Dmitrij Siemens

Combined analysis of πN→πN and πN→ππN in chiral effective field theory at one-loop level

- Formal Aspects
- Combined Fit
- Predictions

Motivation and Methodology

Why?

Aim	Theoretical description of $\pi N \to \pi N$ and $\pi N \to \pi\pi N$ above threshold
Problem I	QCD is non-perturbative for low energies
Solution I	Effective Field Theory ⇒ Chiral Perturbation Theory
Problem II	Resonances play an important role
Solution II	Inclusion of the most dominant resonance $\Delta(1232)$ as an explicit degree of freedom

How To

1. Pick Lagrangian
2. Derive Feynman rules
3. Draw all graphs up to specified order
4. Calculate amplitudes in specified decomposition
5. Calculate T-matrix and matrix element squared
6. Calculate observables like cross sections and phase shifts

2 Chiral Approaches

χPT

- EFT of Standard Model
- Relies upon chiral symmetry of QCD
- DOF are mesons and baryons instead of quarks
- Breakdown scale of theory: $\Lambda_\chi \approx 1 \text{ GeV}$

HBχPT

- Non-relativistic limit of χPT
- Inclusion of $1/m_N$ expansion into power counting
- Original motivation: Allows calculations beyond tree level

Formal Aspects

χPT & HBχPT

Effective Lagrangian

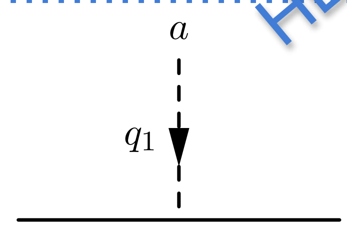
$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(2)}_{\pi\pi} + \mathcal{L}^{(4)}_{\pi\pi} + \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \mathcal{L}^{(4)}_{\pi N}$$

$$Q = \left\{ \frac{q}{\Lambda_\chi}, \frac{M_\pi}{\Lambda_\chi} \right\}$$

χPT

HBχPT

Examples



χPT & HBχPT

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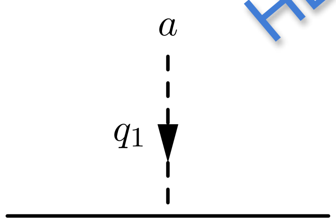
$$\boxed{Q = \left\{ \frac{q}{\Lambda_\chi}, \frac{M_\pi}{\Lambda_\chi} \right\}}$$

Examples

χPT

$$\boxed{-\frac{g_A}{2F_\pi}\, \not{q}_1 \gamma_5\, \tau^a}$$



HBχPT

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χPT & HBχPT

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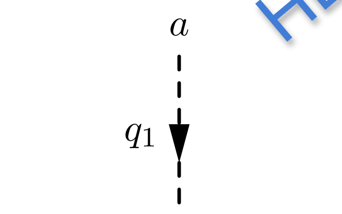

HBχPT

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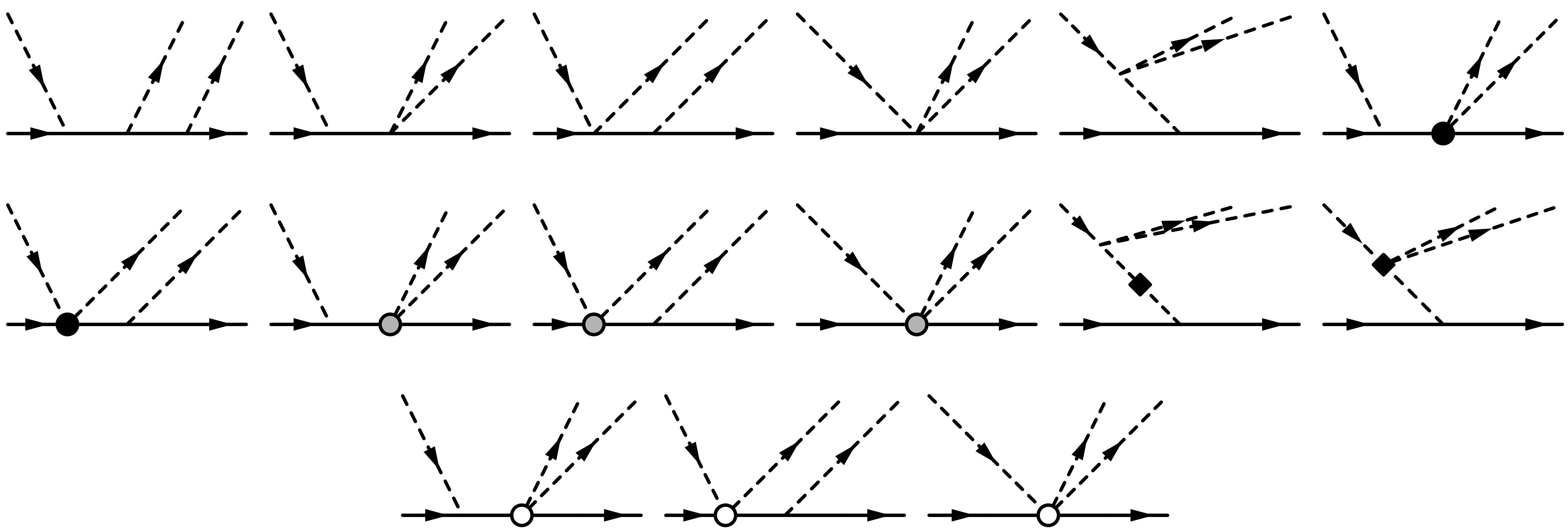
Tree Graphs



χPT & HBχPT

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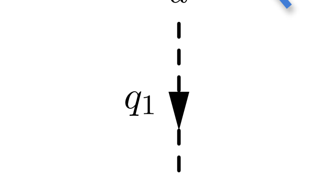

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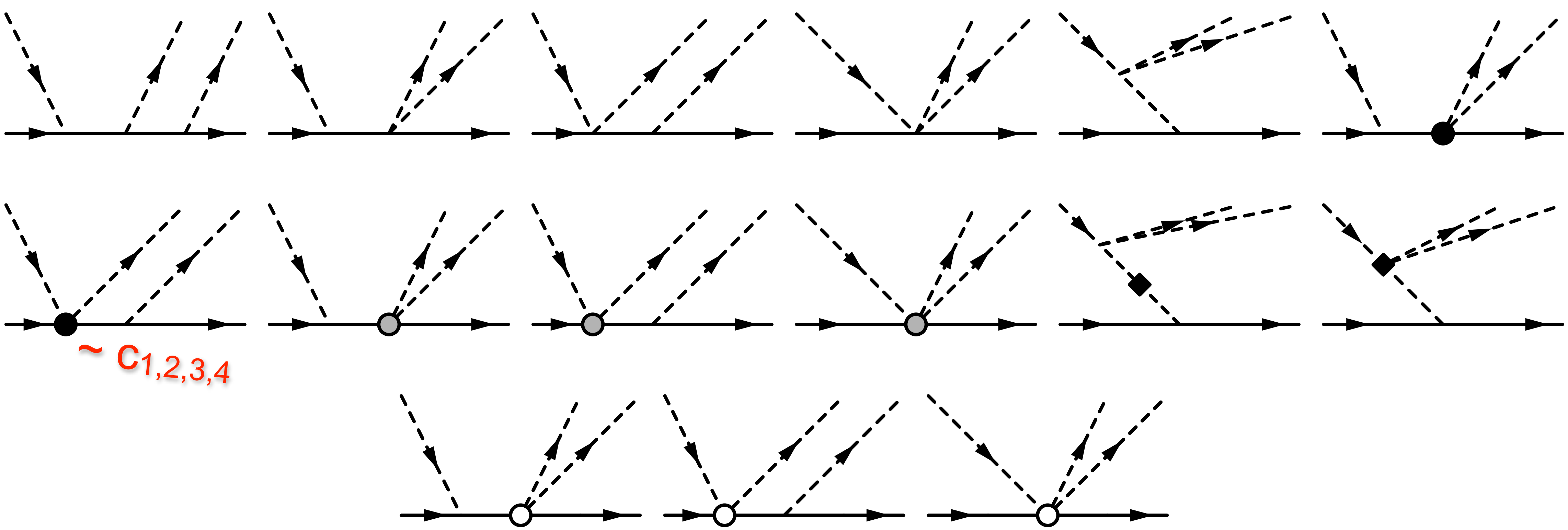

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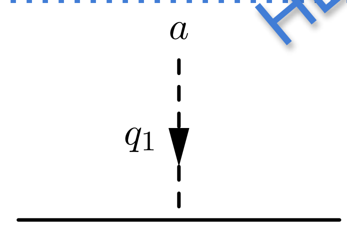

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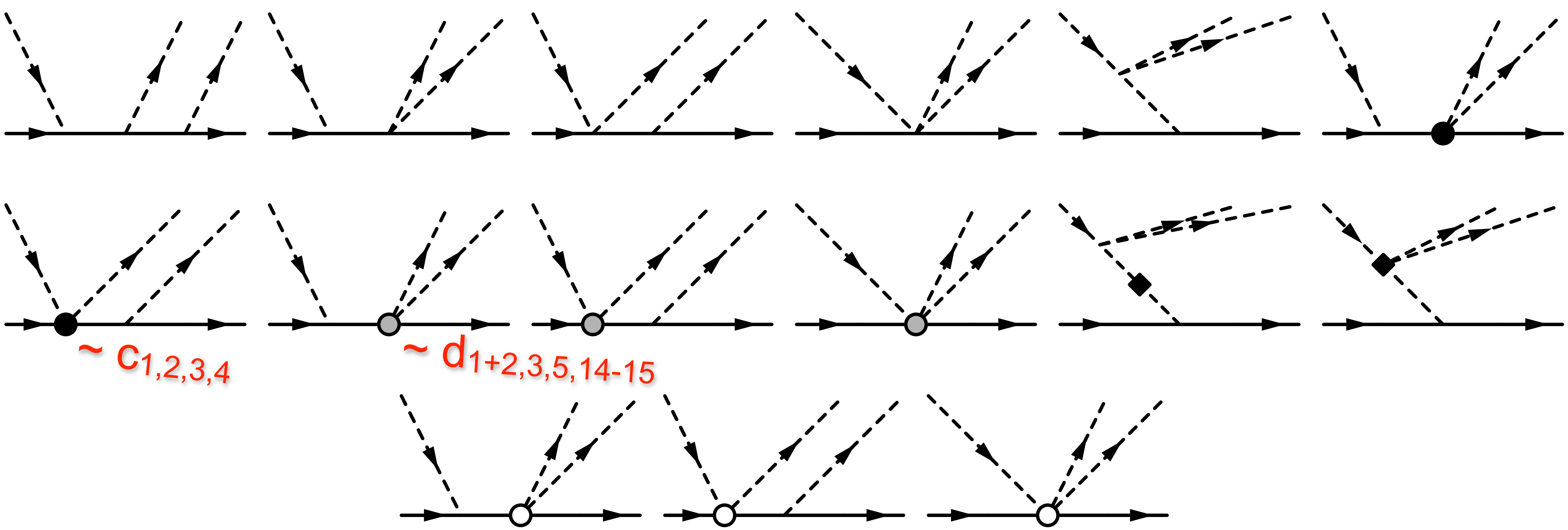

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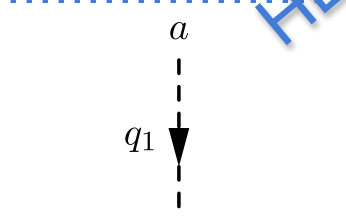

