Hyperons in nuclear matter studied in chiral EFT

Johann Haidenbauer

IAS & JCHP, Forschungszentrum Jülich, Germany

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Introduction

YN interaction in chiral effective field theory

- ΛN and ΣN scattering
  $\rightarrow$ Role of SU(3) flavor symmetry
- Few-body systems with hyperons: ${}^{3}_{\Lambda}\mathrm{H}$, ${}^{4}_{\Lambda}\mathrm{H}$, ${}^{4}_{\Lambda}\mathrm{He}$
  $\rightarrow$ Role of three-body forces
- (Λ, Σ) hypernuclei and hyperons in nuclear matter
  $\rightarrow$ very small spin-orbit splitting: weak spin-orbit force
  $\rightarrow$ repulsive Σ nuclear potential
- implications for astrophysics
  $\rightarrow$ hyperon stars
  stability/size of neutron stars

YN interaction in chiral effective field theory

We follow the scheme of S. Weinberg (1990)
in complete analogy to the χEFT study of the NN interaction by
E. Epelbaum, W. Glöckle, U.-G. Meißner (NPA 671 (2000) 295)

Advantages:

- Power counting
  systematic improvement by going to higher order
- Possibility to derive two- and three baryon forces and external current operators in a consistent way

ΛN, ΣN data base is rather poor $\rightarrow$ impose $SU(3)_f$ constraints

- only about 40 data points
- no polarization data $\Rightarrow$ no phase shift analysis
- constraints from hypernuclei ($^3_\Lambda$H binding energy) are needed

consider the YN interaction in χEFT up to NLO
(contact terms, one- and two-pseudoscalar-meson exchange)

H. Polinder, J.H., U. Meißner, NPA 779 (2006) 244

J.H., N. Kaiser, U. Meißner, A. Nogga, S. Petschauer, W. Weise, NPA 915 (2013) 24

Contact terms for YN – partial-wave projected

spin-momentum structure up to NLO

$$
\begin{aligned}
V(^1S_0) &= \tilde{C}_{^1S_0} + C_{^1S_0}(p^2 + p'^2) \\
V(^3S_1) &= \tilde{C}_{^3S_1} + C_{^3S_1}(p^2 + p'^2) \\
V(^1P_1) &= C_{^1P_1}\, p\, p' \\
V(^3P_1) &= C_{^3P_1}\, p\, p' \\
V(^3P_0) &= C_{^3P_0}\, p\, p' \\
V(^3P_2) &= C_{^3P_2}\, p\, p' \\
V(^3D_1 - {}^3S_1) &= C_{^3S_1 - {}^3D_1}\, p'^2 \\
\\
V(^1P_1 - {}^3P_1) &= C_{^1P_1 - {}^3P_1}\, p\, p' \\
V(^3P_1 - {}^1P_1) &= C_{^3P_1 - {}^1P_1}\, p\, p'
\end{aligned}
$$

(antisymmetric spin-orbit force: $(\vec{\sigma}_1 - \vec{\sigma}_2) \cdot (\vec{q} \times \vec{k})$)

- fix the LECs by a fit directly to YN data

$SU(3)$ structure of contact terms for BB

$SU(3)$ structure for scattering of two octet baryons $\rightarrow$
$8 \otimes 8 = 1 \oplus 8_a \oplus 8_s \oplus 10^* \oplus 10 \oplus 27$

BB interaction can be given in terms of LECs corresponding to the $SU(3)_f$ irreducible representations: C^1, C^{8a}, C^{8s}, C^{10^*}, C^{10}, C^{27}

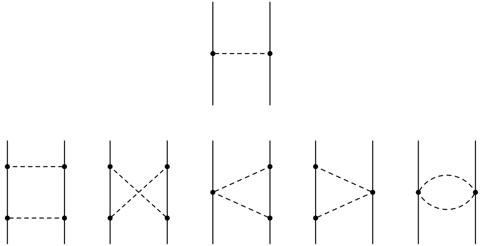
	Channel	I	V_α	V_β	$V_{\beta\to\alpha}$
$S = 0$	$NN \rightarrow NN$	0	–	$C^{10^*}_\beta$	–
	$NN \rightarrow NN$	1	C^{27}_α	–	–
$S = -1$	$\Lambda N \rightarrow \Lambda N$	$\frac{1}{2}$	$\frac{1}{10}\left(9C^{27}_\alpha + C^{8_s}_\alpha\right)$	$\frac{1}{2}\left(C^{8_a}_\beta + C^{10^*}_\beta\right)$	$-C^{8_{sa}}$
	$\Lambda N \rightarrow \Sigma N$	$\frac{1}{2}$	$\frac{3}{10}\left(-C^{27}_\alpha + C^{8_s}_\alpha\right)$	$\frac{1}{2}\left(-C^{8_a}_\beta + C^{10^*}_\beta\right)$	$-3C^{8_{sa}}$
					$C^{8_{sa}}$
	$\Sigma N \rightarrow \Sigma N$	$\frac{1}{2}$	$\frac{1}{10}\left(C^{27}_\alpha + 9C^{8_s}_\alpha\right)$	$\frac{1}{2}\left(C^{8_a}_\beta + C^{10^*}_\beta\right)$	$3C^{8_{sa}}$
	$\Sigma N \rightarrow \Sigma N$	$\frac{3}{2}$	C^{27}_α	C^{10}_β	–

