

Strong dynamics beyond the hierarchy problem

Michele Redi

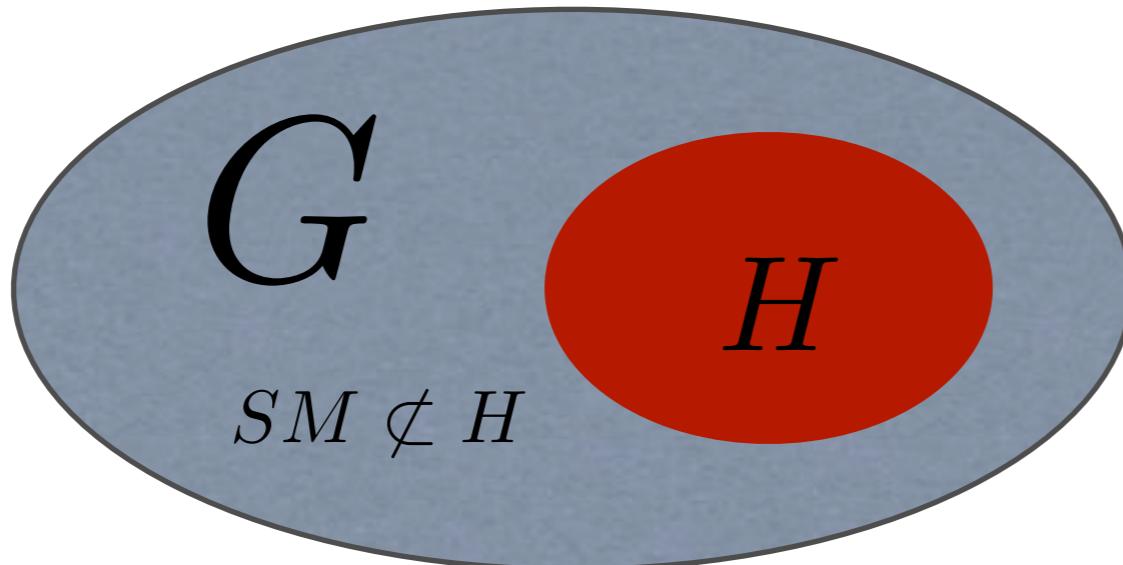
Based on: arxiv 1503.08749 + [1707.05380](#)
with Antipin, Mitridate, Smirnov, Strumia, Vigiani

Baryonic Dark Matter

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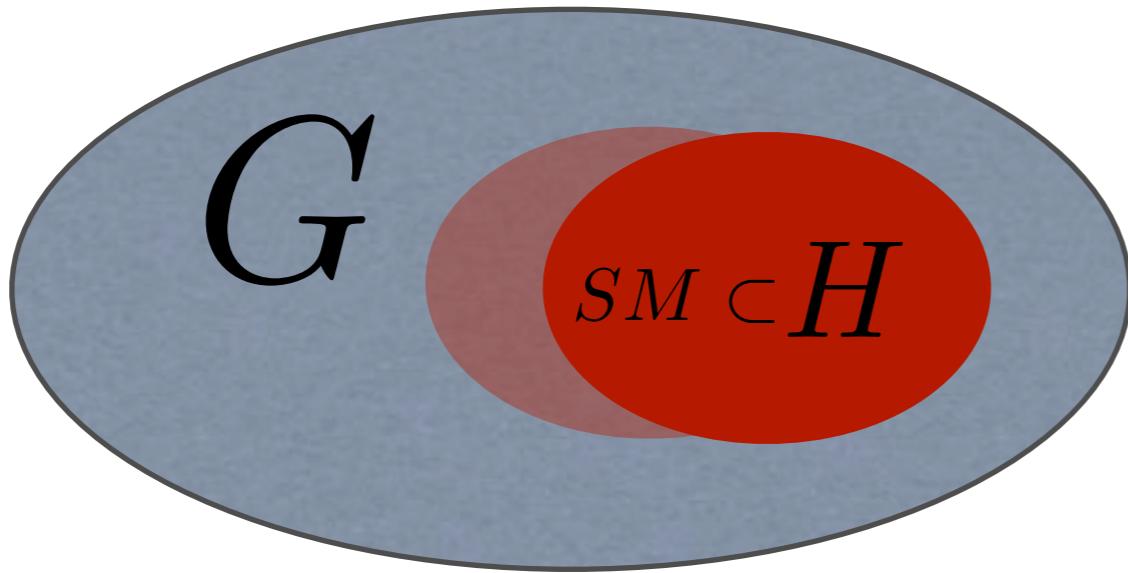
- Technicolor



$$f = v$$



- Composite Higgs



$$f > v$$

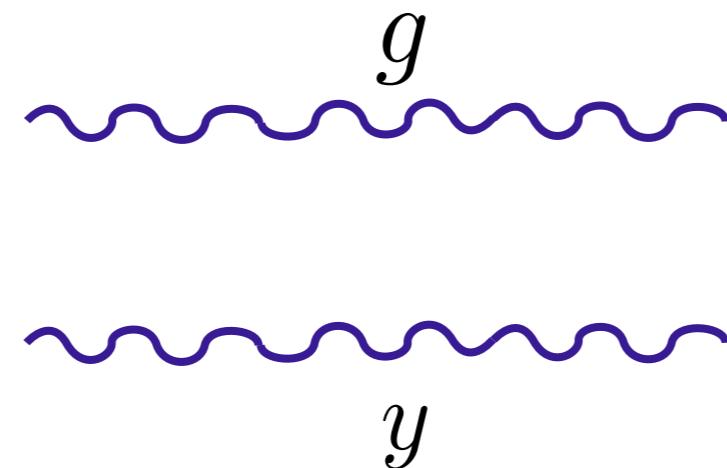
Higgs is a pseudo-Goldstone boson. Electro-weak scale determined by vacuum alignment.

$$\text{DEVIATION SM} \sim \text{TUNING} \sim \frac{v^2}{f^2}$$

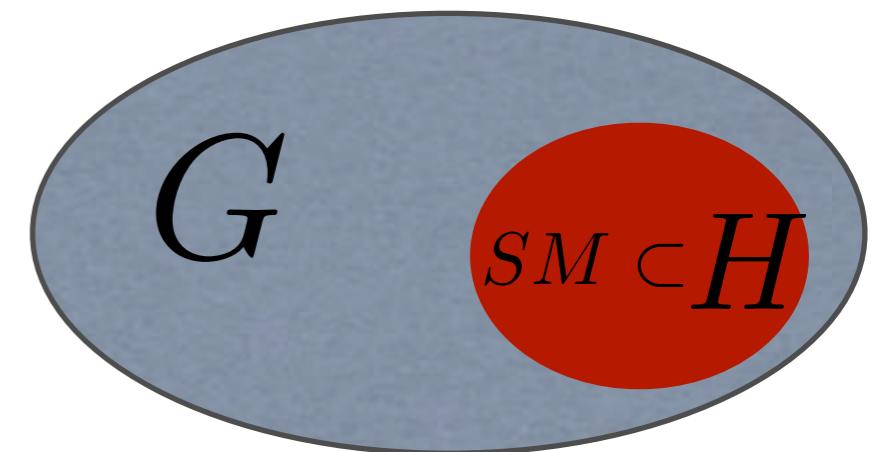


- SM preserving strong dynamics:

$SM + H$



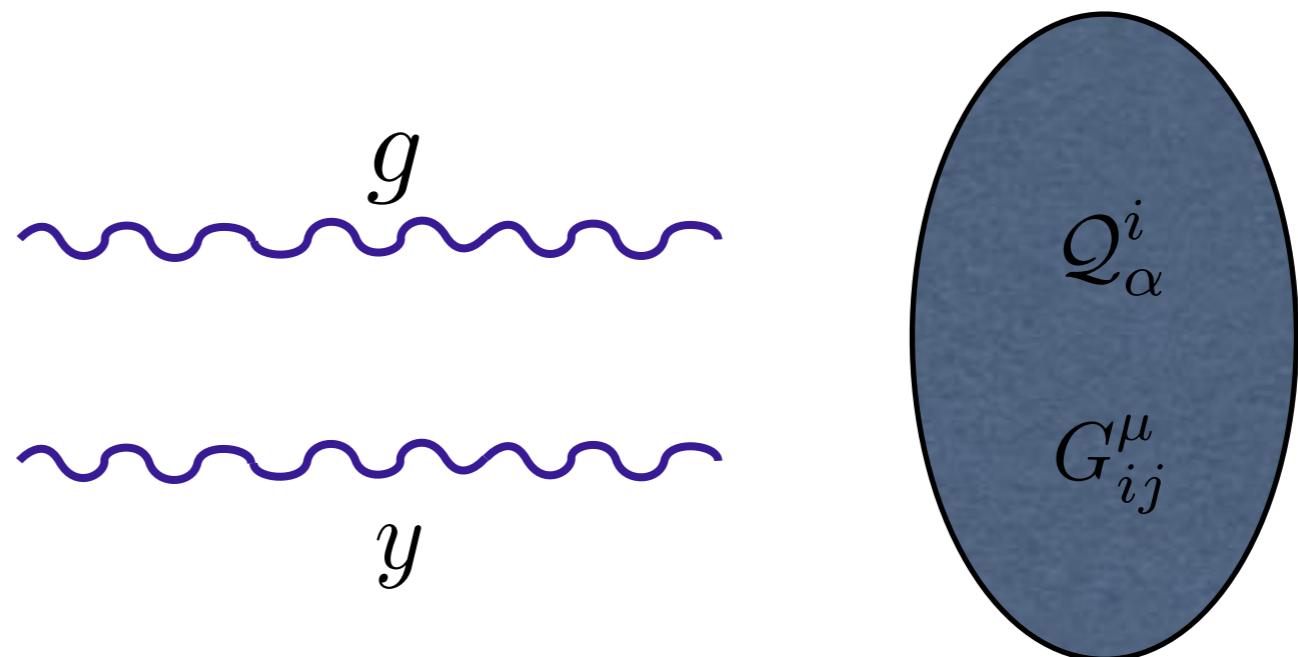
see Contino's talk



Higgs is elementary.

Confining gauge theory with fermions vectorial under SM

$SM + H$



$$Q = (N_{\text{DC}}, SM) + (\overline{N}_{\text{DC}}, \overline{SM})$$

SM including elementary Higgs couples to the strong sector with renormalizable couplings:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \bar{Q}_i (i\gamma^\mu D_\mu - m_i) Q_i - \frac{\mathcal{G}_{\mu\nu}^{A2}}{4g_{\text{DC}}^2} + \frac{\theta_{\text{DC}}}{32\pi^2} \mathcal{G}_{\mu\nu}^A \tilde{\mathcal{G}}_{\mu\nu}^A + [H \bar{Q}_i (y_{ij}^L P_L + y_{ij}^R P_R) Q_j + \text{h.c.}]$$

Very weak bounds:

- Automatic MFV
- Precision tests ok
- LHC: $\Lambda > 1 - 2 \text{ TeV}$

Interesting phenomenology:

- Accidental dark matter candidates
- Plausible signatures for LHC and cosmology
- Composite Higgs  see Contino's talk
- (Rel)axions

Accidental symmetries:

- Dark-Baryon number

$$\mathcal{Q}^i \rightarrow e^{i\alpha} \mathcal{Q}^i \quad \longrightarrow \quad B = \epsilon^{i_1 i_2 \dots i_n} Q_{i_1}^{\{\alpha_1} Q_{i_2}^{\alpha_2} \dots Q_{i_n\}}^{\alpha_n\}}$$

- Dark-Species number

$$Q^i \rightarrow e^{i\alpha_i} Q^i \quad \longrightarrow \quad M = \bar{Q}^i Q^j$$

- G-parity

$$Q \rightarrow e^{-i\pi J_2} Q^c \quad \longrightarrow \quad M = (\bar{Q} Q)_{\text{triplet}}$$

Dark baryons robustly cosmologically stable as the proton.

Majorana vs. Dirac

- Q-complex ($SU(N)$)

Baryons are Dirac particles that can be produced thermally or asymmetrically. Strong direct detection constraints from tree level Z -couplings.

- Q-real ($SO(N)$)

Baryon and anti-baryons are the same particle so 2 DM particles can annihilate. Only thermally produced. Direct detections constraints avoided.

- Light Dark Quarks: $(m_Q < \Lambda_{DC})$

Strongly coupled dynamics, DM simple.

