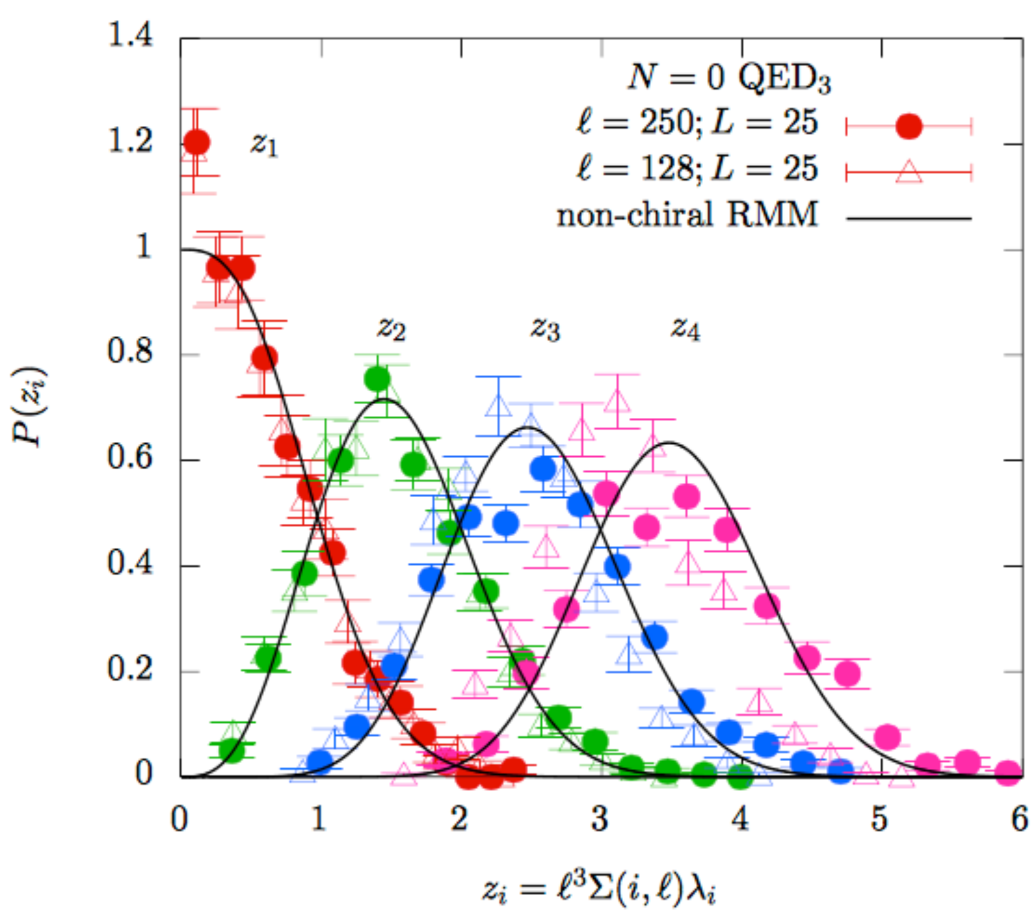
Quenched QED (N = 0) has a non-zero condensate.

Random Matrix Model

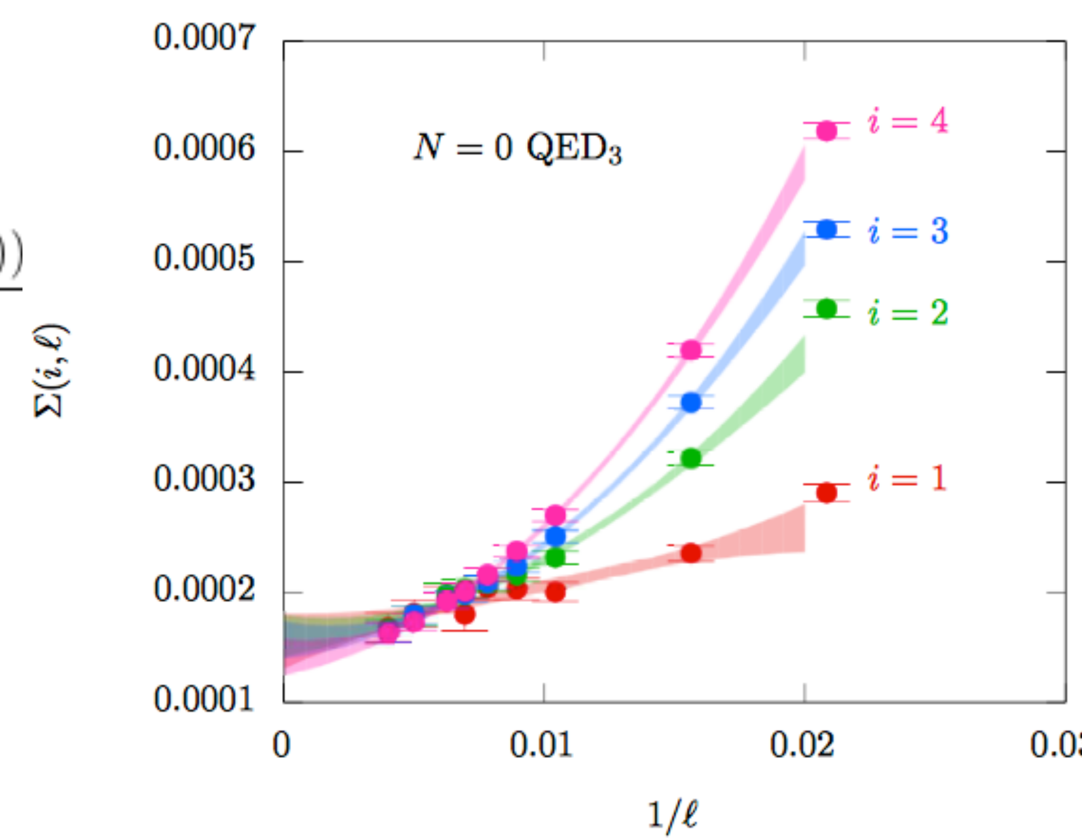
$$K_N(x,y) = \frac{\pi\sqrt{xy}}{2(x-y)}\left[J_{N+\frac{1}{2}}(\pi x)J_{N-\frac{1}{2}}(\pi y) - J_{N-\frac{1}{2}}(\pi x)J_{N+\frac{1}{2}}(\pi y)\right]$$

- N=0
- Distributions of low lying eigenvalues can be obtained from Fredholm determinants.
- Numerical evaluation of the eigenvalues of the kernel and the subsequent computation of determinants and traces of resolvents is the most efficient method to obtain the distributions.

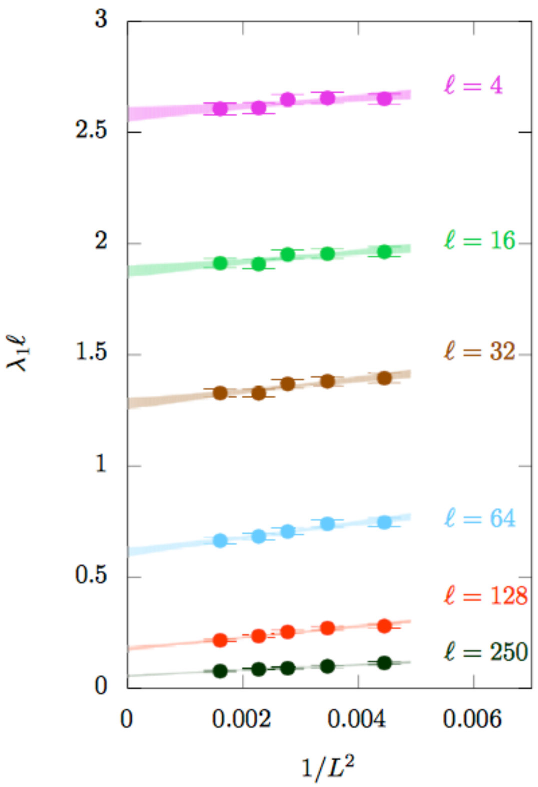




$$K(x,y) = \frac{\sin(\pi(x-y))}{\pi(x-y)}$$



Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

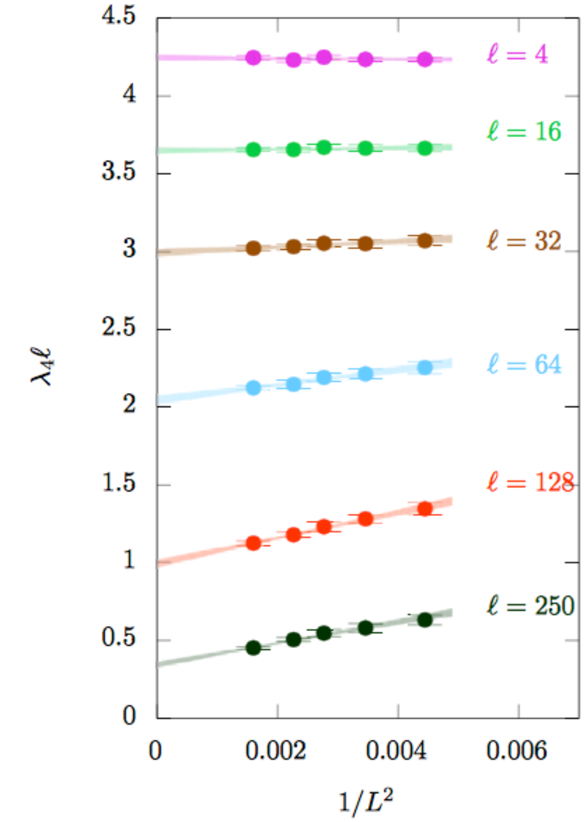


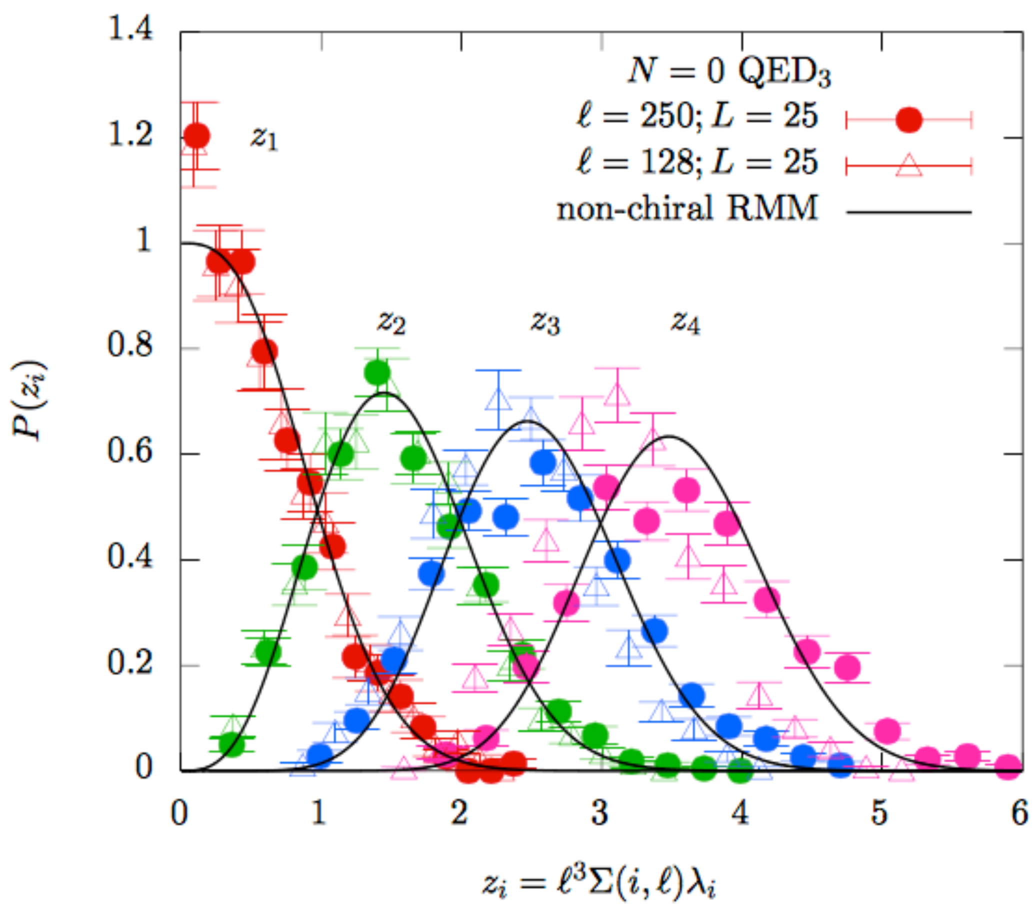
Quenched QED (N = 0) has a non-zero condensate.

Random Matrix Model

$$K_N(x,y) = \frac{\pi\sqrt{xy}}{2(x-y)}\left[J_{N+\frac{1}{2}}(\pi x)J_{N-\frac{1}{2}}(\pi y) - J_{N-\frac{1}{2}}(\pi x)J_{N+\frac{1}{2}}(\pi y)\right]$$

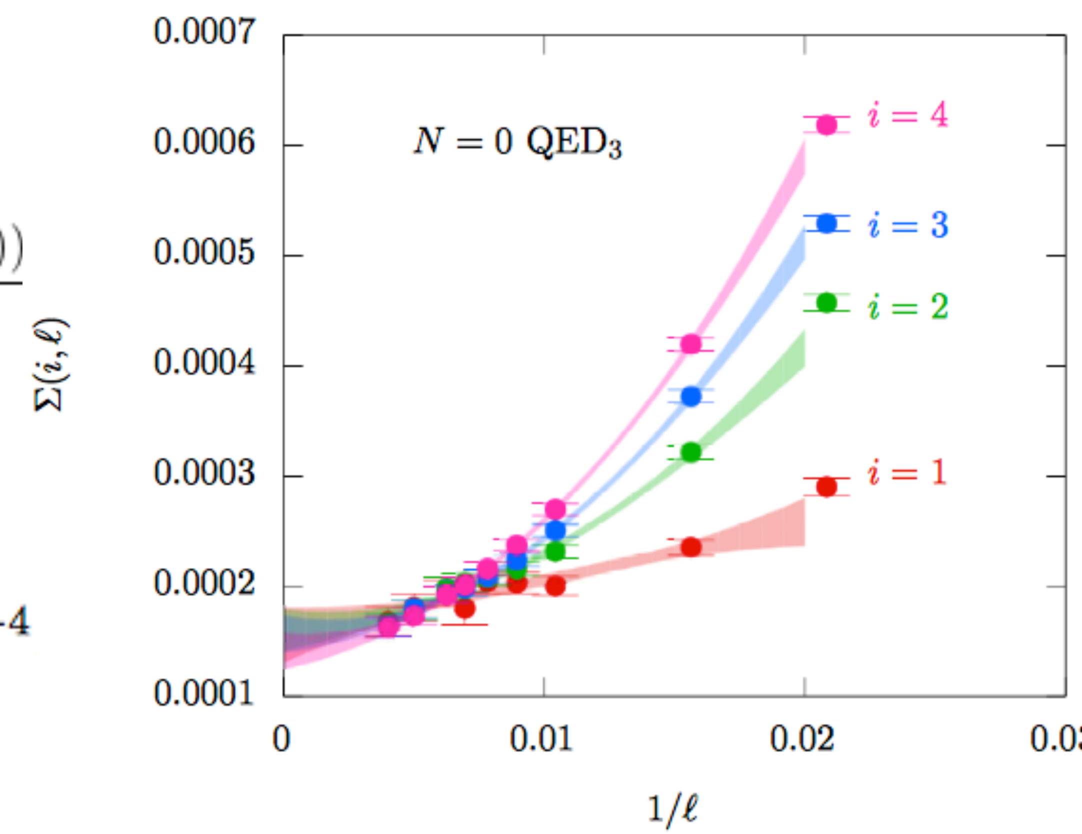
- N=0
- Distributions of low lying eigenvalues can be obtained from Fredholm determinants.
- Numerical evaluation of the eigenvalues of the kernel and the subsequent computation of determinants and traces of resolvents is the most efficient method to obtain the distributions.





$$K(x,y) = \frac{\sin(\pi(x-y))}{\pi(x-y)}$$

$$\Sigma = 1.5(1)\times 10^{-4}$$



Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

N_f=1: Flow of the flavor triplet fermion current central charge from ultraviolet to infrared

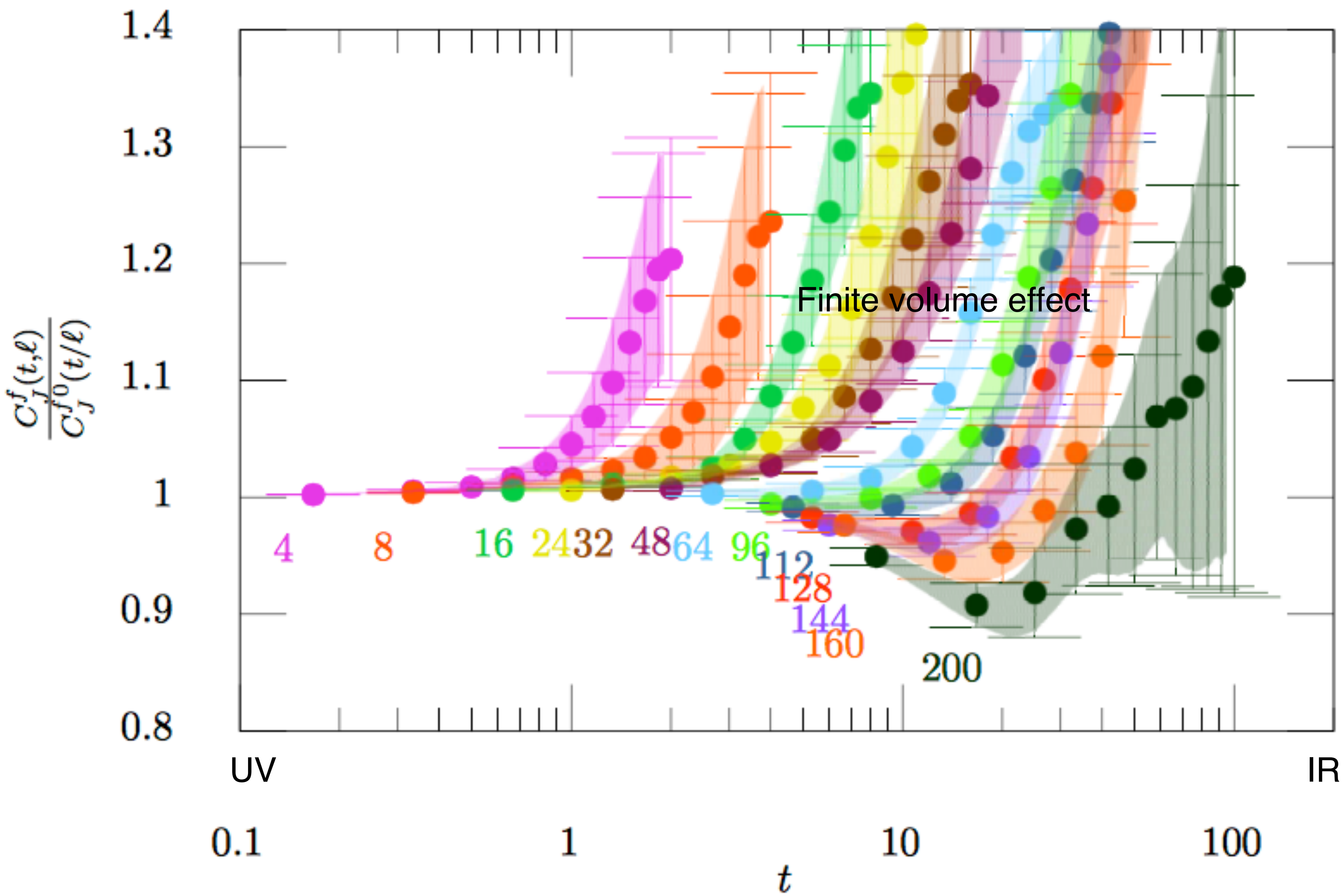
Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

