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Review of Laser-Plasma Accelerator Scaling in Quasi-Linear and Blowout Regimes

European Advanced Accelerator Concepts Workshop, La Biodola, Isola d’Elba, 24-30 September 2017

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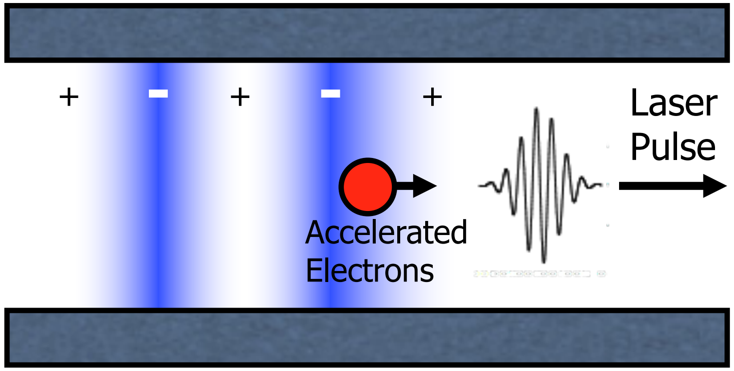
References on LWFA Scaling

- R.F. Hubbard et al., Phys. Rev. E 63, 036502 (2001)
  - Scalings for linear channel guided LWFA
- S. Gordienko and A. Pukhov, Phys. Plasmas 12, 043109 (2005)
  - General scaling law for strong blowout, similarity parameter $n/n_c a_0$
- W. Lu et al., Phys. Rev. ST/AB 10, 061301 (2007)
  - Phenomenological theory for blowout LWFA
- C.B. Schroeder et al., Phys. Rev. ST/AB 13, 101301 (2010)
  - Quasi-linear scalings relevant to a vision of a LWFA based collider
- I.V. Pogorelsky et al., Phys. Rev. ST/AB 19, 091001 (2016)
  - Roles for alternative (long) wavelength drivers

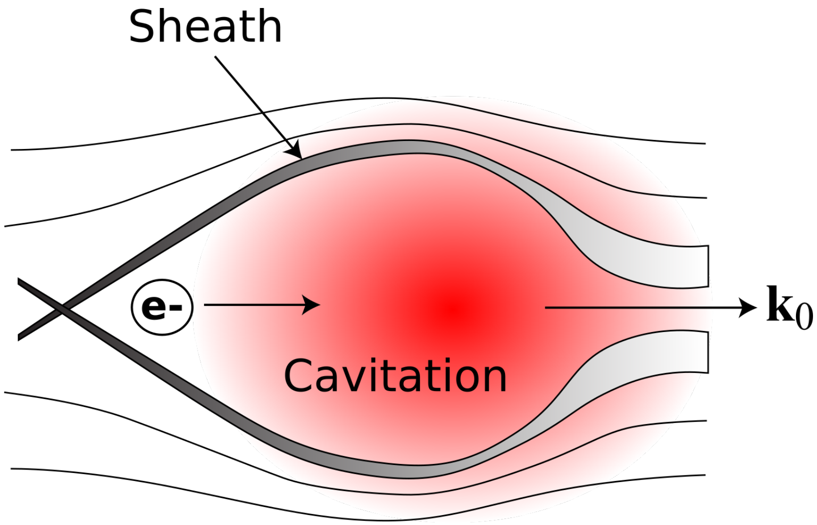
Chronological order. Not exhaustive.

Linear vs. Blowout Regime of LWFA

- Quasi-Linear: wakes given by linear theory, no self-trapping, no self-guiding, pump energy barely perturbed in a dephasing length
- Blowout: wakes highly nonlinear, electrons self-trapped and self-injected, laser self-guided, efficient pump depletion



Structure to impose guiding



Self-guided

This talk will not consider self-modulated or multi-pulse schemes

Limiting Processes in Laser Plasma Accelerators

- Energy Limitation
  - Depletion of laser pulse
  - Dephasing in plasma wave
  - Diffraction of laser pulse
- Charge Limitation
  - Beamloading of plasma waves
- Emittance Limitation
  - Injection mechanism (initial emittance)
  - Growth due to Coulomb scattering
- Energy Spread Limitation
  - Bunch length too long (spatio-temporal phase loading)
  - Space Charge (can be beneficial)
- There are further issues for collider application
  - Spin polarization, positrons, final focus issues

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We cannot hope to examine all of this physics, nor the many clever schemes to harness it. We will review only depletion and dephasing.

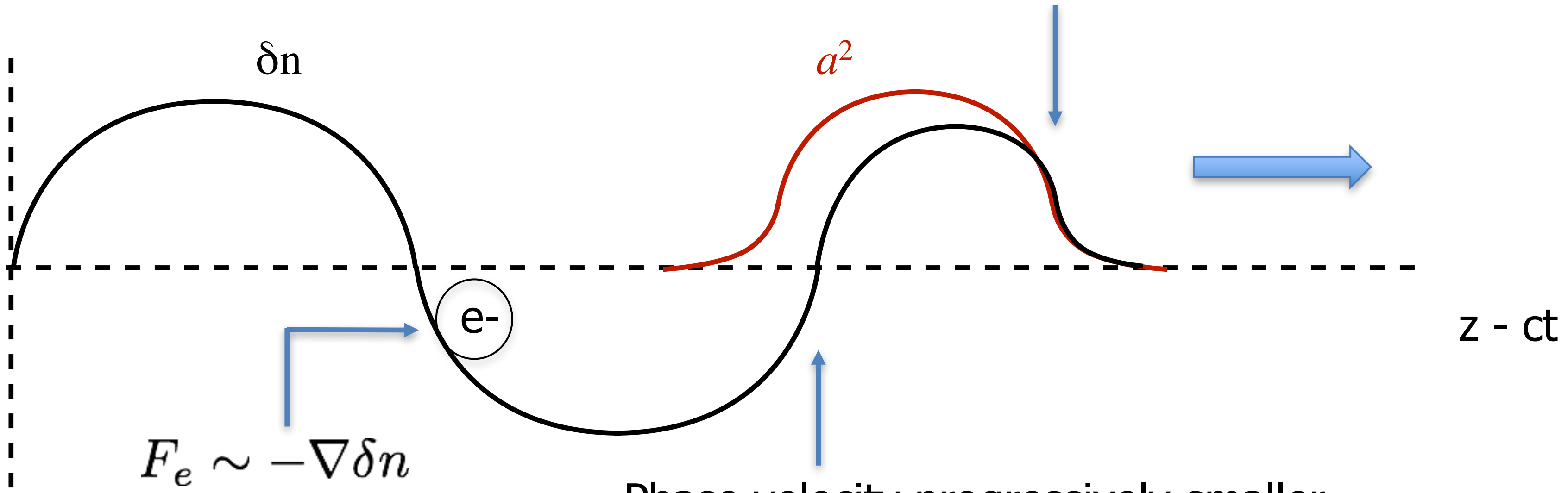
Depletion & Dephasing in the Quasi-linear Regime*

