



Phenomenology of TMDs

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TAURINENSIS

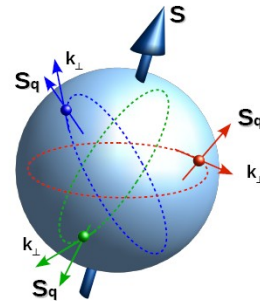


3D Structure of the Nucleon



The exploration of the **3-dimensional structure of the nucleon**, both in momentum and in configuration space, is one of the major issues in high energy hadron physics.

Information on the 3-dimensional structure of the nucleon is embedded in the **Transverse Momentum Dependent** distribution and fragmentation functions (**TMDs**).

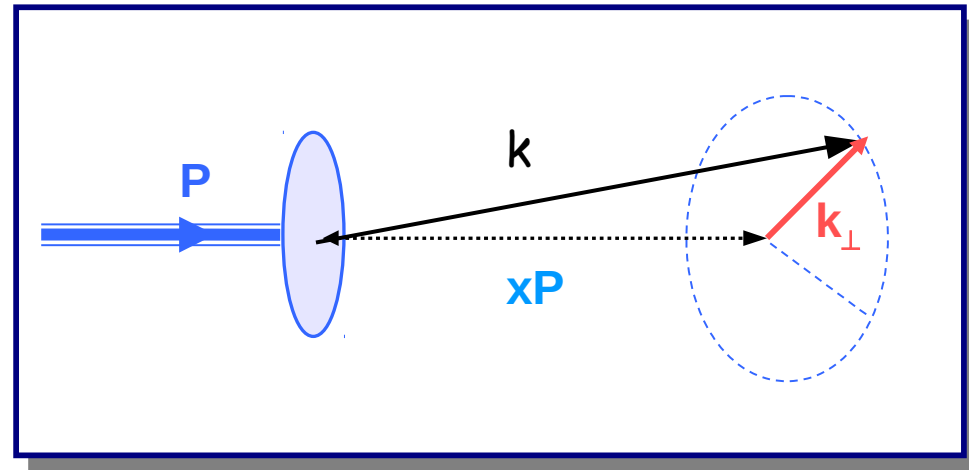
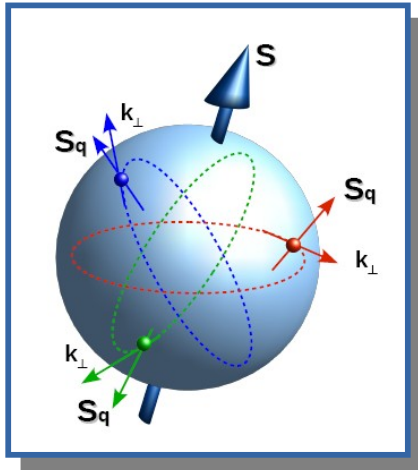


In a very simple **phenomenological** approach, hadronic cross sections and spin asymmetries are generated, within a **QCD factorization** framework, as convolutions of **distribution** and (or) **fragmentation** functions with elementary cross sections.



This simple approach can successfully describe a wide range of experimental data.

Intrinsic Transverse Momentum



We cannot learn about the spin structure of the nucleon without taking into account the **transverse motion** of the partons inside it

Transverse motion is usually integrated over, but there are important **spin- $k_{\perp}$** correlations which should not be neglected

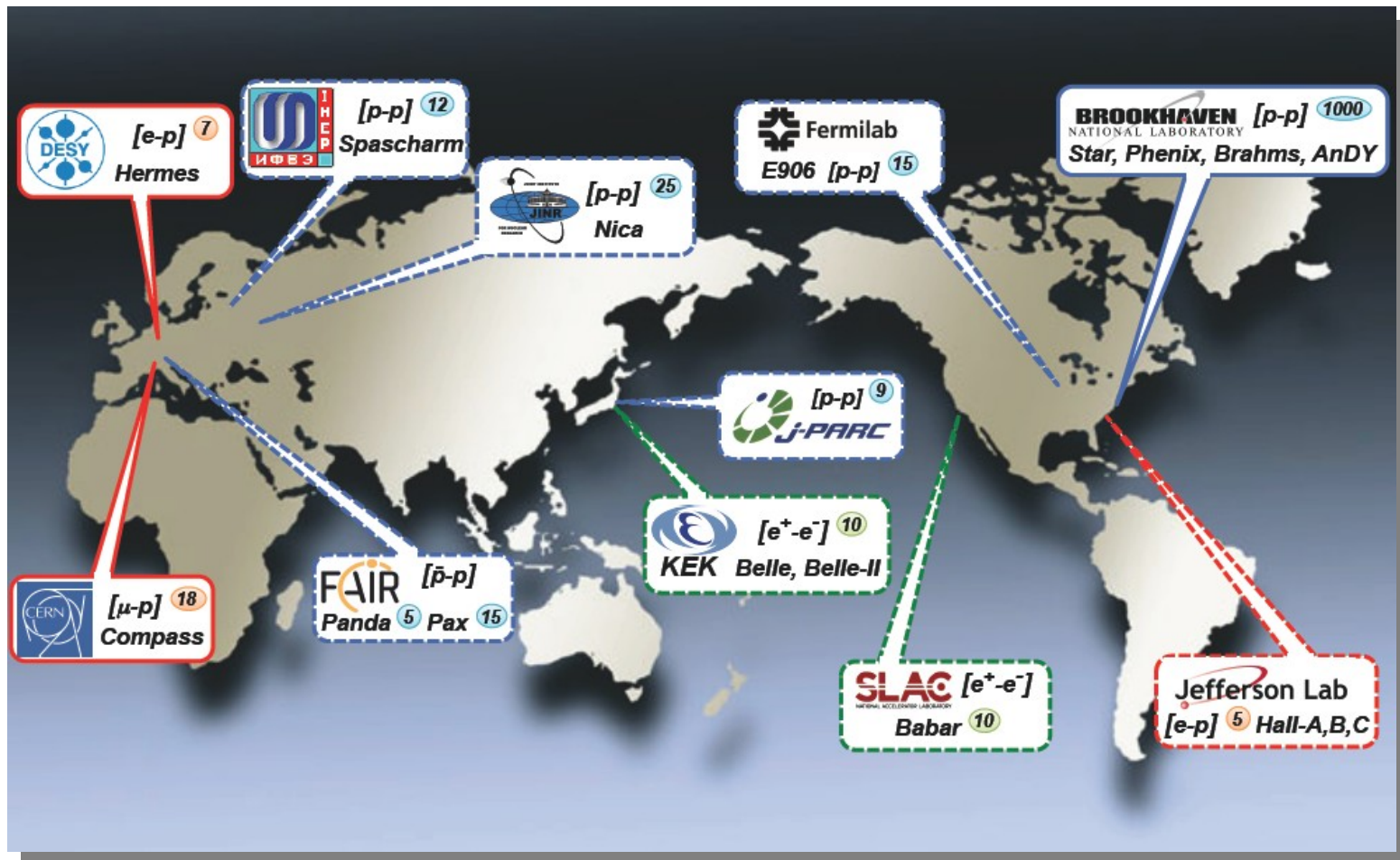


Several theoretical and experimental evidences for transverse motion of partons within nucleons, and of hadrons within fragmentation jets.



***Where can we learn about
the 3D structure of matter ?***

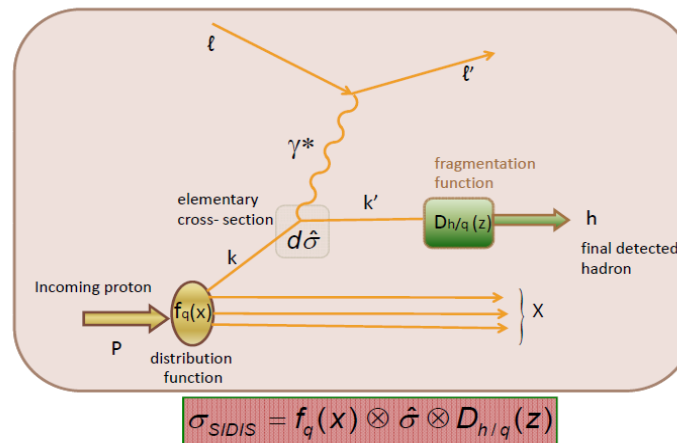
Where can we learn about the 3D structure of matter ?



Experimental data for TMS studies



Unpolarized and Polarized SIDIS scattering



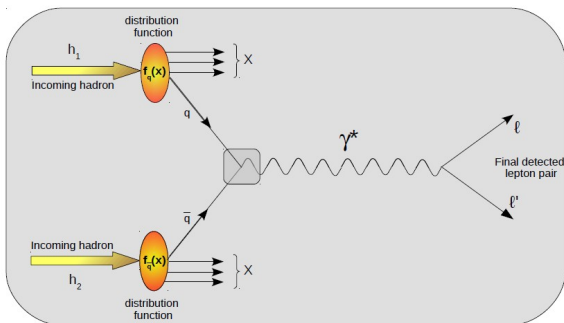
Allows the extraction of
TMD distribution and
fragmentation functions



Experimental data for TMS studies



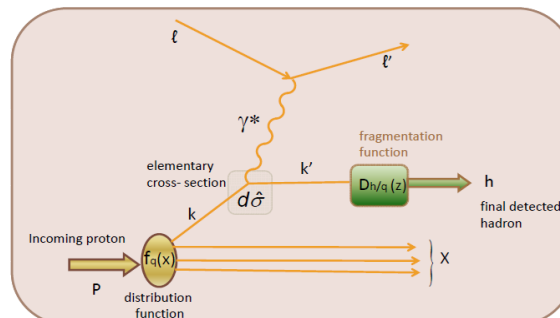
Unpolarized and Polarized Drell-Yan scattering



$$\sigma_{\text{Drell-Yan}} = f_q(x, k_{\perp}) \otimes f_{\bar{q}}(x, k_{\perp}) \otimes \hat{\sigma}_{q\bar{q} \rightarrow \ell\ell'}$$

Allows extraction of **distribution** functions

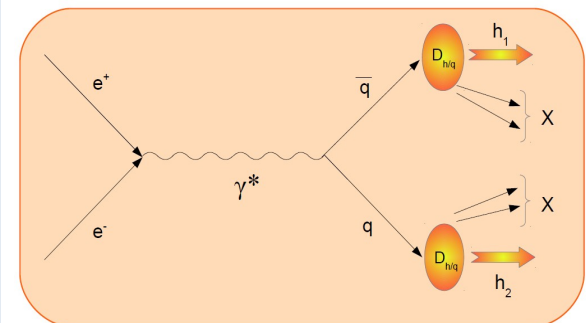
Unpolarized and Polarized SIDIS scattering



$$\sigma_{\text{SIDIS}} = f_q(x) \otimes \hat{\sigma} \otimes D_{h/q}(z)$$

Allows extraction of **distribution** and **fragmentation** functions

$$e^+ e^- \rightarrow h_1 h_2 X$$



$$\sigma_{h_1 h_2} \propto D(z_1) \otimes D(z_2) \otimes \hat{\sigma}$$

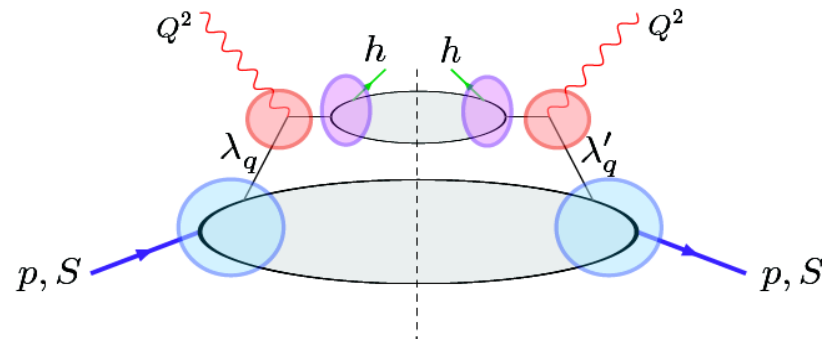
Allows extraction of **fragmentation** functions



From the theory point of view ...



	Perturbative QCD Parton model Factorization ...
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TMD factorization holds at large Q^2 and $P_T \approx k_\perp \approx \lambda_{\text{QCD}}$

Two scales: $P_T \ll Q^2$

(Collins, Soper, Ji, Ma, Yuan, Qiu, Vogelsang, Collins, Metz)

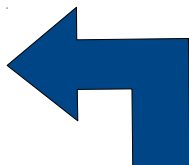
$$\begin{aligned}
 & \frac{d\sigma^{\ell(S_\ell)+p(S)\rightarrow\ell'+h+X}}{dx_B dQ^2 dz_h d^2\mathbf{P}_T d\phi_S} \\
 = & \rho_{\lambda_\ell, \lambda'_\ell}^{\ell, S_\ell} \otimes \rho_{\lambda_q, \lambda'_q}^{q/p, S} \underbrace{f_{q/p, S}(x, \mathbf{k}_\perp)}_{\text{TMD-PDF}} \otimes \underbrace{\hat{M}_{\lambda_\ell, \lambda_q; \lambda_\ell, \lambda_q} \hat{M}_{\lambda'_\ell, \lambda'_q; \lambda'_\ell, \lambda'_q}^*}_{\text{hard scattering}} \otimes \underbrace{\hat{D}_{\lambda_q, \lambda'_q}^h(z, \mathbf{p}_\perp)}_{\text{TMD-FF}}
 \end{aligned}$$

Phenomenology



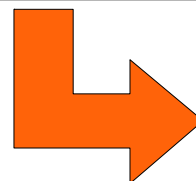
THEORY

- ◆ Perturbative QCD
- ◆ Factorization theorem
- ◆ ...



PHENOMENOLOGY

Mission: devise simple flexible and efficient models to link THEORY with EXPERIMENTS



EXPERIMENTS

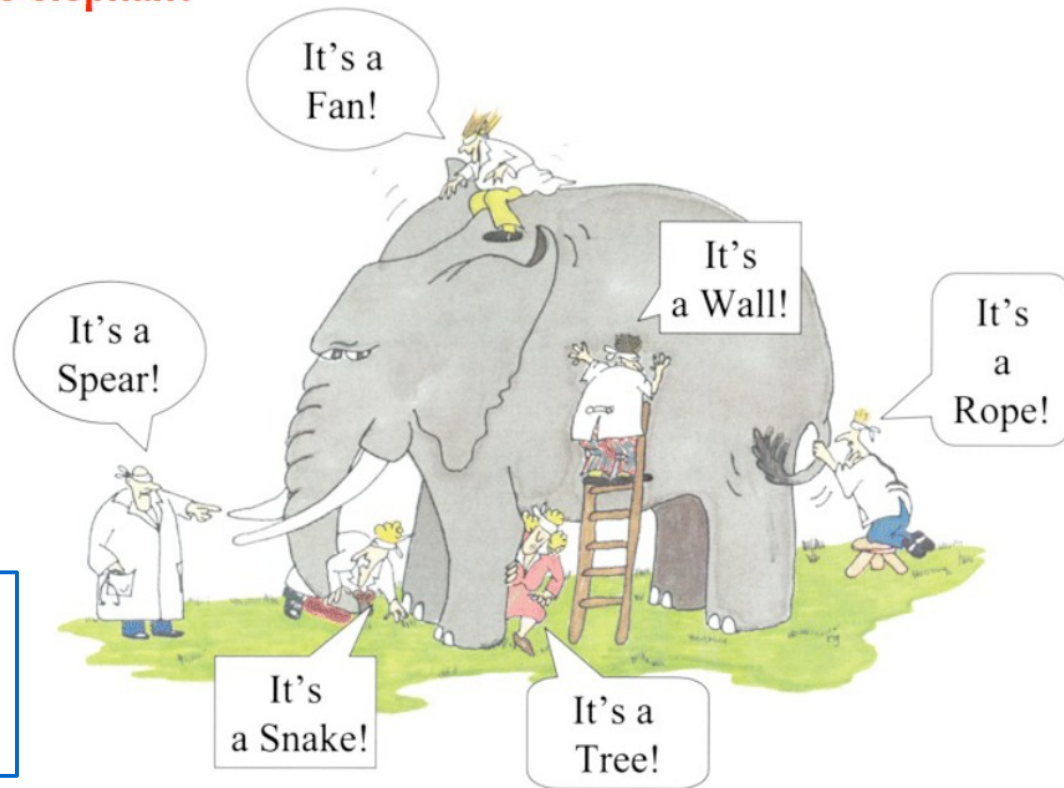
- Drell-Yan scattering
- Di-hadron production from e^+e^- scattering
- DIS and SIDIS processes
- Inclusive single particle production from hadronic scattering

Phenomenology



The blind men and the elephant

from H. Avakian



Experiments → Blind Men
Several different experiments
Measuring the same observable,
with limited coverage