$N_f=1$: Flow of the flavor triplet fermion current central charge from ultraviolet to infrared

$$G_V(t,\ell) = \int dx\; dy\; \left\langle \sum_{k=1}^{2} V_k(0,0,0) V_k(x,y,t) \right\rangle \equiv \frac{C_J^f(t,\ell)}{t^2}$$

N_f=1: Flow of the flavor triplet fermion current central charge from ultraviolet to infrared

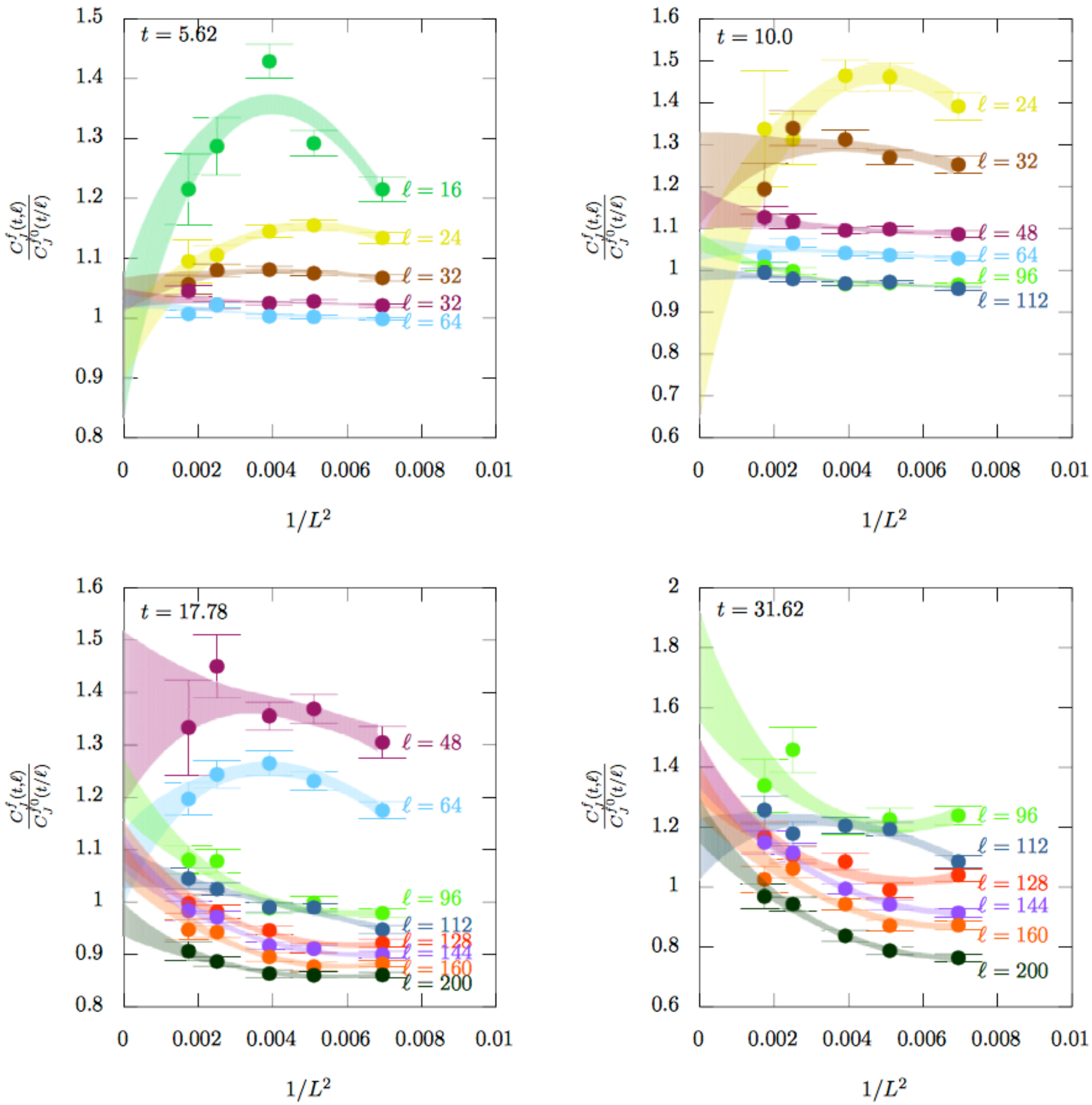
$$G_V(t,\ell) = \int dx\, dy \left\langle \sum_{k=1}^{2} V_k(0,0,0)V_k(x,y,t) \right\rangle \equiv \frac{C_J^f(t,\ell)}{t^2}$$



Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

N_f=1: Flow of the flavor triplet fermion current central charge from ultraviolet to infrared

$$G_V(t,\ell) = \int dx\, dy \left\langle \sum_{k=1}^{2} V_k(0,0,0)\, V_k(x,y,t) \right\rangle \equiv \frac{C_J^f(t,\ell)}{t^2}$$

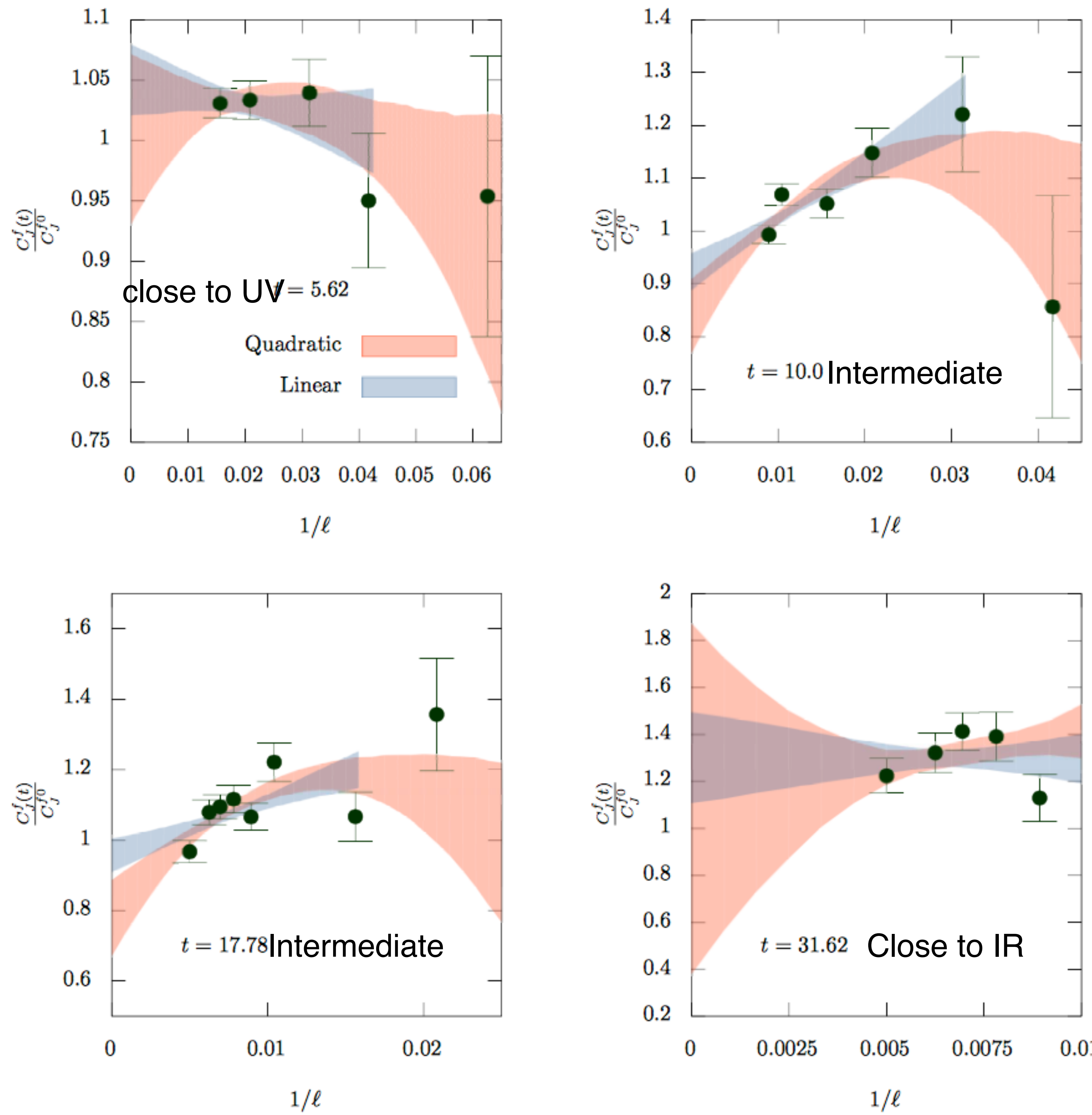


Continuum extrapolation at a fixed separation and volume

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

N_f=1: Flow of the flavor triplet fermion current central charge from ultraviolet to infrared

$$G_V(t,\ell) = \int dx\, dy \left\langle \sum_{k=1}^{2} V_k(0,0,0) V_k(x,y,t) \right\rangle \equiv \frac{C_J^f(t,\ell)}{t^2}$$

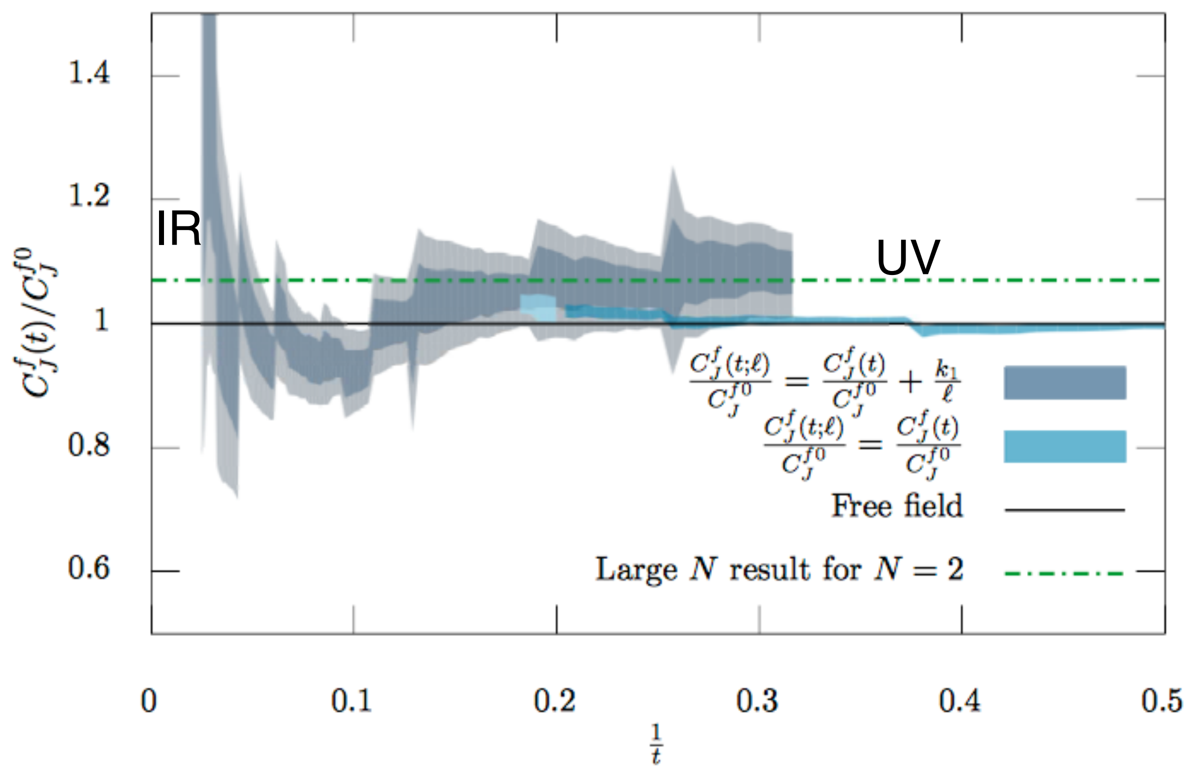


Extrapolation to infinite volume at a fixed separation

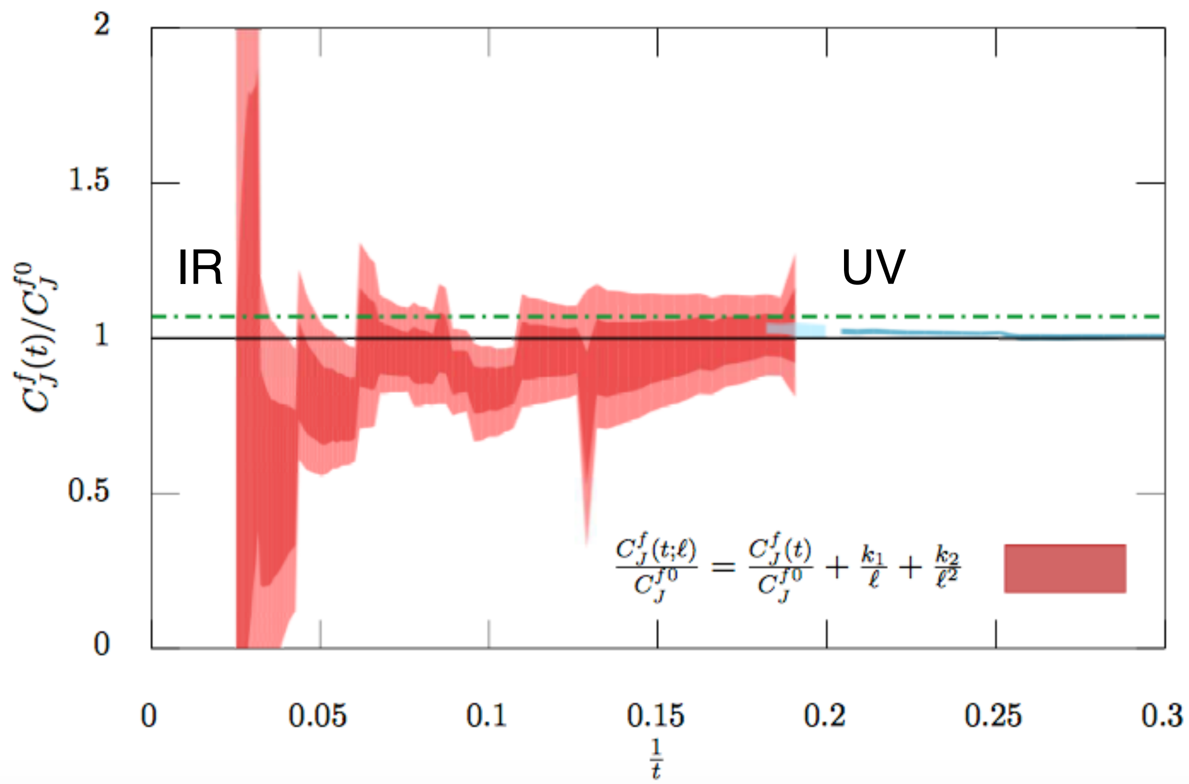
Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

N_f=1: Flow of the flavor triplet fermion current central charge from ultraviolet to infrared

$$G_V(t,\ell) = \int dx\, dy \left\langle \sum_{k=1}^{2} V_k(0,0,0)\, V_k(x,y,t) \right\rangle \equiv \frac{C_J^f(t,\ell)}{t^2}$$



There is a tendency for the flow to decrease from UV to IR



Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Strong self-duality of N=2 parity invariant QED

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Strong self-duality of N=2 parity invariant QED

The fermions in the dual theory are coupled to the topological current, $\tilde{A}_\mu = \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda$

Strong self-duality of N=2 parity invariant QED

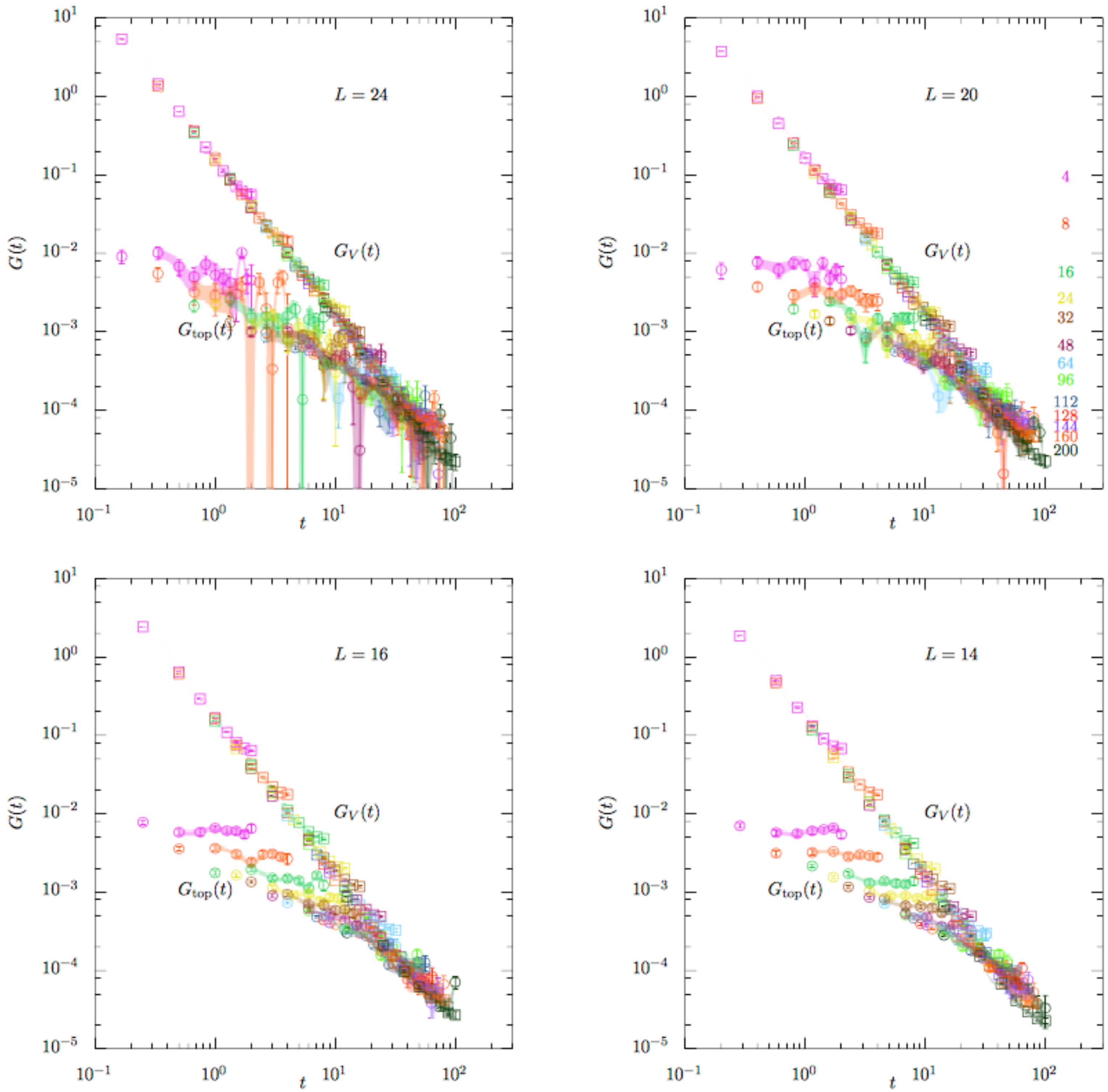
The fermions in the dual theory are coupled to the topological current, $\tilde{A}_\mu = \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda$

Strong self duality implies that the correlator of the flavor triplet current and the topological current should match in IR

Strong self-duality of N=2 parity invariant QED

The fermions in the dual theory are coupled to the topological current, $\tilde{A}_\mu = \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda$