NRL PPD

$$\frac{v_\phi}{c} = 1 + \frac{1}{2}\frac{\omega_p^2}{\omega_0^2}\left(1 + \frac{\delta n}{n} - \frac{\langle a^2 \rangle}{2} - 2\frac{\delta\omega_0}{\omega_0}\right)$$

$$v_g \approx c^2/v_\phi$$

$$v_e > v_g \approx v_{\mathrm{wake}}$$



δn and a^2 terms cancel in front

$F_e \sim -\nabla \delta n$

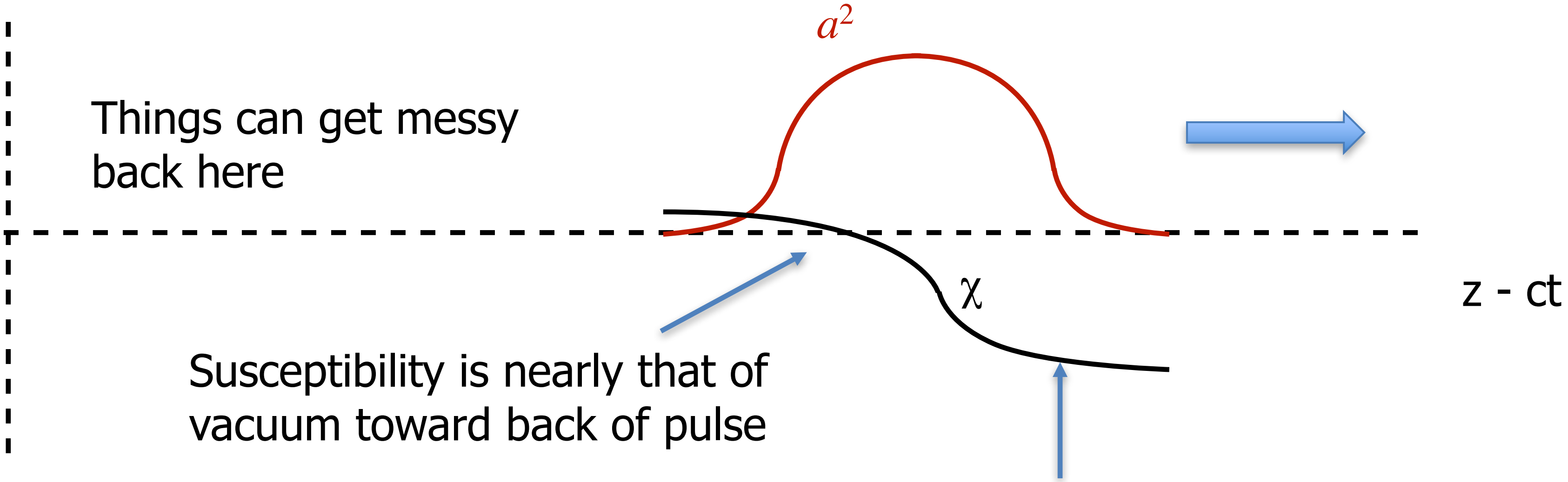
Roll downhill, but defocus when δn>0

Phase velocity progressively smaller moving toward back of pulse : photon deceleration and red-shifting

Pump depletion in blowout regime*

- Front etches away due to diffraction and photon deceleration
- Back is in cavitation region, so is not strongly perturbed

$$\chi = \frac{-n/n_c}{1 + e\phi/mc^2 - eA_z/mc^2}$$ (Quasistatic susceptibility)



