The parity-violating asymmetry in the $^3\mathrm{He}(\vec{n},p)^3\mathrm{H}$ reaction

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Outline

1. Introduction
2. PV potential from EFT
3. $p-p$ longitudinal symmetry
4. $\vec{n} - {}^3\mathrm{He} \to p - {}^3\mathrm{H}$ longitudinal asymmetry

Collaborators

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PV Effects in Few-Nucleon Systems

- PV in nuclei: extraction of the fundamental PV coupling constants
  - πNN coupling constants h_π^1
  - [Ramsey-Musolf & Page, 2006]
- $A = 2, \ldots, 4$ (and more): accurate calculation of bound/scattering states
  - "Realistic" models of NN & 3N interactions
  - Precise numerical techniques for solving the Schroedinger equation
  - PV effects $\sim 10^{-7}$: experimental challenge!
- PV observables
  - $\vec{p}p$ and ${}^3\mathrm{He}(\vec{n},p){}^3\mathrm{H}$ longitudinal asymmetry
  - $\vec{n} + p \rightarrow d + \gamma$ "NPDgamma"
  - neutron and proton spin rotation: $\vec{n}H$, $\vec{n}{}^4\mathrm{He}$, $\vec{p}{}^4\mathrm{He}$

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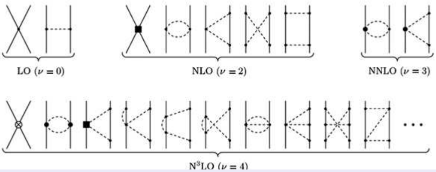
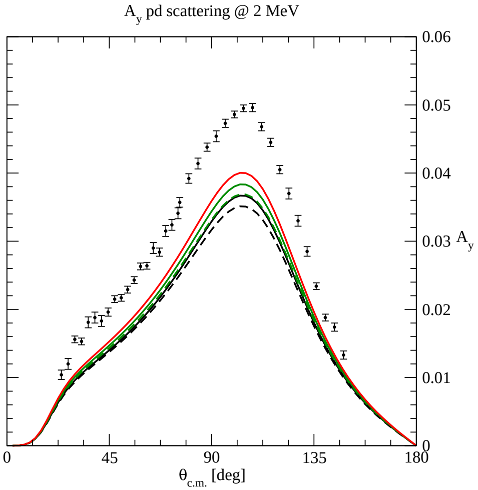
NN interaction

- Effective field theory: $N - \pi$ interactions "dictated" by chiral symmetry [Weinberg (1990), Bernard, Kaiser, & Meissner, (1995); Ordonéz, Ray, & U. van Kolck (1996), . . .]
- NN interaction:
  - N3LO-Jülich [Epelbaum and Coll, 1998-2006]
  - N3LO-Idaho (I-N3LO) [Entem & Machleidt, 2003]



- 3N interaction
  - J-N2LO [Epelbaum *et al*, 2002]
  - N-N2LO [Navratil, 2007]
  - Under progress: N3LO, Δ, CSB, . . .

A_y "puzzle"



A_y pd scattering @ 2 MeV
0.06
0.05
0.04
0.03
A_y
0.02
0.01
0
0
45
90
135
180
θ_c.m. [deg]

Theoretical methods

Kamada *et al*, PRC **64**, 044001 (2001)

Faddeev-Yakubovsky equations

Solved in configuration or momentum space

- Bochum-Cracow-Jülich, Lisboa, Grenoble, LANL, . . .

GFMC

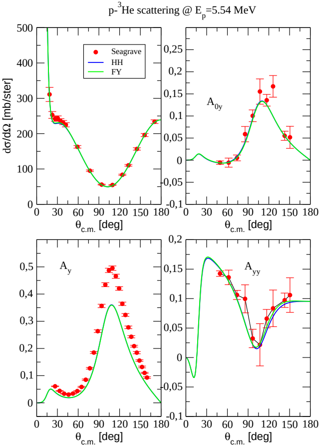
- Argonne-Los Alamos

Variational methods

1. Gaussian basis [Kamimura, Varga]
2. HH [Pisa, Trento-Jerulasem]
3. HO (NCSM) [Navratil and Coll.]

$A = 4$ scattering

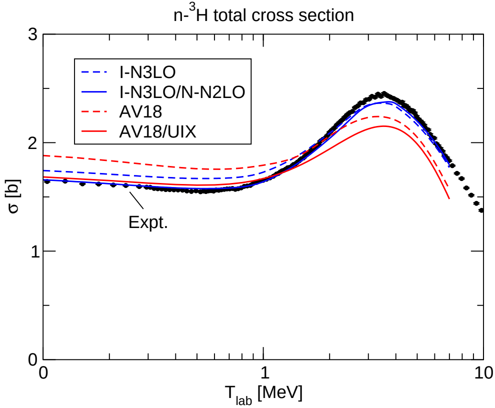
Comparison between FY & HH results
I-N3LO interaction