Strong self duality implies that the correlator of the flavor triplet current and the topological current should match in IR

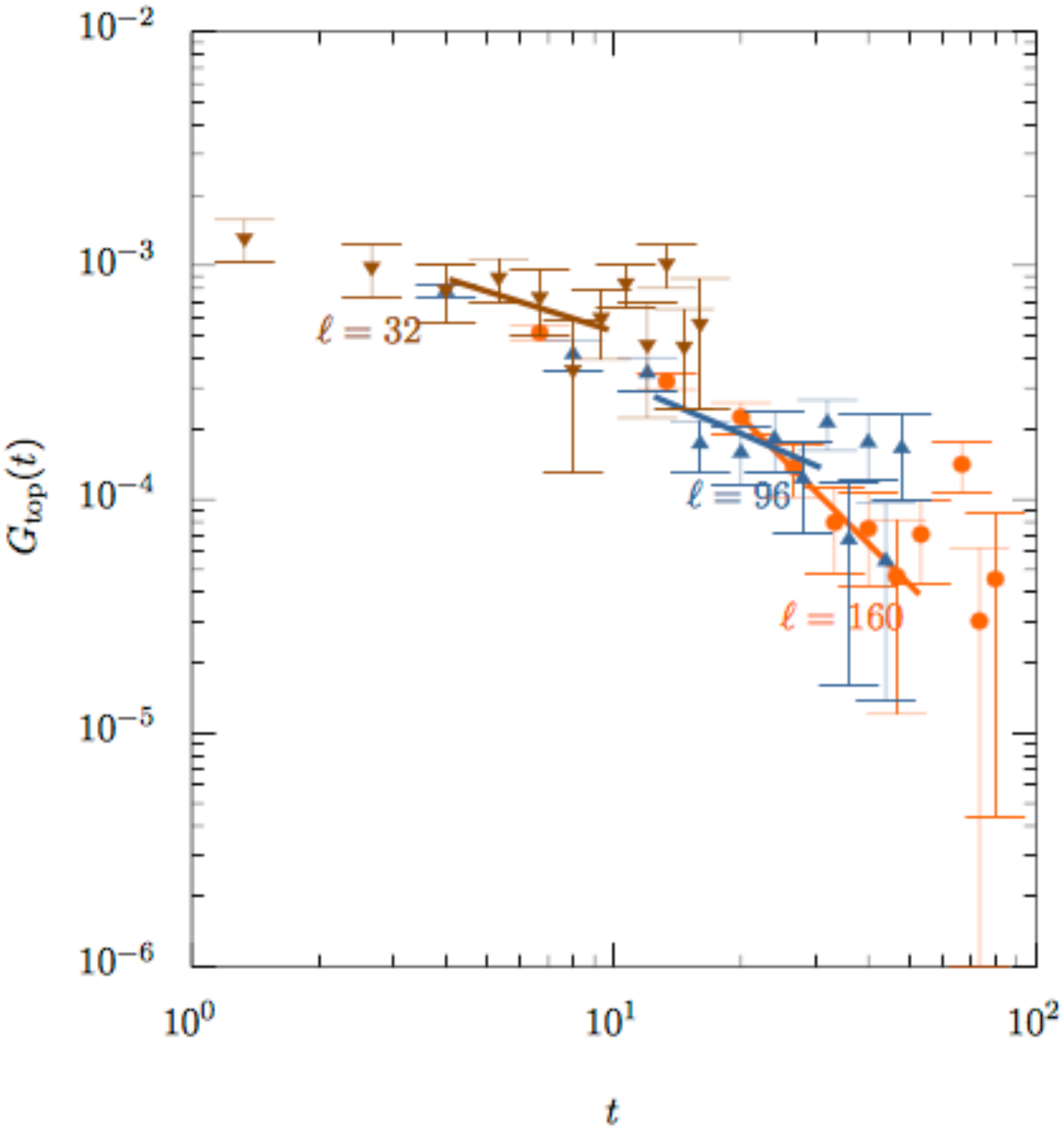


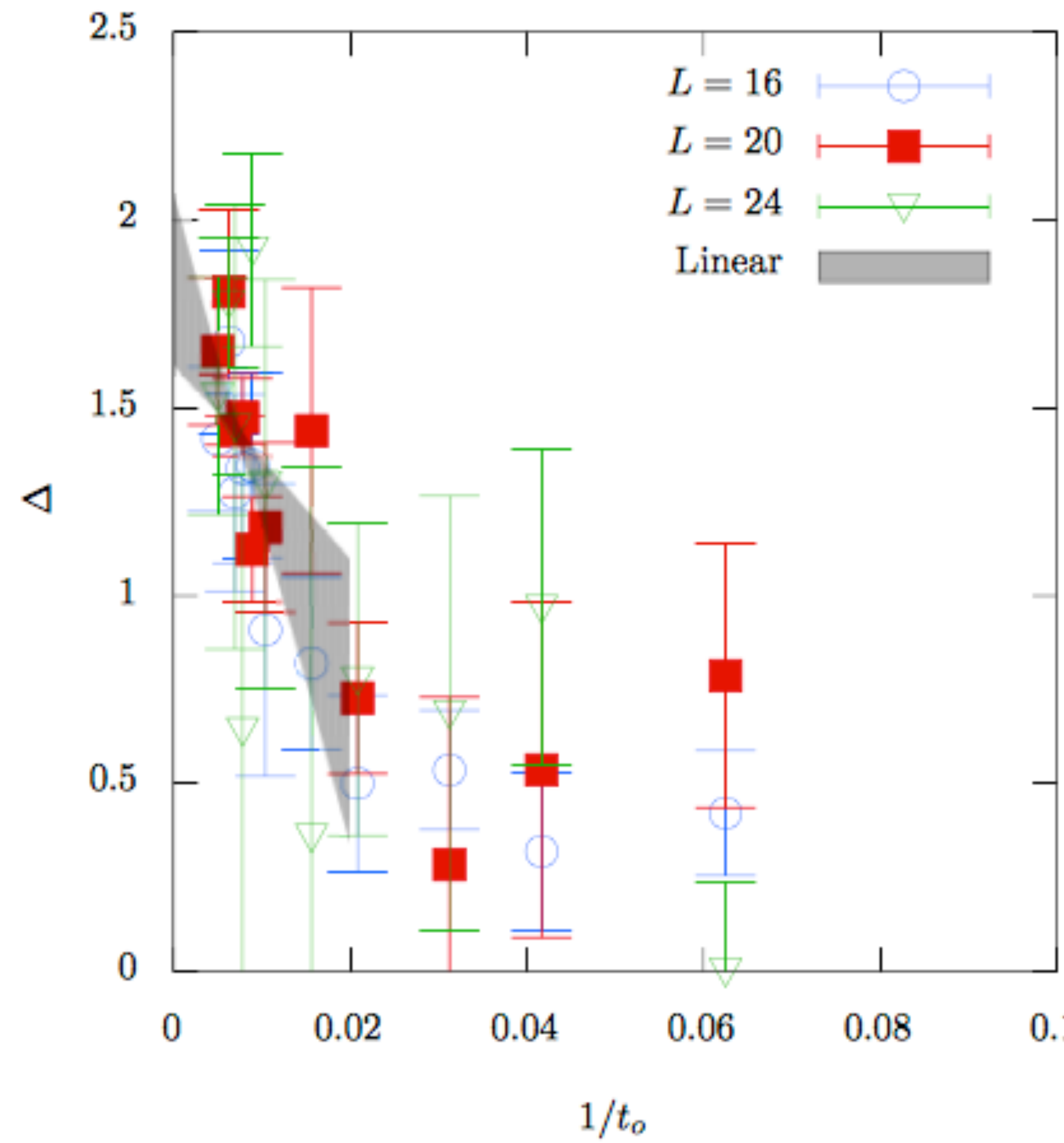
Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Strong self-duality of N=2 parity invariant QED

The fermions in the dual theory are coupled to the topological current, $\tilde{A}_\mu = \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda$

Strong self duality implies that the correlator of the flavor triplet current and the topological current should match in IR



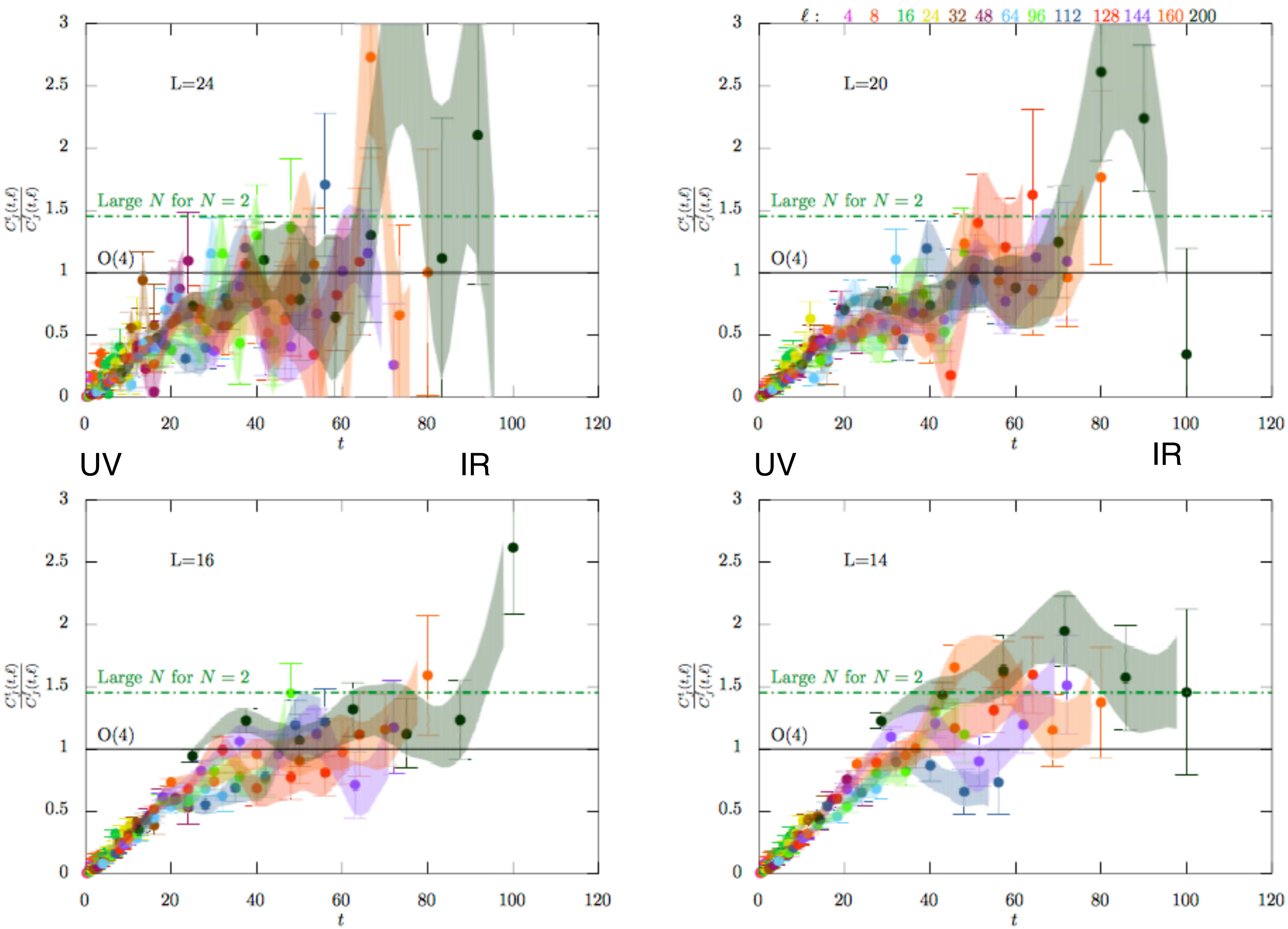


Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Strong self-duality of N=2 parity invariant QED

The fermions in the dual theory are coupled to the topological current, $\tilde{A}_\mu = \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda$

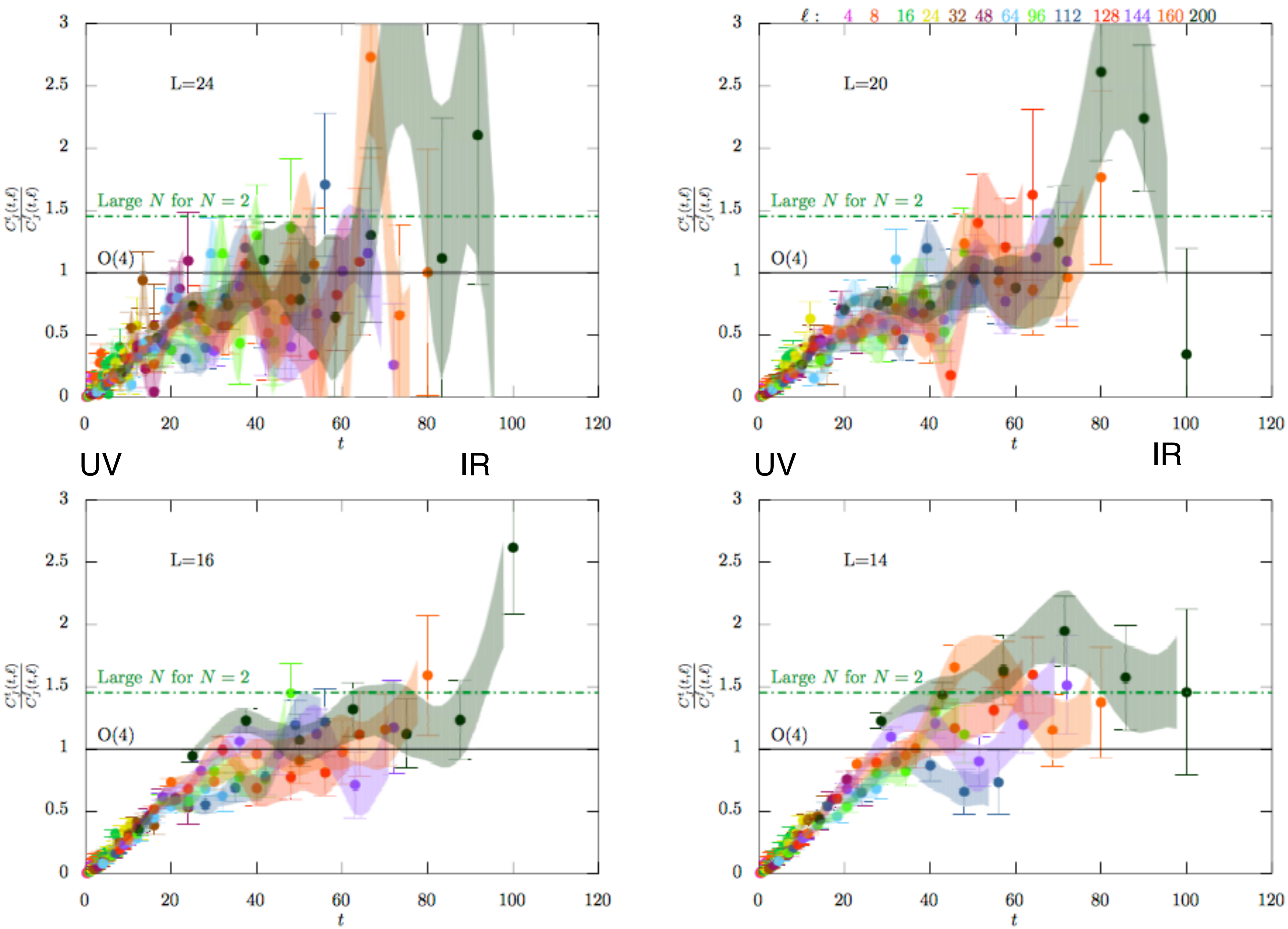
Strong self duality implies that the correlator of the flavor triplet current and the topological current should match in IR



Strong self-duality of N=2 parity invariant QED

The fermions in the dual theory are coupled to the topological current, $\tilde{A}_\mu = \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda$

Strong self duality implies that the correlator of the flavor triplet current and the topological current should match in IR

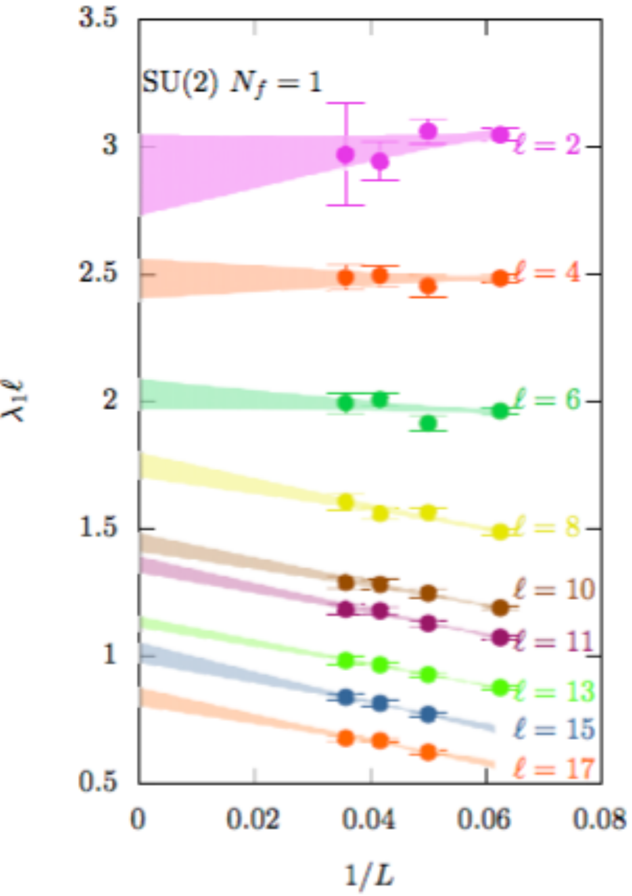


SU(2) X SU(2) symmetry becomes an emergent O(4) symmetry

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

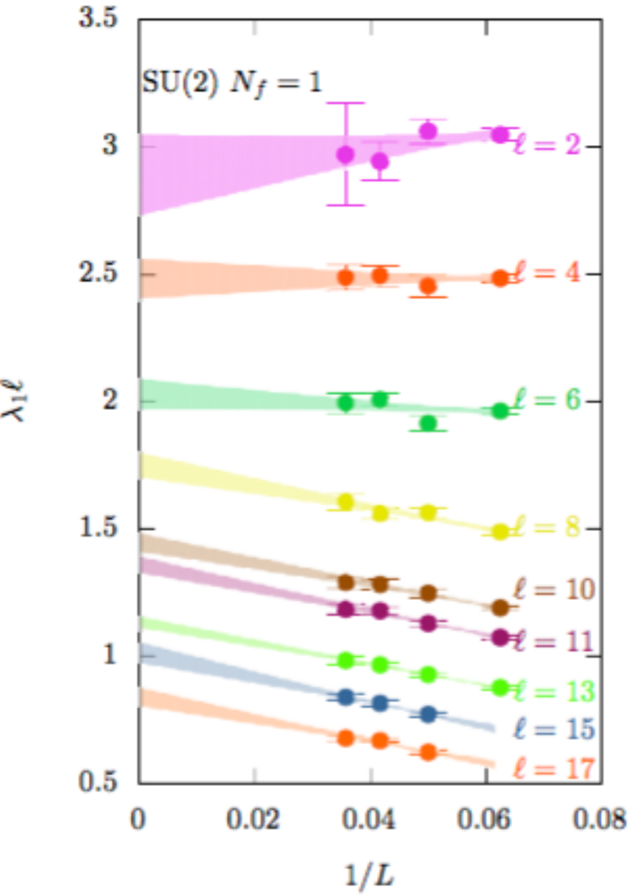
SU(2) with one flavor of four component fermion

