Aspects of Entanglement in Quantum Field Theory

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Plan of the talk

Quantum Field Theory

CFT

Renormalization Group

AdS/CFT

Black Holes physics

Condensed Matter Theory

Lattice models

Entanglement

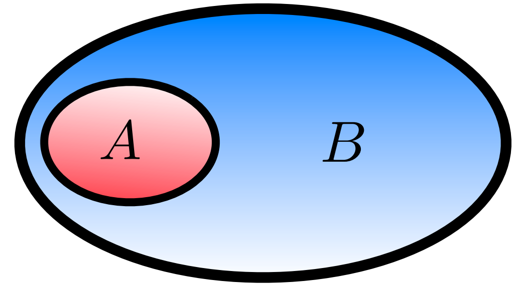
Quantum Information Theory

JPA special issue vol. 42 (2009)   eds.: P. Calabrese, J. Cardy and B. Doyon

- Entanglement in 2D CFT:
  - Entanglement entropies for disjoint intervals
  - Entanglement for mixed states: Negativity
- Entanglement in AdS/CFT:
  - Holographic entanglement entropy
  - Causal Holographic Information

Entanglement entropies: definition

- Quantum system ($\mathcal{H}$) in the ground state $|\Psi\rangle$
  Density matrix $\rho = |\Psi\rangle\langle\Psi| \quad \Longrightarrow \quad \mathrm{Tr}\rho^n = 1$



e.g.: spatial bipartition

- Hilbert space $\boxed{\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B}$

- A's **reduced density matrix** $\boxed{\rho_A = \mathrm{Tr}_B \rho}$ $(\mathrm{Tr}_A \rho_A = 1)$

  if ρ describes a pure state then ρ_A describes a mixed state

- Entanglement entropy $\equiv$ Von Neumann entropy of ρ_A

$$\boxed{S_A = -\mathrm{Tr}_A(\rho_A \log \rho_A)}$$

- Replica trick $$\boxed{S_A = \lim_{n\to 1} \boxed{\frac{\log(\mathrm{Tr}\rho_A^n)}{1-n}} = -\lim_{n\to 1} \frac{\partial}{\partial n} \mathrm{Tr}\,\rho_A^n}$$

Rényi entropies $S_A^{(n)}$

Entanglement entropy: some properties

- Bipartition $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ $\rho = |\Psi\rangle\langle\Psi|$ pure state

$$|\Psi\rangle = \sum_k c_k \, |\Psi_k\rangle_A |\Psi_k\rangle_B$$

$$c_k \geqslant 0 \qquad \sum_k c_k^2 = 1$$

Schmidt decomposition

$$\longrightarrow \quad \boxed{S_A = -\sum_k c_k^2 \log c_k^2 = S_B}$$

$\Longrightarrow$ S_A is not extensive

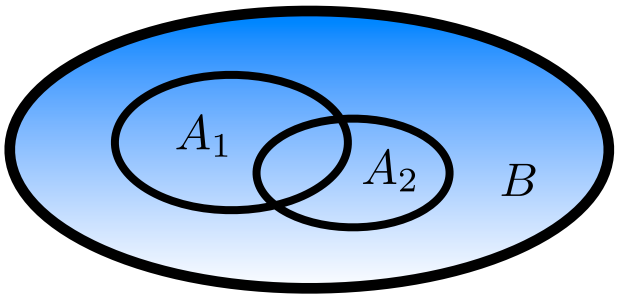
Araki-Lieb inequality

- ρ mixed state $\delta S_A \equiv S_A - S_B$

$$\boxed{|\delta S_A| \leqslant S_{A \cup B} = S_\rho}$$

- Subadditivity $A_1 \cap A_2 = \emptyset$ $S_{A_1} + S_{A_2} \geqslant S_{A_1 \cup A_2}$

- Strong Subadditivity

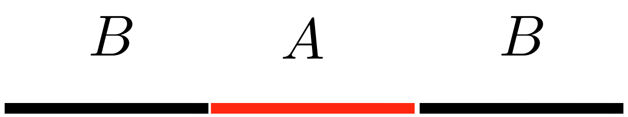


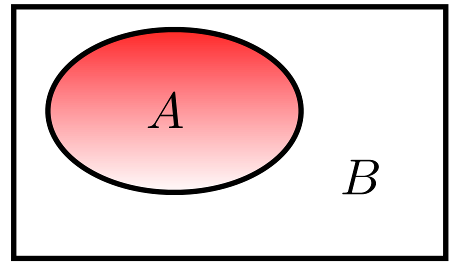
$$S_{A_1} + S_{A_2} \geqslant S_{A_1 \cup A_2} + S_{A_1 \cap A_2}$$

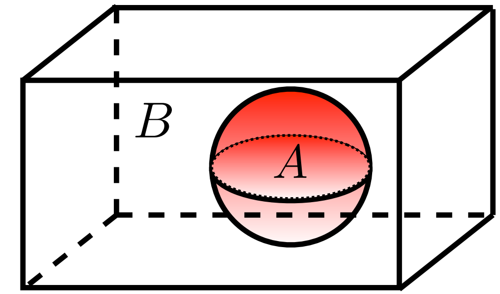
$$S_{A_1} + S_{A_2} \geqslant S_{A_1 \setminus A_2} + S_{A_2 \setminus A_1}$$

Geometric entropy: area law

- Assume that A and B correspond to a *spatial bipartition* of the system







- *Area law:* In d spatial dimensions when $\rho = |\Psi\rangle\langle\Psi|$ $(S_A = S_{A^c})$

$$\boxed{S_A \propto \frac{\text{Area}(\partial A)}{a^{d-1}} + \dots}$$

[Bombelli, Koul, Lee, Sorkin, (1986)]
[Srednicki, (1993)]

In $1+1$ CFTs at $T = 0$

[Holzhey, Larsen, Wilczek, (1994)]
[Calabrese, Cardy, (2004)]

$$\boxed{S_A = \frac{c}{3} \log \frac{\ell}{a} + \text{const}}$$

In $2+1$ CFTs for a circle

[Myers, Sinha, (2011)]
[Klebanov, Pufu, Safdi, (2011)]
[Casini, Huerta, (2012)]

$$\boxed{S_A = \gamma \frac{2\pi R}{a} - f}$$

- Area law violated in presence of Fermi surfaces: $S_A \sim L^{d-1} \log L$

[Wolf, (2005)] [Gioev, Klich, (2005)]

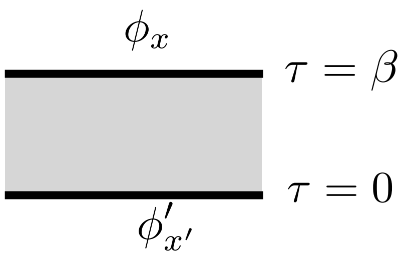
Replica trick for the entanglement entropy

$$S_A = -\mathrm{Tr}(\rho_A \log \rho_A) = \lim_{n\to 1} \frac{\log(\mathrm{Tr}\rho_A^n)}{1-n} = -\lim_{n\to 1} \frac{\partial}{\partial n} \mathrm{Tr}\,\rho_A^n \qquad \mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

Rényi entropies

[Callan, Wilczek, (1994)]
[Holzhey, Larsen, Wilczek, (1994)]
[Calabrese, Cardy, (2004)]

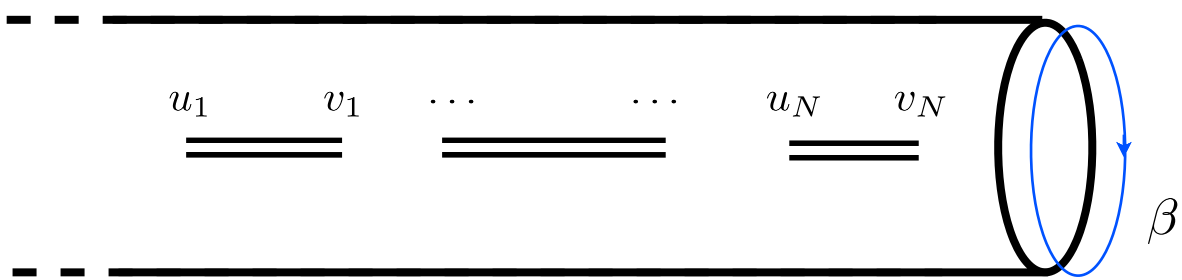
- Example: density matrix ρ in a thermal state at temperature $T = 1/\beta$



$$\mathcal{Z} = \mathrm{Tr}\, e^{-\beta H}$$

The trace sews together the edges at $\tau = 0$ and $\tau = \beta$ providing a cylinder with circumference of length β.