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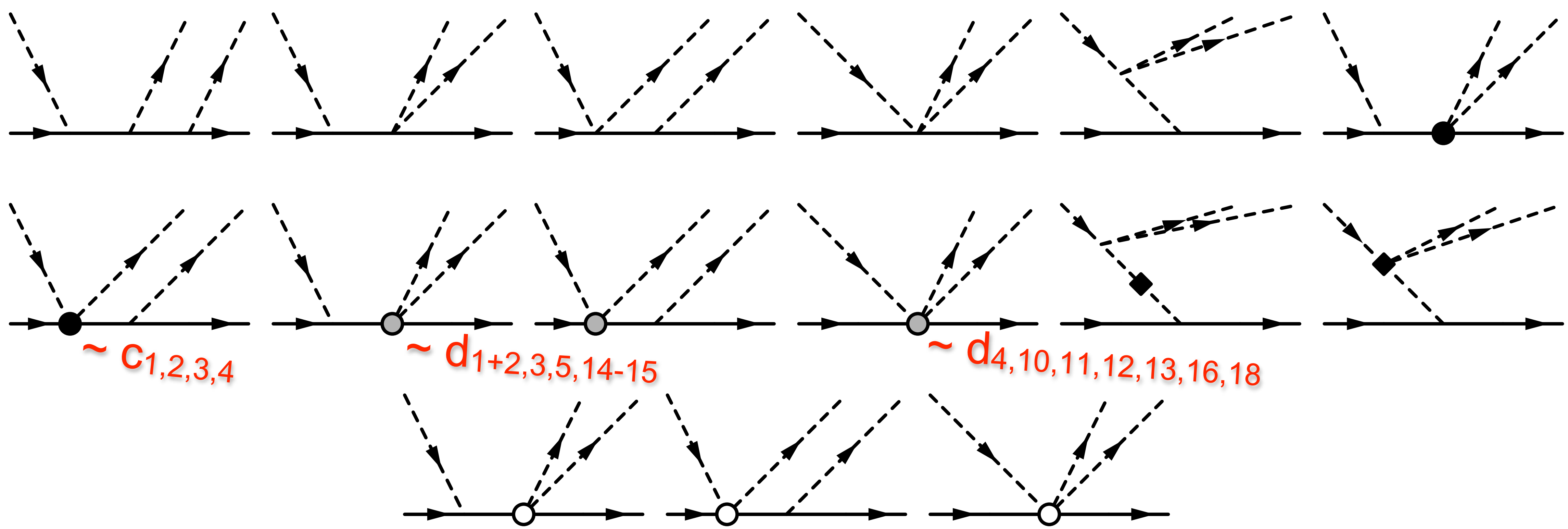

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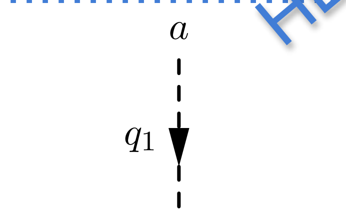

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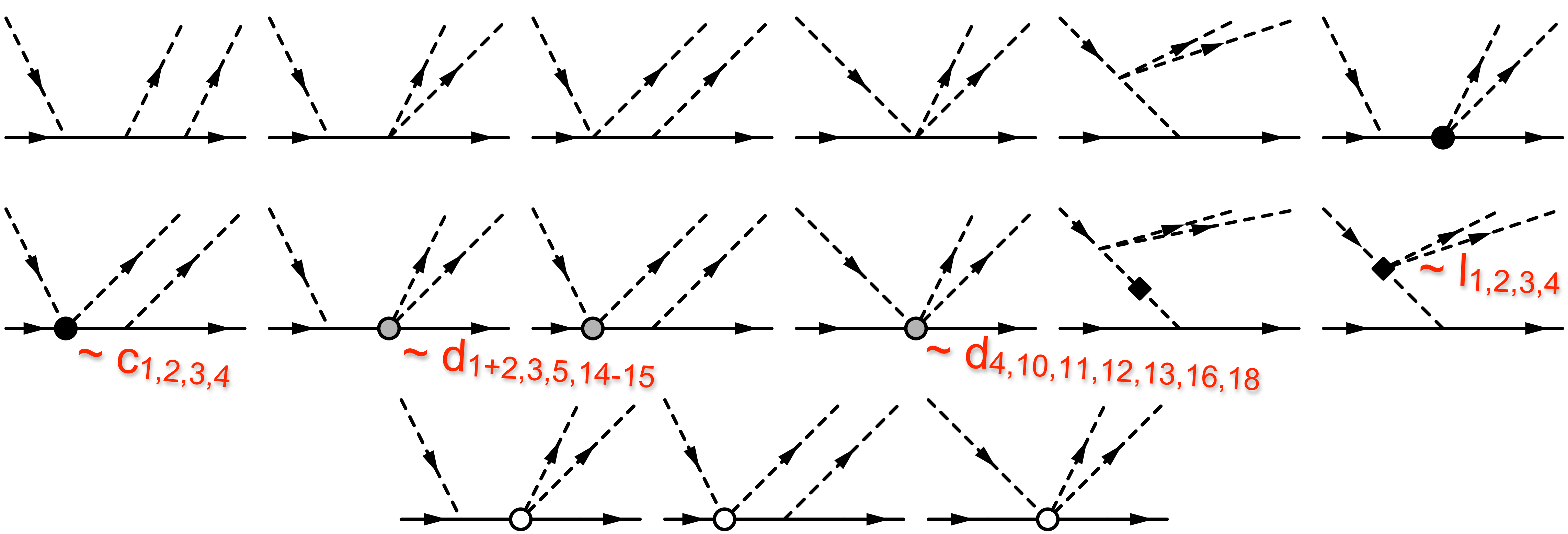

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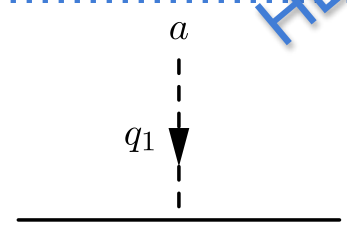

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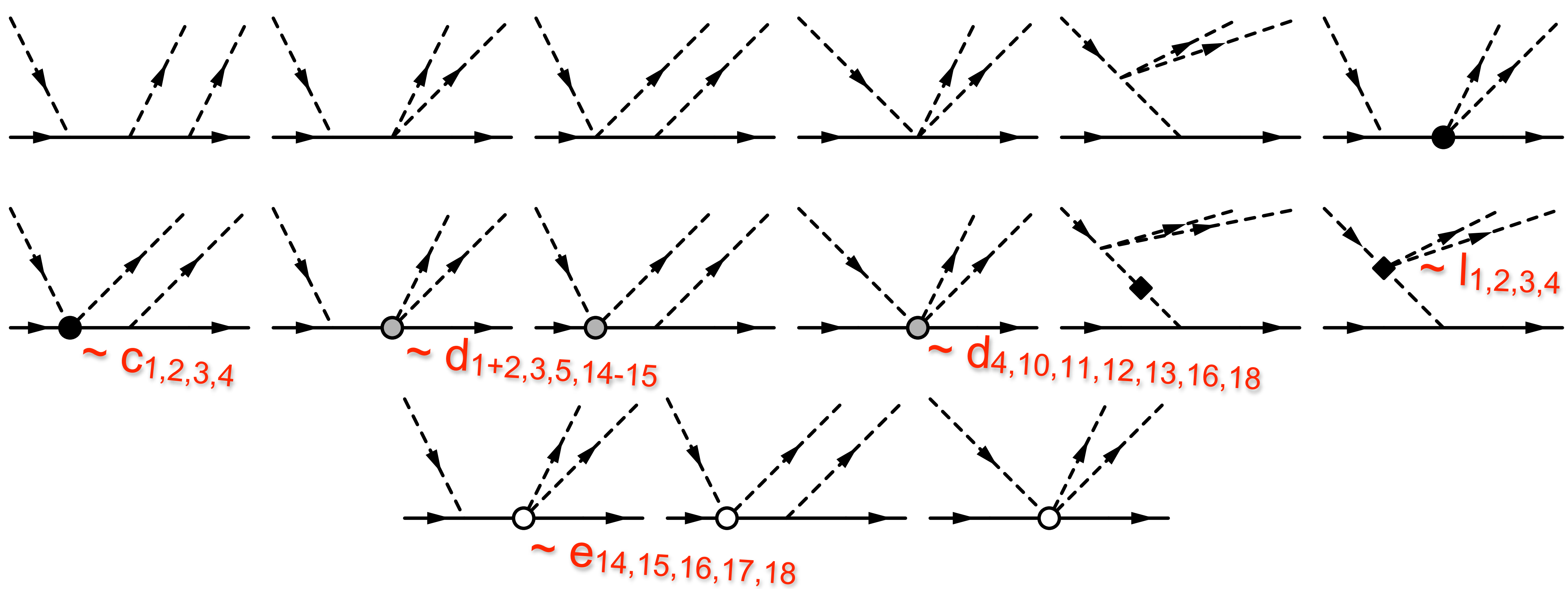

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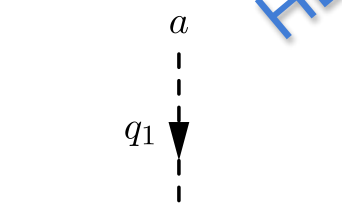

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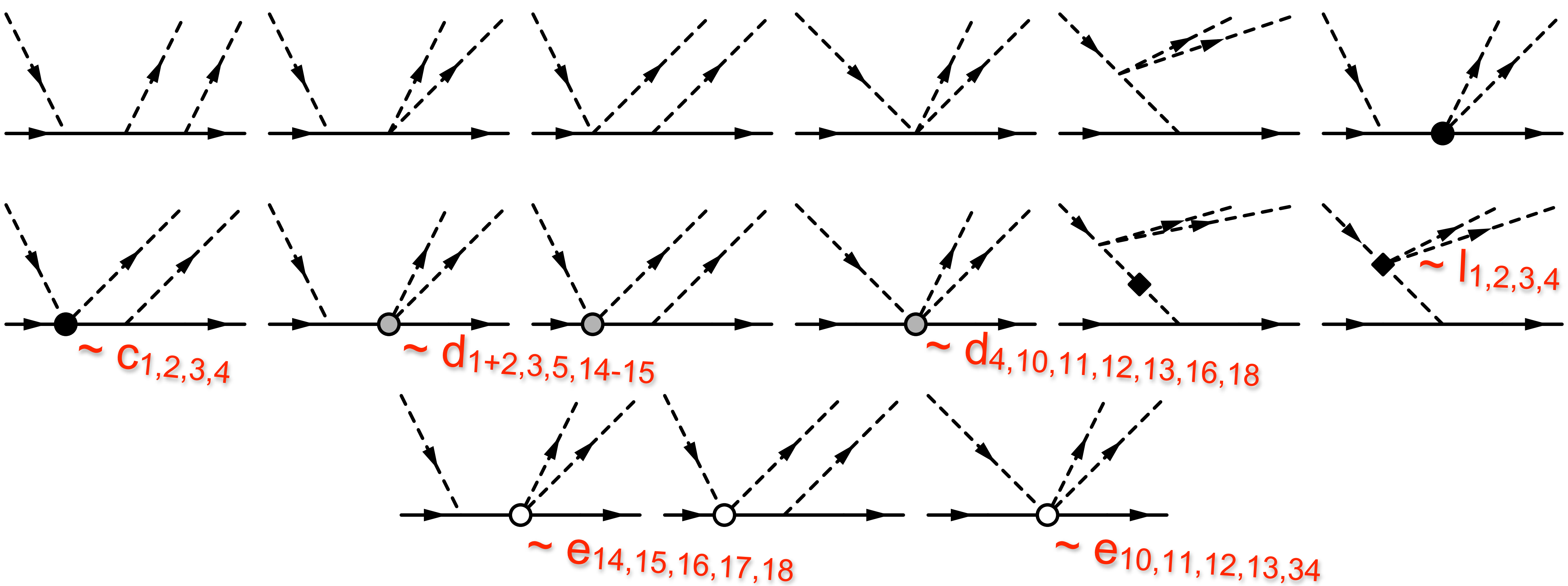