$\alpha = {}^1S_0, {}^3P_0, {}^3P_1, {}^3P_2$, $\beta = {}^3S_1, {}^3S_1 - {}^3D_1, {}^1P_1$

No. of contact terms: LO: 2 (NN) + 3 (YN) + 1 (YY)
NLO: 7 (NN) + 11 (YN) + 4 (YY)

(No. of spin-isospin channels in NN+YN: 10)

One- plus two-ps-meson exchange diagrams



$$V^{OBE}_{B_1 B_2 \to B_1' B_2'} = -f_{B_1 B_1' P} f_{B_2 B_2' P} \frac{(\vec{\sigma}_1 \cdot \vec{q})\,(\vec{\sigma}_2 \cdot \vec{q})}{\vec{q}^{\,2} + m_P^2}$$

$f_{B_1 B_1' P}$ ... coupling constants fulfil standard SU(3) relations
m_P ... mass of the exchanged pseudoscalar meson

$SU(3)$ symmetry breaking due to the mass splitting of the ps mesons
(m_π = 138.0 MeV, m_K = 495.7 MeV, m_η = 547.3 MeV)
taken into account already at LO!

$\Rightarrow$ J.H., S. Petschauer, N. Kaiser, U.-G. Meißner, A. Nogga, W. Weise, NPA 915 (2013) 24

Coupled channels Lippmann-Schwinger Equation

$$
\begin{aligned}
T^{\nu'\nu,J}_{\rho'\rho}(p',p) &= V^{\nu'\nu,J}_{\rho'\rho}(p',p) \\
&+ \sum_{\rho'',\nu''} \int_0^\infty \frac{dp''p''^2}{(2\pi)^3}\, V^{\nu'\nu'',J}_{\rho'\rho''}(p',p'')\frac{2\mu_{\rho''}}{p^2-p''^2+i\eta}\, T^{\nu''\nu,J}_{\rho''\rho}(p'',p)
\end{aligned}
$$

$\rho',\ \rho = \Lambda N,\ \Sigma N$

LS equation is solved for particle channels (in momentum space)

Coulomb interaction is included via the Vincent-Phatak method

The potential in the LS equation is cut off with the regulator function:

$$
V^{\nu'\nu,J}_{\rho'\rho}(p',p) \to f^{\Lambda}(p')V^{\nu'\nu,J}_{\rho'\rho}(p',p)f^{\Lambda}(p); \quad f^{\Lambda}(p) = e^{-(p/\Lambda)^4}
$$

consider values Λ = 550 - 700 MeV [500 - 650 MeV]

NLO calculation

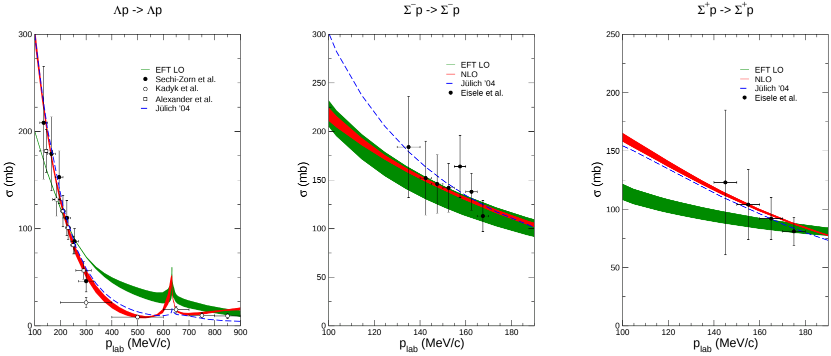
Pseudoscalar-meson exchange

- All one- and two-pseudoscalar-meson exchange diagrams are included
- $SU(3)$ symmetry is broken by using the physical m_π, m_K, and m_η
- $SU(3)$ breaking in the coupling constants is ignored
  $F_\pi = F_K = F_\eta = F_0 = 93$ MeV; $\alpha = F/(F+D) = 0.4$
- Correction to V^{OBE} due to baryon mass differences are ignored