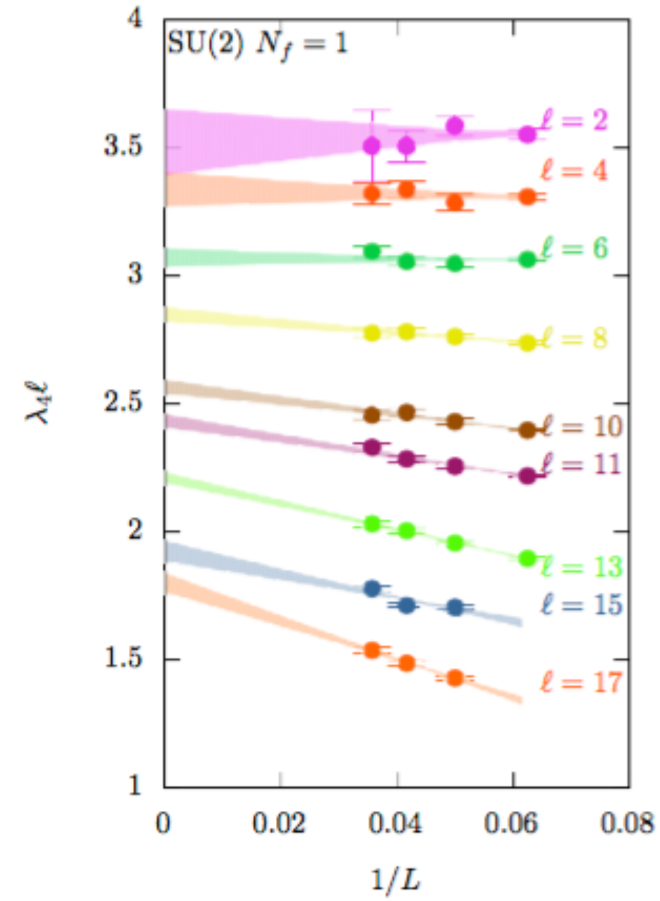
SU(2) with one flavor of four component fermion



Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

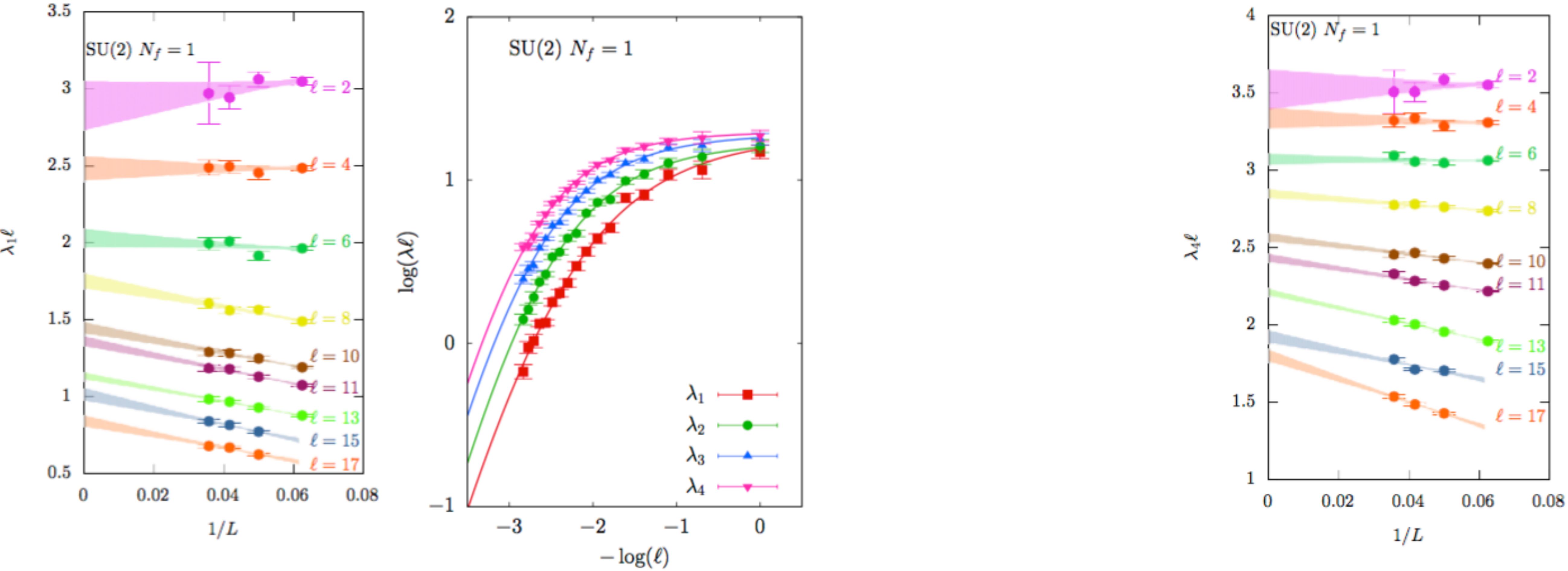
SU(2) with one flavor of four component fermion



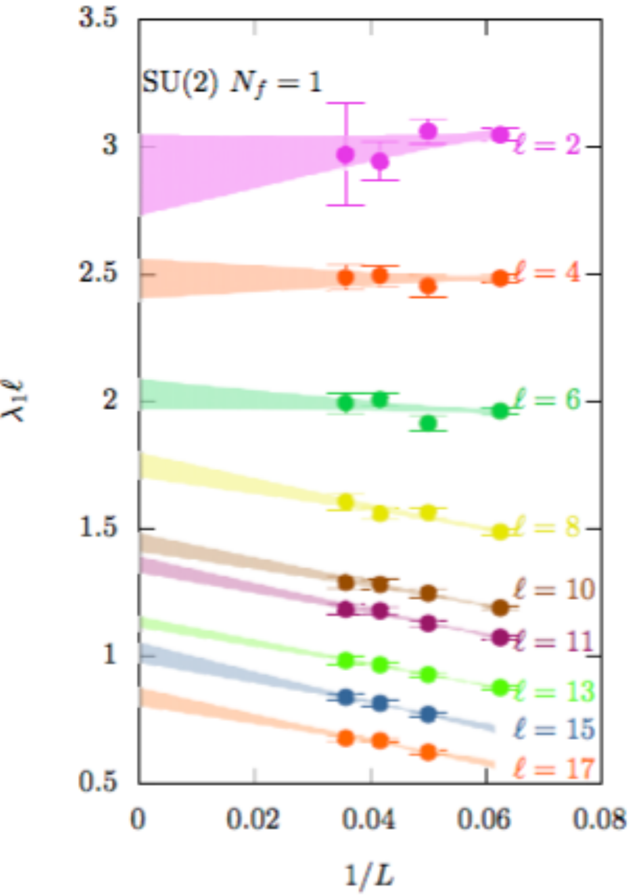


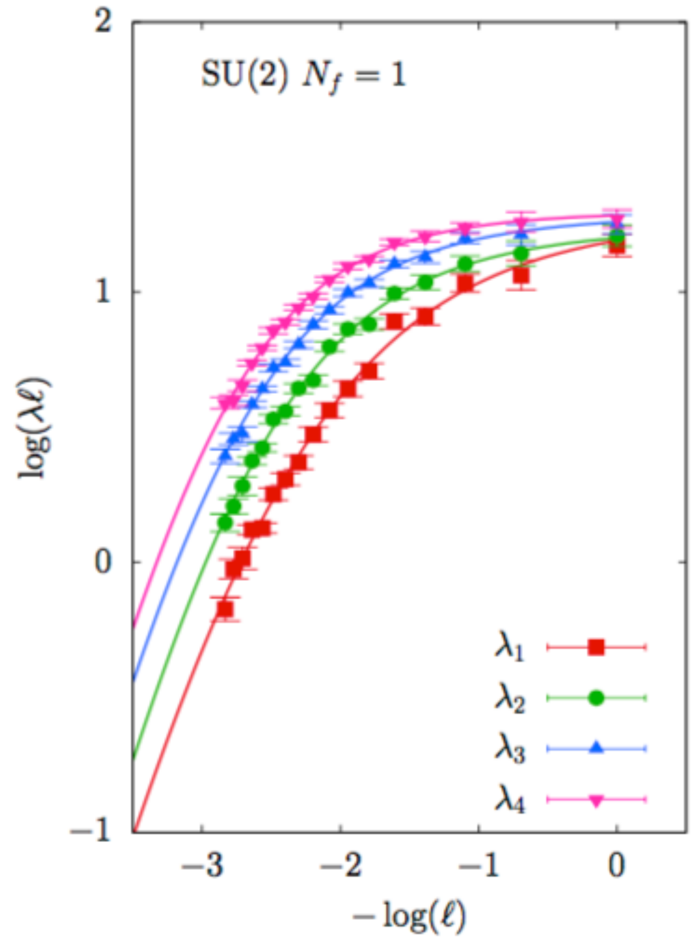
Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

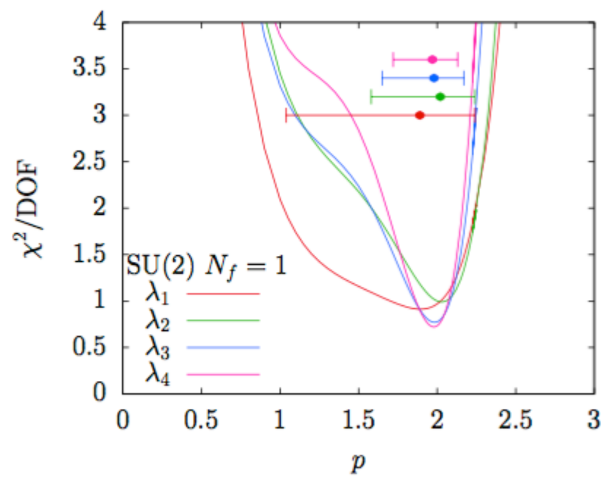
SU(2) with one flavor of four component fermion

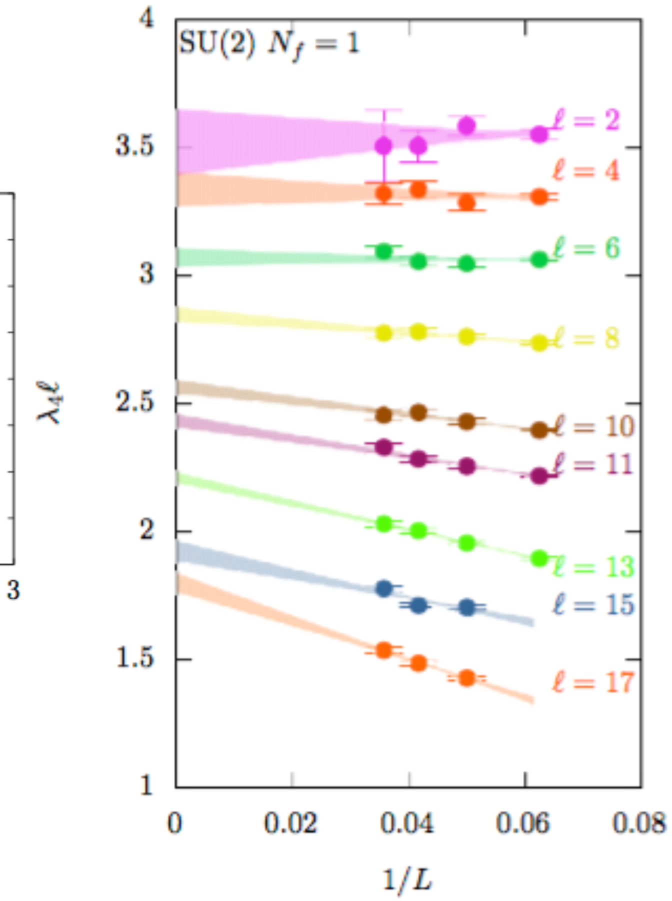


SU(2) with one flavor of four component fermion



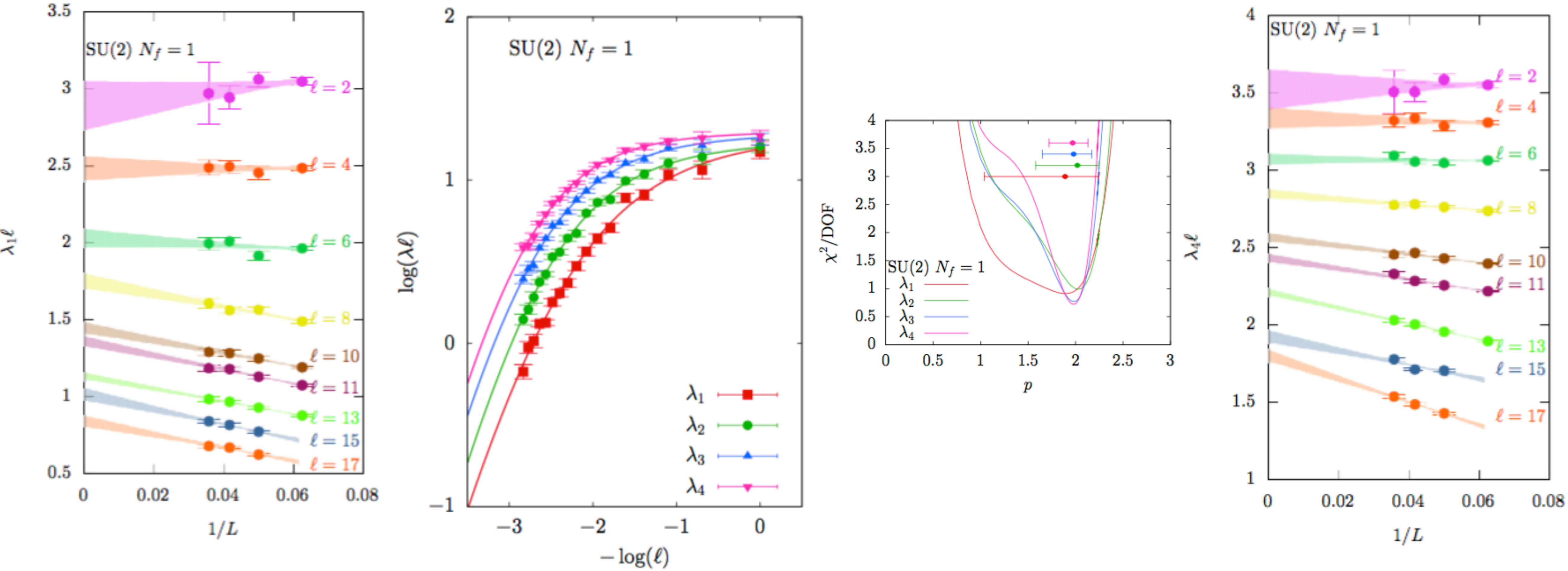






Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

SU(2) with one flavor of four component fermion

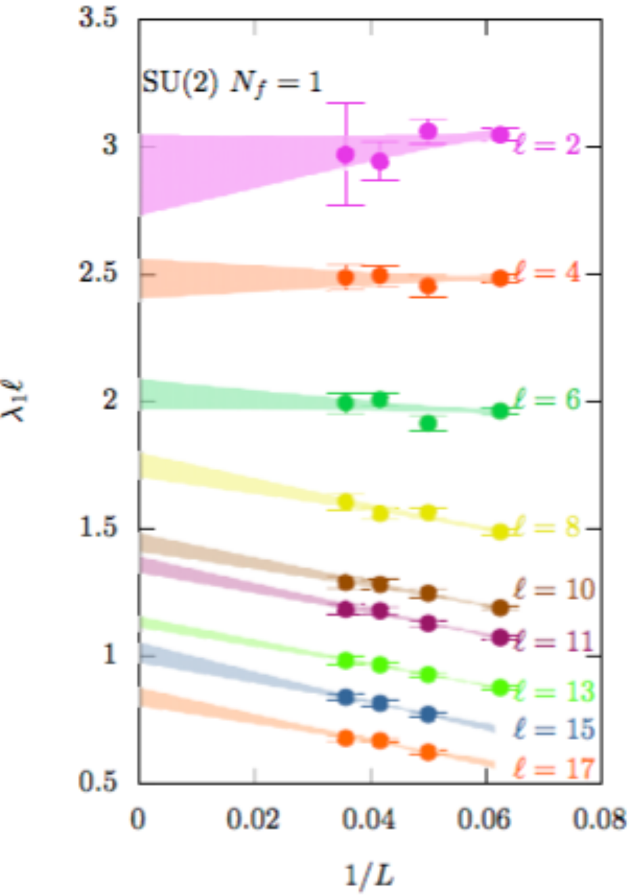


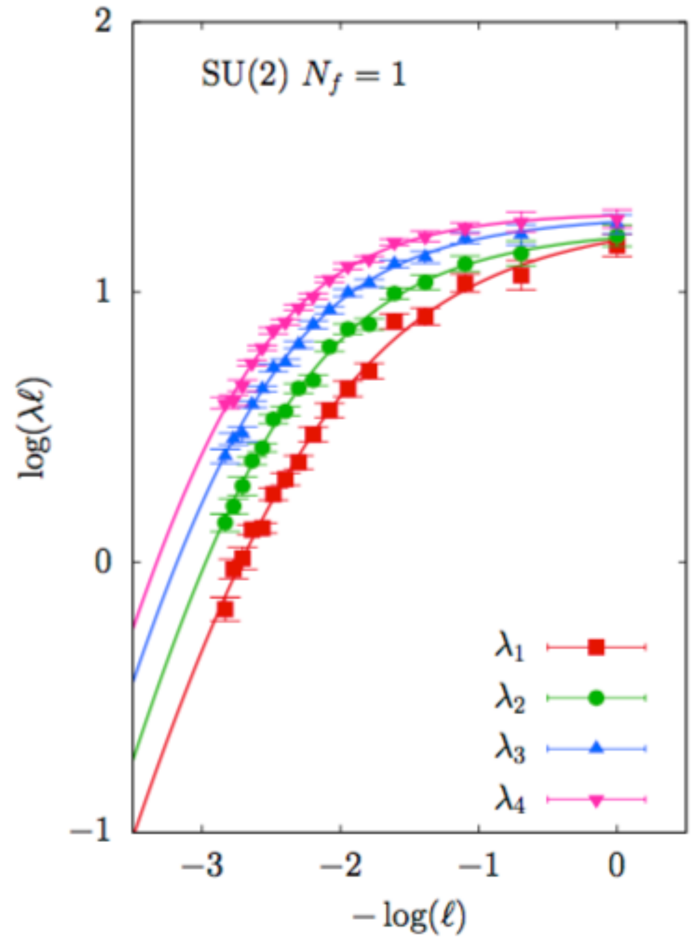
$$K(x,y) = \frac{\sin(\pi(x-y))}{\pi(x-y)} - \frac{\sin(\pi x)}{\pi x}\frac{\sin(\pi y)}{\pi y}$$

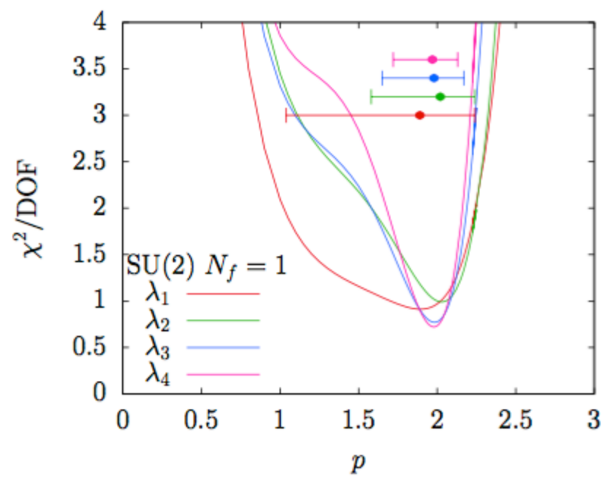
Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