Renormalization I

Meson Sector

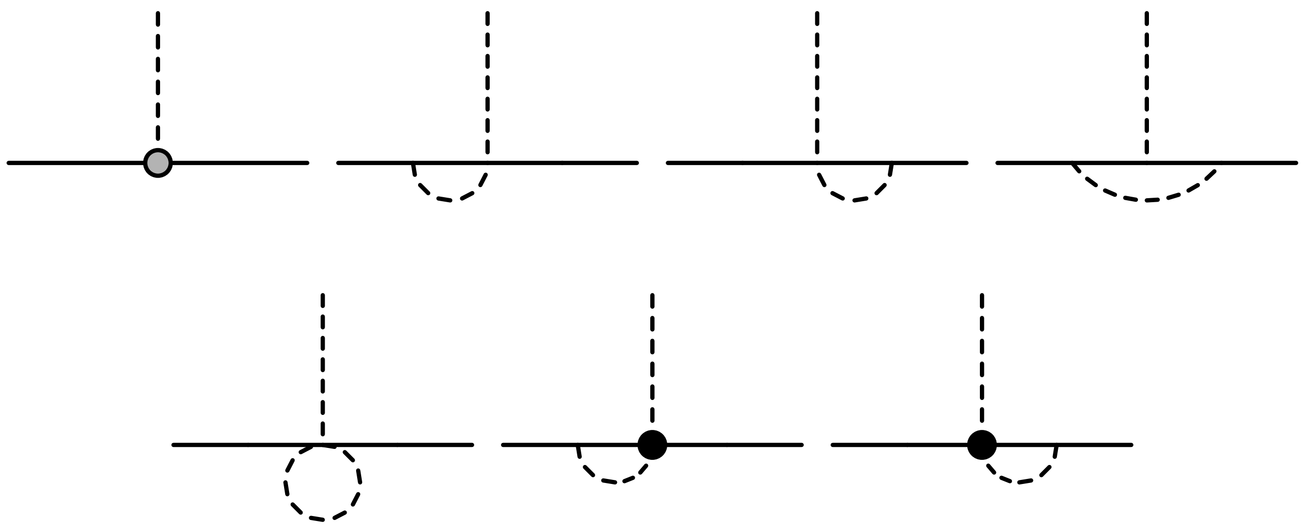
$$M^2 = M_\pi^2 + \delta M^{(4)}$$
$$Z_\pi = 1 + \delta Z_\pi^{(4)}$$
$$F = F_\pi + \delta F_\pi^{(4)}$$

Nucleon Self Energy

$$m = m_N + \delta m^{(2)} + \delta m^{(3)} + \delta m^{(4)}$$

$$Z_N = 1 + \delta Z_N^{(3)} + \delta Z_N^{(4)}$$

Axial-coupling constant

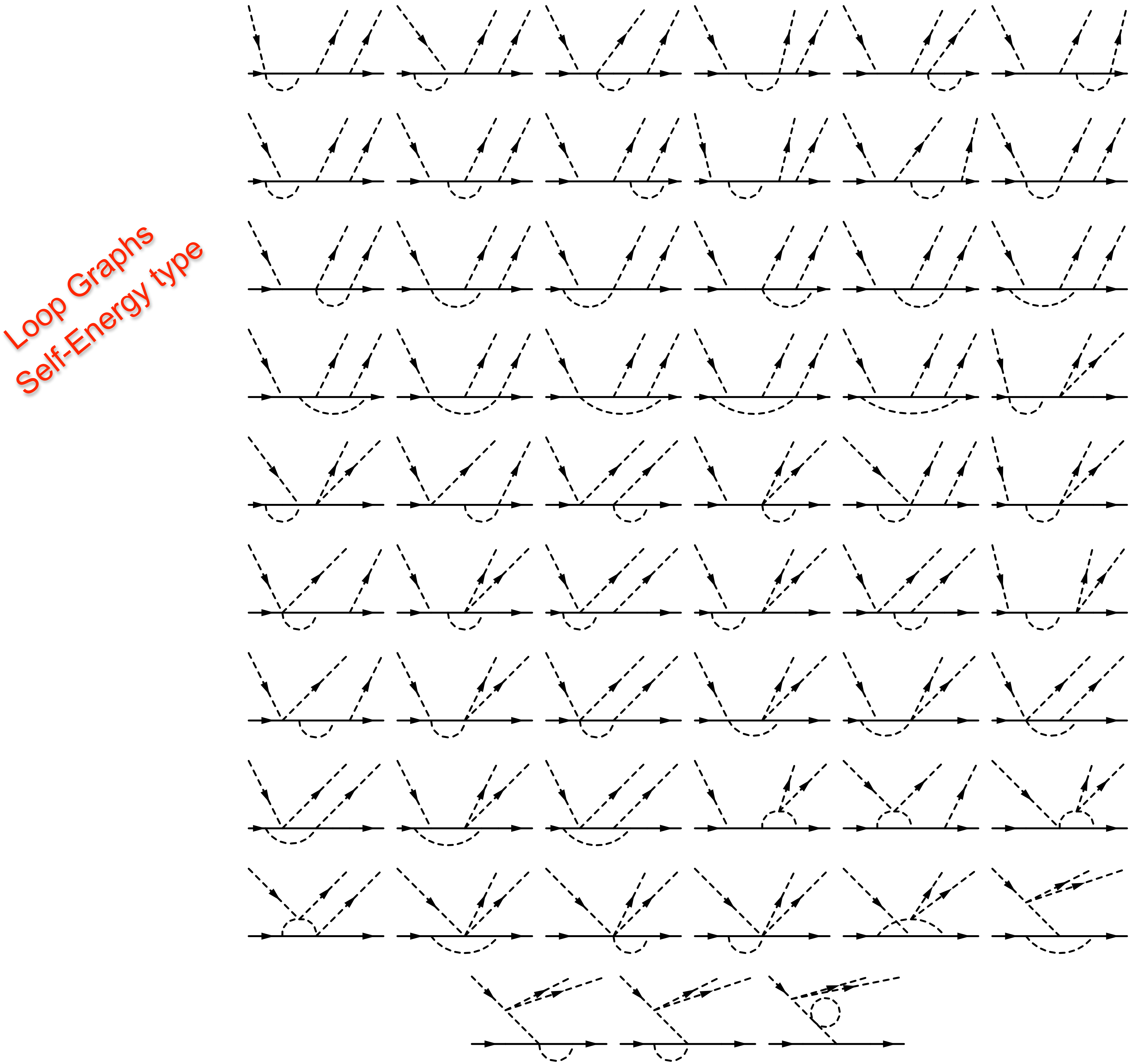


$$g = g_A + \delta g^{(3)} + \delta g^{(4)}$$

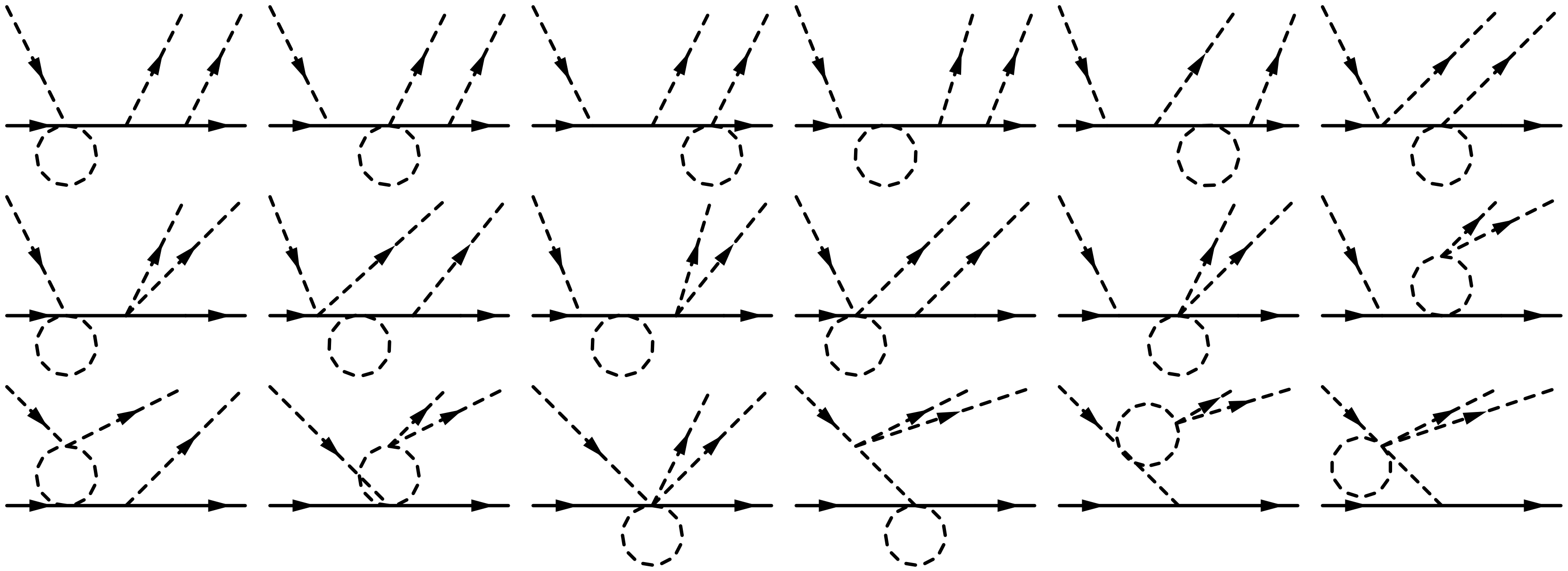
GTR $$\frac{g_{\pi NN} F_\pi}{m_N} = g_A - 2M_\pi^2 d_{18} + \mathcal{O}(Q^5)$$

Linear Combinations

$$c_1 \to c_1 + 2M_\pi^2(e_{22} - 4e_{38})$$
$$c_2 \to c_2 - 8M_\pi^2(e_{20} + e_{35})$$
$$c_3 \to c_3 - 4M_\pi^2(2e_{19} - e_{22} - e_{36})$$
$$c_4 \to c_4 - 4M_\pi^2(2e_{21} - e_{37})$$