- $\rho_A = \mathrm{Tr}_B \rho$ $\qquad A = (u_1, v_1) \cup \cdots \cup (u_N, v_N)$



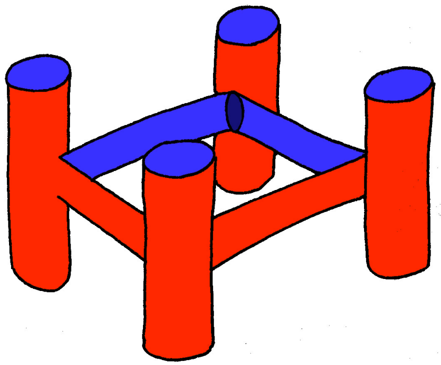
The trace Tr_B sews together only the points $\notin A$.
Open cuts are left along the disjoint intervals (u_j, v_j).

Entanglement entropies & Riemann surfaces

- n copies sewed together cyclically along the cuts

$$\boxed{\mathrm{Tr}\rho_A^n = \frac{\mathcal{Z}_{N,n}}{\mathcal{Z}^n}}$$

$$\text{"}\rho_A^{ij}\,\rho_A^{jk}\,\rho_A^{kl}\,\rho_A^{li}\text{"} \;=$$



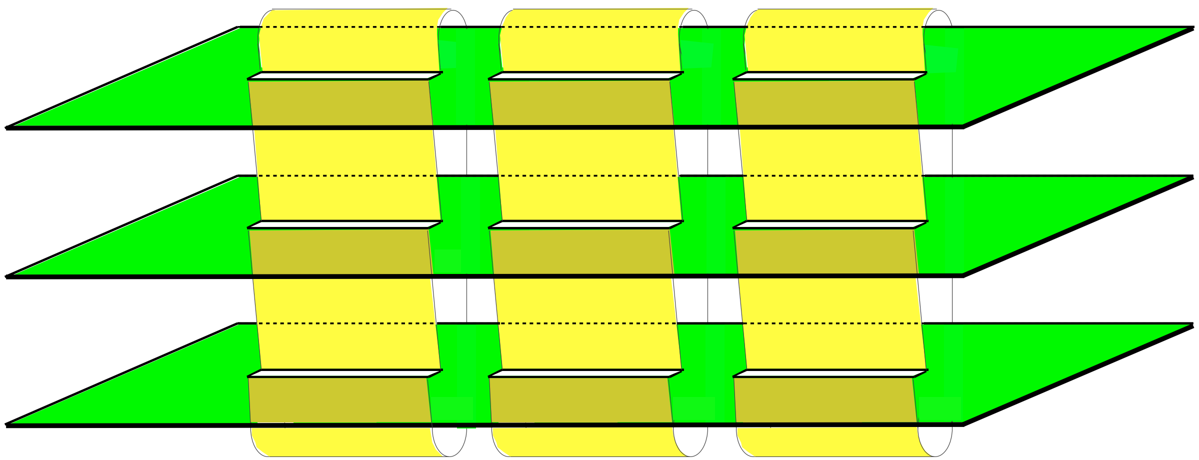
- Partition function on the n sheeted Riemann surface $\mathcal{R}_{N,n}$

$$\mathcal{Z}_{N,n} = \int_{\mathcal{C}_A} [d\varphi_1 \cdots d\varphi_n]_{\mathbf{C}} \; \exp\left[-\int_{\mathbf{C}} dz d\bar{z}\,(\mathcal{L}[\varphi_1](z,\bar{z}) + \ldots + \mathcal{L}[\varphi_n](z,\bar{z}))\right]$$

$$\mathcal{C}_A: \quad \varphi_i(x,0^+) = \varphi_{i+1}(x,0^-) \qquad x \in A = \cup_{j=1}^N A_j \qquad i = 1,\ldots,n$$

N cuts
n sheets

$\mathcal{R}_{3,3}$



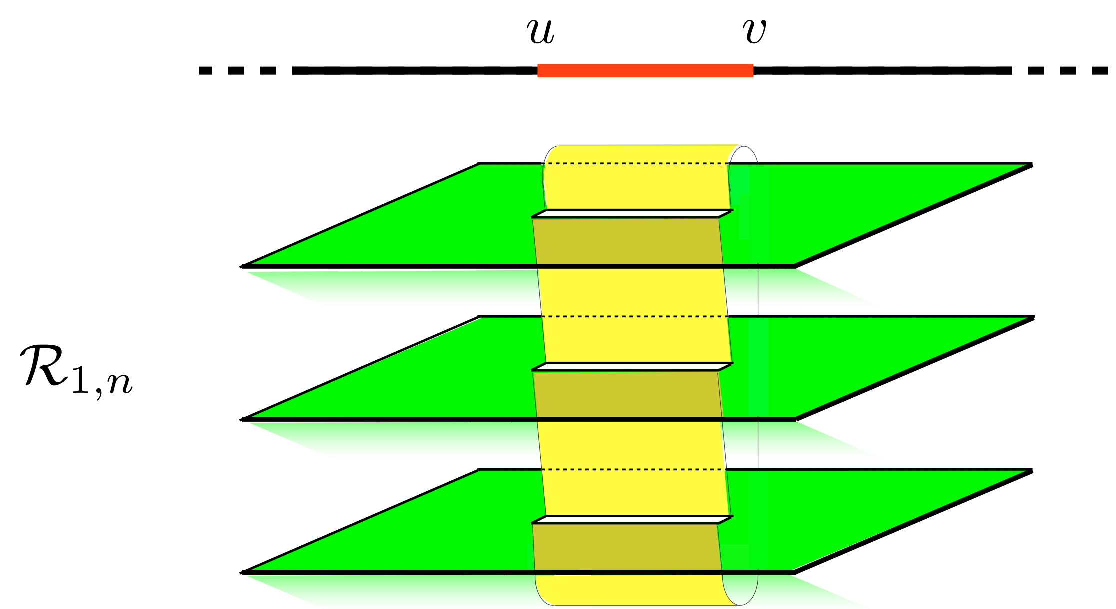
$$\boxed{g = (N-1)(n-1)}$$

- Global symmetry $i \mapsto i+1 \bmod n$ from replication

2D CFT: Entanglement entropies as correlation functions

- For one interval ($N = 1$) the Renyi entropies can be written as a two point function of *twist fields* $\mathcal{T}_n$, $\bar{\mathcal{T}}_n$ on the sphere

  [Calabrese, Cardy, (2004)]
  [Casini, Fosco, Huerta, (2005)] [Ryu, Takayanagi, (2006)] [Cardy, Castro-Alvaredo, Doyon, (2008)]



$$\Delta_n = \frac{c}{12}\left(n - \frac{1}{n}\right)$$

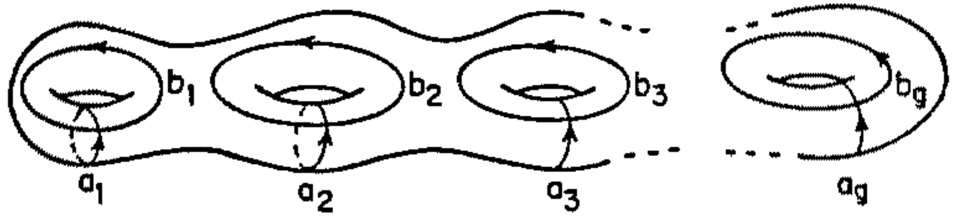
$$\mathrm{Tr}\rho_A^n = \frac{\mathcal{Z}_{1,n}}{\mathcal{Z}^n} = \langle \mathcal{T}_n(u)\bar{\mathcal{T}}_n(v)\rangle = \frac{c_n}{|u-v|^{2\Delta_n}} \quad \Longrightarrow \quad S_A = \frac{c}{3}\log\frac{\ell}{\epsilon} + c'_1$$

- Twist fields have been studied long ago in string theory

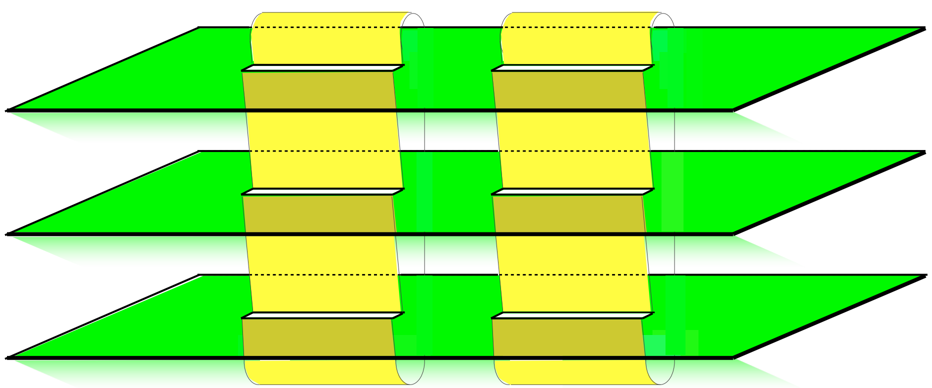
  [Zamolodchikov, (1987)] [Dixon, Friedan, Martinec, Shenker, (1987)]
  [Knizhnik, (1987)] [Bershadsky, Radul, (1987)]