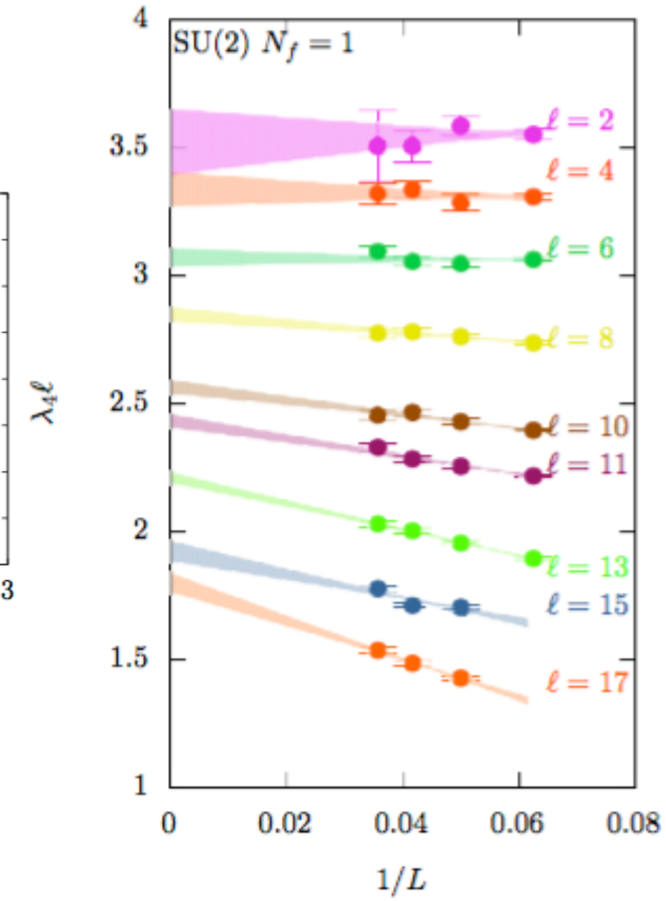
Three dimensional QED and QCD with massless fermions

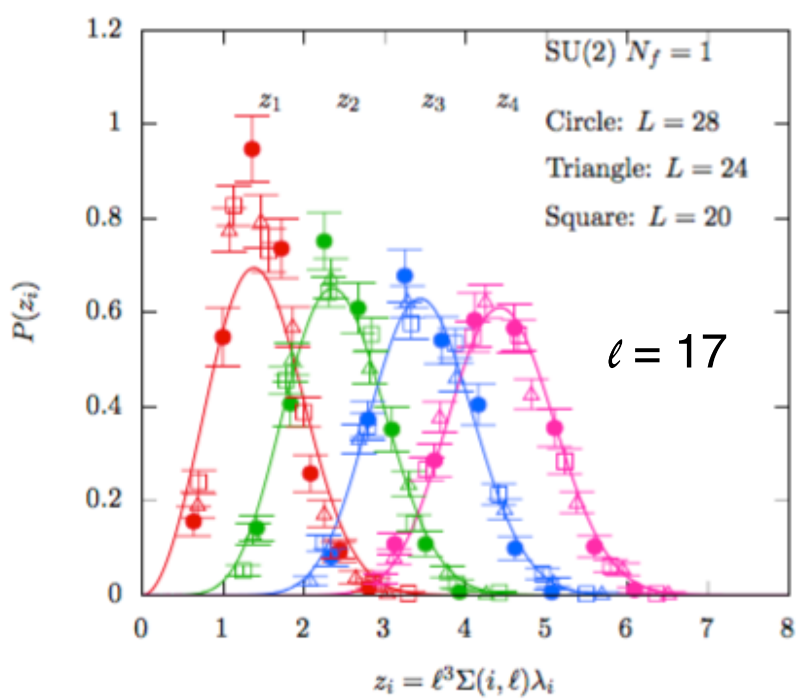
SU(2) with one flavor of four component fermion







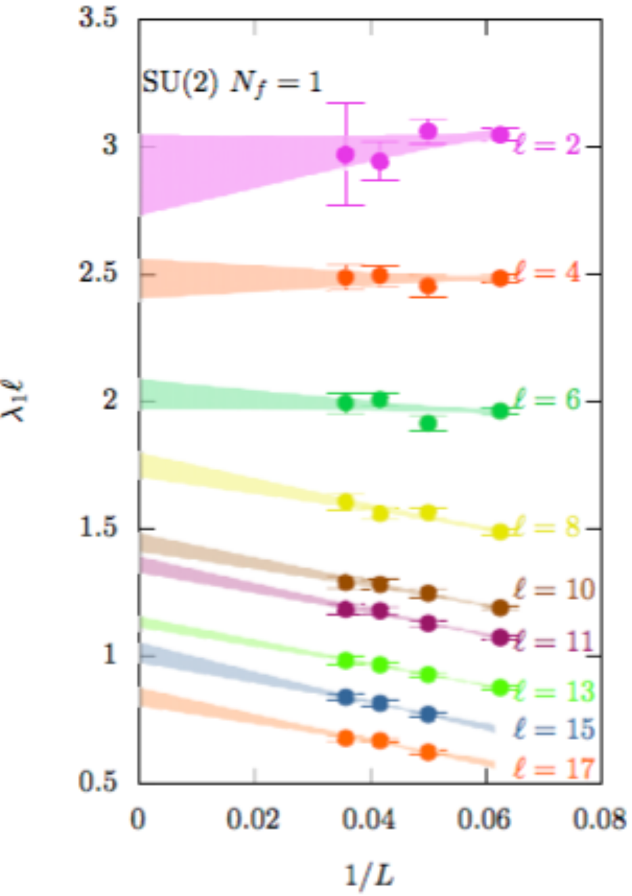


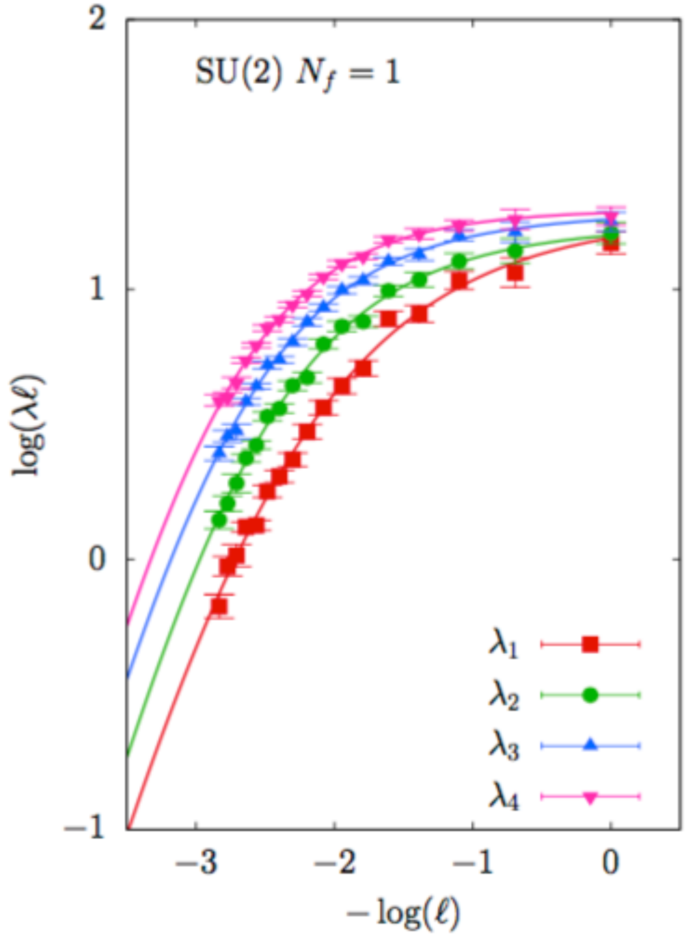


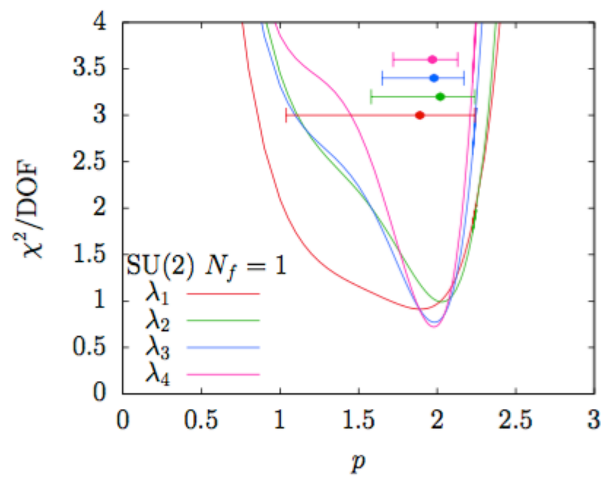
$$K(x,y) = \frac{\sin(\pi(x-y))}{\pi(x-y)} - \frac{\sin(\pi x)}{\pi x}\frac{\sin(\pi y)}{\pi y}$$

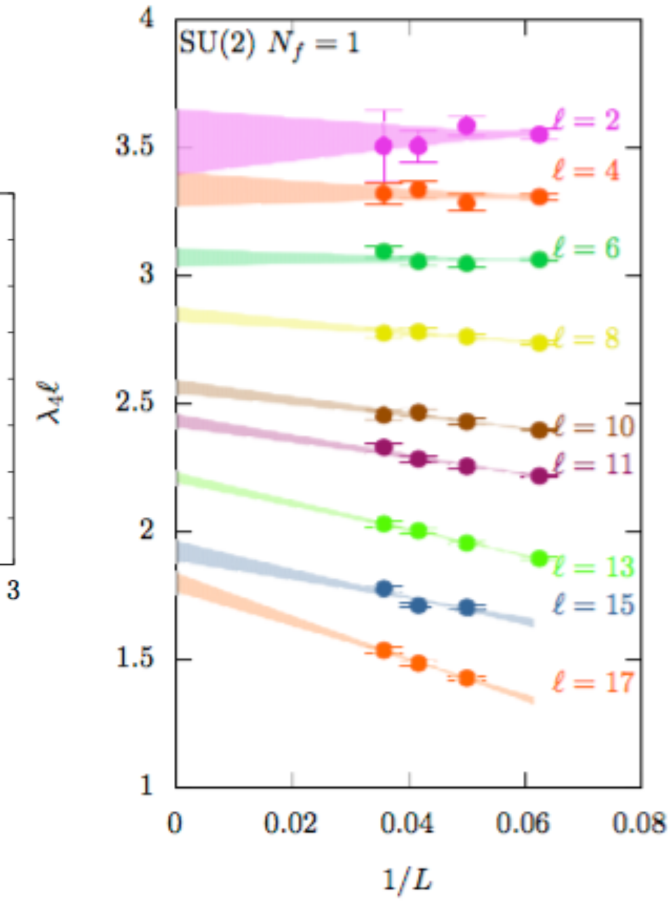
Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

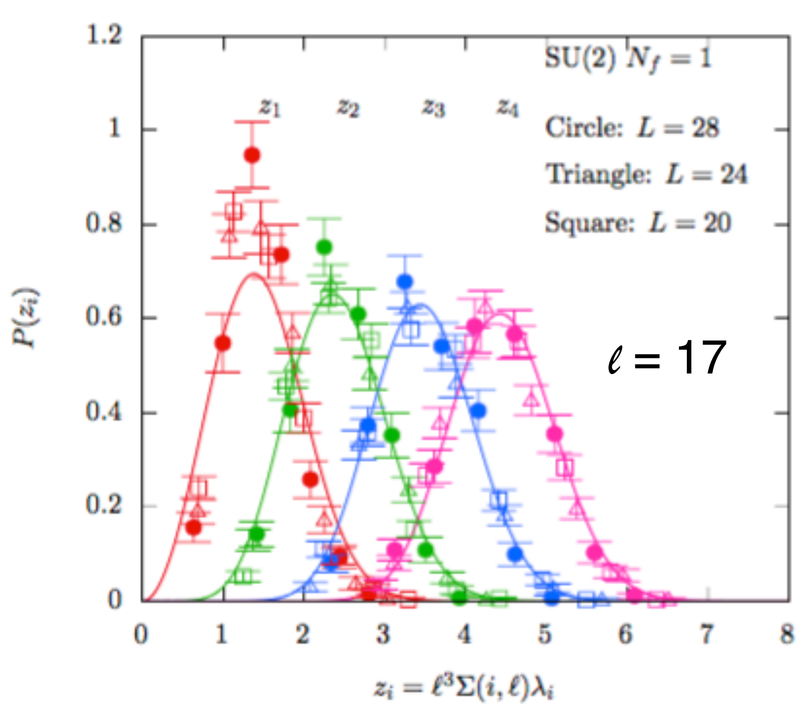
SU(2) with one flavor of four component fermion

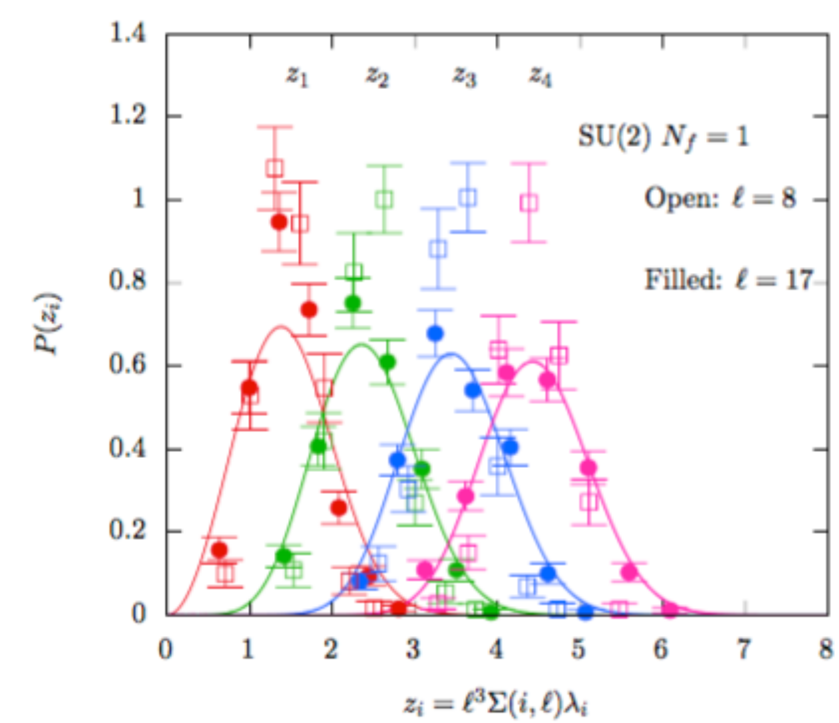










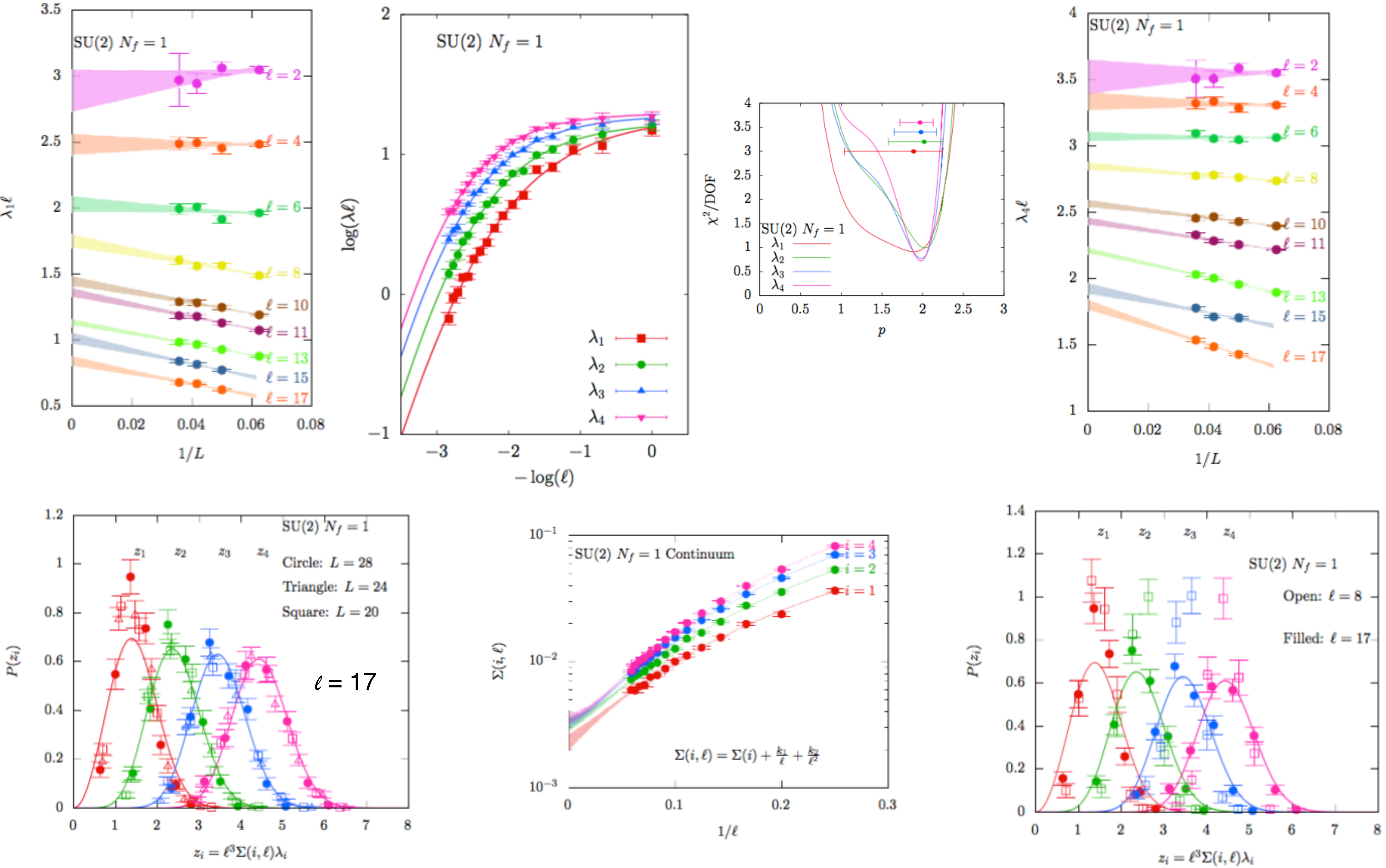


$$K(x,y) = \frac{\sin(\pi(x-y))}{\pi(x-y)} - \frac{\sin(\pi x)}{\pi x}\frac{\sin(\pi y)}{\pi y}$$

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Three dimensional QED and QCD with massless fermions

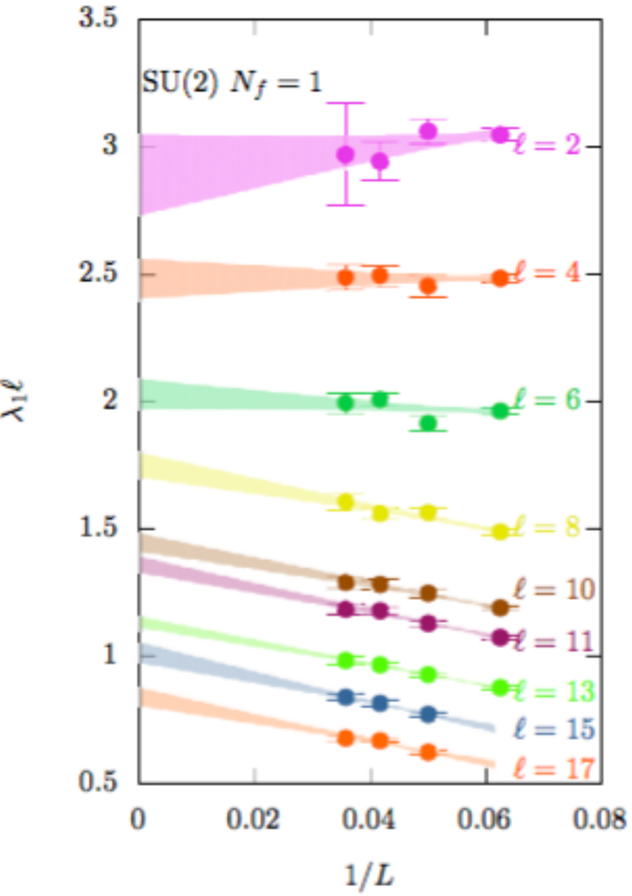
SU(2) with one flavor of four component fermion