Loop Graphs
Self-Energy type

Loop graphs - Tadpole type



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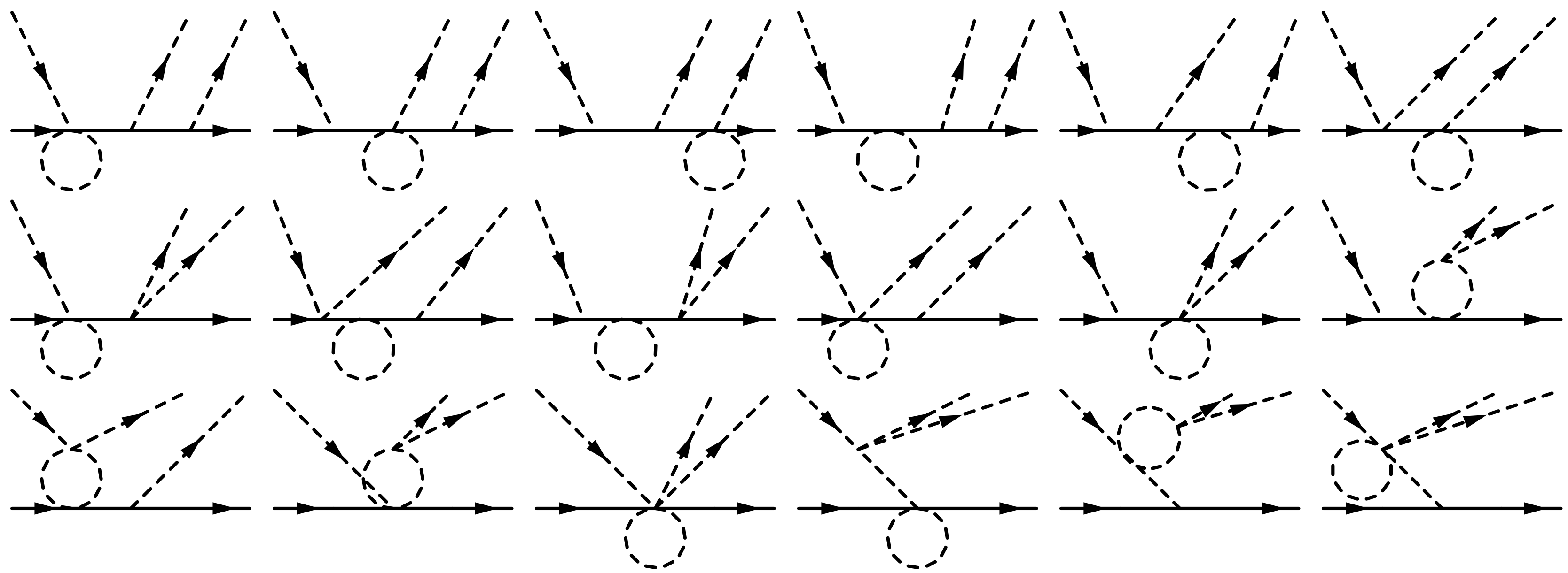
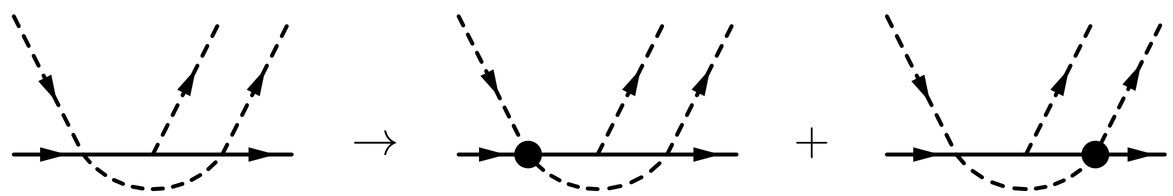


Transition from LO loops to NLO loops



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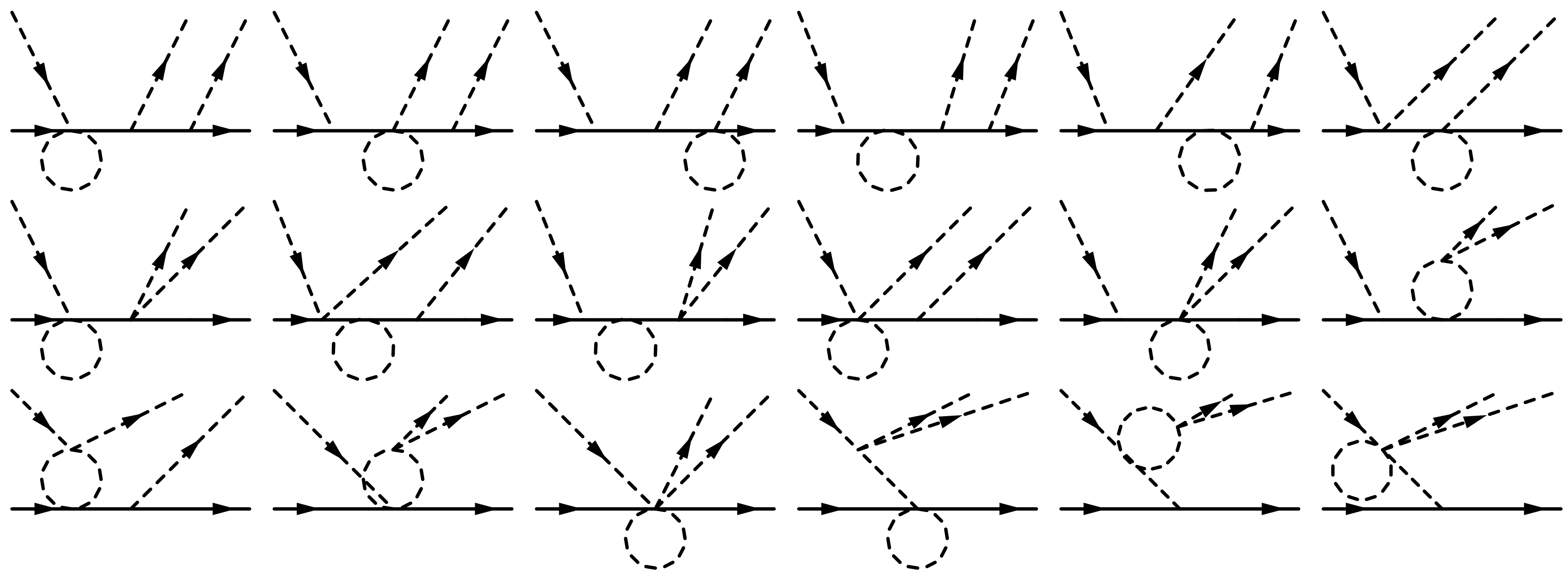

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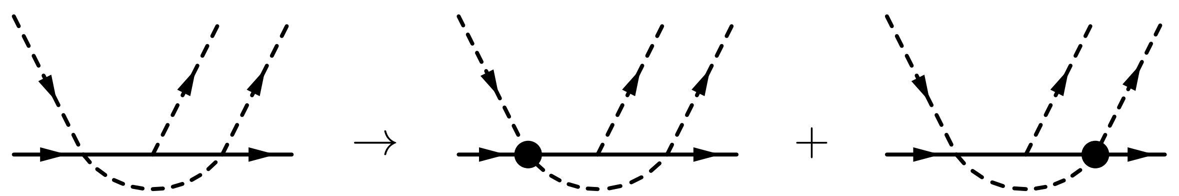
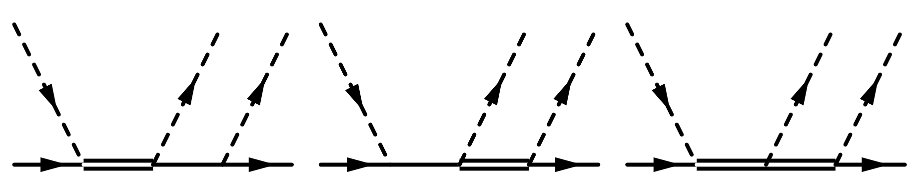
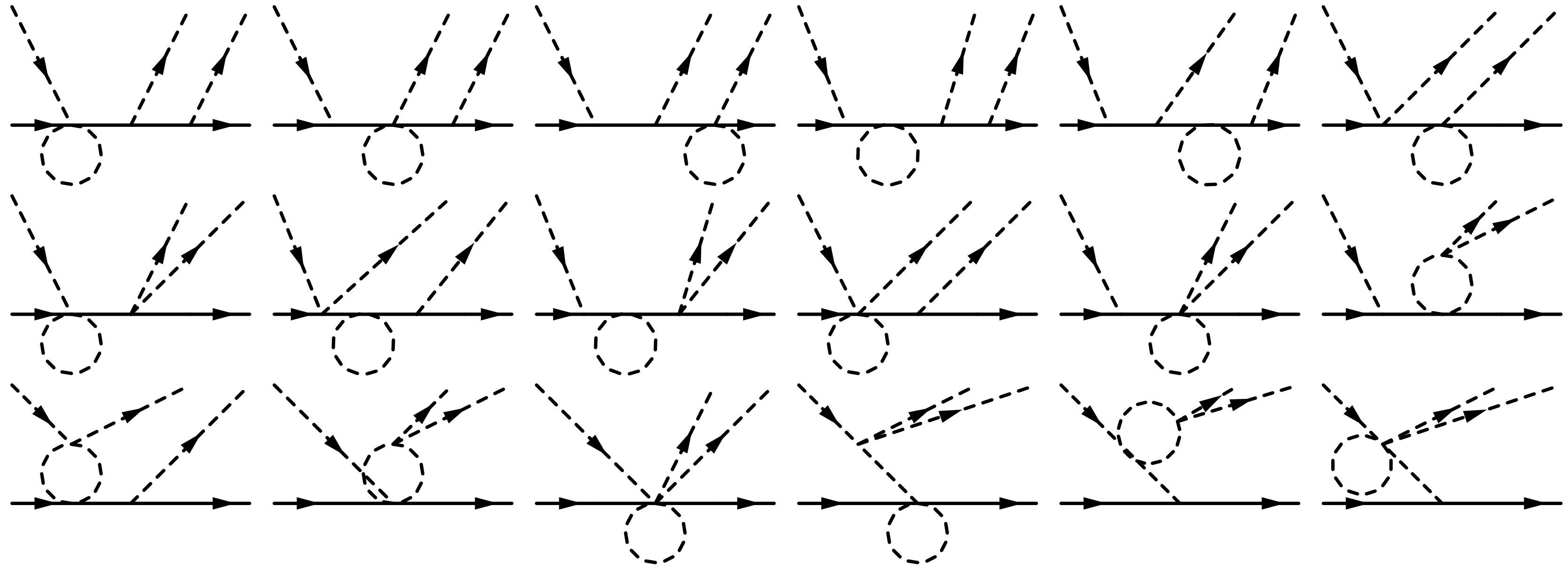


LO Δ graphs



phenomenological

Loop graphs - Tadpole type



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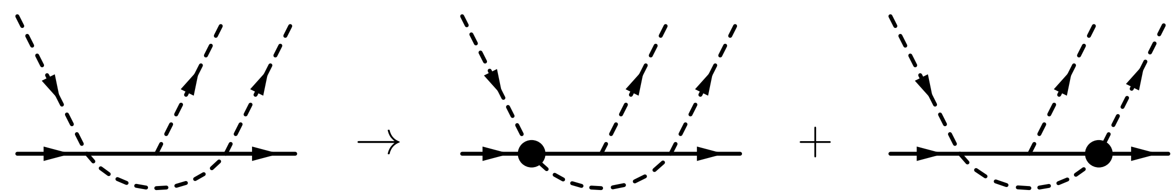

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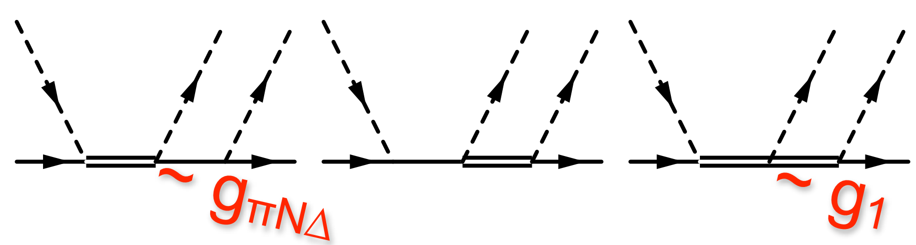