Contact terms

- $SU(3)$ symmetry is assumed
  (at NLO $SU(3)$ breaking corrections to the LO contact terms arise!)
- 10 contact terms in S-waves
  fixed from fit to ΛN and ΣN data
  no SU(3) constraints from the NN sector are imposed!
- 12 contact terms in P-waves and in ${}^3S_1 - {}^3D_1$
  SU(3) constraints from the NN sector are imposed!
- 1 contact term in ${}^1P_1 - {}^3P_1$ (singlet-triplet mixing)
  is fixed from considering Λ-nuclear spin-orbit force in medium

YN integrated cross sections



Jülich '04: J.H., U.-G. Meißner, PRC 72 (2005) 044005

Brueckner reaction-matrix formalism

conventional non-relativistic lowest order Brueckner theory:

$$\begin{aligned}\langle YN|G_{YN}(\zeta)|YN\rangle &= \langle YN|V|YN\rangle \\ &+ \sum_{Y'N} \langle YN|V|Y'N\rangle \, \langle Y'N|\frac{Q}{\zeta - H_0}|Y'N\rangle \, \langle Y'N|G_{YN}(\zeta)|YN\rangle\end{aligned}$$

Q ... Pauli projection operator

$$\zeta = E_Y(p_Y) + E_N(p_N)$$

$$E_\alpha(p_\alpha) = M_\alpha + \frac{p_\alpha^2}{2M_\alpha} + U_\alpha(p_\alpha), \qquad \alpha = Y,\ N$$

$$U_Y(p_Y) = \int_{p_N \le k_F} d^3 p_N \, \langle YN|G_{YN}(\zeta(U_Y))|YN\rangle$$

$B_Y(\infty) = -U_Y(p_Y = 0)$ - evaluated at saturation point of nuclear matter

⇒ J.H, U.-G. Meißner, NPA 936 (2015) 29

Nuclear matter properties

Partial-wave contributions to $-U_\Lambda(p_\Lambda = 0)$ [in MeV] at $k_F = 1.35\ \mathrm{fm}^{-1}$
(for Λ = 600 MeV)

	$^1S_0$	$^3S_1 + ^3D_1$	$^3P_0$	$^1P_1 + ^3P_1$	$^3P_2 + ^3F_2$	Total
EFT LO	12.0	25.5	1.7	−3.3	0.4	36.5
EFT NLO	12.5	12.0	−0.9	−2.1	1.1	22.9
Jülich '04	9.9	35.0	0.7	0.2	3.3	49.7
Jülich '94	3.6	27.2	−0.6	−2.0	0.8	29.8

"Empirical" value for the Λ binding energy in nuclear matter:
$\approx$ 30 MeV

Jülich '94: A. Reuber, K. Holinde, J. Speth, NPA 570 (1994) 543

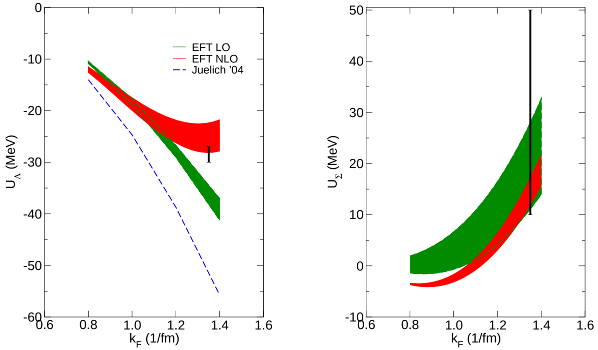
Nuclear matter properties

	EFT LO	EFT NLO	Jülich '04	Jülich '94
Λ [MeV]	$550 \cdots 700$	$500 \cdots 650$		
$-U_\Lambda(0)$	$38.0 \cdots 34.4$	$29.3 \cdots 22.9$	49.7	29.8
$-U_\Sigma(0)$	$-28.0 \cdots -11.1$	$-17.4 \cdots -12.1$	22.2	71.45

ΣN (I=3/2): ${}^3S_1$–${}^3D_1$: decisive for Σ properties in nuclear matter

- A description of YN data is possible with an attractive as well as a repulsive ${}^3S_1$–${}^3D_1$ interaction
- adopt the repulsive solution in accordance with evidence from
  - level shifts and widths of Σ^- atoms
  - (π^-, K^+) inclusive spectra related to Σ^- formation in heavy nuclei

k_F dependence of s.p. potentials



Spin-orbit interaction in Λ-hypernuclei

- Central potential: $U_\Lambda \approx \frac{1}{2} U_N$
- Spin-orbit potential: $U_\Lambda^{\ell s} \leq \frac{1}{20} U_N^{\ell s}$