$A = 4$ scattering problem

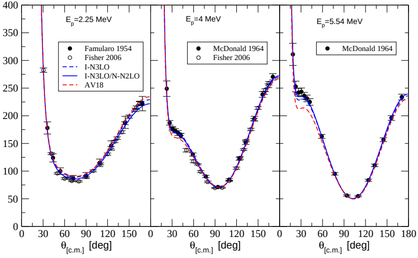
- First accurate solutions with a realistic NN interaction
  - Deltuva & Fonseca, 2007
  - Lazauskas & Carbonell, 2009
- Accurate solution with the HH method (2010)
- inclusion of the 3N interaction

$n - {}^3\mathrm{H}$ total cross section

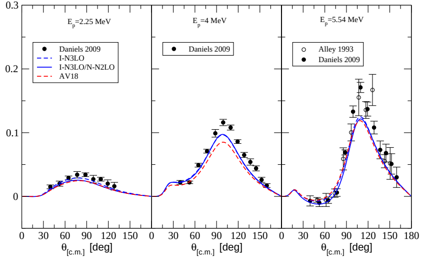


$p - {}^3\mathrm{He}$ scattering

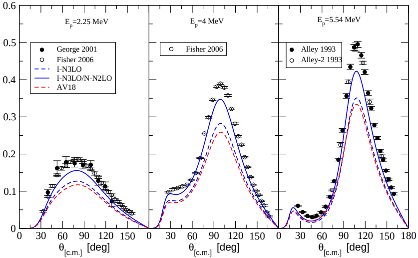
cross section



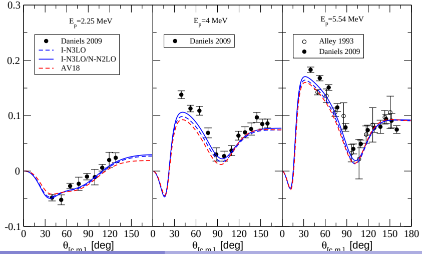
A_{0y} : $p + {}^3\vec{\mathrm{He}} \to p + {}^3\mathrm{He}$



A_y : $\vec{p} + {}^3\mathrm{He} \to p + {}^3\mathrm{He}$



A_{yy} : $\vec{p} + {}^3\vec{\mathrm{He}} \to p + {}^3\mathrm{He}$

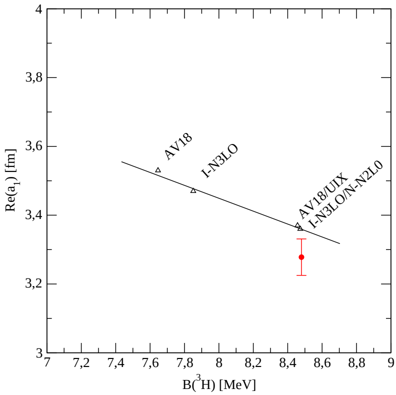


$n - {}^3\mathrm{He}$ scattering lengths [fm]

Int.	Method	a_0 (fm)	a_1 (fm)
AV18	HH	$7.69 - i5.70$	$3.56 - i0.0077$
	RGM	$7.79 - i4.98$	$3.47 - i0.0066$
	FY	$7.71 - i5.25$	$3.43 - i0.0082$
AV18/UIX	HH	$7.89 - i3.44$	$3.39 - i0.0059$
	RGM	$7.63 - i4.05$	$3.31 - i0.0051$
I-N3LO	HH	$7.57 - i4.97$	$3.46 - i0.0048$
	FY		$3.56 - i0.0070$
	AGS	$7.82 - i4.51$	$3.47 - i0.0068$
I-N3LO/N-N2LO	HH	$7.61 - i4.32$	$3.37 - i0.0042$
Exp.[ILL-1]		$7.370(58) - i4.448(5)$	$3.278(53) - i0.001(2)$
Exp.[ILL-2]		7.46(2)	3.36(1)
Exp.[NIST]		7.57(3)	3.48(2)