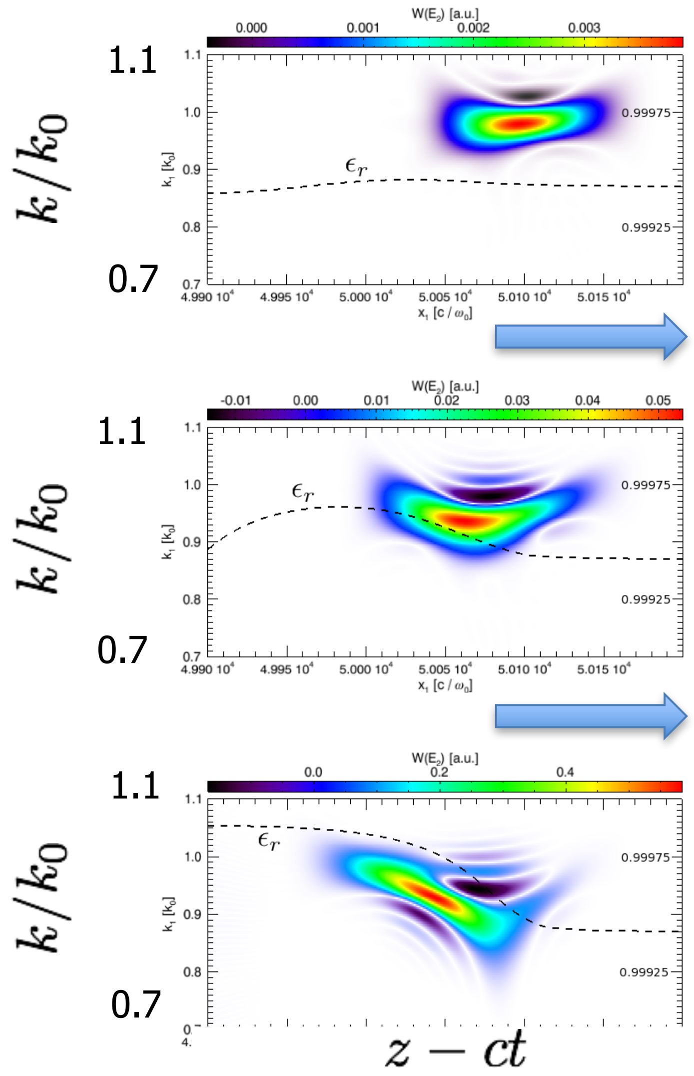
Things can get messy back here

Susceptibility is nearly that of vacuum toward back of pulse

Increasing susceptibility in time red-shifts photons

* Following W. Lu et al. , 2007

Examples of Pump Depletion from PIC Simulations*



Quasi-Linear (a_0=2)

Non-Linear (a_0=3)

Blowout (a_0=4.44)

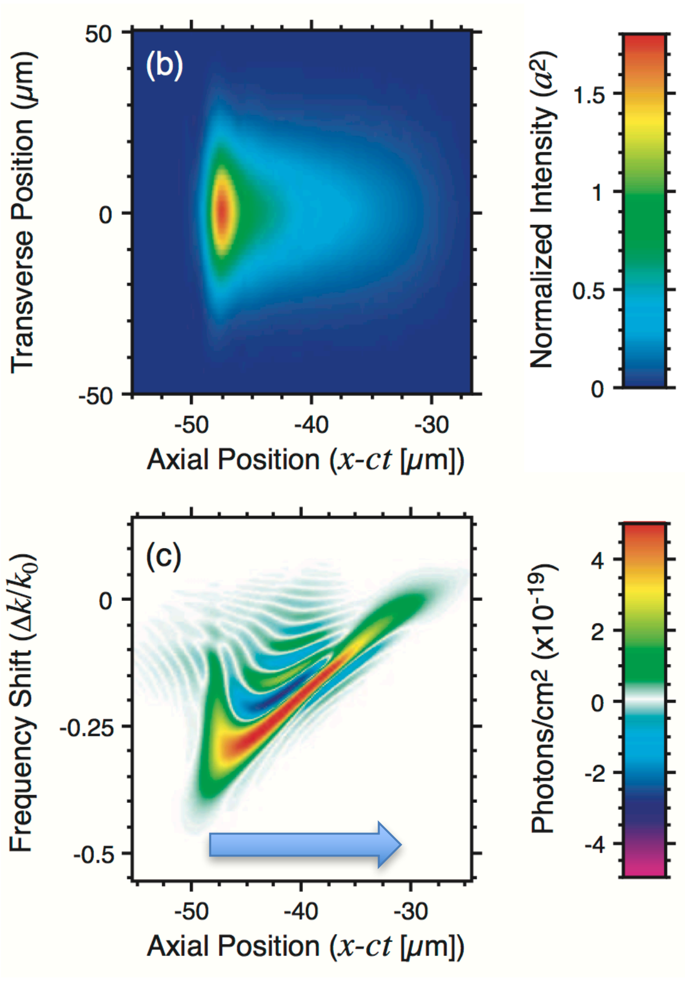
Numerical Method:
Quasi-3D OSIRIS

Pulse is etched backward.
This affects dephasing.

* A. Davidson, Ph.D. Dissertation (2016)

Deep Pump Depletion in Quasi-Linear Regime*

Can only be observed with channel guiding over distance of ~10 cm.



Ponderomotive potential builds in the pulse tail due to photon deceleration and GVD.
This affects dephasing.

Front of the pulse is essentially non-evolving, while middle and rear of pulse are red shifted.

Numerical method:
TurboWAVE PGC Fluid

* D.F. Gordon *et al.*, Phys. Rev. Lett. **90**, 215001 (2003)

List of Symbols

We will be displaying many simple equations. Symbols:

R = blowout radius

r_0 = laser spot size

n = electron density

n_c = critical density

r_e = classical electron radius

ΔE = LPA energy gain

N = electrons in a bunch

a_0 = normalized vector potential

τ = laser pulse length

$\mathcal{E}_L$ = laser energy

$\mathcal{E}_b$ = beam energy

Phenomenological Blowout Scalings*

- Useful for moderate values of a_0 compared with similarity theory

Fundamental relation is the guiding condition: $k_p R \approx k_p r_0 \approx 2\sqrt{a_0}$

Much follows from this; yet it is not rigorously derived!

Suppose $L_{\mathrm{etch}} \approx \frac{n_c}{n} c\tau_{\mathrm{las}}$ > $L_d \approx \frac{2}{3}\frac{n_c}{n} R$

Pump depletion Dephasing

Note dephasing length estimate uses spherical bubble picture (a_0>4). Then,

$$\Delta E \approx \frac{2}{3} mc^2 \frac{n_c}{n} a_0 \qquad N \approx \frac{k_p^3 R^3}{30 k_p r_e} \approx \frac{8 a_0^{3/2}}{30 k_p r_e}$$

(good absolute accuracy) (only for relative comparisons)

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* W. Lu et al., Phys. Rev. ST/AB 10, 061301 (2007)

Implications of blowout scalings for efficiency

$$\Delta E \approx \frac{2}{3} mc^2 \frac{n_c}{n} a_0 \qquad N \approx \frac{k_p^3 R^3}{30 k_p r_e} \approx \frac{8 a_0^{3/2}}{30 k_p r_e}$$