Phenomenology → “where everything comes together nicely”
Combine different sources of information to get the whole picture

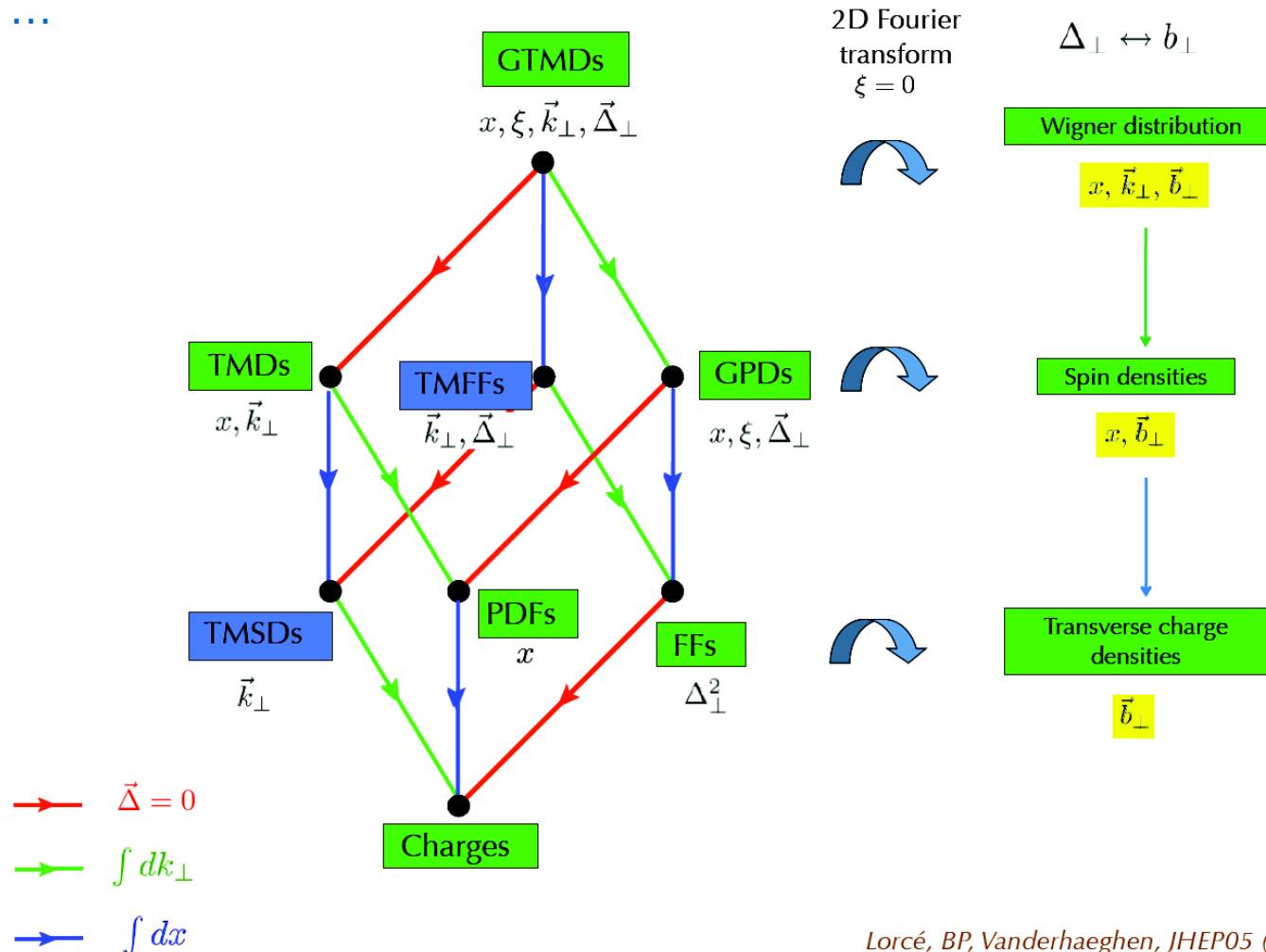


How can we learn about the 3D structure of matter ?

How can we learn about the 3D structure of matter ?



As Barbara Pasquini nicely explained yesterday ...

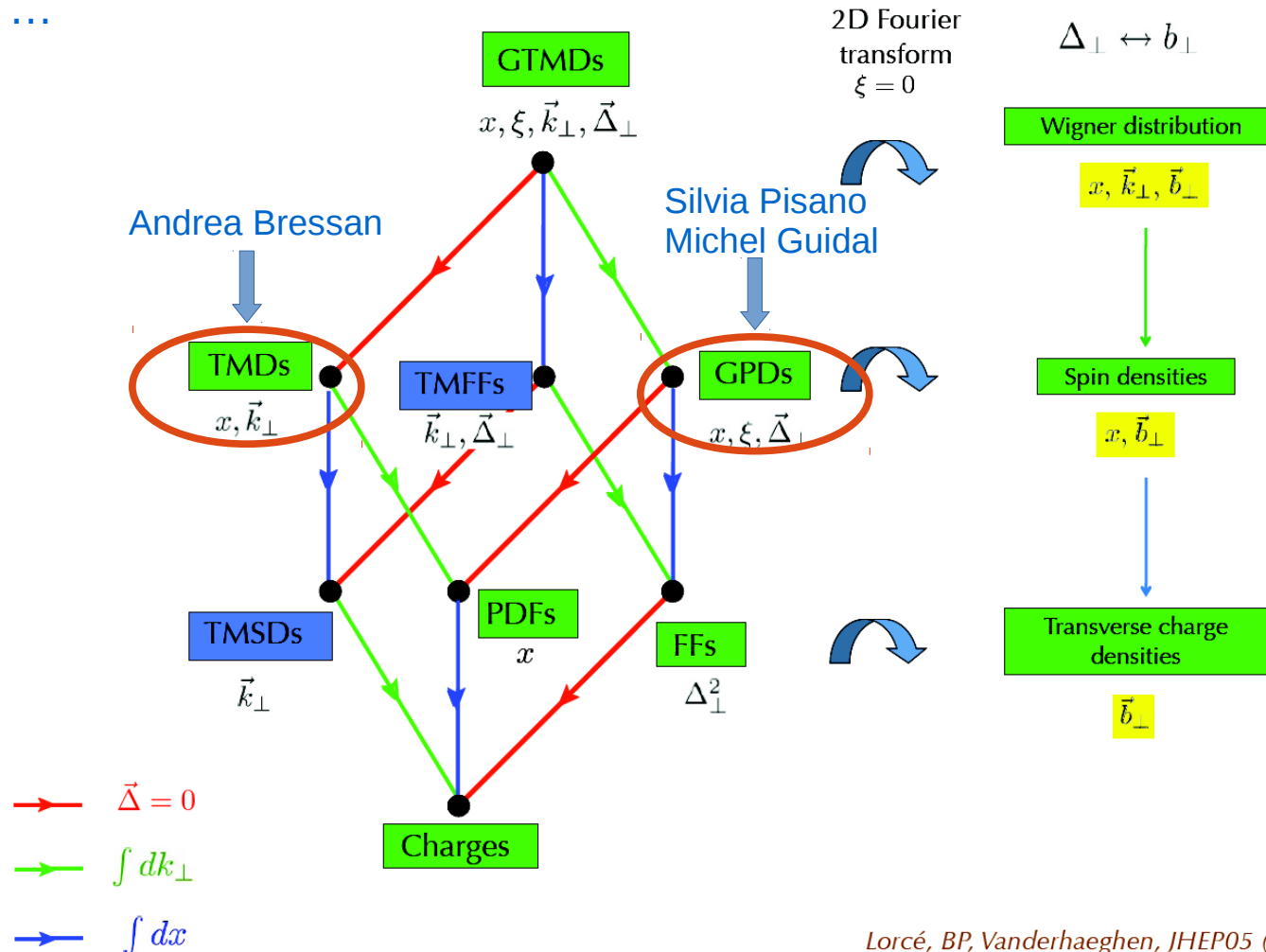


Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041

How can we learn about the 3D structure of matter ?



As Barbara Pasquini nicely explained yesterday ...



Lorcé, BP, Vanderhaeghen, JHEP05 (2011) 041



Wigner function

5D

Transverse
Momentum
Dependent
distributions

$$W(x, k_{\perp}, r_{\perp})$$

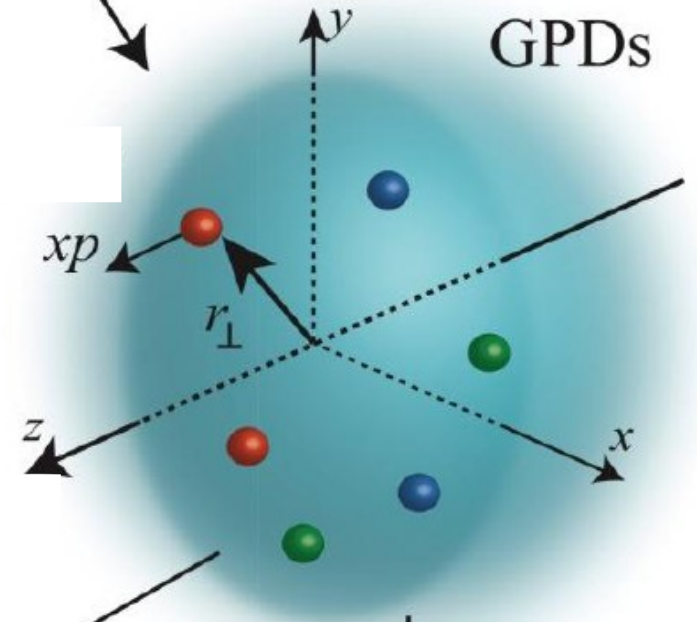
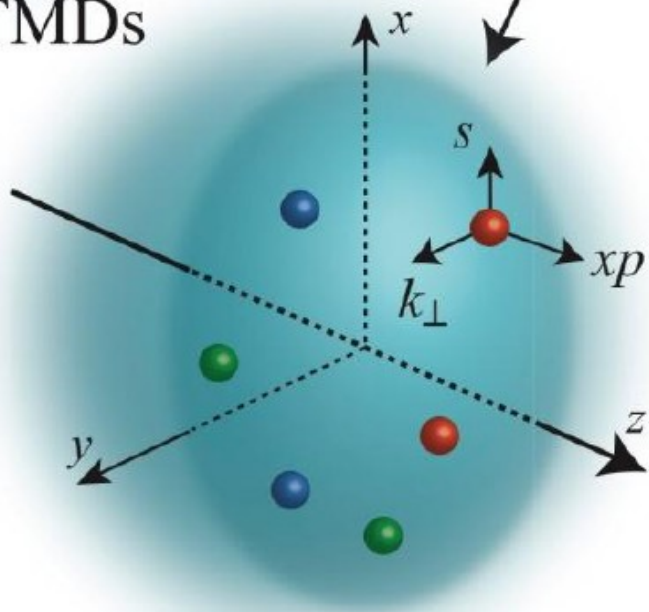
$d^2 r_{\perp}$

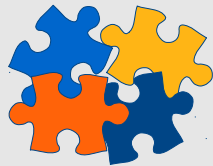
$d^2 k_{\perp}$

TMDs

GPDs

3D

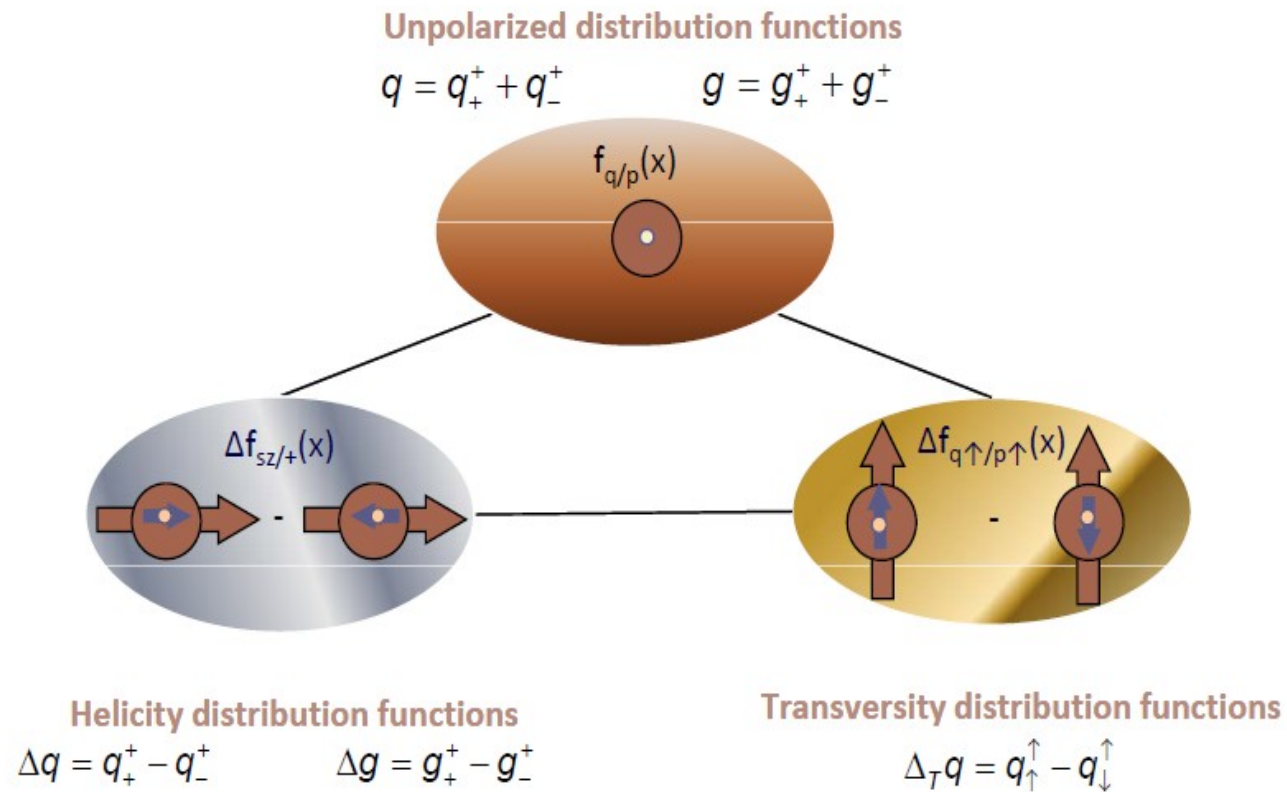




The mechanism which describes how quarks and gluons are bound into hadrons is embedded in the parton distribution and fragmentation functions (PDFs and FFs), the so-called “soft parts” of hadronic scattering processes. These are non-perturbative objects which connect the ideal world of pointlike and massless particles (pQCD) to our much more complex real world, made of nucleons, nuclei and atoms.

PDFs versus TMDs

Collinear parton distribution functions





Transversity

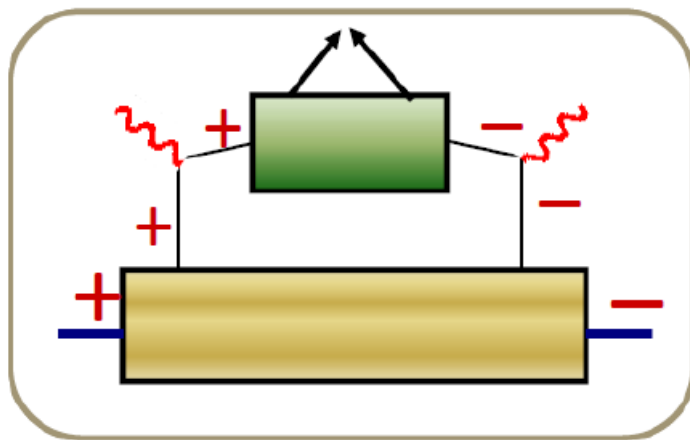
- The **transversity distribution function** contains basic information on the spin structure of the nucleons.
- Being related to the expectation value of a chiral odd operator, it appears in physical processes which require a quark helicity flip; therefore it cannot be measured in usual DIS.
- Drell-Yan → planned experiments in polarized pp at PAX.
- At present, the only chance of gathering information on transversity is **SIDIS**, where it appears associated to the Collins fragmentation function.
- **DOUBLE PUZZLE**: we cannot determine transversity if we do not know the Collins fragmentation function.



