Lattice QCD in view of the SuperB

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lattice QCD forecasts

V. Lubicz @ XI SuperB Workshop LNF 2009

hadronic mel	2006 error	2009 error	2011 error	2015 error
$F_+^{K\pi}(q^2=0)$	0.9%	0.5%	0.4%	$< 0.1\%$
$\hat{B}_K$	11%	5%	3%	1%
F_B	14%	5%	$2.5\% - 4\%$	$1\% - 1.5\%$
$F_{B_s}\sqrt{B_{B_s}}$	13%	5%	$3\% - 4\%$	$1\% - 1.5\%$
ξ_B	5%	2%	$1.5\% - 2\%$	$0.5\% - 0.8\%$
$F^{BD^{(\star)}}(\omega=1)$	4%	2%	1.2%	0.5%
$F_+^{B\pi}(q^2=0)$	11%	11%	$4\% - 5\%$	$2\% - 3\%$
$T_1^{B\rho(K^\star)}(q^2=0)$	13%	13%	$- - -$	$3\% - 4\%$

- Lubicz has shown that, a part from $F_+^{B\pi}(q^2=0)$, 2009 goals had been reached
- in this talk we shall see if Vittorio's prediction for 2011 it has been confirmed
- discuss how (and if!) we can confirm 2015 predictions. . .
- eventually make weather forecasts for 2015!

lattice QCD forecasts: the budget

- in order to improve errors on hadronic matrix elements by using lattice techniques one has to pay (the currency is $TFlops \times year$)

  L.Del Debbio, L.Giusti, M.Lüscher, R.Petronzio, N.T. JHEP 0702 (2007) 056

$$
\begin{aligned}
TFlops \times year \quad &= \quad 0.03 \left(\frac{N_{conf}}{100}\right) \left(\frac{20\ MeV}{m_{ud}}\right) \left(\frac{L_t}{2L_s}\right) \left(\frac{L_s}{3\ fm}\right)^5 \left(\frac{0.1\ fm}{a}\right)^6 \\
&\sim \quad 0.03 \left(\frac{N_{conf}}{100}\right) \left(\frac{20\ MeV}{m_{ud}}\right) \left(\frac{N_t \times N_s}{64 \times 32}\right)^{\sim 3}
\end{aligned}
$$

- i.e., as a rule of thumb, we can say that fixed the pion mass and given a supercomputer we have a budget quantified in terms of number of points of our lattice...
- then we have to decide if to spend this budget in light quark physics (big volumes) or in heavy quark physics (small lattice spacings)
- important:
  - using this formula today is a conservative estimate: several other algorithmic improvements since 2007 (Lüscher deflation acceleration, etc.)
  - on the other hand sampling errors do enter our game and we are neglecting them to obtain our estimates
  - for a detailed discussion of these problems and for a proposal to solve them see (and references therein)

    M. Lüscher, S. Schaefer arXiv:1105.4749

lattice QCD forecasts: the method

- let's play the "lattice effective theory" game invented by: S.Sharpe @ Orsay 2004 "LQCD, present and future"
- concerning continuum extrapolations, we imagine to simulate an $O(a)$ improved theory at a_{min} and $\sqrt{2}a_{min}$ and to extrapolate linearly in a^2

$$\mathcal{O}^{phys} = \mathcal{O}^{latt} \left\{1 + c_2(a\Lambda_{QCD})^2 + c_3(a\Lambda_{QCD})^3 + \ldots\right\} \quad \rightarrow \quad \frac{\Delta O}{O} = (2^{3/2} - 1)\, c_3\, (a_{light}\Lambda_{QCD})^3$$