N=2: higher genus Riemann surfaces from replication

- For many disjoint intervals higher genus Riemann surfaces occur



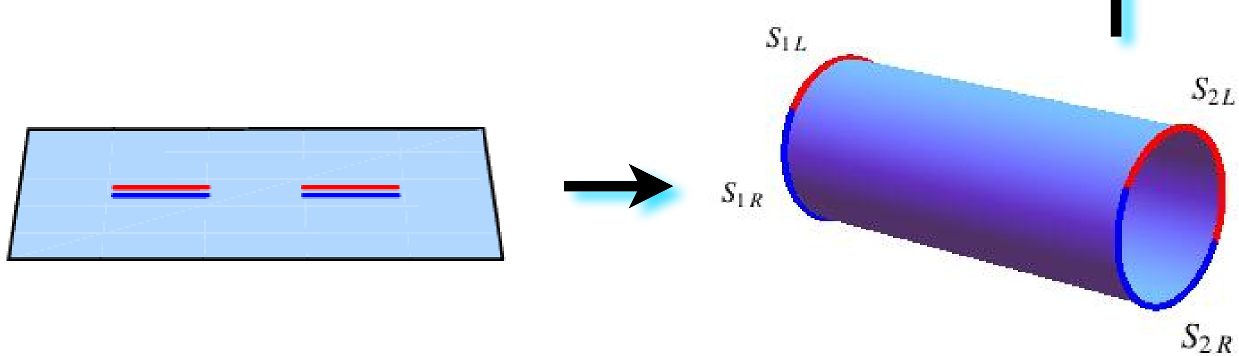
$3g - 3$ complex moduli for $g \geqslant 2$

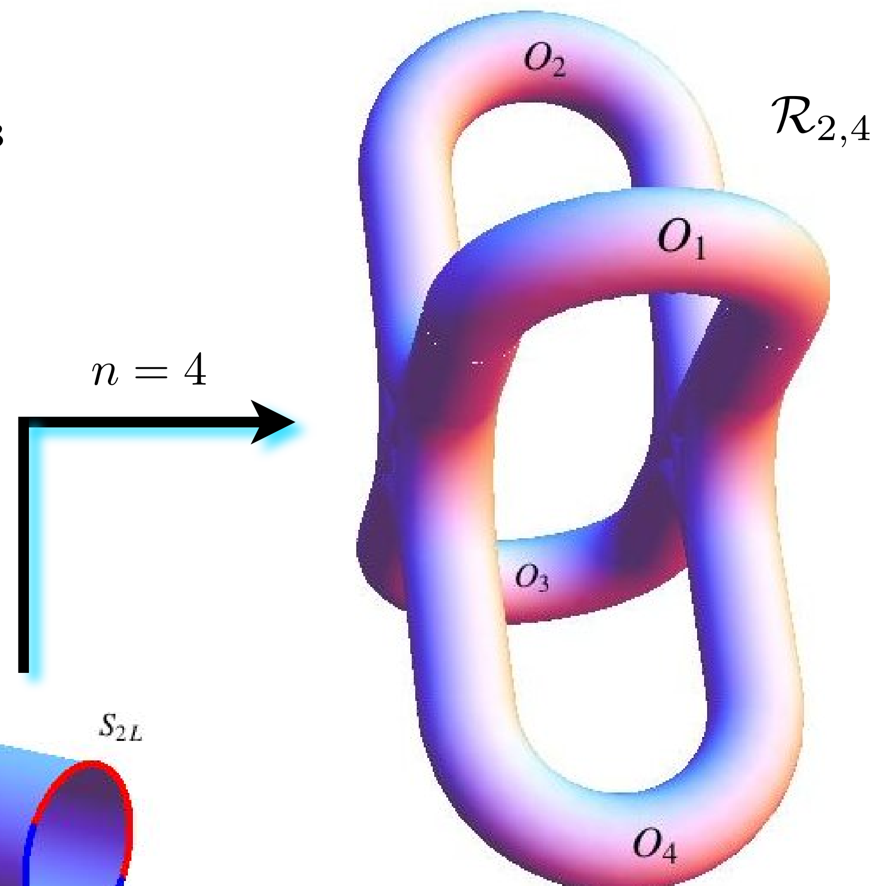
- 

$\mathcal{R}_{2,3}$

We are dealing with a subclass of Riemann surfaces of genus $g = (N-1)(n-1)$ obtained from replication

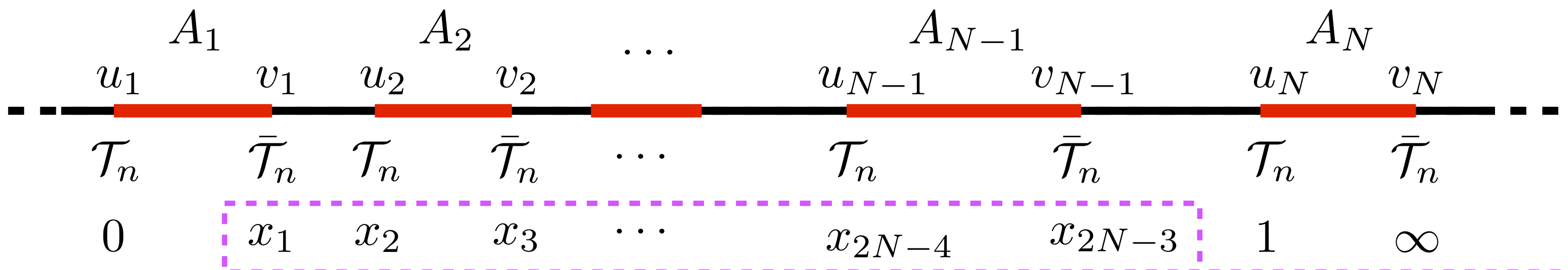
Indeed $\tau = \tau(\boldsymbol{x})$





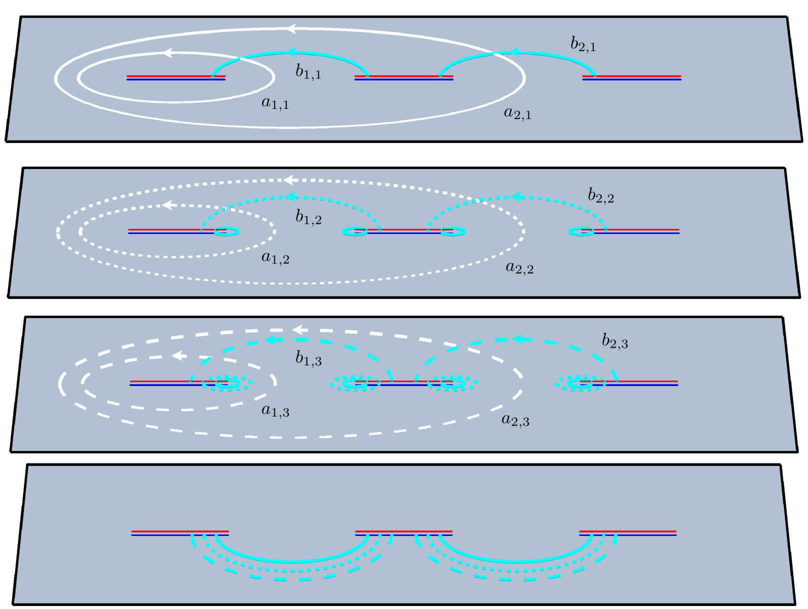
2D CFT: Renyi entropies for many disjoint intervals