Appears we want a_0 large, but be careful:

Beam energy $\mathcal{E}_b = \Delta E N \sim \omega^2 a_0^{5/2} / n^{3/2}$

Laser energy $\mathcal{E}_L \sim I r_0^2 c\tau \sim (\omega^2 a_0^2)(a_0/n)\sqrt{a_0/n} \sim \omega^2 a_0^{7/2} / n^{3/2}$

Guiding condition

Controls etching length and self-modulation

Efficiency goes as $\dfrac{\mathcal{E}_b}{\mathcal{E}_L} \sim \dfrac{1}{a_0}$

(for applications may be worse due to energy in marginal particles)

Current Status of LWFA Electron Bunch Properties

Adapted from Mike Downer, U. Texas, "Overview of LWFA Science", presented at ANAR 2017

Property	**State of Art**	**Reference** Full refs in backups	**Remarks**
Energy	**2 GeV** (±5%, 0.1 nC) **3 GeV** (±15%, ~.05 nC) **4 GeV** (±5%, 0.006 nC)	Wang (2013) – Texas Kim (2013) – GIST Leemans (2014) - LBNL	Accelerates from E ~ 0
Energy Spread	**1%** (0.01 nC, 0.2 GeV) **5-10%**	Rechatin (2009a) – LOA More typical (many)	0.1% desirable for collider/FEL
Normalized Transverse Emittance	**~0.1 π mm-mrad**	Geddes (2008) – LBNL Brunetti (2010) –Strathclyde Plateau (2012) - LBNL	Measurements at resolution limit
Bunch Duration	**~few fs**	Kaluza (2010) – Jena (Faraday) Lundh (2011) – LOA; Heigoldt (2015) – MPQ/Oxford (OTR) Zhang (2016) - Tsinghua	Measurements at resolution limit
Charge	**0.02 nC** @ 0.19 GeV ±5% **0.5 nC** @ 0.25 GeV ±14%	Rechatin (2009b) – LOA Couperus (2017) - HZDR	Beam-loading achieved. FOM: Q/ΔE?
Rep-Rate & Repeatability	**~1 Hz** @ > 1 GeV **1 kHz** @ ~ 1 MeV	Leemans (2014) – LBNL He (2015) - UMich; Salehi (2017) – Umd; Guenot (2017) - LOA	Limited by lasers and gas targets

No one achieves all of these simultaneously

Issues with Experiments in Blowout Regime

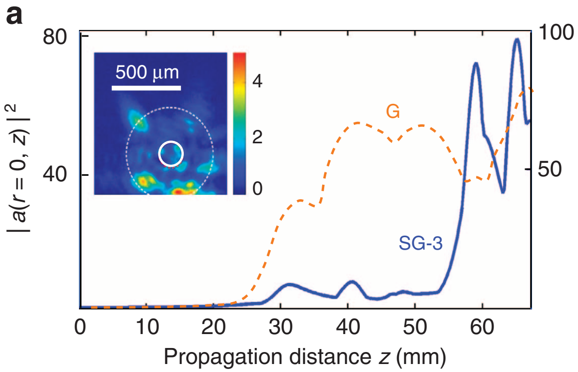
- If a plasma channel is used, predictions difficult.
  - We are missing a theory of blowout wakefields in a channel
- Experimental laser pulses are not fundamental Gaussians
- Low density gives high energy. Low density gives large matched spot size. Therefore high energy electrons need high laser power.
  - Power scaling is weak, $\Delta E \sim P^{1/3}$

Let us examine two famous petawatt experiments:

1. Texas Petawatt 2 GeV Result
2. BELLA 4 GeV Result

Texas Petawatt 2 GeV Experiment

Initial laser parameters: Spot size ~ 275 um, a_0 ~ 0.6, P_0 ~ 0.65 PW
Note that a self guided mode is only expected for $a_0 > 2$.



Measured laser profile and WAKE simulation* from Wang et al., Nat. Comm. 2013.

Due to spot size mismatch, acceleration does not develop until end of plasma. X-rays confirm.

Let us use a_0~8 based on WAKE simulation. This gives

L_{etch} ~ 9 cm, $L_{dephase}$ ~ 6 cm, and results in:

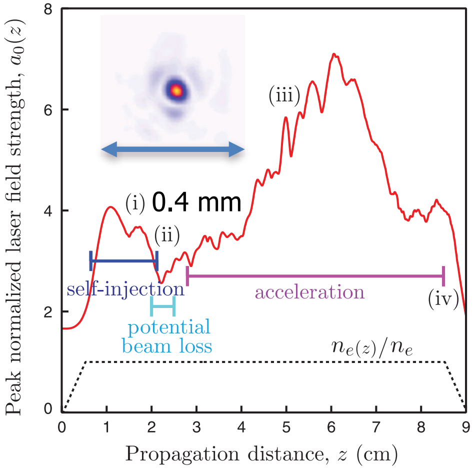
theory: 5.5 GeV, 2.6 nC
actual: 2 GeV, 0.1 nC

Comments:
Laser mode is far from fundamental Gaussian.
Mismatch: matched laser parameters are R ~ 40 um, PL ~ 4.5 PW.
Useful region of plasma is shorter than min(L_{etch},$L_{dephase}$)

* WAKE is a reduced model. Note that 3D PIC would be exceedingly demanding for this experiment.

BELLA 4 GeV Experiment

Initial laser parameters: Spot size ~ 52 um, a_0 ~ 1.6, P_0 ~ 0.3 PW
No self guiding initially, but **there is a channel**.



Measured laser profile and INF&RNO simulation from Leemans et al., PRL 2014.

Initial parameters between quasi-linear and blowout, but evolves into blowout wakefield.