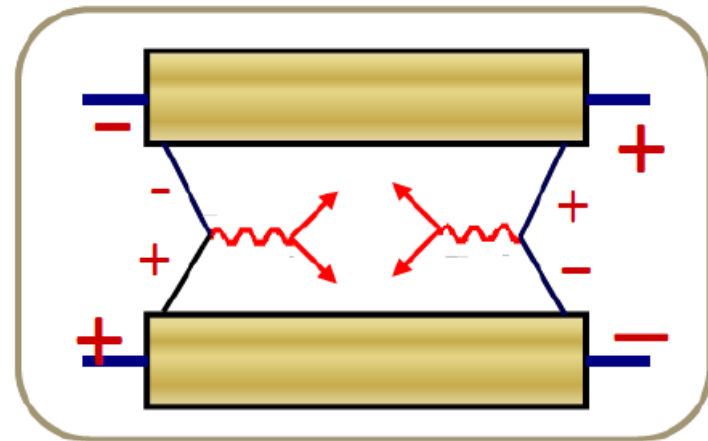
Transversity

- There is **no gluon** transversity distribution function
- Transversity cannot be studied in deep inelastic scattering because it is **chirally odd**
- Transversity can only appear in a cross-section convoluted to another **chirally odd function**

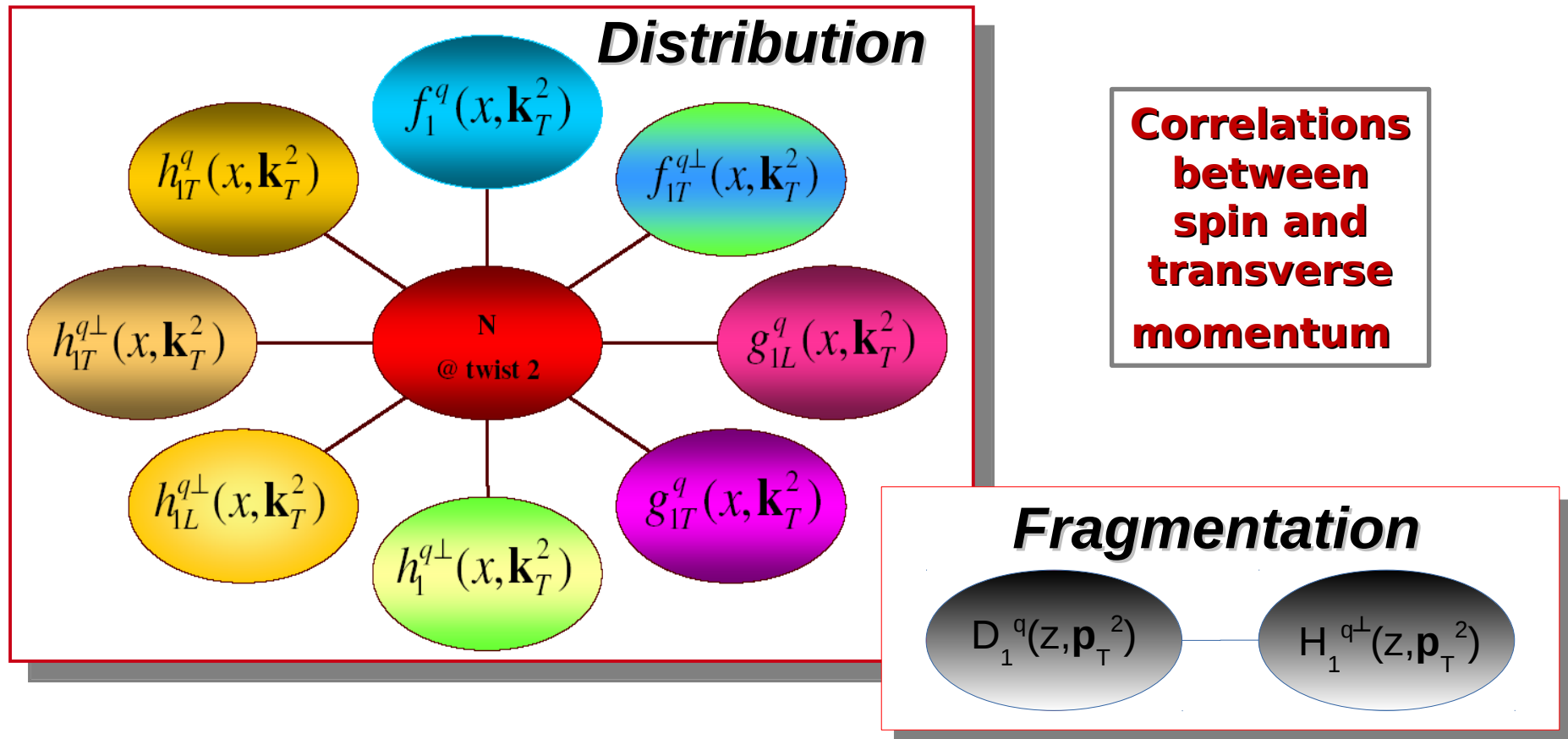
SIDIS



Drell -Yan



TMD distribution and fragmentation functions



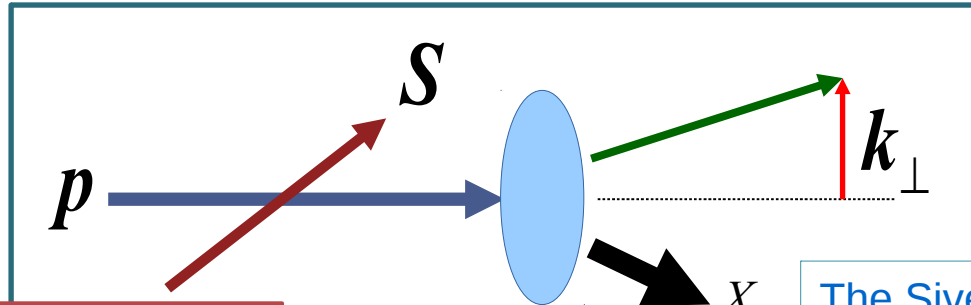


The Sivers function

$$f_{q/p,S}(x, k_{\perp}) = f_{q/p}(x, k_{\perp}) + \frac{1}{2} \Delta^N f_{q/p\uparrow}(x, k_{\perp}) S \cdot (\hat{p} \times \hat{k}_{\perp})$$
$$= f_{q/p}(x, k_{\perp}) - \frac{k_{\perp}}{M} f_{1T}^{\perp q}(x, k_{\perp}) S \cdot (\hat{p} \times \hat{k}_{\perp})$$

The Sivers function is related to the probability of finding an unpolarized quark inside a transversely polarized proton

The Sivers function is T-odd



The Sivers function, is particularly interesting, as it provides information on the partonic orbital angular momentum

The Sivers function embeds the correlation between the proton spin and the quark transverse momentum



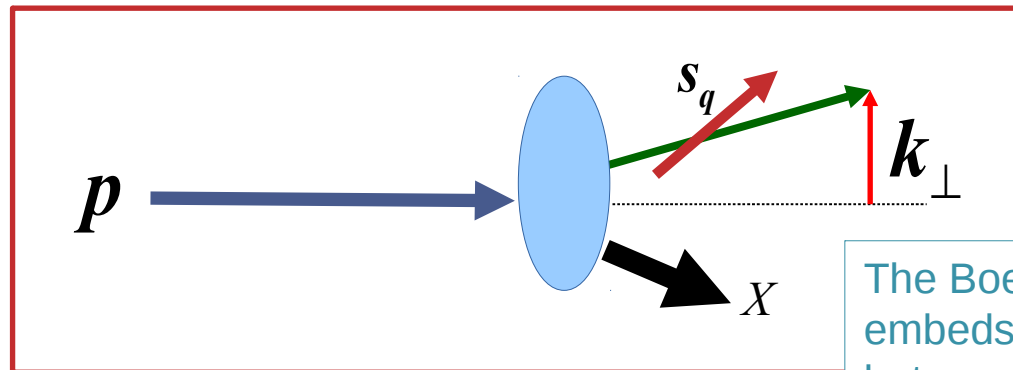
The Boer-Mulders function

$$f_{q,s_q/p}(x, \mathbf{k}_\perp) = \frac{1}{2} f_{q/p}(x, k_\perp) + \frac{1}{2} \Delta f_{q^\uparrow/p}(x, k_\perp) s_q \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp)$$

$$= \frac{1}{2} f_{q/p}(x, k_\perp) - \frac{1}{2} \frac{k_\perp}{M} h_1^{\perp q}(x, k_\perp) s_q \cdot (\hat{\mathbf{p}} \times \hat{\mathbf{k}}_\perp)$$

The Boer-Mulders function is related to the probability of finding a polarized quark inside an unpolarized proton

The Boer Mulders function is chirally odd



The Boer-Mulders function embeds the correlation between the quark spin and its transverse momentum

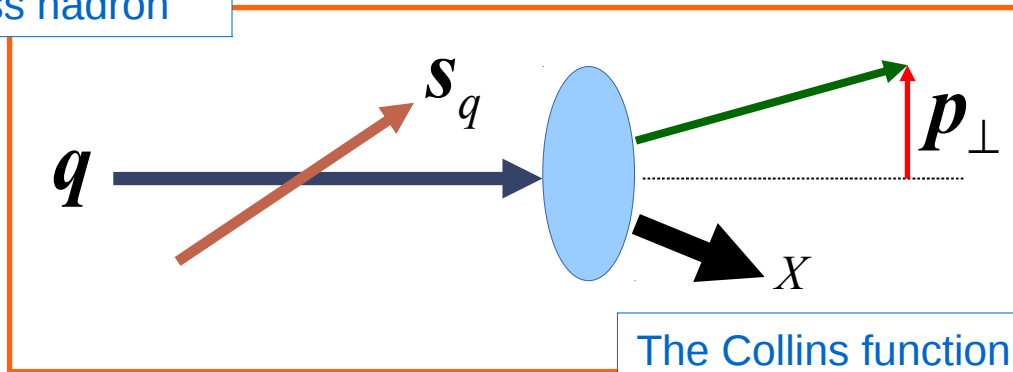


The Collins function

$$D_{h/q,s_q}(z, \mathbf{p}_\perp) = D_{h/q}(z, p_\perp) + \frac{1}{2} \Delta^N D_{h/q\uparrow}(z, p_\perp) \mathbf{s}_q \cdot (\hat{\mathbf{p}}_q \times \hat{\mathbf{p}}_\perp) \\ = D_{h/q}(z, p_\perp) + \frac{p_\perp}{z M_h} H_1^{\perp q}(z, p_\perp) \mathbf{s}_q \cdot (\hat{\mathbf{p}}_q \times \hat{\mathbf{p}}_\perp)$$

The Collins function is related to the probability that a transversely polarized struck quark will fragment into a spinless hadron

The Collins function is chirally odd

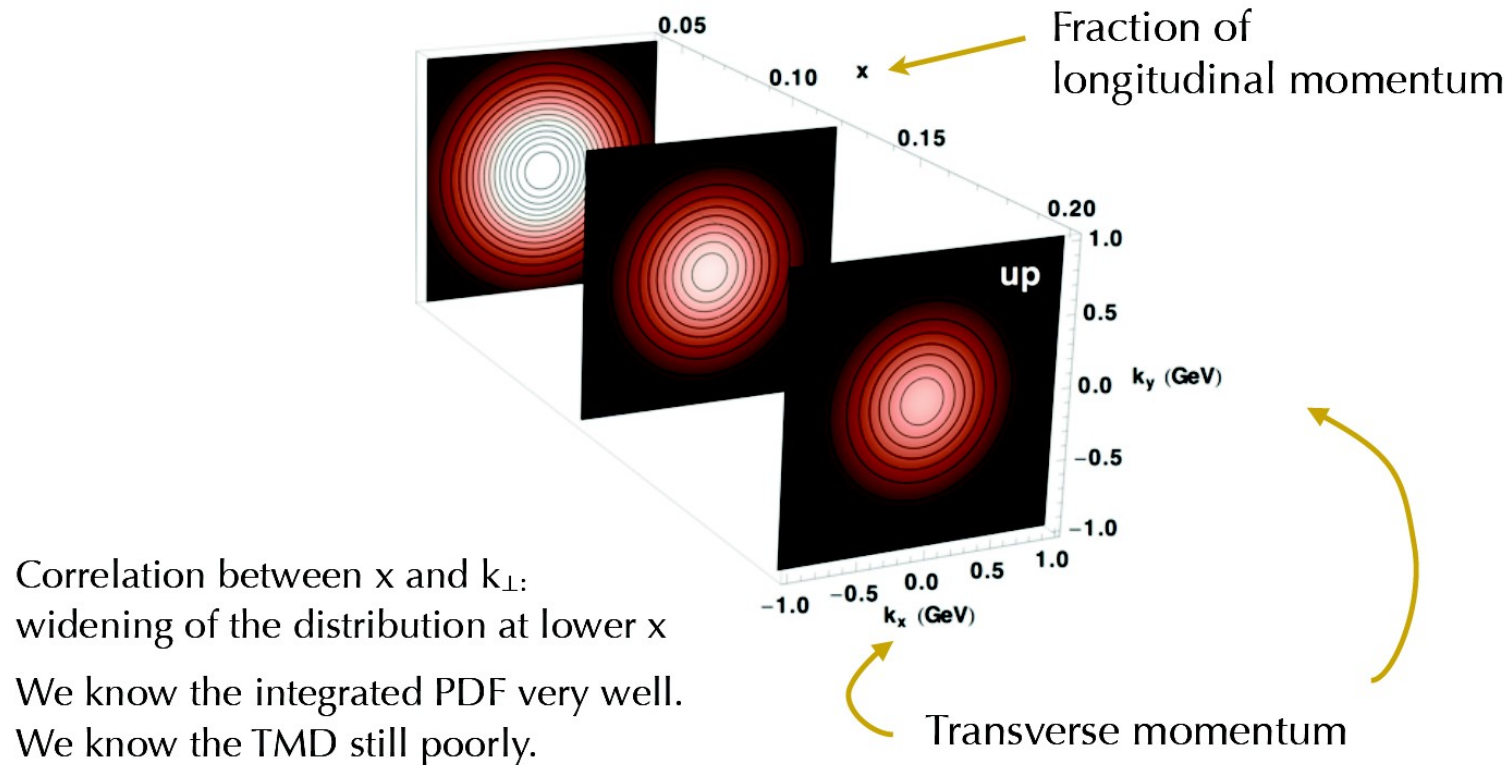


The Collins function embeds the correlation between the fragmenting quark spin and the transverse momentum of the produced hadron



An example: unpolarized TMD

From B. Pasquini talk ...



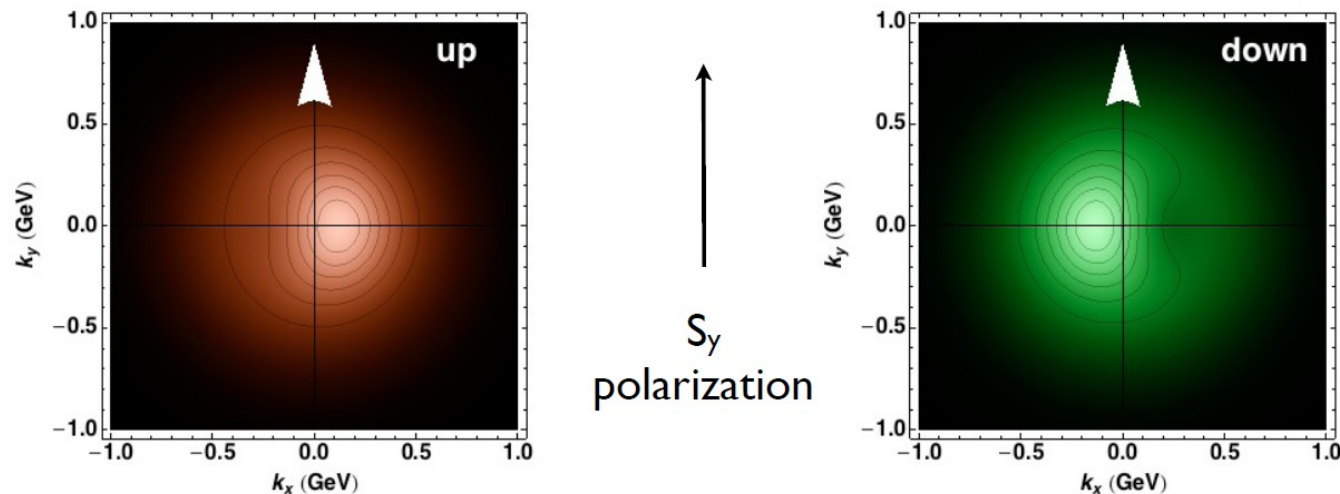
An example: the Sivers function



From B. Pasquini talk ...

distribution of unpolarized q in $\perp$ polarized $p^\uparrow$

$$f_{q/p^\uparrow}(x, \mathbf{k}_\perp) = f_1^q(x, \mathbf{k}_\perp^2) - f_{1T}^\perp(x, \mathbf{k}_\perp^2) \frac{(\hat{\mathbf{P}} \times \mathbf{k}_\perp) \cdot \mathbf{S}}{M}$$



deformation induced by Sivers function

*Bacchetta & Contalbrigo, The proton in 3D
Il Nuovo Saggiatore* **28** (12) n.1,2

Spin and angular momentum



*Proton and quarks are **spin**- $\frac{1}{2}$ particles*

*Quarks are confined inside an extended proton and move – the motion creates **Orbital Angular Momentum***

Can this motion be correlated with the spin of the proton?