N disjoint intervals $\Longrightarrow$ $2N$ point function of twist fields

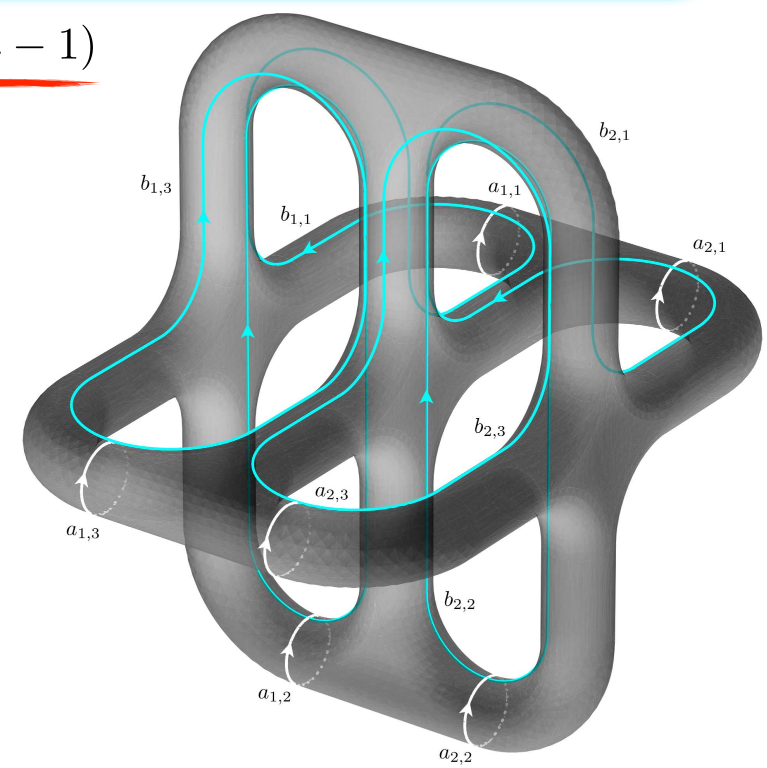


$$\mathrm{Tr}\rho_A^n = \frac{\mathcal{Z}_{N,n}}{\mathcal{Z}^n} = \langle \prod_{i=1}^{N} \mathcal{T}_n(u_i)\bar{\mathcal{T}}_n(v_i)\rangle = c_n^N \left| \frac{\prod_{i<j}(u_j-u_i)(v_j-v_i)}{\prod_{i,j}(v_j-u_i)} \right|^{2\Delta_n} \mathcal{F}_{N,n}(\boldsymbol{x})$$

$\mathcal{Z}_{N,n}$ is the partition function of $\mathcal{R}_{N,n}$ of genus $g=(N-1)(n-1)$



$\mathcal{R}_{3,4}$



Free compactified boson & Ising model

- $\mathcal{R}_{N,n}$ is $\boxed{y^n = \prod_{\gamma=1}^{N}(z - x_{2\gamma-2}) \left[\prod_{\gamma=1}^{N-1}(z - x_{2\gamma-1})\right]^{n-1}}$ $\quad g = (N-1)(n-1)$ [Enolski, Grava, (2003)]

- Partition function for a generic Riemann surface studied long ago in string theory [Zamolodchikov, (1987)] [Alvarez-Gaume, Moore, Vafa, (1986)] [Dijkgraaf, Verlinde, Verlinde, (1988)]

- Free compactified boson ($\eta \propto R^2$) [Coser, Tagliacozzo, E.T., (2013)]

$$\boxed{\mathcal{F}_{N,n}(\boldsymbol{x}) = \frac{\Theta(\mathbf{0}|T_\eta)}{|\Theta(\mathbf{0}|\tau)|^2} \qquad T_\eta = \begin{pmatrix} \mathrm{i}\,\eta\,\mathcal{I} & \mathcal{R} \\ \mathcal{R} & \mathrm{i}\,\mathcal{I}/\eta \end{pmatrix} \qquad \begin{matrix} \tau = \mathcal{R} + \mathrm{i}\,\mathcal{I} \\ \text{period matrix} \end{matrix}}$$

- Riemann theta function with characteristic $\quad \Theta[\boldsymbol{e}](\mathbf{0}|\Omega) = \sum_{\boldsymbol{m}\,\in\,\mathbb{Z}^p} \exp\left[\mathrm{i}\pi(\boldsymbol{m}+\boldsymbol{\varepsilon})^{\mathrm{t}} \cdot \Omega \cdot (\boldsymbol{m}+\boldsymbol{\varepsilon}) + 2\pi\mathrm{i}(\boldsymbol{m}+\boldsymbol{\varepsilon})^{\mathrm{t}} \cdot \boldsymbol{\delta}\right]$

- Ising model $\boxed{\mathcal{F}^{\text{Ising}}_{N,n}(\boldsymbol{x}) = \frac{\sum_{\boldsymbol{e}} |\Theta[\boldsymbol{e}](\mathbf{0}|\tau)|}{2^g\,|\Theta(\mathbf{0}|\tau)|}}$

Nasty n dependence

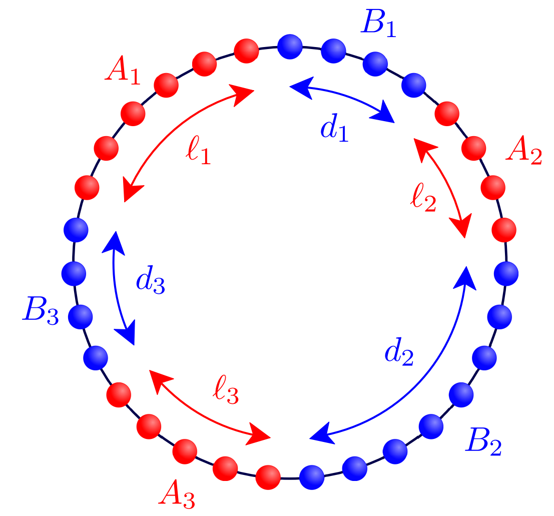
- Two intervals case: [Caraglio, Gliozzi, (2008)] [Furukawa, Pasquier, Shiraishi, (2009)] [Calabrese, Cardy, E.T., (2009), (2011)] [Fagotti, Calabrese, (2010)] [Alba, Tagliacozzo, Calabrese, (2010), (2011)]

The periodic harmonic chain

- Periodic chain of harmonic oscillators

$$H = \sum_{n=0}^{L-1} \left(\frac{1}{2M}\, p_n^2 + \frac{M\omega^2}{2}\, q_n^2 + \frac{K}{2} (q_{n+1} - q_n)^2 \right)$$

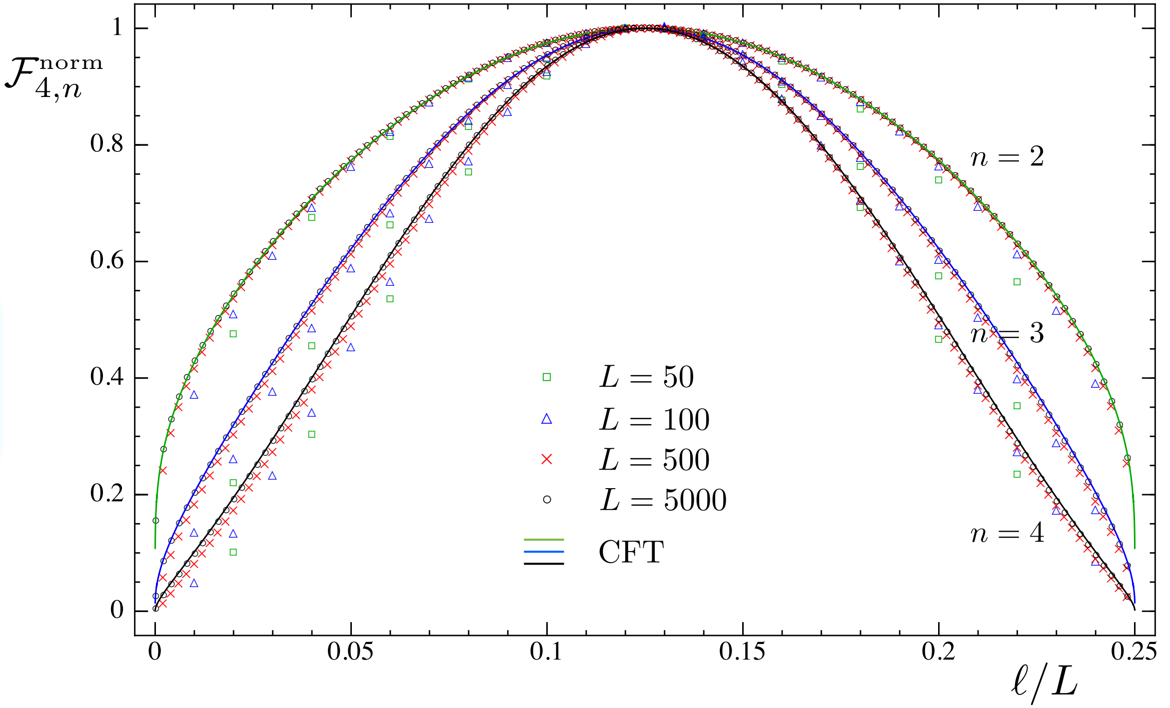
The massless case in the continuum limit is the $c = 1$ free boson on the line
[Peschel, Chung, (1999)] [Botero, Reznik, (2004)]
[Audenaert, Eisert, Plenio, Werner,(2002)]



- Decompactification regime (large η)

[Coser, Tagliacozzo, E.T., (2013)]

$$\boxed{\mathcal{F}_{N,n}^{\mathrm{dec}}(\boldsymbol{x}) = \frac{\eta^{g/2}}{\sqrt{\det(\mathcal{I})}\, |\Theta(\mathbf{0}|\tau)|^2}}$$