- Heavy Dark Quarks: $(m_Q > \Lambda_{DC})$

$$\Lambda_{DC} \sim m_Q \exp \left[-\frac{6\pi}{11C_2(G)\alpha_{DC}(m_Q)} \right] \quad r_{DC} \sim (\alpha_{DC} m_Q)^{-1}$$

Effective DM coupling is perturbative.

Cosmology non-standard and model dependent:

- $r_{DC} < \Lambda_{DC}^{-1}$

“Coulomb”

- $r_{DC} > \Lambda_{DC}^{-1}$

“Quarkonium”

Light Quarks

- with O.Antipin, A. Strumia, E.Vigiani '15

SU(N)

SU(N) gauge theory with NF light flavors.
Dark-quarks are vectorial with respect to SM.

Fermions	SM	$SU(n)_{\text{TC}}$	
Ψ_L	$\sum_i r_i$	n	$\sum_i d[r_i] = N_F$
Ψ_R	$\sum_i \bar{r}_i$	\bar{n}	

$$\langle \bar{\Psi}^i \Psi^j \rangle \sim 4\pi f^3 \delta^{ij}$$

Vacuum does not break electro-weak symmetry.

Nambu-Goldstone bosons:

$$\frac{SU(N_F) \times SU(N_F)}{SU(N_F)}$$

$$\text{Adj}_{SU(N_F)} = \sum_{i=1}^K r_i \times \sum_{i=1}^K \bar{r}_i - 1$$

- **Dark-Pions**

Pions behave as elementary minimal dark matter candidates.

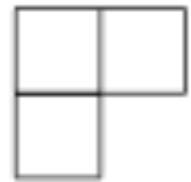
Strumia, Cirelli '05

$$m_{I=1} \sim 2.5 \text{ TeV} \quad \sigma_{SI} = 0.12 \pm 0.03 \times 10^{-46} \text{ cm}^2$$

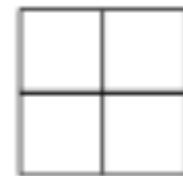
- **Dark-Baryons**

Lightest multiplet has minimum spin. Flavor rep:

$$N_{DC} = 3$$



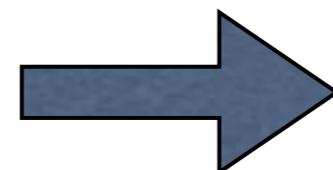
$$N_{DC} = 4$$



DM candidate:

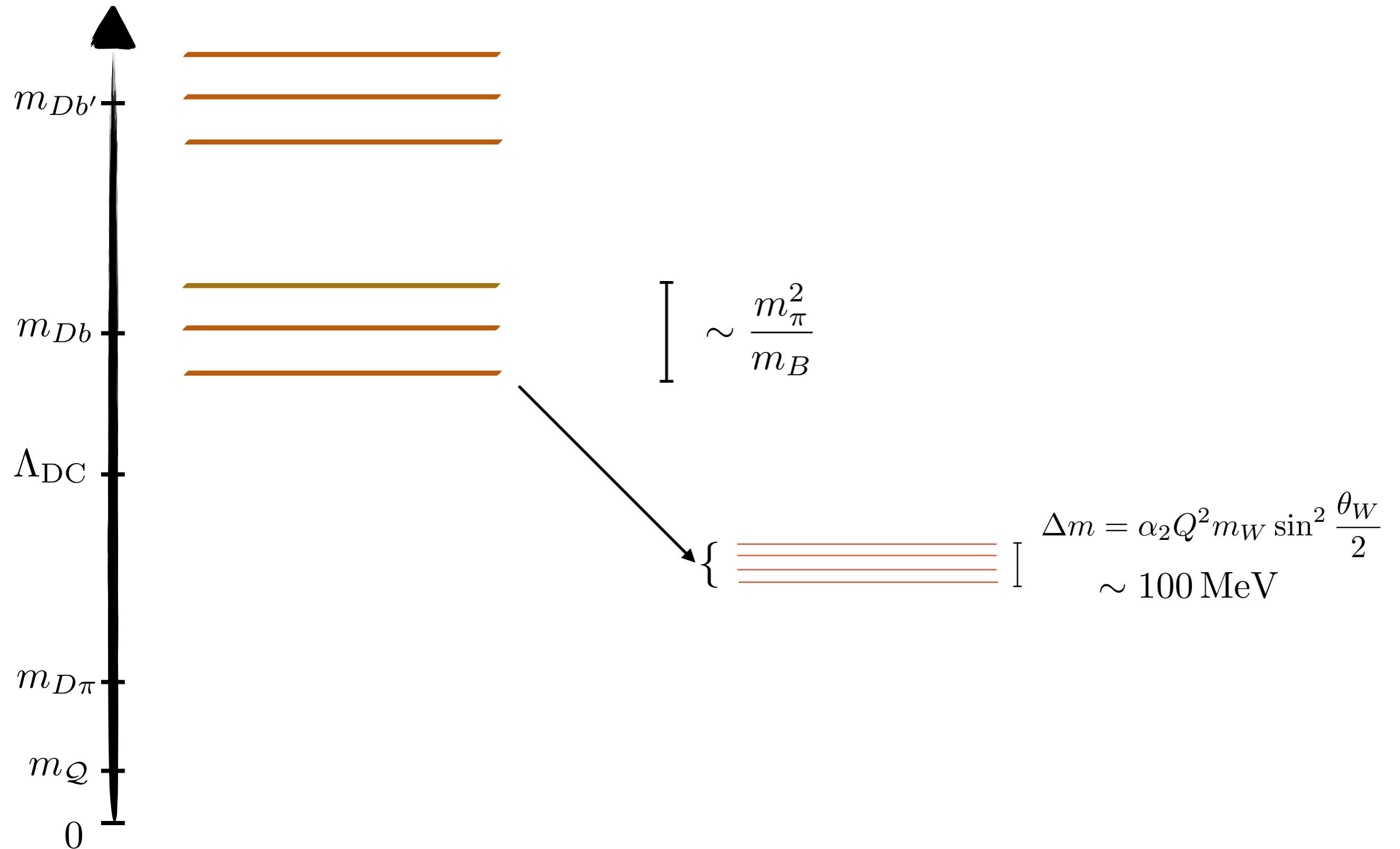
$$Q_{DB} = T_{DB}^3 + Y_{DB} = 0$$

$$Y_{DB} = 0$$



$|=0, 1, 2, \dots$

Flavor multiplets are split by fermion masses and gauge interactions:



Classification:

$$R = (N, SM) \oplus (\bar{N}, S\bar{M})$$

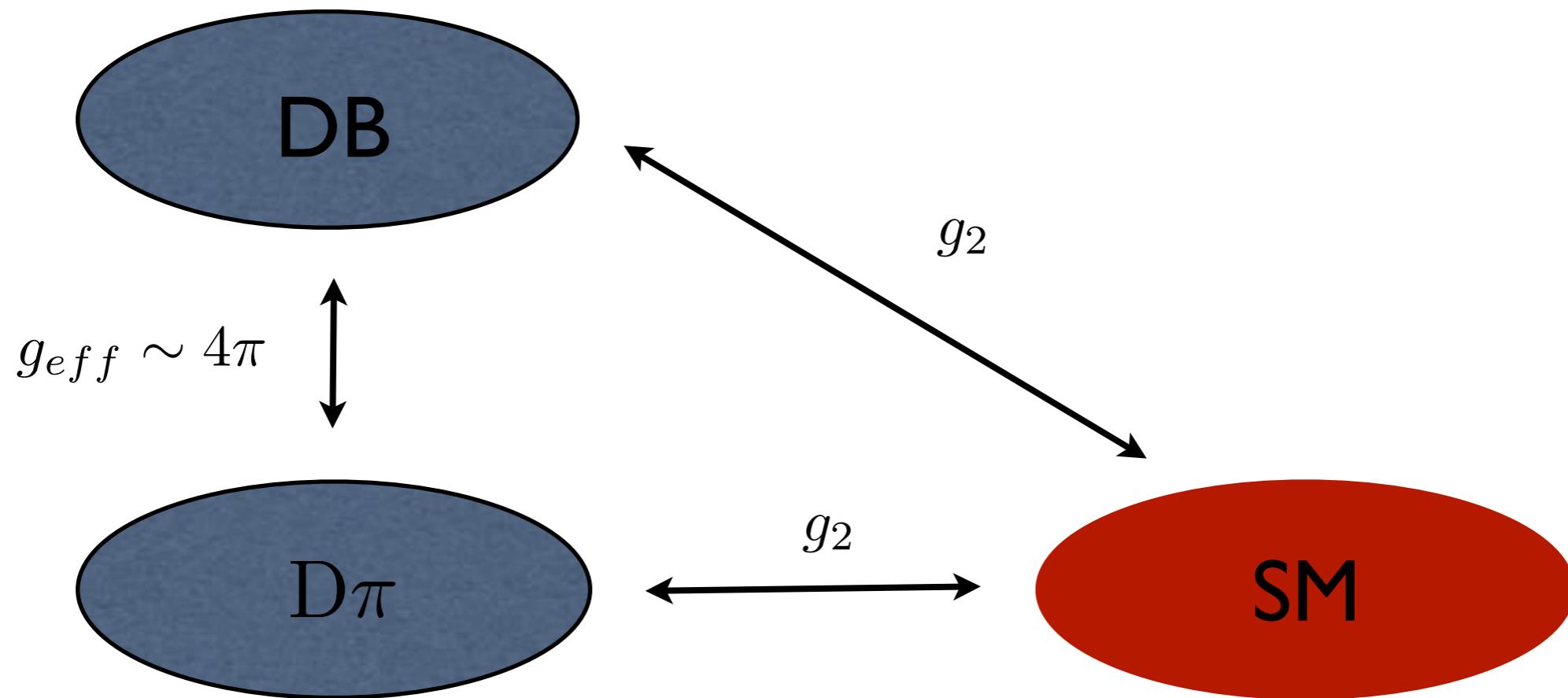
SU(5)	SU(3) _c	SU(2) _L	U(1) _Y	charge	name
1	1	1	0	0	N
$\bar{5}$	$\bar{3}$	1	-1/3	-1/3	D
	1	2	1/2	0, 1	L
10	$\bar{3}$	1	-2/3	-2/3	U
	1	1	1	1	E
	3	2	1/6	-1/3, 2/3	Q
15	3	2	1/6	-1/3, 2/3	Q
	1	3	1	0, 1, 2	T
	6	1	-2/3	-2/3	S
24	1	3	0	-1, 0, 1	V
	8	1	0	0	G
	$\bar{3}$	2	5/6	1/3, 4/3	X
	1	1	0	0	N

- SU(N) asymptotically free
- No Landau poles below the Planck scale.
- Lightest dark-baryon with $Q=Y=0$
- No unwanted stable particles

Golden models:

$SU(N)$ techni-color. Techni-quarks	Yukawa couplings	Allowed N	Techni- pions	Techni- baryons	under
$N_{TF} = 3$			8	$8, \bar{6}, \dots$ for $N = 3, 4, \dots$	$SU(3)_{TF}$
$\Psi = V$	0	3	3	$VVV = 3$	$SU(2)_L$
$\Psi = N \oplus L$	1	3, .., 14	unstable	$N^{N*} = 1$	$SU(2)_L$
$N_{TF} = 4$			15	$\bar{20}, 20', \dots$	$SU(4)_{TF}$
$\Psi = V \oplus N$	0	3	3×3	$VVV, VNN = 3, VVN = 1$	$SU(2)_L$
$\Psi = N \oplus L \oplus \tilde{E}$	2	3, 4, 5	unstable	$N^{N*} = 1$	$SU(2)_L$
$N_{TF} = 5$			24	$\bar{40}, \bar{50}$	$SU(5)_{TF}$
$\Psi = V \oplus L$	1	3	unstable	$VVV = 3$	$SU(2)_L$
$\Psi = N \oplus L \oplus \tilde{L}$	2	3	unstable	$NLL = 1$	$SU(2)_L$
=	2	4	unstable	$NNLL\tilde{L}, L\tilde{L}LL = 1$	$SU(2)_L$
$N_{TF} = 6$			35	$70, \bar{105}'$	$SU(6)_{TF}$
$\Psi = V \oplus L \oplus N$	2	3	unstable	$VVV, VNN = 3, VVN = 1$	$SU(2)_L$
$\Psi = V \oplus L \oplus \tilde{E}$	2	3	unstable	$VVV = 3$	$SU(2)_L$
$\Psi = N \oplus L \oplus \tilde{L} \oplus \tilde{E}$	3	3	unstable	$NLL, \tilde{L}\tilde{L}\tilde{E} = 1$	$SU(2)_L$
=	3	4	unstable	$NNLL\tilde{L}, L\tilde{L}LL\tilde{L}, N\tilde{E}\tilde{L}\tilde{L} = 1$	$SU(2)_L$
$N_{TF} = 7$			48	112	$SU(7)_{TF}$
$\Psi = L \oplus \tilde{L} \oplus E \oplus \tilde{E} \oplus N$	4	3	unstable	$LLE, \tilde{L}\tilde{L}\tilde{E}, L\tilde{L}N, E\tilde{E}N = 1$	$SU(2)_L$
$\Psi = N \oplus L \oplus \tilde{E} \oplus V$	3	3	unstable	$VVV, VNN = 3, VVN = 1$	$SU(2)_L$
$N_{TF} = 9$			80	240	$SU(9)_{TF}$
$\Psi = Q \oplus \tilde{D}$	1	3	unstable	$QQ\tilde{D} = 1$	$SU(2)_L$
$N_{TF} = 12$			143	572	$SU(12)_{TF}$
$\Psi = Q \oplus \tilde{D} \oplus \tilde{U}$	2	3	unstable	$QQ\tilde{D}, \tilde{D}\tilde{D}\tilde{U} = 1$	$SU(2)_L$