RGM: Hofmann & Hale, PRC **77**, 044002 (2008)
FY: Lazauskas, `nucl-th arXiv:0905.3119`
ILL-1: Zimmer *et al.*, EPJA **4**, 1 (2002)
ILL-2: Ketter *et al.*, EPJA **27**, 243 (2006)
NIST: Huffman *et al.*, PRC **70**, 014004 (2004)

Triplet $n - {}^3\mathrm{He}$ scattering lenght vs. $B({}^3\mathrm{H})$



Also calculated by Deltuva & Fonseca, (2007)

PV Lagrangian

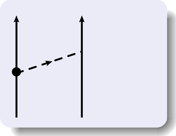
- $\mathcal{L}^{PV}_{\rm Hadrons}$ constructed so that it violates chiral symmetry as the "standard model" Lagrangian $\mathcal{L}^{\rm weak}_q$ ($\mathcal{P}$-odd but $\mathcal{CP}$ even terms)
- "Building blocks": $u = \exp(i\vec{\tau}\cdot\vec{\pi}/2f_\pi)$ and related quantities (u_μ, $X^a_\pm$, ...)
  - $u_\mu = i\left(u\partial_\mu u^\dagger - u^\dagger\partial_\mu u\right)$
  - $X^a_L = u\tau_a u^\dagger$, $X^a_R = u^\dagger\tau_a u$, $X^a_\pm = X^a_L \pm X^a_R$
- Expression up to one four-gradient [Kaplan & Savage, 1992]

$$
\begin{aligned}
\mathcal{L}^{PV,-1}_{\Delta T=1} &= -\frac{h^1_\pi f_\pi}{2\sqrt{2}}\overline{N}X^3_- N \sim \frac{h^1_\pi}{\sqrt{2}}\overline{N}(\vec{\tau}\times\vec{\pi})_z N \\
\mathcal{L}^{PV,0}_{\Delta T=0} &= -h^0_V \overline{N} u_\mu \gamma^\mu N \\
\mathcal{L}^{PV,0}_{\Delta T=1} &= +\frac{h^1_V}{2}\overline{N}\gamma^\mu N Tr(u_\mu X^3_+) - \frac{h^1_A}{2}\overline{N}\gamma^\mu\gamma^5 N Tr(u_\mu X^3_-) \\
\mathcal{L}^{PV,0}_{\Delta T=2} &= h^2_V I^{ab}\overline{N}(X^a_R u_\mu X^b_R + X^a_L u_\mu X^b_L)\gamma^\mu N \\
&\quad -\frac{h^2_A}{2} I^{ab}\overline{N}(X^a_R u_\mu X^b_R - X^a_L u_\mu X^b_L)\gamma^\mu\gamma^5 N
\end{aligned}
$$

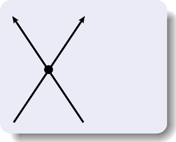
+ 5 contact terms of order Q [Girlanda, 2008]   LEC'c $\sim G_F f^2_\pi \approx 10^{-7}$

PV potential at order $\mathcal{O}(Q)$

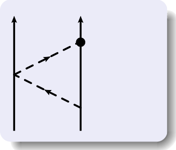
$$\boldsymbol{k} = \boldsymbol{p}_1' - \boldsymbol{p}_1 = -(\boldsymbol{p}_2' - \boldsymbol{p}_2) \qquad \boldsymbol{Q} = (\boldsymbol{p}_1' + \boldsymbol{p}_1 - \boldsymbol{p}_2' - \boldsymbol{p}_2)/2$$



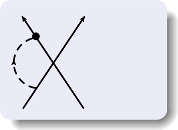
$$-\frac{g_A h_\pi^1}{2\sqrt{2} f_\pi} i (\vec{\tau}_1 \times \vec{\tau}_2)_z \frac{(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\cdot \boldsymbol{k}}{k^2 + m_\pi^2} \qquad (\sim Q^{-1})$$