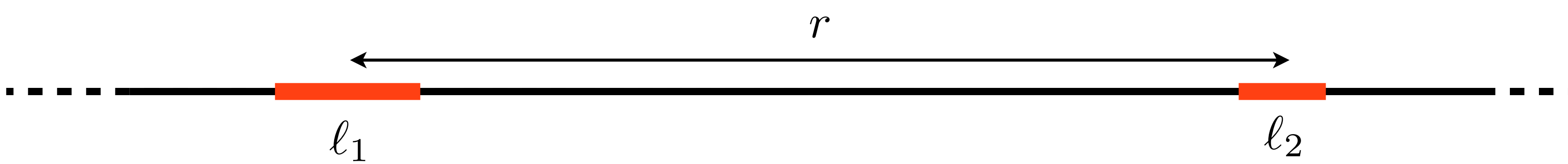
- Numerical checks for the Ising model through Matrix Product States (MPS)

Short intervals expansion

[Headrick, (2010)]
[Calabrese, Cardy, E.T., (2011)]

- $\mathrm{Tr}\rho_A^n$ when the lengths ℓ_p of the intervals are small w.r.t. to other characteristc lengths of the system

- E.g.: two intervals



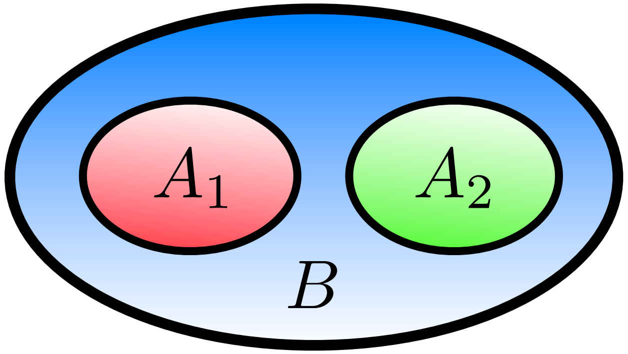
$$\mathrm{Tr}\rho_A^n = c_n^2 (\ell_1 \ell_2)^{-c/6(n-1/n)} \sum_{\{k_j\}} \left(\frac{\ell_1 \ell_2}{n^2 r^2} \right)^{\sum_j (\Delta_j + \bar{\Delta}_j)} \langle \prod_{j=1}^{n} \phi_{k_j} \left(e^{2\pi i j/n} \right) \rangle_{\mathbf{C}}^2$$

$\mathrm{Tr}\rho_A^n$ for disjoint intervals contains *all* the data of the CFT
(conformal dimensions and OPE coefficients)

The vacuum
is not
empty

Negativity & partial transpose: motivations & definitions

- *Tripartite* system $\mathcal{H} = \mathcal{H}_{A_1} \otimes \mathcal{H}_{A_2} \otimes \mathcal{H}_B \quad \Longrightarrow \quad \rho_{A_1 \cup A_2}$ is mixed



$S_{A_1 \cup A_2}$: entanglement between $A_1 \cup A_2$ and B

Entanglement between A_1 and A_2?

A *computable* measure of the entanglement is the *logarithmic negativity*

- ρ^{T_2} is the *partial transpose* of ρ

$$\boxed{\langle e_i^{(1)} e_j^{(2)} | \, \rho^{T_2} \, | e_k^{(1)} e_l^{(2)} \rangle \; = \; \langle e_i^{(1)} e_l^{(2)} | \, \rho \, | e_k^{(1)} e_j^{(2)} \rangle} \qquad (|e_i^{(k)}\rangle \text{ base of } \mathcal{H}_{A_k})$$

[Peres, (1996)] [Zyczkowski, Horodecki, Sanpera, Lewenstein, (1998)]
[Vidal, Werner, (2002)] [Eisert, (2001)]

Trace norm

$$\boxed{||\rho^{T_2}|| \equiv \mathrm{Tr}|\rho^{T_2}| = \sum_i |\lambda_i| = 1 - 2 \sum_{\lambda_i < 0} \lambda_i \qquad (\mathrm{Tr}\, \rho^{T_2} = 1)}$$

Logarithmic negativity

$$\mathcal{E}_{A_2} \equiv \ln ||\rho^{T_2}||$$

- Bipartite system $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ in a *generic* state $\rho \longrightarrow \mathcal{E}_A = \mathcal{E}_B$

Replica approach to Negativity

[Calabrese, Cardy, E.T., (2012)]

- A *parity effect* for $\boxed{\mathrm{Tr}(\rho^{T_2})^n}$

$$\mathrm{Tr}(\rho^{T_2})^{n_e} = \sum_i \lambda_i^{n_e} = \sum_{\lambda_i>0} |\lambda_i|^{n_e} + \sum_{\lambda_i<0} |\lambda_i|^{n_e}$$

$$\mathrm{Tr}(\rho^{T_2})^{n_o} = \sum_i \lambda_i^{n_o} = \sum_{\lambda_i>0} |\lambda_i|^{n_o} - \sum_{\lambda_i<0} |\lambda_i|^{n_o}$$

- Replica limit

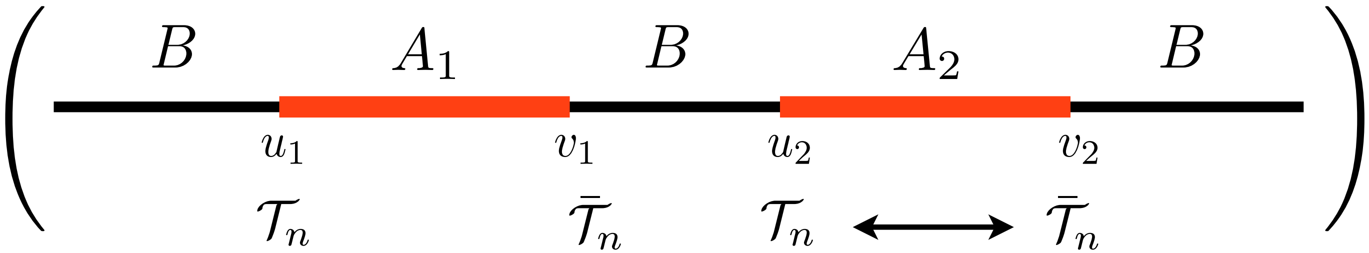
$$\boxed{\mathcal{E}_A = \log ||\rho^{T_2}|| = \lim_{n_e \to 1} \log \left[\mathrm{Tr}(\rho^{T_2})^{n_e} \right]}$$

$$\lim_{n_o \to 1} \mathrm{Tr}(\rho^{T_2})^{n_o} = \mathrm{Tr}\, \rho^{T_2} = 1$$

Analytic continuation on the *even* sequence $\mathrm{Tr}(\rho^{T_2})^{n_e}$ (*make 1 an even number*)

Partial Transposition for bipartite systems: pure states

$\mathcal{H} = \mathcal{H}_{A_1} \otimes \mathcal{H}_{A_2}$ and the whole system in the ground state

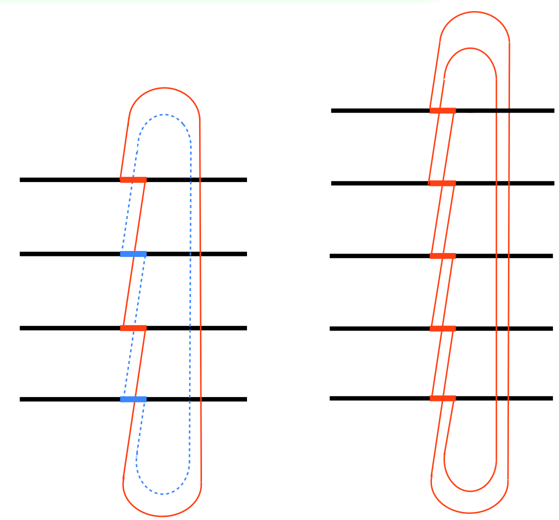
- $\lim_{B \to \emptyset}$ 

Partial Transposition = exchange two twist fields

$$\mathrm{Tr}(\rho_A^{T_2})^n = \langle \mathcal{T}_n^2(u_2) \bar{\mathcal{T}}_n^2(v_2) \rangle$$

- $\mathcal{T}_n^2$ connects the j-th sheet with the $(j+2)$-th one
  Even $n = n_e \implies$ decoupling

- $$\mathrm{Tr}(\rho_A^{T_2})^{n_e} = \left(\langle \mathcal{T}_{n_e/2}(u_2) \bar{\mathcal{T}}_{n_e/2}(v_2) \rangle \right)^2 = \left(\mathrm{Tr}\, \rho_{A_2}^{n_e/2} \right)^2$$
  $$\mathrm{Tr}(\rho_A^{T_2})^{n_o} = \langle \mathcal{T}_{n_o}(u_2) \bar{\mathcal{T}}_{n_o}(v_2) \rangle = \mathrm{Tr}\, \rho_{A_2}^{n_o}$$