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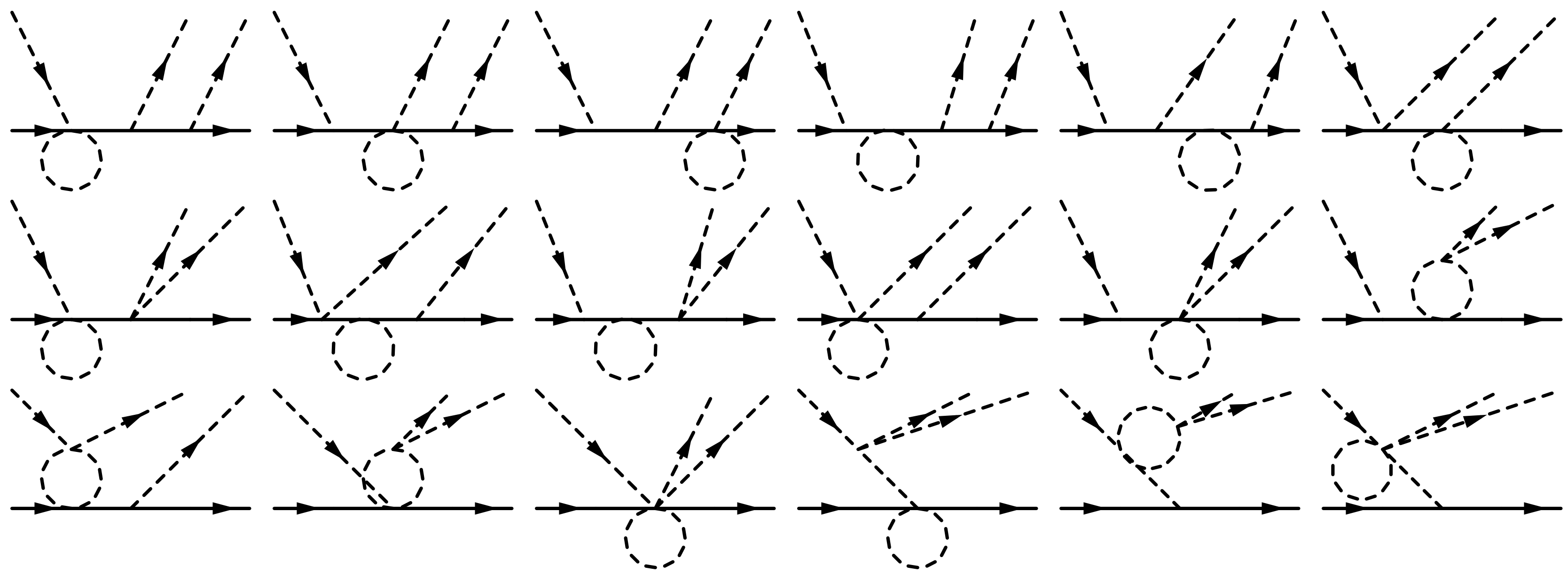

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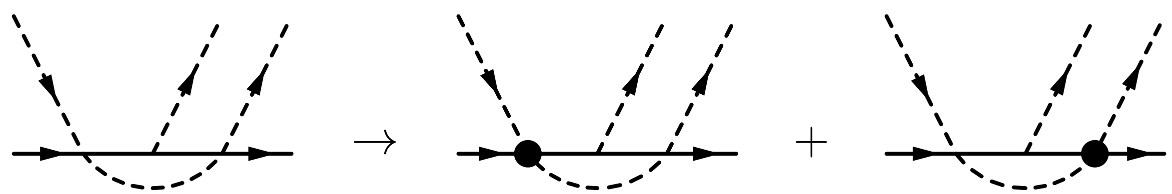

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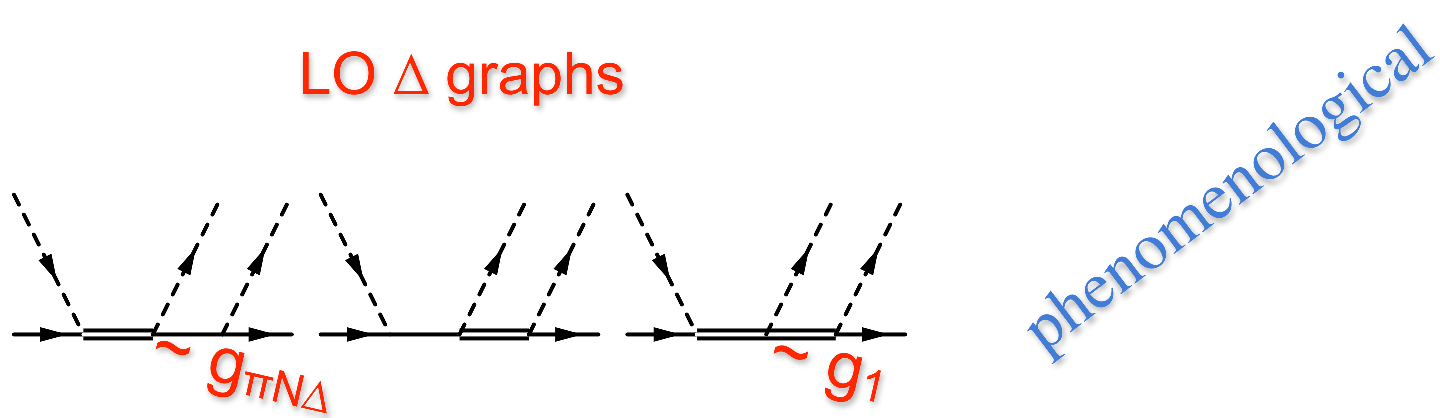
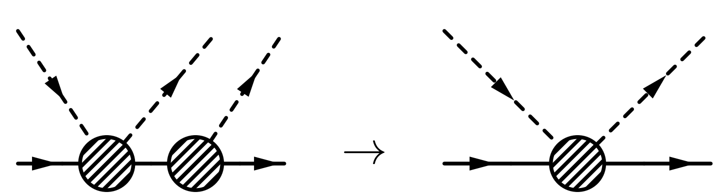


Transition from πN→ππN graphs to πN→πN graphs



Renormalization II

Meson Sector

$$l_i = \frac{\beta_{l_i}}{32\pi^2}\bar{l}_i + \beta_{l_i}\left(\bar{\lambda} + \frac{1}{32\pi^2}\log\left(\frac{M_\pi^2}{\mu^2}\right)\right)$$

$$\bar{\lambda} = \frac{1}{16\pi^2}\left(\frac{1}{d-4} + \frac{1}{2}(\gamma_E - 1 - \ln 4\pi)\right)$$

HB approach

$$d_i = \bar{d}_i + \frac{\beta_{d_i}}{F_\pi^2}\left(\bar{\lambda} + \frac{1}{32\pi^2}\log\left(\frac{M_\pi^2}{\mu^2}\right)\right)$$

$$e_i = \bar{e}_i + \frac{\beta_{e_i}}{F_\pi^2}\left(\bar{\lambda} + \frac{1}{32\pi^2}\log\left(\frac{M_\pi^2}{\mu^2}\right)\right)$$

Covariant “Modified“ EOMS

$$c_i = \bar{c}_i + \delta c_i^{(3)} + \delta c_i^{(4)}$$

$$d_i = \bar{d}_i + \delta d_i^{(3)} + \delta d_i^{(4)}$$

$$e_i = \bar{e}_i + \delta e_i^{(4)}$$

$$x \in \{c, d, e\}$$

$$\delta x_i^{(n)} = \bar{x}_{i,f}^{(n)} + \frac{\beta_{x_i,B}^{(n)}}{F_\pi^2}\left(\bar{\lambda} + \frac{1}{32\pi^2}\log\left(\frac{m_N^2}{\mu^2}\right)\right) + \frac{\beta_{x_i,M}^{(n)}}{F_\pi^2}\left(\bar{\lambda} + \frac{1}{32\pi^2}\log\left(\frac{M_\pi^2}{\mu^2}\right)\right)$$

Combined Fit

Phase Shifts - πN→πN

$$T^{ba} = \chi^{\dagger}_{N'} \left(\delta^{ab} T^{+} + \mathrm{i}\epsilon^{bac} \tau_c T^{-} \right) \chi_N$$

Phase Shifts - πN→πN

$$T^{ba} = \chi^\dagger_{N'}\left(\delta^{ab}T^+ + \mathrm{i}\epsilon^{bac}\tau_c T^-\right)\chi_N$$

χPT

$$T^\pm = \bar{u}^{(s')}\left(A^\pm + \not{q}B^\pm\right)u^{(s)}$$

$$f^I_{l\pm}(s) = \frac{1}{16\pi\sqrt{s}}\Big((E+m_N)\left(A^I_l(s) + (\sqrt{s}-m_N)B^I_l(s)\right)$$
$$+(E-m_N)\left(-A^I_{l\pm}(s) + (\sqrt{s}+m_N)B^I_{l\pm}\right)\Big)$$

$$X^I_l(s) = \int_{-1}^{1}\mathrm{d}z\, X^I(s,t)P_l(z)$$

$$X \in \{A, B\}$$

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HBχPT

$$T^{\pm} = \bar{u}^{(s')}_v \left(g^{\pm} + 2\mathrm{i}\, S \cdot q \times q' h^{\pm}\right) u^{(s)}_v$$

$$f^I_{l\pm}(s) = \frac{E + m_N}{16\pi\sqrt{s}} \int_{-1}^{1} \mathrm{d}z \, \left(g^I P_l(z) + \boldsymbol{q}^2 h^I (P_{l\pm}(z) - z P_l(z))\right)$$