Sivers function and angular momentum



quark and gluon contribution to the nucleon spin

$$J^{q,g} = \frac{1}{2} \int_{-1}^1 dx x \left(\underset{\substack{\downarrow \\ \text{unpolarized PDF}}}{H^{q,g}(x, 0, 0)} + E^{q,g}(x, 0, 0) \right)$$

not directly accessible

A. Bacchetta, M. Radici, Phys. Rev. Lett. 107 (2011) 212001

Assume

$$\begin{aligned} f_{1T}^{\perp(0)a}(x; Q_L^2) &= -L(x) E^a(x, 0, 0; Q_L^2) \\ f_{1T}^{\perp(0)a}(x, Q) &= \int d^2 \mathbf{k}_\perp \hat{f}_{1T}^{\perp a}(x, \mathbf{k}_\perp; Q) \end{aligned}$$

$L(x)$ = Lensing function

- Parameterize the Sivers and Lensing function
(can be extracted from models or by fitting data)
- Obtain $E^q(x)$
- Estimate J: $J^u \sim 0.23$, $J^{q \neq u} \sim 0$ at $Q^2 = 2.4 \text{ GeV}^2$

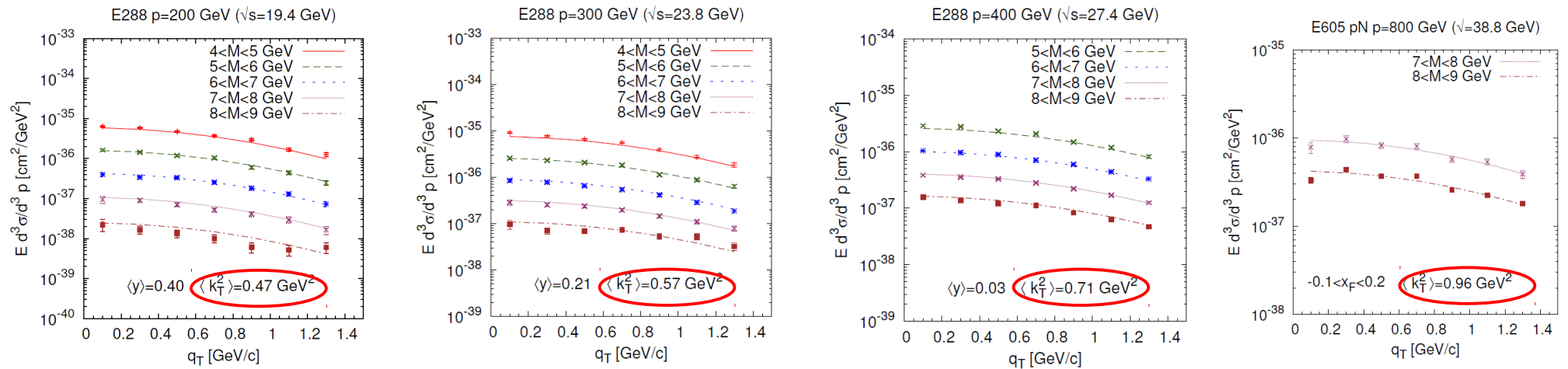


Is the $k_{\perp}$ TMD distribution Gaussian ?

Drell-Yan phenomenology



Stefano Melis preliminary studies



The fit on E288 and E605 Drell-Yan data is performed by assuming a gaussian $k_{\perp}$ dependence with a DGLAP evolution of the factorized

$$\hat{f}_{q/p}(x, k_{\perp}; Q) = f_{q/p}(x; Q) \frac{e^{-k_{\perp}^2 / \langle k_{\perp}^2 \rangle}}{\pi \langle k_{\perp}^2 \rangle}$$

The gaussian width is fitted independently for each different energy data set.

Notice that $\langle k_{\perp}^2 \rangle$ grows as energy grows

Schweitzer, Teckentrup, Metz, Phys.Rev. D81 (2010) 094019
D'Alesio, Murgia, Phys. Rev. D70 (2004) 074009



Extracting the unpolarized TMD Gaussian widths from SIDIS data

Let's see whether the Gaussian approximation works in the most simple case ...

Extracting the unpolarized TMD Gaussian widths from SIDIS multiplicities



M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, JHEP 1404 (2014) 005, ArXiv:1312.6261

- Data: **Hermes** (p and d targets, π^+ , π^- , K^+ , K^- production)

2660 data points in (x , z , P_T , Q^2 bins)

A. Airapetian et al.,
Phys. Rev. D87
(2013) 074029

Compass (d target, h^+ , h^- production)

18627 data points in (x , z , P_T , Q^2 bins)

C. Adolph et al.,
Eur. Phys. J. C73,
2531 (2013)

- Parameterizations:

$$\hat{f}_{q/p}(x, k_\perp; Q) = f_{q/p}(x; Q) \frac{e^{-k_\perp^2 / \langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}$$

CTEQ6L (DGLAP evolution)

1 free parameter
(no evolution)

$$D_{h/q}(z, p_\perp) = D_{h/q}(z) \frac{e^{-p_\perp^2 / \langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle},$$

DSS (DGLAP evolution)

1 free parameter
(no evolution)

Extracting the unpolarized TMD Gaussian widths from SIDIS multiplicities



In the simplest form of this model:

Flavor-independent average transverse momenta

No x-dependence

No z-dependence

Two parameters in total

Gaussian model:

$$\langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle.$$

$$\sigma \propto \frac{1}{\pi \langle P_T^2 \rangle} e^{-P_T^2 / \langle P_T^2 \rangle}$$

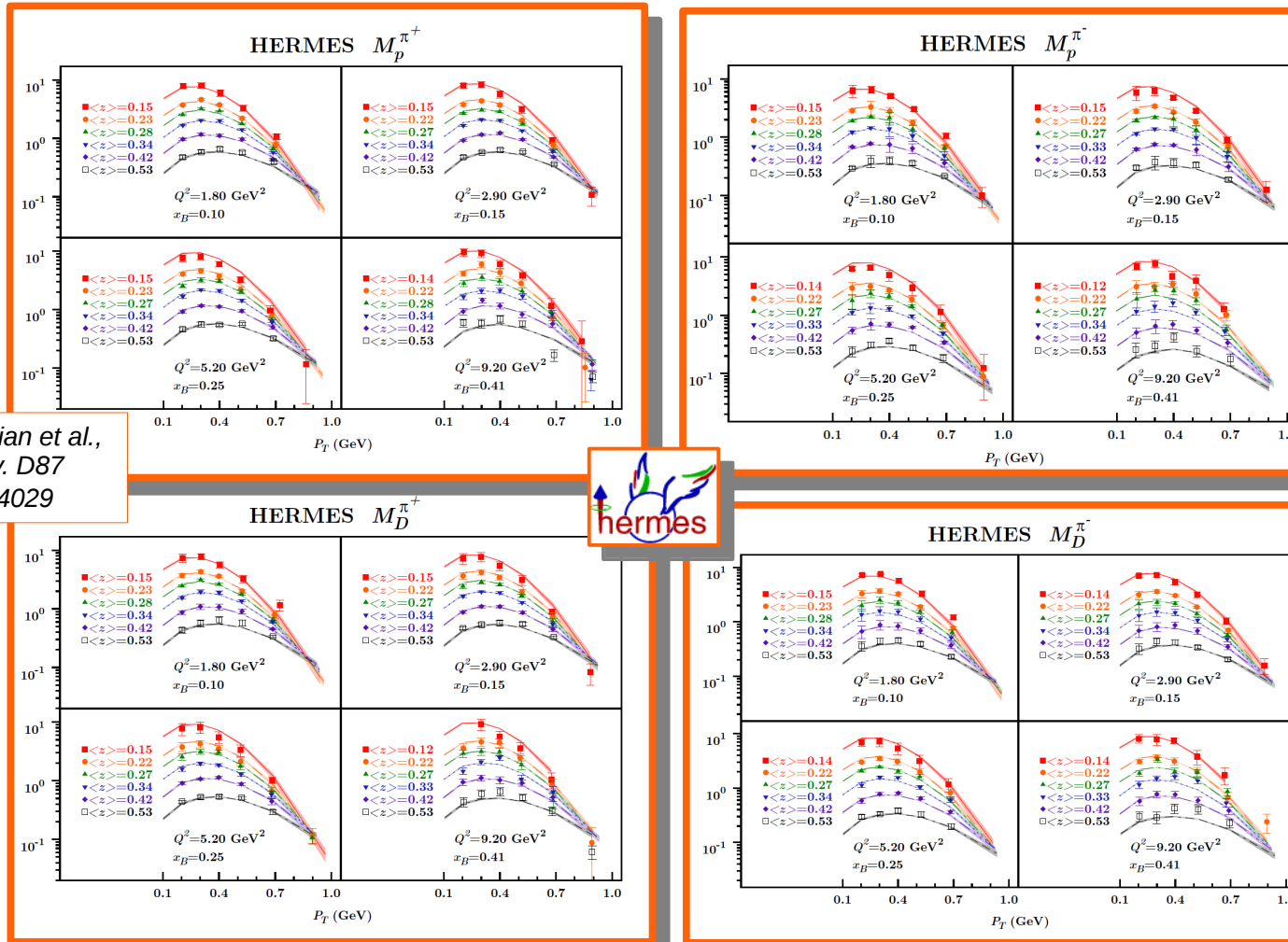
Normalization

Gaussian width

Extracting the unpolarized TMD Gaussian widths from SIDIS multiplicities



M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, JHEP 1404 (2014) 005, ArXiv:1312.6261



NO flavour dep.
In distr. fns.,
MILD flavour dep.
in fragm. fns,

Results agree with
A. Signori et al.,
JHEP 1311
(2013) 194

$$\langle k_{\perp}^2 \rangle = 0.57 \pm 0.08 \text{ GeV}^2$$

$$\langle p_{\perp}^2 \rangle = 0.12 \pm 0.01 \text{ GeV}^2$$

$$\chi_{\text{dof}}^2 = 1.69$$

Extracting the unpolarized TMD Gaussian widths from SIDIS multiplicities



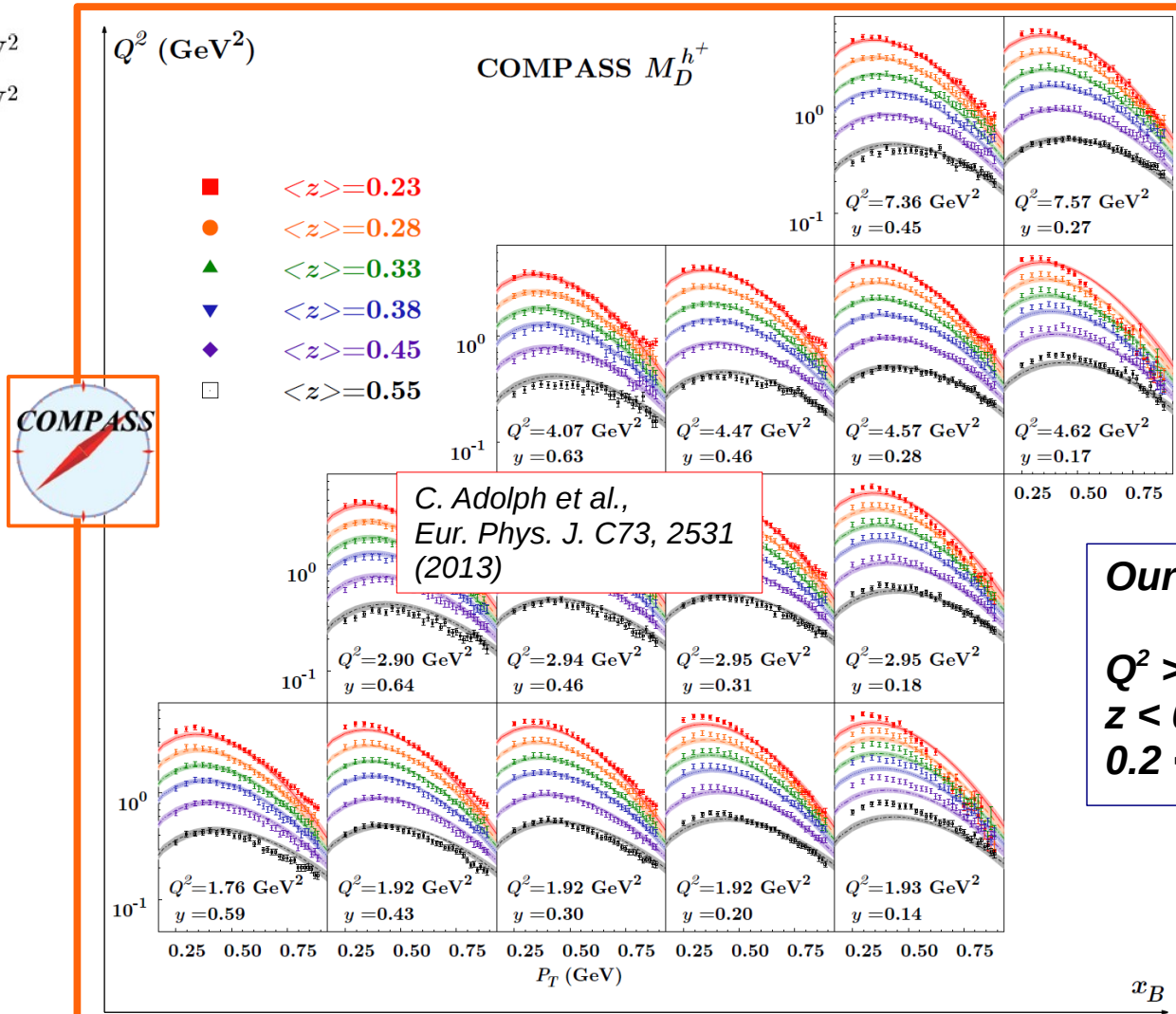
M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, JHEP 1404 (2014) 005, ArXiv:1312.6261

$$\langle k_{\perp}^2 \rangle = 0.60 \pm 0.14 \text{ GeV}^2$$

$$\langle p_{\perp}^2 \rangle = 0.20 \pm 0.02 \text{ GeV}^2$$

$$\chi_{\text{dof}}^2 = 3.42$$

$$N_y = A + B y$$



NO flavour dep.
In distr. fns.,
NO flavour dep.
in fragm. fns,

Our cuts:

$$Q^2 > 1.6 \text{ GeV}^2$$

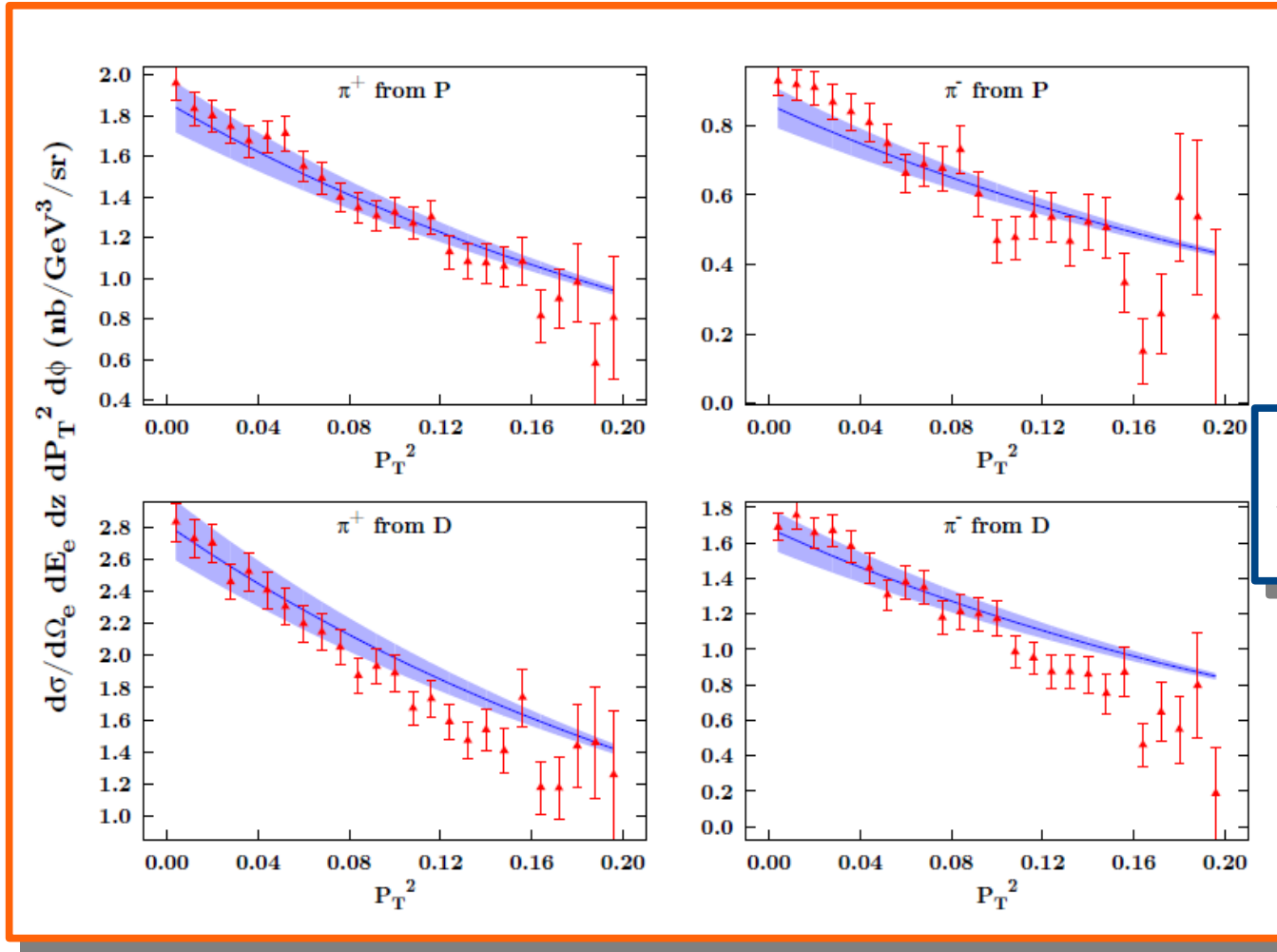
$$z < 0.6$$

$$0.2 < P_T < 0.9$$

Comparison with Jlab data HALL C



M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, JHEP 1404 (2014) 005, ArXiv:1312.6261