Matched spot size for external guiding ~ 70 um

Let us use a_0~5 based on INF&RNO simulation. This gives

L_{etch} ~ 3 cm, $L_{dephase}$ ~ 5 cm, and results in:

theory: 2.7 GeV, 1.1 nC
actual: 4.2 GeV, 0.006 nC

Comments:
Laser mode is more top-hat than Gaussian. PIC uses actual profile and gives 4.3 GeV, .05 nC.
Mismatch: Matched laser parameters are R ~ 30 um, P_0 ~ 1.4 PW.
Plasma source is fairly well matched to min(L_{etch},$L_{dephase}$)

Issues in simulation of blowout regime

- Scale separation : optical vs. plasma
- Strong transverse gradients impact cell aspect ratio requirements
- Strong nonlinearity and pump depletion stress envelope approximations
- Overlap of accelerated beam with laser radiation stresses validity of relativistic ponderomotive force (even for PGC)

Solutions*:

1. Exploit near axisymmetry and expand in a small number of azimuthal modes.
2. Transform to a boosted frame.
3. Exploit underdense and/or quasistatic approximations – but approximations may be dubious.

*A.F. Lifschitz et al., J. Comp. Phys. 228 1803 (2009); V.-L. Vay, Phys. Rev. Lett. 98, 130405 (2007); P. Mora and T.M. Antonsen, Phys. Rev. E 53, 2068 (1996)

Example of Quasi-3D Blowout Simulations*

Azimuthal mode decomposition has been put into OSIRIS.
This enables full scale modeling of many cases of blowout LPA.
Otherwise, a 10 GeV run would consume 20-30 million hours.

Calculated								Simulated	
a0	P	τ	n_p	Z_R	W_0	L_d	Est. E	Q_{mono}	Max E
	(TW)	(fs)	(cm^{-3})	(cm)	(μm)	(cm)	(GeV)	(pC)	(GeV)
4.44	324	50	1.00e18	0.19	22.0	2.62	2.52	306	3.46
4.44	649	72.0	5.0e17	0.394	31.7	7.37	5.28	255	6.63
4.44	1298	102.5	2.5e17	0.788	44.8	20.8	10.57	146	13.6

Petawatt laser is predicted to reach 10 GeV.
Simulated charge is much greater than anyone gets in experiment.
Analytical scaling reasonably tracks PIC simulation energy.

* A. Davidson, Ph.D. Dissertation (2016)

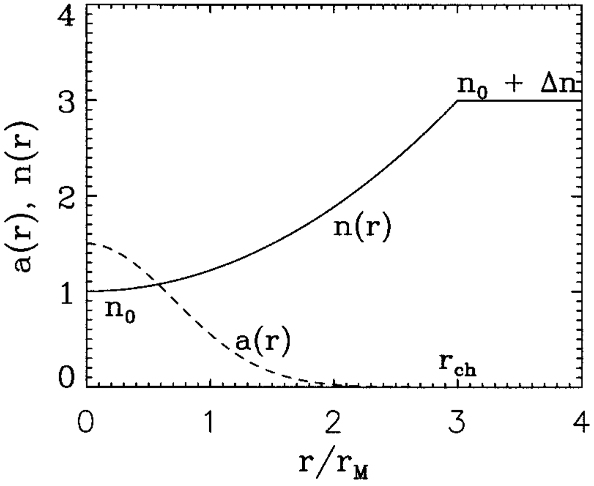
Quasi-linear regime and channel guiding

- Defining Quasi-linearity
  - Perturbation expansion in a_0 leads to condition $a_0^2 << 4$
  - Spot size is such that power is below self-focusing threshold

Without guiding structure weakly nonlinear interaction is limited to

$$z_R = \pi r_0^2/\lambda$$

This diffraction length is typically shorter than the dephasing length. The diffraction limitation is overcome using plasma channels:



Parabolic density guides the fundamental Gaussian laser beam mode

$$n(r) = n_0 + \Delta n r^2/r_{\rm ch}^2$$

$$r_M = \left(r_{\rm ch}^2/\pi r_e \Delta n\right)^{1/4}$$

Compact High Energy Single Stage

- In an early paper, Hubbard et al. considered a set of performance criteria not tied to applications requiring high charge.
- Optimization of single-stage channel guided LWFA

Performance criteria	Accounting for
Acceleration Length, L_d	Off-resonance effects (pulse duration)
Final Energy, $\Delta\mathrm{E}$	Mode dispersion (spot size correction)

Inputs:

Laser Power, P_0

Laser Wavelength, λ

Resonance Ratio, $\alpha_r = \tau_L \omega_p / 2\pi$

Channel Parameters, $n_0, \Delta n, r_{\mathrm{ch}}$

Compact High Energy Single Stage Performance

The accelerating gradient is

$$E_m = \underset{\uparrow}{0.4} E_{\text{br}} \frac{\sin(\pi \alpha_r) a_0^2}{\sqrt{1 + a_0^2/2}}$$

Semi-empirical factor

Dephasing length:

$$L_d = \frac{c^3 \tau_L^3 / \alpha_r^3}{2\lambda^2 (1 + \alpha_s)}$$

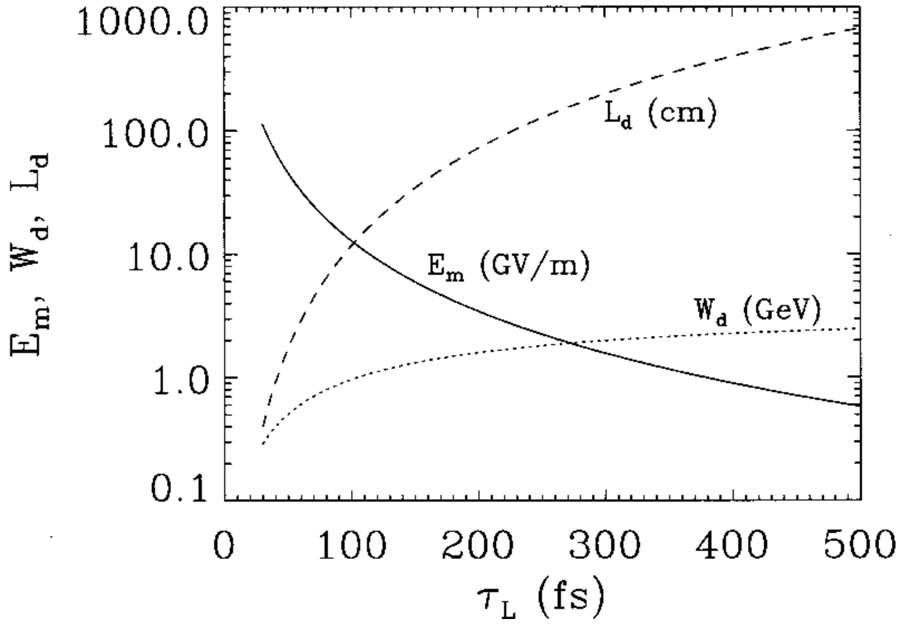
$$\alpha_s = 4 / k_p^2 r_M^2$$

Spot size correction

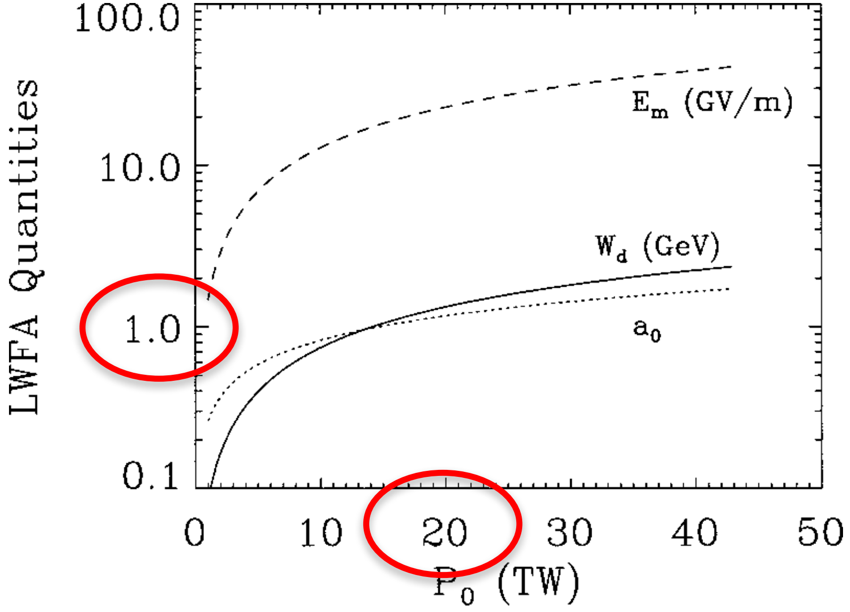
Dephasing limited energy gain is:

$$\gamma_d = 3.2 \frac{P_0 \tau_L}{mc^2} \frac{r_e}{r_{\text{ch}}} \sqrt{\frac{\Delta n}{n_0}} \frac{\operatorname{sinc}(\pi \alpha_r)}{(1 + \alpha_s)\sqrt{1 + a_0^2/2}}$$

Performance Results Reproduced from Hubbard et al.



Pulse width affects length more strongly than energy



GeV possible (in principle) with modest laser power

Electron-Positron Collider Considerations

- Studied by Schroeder et al. (PRSTAB 2010 & 2012)
  - Assuming quasi-linear regime with channel guiding
- Two major issues are added
  - We need high luminosity
  - We need positrons

$$\mathcal{L} = \frac{P_b N}{4\pi U_b \sigma_x^* \sigma_y^*}$$

The overwhelming issue with achieving suitable luminosity is the lack of suitable laser technology. Schroeder et al. 2012 conclude:

Wall Power > 100 MW
Total efficiency ~ 6%

→

COM energy = 1 TeV
Luminosity = $2\mathrm{x}10^{34}\ \mathrm{s}^{-1}\mathrm{cm}^{-2}$

These conclusions may be strongly affected by details of the configuration of final focus and guiding structure.

Collider Stage Scalings

Quasi-linear regime essentially fixes the following:

$a_0 \sim 1$	Weak to moderate nonlinearity
$\omega_p \tau_L \sim \pi$	Resonance condition
$k_p r_0 \sim 4$	Simultaneously suppress blowout and SF

Fixing these leads to scaling for laser energy and bunch charge

$$\mathcal{E}_L \sim \frac{\omega^2}{\omega_p^2} \frac{1}{\sqrt{n_0}} \qquad N \sim \frac{1}{\sqrt{n_0}}$$

Increasing charge means increasing laser energy

Quasi-Linear Energy Gain Scaling

Schroeder et al. consider pump depletion to be the limiting factor.
The following scaling is given for energy per stage:

$$\Delta E \sim \frac{\omega^2}{\omega_p^2}$$

For fixed density (and therefore scale length) high frequency preferred.
Alternatively, insert quasi-linear fixed parameters into Hubbard et al.:

$$\Delta E \approx 0.17 \mathcal{E}_L r_e k_p$$

(there is no inconsistency)

In this view, energy gain and charge are trade-off parameters.

→ High frequency optimizes energy, low frequency optimizes charge.

Issues with experiments in Quasi-linear regime

- Requires external source of ultra-short bunches
- Requires long, stable, plasma channel with closely prescribed longitudinal density profile (uniform or tapered)
- Lack of diagnostics (what goes on inside channel?)