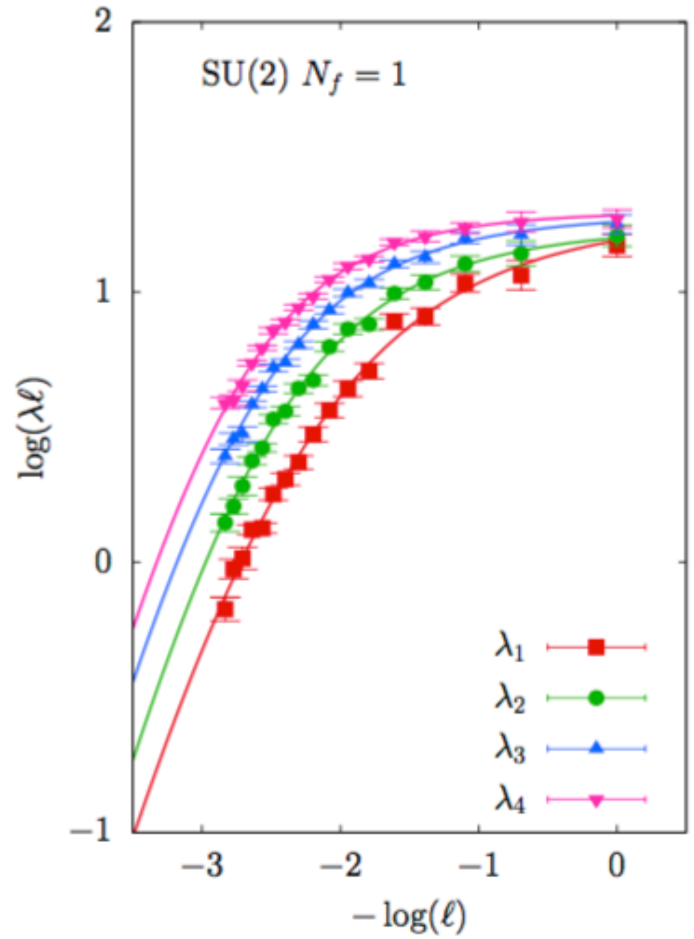


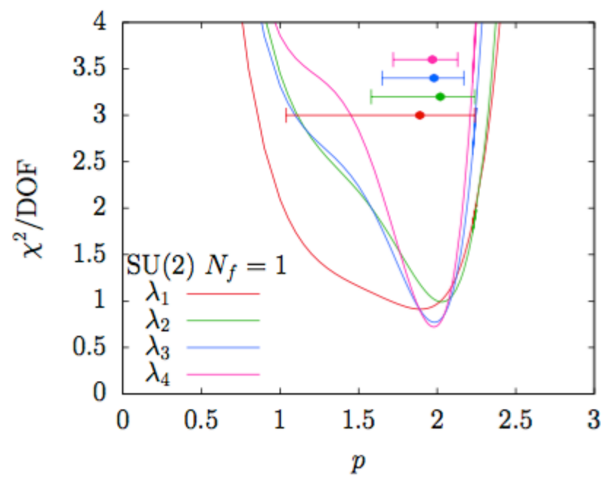
$$K(x,y) = \frac{\sin(\pi(x-y))}{\pi(x-y)} - \frac{\sin(\pi x)}{\pi x}\frac{\sin(\pi y)}{\pi y}$$

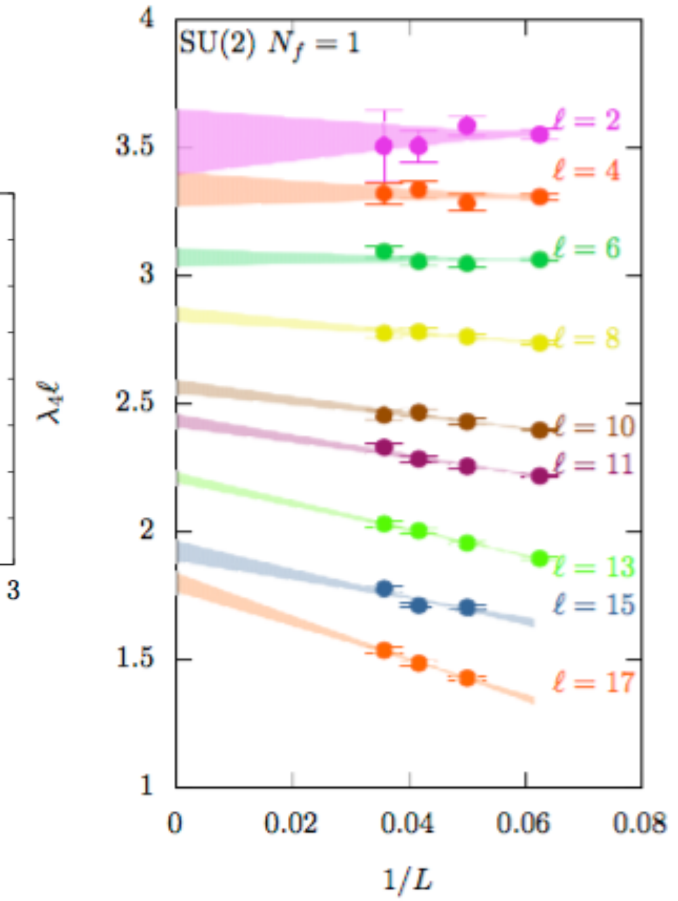
Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

SU(2) with one flavor of four component fermion

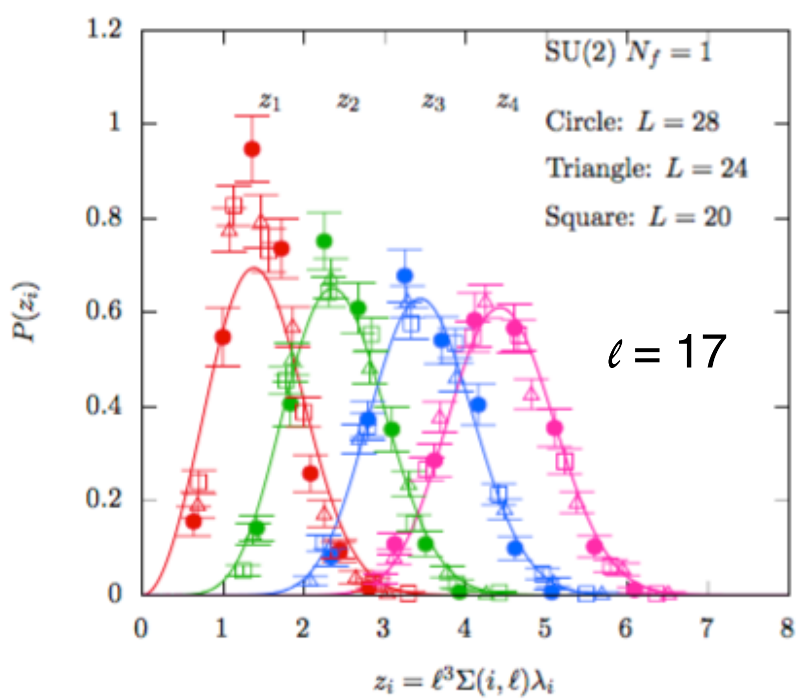


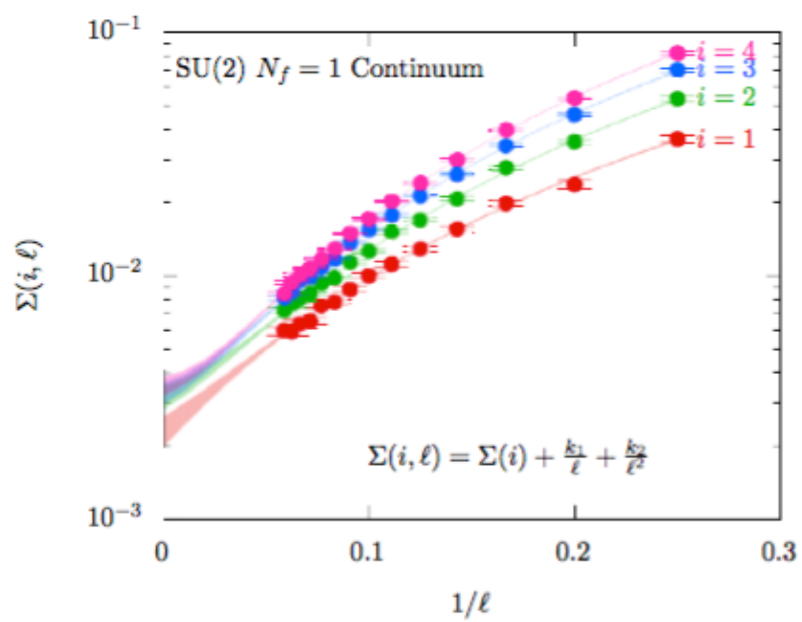


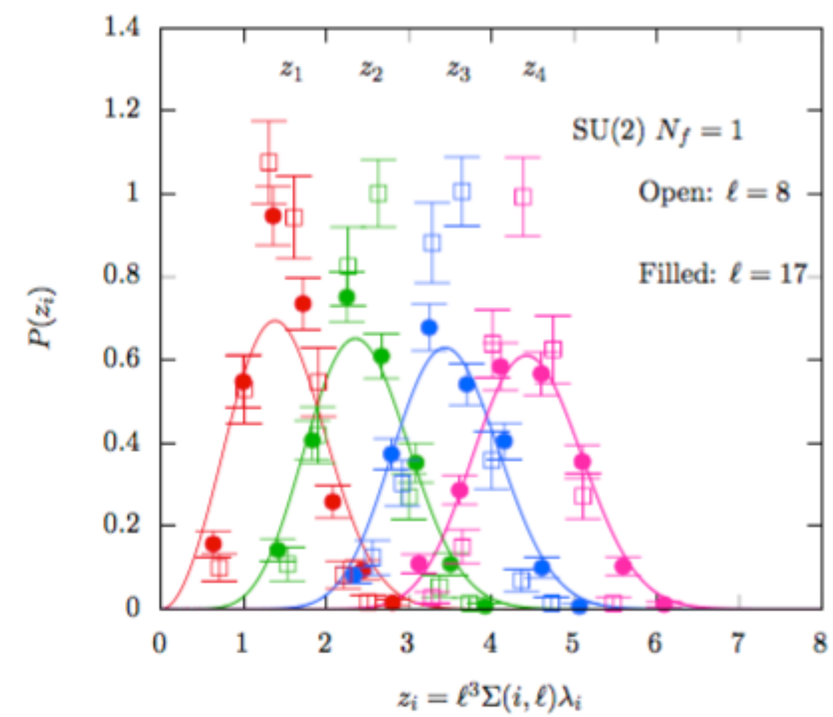




$\Sigma = 0.0034(4)$







$$K(x,y) = \frac{\sin(\pi(x-y))}{\pi(x-y)} - \frac{\sin(\pi x)}{\pi x}\frac{\sin(\pi y)}{\pi y}$$

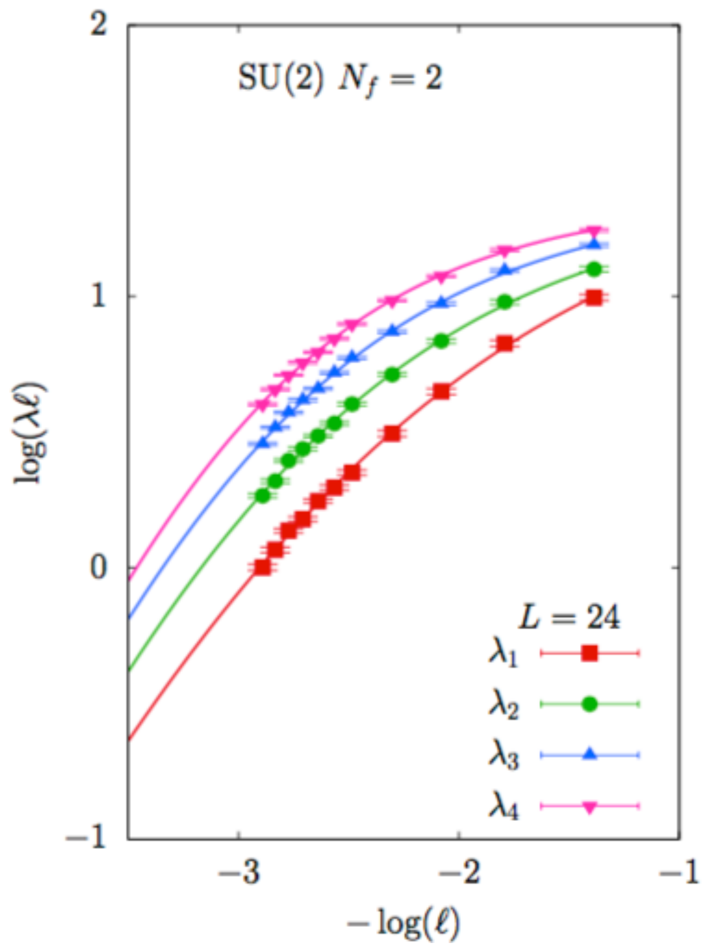
Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Three dimensional QED and QCD with massless fermions

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

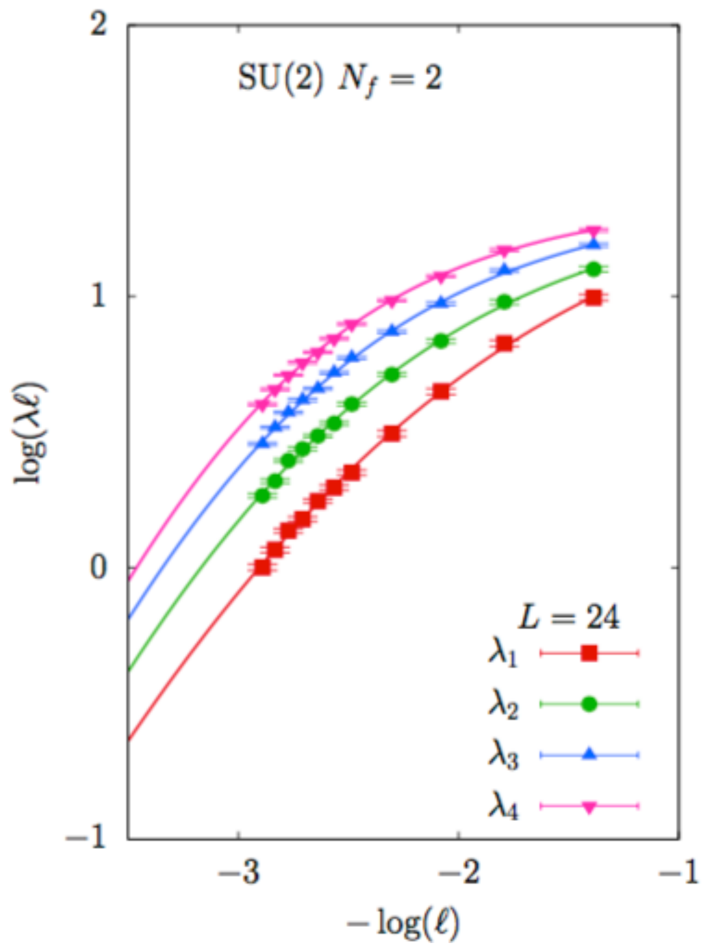
SU(2) with two flavors of four component fermions

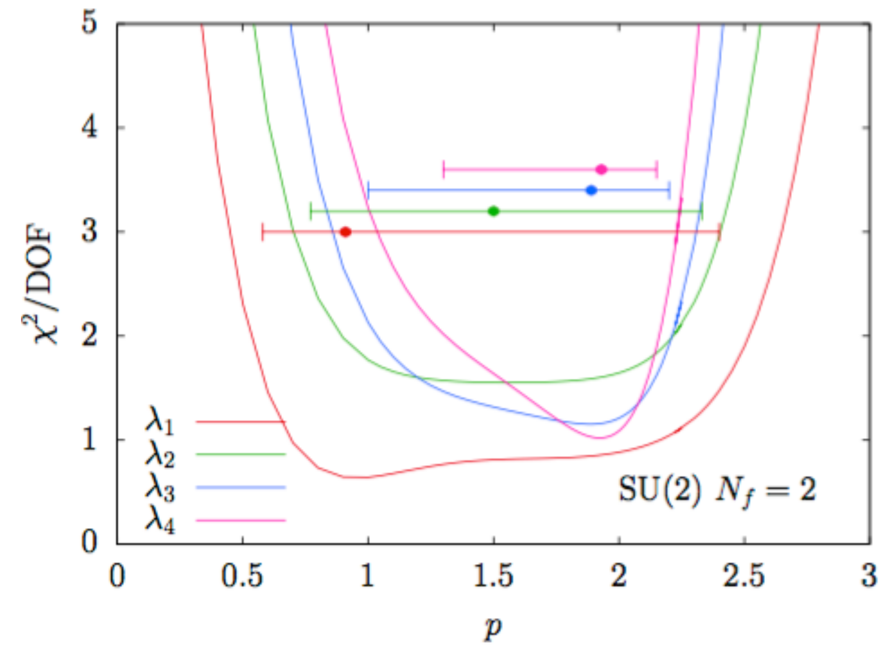
SU(2) with two flavors of four component fermions



Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

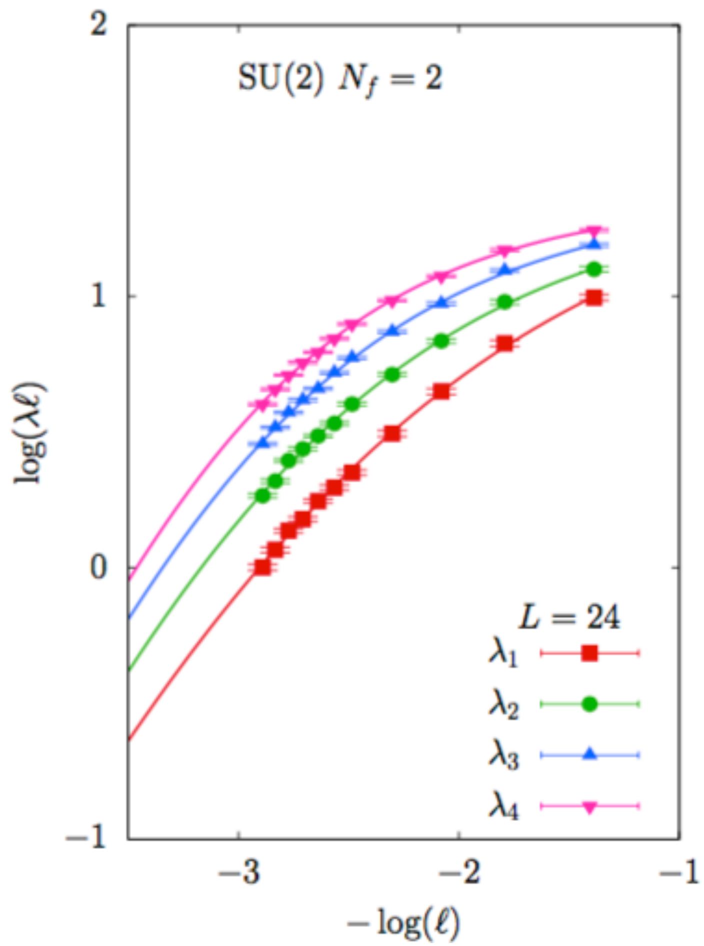
SU(2) with two flavors of four component fermions