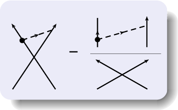
$$\frac{C_1}{4\pi f_\pi m_\pi^2} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\cdot \boldsymbol{Q} + \frac{C_2}{4\pi f_\pi m_\pi^2} i (\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2)\cdot \boldsymbol{k} + \ldots \qquad (\sim Q^1)$$



$$+\frac{g_A h_\pi^1}{2\sqrt{2} f_\pi} \frac{m_\pi^2}{(4\pi f_\pi)^2} i (\vec{\tau}_1 \times \vec{\tau}_2)_z (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\cdot \boldsymbol{k} L(k) \qquad (\sim Q^1)$$



$$\sim \int d^3\boldsymbol{q} f(q^2)\boldsymbol{q}\cdot\sigma \to 0$$



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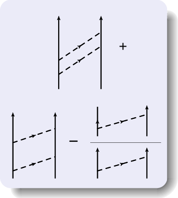
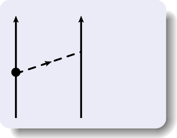


$$+\frac{g_A h_\pi^1}{2\sqrt{2}f_\pi}\frac{g_A^2 m_\pi^2}{(4\pi f_\pi)^2}\Big(4(\tau_1^z+\tau_2^z)_z(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{k}L(k)$$
$$+i(\vec{\tau}_1\times\vec{\tau}_2)_z(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{k}\Big[H(k)-3L(k)\Big]\Big) \qquad (\sim Q^1)$$

$$s=\sqrt{k^2+4m_\pi^2} \qquad L(k)=\frac{1}{2}\frac{s}{k}\log\left(\frac{s+k}{s-k}\right) \qquad H(k)=\frac{s^2-k^2}{s^2}L(k)$$

Further terms. . .



The OPE diagram at LO is of order $\sim Q^{-1}$. We need to consider also high order πNN vertices + relativistic corrections

[Zhu *et al.*, 2005] include an addidional term in the potential, giving a correction $\mathcal{O}(Q)$ to the OPE

$$\sim -\frac{k_\pi^1 g_A}{2\sqrt{2}} i(\vec{\tau}_1 \times \vec{\tau}_2)_z \frac{k^2\ (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)\cdot \boldsymbol{k}}{k^2 + m_\pi^2}$$

PV interactions with more than one four-gradients

Method: list all possible πNN vertices and then neglect the "linear dependent" ones

1. using EOM
2. performing a non-relativistic reduction of the "vertex functions" and searching for the independent operators order-by-order

Under progress PRELIMINARY: no independent πNN vertices at this order

Summary

$$V_{PV} = \underbrace{V_{\text{OPE}-\text{LO}} + V_{\text{OPE}-\text{NNLO}} + V_{\text{TPE}-\text{triangle}} + V_{\text{TPE}-\text{box}}}_{\sim h^1_\pi} + V_{CT}$$

$$\begin{aligned}
V_{CT} &= \frac{C_1}{4\pi f_\pi m_\pi^2}\boldsymbol{Q}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2) + \frac{C_2}{4\pi f_\pi m_\pi^2} i\boldsymbol{k}\cdot(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2) \\
&+ \frac{C_3}{4\pi f_\pi m_\pi^2}(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z i\boldsymbol{k}\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2) + \frac{C_4}{4\pi f_\pi m_\pi^2}(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2)_z\boldsymbol{Q}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2) \\
&+ \frac{C_5}{4\pi f_\pi m_\pi^2}\left[3(\boldsymbol{\tau}_1)_z(\boldsymbol{\tau}_2)_z - \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right]\boldsymbol{Q}\cdot(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2) \\
V_{\text{OPE}-\text{NNLO}} &= \left(\frac{g_A h^1_\pi}{8\sqrt{2} f_\pi m_\pi^2}\right)(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z \frac{1}{k^2+m_\pi^2}\times \\
&\left\{-4iQ^2\boldsymbol{k}\cdot(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2) + \boldsymbol{k}\cdot\boldsymbol{\sigma}_1\,(\boldsymbol{k}\times\boldsymbol{Q})\cdot\boldsymbol{\sigma}_2 + \boldsymbol{k}\cdot\boldsymbol{\sigma}_2\,(\boldsymbol{k}\times\boldsymbol{Q})\cdot\boldsymbol{\sigma}_1\right\}
\end{aligned}$$

additional parameter: cutoff Λ ($= 500 - 700$ MeV)

→ [Desplanques *et al.*, 2005], [Zhu *et al.*, 2005]

$p - p$ longitudinal symmetry at energy E

$$A_z^{pp}(\theta,E)=\frac{\sigma_{+1/2}(\theta,E)-\sigma_{-1/2}(\theta,E)}{\sigma_{+1/2}(\theta,E)+\sigma_{-1/2}(\theta,E)} \qquad \overline{A}_z^{pp}(E)=\frac{\int_{\theta_1\le\theta\le\theta_2} d\hat{r}\; A_z^{pp}(\theta,k)\sigma(\theta,E)}{\int_{\theta_1\le\theta\le\theta_2} d\hat{r}\;\sigma(\theta,E)}\,,$$