Relic abundance:

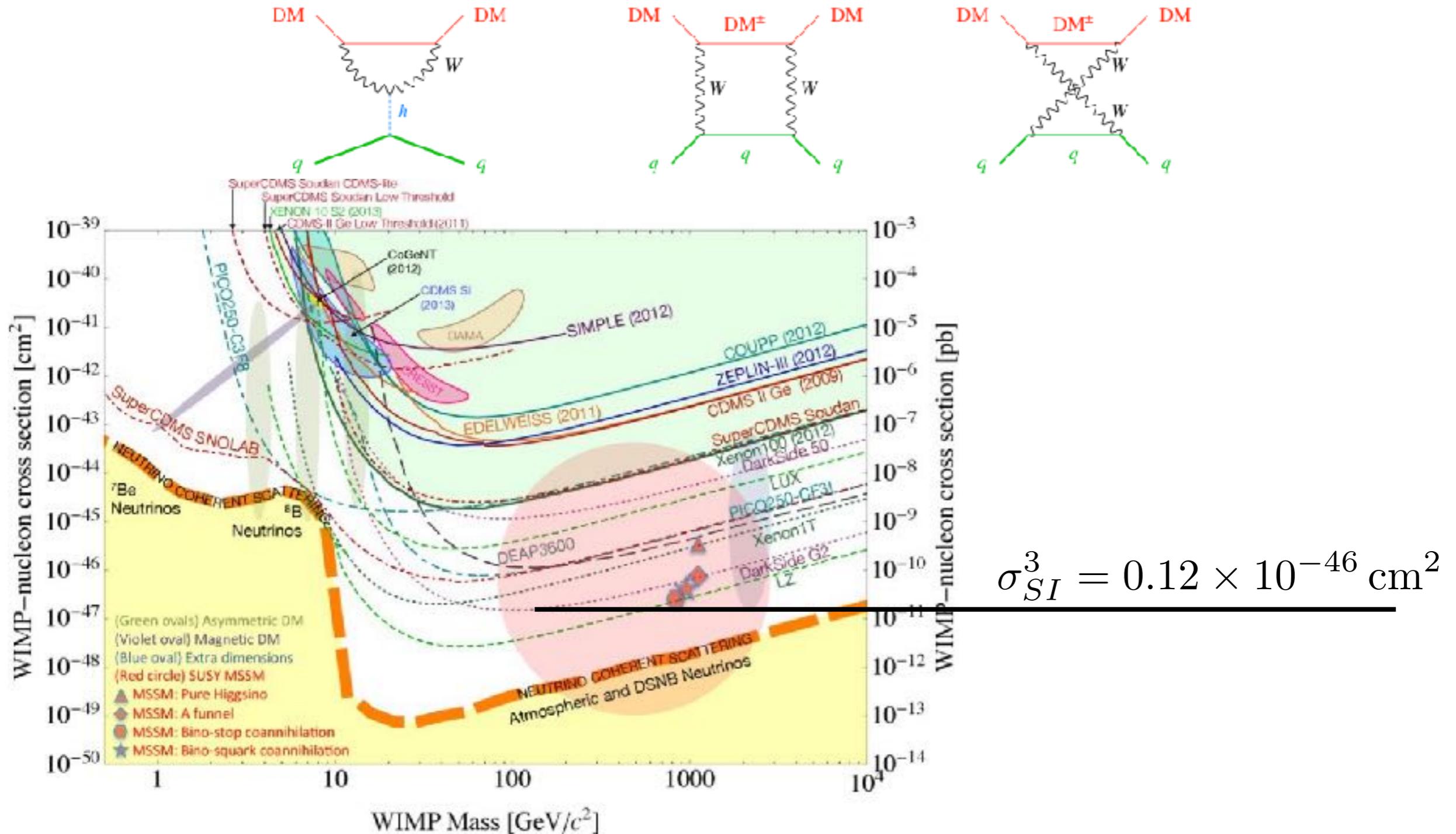


$$\langle \sigma_{B\bar{B}}^{ANN} v \rangle \sim \frac{4\pi}{m_B^2}$$

THERMAL ABUNDANCE
 $m_B \sim 100 \text{ TeV}$

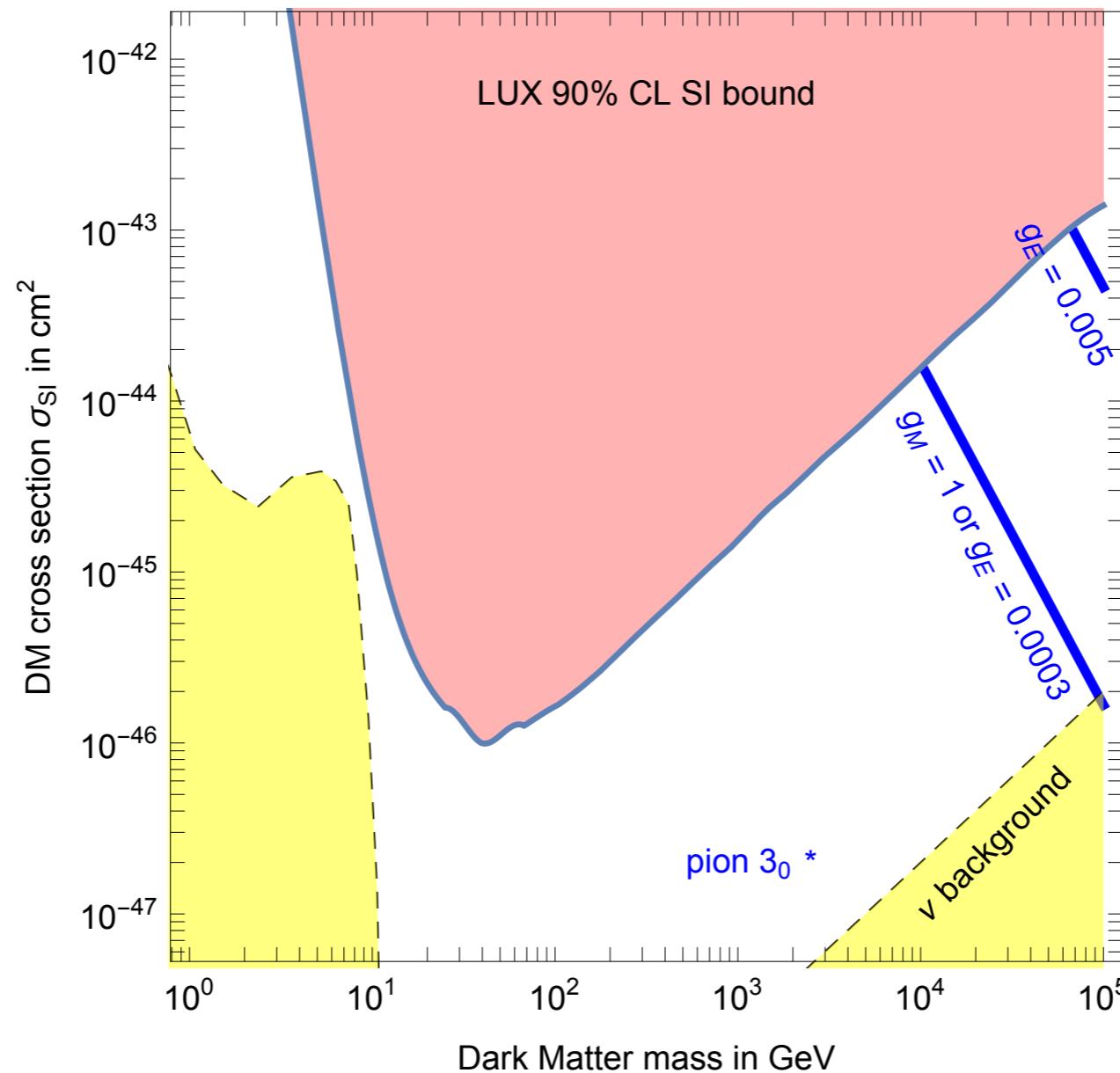
DM could also be produced asymmetrically.

If DB has SM charges it interacts as WIMPS.



Yukawa couplings very constrained.

Dirac baryon DM



Dipole interactions:

$$\frac{d\sigma}{dE_R} \approx \frac{e^2 Z^2}{16\pi m_B^2 E_R} \left(g_M^2 + \frac{g_E^2}{v^2} \right) \longrightarrow g_M^2 + 10^7 g_E^2 < \left(\frac{m_B}{5 \text{ TeV}} \right)^3$$

$$g_M = \mathcal{O}(1)$$

$$g_E \sim \frac{e \theta_{\text{TC}} \min[m_Q]}{M_{\text{DM}}}$$

SO(N)

With N_F flavors in the vector rep:

$$\langle 0 | q_i^a q_i^b | 0 \rangle \sim 4\pi f^3 \delta^{ab} \longrightarrow \frac{SU(N_F)}{SO(N_F)}$$

Fermions are in a real dark color rep:

- No difference between baryons and anti-baryons.

Two baryons can annihilate into N pions

$$\epsilon^{i_1 i_2 \dots i_N} \epsilon^{j_1 j_2 \dots j_N} = (\delta_{i_1 j_1} \delta_{i_2 j_2} \dots \delta_{i_N j_N} \pm \text{permutations})$$

- Real SM fermions have Majorana masses

NN

VV

GG

After electro-weak symmetry breaking neutral mass eigenstates are Majorana particles. Analogous to SUSY neutralinos.

SO(N) DM candidates are Majorana fermions or real scalars:

- **production**

Cannot be produced through an asymmetry.

Thermal abundance:

$$m_B \sim 100 \text{ TeV}$$

- **detection**

There are no vector couplings with Z eliminating spin independent bounds. No dipole interactions.