Phase Shifts - πN→πN

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Isospin basis

$$X^{I=1/2} = X^+ + 2X^-\,, \quad X^{I=3/2} = X^+ - X^-$$

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Isospin basis

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Unitarization prescription

$$\delta^I_{l\pm}(s) = \arctan(|\boldsymbol{q}|\Re\, f^I_{l\pm}(s))$$

Observables - $\pi N \rightarrow \pi\pi N$

Unpolarized Observables

Observables - πN→ππN

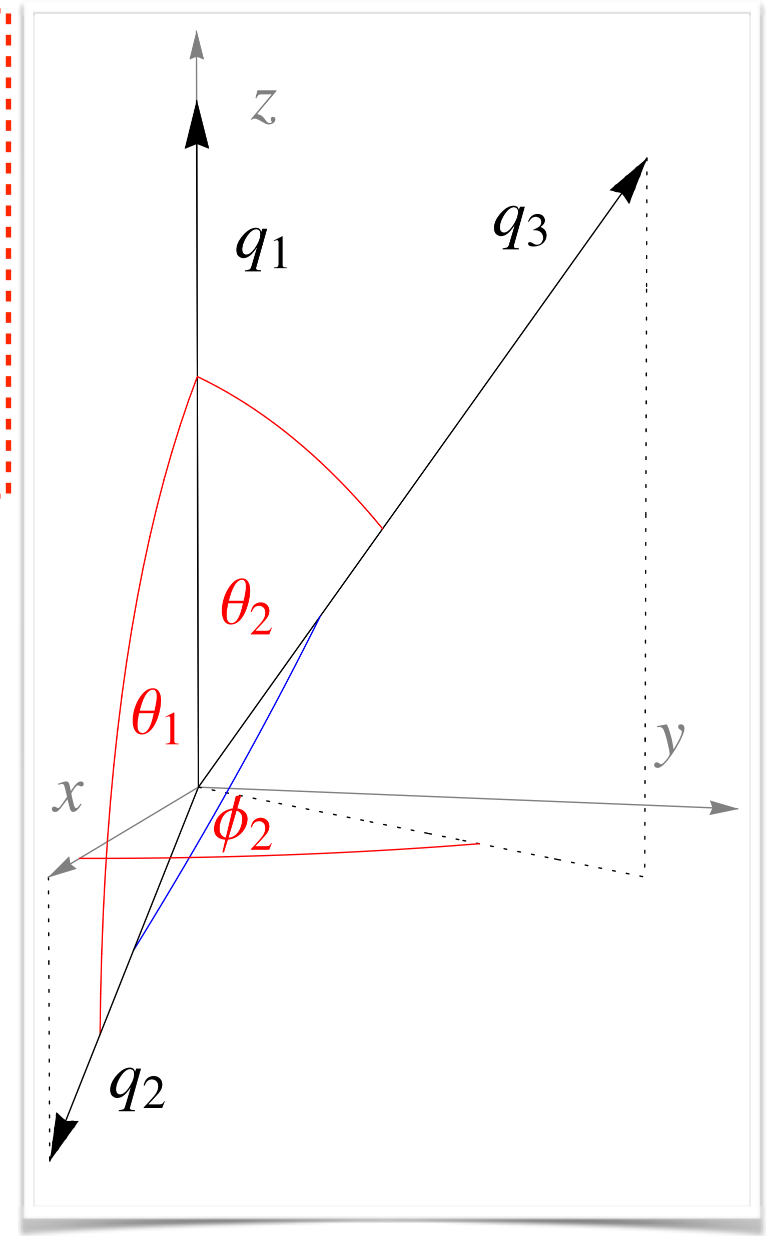
Unpolarized Observables

- σ_{tot}

Observables - πN→ππN

Unpolarized Observables

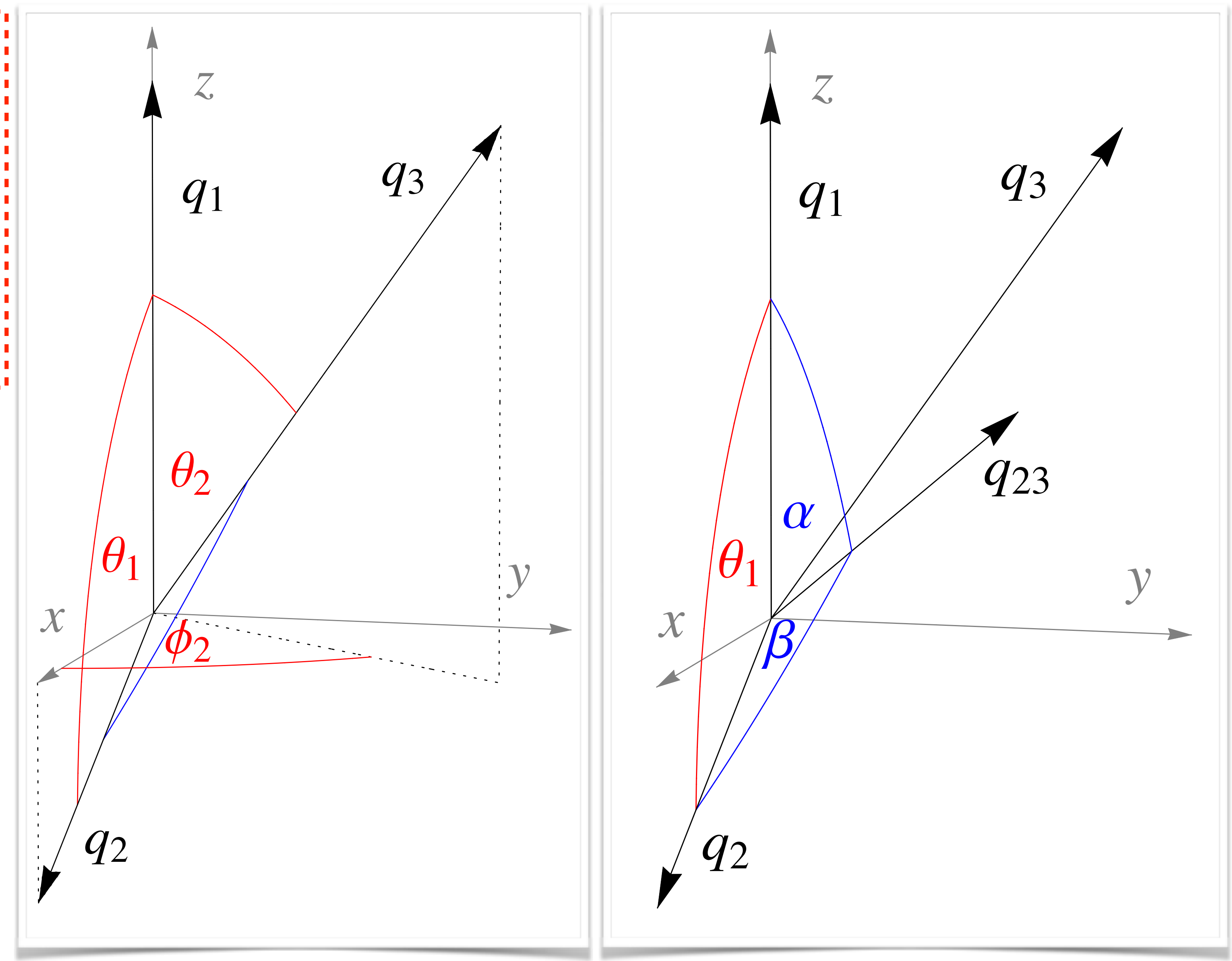
- σ_{tot}
- $\frac{\mathrm{d}^2\sigma}{\mathrm{d}\omega_2\mathrm{d}\Omega_2}$, $\frac{\mathrm{d}^3\sigma}{\mathrm{d}\omega_2\mathrm{d}\Omega_2\mathrm{d}\Omega_3}$ and W



Observables - πN→ππN

Unpolarized Observables

- σ_{tot}
- $\dfrac{\mathrm{d}^2\sigma}{\mathrm{d}\omega_2\mathrm{d}\Omega_2}$, $\dfrac{\mathrm{d}^3\sigma}{\mathrm{d}\omega_2\mathrm{d}\Omega_2\mathrm{d}\Omega_3}$ and W
- $\dfrac{\mathrm{d}\sigma}{dM^2_{\pi\pi}}$, $\dfrac{\mathrm{d}\sigma}{\mathrm{d}t}$, $\dfrac{\mathrm{d}\sigma}{\mathrm{d}t\,\mathrm{d}M^2_{\pi\pi}}$ and $\dfrac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}$



Observables - πN→ππN

Unpolarized Observables

- σ_{tot}
- $\frac{\mathrm{d}^2\sigma}{\mathrm{d}\omega_2\mathrm{d}\Omega_2}$, $\frac{\mathrm{d}^3\sigma}{\mathrm{d}\omega_2\mathrm{d}\Omega_2\mathrm{d}\Omega_3}$ and W
- $\frac{\mathrm{d}\sigma}{dM^2_{\pi\pi}}$, $\frac{\mathrm{d}\sigma}{\mathrm{d}t}$, $\frac{\mathrm{d}\sigma}{\mathrm{d}t\,\mathrm{d}M^2_{\pi\pi}}$ and $\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}$

Observables - πN→ππN

Unpolarized Observables

- $\sigma_{\rm tot}$
- $\dfrac{\mathrm{d}^2\sigma}{\mathrm{d}\omega_2\mathrm{d}\Omega_2}$, $\dfrac{\mathrm{d}^3\sigma}{\mathrm{d}\omega_2\mathrm{d}\Omega_2\mathrm{d}\Omega_3}$ and W
- $\dfrac{\mathrm{d}\sigma}{dM^2_{\pi\pi}}$, $\dfrac{\mathrm{d}\sigma}{\mathrm{d}t}$, $\dfrac{\mathrm{d}\sigma}{\mathrm{d}t\,\mathrm{d}M^2_{\pi\pi}}$ and $\dfrac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}$

Unpolarized Matrix Element Squared

$$|\mathcal{M}|^2 = \frac{1}{2}\sum_{s,s'} T^\dagger_{ss'} T_{ss'}$$

Observables - πN→ππN

Unpolarized Observables

- $\sigma_{\rm tot}$
- $\dfrac{\mathrm{d}^2\sigma}{\mathrm{d}\omega_2\mathrm{d}\Omega_2}$, $\dfrac{\mathrm{d}^3\sigma}{\mathrm{d}\omega_2\mathrm{d}\Omega_2\mathrm{d}\Omega_3}$ and W
- $\dfrac{\mathrm{d}\sigma}{dM^2_{\pi\pi}}$, $\dfrac{\mathrm{d}\sigma}{\mathrm{d}t}$, $\dfrac{\mathrm{d}\sigma}{\mathrm{d}t\,\mathrm{d}M^2_{\pi\pi}}$ and $\dfrac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}$

⟺

Unpolarized Matrix Element Squared

$$|\mathcal{M}|^2 = \frac{1}{2}\sum_{s,s'} T^\dagger_{ss'} T_{ss'}$$

χPT

$$T^{abc}_{ss'} = \mathrm{i}\bar{u}^{(s')}\gamma_5 \left(F^{abc}_1 + (\not{q}_2 + \not{q}_3) F^{abc}_2 + (\not{q}_2 - \not{q}_3) F^{abc}_3 + (\not{q}_2\not{q}_3 - \not{q}_3\not{q}_2) F^{abc}_4 \right) u^{(s)}$$

Observables - πN→ππN

Unpolarized Observables

- $\sigma_{\rm tot}$
- $\dfrac{\mathrm{d}^2\sigma}{\mathrm{d}\omega_2\mathrm{d}\Omega_2}$, $\dfrac{\mathrm{d}^3\sigma}{\mathrm{d}\omega_2\mathrm{d}\Omega_2\mathrm{d}\Omega_3}$ and W
- $\dfrac{\mathrm{d}\sigma}{dM^2_{\pi\pi}}$, $\dfrac{\mathrm{d}\sigma}{\mathrm{d}t}$, $\dfrac{\mathrm{d}\sigma}{\mathrm{d}t\,\mathrm{d}M^2_{\pi\pi}}$ and $\dfrac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}$

$\Longleftrightarrow$

Unpolarized Matrix Element Squared

$$|\mathcal{M}|^2 = \frac{1}{2}\sum_{s,s'} T^{\dagger}_{ss'} T_{ss'}$$

χPT

$$T^{abc}_{ss'} = \mathrm{i}\bar{u}^{(s')}\gamma_5\left(F^{abc}_1 + (\not{q}_2 + \not{q}_3)F^{abc}_2 + (\not{q}_2 - \not{q}_3)F^{abc}_3 + (\not{q}_2\not{q}_3 - \not{q}_3\not{q}_2)F^{abc}_4\right)u^{(s)}$$

HBχPT

$$T^{abc}_{ss'} = \bar{u}^{(s')}_v\left(S\cdot q_1\,A^{abc} + S\cdot q_2\,B^{abc} + S\cdot q_3\,C^{abc} + \mathrm{i}\epsilon_{\mu\nu\alpha\beta}\,q_1^\mu q_2^\nu q_3^\alpha v^\beta\,D^{abc}\right)u^{(s)}_v$$

Observables - πN→ππN

Unpolarized Observables

- σ_{tot}
- $\dfrac{\mathrm{d}^2\sigma}{\mathrm{d}\omega_2\mathrm{d}\Omega_2}$, $\dfrac{\mathrm{d}^3\sigma}{\mathrm{d}\omega_2\mathrm{d}\Omega_2\mathrm{d}\Omega_3}$ and W
- $\dfrac{\mathrm{d}\sigma}{dM^2_{\pi\pi}}$, $\dfrac{\mathrm{d}\sigma}{\mathrm{d}t}$, $\dfrac{\mathrm{d}\sigma}{\mathrm{d}t\,\mathrm{d}M^2_{\pi\pi}}$ and $\dfrac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}$

$\Longleftrightarrow$

Unpolarized Matrix Element Squared

$$|\mathcal{M}|^2 = \frac{1}{2}\sum_{s,s'} T^\dagger_{ss'} T_{ss'}$$

χPT

$$T^{abc}_{ss'} = \mathrm{i}\bar{u}^{(s')}\gamma_5\left(F_1^{abc} + (\not{q}_2 + \not{q}_3)F_2^{abc} + (\not{q}_2 - \not{q}_3)F_3^{abc} + (\not{q}_2\not{q}_3 - \not{q}_3\not{q}_2)F_4^{abc}\right)u^{(s)}$$

HBχPT

$$T^{abc}_{ss'} = \bar{u}_v^{(s')}\left(S\cdot q_1\,A^{abc} + S\cdot q_2\,B^{abc} + S\cdot q_3\,C^{abc} + \mathrm{i}\epsilon_{\mu\nu\alpha\beta}\,q_1^\mu q_2^\nu q_3^\alpha v^\beta\,D^{abc}\right)u_v^{(s)}$$

$$X^{abc} = \chi^\dagger_{N'}\left(\tau^a\delta^{bc}X_1 + \tau^b\delta^{ac}X_2 + \tau^c\delta^{ab}X_3 + \mathrm{i}\epsilon^{abc}X_4\right)\chi_N$$

Observables - πN→ππN

Unpolarized Observables

- σ_{tot}
- $\dfrac{d^2\sigma}{d\omega_2 d\Omega_2}, \dfrac{d^3\sigma}{d\omega_2 d\Omega_2 d\Omega_3}$ and W
- $\dfrac{d\sigma}{dM^2_{\pi\pi}}, \dfrac{d\sigma}{dt}, \dfrac{d\sigma}{dt\, dM^2_{\pi\pi}}$ and $\dfrac{d\sigma}{d\cos\theta}$