$n = 4$ $\quad$ $n = 5$

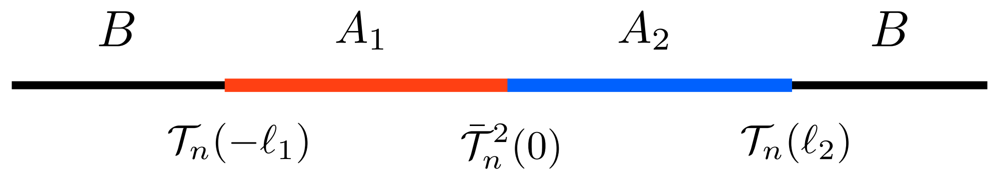
- 2D CFT: these identities give $\boxed{\Delta_{\mathcal{T}_n^2}}$ $\implies$ $\boxed{\mathcal{E} = \frac{c}{2} \log(\ell/a) + \mathrm{const}}$

- For $n_e \to 1$ we find $\mathcal{E} = 2 \log \mathrm{Tr} \rho_2^{1/2}$ (Renyi entropy 1/2)

2D CFT: two adjacent & disjoint intervals

$$\rho^{T_2}_{A_1 \cup A_2}$$

- Adjacent intervals

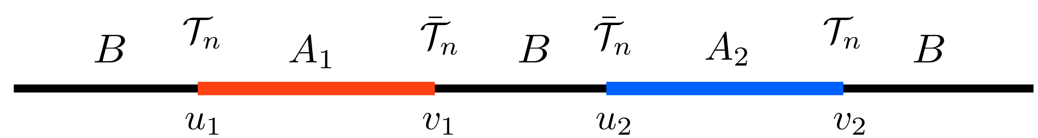


$$\mathrm{Tr}(\rho_A^{T_2})^n = \langle \mathcal{T}_n(-\ell_1)\bar{\mathcal{T}}^2_n(0)\mathcal{T}_n(\ell_2)\rangle$$

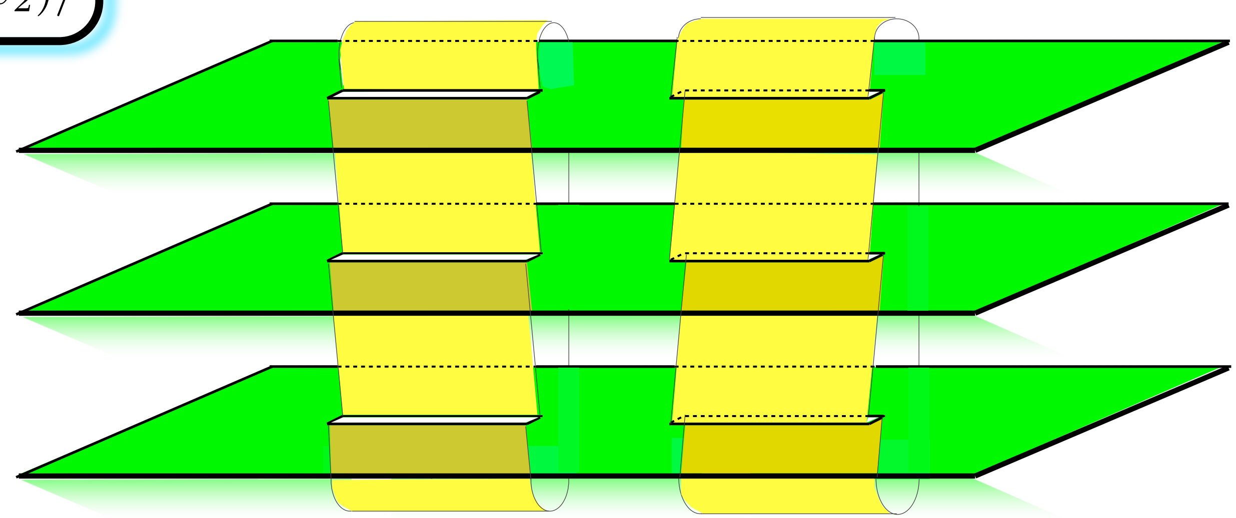
$$\mathcal{E} = \frac{c}{4} \ln \left(\frac{\ell_1 \ell_2}{\ell_1 + \ell_2} \right) + \mathrm{const}$$

- *Analytic continuation* $n_e \to 1$

- Disjoint intervals



$$\mathrm{Tr}(\rho_A^{T_2})^n = \langle \mathcal{T}_n(u_1)\bar{\mathcal{T}}_n(v_1)\bar{\mathcal{T}}_n(u_2)\mathcal{T}_n(v_2)\rangle$$



Renyi entropies vs traces of the Partial Transpose

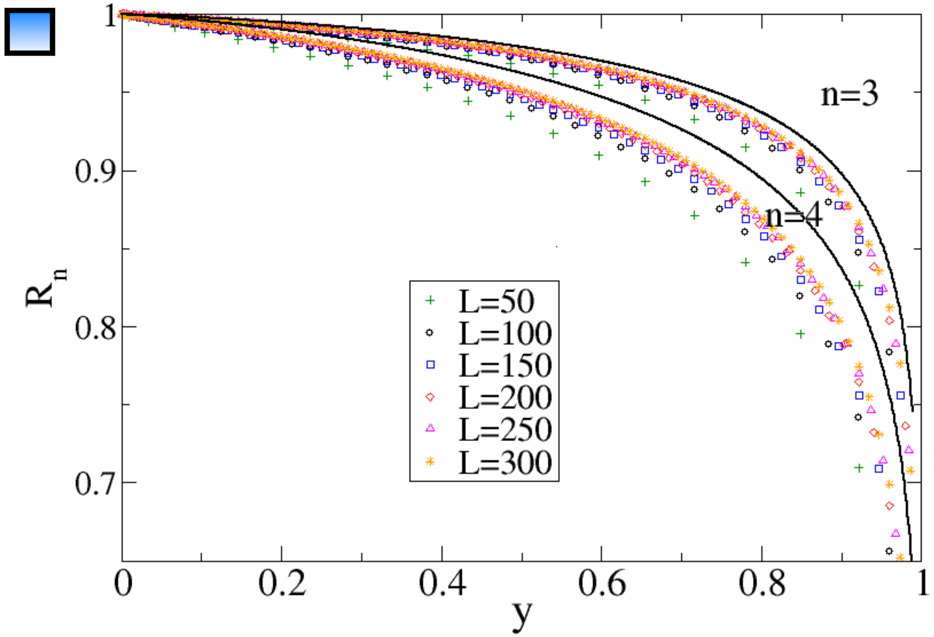
$\mathrm{Tr}\,\rho_A^n$

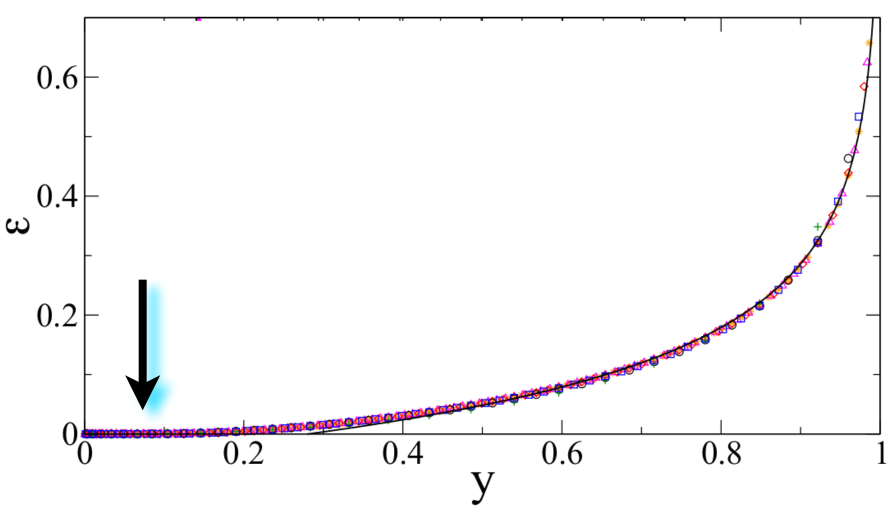
$\mathrm{Tr}(\rho_A^{T_2})^n$

Two disjoint intervals

- Previous numerical results for $\mathcal{E}$: Ising (DMRG) and harmonic chains

[Wichterich, Molina-Vilaplana, Bose, (2009)]
[Marcovitch, Retzker, Plenio, Reznik, (2009)]