Golden models:

SO(N) techni-color. Techni-quarks	Yukawa couplings	Allowed N	Techni- pions	Techni- baryons	under
$N_{\text{TF}} = 3$			5	3, 1, ... for $N = 3, 4, \dots$	$\text{SO}(3)_{\text{TF}}$
$\Psi = V$	0	3, 4, .., 7	unstable	$V^N = 3, 1, \dots$	$\text{SU}(2)_L$
$N_{\text{TF}} = 4$			9	4, 1, ...	$\text{SO}(4)_{\text{TF}}$
$\Psi = N \oplus V$	0	3, 4, .., 7	3	$VVN = 1, V(VV + NN) = 3,$ $VV(VV + NN) = 1, \dots$	$\text{SU}(2)_L$ $\text{SU}(2)_L$
$N_{\text{TF}} = 5$			14	5, 1...	$\text{SO}(5)_{\text{TF}}$
$\Psi = L \oplus N$	1	3, 4, .., 14	unstable	$L\bar{L}N = 1,$ $L\bar{L}(L\bar{L} + NN) = 1, \dots$	$\text{SU}(2)_L$ $\text{SU}(2)_L$
$N_{\text{TF}} = 7$			27	1, ...	$\text{SO}(7)_{\text{TF}}$
$\Psi = L \oplus V$	1	4	unstable	$(L\bar{L} + VV)^2 = 1$	$\text{SU}(2)_L$
$\Psi = L \oplus E \oplus N$	2	4, 5	unstable	$(E\bar{E} + L\bar{L})^2 + NN(L\bar{L} + E\bar{E}) = 1$	$\text{SU}(2)_L$
$N_{\text{TF}} = 8$			35	1	$\text{SO}(8)_{\text{TF}}$
$\Psi = G$	0	4	unstable	$GGGG = 1$	$\text{SU}(2)_L$
$\Psi = L \oplus N \oplus V$	2	4	unstable	$(L\bar{L} + VV)^2 + NN(L\bar{L} + VV) = 1$	$\text{SU}(2)_L$
$N_{\text{TF}} = 9$			44	1	$\text{SO}(9)_{\text{TF}}$
$\Psi = L \oplus E \oplus V$	2	4	unstable	$(E\bar{E} + L\bar{L} + VV)^2 = 1$	$\text{SU}(2)_L$
$N_{\text{TF}} = 10$			54	1	$\text{SO}(10)_{\text{TF}}$
$\Psi = L \oplus E \oplus V \oplus N$	3	4	unstable	as $L \oplus E \oplus V + NN(L\bar{L} + E\bar{E} + VV) = 1$	$\text{SU}(2)_L$

Q=L+N

$$m_L \bar{L}L + \frac{m_N}{2} NN + y_L H^\dagger L N + y_R^* H \bar{L} N + h.c$$

$$N = 3 : \quad \left(\begin{array}{c|c} \square & \square \\ \square & \square \end{array} \right)_{SU(N_{\text{TF}})} = \left(\begin{array}{c|c} \square & \square \\ \square & \square \end{array} \oplus \square \right)_{SO(N_{\text{TF}})}$$

Lightest baryon is a quintuplet of $SO(5)$
containing “Higgsino” + “bino” states

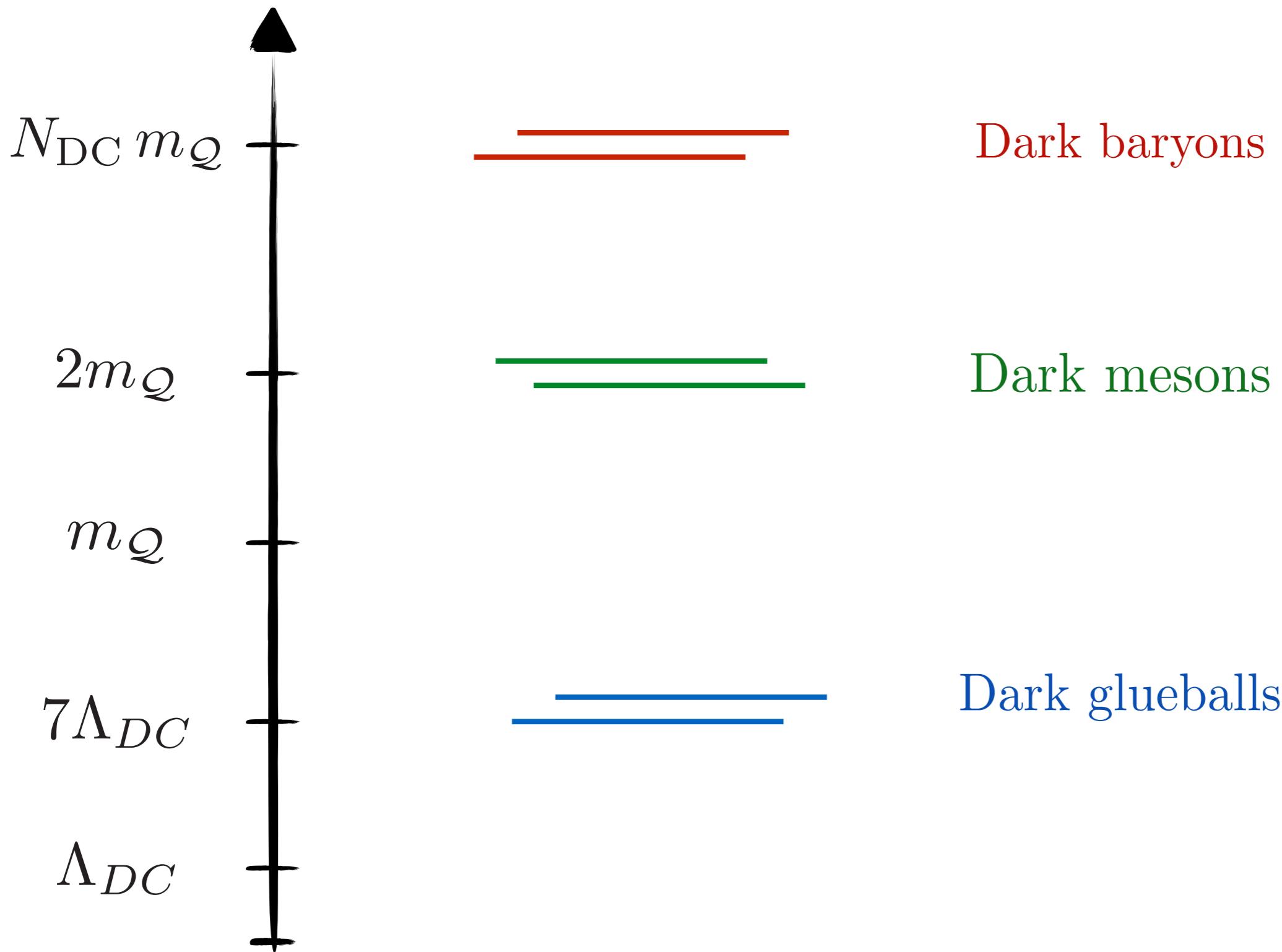
$$\begin{matrix} & 1_0 & 2_{1/2} & 2_{-1/2} & \cdots \\ 1_0 & \begin{pmatrix} m_{1_0} & y_L v & y_R v & \cdots \\ y_L^* v & 0 & m_{2_{1/2}} & \cdots \\ y_R^* v & m_{2_{1/2}} & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \\ 2_{1/2} \\ 2_{-1/2} \\ \vdots \end{matrix}$$

- “Higgsino DM” $m_L \ll m_N$

$$\Delta m_M \sim \frac{y^2 v^2}{m_N}$$

Heavy quarks

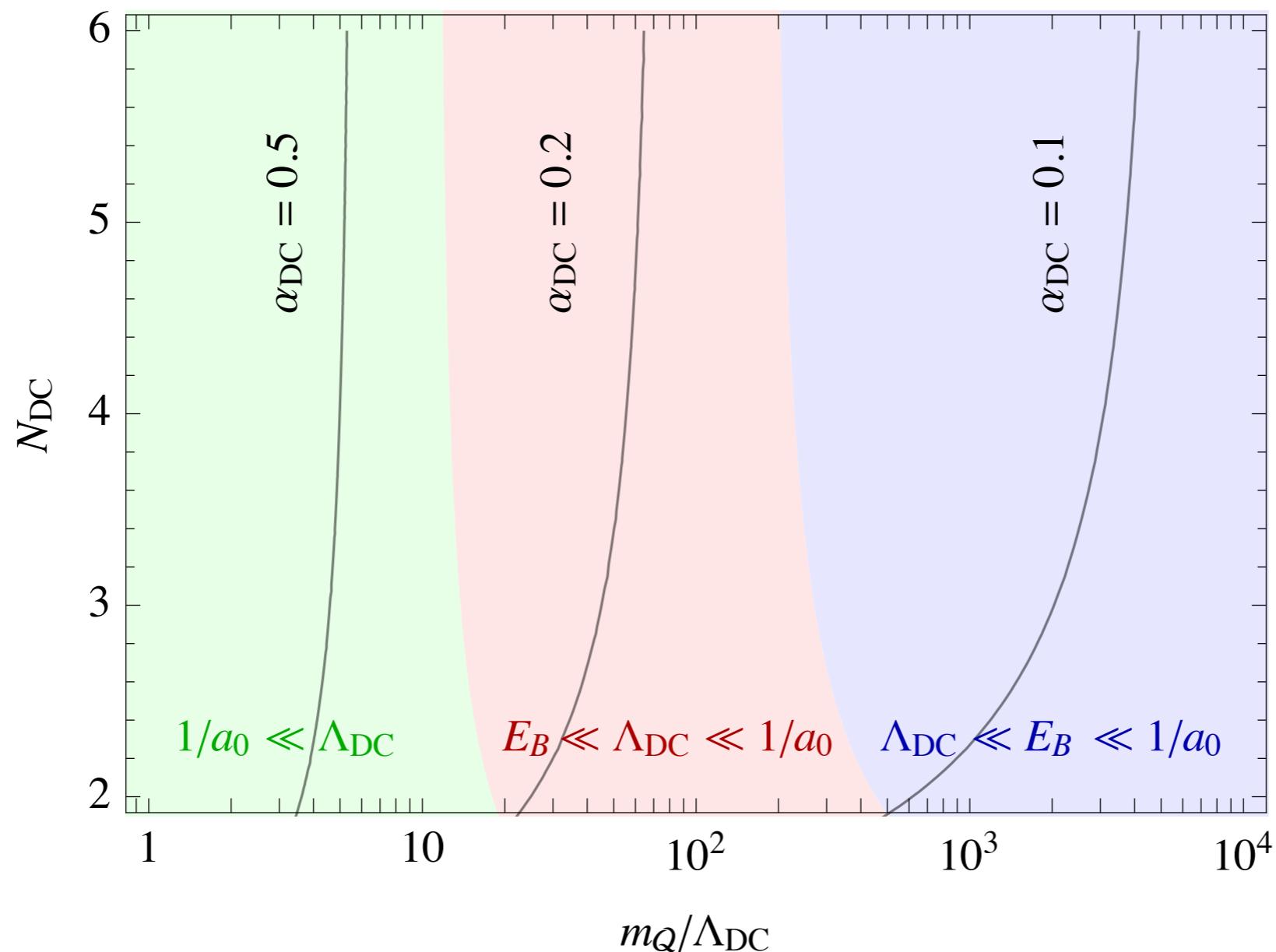
- [1707.05380](#) with A. Mitridate, J. Smirnov, A. Strumia



Non-relativistic bound states N fermions:

$$V \sim -\frac{\alpha_{DC}}{r} + \Lambda_{DC}^2 r$$

$SU(N_{DC})$



Lightest baryons are naturally made of a single specie.

- $SU(3)$

$$\Psi = N \oplus \dots$$

$$DM = NNN,$$

$$I(J^P) = 0 \left(\frac{3}{2}^+ \right)$$

$$\Psi = V \oplus \dots$$

$$DM = VVV,$$

$$I(J^P) = 1 \left(\frac{1}{2}^+ \right)$$

- $SO(3)$

$$\Psi = L \oplus N \oplus V + \dots$$

$$DM = L L \bar{L}$$

$$I(J^P) = \frac{1}{2} \left(\frac{1}{2}^+ \right)$$

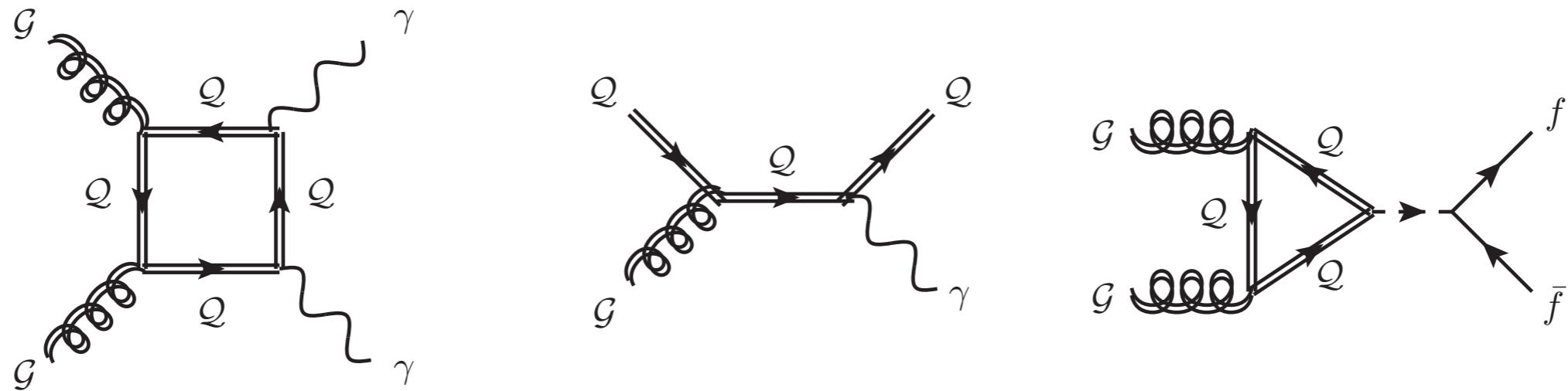
Static properties of bound states:

$$m_B \approx N_{\text{DC}} m_Q$$

$$E_B \sim \alpha_{\text{DC}}^2 N_{\text{DC}}^3 m_Q$$

Glueballs:

$$M_{0^{++}} \approx 7\Lambda_{DC}$$



$$\tau_{DG}^{\gamma\gamma} \sim 10 \text{sec} \left(\frac{10 \text{ GeV}}{m_{DG}} \right)^9 \left(\frac{m_Q}{\text{TeV}} \right)^8$$

$$\tau_{DG}^{b\bar{b}} \sim 10^{-3} \text{sec} \left(\frac{0.1}{y} \right)^4 \left(\frac{10 \text{ GeV}}{m_{DG}} \right)^7 \left(\frac{m_Q}{\text{TeV}} \right)^4$$

Glueballs need to decay before BBN:

$$\tau_{DG} + t_{\Lambda_{DC}} < 1 \text{s}$$

Non standard cosmological histories:

- $\Lambda_{DC} > \frac{m_Q}{25}$

Baryons form before thermal free-out.