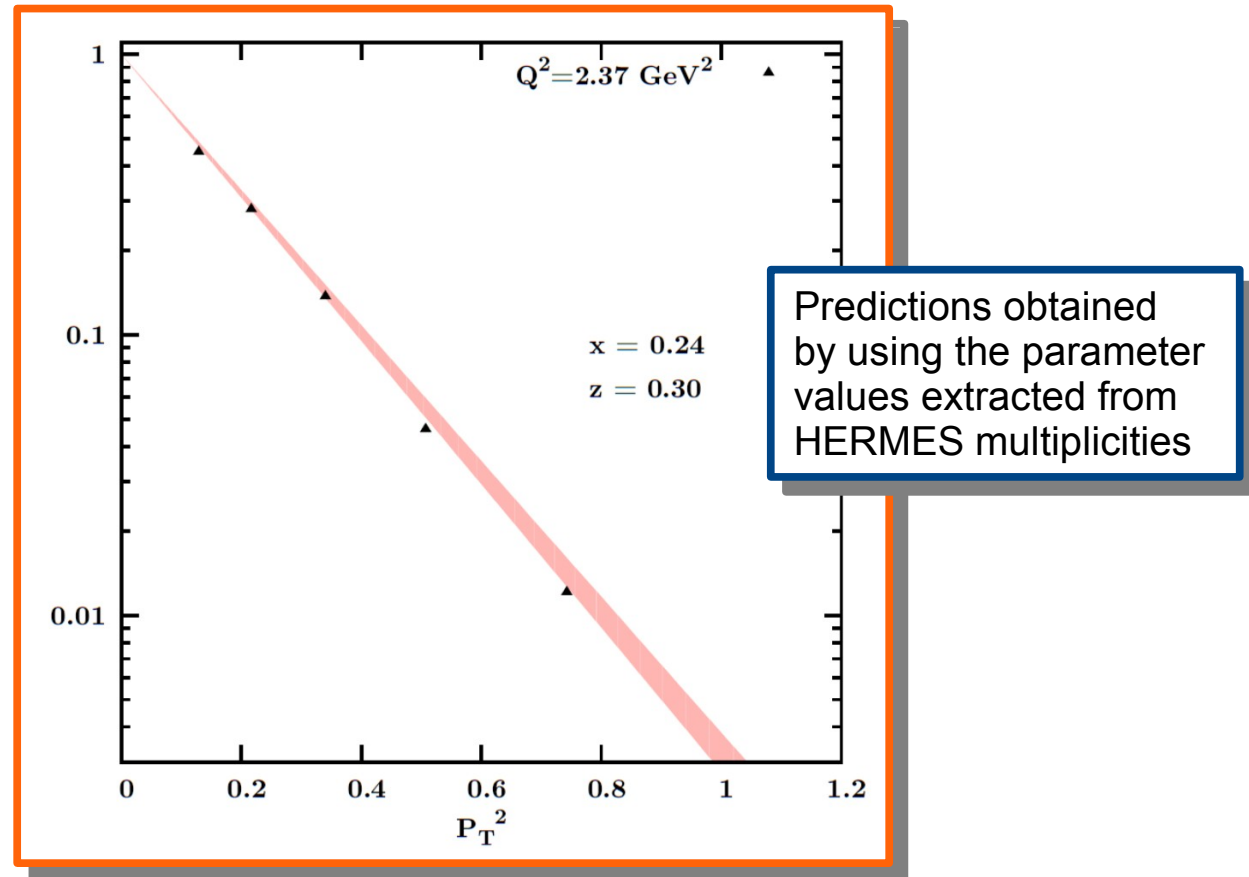
Predictions obtained by using the parameter values extracted from HERMES multiplicities

R. Asaturyan et al., Phys. Rev. C85, 015202 (2012)

Comparison with Jlab data CLAS 6



M. Anselmino, M. Boglione, O. Gonzalez, S. Melis, A. Prokudin, *JHEP* 1404 (2014) 005, ArXiv:1312.6261



M. Osipenko et al., *Phys. Rev. D* 80, 032004 (2009)

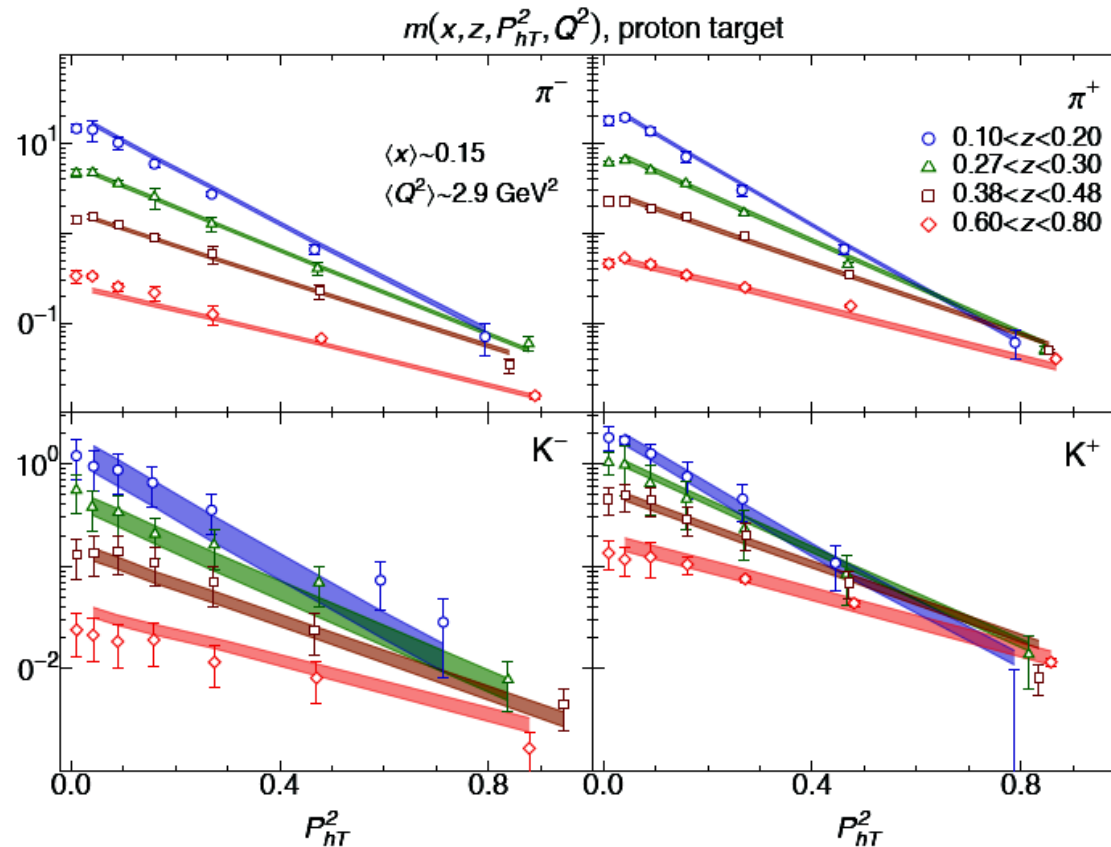


proton target global $\chi^2 / \text{d.o.f.} = 1.63 \pm 0.12$
 no flavor dep. 1.72 ± 0.11

5/7 parameters
 Much more complex
 parametrization of x
 and z dependence

π^-
 1.80 ± 0.27
 1.83 ± 0.25

K^-
 0.78 ± 0.15
 0.87 ± 0.16



π^+
 2.64 ± 0.21
 2.89 ± 0.23

K^+
 0.46 ± 0.07
 0.43 ± 0.07

A. Signori, A. Bacchetta, M. Radici, G. Schnell, JHEP 1311 (2013) 194

What about $k_{\perp}$ Gaussians then ?



- Gaussian $k_{\perp}$ distributions seem to describe the data pretty well.
- However, one could legitimately wonder whether other distributions could do better.



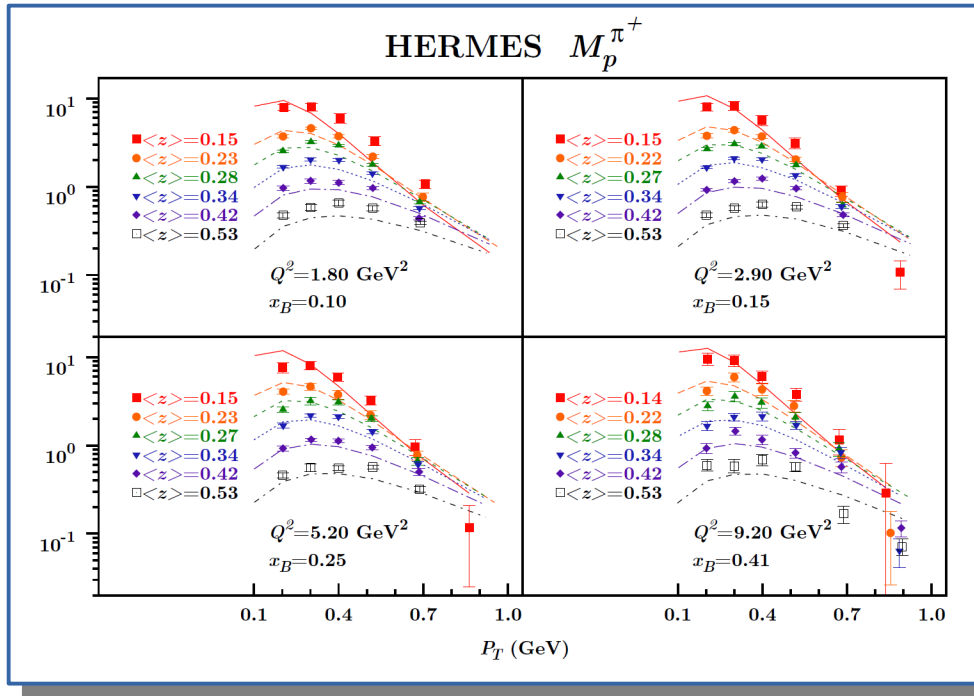
- Drop the $x - k_{\perp}$ factorization hypothesis and use a quark-diquark like model for the unpolarized TMDs
- Perform the same fits again and compare with the results obtained with Gaussian distributions
- **We find that the description of data significantly worsen !**

What about $k_{\perp}$ Gaussians then ?



V. Barone, M. Boglione, O. Gonzalez, S. Melis, PRELIMINARY

Vaguely inspired to A. Bacchetta, F. Conti, M. Radici,
Phys.Rev. D78 (2008) 074010



$$\chi^2_{dof} = 6.96$$

$$M_s^2 = 0.43 \quad \Lambda^2 = 0.25^\dagger$$

$$\tilde{M}_s^2 = 1.11 \quad \tilde{\Lambda}^2 = 0.00^*$$

$$f(x, k_{\perp}) = f(x) \frac{6L^6}{\pi(2\mu^2 + L^2)} \frac{k_{\perp}^2 + \mu^2}{(k_{\perp}^2 + L^2)^4}$$

$$L^2 = xM_s^2 + (1-x)\Lambda^2 - x(1-x)M_p^2$$

$$\mu^2 = (m + xM_p)^2$$

$$D(z, p_{\perp}) = D(z) \frac{6\tilde{L}^6}{\pi(2\nu^2 + \tilde{L}^2)} \frac{p_{\perp}^2 + \nu^2}{(p_{\perp}^2 + \tilde{L}^2)^4}$$

$$\tilde{L}^2 = (1-z)M_h^2 + z(\tilde{M}_s^2 - \tilde{\Lambda}^2) + z^2\tilde{\Lambda}^2$$

$$\nu^2 = (\tilde{M}_s - (1-z)\tilde{m})^2$$

$$\langle k_{\perp}^2 \rangle = \frac{L^2(2L^2 + \mu^2)}{L^2 + 2\mu^2}$$

$$\langle p_{\perp}^2 \rangle = \frac{\tilde{L}^2(2\tilde{L}^2 + \nu^2)}{\tilde{L}^2 + 2\nu^2}$$



Simultaneous extraction of transversity and the Collins function

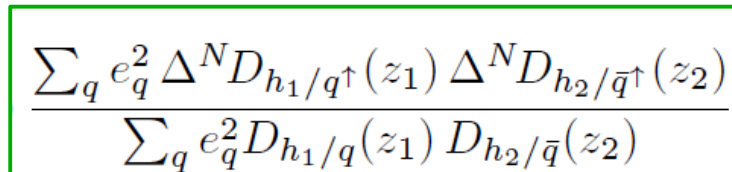
A 3D rendering of four interlocking puzzle pieces in red, blue, yellow, and green, arranged in a small cluster.