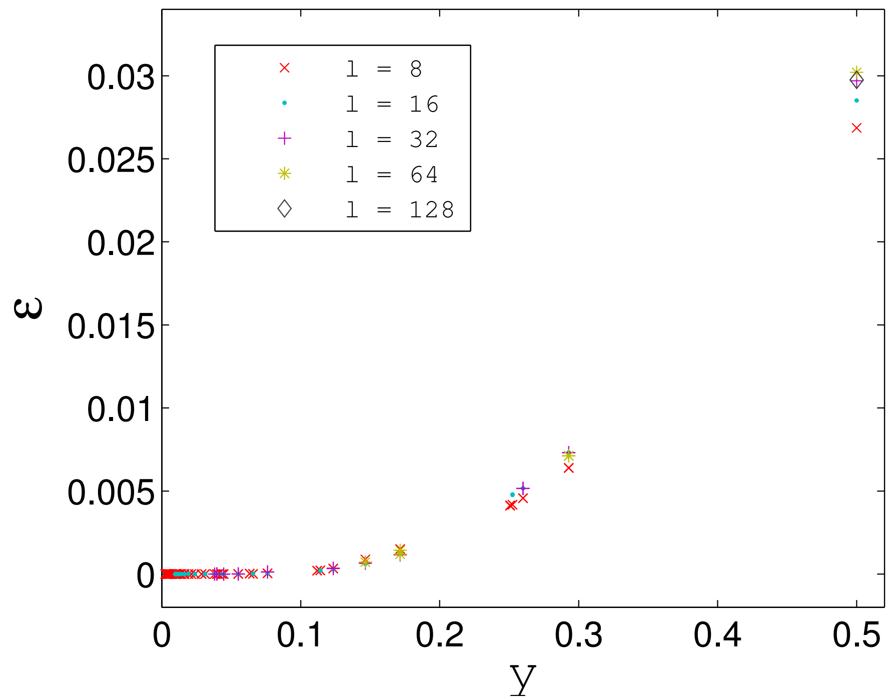
- Periodic harmonic chain:

[Calabrese, Cardy, E.T., (2012)]

$$R_n = \frac{\mathrm{Tr}(\rho_A^{T_2})^n}{\mathrm{Tr}\,\rho_A^n}$$

- Ising model: (Tree Tensor Network)

[Calabrese, Tagliacozzo, E.T., (2013)]

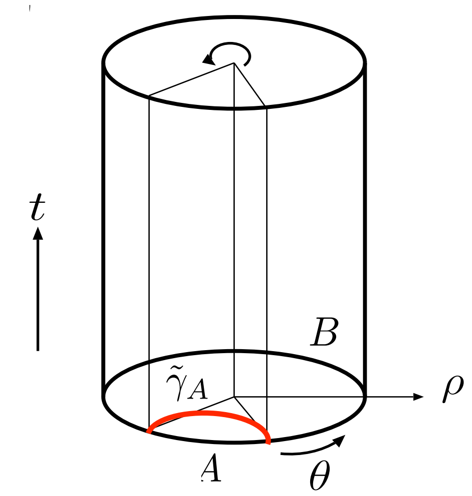


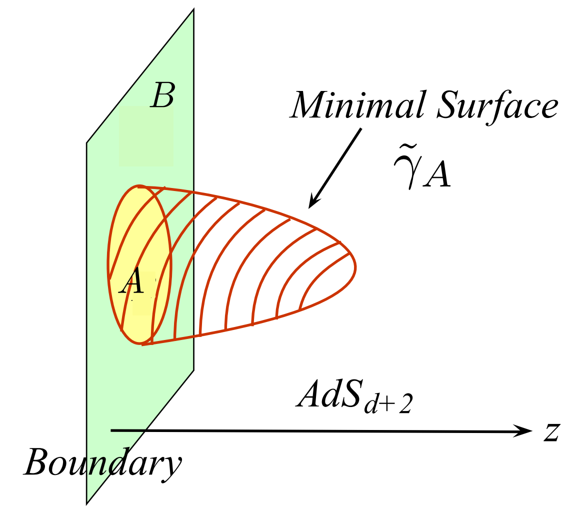
Holographic entanglement entropy

[Ryu, Takayanagi, (2006)]

AdS_{d+2}/CFT_{d+1} correspondence

- Prescription: in regularized AdS_{d+2}
  - Constant time slice
  - Consider the surfaces γ_A s.t. $\partial\gamma_A = \partial A$ which are *homologous* to A
    (it exists a bulk region $\mathcal{R}$ s.t. $\partial\mathcal{R} = A \cup \gamma_A$)
    [Headrick, Takayanagi, (2007)] [Fursaev, (2006)]
    [Azeyanagi, Nishioka, Takayanagi,(2008)]
  - Find the surface $\tilde{\gamma}_A$ with *minimal area*
  - $$\boxed{S_A = \frac{\mathrm{Area}(\tilde{\gamma}_A)}{4G_N^{(d+2)}}}$$





- $d = 1$ formula $S_A = \frac{c}{3}\log\frac{\ell}{a} + c_0$ (with Brown-Hennaux central charge $c = 3R/(2G_N) \gg 1$)
- Area law for $d > 1$ recovered $S_A \propto \mathrm{Area}(\partial A)/a^{d-1}$

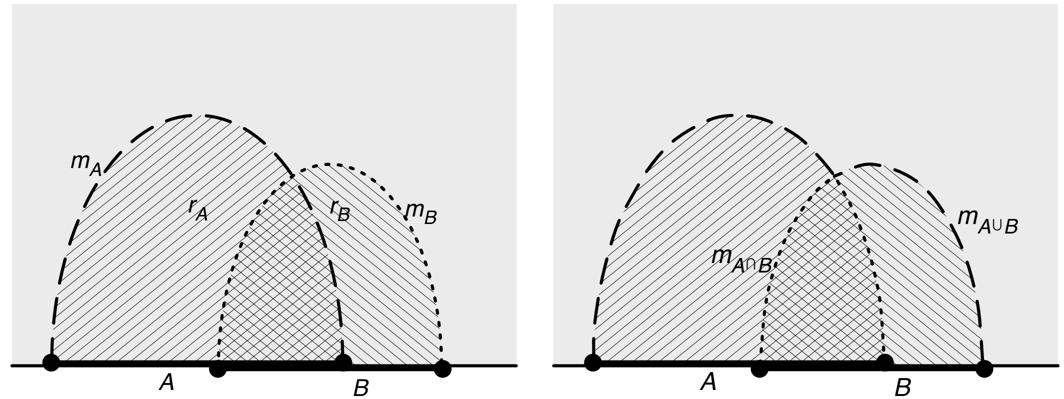
Holographic entanglement entropy

- Holographic proof of strong subadditivity

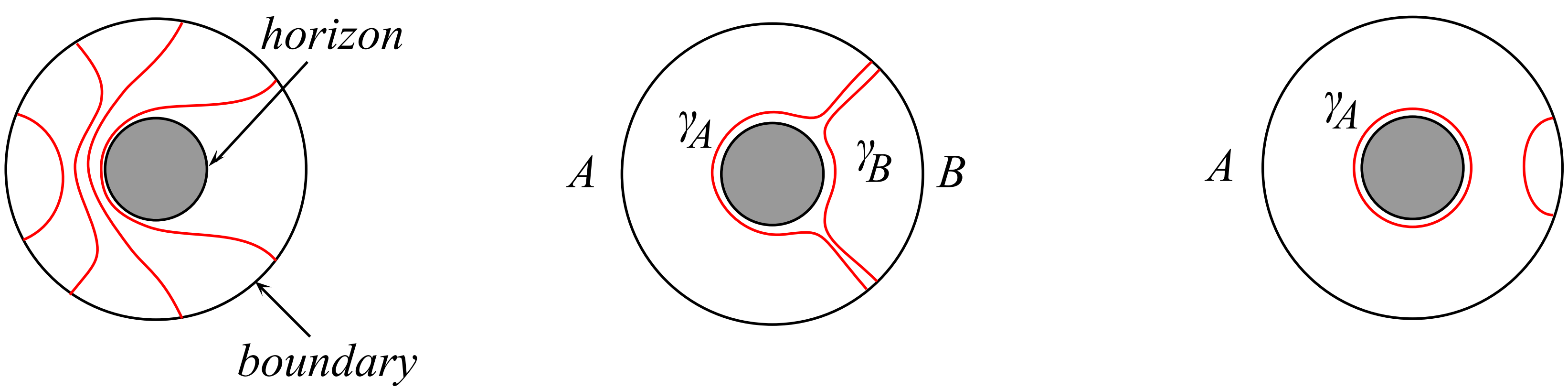
  $S_{A_1} + S_{A_2} \geqslant S_{A_1 \cup A_2} + S_{A_1 \cap A_2}$

  $S_{A_1} + S_{A_2} \geqslant S_{A_1 \setminus A_2} + S_{A_1 \setminus A_2}$

  [Headrick, Takayanagi, (2007)]



- Thermal state ($S_A \neq S_B$) [Headrick, Takayanagi, (2007)]
  [Azeyanagi, Nishioka, Takayanagi, (2008)] [Hubeny, Maxfield, Rangamani, Tonni, (2013)]