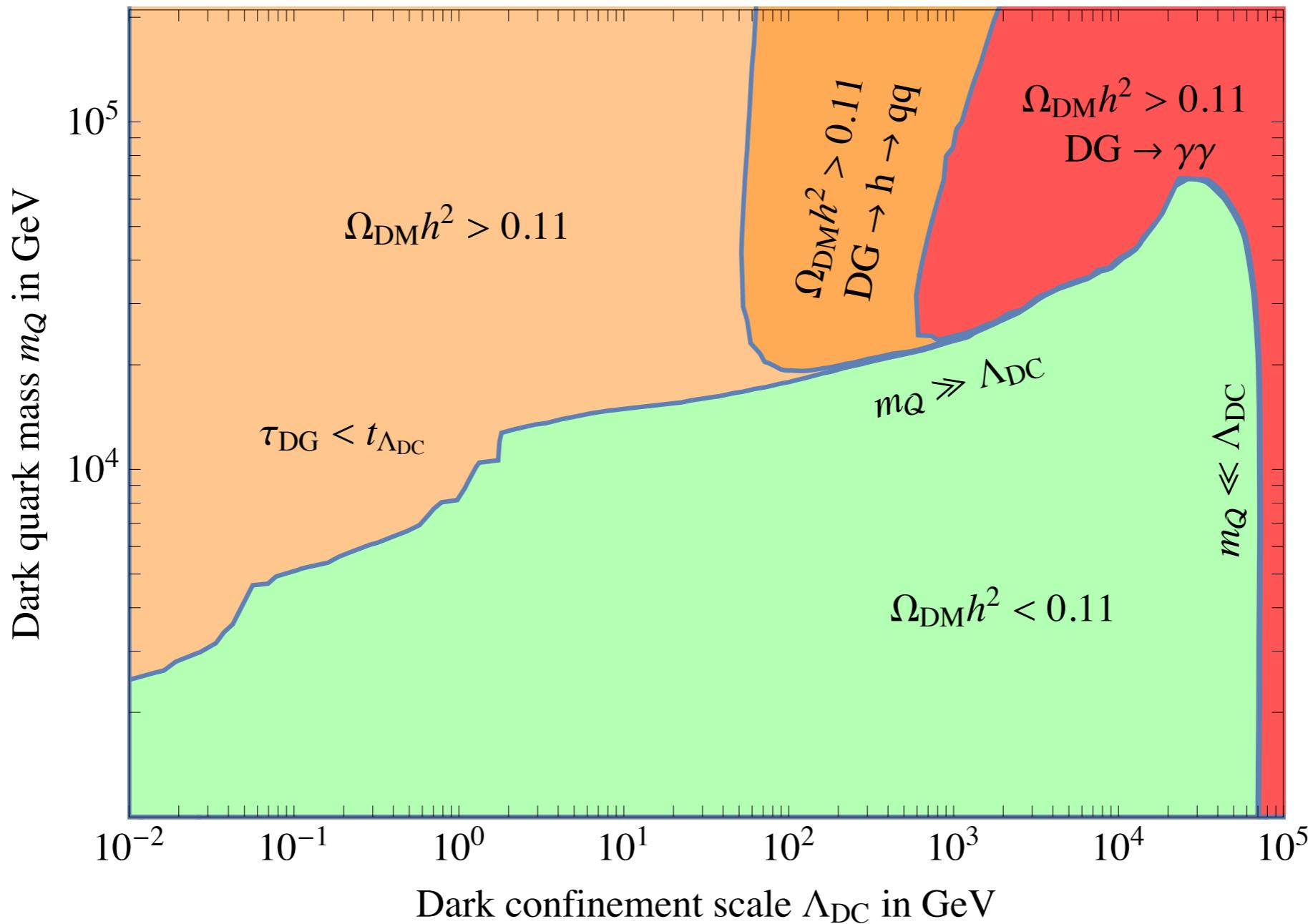
$$\langle\sigma v\rangle = \frac{4\pi\alpha_{eff}^2}{m_B^2} + \langle\sigma v\rangle_{SM} \quad m_Q < 100 \text{ TeV}$$

- $\Lambda_{DC} < \frac{m_Q}{25}$

Dark quarks free-out in the perturbative regime. A fraction recombines into baryons after dark confinement.

$$\Omega_{DM} = p_B \Omega_{Q+\bar{Q}} \quad p_B = \mathcal{O}(1)$$

Glueball decays may dilute abundance, late time annihilations, thermal excitement...

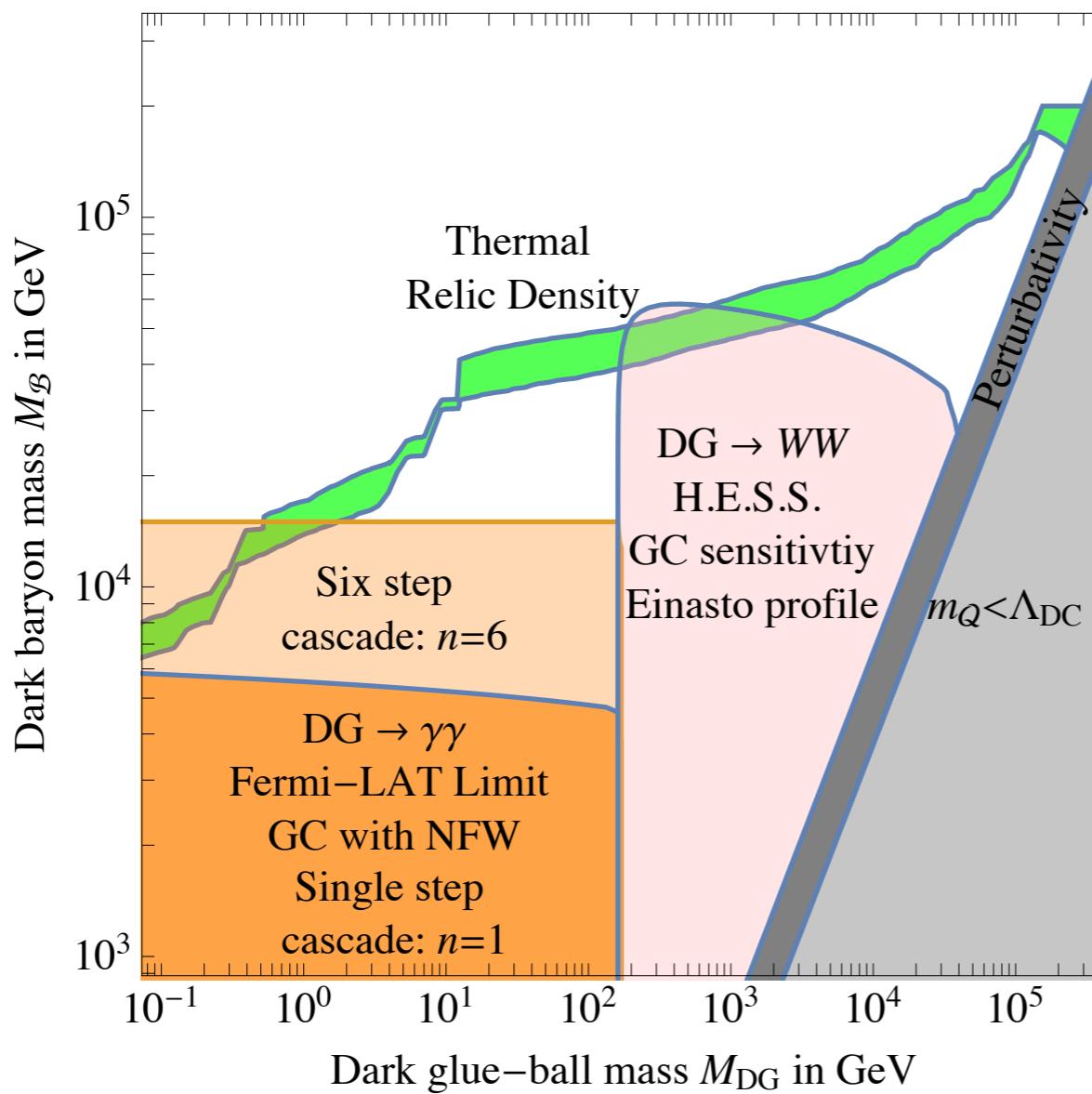


In the non-relativistic regime thermal abundance of DM can be obtained for masses down to few TeV.

Indirect detection:

At low energies annihilation cross-section can be huge for extended objects:

$$(Q^{N_{DC}}) + (\bar{Q}^{N_{DC}}) \rightarrow (Q\bar{Q}) + (Q^{N_{DC}-1})(\bar{Q}^{N_{DC}-1})$$



$$\sigma_{B\bar{B}} v_{\text{rel}} \sim \frac{1}{\alpha_{DC}} \frac{\pi}{m_Q^2}$$

OTHER PHENO

(O.Antipin, MR, arxiv:1508.01112
Aguagliaro, Antipin, Becciolini, De Curtis, MR 1609.07122)

COLLIDER SIGNATURES

- $m_Q < \Lambda_{DC}$

Kilic, Okui, Sundrum '09

Goldstone bosons and vector bosons with SM charges:

$$\langle 0 | \bar{\Psi} \gamma^\mu T^a \Psi | \rho^b \rangle = -\delta^{ab} m_\rho f_\rho \epsilon^\mu$$

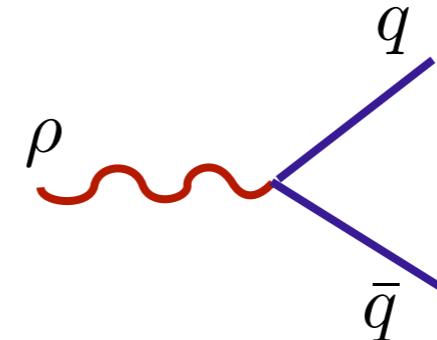
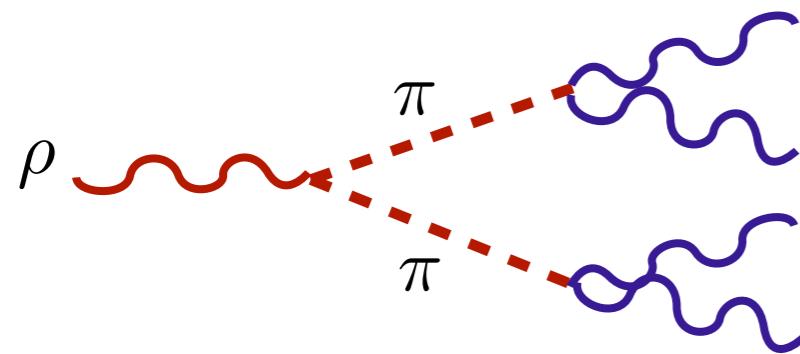
$$\langle 0 | \bar{\Psi} \gamma^\mu \gamma^5 T^a \Psi | \pi^b \rangle = -i \delta^{ab} f p^\mu$$

Heavy vectors mix with SM gauge bosons



Unlike composite Higgs fermions are elementary.