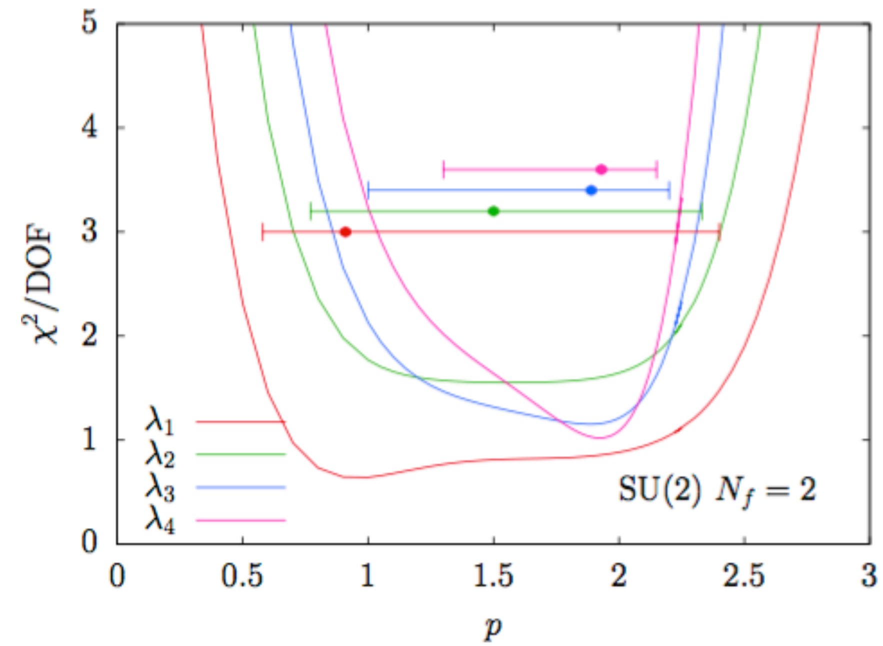


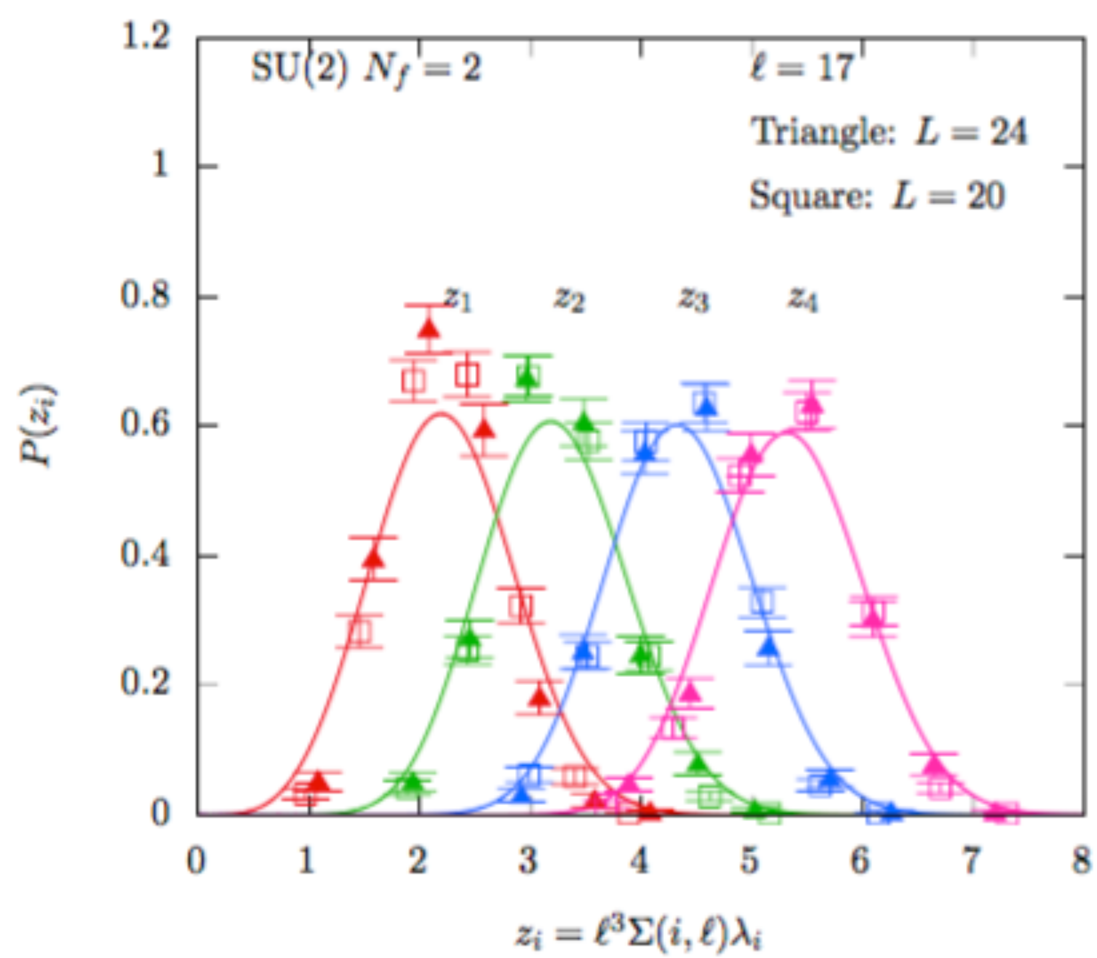


Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

SU(2) with two flavors of four component fermions

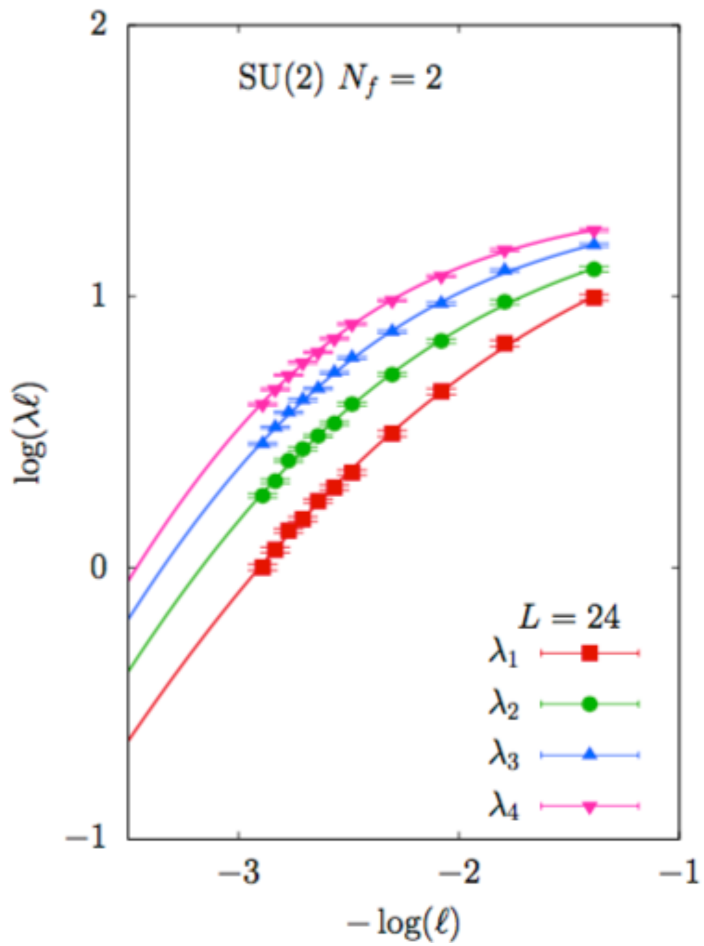


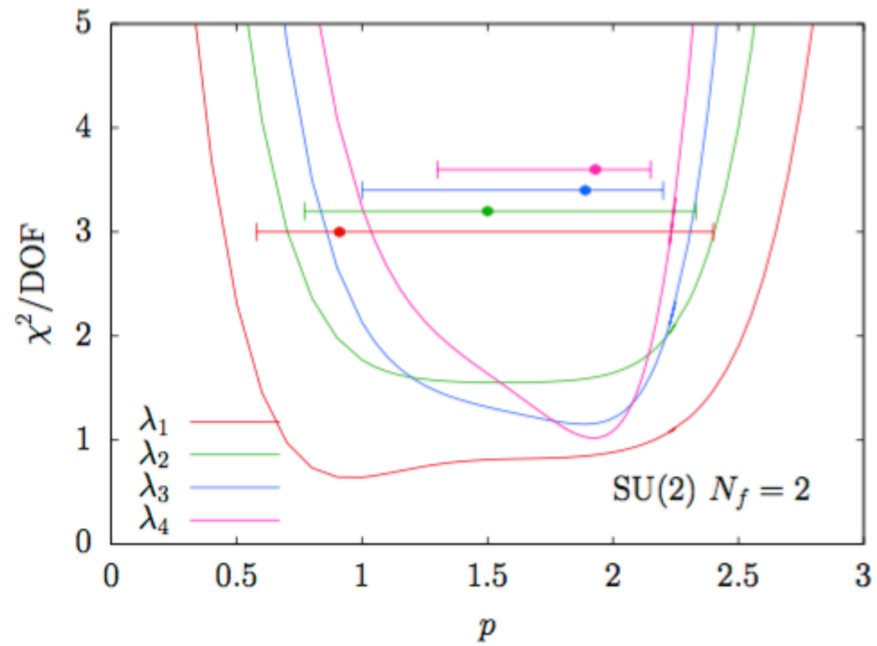


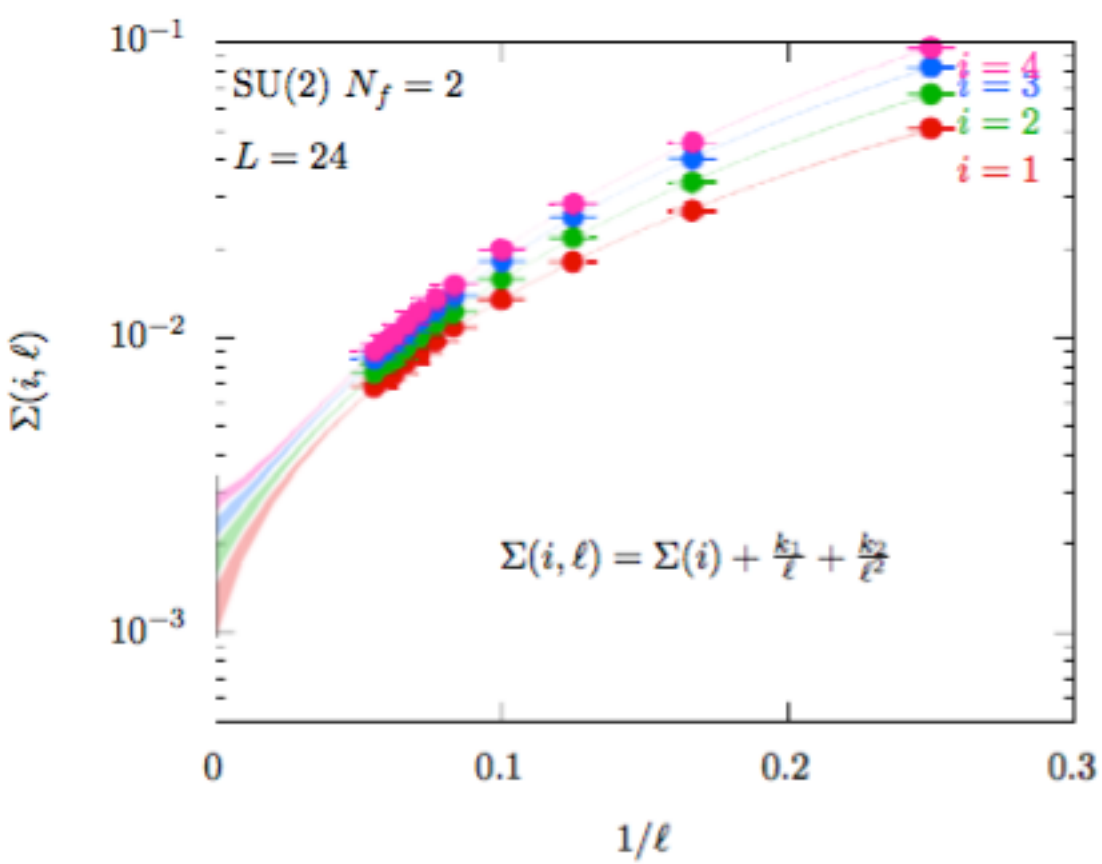


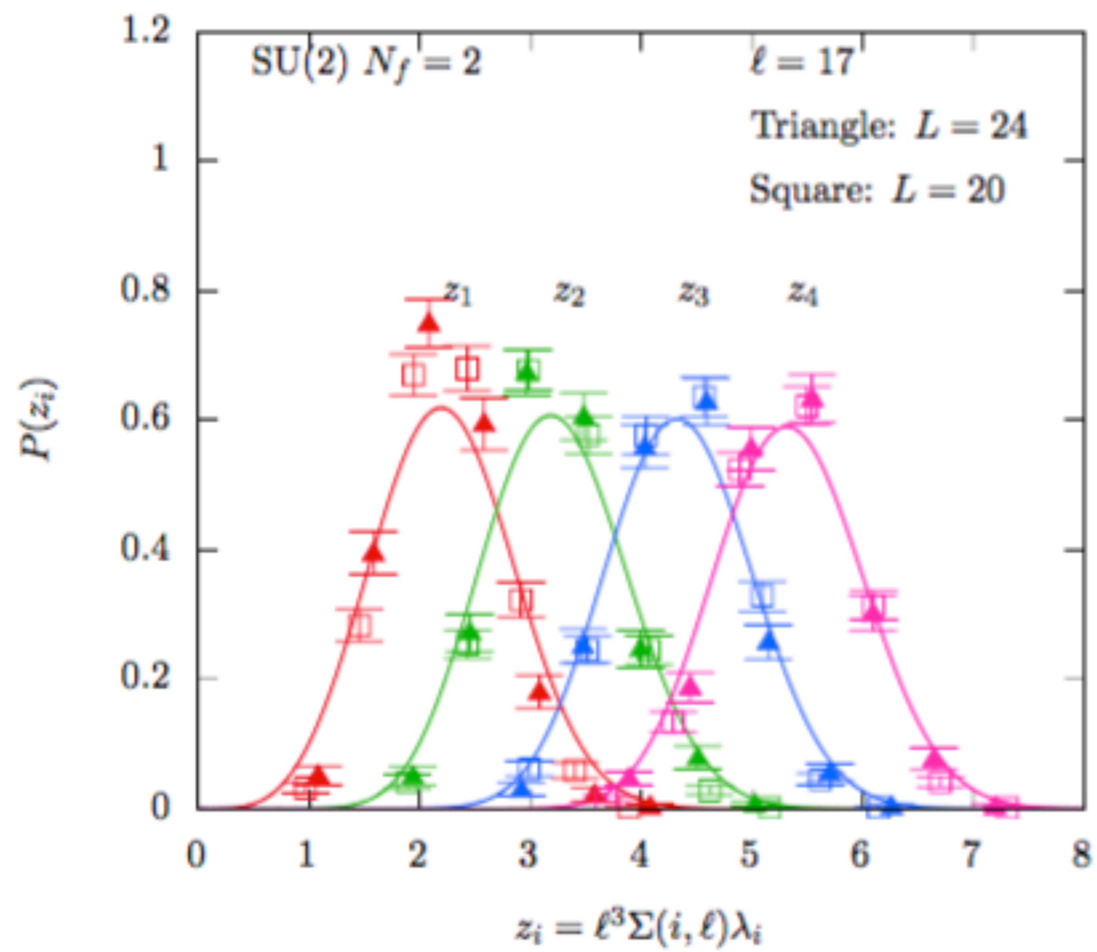
Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