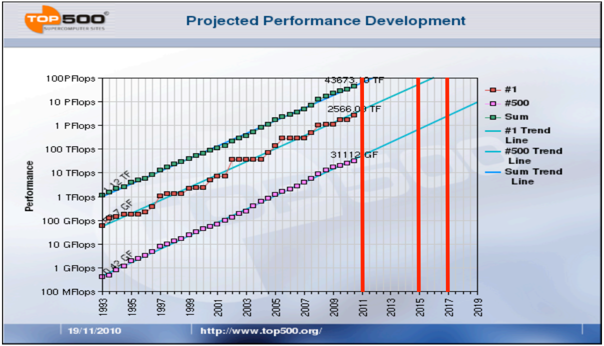
$$\mathcal{O}^{phys} = \mathcal{O}^{latt} \left\{1 + c_2(am_h)^2 + c_3(am_h)^3 + \ldots\right\} \quad \rightarrow \quad \frac{\Delta O}{O} = (2^{3/2} - 1)\, c_3\, (a_{heavy}m_h)^3$$

- we assume $c_3 \sim 1$ (if $c_3 = 0$ usually c_4 large) and set the goal precision to 1%, getting

scale (GeV)	a (fm)	$N_t \times N_s$ @ 3fm	$Pflops \times y$	$N_t \times N_s$ @ 4fm	$Pflops \times y$
0.5	0.069	96×48	10^{-3}	128×64	2×10^{-3}
2.0	0.017	360×180	1	480×240	5
4.0	0.009	720×360	60	960×480	340

- this estimates have to be considered conservative

lattice QCD forecasts: how much can we pay?



scale (GeV)	a (fm)	$N_t \times N_s$ @ 3fm	$Pflops \times y$	$when$	$N_t \times N_s$ @ 4fm	$Pflops \times y$	$when$
0.5	0.069	96×48	10^{-3}	2006	128×64	2×10^{-3}	2007
2.0	0.017	360×180	1	2013	480×240	5	2015
4.0	0.009	720×360	60	2017	960×480	340	future

BMW, arXiv:1011.2711

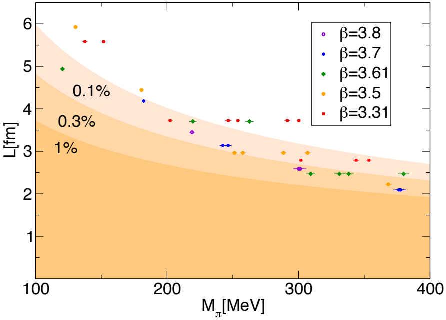
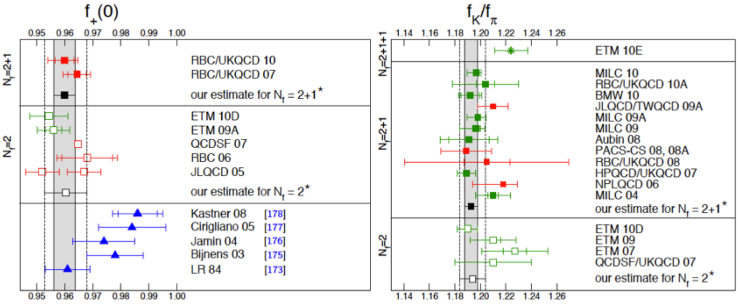


Figure 1: *Summary of our simulation points. The pion masses and the spatial sizes of the lattices are shown for our five lattice spacings. The percentage labels indicate regions, in which the expected finite volume effect [3] on M_π is larger than 1%, 0.3% and 0.1%, respectively. In our runs this effect is smaller than about 0.5%, but we still correct for this tiny effect.*

- from the previous slide we learn that light meson's observable should be under control now!
- chiral extrapolations are no more a source of concern in 2011 (not only BMW collaboration,. . .)
- . . . at least if one is spending his own budget for simulating big volumes

F_K/F_π & $F_+^{K\pi}(0)$ summary from FLAG

G.Colangelo et al. arXiv:1011.4408



$$F_+^{K\pi}(0) = 0.956(3)(4) \qquad \sim 0.5\% \qquad\qquad \frac{F_K}{F_\pi} = 1.193(5) \qquad \sim 0.5\%$$

predicted 2011 $\sim 0.4\%$

- error prediction for 2015 is $\sim 0.1\%$... can we reach this goal?
- a step backward: are these error estimates reliable? i.e. can we trust our predictions?
- within the lattice community we could discuss all the life about that, but...