Anselmino, Boglione, D'Alesio, Gonzalez, Melis, Murgia, Prokudin, in preparation



$$d\sigma^\uparrow - d\sigma^\downarrow = \sum_q h_{1q}(x, k_\perp) \otimes d\Delta\hat{\sigma}(y, \mathbf{k}_\perp) \otimes \Delta^N D_{h/q^\uparrow}(z, \mathbf{p}_\perp)$$

Collins

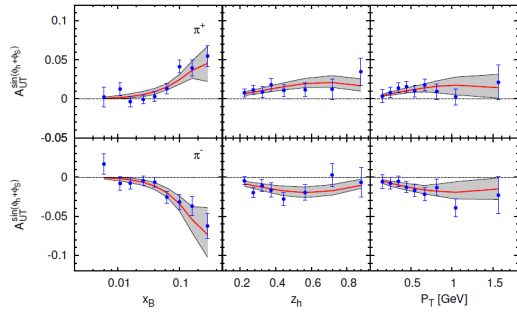
$$e^+ e^- \rightarrow h_1 h_2 X$$


Simultaneous extraction of transversity and the Collins function



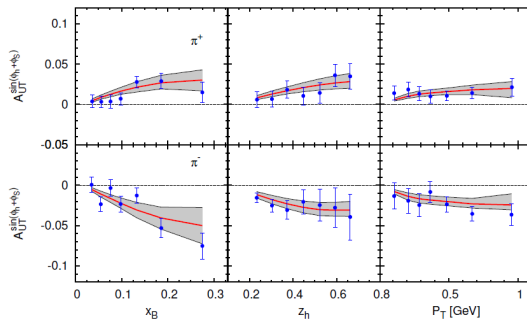
Anselmino, Boglione, D'Alesio, Melis, Murgia, Prokudin, *Phys.Rev. D87 (2013) 094019*

COMPASS PROTON



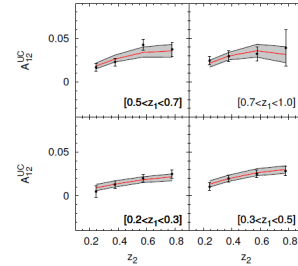
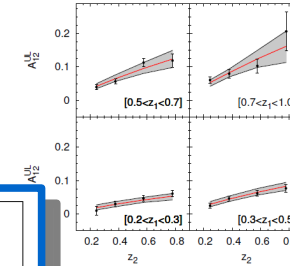
SIDIS

HERMES PROTON

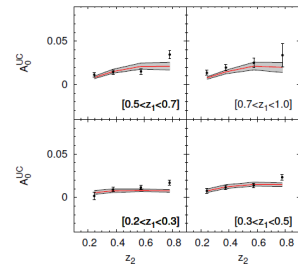
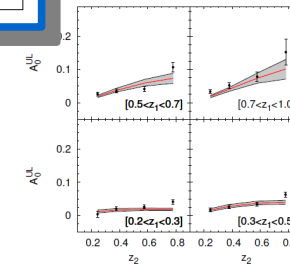


SIDIS data are fitted with excellent χ^2

	FIT DATA 178 points	SIDIS 146 points	A_{12}^{UL} 16 points	A_{12}^{UC} 16 points	A_0^{UL} 16 points	A_0^{UC} 16 points
Standard Parameterization $\chi^2_{d.o.f} = 0.80$	$\chi^2_{tot} = 135$	$\chi^2 = 123$	$\chi^2 = 7$	$\chi^2 = 5$	$\chi^2 = 44$ NO FIT	$\chi^2 = 39$ NO FIT
Standard Parameterization $\chi^2_{d.o.f} = 1.12$	$\chi^2_{tot} = 190$	$\chi^2 = 125$	$\chi^2 = 20$ NO FIT	$\chi^2 = 12$ NO FIT	$\chi^2 = 35$	$\chi^2 = 30$
Polynomial Parameterization $\chi^2_{d.o.f} = 0.81$	$\chi^2_{tot} = 136$	$\chi^2 = 123$	$\chi^2 = 8$	$\chi^2 = 5$	$\chi^2 = 45$ NO FIT	$\chi^2 = 39$ NO FIT
Polynomial Parameterization $\chi^2_{d.o.f} = 1.01$	$\chi^2_{tot} = 171$	$\chi^2 = 141$	$\chi^2 = 44$ NO FIT	$\chi^2 = 27$ NO FIT	$\chi^2 = 15$	$\chi^2 = 15$



$e^+e^- \rightarrow h_1 h_2 X$



There seems to be some tension
In the fit between BELLE A_{12} and A_0

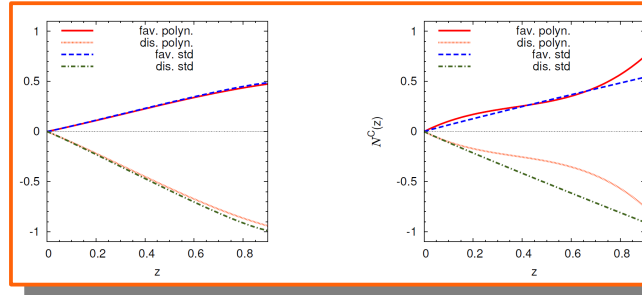
Tension between *BELLE* A_0 and A_{12} ?



Anselmino, Boglione, D'Alesio, Melis, Murgia, Prokudin, *Phys.Rev. D87* (2013) 094019

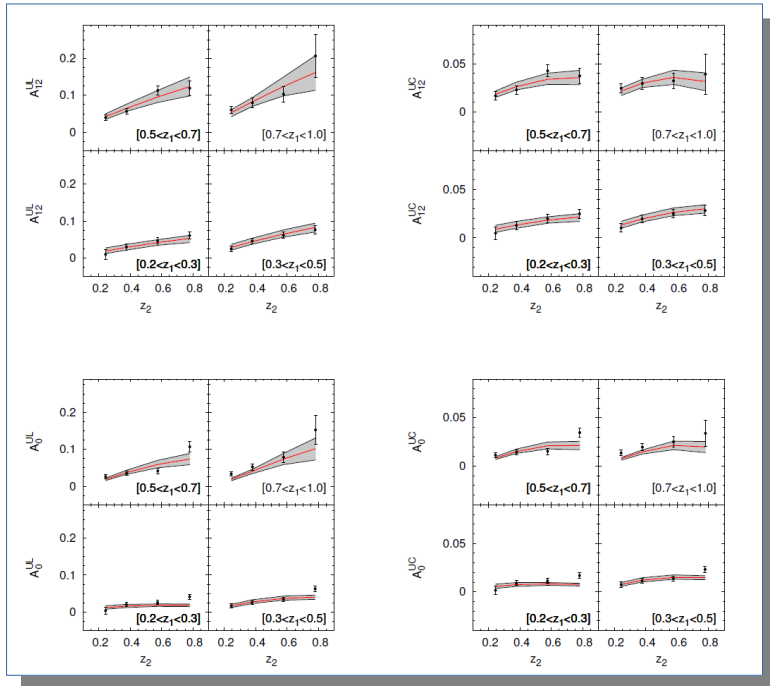
**Standard parameterization
of the Collins function**

$$\mathcal{N}_q^C(z) = N_q^C z^\gamma (1-z)^\delta \frac{(\gamma+\delta)^{(\gamma+\delta)}}{\gamma^\gamma \delta^\delta}$$

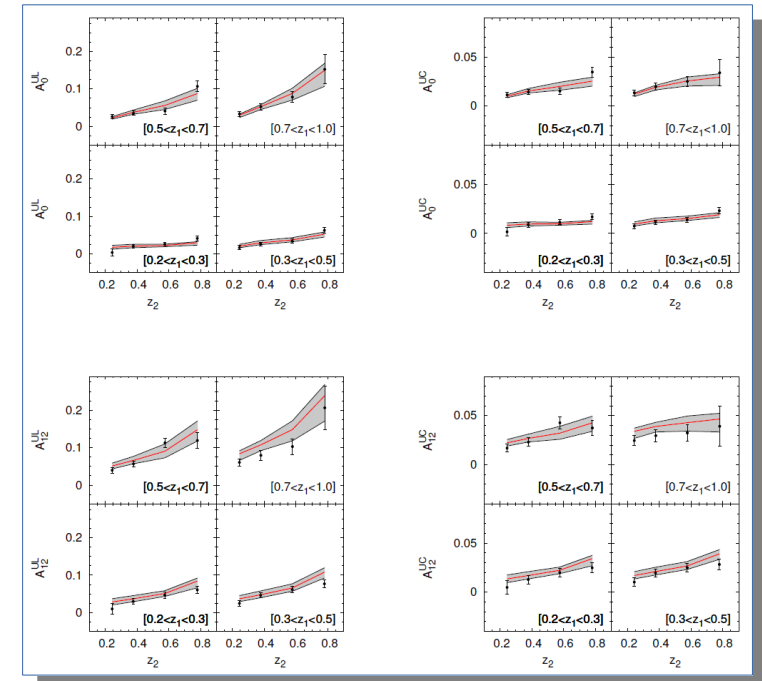


**Polynomial parameterization
of the Collins function**

$$\mathcal{N}_q^C(z) = N_q^C z [(1-a-b) + az + bz^2]$$



Fit of A_{12} with standard param.



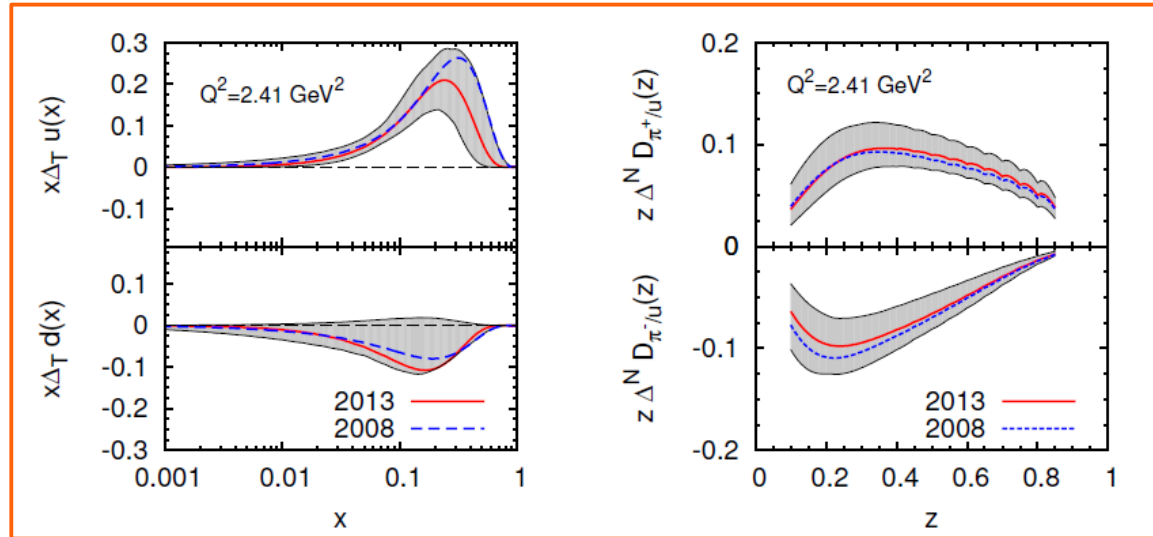
Fit of A_0 with polynomial param.

Simultaneous extraction of transversity and the Collins function

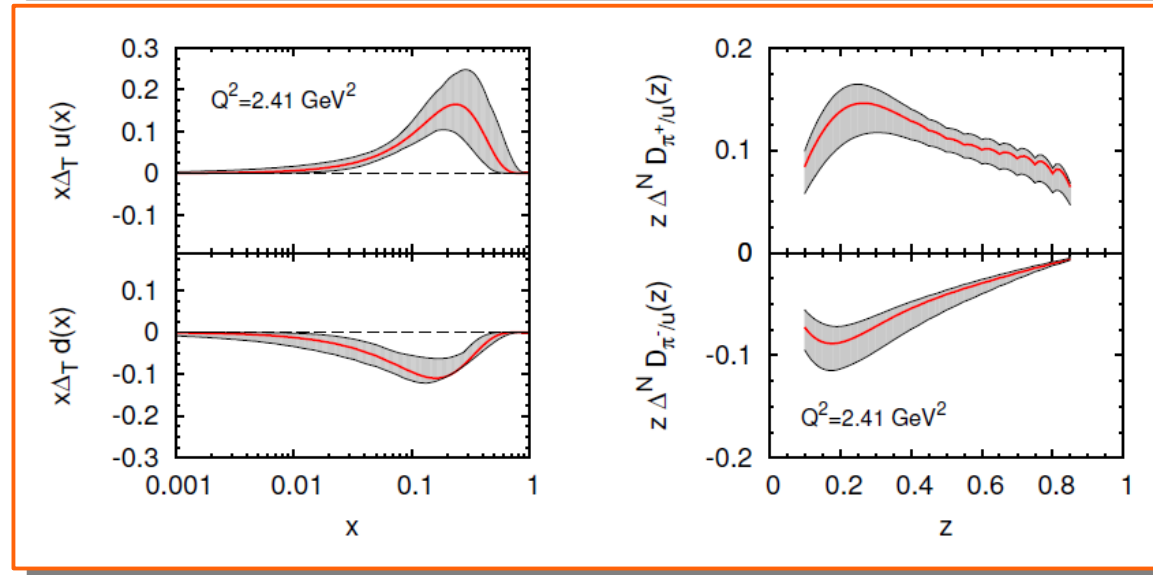


Anselmino, Boglione, D'Alesio, Melis, Murgia, Prokudin, *Phys.Rev. D87 (2013) 094019*

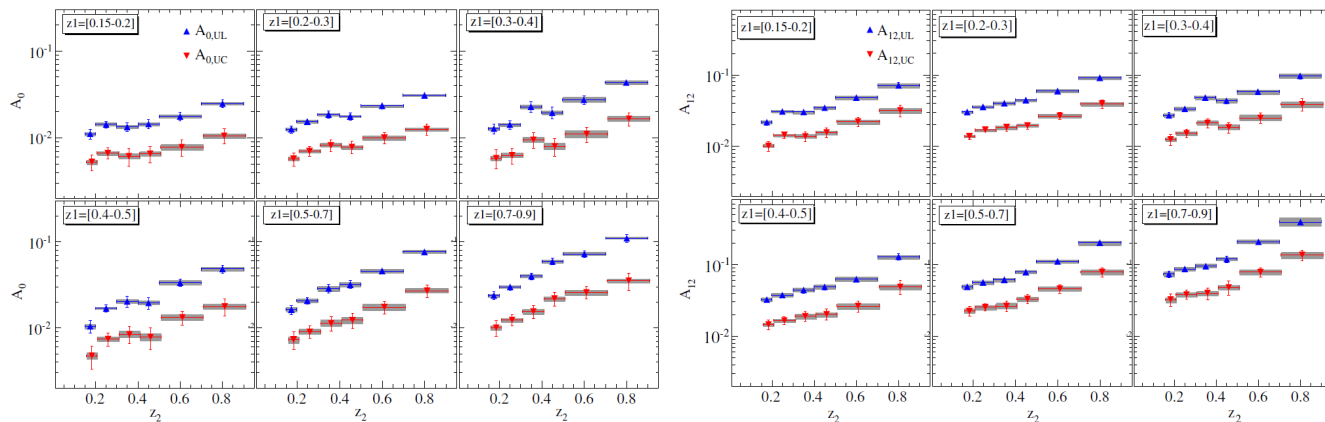
Standard.
param.
of the
Collins
function



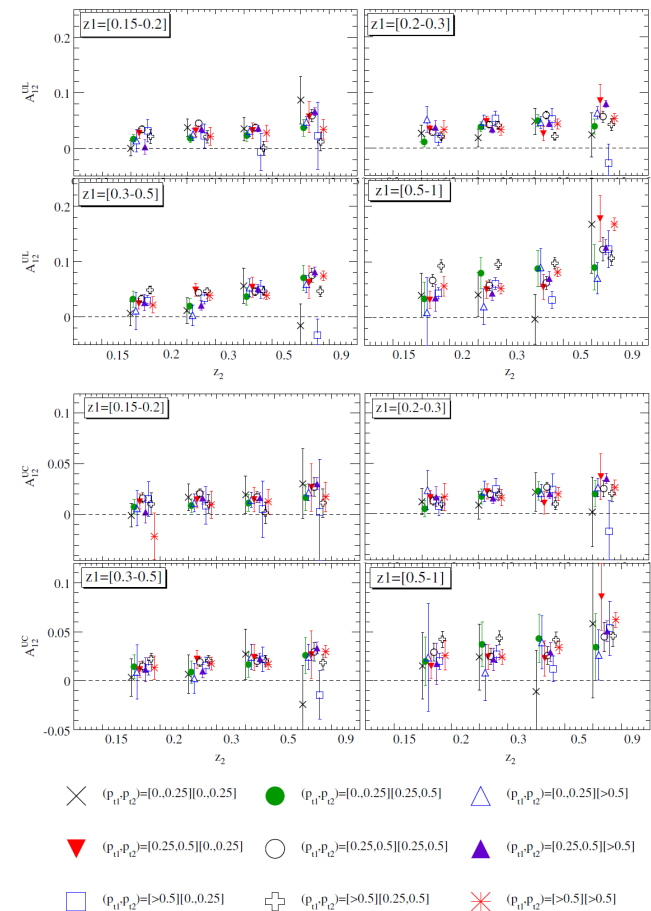
Polynom.
param.
of the
Collins
function



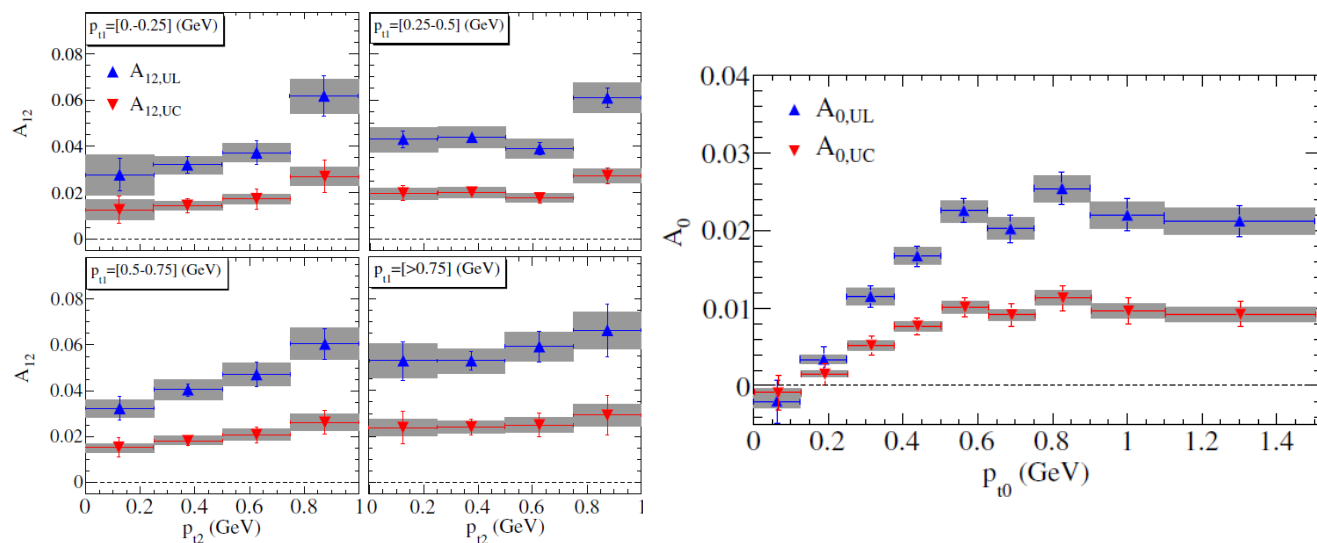
BaBar measurements of A_0 and A_{12} as a function of z_1 and z_2