Decay to hidden pions and back to SM gauge bosons through anomalies or quarks



$$\text{Br}(\rho \rightarrow q\bar{q}) \propto \frac{g_2^4}{g_\rho^4}$$

Pions can also be collider stable or long lived.

Pions can also be produced through SM interactions

$$pp \rightarrow W^\pm \rightarrow \pi_3^\pm \pi_3^0 \rightarrow 3\gamma + W^\pm$$

Best bound from CDF!

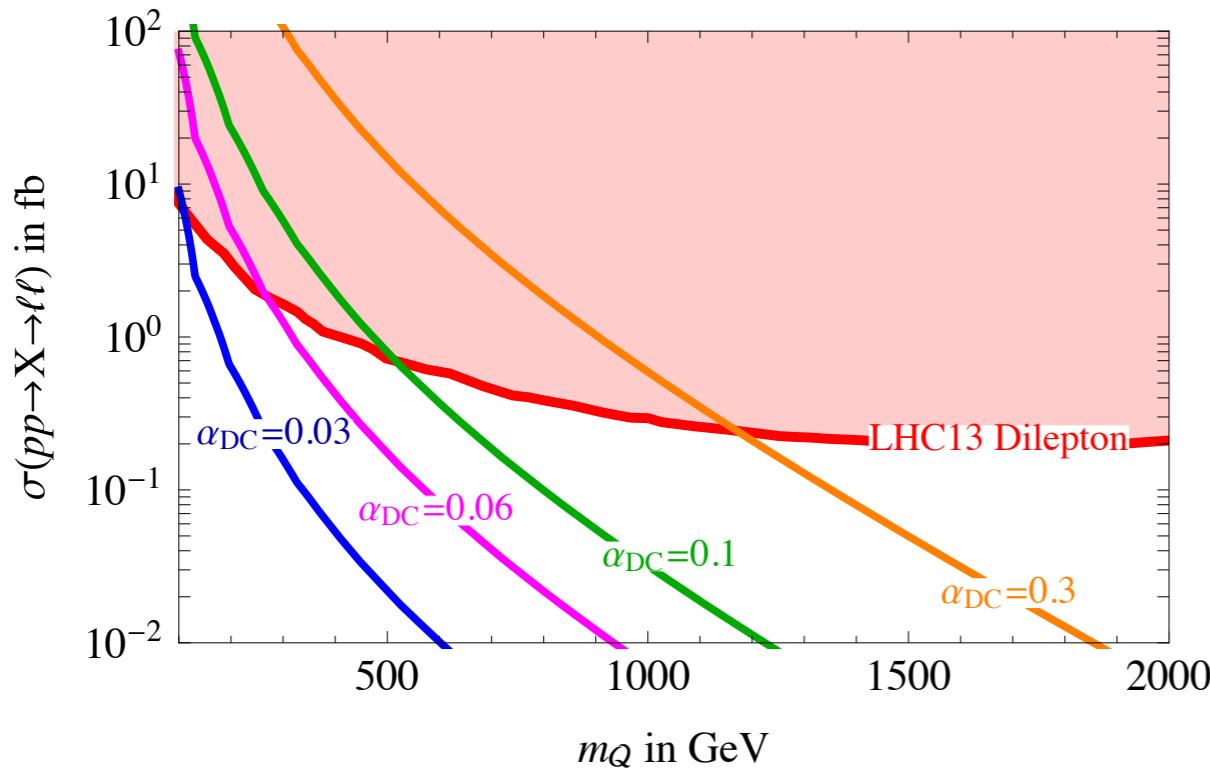
$$m_K > 230 \text{ GeV}$$

- $m_Q > \Lambda_{DC}$

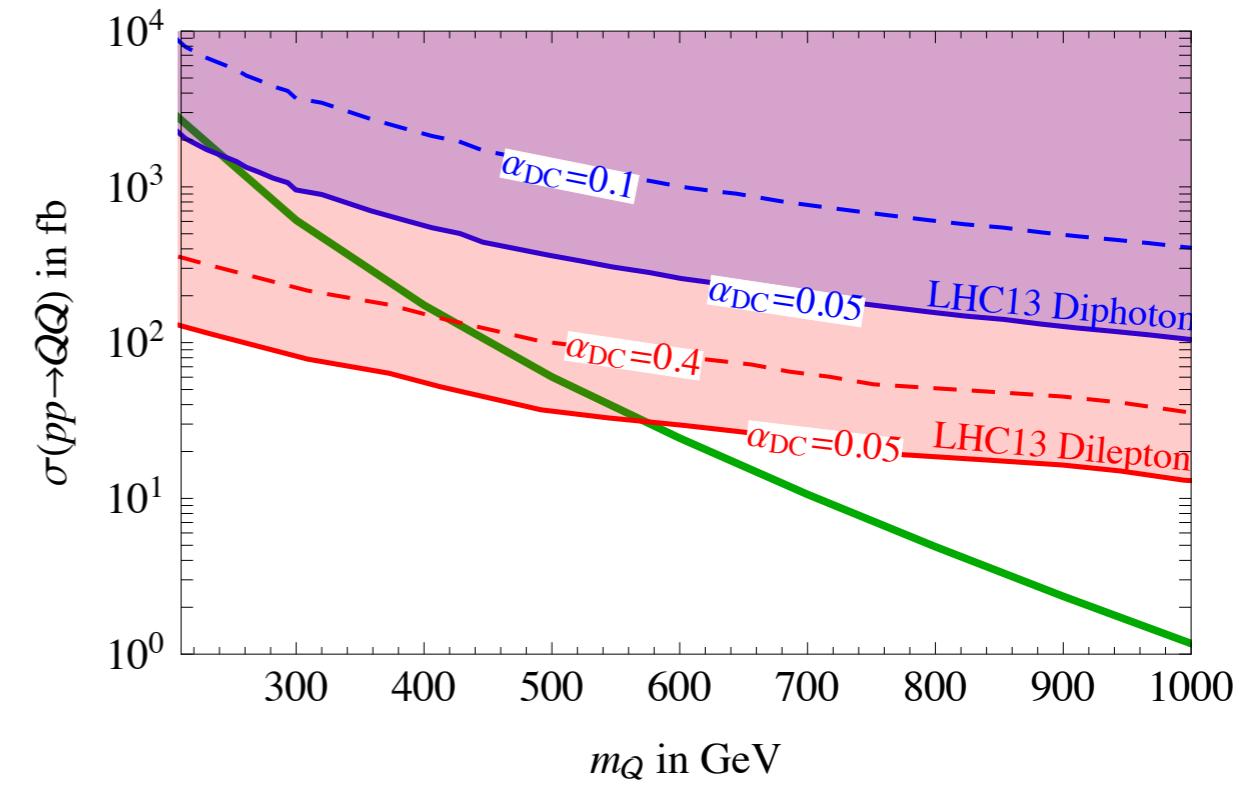
Mitridate, MR, Smirnov, Strumia '17

Mesons can be produced singly or through hadronization.
Spin-0 resonances decay to SM gauge bosons and spin-1 to fermions and scalars.

Most significant bound from resonant production of spin-1 particles decaying to leptons.



Single



Hadronization

PARTIALLY COMPOSITE HIGGS

Antipin, MR '15

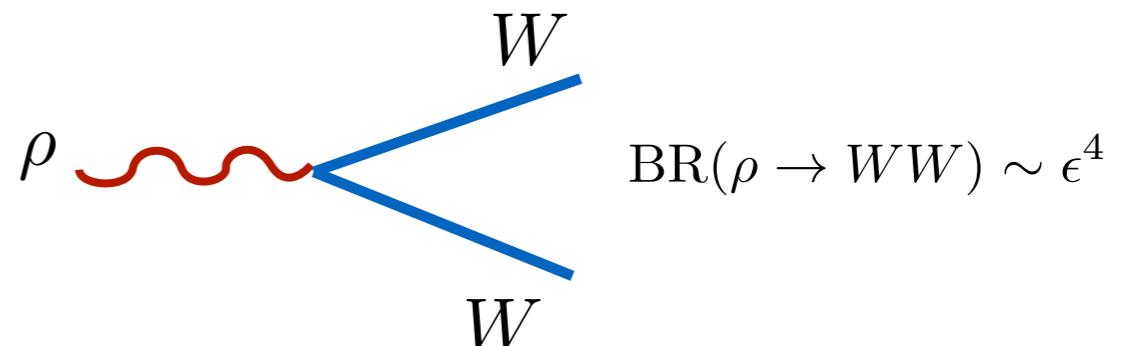
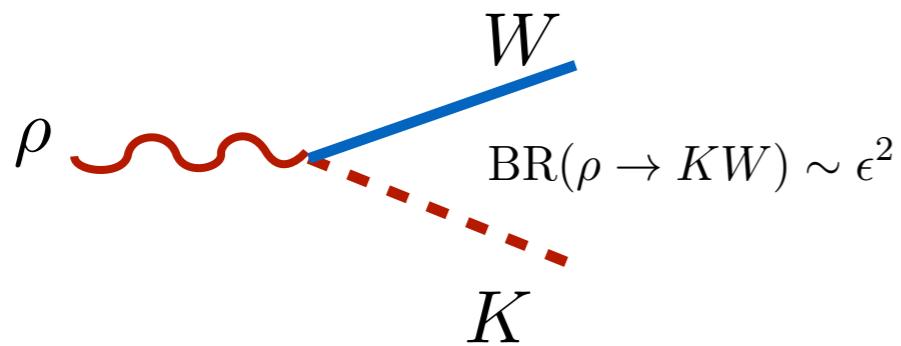
$$H \bar{Q}_i (y_{ij}^L P_L + y_{ij}^R P_R) Q_j \longrightarrow (y_L - y_R) m_\rho f H K + \dots$$

$$M^2 = \begin{pmatrix} m_H^2 & \epsilon m_K^2 \\ \epsilon^* m_K^2 & m_K^2 \end{pmatrix} \quad \epsilon \sim (y_L - y_R) \frac{m_\rho f}{m_K^2}$$

Higgs interpolates between elementary and composite.

- $\epsilon < 1$

Elementary Higgs



- Small effects in precision tests, Higgs couplings etc...

$$\Delta \hat{T} \sim \frac{v^2}{f^2} \epsilon^4$$

$$\Delta \hat{S} \sim \frac{m_W^2}{m_\rho^2} \epsilon^2$$

$$\frac{\Delta h_{WW}}{h_{WW}^{SM}} \sim \frac{v^2}{f^2} \epsilon^3$$

- Pions with species number decay through Higgs:

$$K \rightarrow H + \eta$$

- $\epsilon > 1$

Composite Higgs

Kaplan Georgi '84
Aguilaro, Becciolini,
De Curtis, MR '16

$$m_H^2 - |\epsilon|^2 m_K^2 \approx 0$$

$$m_h^{gauge} \approx 150 \sqrt{\frac{3}{N_{DC}}} \text{ GeV}$$

Elementary Higgs generates vacuum misalignment of composite Higgs!