F_K/F_π & $F_+^{K\pi}(q^2)$ can be measured (within SM)

we do have a lot of precise experimental measurements in the quark flavour sector of the standard model that, combined with CKM unitarity (first row), allow us to measure hadronic matrix elements

a simple example from FLAVIAnet kaon working group M.Antonelli et al. Eur.Phys.J.C69

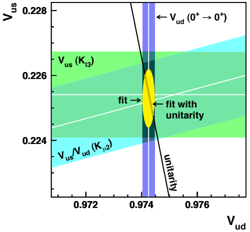
$$\begin{cases} \left|\frac{V_{us}F_K}{V_{ud}F_\pi}\right| = 0.27599(59) \\ \\ \left|V_{us}F_+^{K\pi}(0)\right| = 0.21661(47) \end{cases} \qquad \begin{cases} |V_{ud}|^2 + |V_{us}|^2 = 1 \\ \\ |V_{ud}| = 0.97425(22) \end{cases}$$

where $|V_{ud}|$ comes by combining 20 super-allowed nuclear β-decays and $|V_{ub}|$ has been neglected because smaller than the uncertainty on the other terms, combine to give

$$|V_{us}| = 0.22544(95)$$

$$F_+^{K\pi}(0) = 0.9608(46) \qquad F_+^{K\pi}(0)\Big|_{lattice} = 0.956(3)(4)$$

$$\frac{F_K}{F_\pi} = 1.1927(59) \qquad \frac{F_K}{F_\pi}\Big|_{lattice} = 1.193(5)$$



F_K/F_π & $F_+^{K\pi}(q^2)$ reducing the error

there are two sources of isospin breaking effects,

$$\underbrace{m_u \neq m_d}_{QCD} \qquad\qquad \underbrace{q_u \neq q_d}_{QED}$$

in the particular and (lucky) case of these observables, the correction to the isospin symmetric limit due to the difference of the up and down quark masses (QCD) can be estimated in *chiral perturbation theory*,

$$\begin{cases} F_+^{K\pi}(0) = 0.956(3)(4) \qquad \sim 0.5\% \\ \\ \left(\dfrac{F_+^{K^+\pi^0}(q^2)}{F_+^{K^0\pi^-}(q^2)} - 1\right)_{QCD} = 0.029(4) \end{cases}$$

A. Kastner, H. Neufeld Eur.Phys.J.C57 (2008)

$$\begin{cases} \dfrac{F_K}{F_\pi} = 1.193(5) \qquad \sim 0.5\% \\ \\ \left(\dfrac{F_{K^+}/F_{\pi^+}}{F_K/F_\pi} - 1\right)_{QCD} = -0.0022(6) \end{cases}$$

V. Cirigliano, H. Neufeld arXiv:1102.0563

reducing the error on these quantities without taking into account isospin breaking is useless. . .

QCD isospin breaking on the lattice

RM123 collaboration

- the idea is to calculate QCD isospin corrections at first order in $m_d - m_u$:

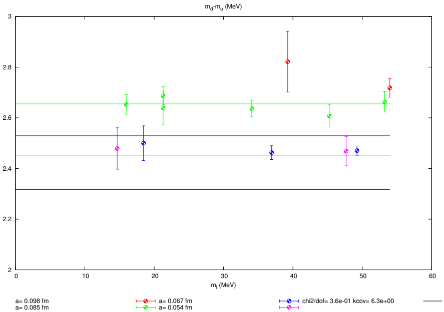
$$
\begin{aligned}
S_f &= \bar{u}\,(D[U] + m_u)\,u + \bar{d}\,(D[U] + m_d)\,d \\
&= \underbrace{\bar{u}\,(D[U] + \bar{m})\,u + \bar{d}\,(D[U] + \bar{m})\,d}_{S_f^0} - \underbrace{(m_d - m_u)\,\frac{\bar{u}u - \bar{d}d}{2}}_{\Delta m\, S^3}
\end{aligned}
$$