  ${}^{13}_{\Lambda}\mathrm{C} \;\rightarrow\; E_\Lambda(p_{1/2^-}) - E_\Lambda(p_{3/2^-}) = (152 \pm 54 \pm 36)\ \mathrm{keV}$

  (S. Ajimura et al., PRL 86 (2001) 4255)

  ${}^{9}_{\Lambda}\mathrm{Be} \;\rightarrow\; E_\Lambda(p_{3/2^+}) - E_\Lambda(p_{5/2^+}) = (43 \pm 5)\ \mathrm{keV}$

  (H. Akikawa et al., PRL 88 (2002) 82501)

Strength of the spin-orbit potential is usually quantified in terms of the so-called Scheerbaum factor S_Λ:

$$U_\Lambda^{\ell s}(r) = -\frac{\pi}{2} S_\Lambda \frac{1}{r} \frac{d\rho(r)}{dr} \boldsymbol{\ell} \cdot \boldsymbol{\sigma}$$

$\rho(r)$ ... nucleon density distribution
ℓ ... single-particle orbital angular momentum operator

phenomenological analyses (Hiyama, Fujiwara, Kohno) of ${}^{9}_{\Lambda}\mathrm{Be}$:

$\Rightarrow -4.6 \leq S_\Lambda \leq -3.0$ MeV fm^5

- aim at $S_\Lambda \approx -3.7$ MeV fm^5 (and fix $C^{8_{as}}$ accordingly)

Partial-wave contributions to S_Λ [MeV fm^5]

$$S_\Lambda(p_\Lambda) \propto \sum_J \Big\{ (J+2)\, G^J_{1\,J+1,\,1\,J+1}(p_\Lambda) + G^J_{1J,1J}(p_\Lambda) - (J-1)\, G^J_{1\,J-1,\,1\,J-1}(p_\Lambda)$$
$$-\sqrt{J(J+1)}\, \Big[G^J_{1J,0J}(p_\Lambda) + G^J_{0J,1J}(p_\Lambda) \Big] \Big\}$$

(notation: $G^J_{S\,L,S'\,L'}$)

	$^3P_0$	$^3D_1$	$^3P_1$	$^1P_1 \leftrightarrow {^3P_1}$	$^3P_2$	Total
LO (550)	8.7	−0.2	−5.2	0.0	−0.9	2.7
LO (700)	10.4	−0.2	−4.7	0.0	−1.4	4.4
NLO (500)	−5.9	−0.6	−3.3	8.9	−3.4	−3.7
NLO (650)	−4.4	−0.6	−2.0	5.8	−3.0	−3.7
NLO†(650)	−4.4	−0.6	−4.6	0.0	−3.0	−12.0
Jülich '04	4.0	0.5	−1.3	4.6	−9.2	−1.7
Jülich '94	−3.3	1.2	−3.8	8.4	−2.5	−0.4

NLO†(650) ... with $C^{8as} = 0$

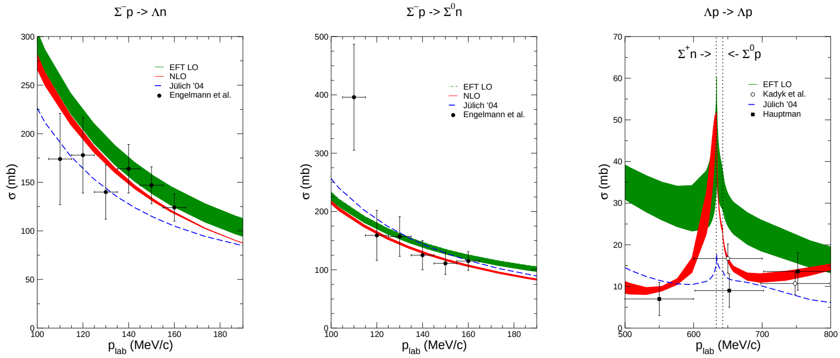
$\Rightarrow$ cancellation between spin-orbit and antisymmetric spin-orbit components

Summary

YN interaction based on chiral *EFT*

- approach is based on a modified Weinberg power counting, analogous to the NN case
- The potential (contact terms, pseudoscalar-meson exchanges) is derived imposing $SU(3)_f$ constraints
- Excellent results at next-to-leading order (NLO)
  YN data are reproduced with a quality comparable to phenomenological models
- Λ and Σ in nuclear matter:
  Λ single-particle potential at nuclear matter saturation density is in line with "empirical" value
  a repulsive Σ single-particle potential is achieved
  a weak Λ-nuclear spin-orbit potential is achieved

YN integrated cross sections



Changes in $^1P_1 - {}^3P_1$ Λp phase shifts