Unpolarized Matrix Element Squared

$$|\mathcal{M}|^2 = \frac{1}{2}\sum_{s,s'} T^\dagger_{ss'} T_{ss'}$$

χPT

$$T^{abc}_{ss'} = i\bar{u}^{(s')}\gamma_5 \left(F^{abc}_1 + (\not{q}_2 + \not{q}_3)F^{abc}_2 + (\not{q}_2 - \not{q}_3)F^{abc}_3 + (\not{q}_2\not{q}_3 - \not{q}_3\not{q}_2)F^{abc}_4\right) u^{(s)}$$

HBχPT

$$T^{abc}_{ss'} = \bar{u}^{(s')}_v \left(S\cdot q_1\, A^{abc} + S\cdot q_2\, B^{abc} + S\cdot q_3\, C^{abc} + i\epsilon_{\mu\nu\alpha\beta}\, q_1^\mu q_2^\nu q_3^\alpha v^\beta\, D^{abc}\right) u^{(s)}_v$$

$$X^{abc} = \chi^\dagger_{N'}\left(\tau^a\delta^{bc}X_1 + \tau^b\delta^{ac}X_2 + \tau^c\delta^{ab}X_3 + i\epsilon^{abc}X_4\right)\chi_N$$

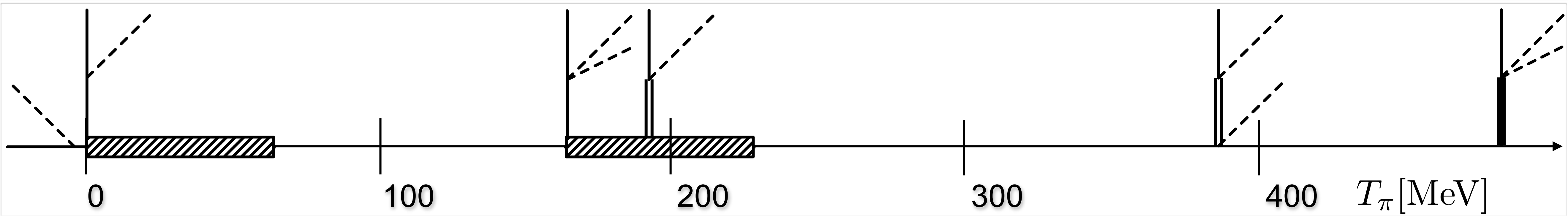
Physically Accessible Channels

I.	$\pi^- p \to \pi^0\pi^0 n$
II.	$\pi^- p \to \pi^+\pi^- n$
III.	$\pi^+ p \to \pi^+\pi^+ n$
IV.	$\pi^+ p \to \pi^+\pi^0 p$
V.	$\pi^- p \to \pi^0\pi^- p$

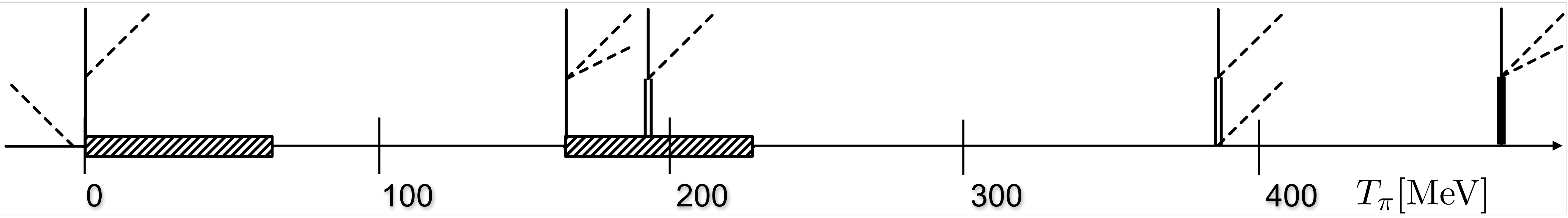
T-matrix Reduction

I.	$X = \sqrt{2}X_1$
II.	$X = \sqrt{2}(X_1 + X_2)$
III.	$X = \sqrt{2}(X_2 + X_3)$
IV.	$X = X_3 + X_4$
V.	$X = X_2 + X_4$

Fitting Procedure



Fitting Procedure



Combined Fit

$$\chi^2 = \chi^2_{\pi N} + \chi^2_{\pi\pi N} + \chi^2_c$$

$$\chi^2_{\pi N} = \sum_i \frac{(\delta_i^{exp} - \delta_i^{th})^2}{\Delta\delta_i^2}$$

$\Delta\delta_{S,P} = 5\%$ $\Delta\delta_D = 20\%$

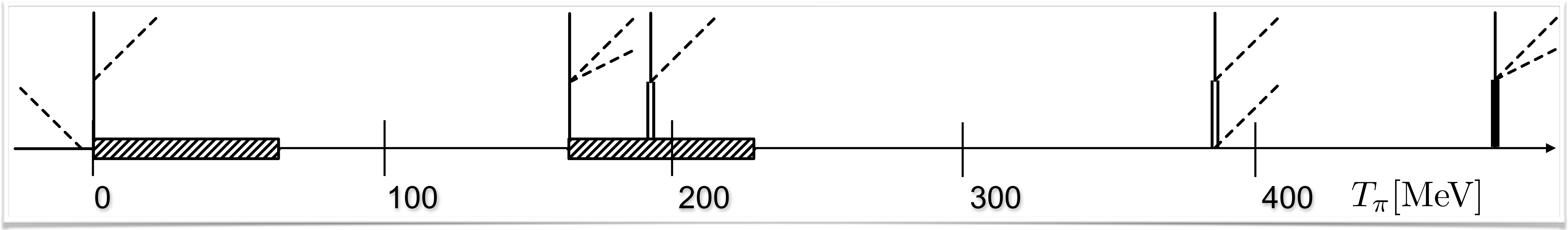
$$\chi^2_{\pi\pi N} = \sum_i \frac{(\sigma_i^{exp} - \sigma_i^{th})^2}{\Delta\sigma_i^2}$$

$\Delta\sigma = \Delta\sigma^{exp}$

$$\chi^2_c = \sum_i \frac{(x_i - \bar{x}_i)^2}{R_i^2}$$

$R_{d_i} = 3$ $R_{e_i} = 5$

Fitting Procedure



Combined Fit

$$\chi^2 = \chi^2_{\pi N} + \chi^2_{\pi\pi N} + \chi^2_c$$

$$\chi^2_{\pi N} = \sum_i \frac{(\delta_i^{exp} - \delta_i^{th})^2}{\Delta\delta_i^2}$$

$\Delta\delta_{S,P} = 5\%$ $\quad$ $\Delta\delta_D = 20\%$

$$\chi^2_{\pi\pi N} = \sum_i \frac{(\sigma_i^{exp} - \sigma_i^{th})^2}{\Delta\sigma_i^2}$$

$\Delta\sigma = \Delta\sigma^{exp}$

$$\chi^2_c = \sum_i \frac{(x_i - \bar{x}_i)^2}{R_i^2}$$

$R_{d_i} = 3$ $\quad$ $R_{e_i} = 5$

Naturalness Condition

$$|x_i| = \frac{|\tilde{x}_i|}{\Lambda^n} \sim \frac{Q^0}{\Lambda^n} < \frac{Q^{-1}}{\Lambda^n} = \frac{1}{q\Lambda^{n-1}}$$

$\Lambda \sim m_\rho \sim 770$ MeV

$\omega^{CMS}_{\pi N} < 190$ MeV

$\omega^{CMS}_{\pi\pi N} < 280$ MeV

$|c_i| \sim 1.0 < 3.0 < 5.5 \quad \mathrm{GeV}^{-1}$

$|d_i| \sim 1.5 < 4.0 < 7.0 \quad \mathrm{GeV}^{-2}$

$|e_i| \sim 2.0 < 5.5 < 9.0 \quad \mathrm{GeV}^{-3}$

Fits

Input

m_N	M_π	F_π	g_A	l_1	l_2	l_3	l_4
938.27	139.57	92.4	1.27	-0.4 ± 0.6	4.3 ± 0.1	2.9 ± 2.4	4.4 ± 0.2

Bijnens, Ecker 2014

LECs	HB			Cov		
	KH	GW	RS	KH	GW	RS
c_1	-1.27 ± 0.08	-1.60 ± 0.07	-1.39 ± 0.02	-1.12 ± 0.08	-1.43 ± 0.07	-1.25 ± 0.02
c_2	3.56 ± 0.12	3.35 ± 0.11	3.42 ± 0.04	3.49 ± 0.11	3.38 ± 0.10	3.57 ± 0.04
c_3	-6.29 ± 0.08	-6.43 ± 0.07	-6.19 ± 0.03	-5.94 ± 0.08	-6.15 ± 0.07	-6.08 ± 0.03
c_4	3.60 ± 0.04	3.64 ± 0.04	3.61 ± 0.02	3.35 ± 0.04	3.44 ± 0.04	3.48 ± 0.02
$d_1 + d_2$	3.67 ± 0.15	3.34 ± 0.13	3.30 ± 0.06	3.06 ± 0.12	2.98 ± 0.11	3.15 ± 0.05
d_3	-4.14 ± 0.29	-3.10 ± 0.28	-3.30 ± 0.10	-2.46 ± 0.18	-1.97 ± 0.17	-2.48 ± 0.06
d_4	-0.86 ± 2.15	-1.01 ± 2.14	-0.97 ± 2.18	4.44 ± 1.70	4.43 ± 1.70	4.48 ± 1.67
d_5	0.66 ± 0.18	-0.02 ± 0.17	0.11 ± 0.05	0.00 ± 0.15	-0.49 ± 0.14	-0.26 ± 0.05
d_{10}	-0.62 ± 1.84	-0.26 ± 1.86	-0.44 ± 1.86	-1.80 ± 1.91	-1.17 ± 1.93	-1.98 ± 1.88
d_{11}	-2.65 ± 1.99	-2.30 ± 2.00	-2.46 ± 2.00	-2.24 ± 2.07	-1.99 ± 2.07	-2.41 ± 2.07
d_{12}	3.85 ± 1.96	3.40 ± 1.99	3.38 ± 1.98	5.41 ± 1.80	4.73 ± 1.82	5.62 ± 1.77
d_{13}	1.21 ± 2.06	1.08 ± 2.06	1.02 ± 2.07	-0.78 ± 2.02	-0.81 ± 2.02	-0.69 ± 2.02
$d_{14} - d_{15}$	-6.92 ± 0.28	-5.95 ± 0.25	-5.88 ± 0.12	-5.02 ± 0.21	-4.50 ± 0.19	-4.92 ± 0.10
d_{16}	1.62 ± 0.74	1.34 ± 0.74	1.55 ± 0.73	1.76 ± 0.70	1.64 ± 0.71	1.73 ± 0.69
$\chi^2_{\pi N}$	170	131	159	242	98	166
$\chi^2_{\pi\pi N}$	172	169	167	176	171	176

$$|c_i| \sim 1.0 < 3.0 < 5.5 \qquad\qquad |d_i| \sim 1.5 < 4.0 < 7.0$$

Fits

Input

m_N	M_π	F_π	g_A	l_1	l_2	l_3	l_4
938.27	139.57	92.4	1.27	-0.4 ± 0.6	4.3 ± 0.1	2.9 ± 2.4	4.4 ± 0.2