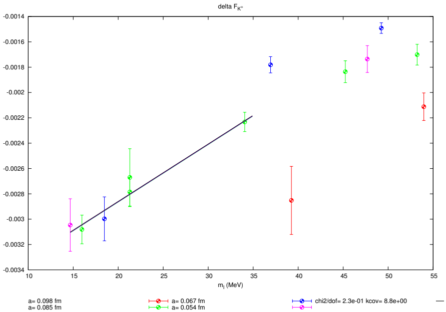
- the calculation of an observable proceeds as follows

$$
\begin{aligned}
\langle \mathcal{O} \rangle + \Delta\langle \mathcal{O} \rangle &= \frac{\int DU\, e^{-S_g[U]-S_f[U]}\ \mathcal{O}}{\int DU\, e^{-S_g[U]-S_f[U]}} = \frac{\int DU\, e^{-S_g[U]-S_f^0[U]}\ (1 + \Delta m S^3)\ \mathcal{O}}{\int DU\, e^{-S_g[U]-S_f^0[U]}\ (1 + \Delta m S^3)} \\
&= \langle \mathcal{O} \rangle + \Delta m \langle S^3\ \mathcal{O} \rangle - \underbrace{\Delta m \langle S^3 \rangle}_{=0}
\end{aligned}
$$

QCD isospin breaking on the lattice

RM123 collaboration, PRELIMINARY!





taking as input

$$\Delta M_K = M_{K^0} - M_{K^+} - \Delta M_K^{QED} = -6.0(6)\ MeV$$

we get

$$(m_d - m_u)^{\bar{M}S,2GeV} = 2.32(11)(22)\ MeV$$

$$\left(\frac{F_{K^+}/F_{\pi^+}}{F_K/F_\pi} - 1 \right)_{QCD} = -0.0038(3)$$

to be compared with the χ-pt estimate $-0.0022(6)$

we have data also for $F_+^{K\pi}(q^2)$ but are too preliminary to be discussed here.

B_K summary from FLAG

G.Colangelo et al. arXiv:1011.4408

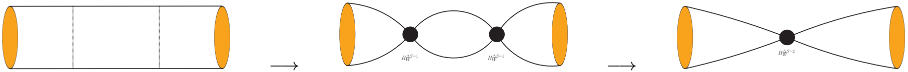
Collaboration	Ref.	N_f	publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	running	B_K	$\hat{B}_K$
Kim 09	[252]	2+1	C	★	●	●	■	●	0.512(14)(34)	0.701(19)(47)
Aubin 09	[240]	2+1	A	●	★□	●	★	●	0.527(6)(21)	0.724(8)(29)
RBC/UKQCD 09	[253]	2+1	C	●	●	★	★	●	0.537(19)	0.737(26)
RBC/UKQCD 07A, 08	[84, 254]	2+1	A	■	●	★	★	●	0.524(10)(28)	0.720(13)(37)
HPQCD/UKQCD 06	[255]	2+1	A	■	●*	★	■	●	0.618(18)(135)	0.83(18)
ETM 09D	[256]	2	C	★	●	●	★	●	0.52(2)(2)	0.73(3)(3)
JLQCD 08	[250]	2	A	■	●	■	★	●	0.537(4)(40)	0.758(6)(71)
RBC 04	[257]	2	A	■	■	■†	★	●	0.495(18)	0.699(25)
UKQCD 04	[258]	2	A	■	■	■†	■	●	0.49(13)	0.69(18)

the average is obtained by considering $n_f = 2 + 1$ results only (no debate!) and is

$$B_K(2GeV) = 0.527(6)(21) \qquad \hat{B}_K = 0.724(8)(29) \ \sim 4\% \qquad \text{predicted 2011} \ \sim 3\%$$

the error is bigger than 1% because of the systematics due to the renormalization of the four fermion operator is $\sim 3\%$

can we do better?