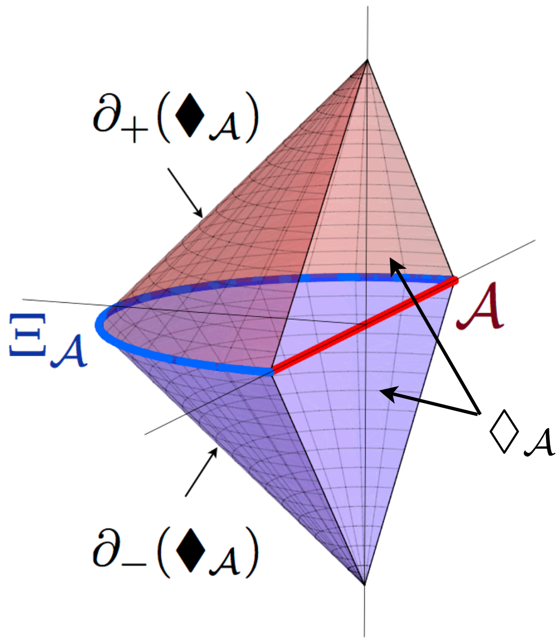


- Holographic Renyi entropies [Casini, Huerta, Myers, (2011)] [Hung, Myers, Smolkin, Yale, (2011)]

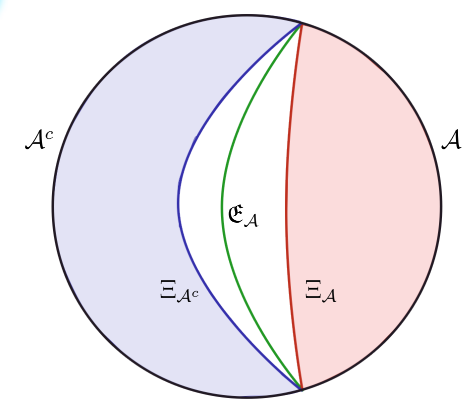
- Quantum correction to Ryu-Takayanagi formula
  [Faulkner, Lewkowycz, Maldacena, (2013)]

Causal holographic information

[Hubeny, Takayanagi, Rangamani, (2007)]
[Hubeny, Rangamani, (2012)]



- Does holography suggest new quantities?
- Natural construction starting from the *domain of dependence* of A ($\lozenge_\mathcal{A}$) *causal wedge* $\blacklozenge_\mathcal{A}$ associated to $\mathcal{A}$
- $$\boxed{\chi_A = \frac{\text{Area}(\Xi_A)}{4G_N^{(d+2)}}}$$



- $\chi_A = S_A$ for AdS_3, BTZ and rotating BTZ black hole
- $\chi_A = S_A$ for spherical A in the boundary CFT$_{d+1}$
- for the complementar region: $\chi_{A^c} \neq \chi_A$
- χ_A does not satisfy strong subadditivity

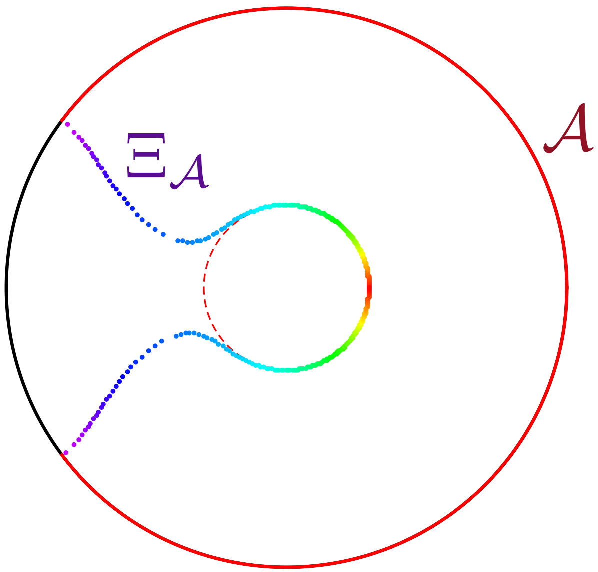
- A proposal for the CFT dual of the causal holographic information: One-point entropy [Kelly, Wall, (2013)]

CHI: Schwarzschild-AdS black hole

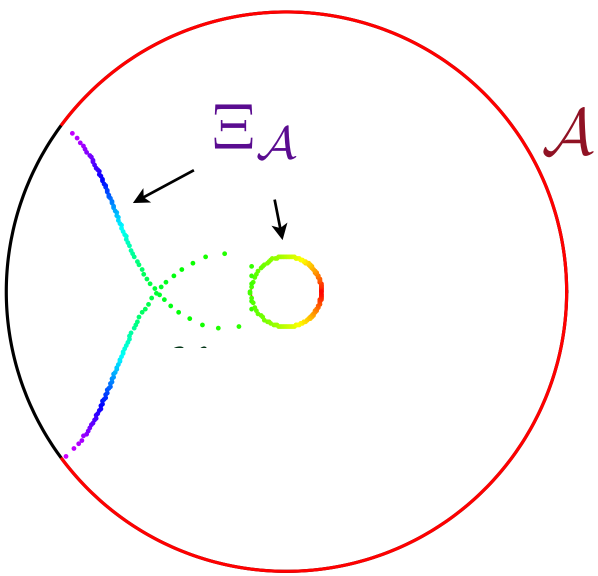
[Hubeny, Rangamani, E.T., (2013)]

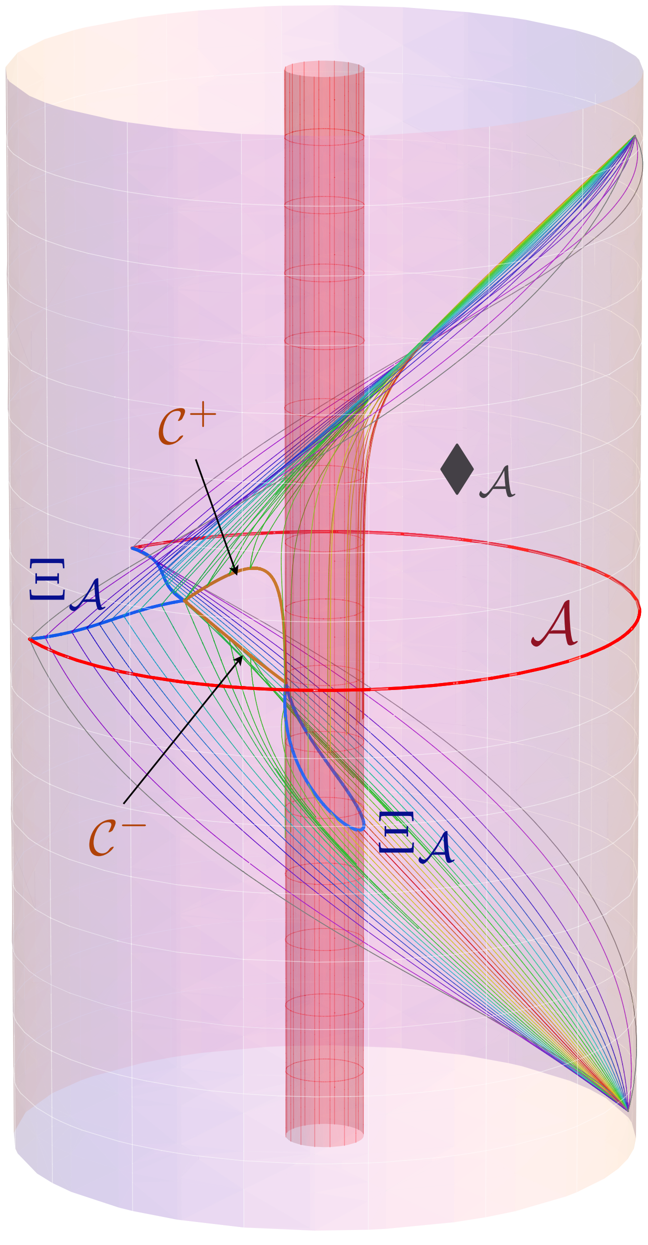
Schwarzschild-AdS$_5$

$r_+ = 1$



$r_+ = 0.2$





Conclusions

- Entanglement entropies for *disjoint intervals* contain *all the CFT data*
- *Entanglement for mixed states*: negativity in QFT
- Holography can suggest new ways to quantify entanglement (e.g. CHI)

→ Analytic continuations, Negativity for fermions,
Higher dimensions, Interactions, Holographic interpretations, . . .

- Out of equilibrium dynamics (thermalization, quantum quenches)
  [Calabrese, Cardy, (2005), (2007)] [Hubeny, Rangamani, Takayanagi, (2007)] [. . .]
- Entanglement & RG [Casini, Huerta, (2004), (2012)] [Myers, Singh, (2012)] [. . .]
- Quantum Hall Effect [Li, Haldane, (2008)] [. . .]
- Excited states [Berganza, Alcaraz, Sierra, (2011)] [. . .]

Thank you!