BaBar multidimensional data on A_{12} in bins of $(z_1, z_2, p_{t1}, p_{t2})$



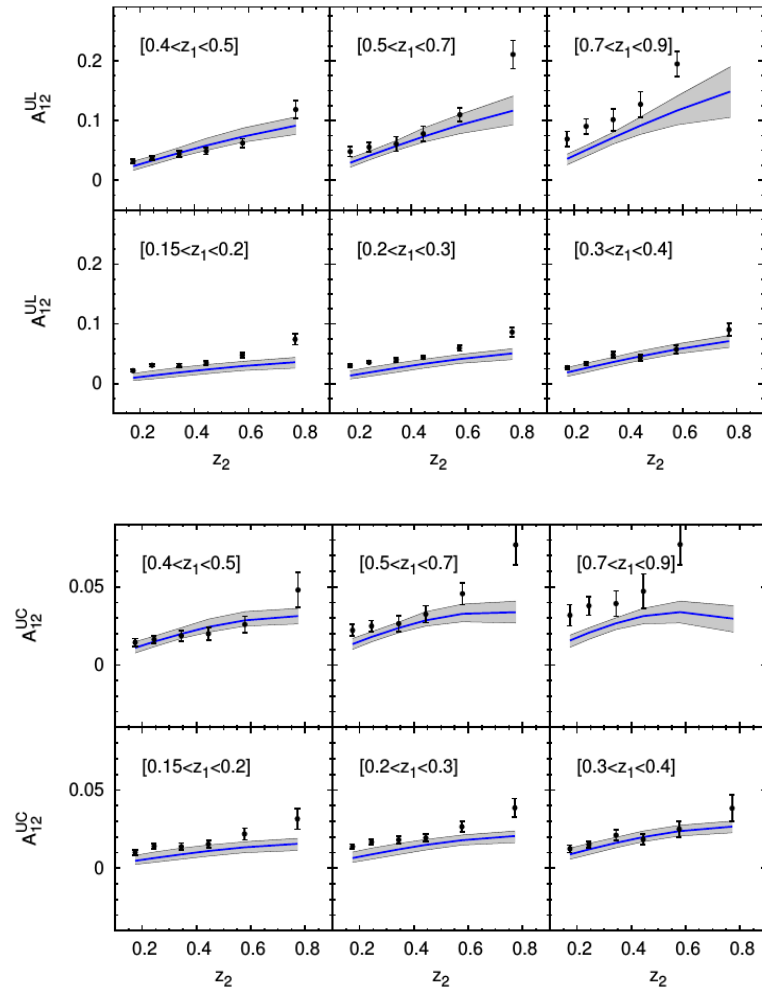
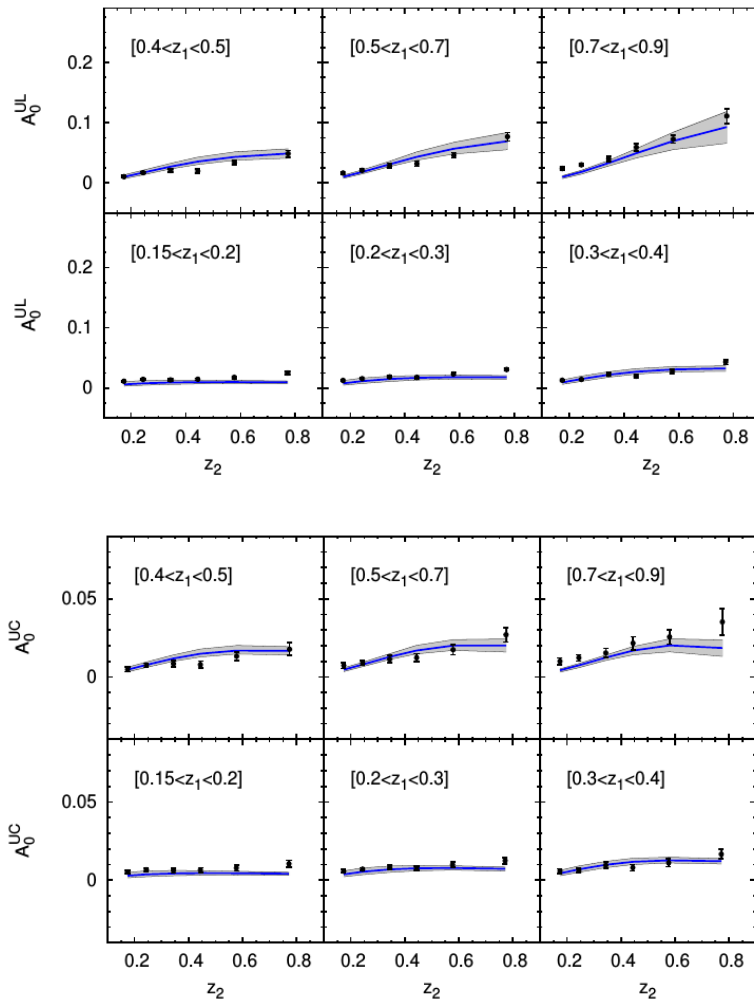
BaBar measurements of A_0 and A_{12} as a function of p_{t0}, p_{t1} and p_{t2}



Our predictions compared to BaBar data



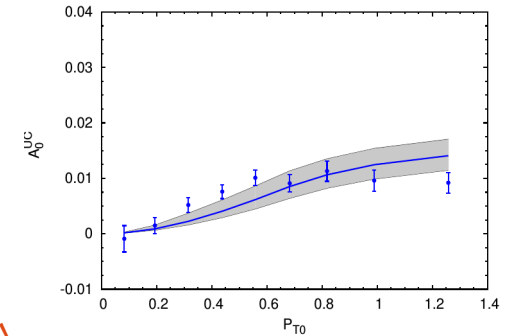
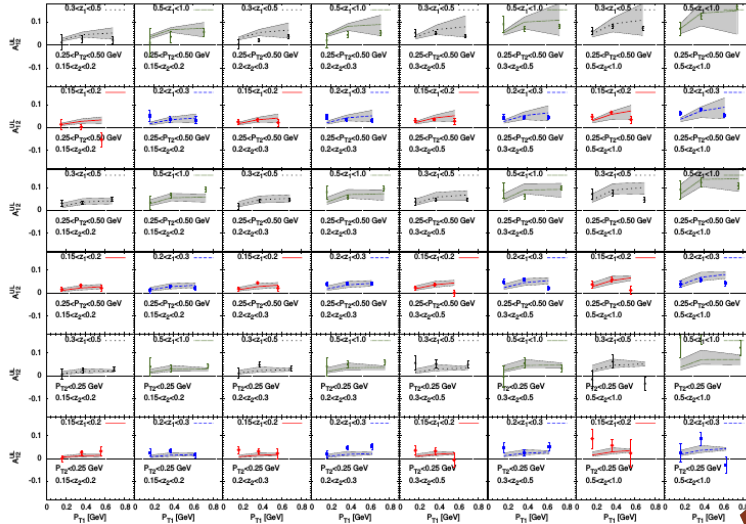
Predictions obtained by using the parameters extracted by best-fitting BELLE A_{12} experimental data with the standard parametrization of the Collins function are compared to BaBar measurements of A_0 and A_{12} as a function of z_1 and z_2



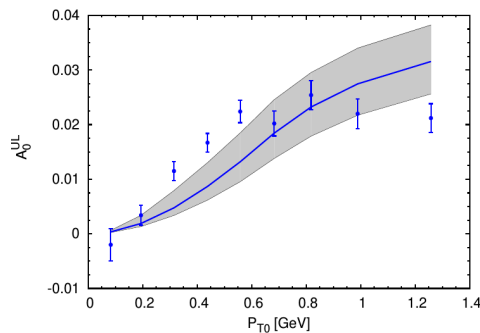
Our predictions compared to BaBar data



Anselmino, Boglione, D'Alesio, Gonzalez, Melis, Murgia, Prokudin, in preparation



Preliminary
Very many thanks to S. Melis and O. Gonzalez!

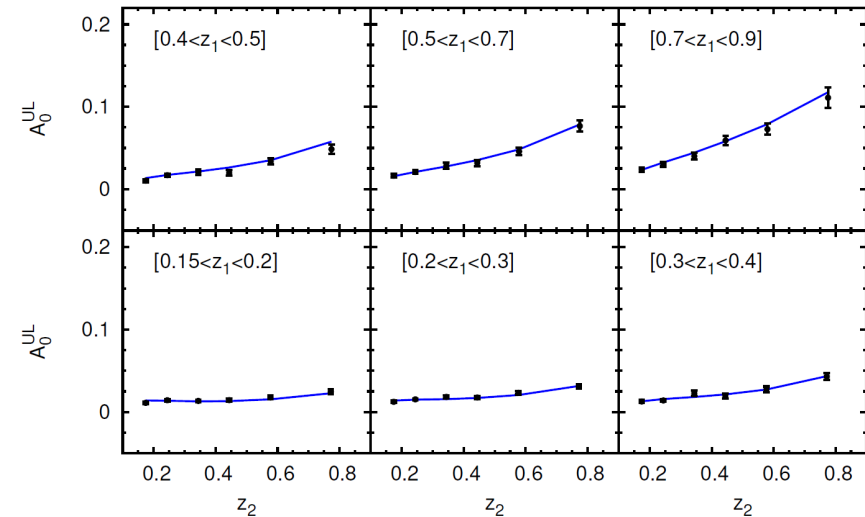
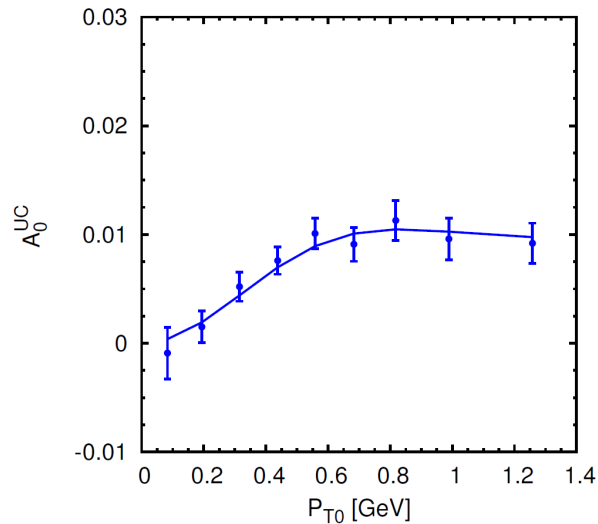




New (preliminary) fits

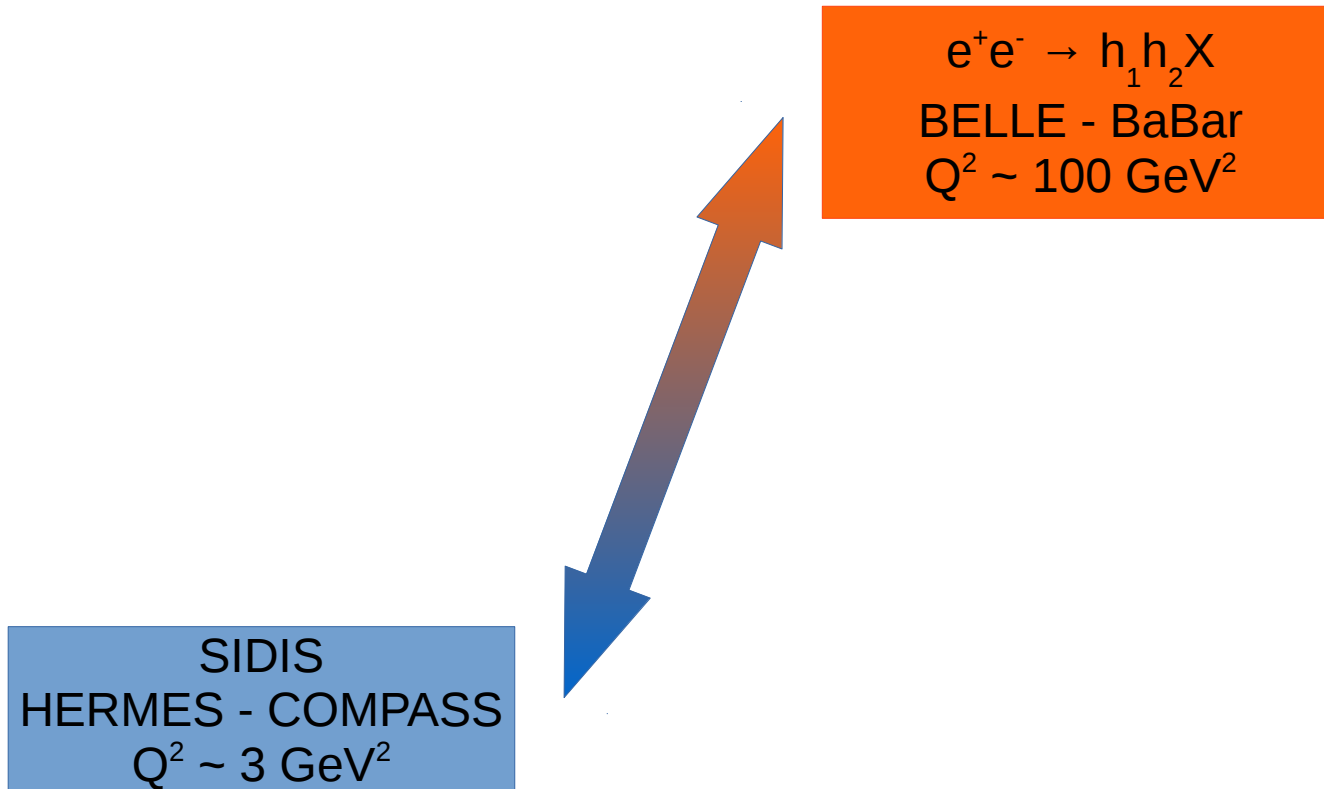
Anselmino, Boglione, D'Alesio, Gonzalez, Melis, Murgia, Prokudin, in preparation

- Preliminary fits of the new data look very promising (new flexible polynomial parameterization)



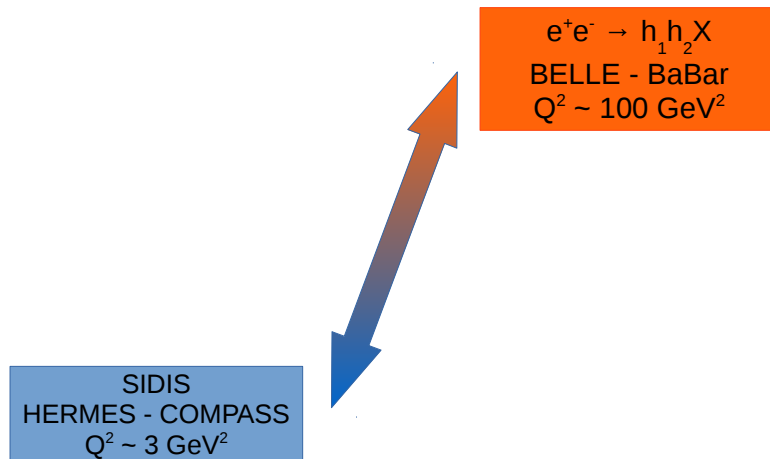
- Moreover, we are studying different approaches to **scale evolution** for the Collins functions

What about Q^2 evolution ?





What about Q^2 evolution ?



Simultaneous fits of SIDIS and $e^+e^- \rightarrow h_1 h_2 X$
Involve data sets at very different Q^2 scales

In our computation the Collins TMD function evolves according to DGLAP evolution equations, through its $D_{h/q}(z, p_t, Q^2)$ component

Could TMD evolution be an issue ?
Could TMD evolution affect our results ?