B_K parametrizes the mixing of the neutral Kaons in the effective theory in which both the W bosons and the up-type quarks have been integrated out,

$$B_K(\mu) = \frac{\langle \bar{K} | \, H_W^{\Delta S=2}(\mu) \, | K \rangle}{\frac{8}{3} F_K^2 M_K^2}$$

in order to be used in ϵ_K formula, the figures in the previous slides have to be corrected for a factor parametrizing long distance contributions estimated phenomenologically

$$\hat{B}_K = \kappa_\epsilon \, \hat{B}_K^{lattice} \qquad\qquad \kappa_\epsilon \simeq 0.92$$

A.Buras, D. Guadagnoli Phys.Rev. D78 (2008)

in order to do better on this process, we should be able to make a step backward and compute on the lattice the long distance contributions,

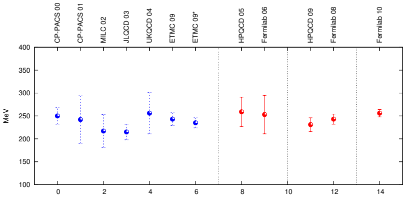
$$\langle \bar{K} | \, \mathrm{T} \left\{ \int d^4x \, H_W^{\Delta S=1}(x;\mu) \, H_W^{\Delta S=1}(0;\mu) \right\} | K \rangle$$

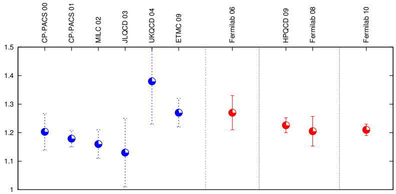
to this end, we should be able to make sense of the previous quantity in euclidean space

G.Isidori, G.Martinelli, P.Turchetti Phys.Lett. B633 (2006)

N. Crist arXiv:1012.6034

F_B & F_{B_s} averages





as a conservative estimate of the error, one can average "uncorrelated" $N_f = 2+1$ results getting

$$F_B^{N_f=2+1} = 205(12)\ MeV \qquad \sim 6\% \qquad \text{predicted 2011} \sim 2.5 \div 4\%$$

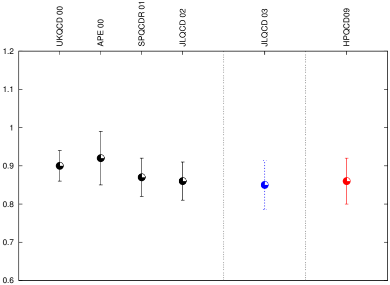
$$F_{B_s}^{N_f=2+1} = 250(12)\ MeV \qquad \sim 5\% \qquad \text{predicted 2011} \sim 3 \div 4\%$$

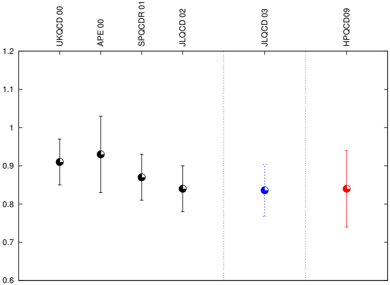
$$\frac{F_{B_s}}{F_B}^{N_f=2+1} = 1.215(19) \qquad \sim 1.5\%$$

these are almost the same figures shown by Lubicz in 2009 (updates by the same lattice collaborations)

the true question is: are these reasonable estimates?

B_B & B_{B_s} averages





a single $N_f = 2+1$ calculation, that combines with F_{B_q} to give

$$F_{B_s}\sqrt{\hat{B}_{B_s}}^{N_f=2+1} = 233(14)\ MeV \qquad \sim 6\% \qquad \text{predicted 2011} \ \sim 3 \div 4\%$$

$$\xi_B^{N_f=2+1} = 1.237(32) \qquad \sim 2.5\% \qquad \text{predicted 2011} \ \sim 1.5 \div 2\%$$

again, these are almost the same figures shown by Lubicz in 2009 (updates by the same lattice collaborations)

the true question is: are these reasonable estimates?

we usually spend all our budget for big volumes

by simulating b-quarks on the same volumes that we use to extract light meson's physics we have to extrapolate in $1/m_h$, (linear extrapolation from m_h and $\sqrt{2}m_h$)

$$\mathcal{O}^{phys} = \mathcal{O}^{latt} \left\{ 1 + b_1 \frac{\Lambda_{QCD}}{m_h} + b_2 \left(\frac{\Lambda_{QCD}}{m_h} \right)^2 + \dots \right\} \rightarrow \frac{\Delta O}{O} = \frac{b_2}{2} \left(\frac{\Lambda_{QCD}}{m_h} \right)^2 \sim 2 \div 3\%$$

$$\rightarrow \frac{\Delta O_B}{O_B} \propto \sqrt{a_n^2 \left(\frac{1}{\Lambda_{QCD} L} \right)^{2n} + b_2^2 \left(\frac{\Lambda_{QCD}}{m_h} \right)^4 + c_3^2 (am_h)^6} \sim 3 \div 4\%$$

this can be considered a rough estimate of the bigger errors on B mesons's observables