Experiments

- Bonn [Eversheim *et al.*, 1991] $\overline{A}_z(13.6\mathrm{MeV})=(-0.97\pm0.20)\times10^{-7}$
- PSI [Kistryn *et al.*, 1987] $\overline{A}_z(45\mathrm{MeV})=(-1.53\pm0.21)\times10^{-7}$
- TRIUMF [Berdozet *al.*, 2003] $\overline{A}_z(221\mathrm{MeV})=(+0.84\pm0.34)\times10^{-7}$

Isospin matrix elements

$$\langle T=1,T_z=+1|(\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2)_z|T=1,T_z=+1\rangle=0$$
$$\langle T=1,T_z=+1|(\boldsymbol{\tau}_1+\boldsymbol{\tau}_2)_z|T=1,T_z=+1\rangle=2$$
$$\langle T=1,T_z=+1|I_{ij}(\boldsymbol{\tau}_1)_i(\boldsymbol{\tau}_2)_j|T=1,T_z=+1\rangle=2$$

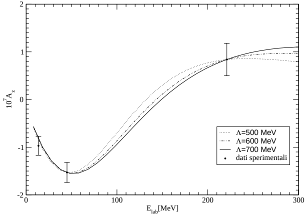
$$\overline{A}_z^{pp}(E)=a_0(E)h_\pi^1+a_1(E)C_1'+a_2(E)C_2 \qquad C_1'=C_1+2C_4+2C_5$$

Note: at low energies the observable is dominated by ${}^1S_0\leftrightarrow{}^3P_0$, then $a_i(E)\sim\sqrt{E}$

Fit of the LEC's

Reasonable range for $h_\pi^1 = 0 \div 12.0 \times 10^{-7}$, "best value" $h_\pi^1 = 4.56 \times 10^{-7}$
[Desplanques, Donogue, & Holstein, 1980]

Λ [MeV]	$C_1' \times 10^7$	$C_2 \times 10^7$	$C_1' \times 10^7$	$C_2 \times 10^7$	$C_1' \times 10^7$	$C_2 \times 10^7$
	$h_\pi^1 = 4.56 \times 10^{-7}$		$h_\pi^1 = 0$		$h_\pi^1 = 12.0 \times 10^{-7}$	
500	-0.97026	9.6319	-0.31731	3.59265	-1.9211	18.426
600	-0.86155	9.4336	$+0.01820$	3.66831	-2.1428	17.829
700	-1.15474	9.5106	-0.01067	4.42059	-2.8207	16.922

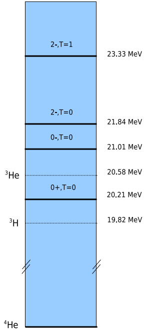


Comparing the EFT potential with the DDH potential (ρ and ω exchanges – resonance saturation) [Desplanques, Donogue, & Holstein, 1980]:

$h_\pi^1 = 4.56$, $C_1 \approx -1$, $C_2 \approx +10$ $C_3 \approx 0$ $C_4 \approx -1$ $C_5 \approx +1$

Longitudinal asymmetry in $\vec{n} + {}^3\mathrm{He} \rightarrow p + {}^3\mathrm{H}$

$$V_{PV} = h^1_\pi O_0^{(1)} + C_1 O_1^{(0)} + C_2 O_2^{(0)} + C_3 O_3^{(1)} + C_4 O_4^{(1)} + C_5 O_5^{(2)}$$



- 0^+ and 0^- scattering dominated by $T = 0$ resonances
- EFT:
  - π: $\sim h^1_\pi (\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_z$ isovector $\Delta T = 1$
  - contact terms 1 and 2: $\Delta T = 0$ ($C_1 \approx -1$, $C_2 \approx 10$)
  - contact terms 3 and 4: $\Delta T = 1$ ($C_3 \approx 0$, $C_4 \approx -1$)
  - contact term 5 : $\Delta T = 2$ ($C_5 \approx +1$)
- $\rightarrow$ dominant contribution from contact term 2

Contributing waves

- initial state ($n-{}^3\mathrm{He}$) $q \approx 0$: ${}^1S_0$, ${}^3S_1$
- final state ($p-{}^3\mathrm{H}$) $q = 0.165$ fm^{-1}:
  - $J=0$: ${}^1S_0$, ${}^3P_0$
  - $J=1$: ${}^3S_1 - {}^3D_1$, ${}^1P_1 - {}^3P_1$