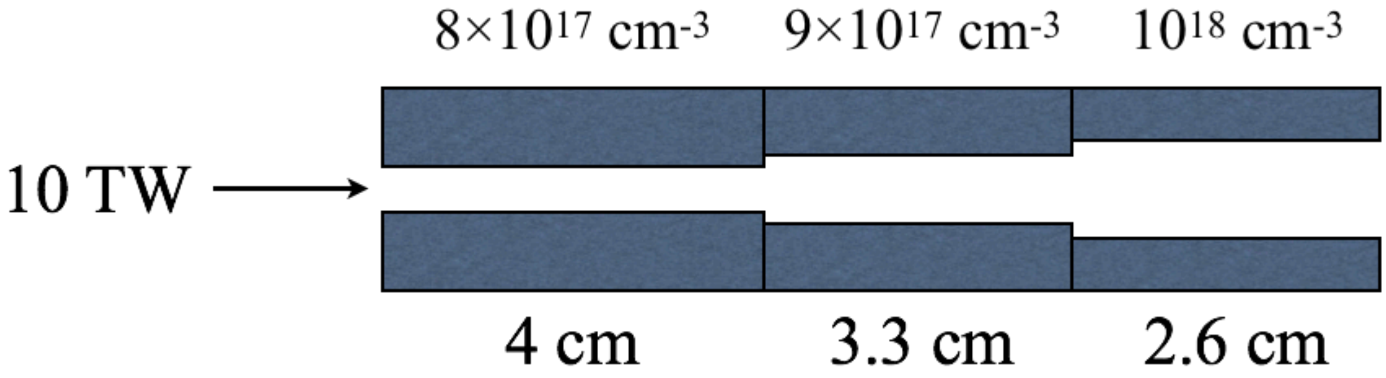
Main problem: No results yet!

Issues in simulation of quasi-linear regime

- Extreme scale separation between length of plasma and laser wavelength
- Weak nonlinearity allows laser field to be enveloped
  - Ponderomotive guiding center
  - Quasi-static treatment
- Deep pump depletion stresses enveloped models for laser fields
- Reduced geometry
  - Axisymmetry may be assumed if envelope model is used for laser fields
  - Near-axisymmetry (m=-1,0,1) may be assumed for fully explicit fields
- Lorentz boosted frame may reduce computational load at expense of numerical stability issues

Simulated GeV Acceleration With Modest Laser*

Numerical method: turboWAVE ponderomotive guiding center, axisymmetric



Externally inject electrons into 5th bucket of quasi-linear wake.

Scaling suggests 20 TW is needed to achieve 1 GeV.
The density taper is a way to "beat the scaling."

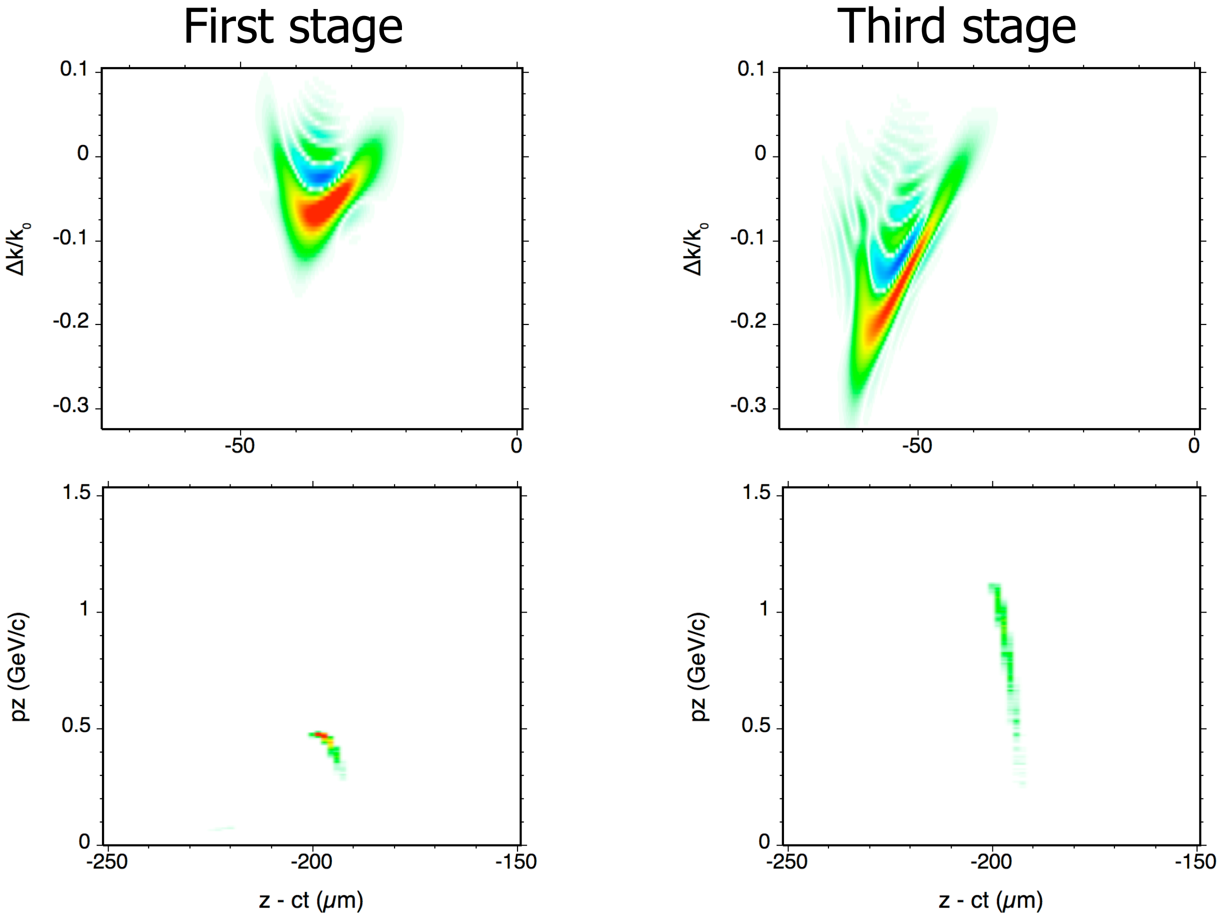
* D.F. Gordon et al., Proc. SPIE 8079, 8079OJ (2011)

Simulated GeV Acceleration With Modest Laser (2)

Numerical method: turboWAVE ponderomotive guiding center, axisymmetric

Photon Phase Space

Electron Phase Space



* D.F. Gordon et al., Proc. SPIE 8079, 8079OJ (2011)

Conclusions

- Trade-off spaces are challenging for both blowout and quasi-linear laser plasma acceleration schemes
  - Incompatibility of high energy, high charge, low emittance, and low energy spread is seen in both scalings and experimental results
- Difficult in realistic experiments to achieve ideal parameters for propagation of self-guided fundamental mode
- Missing some key scalings
  - Blowout wakefields in a plasma channel
  - Accurate estimate of charge in high energy peak, esp. with real lasers
- Electron-positron collider faces major challenges
  - Laser technology
  - Beam quality

Many opportunities remain for outstanding contributions.