$N_t \times N_s$	$Pflops \times y$	scale (GeV)	a (fm)	L (fm)
96×48	10^{-3}	0.5	0.069	3 fm
96×48	10^{-3}	2.0	0.017	0.8 fm
96×48	10^{-3}	4.0	0.009	0.4 fm
360×180	1	0.5	0.069	12 fm
360×180	1	2.0	0.017	3 fm
360×180	1	4.0	0.009	1.5 fm

in case of b-physics it (may be) is convenient to change strategy and, given our budget and the scale we want to "accommodate" eventually to do finite volume calculations

step scaling method

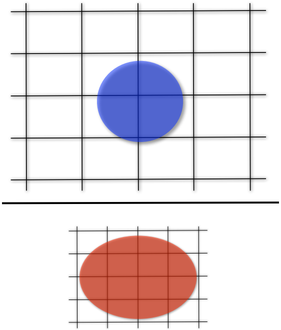
(Guagnelli, Palombi, Petronzio, N.T. Phys.Lett.B546:237,2002)

$$\mathcal{O}(m_b, m_l) = \mathcal{O}(m_b, m_l; L_0) \underbrace{\frac{\mathcal{O}(m_b, m_l; 2L_0)}{\mathcal{O}(m_b, m_l; L_0)}}_{\sigma(m_b, m_l; L_0)} \frac{\mathcal{O}(m_b, m_l; 4L_0)}{\mathcal{O}(m_b, m_l; 2L_0)} \cdots$$

- step scaling functions, the σ's, have to be calculated at lower values of the high energy scale

$$\mathcal{O}(m_b, m_l; L_0) \leftarrow m_b = m_b^{phys}$$

$$\sigma(m_b, m_l; nL_0) \leftarrow m_b \leq \frac{m_b^{phys}}{n}$$



- but extrapolating the step scaling functions is much easier than extrapolating the observable itself

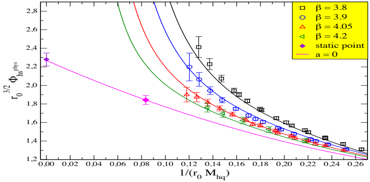
$$\mathcal{O}(m_b, m_l; L) = \mathcal{O}^0(m_l; L) \left[1 + \frac{\mathcal{O}^1(m_l; L)}{m_b} \right]$$

$$\sigma(m_b, m_l; L) = \frac{\mathcal{O}^0(m_l; 2L)}{\mathcal{O}^0(m_l; L)} \left[1 + \frac{\mathcal{O}^1(m_l; 2L) - \mathcal{O}^1(m_l; L)}{m_b} \right]$$

extrapolating $\mathcal{O}$ vs extrapolating finite volume effects

let's take the simplest example, $\Phi_{B_s} = f_{B_s}\sqrt{M_{B_q}}$

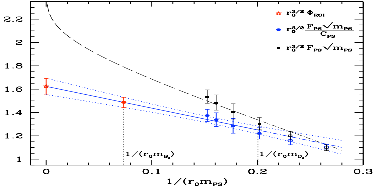
B.Blossier et al. PoS LAT2009 151



the standard approach to b-physics consists in:

- making simulations at "not so heavy" quark masses ($m_h \sim m_c$)
- extrapolating at the physical point ($m_h^{phys} = m_b$)
- constraining extrapolations with HQET (possibly non-perturbatively renormalized and matched)