SU(2) with two flavors of four component fermions

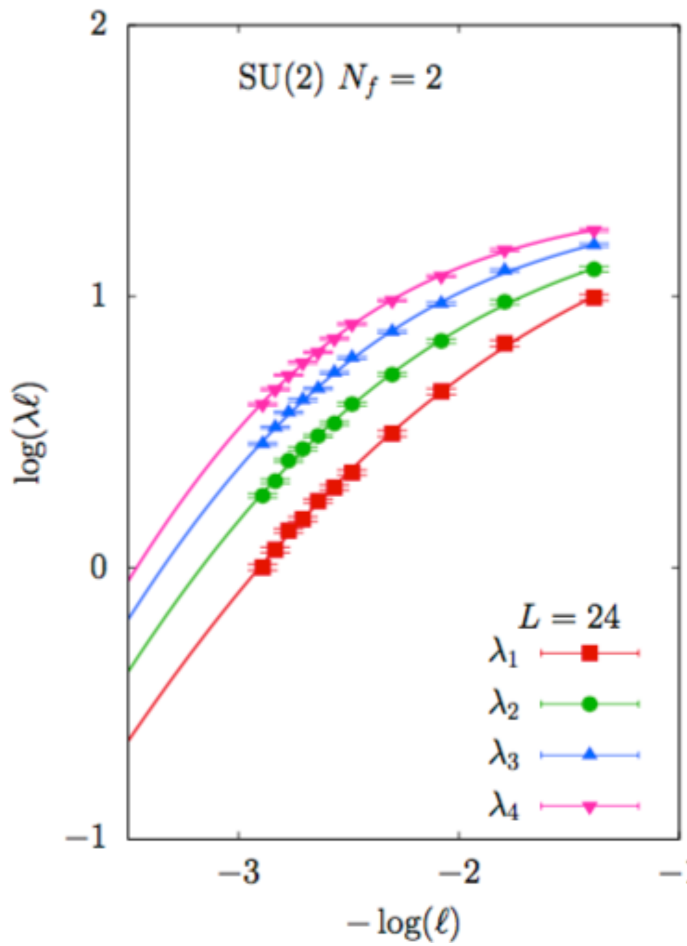


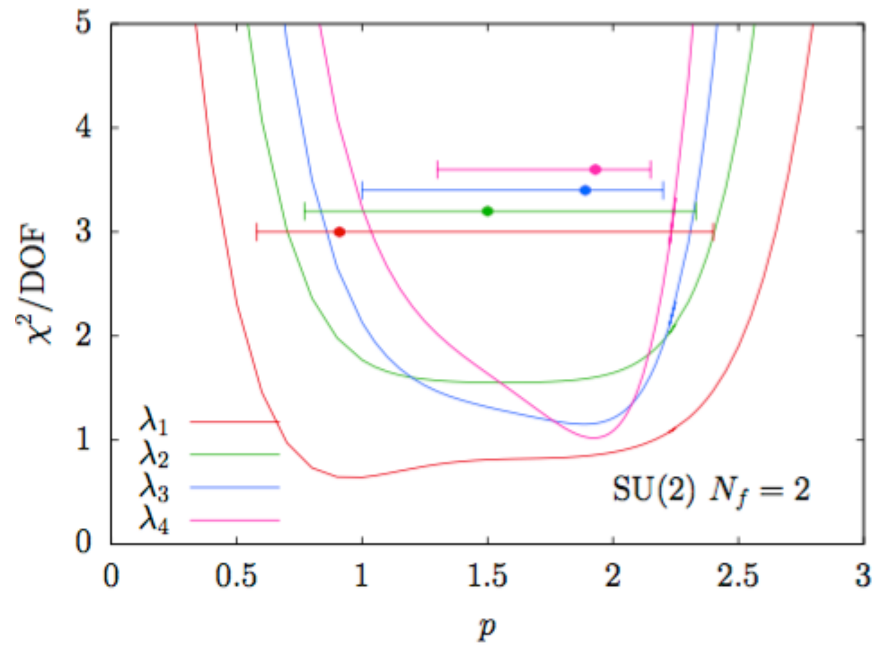




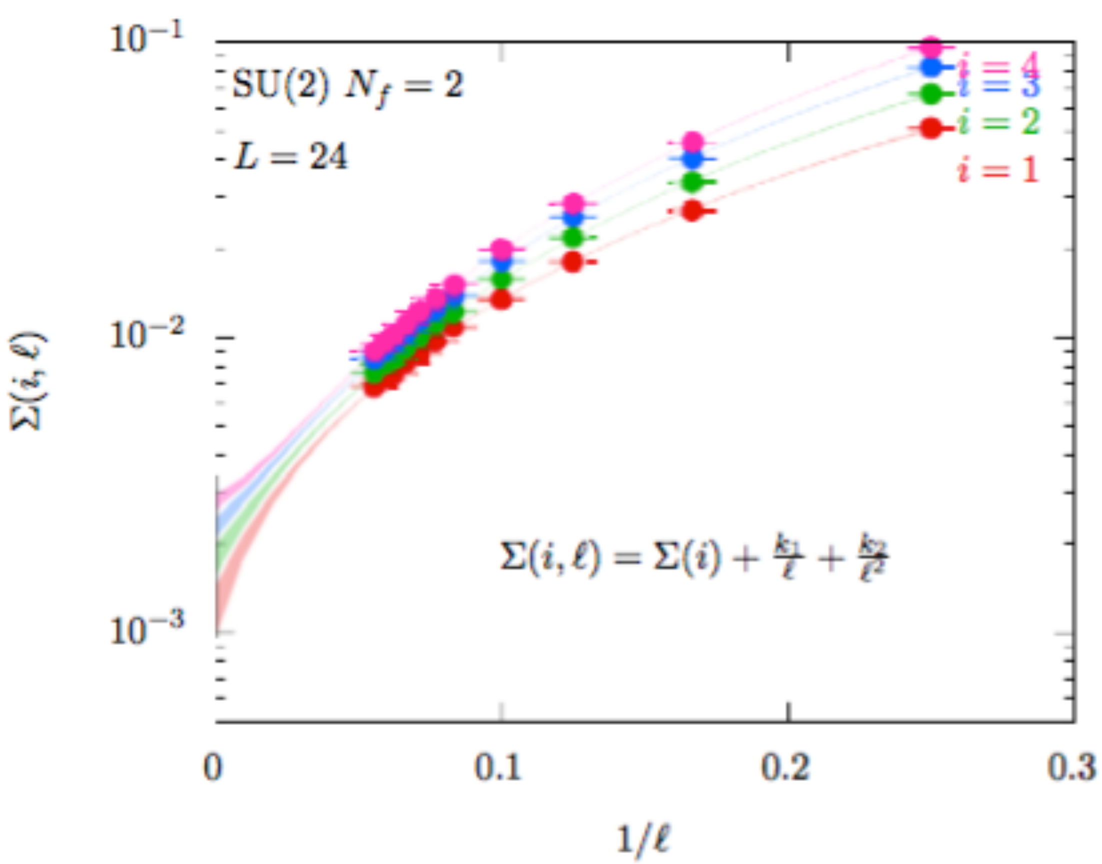


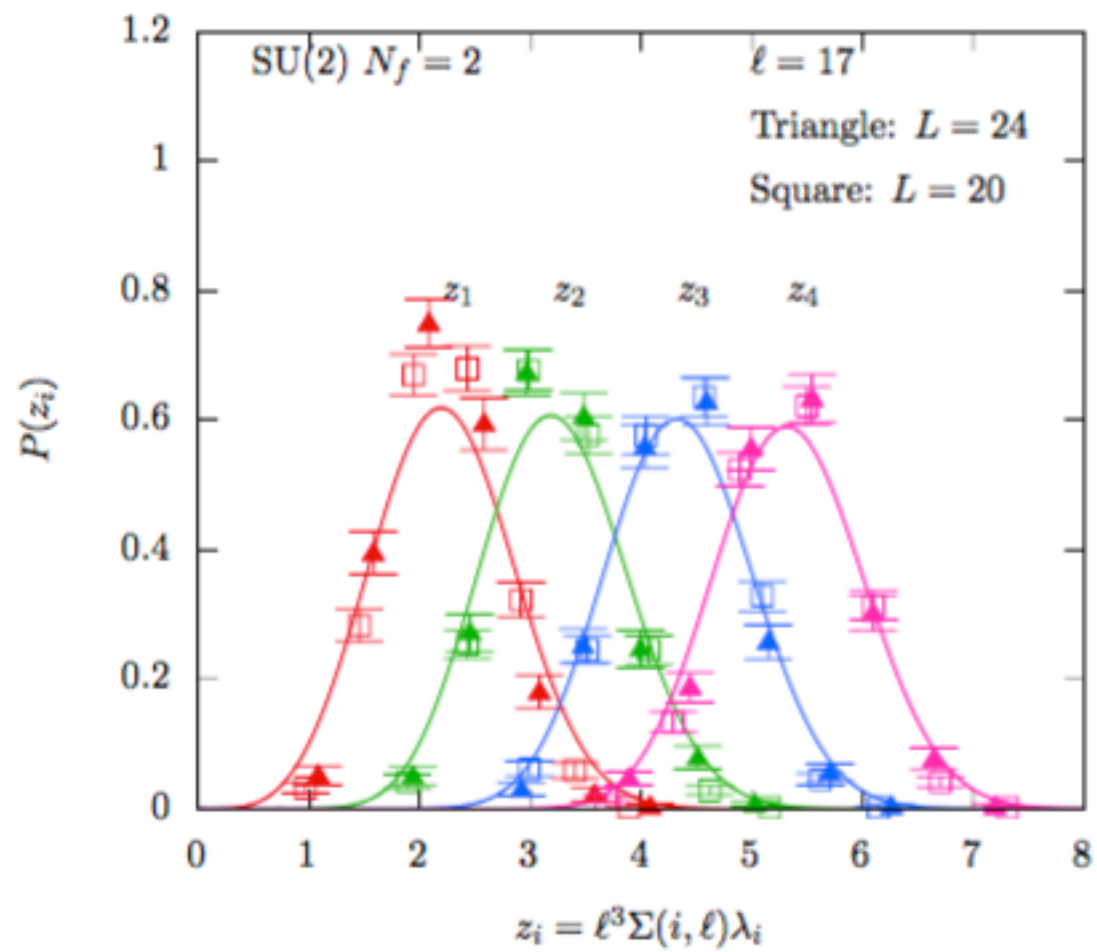
SU(2) with two flavors of four component fermions





Estimate
Σ = 0.0023(7)





Three dimensional QED and QCD with massless fermions

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Son’s model on the lattice

Son’s model on the lattice

V_A: Unitary operator associated with overlap fermions in a background gauge field, A

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Son's model on the lattice

V_A: Unitary operator associated with overlap fermions in a background gauge field, A

Chern-Simons action: $\det V_A = e^{-\frac{i}{4\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda}$

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Son's model on the lattice

V_A: Unitary operator associated with overlap fermions in a background gauge field, A

Chern-Simons action: $\det V_A = e^{-\frac{i}{4\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda}$

Parity: $V_{A_p} = V_A^\dagger$

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Son's model on the lattice

V_A: Unitary operator associated with overlap fermions in a background gauge field, A

Chern-Simons action: $\det V_A = e^{-\frac{i}{4\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda}$

Parity: $V_{A_p} = V_A^\dagger$

Charge conjugation: $V_{-A} = \sigma_2 V_A^t \sigma_2$

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Son's model on the lattice

V_A: Unitary operator associated with overlap fermions in a background gauge field, A

Chern-Simons action: $\det V_A = e^{-\frac{i}{4\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda}$

Parity: $V_{A_p} = V_A^\dagger$

Charge conjugation: $V_{-A} = \sigma_2 V_A^t \sigma_2$

Mixed Chern-Simons action: $\det\left[V_A V_B V_{A+B}^\dagger\right] = \det\left[V_A^\dagger V_B^\dagger V_{A-B}\right] = e^{\frac{i}{2\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu B_\lambda}$

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Son's model on the lattice

V_A: Unitary operator associated with overlap fermions in a background gauge field, A

Chern-Simons action: $\det V_A = e^{-\frac{i}{4\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda}$

Parity: $V_{A_p} = V_A^\dagger$

Charge conjugation: $V_{-A} = \sigma_2 V_A^t \sigma_2$

Mixed Chern-Simons action: $\det\left[V_A V_B V_{A+B}^\dagger\right] = \det\left[V_A^\dagger V_B^\dagger V_{A-B}\right] = e^{\frac{i}{2\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu B_\lambda}$

Non-trivial parity cancellation: $\det\left[V_{2A}^\dagger V_A^4\right] = 1$

Son's model on the lattice

V_A: Unitary operator associated with overlap fermions in a background gauge field, A

Chern-Simons action: $\det V_A = e^{-\frac{i}{4\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda}$

Parity: $V_{A_p} = V_A^\dagger$

Charge conjugation: $V_{-A} = \sigma_2 V_A^t \sigma_2$

Mixed Chern-Simons action: $\det\left[V_A V_B V_{A+B}^\dagger\right] = \det\left[V_A^\dagger V_B^\dagger V_{A-B}\right] = e^{\frac{i}{2\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu B_\lambda}$

Non-trivial parity cancellation: $\det\left[V_{2A}^\dagger V_A^4\right] = 1$

$$Z(C) = \int [dA] e^{S_g(A)} \left[\det \frac{1+V_{2A}}{2}\left(V_A^\dagger\right)^2\right] \det\left[V_A^\dagger V_C^\dagger \det V_{A-C}\right]$$

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Son's model on the lattice

V_A: Unitary operator associated with overlap fermions in a background gauge field, A

Chern-Simons action: $\det V_A = e^{-\frac{i}{4\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda}$

Parity: $V_{A_p} = V_A^\dagger$

Charge conjugation: $V_{-A} = \sigma_2 V_A^t \sigma_2$

Mixed Chern-Simons action: $\det\left[V_A V_B V_{A+B}^\dagger\right] = \det\left[V_A^\dagger V_B^\dagger V_{A-B}\right] = e^{\frac{i}{2\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu B_\lambda}$

Non-trivial parity cancellation: $\det\left[V_{2A}^\dagger V_A^4\right] = 1$

$$Z(C) = \int [dA] e^{S_g(A)} \left[\det \frac{1+V_{2A}}{2}\left(V_A^\dagger\right)^2\right] \det\left[V_A^\dagger V_C^\dagger \det V_{A-C}\right]$$

$Z(C) = Z(C_p) = Z^*(C)$

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Son's model on the lattice

V_A: Unitary operator associated with overlap fermions in a background gauge field, A

Chern-Simons action: $\det V_A = e^{-\frac{i}{4\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda}$

Parity: $V_{A_p} = V_A^\dagger$

Charge conjugation: $V_{-A} = \sigma_2 V_A^t \sigma_2$

Mixed Chern-Simons action: $\det\left[V_A V_B V_{A+B}^\dagger\right] = \det\left[V_A^\dagger V_B^\dagger V_{A-B}\right] = e^{\frac{i}{2\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu B_\lambda}$

Non-trivial parity cancellation: $\det\left[V_{2A}^\dagger V_A^4\right] = 1$

$$Z(C) = \int [dA] e^{S_g(A)} \left[\det \frac{1+V_{2A}}{2}\left(V_A^\dagger\right)^2\right] \det\left[V_A^\dagger V_C^\dagger \det V_{A-C}\right]$$

$Z(C) = Z(C_p) = Z^*(C)$

Questions we are working on

Son's model on the lattice

V_A: Unitary operator associated with overlap fermions in a background gauge field, A

Chern-Simons action: $\det V_A = e^{-\frac{i}{4\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda}$

Parity: $V_{A_p} = V_A^\dagger$

Charge conjugation: $V_{-A} = \sigma_2 V_A^t \sigma_2$

Mixed Chern-Simons action: $\det\left[V_A V_B V_{A+B}^\dagger\right] = \det\left[V_A^\dagger V_B^\dagger V_{A-B}\right] = e^{\frac{i}{2\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu B_\lambda}$

Non-trivial parity cancellation: $\det\left[V_{2A}^\dagger V_A^4\right] = 1$

$$Z(C) = \int [dA] e^{S_g(A)} \left[\det \frac{1+V_{2A}}{2}\left(V_A^\dagger\right)^2\right] \det\left[V_A^\dagger V_C^\dagger \det V_{A-C}\right]$$

$Z(C) = Z(C_p) = Z^*(C)$

Questions we are working on

Set C=0.

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Son's model on the lattice

V_A: Unitary operator associated with overlap fermions in a background gauge field, A

Chern-Simons action: $\det V_A = e^{-\frac{i}{4\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda}$

Parity: $V_{A_p} = V_A^\dagger$

Charge conjugation: $V_{-A} = \sigma_2 V_A^t \sigma_2$

Mixed Chern-Simons action: $\det\left[V_A V_B V_{A+B}^\dagger\right] = \det\left[V_A^\dagger V_B^\dagger V_{A-B}\right] = e^{\frac{i}{2\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu B_\lambda}$

Non-trivial parity cancellation: $\det\left[V_{2A}^\dagger V_A^4\right] = 1$

$$Z(C) = \int [dA] e^{S_g(A)} \left[\det \frac{1+V_{2A}}{2}\left(V_A^\dagger\right)^2\right] \det\left[V_A^\dagger V_C^\dagger \det V_{A-C}\right]$$

$Z(C) = Z(C_p) = Z^*(C)$

Questions we are working on

- Set C=0.
- Non-trivial parity cancellation is not exact on the lattice. What happens in the continuum limit?

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Son's model on the lattice

V_A: Unitary operator associated with overlap fermions in a background gauge field, A

Chern-Simons action: $\det V_A = e^{-\frac{i}{4\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda}$

Parity: $V_{A_p} = V_A^\dagger$

Charge conjugation: $V_{-A} = \sigma_2 V_A^t \sigma_2$

Mixed Chern-Simons action: $\det\left[V_A V_B V_{A+B}^\dagger\right] = \det\left[V_A^\dagger V_B^\dagger V_{A-B}\right] = e^{\frac{i}{2\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu B_\lambda}$

Non-trivial parity cancellation: $\det\left[V_{2A}^\dagger V_A^4\right] = 1$

$$Z(C) = \int [dA] e^{S_g(A)} \left[\det \frac{1+V_{2A}}{2}\left(V_A^\dagger\right)^2\right] \det\left[V_A^\dagger V_C^\dagger \det V_{A-C}\right]$$

$Z(C) = Z(C_p) = Z^*(C)$

Questions we are working on

Set C=0.

- Non-trivial parity cancellation is not exact on the lattice. What happens in the continuum limit?
- Assuming the theory is parity invariant in the continuum, is it scale invariant?

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Son’s model on the lattice

V_A: Unitary operator associated with overlap fermions in a background gauge field, A

Chern-Simons action: $\det V_A = e^{-\frac{i}{4\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda}$

Parity: $V_{A_p} = V_A^\dagger$

Charge conjugation: $V_{-A} = \sigma_2 V_A^t \sigma_2$

Mixed Chern-Simons action: $\det\left[V_A V_B V_{A+B}^\dagger\right] = \det\left[V_A^\dagger V_B^\dagger V_{A-B}\right] = e^{\frac{i}{2\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu B_\lambda}$

Non-trivial parity cancellation: $\det\left[V_{2A}^\dagger V_A^4\right] = 1$

$$Z(C) = \int [dA] e^{S_g(A)} \left[\det \frac{1+V_{2A}}{2}\left(V_A^\dagger\right)^2\right] \det\left[V_A^\dagger V_C^\dagger \det V_{A-C}\right]$$

$Z(C) = Z(C_p) = Z^*(C)$

Questions we are working on

- Set C=0.
  - Non-trivial parity cancellation is not exact on the lattice. What happens in the continuum limit?
  - Assuming the theory is parity invariant in the continuum, is it scale invariant?
- Set C away from zero.

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Son's model on the lattice

V_A: Unitary operator associated with overlap fermions in a background gauge field, A

Chern-Simons action: $\det V_A = e^{-\frac{i}{4\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda}$

Parity: $V_{A_p} = V_A^\dagger$

Charge conjugation: $V_{-A} = \sigma_2 V_A^t \sigma_2$

Mixed Chern-Simons action: $\det\left[V_A V_B V_{A+B}^\dagger\right] = \det\left[V_A^\dagger V_B^\dagger V_{A-B}\right] = e^{\frac{i}{2\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu B_\lambda}$

Non-trivial parity cancellation: $\det\left[V_{2A}^\dagger V_A^4\right] = 1$

$$Z(C) = \int [dA] e^{S_g(A)} \left[\det \frac{1+V_{2A}}{2}\left(V_A^\dagger\right)^2\right] \det\left[V_A^\dagger V_C^\dagger \det V_{A-C}\right]$$

$Z(C) = Z(C_p) = Z^*(C)$

Questions we are working on

- Set C=0.
  - Non-trivial parity cancellation is not exact on the lattice. What happens in the continuum limit?
  - Assuming the theory is parity invariant in the continuum, is it scale invariant?
- Set C away from zero.
  - What is the induced action on C?

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Son's model on the lattice

V_A: Unitary operator associated with overlap fermions in a background gauge field, A

Chern-Simons action: $\det V_A = e^{-\frac{i}{4\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda}$

Parity: $V_{A_p} = V_A^\dagger$

Charge conjugation: $V_{-A} = \sigma_2 V_A^t \sigma_2$

Mixed Chern-Simons action: $\det \left[V_A V_B V_{A+B}^\dagger \right] = \det \left[V_A^\dagger V_B^\dagger V_{A-B} \right] = e^{\frac{i}{2\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu B_\lambda}$

Non-trivial parity cancellation: $\det \left[V_{2A}^\dagger V_A^4 \right] = 1$

$$Z(C) = \int [dA] e^{S_g(A)} \left[\det \frac{1 + V_{2A}}{2} \left(V_A^\dagger \right)^2 \right] \det \left[V_A^\dagger V_C^\dagger \det V_{A-C} \right]$$

$Z(C) = Z(C_p) = Z^*(C)$

Questions we are working on

- Set C=0.
  - Non-trivial parity cancellation is not exact on the lattice. What happens in the continuum limit?
  - Assuming the theory is parity invariant in the continuum, is it scale invariant?
- Set C away from zero.
  - What is the induced action on C?
  - Is it parity invariant for an arbitrary background?

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Son's model on the lattice

V_A: Unitary operator associated with overlap fermions in a background gauge field, A

Chern-Simons action: $\det V_A = e^{-\frac{i}{4\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda}$

Parity: $V_{A_p} = V_A^\dagger$

Charge conjugation: $V_{-A} = \sigma_2 V_A^t \sigma_2$

Mixed Chern-Simons action: $\det\left[V_A V_B V_{A+B}^\dagger\right] = \det\left[V_A^\dagger V_B^\dagger V_{A-B}\right] = e^{\frac{i}{2\pi}\int d^3x \epsilon_{\mu\nu\lambda} A_\mu \partial_\nu B_\lambda}$

Non-trivial parity cancellation: $\det\left[V_{2A}^\dagger V_A^4\right] = 1$

$$Z(C) = \int [dA] e^{S_g(A)} \left[\det \frac{1+V_{2A}}{2}\left(V_A^\dagger\right)^2\right] \det\left[V_A^\dagger V_C^\dagger \det V_{A-C}\right]$$

$Z(C) = Z(C_p) = Z^*(C)$

Questions we are working on

- Set C=0.
  - Non-trivial parity cancellation is not exact on the lattice. What happens in the continuum limit?
  - Assuming the theory is parity invariant in the continuum, is it scale invariant?
- Set C away from zero.
  - What is the induced action on C?
  - Is it parity invariant for an arbitrary background?
  - Do we need to add a Maxwell term for C to make sense of a theory where C is dynamical?

Nikhil Karthik and Rajamani Narayanan, Department of Physics, Florida International University, Miami, FL 33199

Thank you for your attention