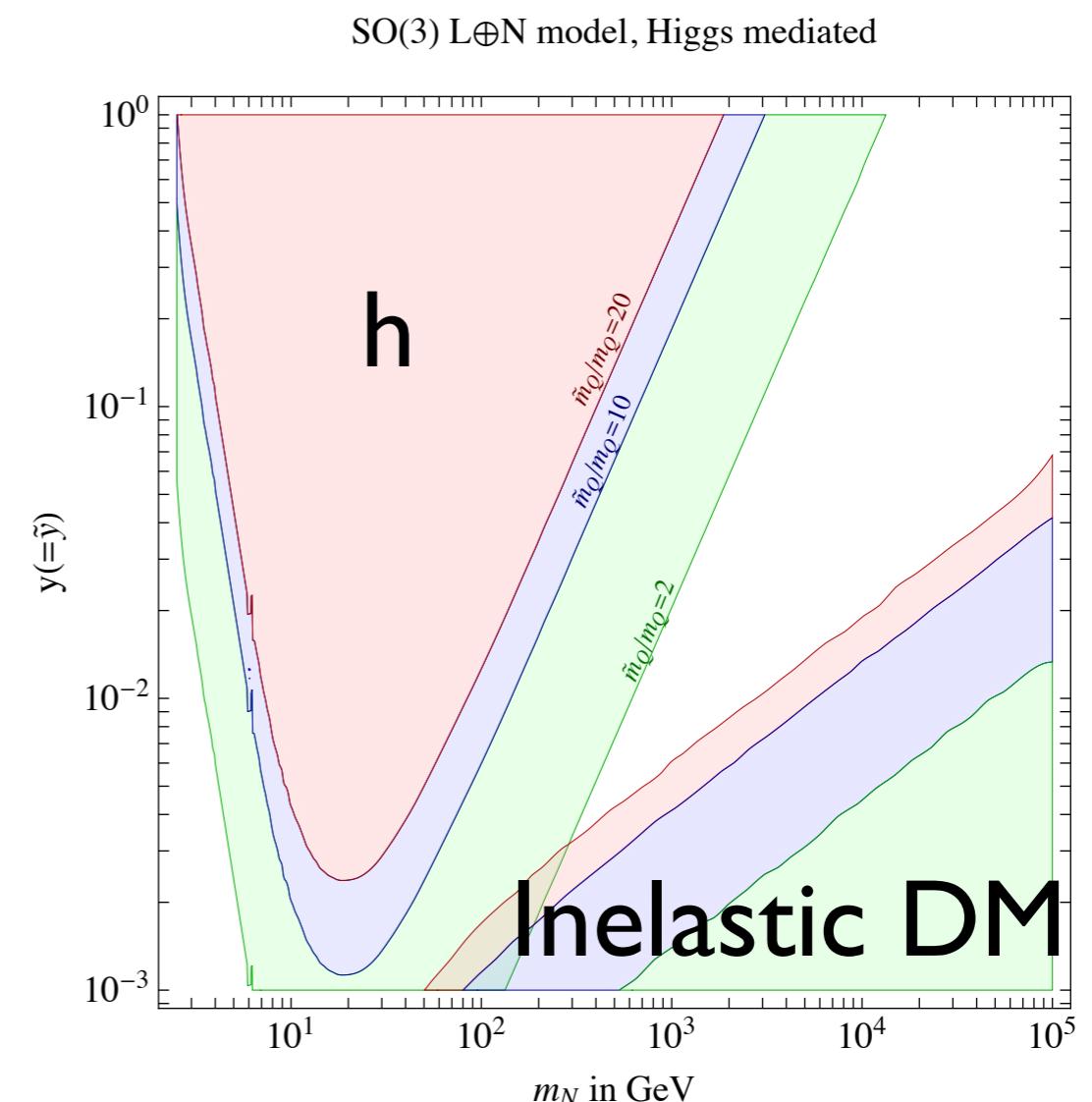
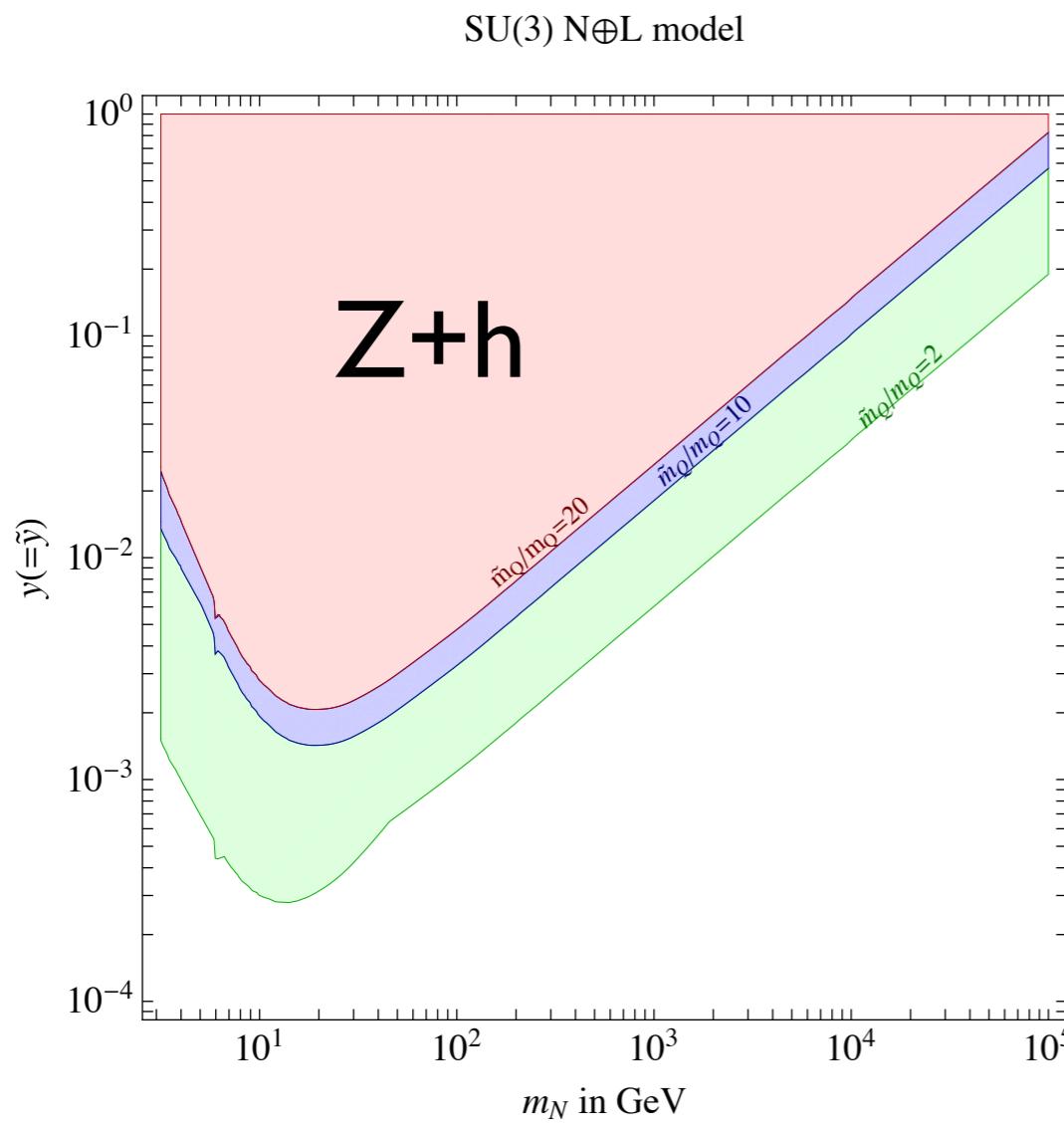
Viable UV completion of composite Higgs.
Not natural... supersymmetry? relaxion?

CONCLUSIONS

- A strongly coupled sector that does not break the SM symmetry is a “natural” extension of the SM compatible with current data and potentially relevant for DM, collider...
- SU(N) models generate complex DM while SO(N) models give real DM with very different phenomenology depending on the confinement scale. Thermal abundance of DM is obtained for masses 1-100 TeV. With heavy quarks cosmology is non standard.
- Dark color could be accessible to colliders. Interesting effects include: resonance production, long lived glueballs, compositeness, EDMs, gravitational waves, unification...

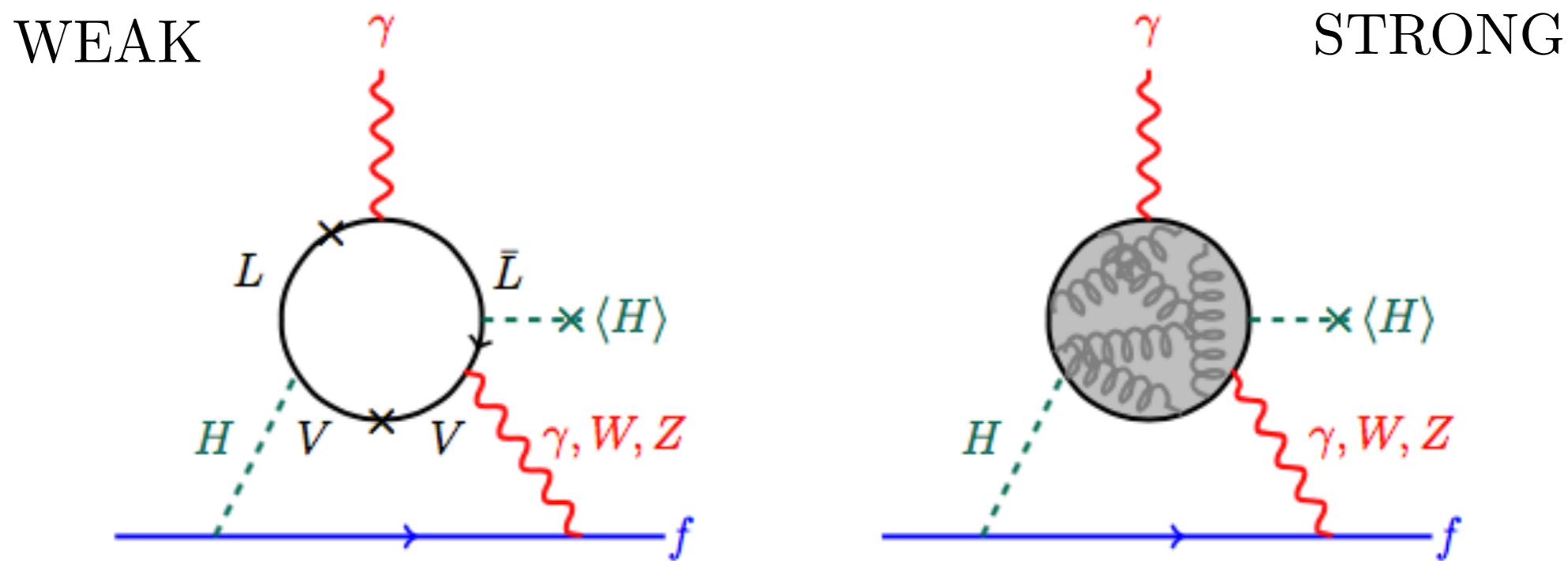
Direct detection:

- **SU(N):** Z and Higgs mediated SI scatterings
- **SO(N):** Higgs SI x-sec and Z inelastic transitions.



ELECTRIC DIPOLE MOMENTS

EDM for SM particles generated with complex Yukawas:



$$d_e \approx 10^{-27} \text{ e} \cdot \text{cm} \times \text{Im}(y_L y_R) \times \frac{N_{DC}}{3} \times \left(\frac{\text{TeV}}{m_{\pi,\eta}} \right)^2 \times \left(\frac{\Lambda_{DC}}{\text{TeV}} \right)^2$$

$$d_e < 8.7 \times 10^{-29} \text{ e cm} \quad @ 90\% \text{ C.L.}$$

GRAVITATIONAL WAVES

Confinement phase transition often 1st order:

$$3 \leq N_F \leq 4N \quad \text{and} \quad N > 3$$

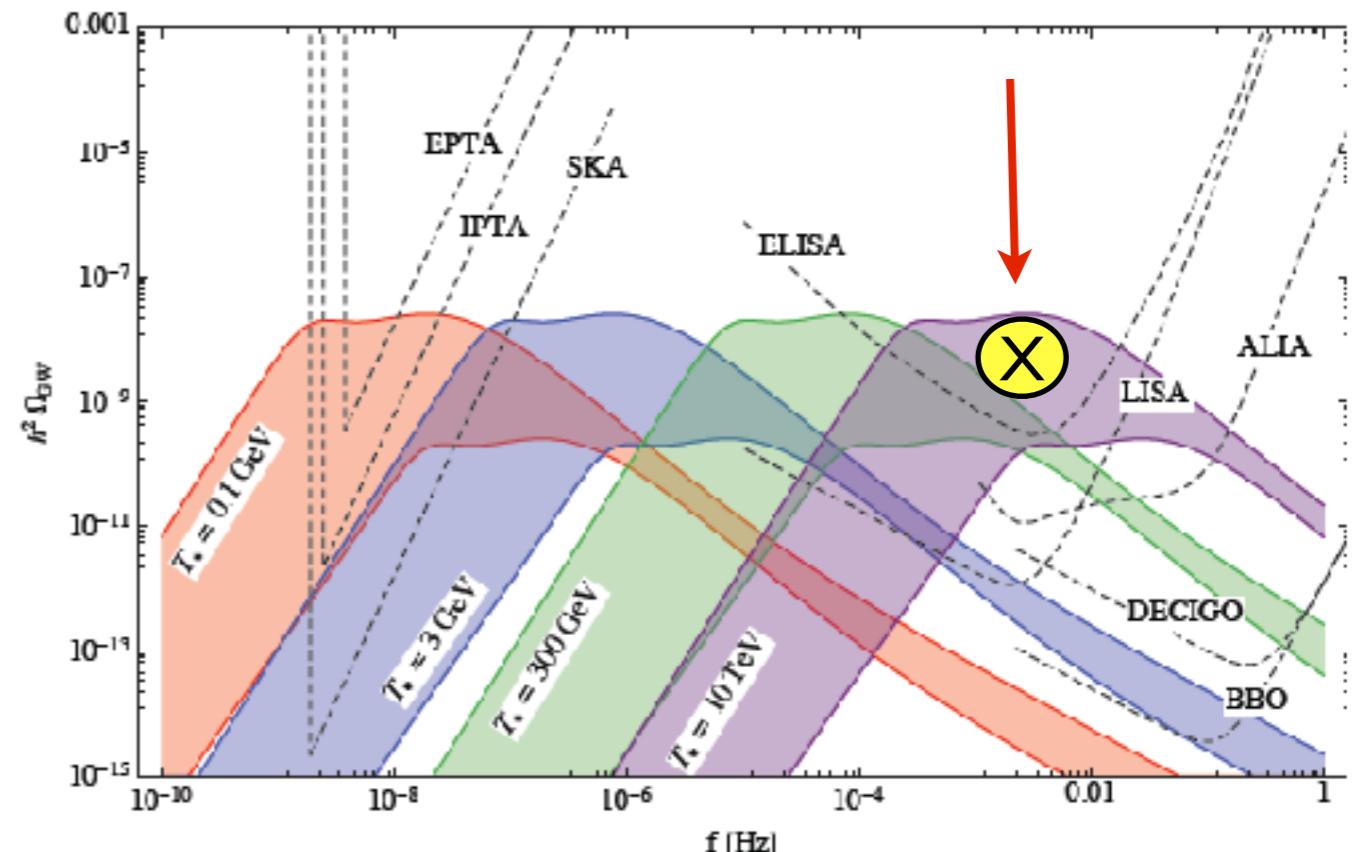
Phase transition : $T \sim \Lambda_{\text{DC}}$