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BACKUP VIEWGRAPHS

D.F. Gordon et al., EAAC 2017

Similarity Theory for Blowout LPA*

Any unbounded collisionless plasma, with given initial conditions, induces a family of systems parametrized by an arbitrary frequency ω. The following are constant for any system in the family:

$$A_\mu(\omega x_\mu) \qquad p_\mu(\omega t) \qquad n_e(\omega x_\mu)/n_c(\omega)$$

For an ultra-relativistic laser excited plasma, there is an additional parameter characterizing families of intial conditions:

$$S = \frac{n_{e0}}{n_c(\omega_0)a_0} \qquad a_0 \gg 1$$

Where a_0 is the peak normalized vector potential of the driving laser pulse, and ω_0 is the laser frequency. One can show for S<<1:

$$\Delta E \approx mc^2 \frac{c\tau}{R}\frac{n_c}{n}a_0^2 \qquad \frac{\mathcal{E}_b}{\mathcal{E}_L} \approx \text{const} \approx 20\%$$

Unclear if this regime has yet been seen experimentally.

*S. Gordienko and A. Pukhov, Phys. Plasmas 12, 043109 (2005)

Implications of blowout scalings for emittance

$$\Delta E \approx \frac{2}{3} m c^2 \frac{n_c}{n} a_0 \qquad N \approx \frac{k_p^3 R^3}{30 k_p r_e} \approx \frac{8 a_0^{3/2}}{30 k_p r_e}$$

$$\epsilon \sim \Delta r \Delta p_\perp \sim \left(\frac{1}{k_p}\right)(a_0)$$

$$\epsilon \sim \sqrt{n}\frac{\Delta E}{\omega^2} \qquad \epsilon \sim \frac{N}{\sqrt{a_0}}$$

External injection (broad sense) appears to be needed for high quality beams

Final Focus Considerations (Emittance)*

- In Quasi-linear regime, external injection is required, so that suitably high quality beam may be assumed as an initial condition
- Therefore the issue is emittance growth in the LPA stages

Beam density is constrained to $n_b<n_0$
Therefore spot size is constained to some minimum (bad for emittance).

$$\epsilon \sim \sqrt{\frac{2Z(Z+1)r_e^2 N_b \Lambda_c}{\sqrt{2\pi}\sigma_s (d\gamma/ds)}(\gamma_f - \gamma_i)}$$

Density is buried in accelerating rate. Puts pressure in direction of high density if a gentle parabolic channel is assumed.

* Following Lebedev&Nagaitsev, PRSTAB 16, 108001 (2013)

Final Focus Considerations (Beamstrahlung)*

Beamstrahlung energy loss scaling $$\delta_b \sim \frac{N^{2/3}\sigma_z^{1/3}}{\sigma_*^{2/3}\gamma^{1/3}}$$

Demoninator cannot be changed or we lose luminosity and COM energy.

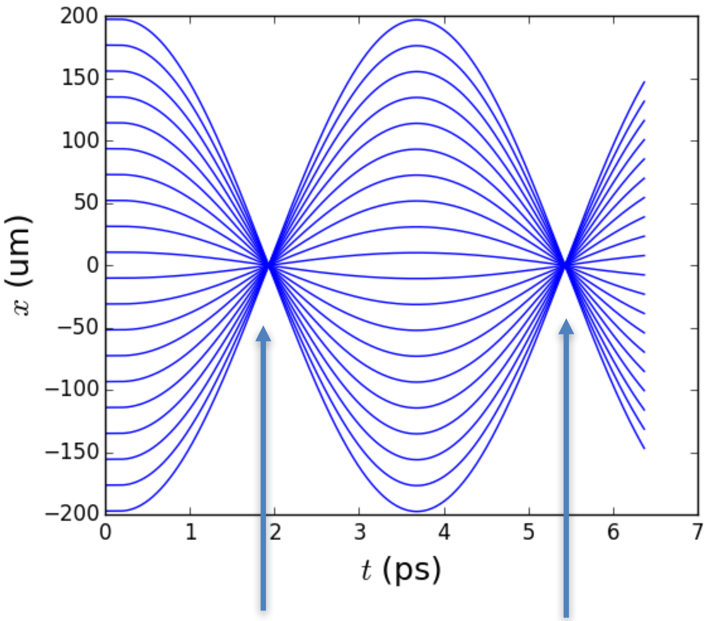
Hence need less bunch charge with proportionately higher rep rate, and/or reduction in bunch length.

Creates pressure in direction of higher plasma density and higher laser frequency.

* Following Schroeder et al., PRSTAB 2012

Characteristics of mismatched plasma channel

Eikonal treatment gives ray evolution as $\frac{d^2r}{dt^2} + \Omega^2 r = 0 \qquad \Omega^2 = \frac{c^2}{r_{\rm ch}^2}\frac{\Delta n}{n_0}\frac{\omega_p^2}{\omega^2}$



Mismatch leads to perfectly periodic focusing in time (but not in space).

Wave theory required near caustics.
However, paraxial theory spuriously distorts temporal structure.