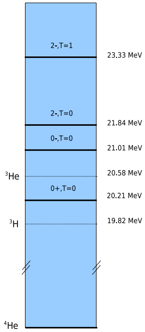
Neglecting ${}^3D_1$, we have to compute the matrix elements $T^J_{LS,L'S'} \equiv T^{ex}_{LS,L'S'}$

PC		PV	
${}^1S_0 \to {}^1S_0$	$T^0_{00,00}$	${}^1S_0 \to {}^3P_0$	$T^1_{00,11}$
${}^3S_1 \to {}^3S_1$	$T^0_{01,01}$	${}^3S_1 \to {}^1P_1$	$T^1_{01,10}$
		${}^3S_1 \to {}^3P_1$	$T^1_{01,11}$

- PC T-matrix elements + $\Psi^{J\pi}_{LS}$: using the KVP+HH method, starting from a NN+3N interaction model (neglecting the PV potential)
- PV T-matrix elements: $T^J_{0J,1S} = \langle \mathcal{T}\Psi^{J-}_{1S'} | V_{PV} | \Psi^{J+}_{0J} \rangle$ (Monte Carlo code by R. Schiavilla)

Results for the PV EFT potential (1) Preliminary

N3LO/N2LO PC potential



PC matrix elements

- ${}^1S_0$: $T^0_{00,00} = 1.0 - i5.0$ large ($T \sim \tan\delta \to \delta > \pi/2$)
- ${}^3S_1$: $T^1_{01,01} = -0.15 + i0.06$ small – no resonant behaviour

PV matrix elements

Transition	$T^J_{LS,L'S'}$	$O_0^{(\Delta T=1)}$	$O_2^{(\Delta T=0)}$
${}^1S_0 \to {}^3P_0$	$T^0_{00,11}$	$2.29 + i0.74$	$-3.75 - i19.5$
${}^3S_1 \to {}^1P_1$	$T^1_{01,10}$	$0.00 - i0.05$	$+0.04 + i0.17$
${}^3S_1 \to {}^3P_1$	$T^1_{01,11}$	$0.01 - i0.01$	$-0.02 + i0.44$

We found a significative contribution from the $\Delta T = 1$ operators
it originates from the difference between $n^3\mathrm{He}$ and $p^3\mathrm{H}$ relative wave function

Results for the PV EFT potential (2) Preliminary

$$V_{PV} = h_\pi^1 O_0^{(1)} + C_1 O_1^{(0)} + C_2 O_2^{(0)} + C_3 O_3^{(1)} + C_4 O_4^{(1)} + C_5 O_5^{(2)}$$

$$a_z = h_\pi^1 a_0 + C_1 a_1 + C_2 a_2 + C_3 a_3 + C_4 a_4 + C_5 a_5$$

Inter.	a_0	a_1	a_2	a_3	a_4	a_5
N3LO/N2LO	-0.153	$+0.027$	0.021	-0.111	-0.023	-0.002
	h_π^1	C_1	C_2	C_3	C_4	C_5
Param. (units 10^{-7})	4.56	-1	10	0	-1	$+1$
product (units 10^{-7})	-0.70	-0.027	$+0.21$	0.0	$+0.023$	-0.002
a_z			-0.50×10^{-7}			

$h_\pi^1 = 0$: $a_z = +0.20 \times 10^{-7}$, $h_\pi^1 = 12 \times 10^{-7}$: $a_z = -1.4 \times 10^{-7}$

Conclusions

Summary

- Re-derivation of the EFT PV potential at $\mathcal{O}(Q)$
  - TPE in agreement with Desplanques *et al.*, 2005
  - number of different LEC: 6 (h^1_π + 5 contact interactions)
- study of $\vec{p} - p$ longitudinal asymmetry (it allows to fix 2 LEC's)
- study of the $\vec{n} + {}^3\mathrm{He} \to p + {}^3\mathrm{H}$ longitudinal asymmetry
- sensitive to both h^1_π, C_2

Future work

- Complete the calculation (in progress)
  - convergence, Monte Carlo, etc.
  - other PC potentials (AV18/UIX, AV18/Illinois, $V_{\mathrm{low-k}}, \ldots$)
- Calculation of other PV observables
  - $n + p \to d + \gamma$: measurement in progress ORNL (h^1_π)
  - n spin rotation in ($n - p$, $n - d$ and $n - \alpha$)