Peak frequency: $f_{\text{peak}} = 3.3 \times 10^{-3} \text{ Hz} \times \left(\frac{T}{10 \text{ TeV}} \right) \times \left(\frac{\beta}{10H} \right)$

Amplitude of
the GW signal :

$$h^2 \Omega_{\text{GW}} \sim 10^{-9}$$

P. Schwaller 15'



UNIFICATION

Incomplete SU(5) reps modify SM running

SU(5)	SU(3) _c	SU(2) _L	U(1) _Y	charge	name	Δb_3	Δb_2	Δb_Y
1	1	1	0	0	N	0	0	0
$\bar{5}$	$\bar{3}$	1	1/3	1/3	D	1/3	0	2/9
	1	2	-1/2	0, -1	L	0	1/3	1/3
10	$\bar{3}$	1	-2/3	-2/3	U	1/3	0	8/9
	1	1	1	1	E	0	0	2/3
	3	2	1/6	2/3, -1/3	Q	2/3	1	1/9
15	3	2	1/6	2/3, -1/3	Q	2/3	1	1/9
	1	3	1	0, 1, 2	T	0	4/3	2
	6	1	-2/3	-2/3	S	5/3	0	8/9
24	1	3	0	-1, 0, 1	V	0	4/3	0
	8	1	0	0	G	2	0	0
	$\bar{3}$	2	5/6	4/3, 1/3	X	2/3	1	25/9
	1	1	0	0	N	0	0	0

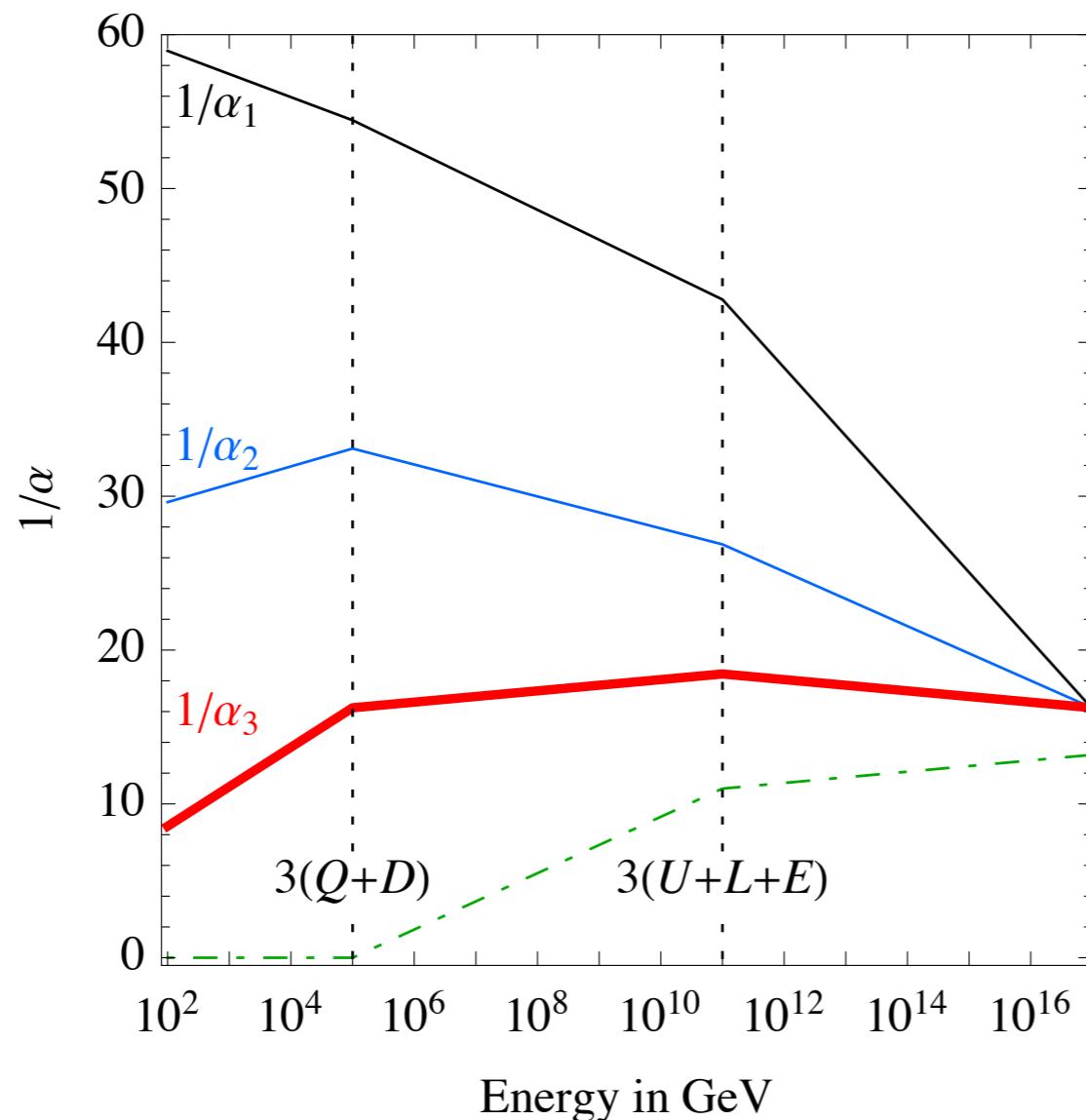
$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_{\text{GUT}}} + \frac{b_i^{\text{SM}}}{2\pi} \log \frac{M_{\text{GUT}}}{M_Z} + \frac{\Delta b_i}{2\pi} \log \frac{M_X}{\Lambda_{\text{TC}}} + \frac{\Delta b}{2\pi} \log \frac{M_{\text{GUT}}}{M_X}$$

$$\ln \frac{M_X}{\Lambda_{\text{TC}}} = \frac{68}{\Delta b_{21} - 1.9\Delta b_{32}}, \quad \ln \frac{M_{\text{GUT}}}{M_X} = \frac{35.3\Delta b_{21} - 49.2\Delta b_{32}}{\Delta b_{21} - 1.9\Delta b_{32}}$$

Ex:

$$Q + \tilde{D}$$

$$\text{DM} = QQ\tilde{D}$$

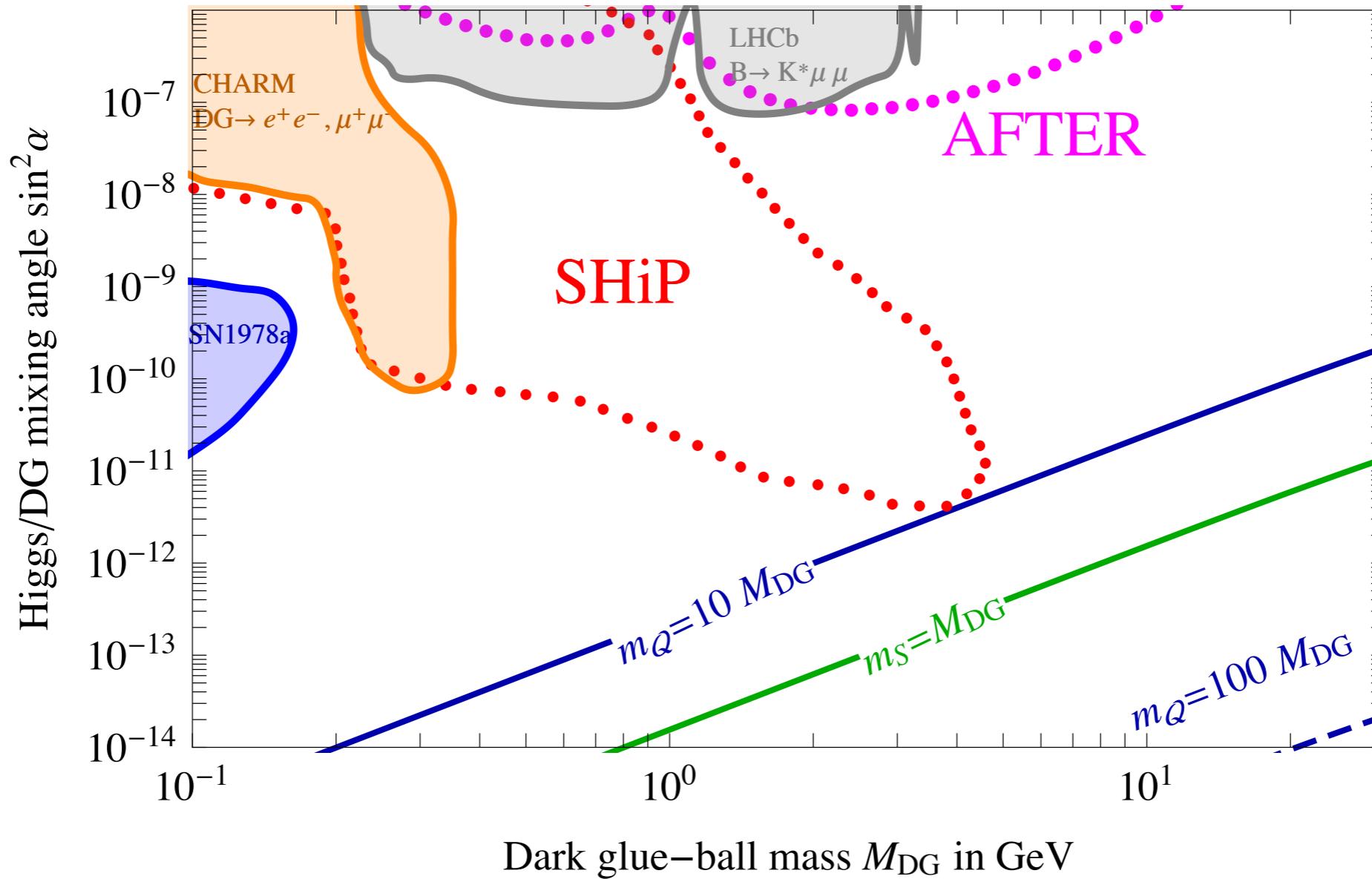


$$\alpha_{\text{GUT}} \approx 0.06$$

$$M_{\text{GUT}} \approx 2 \times 10^{17} \text{ GeV}$$

$$\Lambda_{DC} = 100 \text{ TeV} \quad M_X \approx 2 \times 10^{11} \text{ GeV}$$

SHIP



$$\mathcal{O}_6 = \frac{\alpha_{\text{DC}}}{4\pi} H^\dagger H \mathcal{G}_{\mu\nu}^A \mathcal{G}^{\mu\nu A}$$

$$\sin \alpha \approx c_6 \frac{\alpha_{DC}}{4\pi} \frac{v f_{0S}}{M_h^2}$$