TMD evolution phenomenology

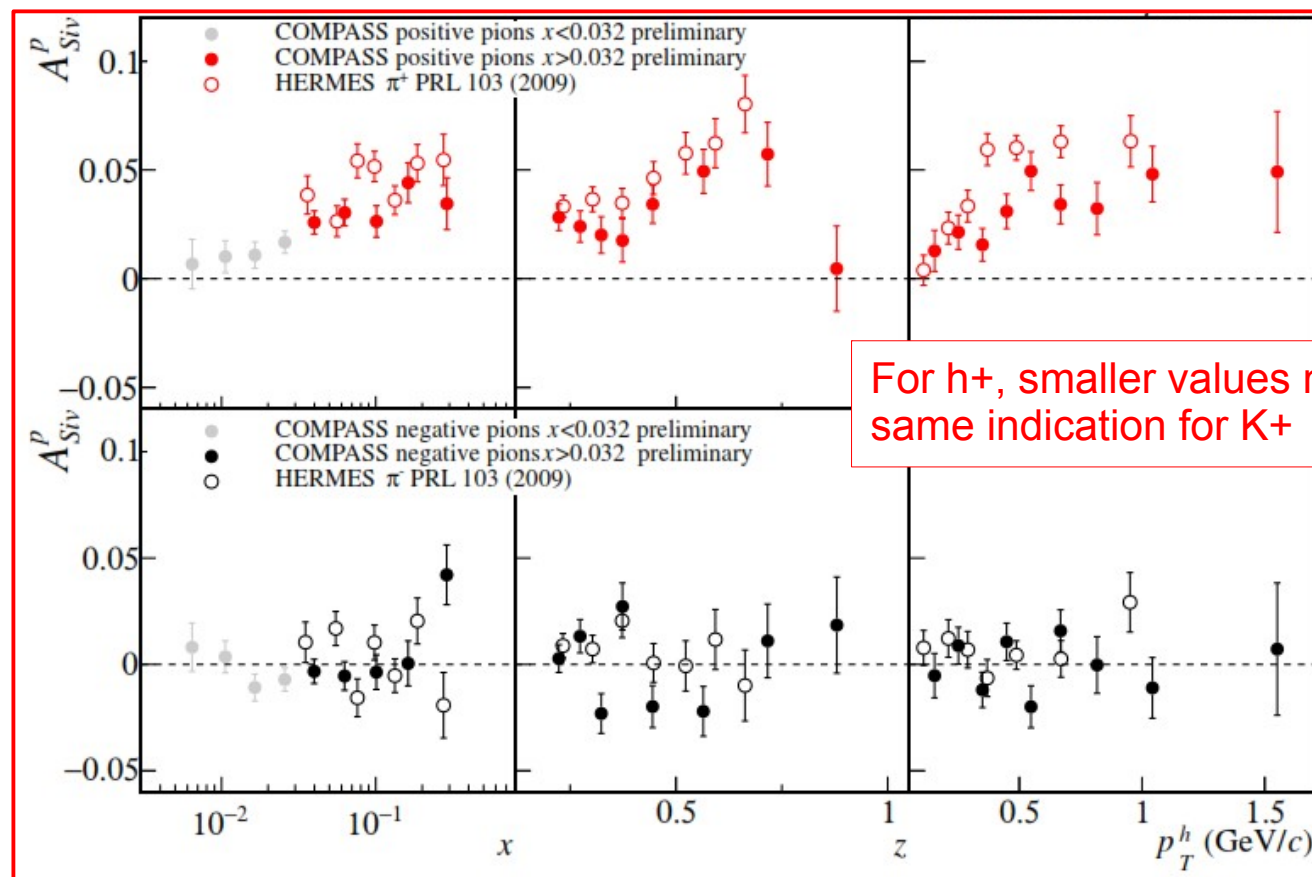
Does most recent SIDIS data suggest TMD evolution ?



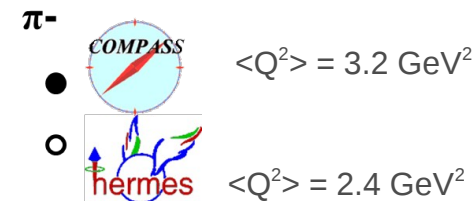
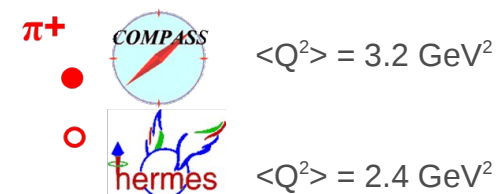
Sivers asymmetry on **proton** ($x > 0.032$)

Charged pions (and kaons), 2010 data

Comparison with HERMES results



For h^+ , smaller values measured by COMPASS;
same indication for K^+



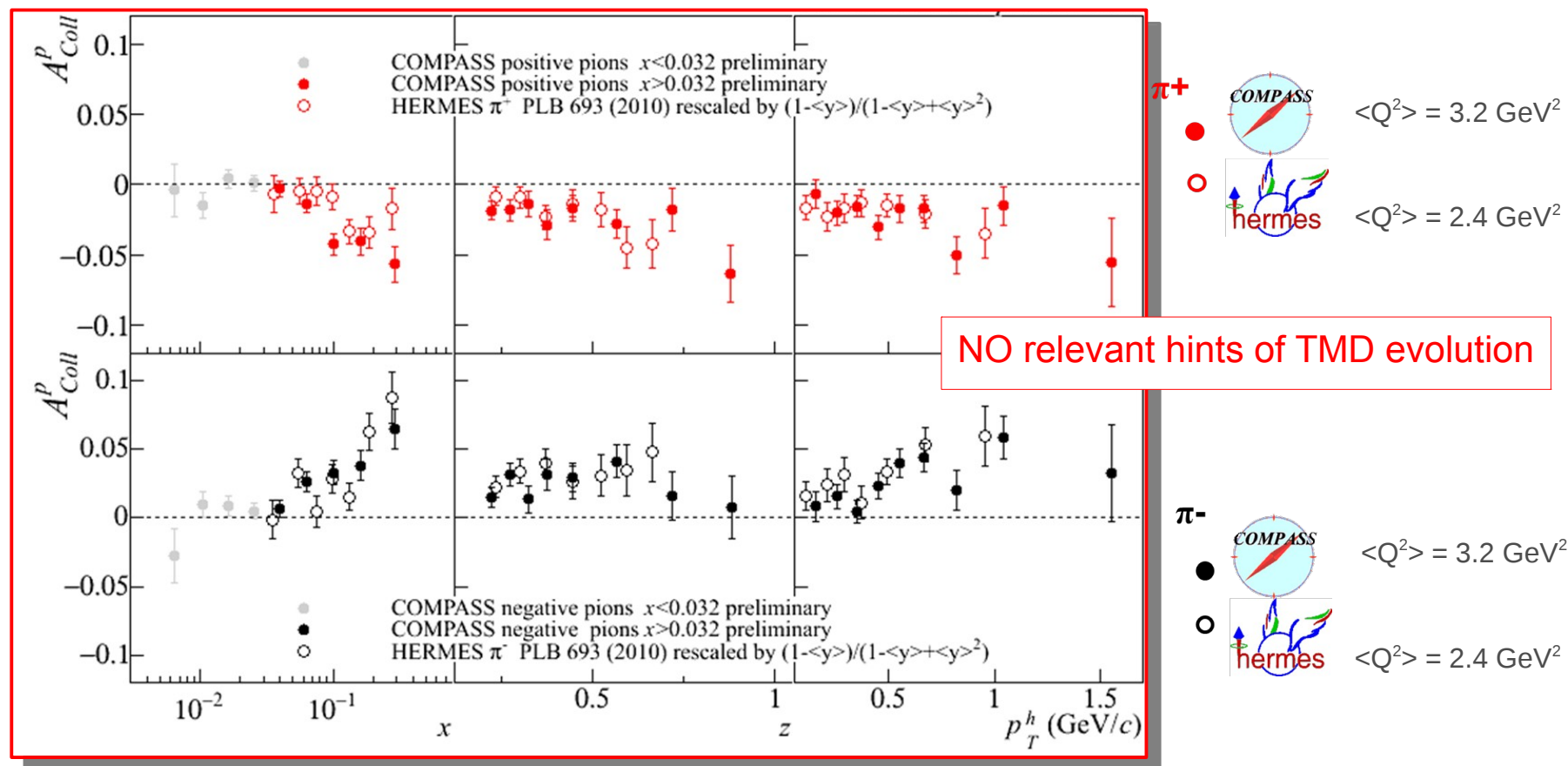
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Collins asymmetry on **proton** ($x > 0.032$)

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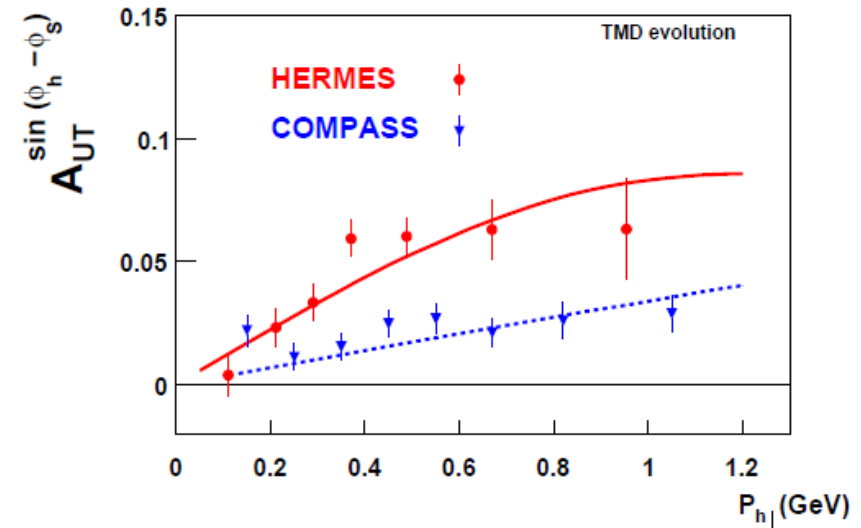
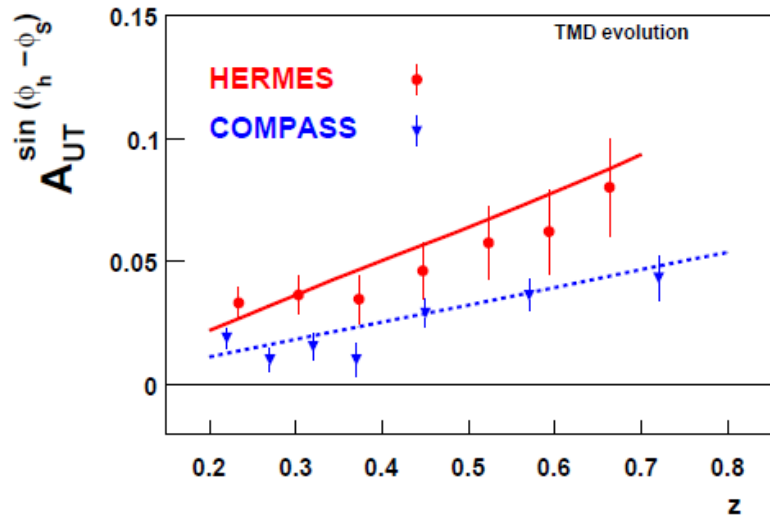
Anna Martin



Sivers TMD evolution: phenomenological results



Aybat, Prokudin, Rogers, Phys. Rev. Lett. 108, (2011) 242003



$\langle Q^2 \rangle = 2.4 \text{ GeV}^2$

Q^2 in the range
[1.3 – 6.2] GeV^2

- No x dependence taken into account
- Sivers A_{UT} calculated at two fixed different values of Q^2 : 2.4 and 3.8 GeV^2
- Evolution effects are then compared.



$\langle Q^2 \rangle = 3.8 \text{ GeV}^2$

Q^2 in the range
[1.3 – 20.5] GeV^2

Sivers function from HERMES and COMPASS SIDIS data



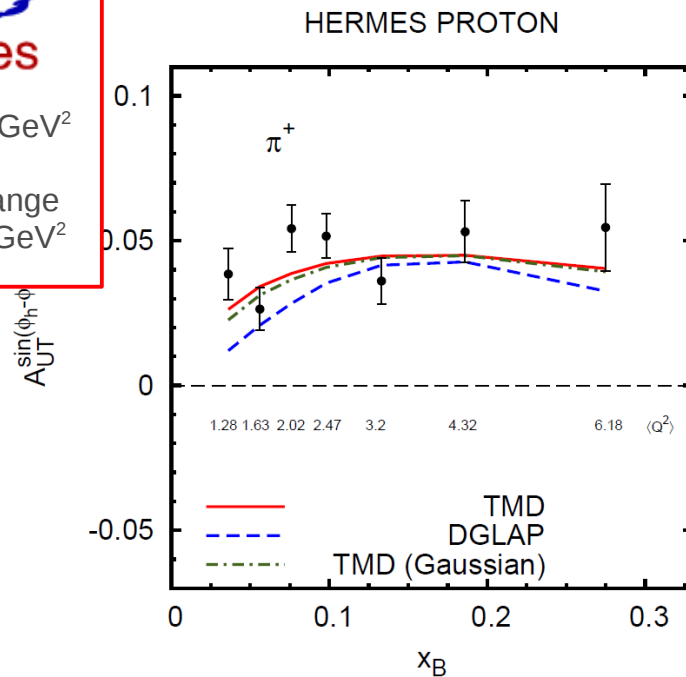
Anselmino, Boglione, Melis, *Phys. Rev. D* 86 (2012) 014028

- Q^2 and x dependence rigorously taken into account
- **2 different fits:**
 - TMD-fit (computing TMD evolution equations numerically)
 - DGLAP evolution equation for the collinear part of the TMD)



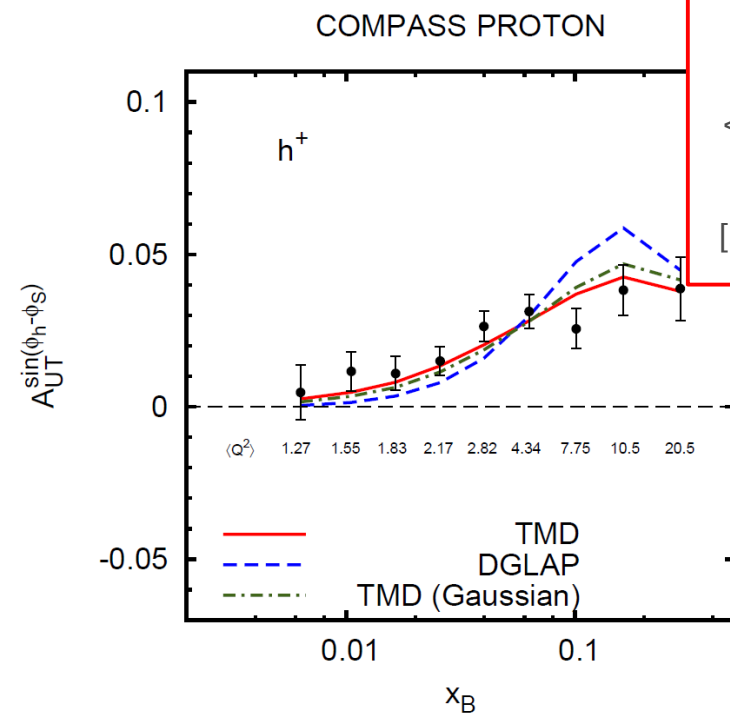
$\langle Q^2 \rangle = 2.4 \text{ GeV}^2$

Q^2 in the range
[1.3 – 6.2] GeV^2



$\langle Q^2 \rangle = 3.8 \text{ GeV}^2$

Q^2 in the range
[1.3 – 20.5] GeV^2



A. Airapetian et al., *Phys. Rev. Lett.* 103, (2009) 152002

C. Adolph et al., *Phys. Lett. B* 717 (2012) 383

Scale Evolution of unpolarized multiplicities



HERMES and COMPASS multiplicities cover the same range in Q^2 ...

$$\langle k_{\perp}^2 \rangle = g_1 + g_2 \ln(Q^2/Q_0^2) + g_3 \ln(10 e x)$$

$$\langle p_{\perp}^2 \rangle = g'_1 + z^2 g'_2 \ln(Q^2/Q_0^2)$$

$$\langle P_T^2 \rangle = g'_1 + z^2 [g_1 + g_2 \ln(Q^2/Q_0^2) + g_3 \ln(10 e x)]$$

- HERMES multiplicities show no sensitivity to these parameters
- COMPASS fitting is much more involved.
After correcting for normalization,
we find that the total χ^2 decreases from 3.42 to 2.69.



Conclusions

- $k_{\perp}$ - Gaussian distributions work reasonably well. However the Gaussian widths appear to depend on the energy (s) of each experiment
- Non perturbative aspects (TMDs) are crucial also for high-energy observable (for instance, Z -production spectrum)
- We have come a long way, but ...

What Next ?

- Find theories/models/prescriptions which simultaneously explain all available experimental data from different experiments (Drell-Yan, SIDIS and $e+e^-$ scattering) with TMD evolution
- More, new, high-quality data are very much needed to be able to perform solid and realistic phenomenological analyses of TMDs.
EIC ...