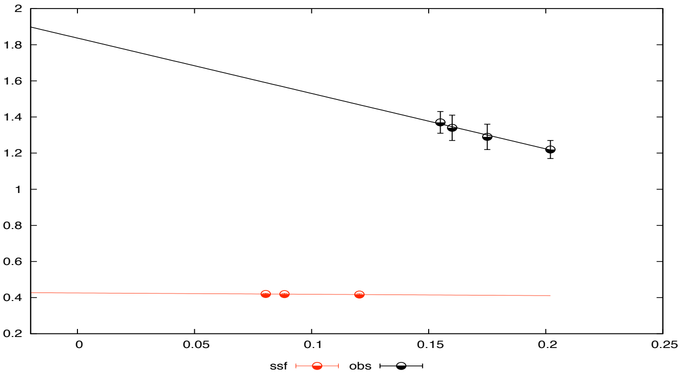
J. Heitger and R. Sommer JHEP 0402:022,2004
M. Della Morte et al. JHEP 0802:07,2008



$$\frac{\Phi_{B_q}}{C_{PS}} = f_q^0\left[1 + \frac{f_q^1}{m_b} + \dots\right]$$

extrapolating $\mathcal{O}$ vs extrapolating finite volume effects

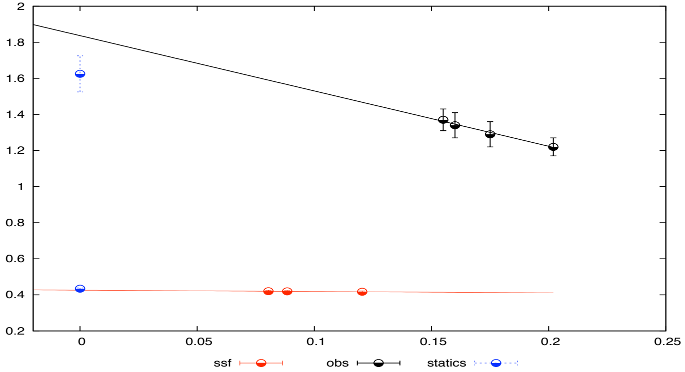
let's take the simplest example, $\Phi_{B_s} = f_{B_s}\sqrt{M_{B_q}}$



G.M.de Divitiis, M.Guagnelli, F.Palombi, R.Petronzio, N.T. Nucl.Phys.B672:372-386,2003

extrapolating $\mathcal{O}$ vs extrapolating finite volume effects

let's take the simplest example, $\Phi_{B_s} = f_{B_s}\sqrt{M_{B_q}}$

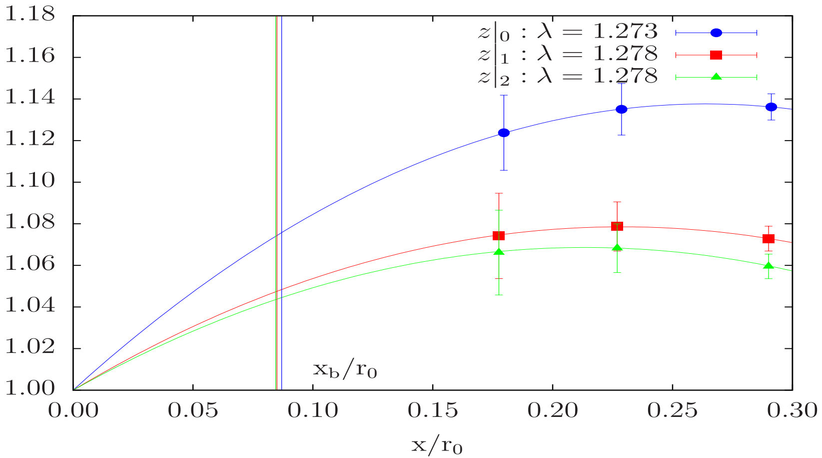


G.M.de Divitiis, M.Guagnelli, F.Palombi, R.Petronzio, N.T. Nucl.Phys.B672:372-386,2003
D.Guazzini, R.Sommer, N.T. JHEP 0801:076 (2008)

similar ideas have been developed in

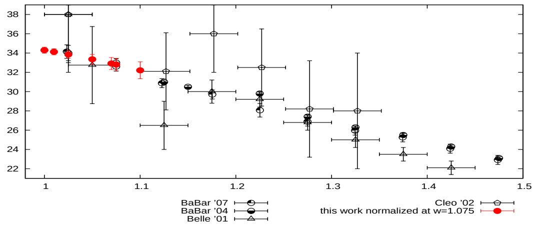
B.Blossier et al. JHEP 1004:049 (2010)

one considers ratios of observables at fixed large volume but at different values of the heavy quark masses in such a way that the static limit is exactly known:

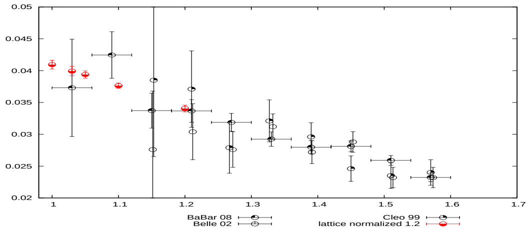


$B \to D^{(\star)}\ell\nu$ at $\omega > 1$

de Divitiis,Petronzio,N.T. Nucl.Phys.B807:373,2009
de Divitiis,Molinaro,Petronzio,N.T. Phys.Lett.B655:45,2007



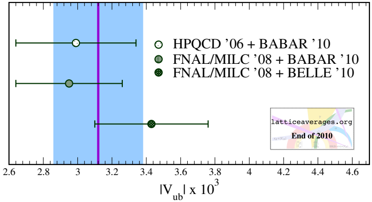
$$V_{cb}(@w = 1.075) \;=\; 37.4(8)(5) \times 10^{-3}$$

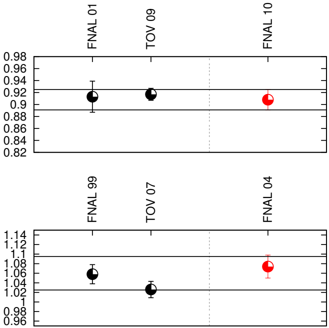


$$V_{cb}(@w = 1.2) \;=\; 38.4(9)(42) \times 10^{-3}$$

$B \to D^{(\star)}\ell\nu$ & $B \to \pi\ell\nu$ at $\omega = 1$

Laiho, Lunghi, Van de Water 2+1 Flavor Lattice QCD Averages





$$|V_{ub}| \times 10^{-3} = 3.12(26) \quad \sim \quad 8\%$$

predicted 2011 $\sim$ 5%

nothing changed on the lattice side since 2009

$$F(1) = 0.908(17) \quad \sim \quad 1.8\%$$
$$G(1) = 1.060(35) \quad \sim \quad 3\%$$

predicted 2011 $\sim$ 1.2%

same analysis of Lubicz, Tarantino, arXiv:0807.4605 except for the updated value of $F(1)$ by Fermilab/MILC collaboration

lattice QCD forecasts

N. Tantalo @ XVII SuperB Workshop La Biodola 2011

hadronic mel	2009 error	2011 prediction	2011 error	2015 error
$F_+^{K\pi}(q^2=0)$	0.5%	0.4%	0.5%	$< 0.1\%$
$\hat{B}_K$	5%	3%	4%	1%
F_B	5%	$2.5\% - 4\%$	6%	$1\% - 1.5\%$
$F_{B_S}\sqrt{B_{B_S}}$	5%	$3\% - 4\%$	6%	$1\% - 1.5\%$
ξ_B	2%	$1.5\% - 2\%$	2.5%	$0.5\% - 0.8\%$
$F^{BD^{(\star)}}(\omega=1)$	2%	1.2%	1.8%	0.5%
$F_+^{B\pi}(q^2=0)$	11%	$4\% - 5\%$	8%	$2\% - 3\%$
$T_1^{B\rho(K^\star)}(q^2=0)$	13%	$- - -$	$- - -$	$3\% - 4\%$

- goals have been reached for light meson's observables
- errors for B meson's quantities are oscillating (big efforts form HPQCD and FNAL/MILC collaborations)
- reducing errors in B physics requires dedicated efforts and, in my opinion, change of strategies...
- sorry, no more time for weather forecasts for 2015!