$g_{\pi N \Delta}$	g_1
1.35	2.29

large N_c

Bijnens, Ecker 2014

$Q^3+\delta^1$

LECs	HB KH	HB GW	HB RS	Cov KH	Cov GW	Cov RS
c_1	-1.29 ± 0.08	-1.61 ± 0.07	-1.35 ± 0.02	-0.93 ± 0.08	-1.26 ± 0.07	-0.98 ± 0.02
c_2	1.50 ± 0.12	1.34 ± 0.11	1.29 ± 0.04	1.44 ± 0.11	1.39 ± 0.10	1.34 ± 0.04
c_3	-2.52 ± 0.08	-2.70 ± 0.08	-2.25 ± 0.03	-2.34 ± 0.08	-2.65 ± 0.08	-2.16 ± 0.03
c_4	1.84 ± 0.04	1.90 ± 0.04	1.77 ± 0.02	1.62 ± 0.04	1.74 ± 0.04	1.61 ± 0.02
$d_1 + d_2$	0.57 ± 0.15	0.32 ± 0.14	-0.13 ± 0.06	0.42 ± 0.13	0.46 ± 0.12	0.05 ± 0.05
d_3	-1.64 ± 0.29	-0.74 ± 0.27	-0.77 ± 0.10	-1.16 ± 0.18	-0.79 ± 0.17	-0.66 ± 0.06
d_4	-1.16 ± 2.37	-1.18 ± 2.36	-0.97 ± 2.40	0.04 ± 2.21	0.24 ± 2.12	0.28 ± 2.15
d_5	0.90 ± 0.18	0.26 ± 0.17	0.55 ± 0.05	0.66 ± 0.15	0.18 ± 0.14	0.32 ± 0.05
d_{10}	-0.59 ± 1.93	-0.32 ± 1.93	-0.51 ± 1.93	0.29 ± 2.09	0.62 ± 2.08	0.62 ± 2.08
d_{11}	-3.07 ± 2.00	-2.83 ± 2.00	-3.14 ± 2.00	-0.20 ± 2.06	-0.09 ± 2.05	-0.07 ± 2.06
d_{12}	1.01 ± 2.05	0.67 ± 2.06	0.51 ± 2.05	0.66 ± 1.95	0.44 ± 1.94	0.06 ± 1.94
d_{13}	-2.51 ± 2.05	-2.61 ± 2.05	-2.80 ± 2.05	-2.53 ± 1.99	-2.56 ± 1.98	-2.59 ± 1.99
$d_{14} - d_{15}$	-1.66 ± 0.28	-0.82 ± 0.26	0.02 ± 0.12	-0.89 ± 0.22	-0.59 ± 0.20	0.11 ± 0.10
d_{16}	-0.32 ± 0.70	-0.43 ± 0.71	-0.39 ± 0.68	0.97 ± 0.70	0.82 ± 0.70	0.88 ± 0.69
$\chi^2_{\pi N}$	123	205	19	126	154	12
$\chi^2_{\pi\pi N}$	183	180	188	189	186	187

$$|c_i| \sim 1.0 < 3.0 < 5.5 \qquad |d_i| \sim 1.5 < 4.0 < 7.0$$

Q^4

LECs	HB KH	HB GW	HB RS	Cov KH	Cov GW	Cov RS
c_1	-0.77 ± 0.11	-0.96 ± 0.11	-0.94 ± 0.08	-0.90 ± 0.14	-1.18 ± 0.13	-1.02 ± 0.09
c_2	2.96 ± 0.32	3.96 ± 0.31	2.84 ± 0.27	3.52 ± 0.32	3.73 ± 0.31	3.35 ± 0.23
c_3	-3.97 ± 0.10	-4.89 ± 0.08	-4.06 ± 0.11	-5.26 ± 0.12	-6.00 ± 0.11	-5.23 ± 0.11
c_4	2.87 ± 0.09	3.39 ± 0.07	2.90 ± 0.12	3.48 ± 0.08	3.83 ± 0.06	3.47 ± 0.10
$d_1 + d_2$	4.46 ± 0.14	4.23 ± 0.13	4.76 ± 0.08	5.18 ± 0.15	4.94 ± 0.14	5.09 ± 0.07
d_3	-4.00 ± 0.21	-2.98 ± 0.20	-3.82 ± 0.08	-5.65 ± 0.28	-5.13 ± 0.25	-5.01 ± 0.12
d_4	0.71 ± 2.04	0.17 ± 1.97	0.61 ± 1.88	-2.26 ± 1.88	-2.87 ± 1.76	-2.32 ± 1.88
d_5	0.18 ± 0.16	-0.57 ± 0.15	-0.37 ± 0.05	0.69 ± 0.18	0.24 ± 0.16	0.07 ± 0.06
d_{10}	-5.94 ± 1.72	-4.17 ± 1.76	-6.08 ± 1.66	-7.19 ± 1.79	-5.65 ± 1.81	-6.22 ± 1.79
d_{11}	-2.39 ± 1.97	-2.50 ± 1.97	-2.43 ± 1.95	-2.47 ± 2.00	-1.34 ± 1.99	-2.14 ± 1.99
d_{12}	6.10 ± 1.71	6.20 ± 1.73	6.32 ± 1.64	8.82 ± 1.78	7.28 ± 1.76	7.75 ± 1.70
d_{13}	-2.27 ± 2.07	-3.69 ± 2.07	-2.32 ± 2.02	-1.14 ± 1.97	-1.32 ± 1.92	-1.30 ± 1.92
$d_{14} - d_{15}$	-8.00 ± 0.24	-6.89 ± 0.23	-8.23 ± 0.12	-9.54 ± 0.26	-8.77 ± 0.24	-8.93 ± 0.12
d_{16}	6.33 ± 0.70	7.55 ± 0.71	6.45 ± 0.69	-0.70 ± 0.65	-0.89 ± 0.63	-0.72 ± 0.64
e_{10}	-3.54 ± 4.58	-4.18 ± 4.54	-4.21 ± 4.52	-3.73 ± 4.42	-4.91 ± 4.33	-3.69 ± 4.42
e_{11}	0.36 ± 4.74	0.41 ± 4.72	0.68 ± 4.65	2.58 ± 4.10	3.30 ± 3.92	2.65 ± 4.09
e_{12}	1.62 ± 3.73	0.61 ± 3.83	1.85 ± 3.66	1.80 ± 3.52	2.27 ± 3.51	1.70 ± 3.51
e_{13}	-0.87 ± 3.80	-1.19 ± 3.85	-1.43 ± 3.75	-2.21 ± 3.36	-3.20 ± 3.27	-2.50 ± 3.34
e_{14}	1.41 ± 0.11	1.42 ± 0.10	1.18 ± 0.10	0.32 ± 0.12	1.09 ± 0.11	0.40 ± 0.12
e_{15}	-12.73 ± 0.64	-6.41 ± 0.56	-13.55 ± 0.61	-5.36 ± 0.39	-3.37 ± 0.36	-5.50 ± 0.34
e_{16}	6.77 ± 1.27	-0.80 ± 1.22	8.29 ± 1.10	0.92 ± 0.60	-1.48 ± 0.55	1.28 ± 0.47
e_{17}	-0.48 ± 0.11	-0.43 ± 0.11	-0.46 ± 0.11	0.47 ± 0.09	0.02 ± 0.09	0.32 ± 0.10
e_{18}	5.05 ± 0.49	1.96 ± 0.39	6.10 ± 0.61	1.15 ± 0.28	0.08 ± 0.22	1.57 ± 0.35
e_{34}	0.29 ± 4.84	0.43 ± 4.85	0.51 ± 4.82	0.86 ± 4.77	1.22 ± 4.75	0.95 ± 4.77
$\chi^2_{\pi N}$	187 + 160	125 + 169	41 + 200	147 + 6	79 + 56	31 + 34
$\chi^2_{\pi\pi N}$	244	250	257	234	238	228

$|c_i| \sim 1.0 < 3.0 < 5.5$ $\quad$ $|d_i| \sim 1.5 < 4.0 < 7.0$ $\quad$ $|e_i| \sim 2.0 < 5.5 < 9.0$

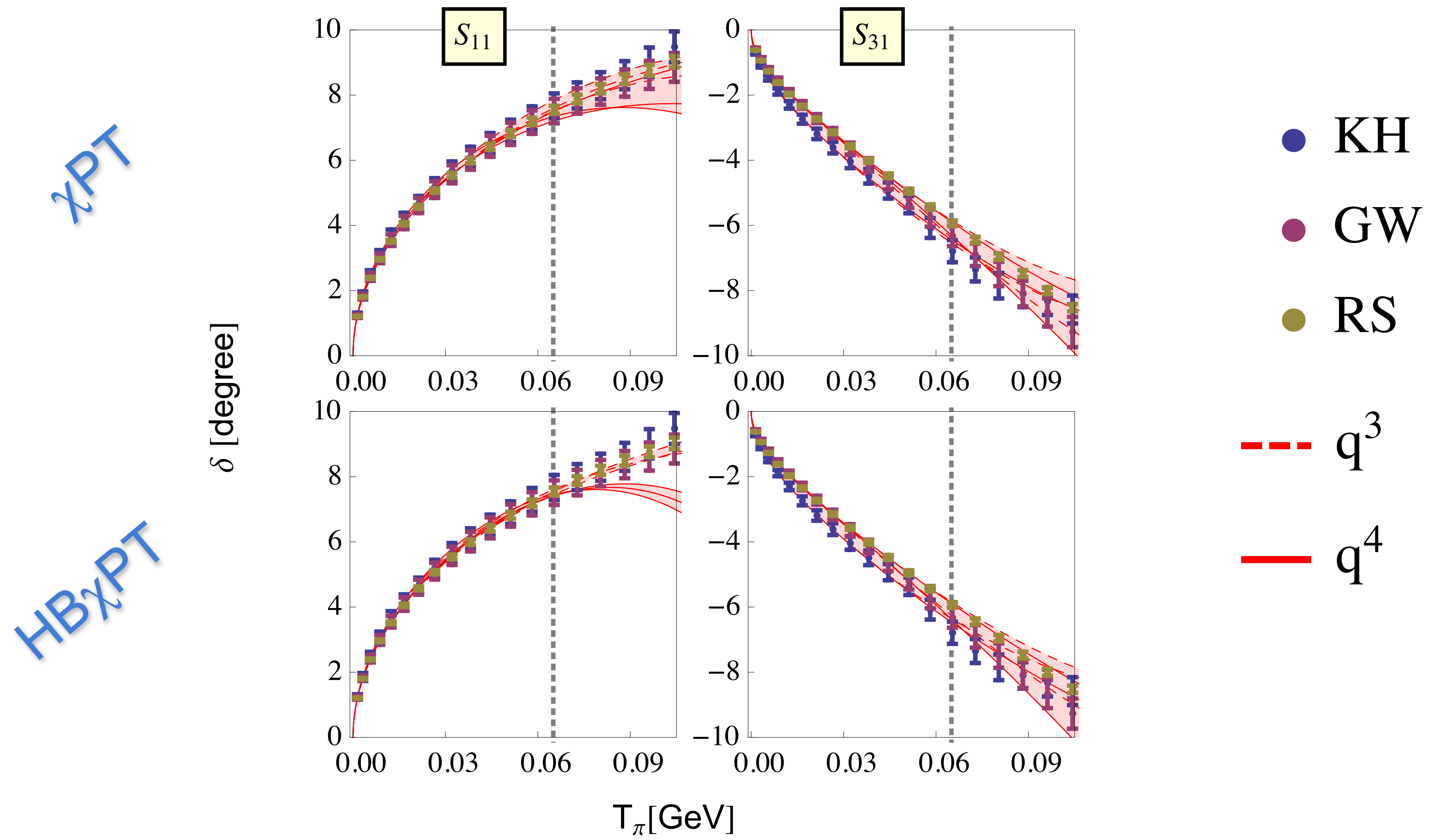
$Q^4+\delta^1$

LECs	HB KH	HB GW	HB RS	Cov KH	Cov GW	Cov RS
c_1	-1.12 ± 0.17	-1.60 ± 0.24	-1.28 ± 0.11	-1.00 ± 0.20	-1.67 ± 0.21	-1.14 ± 0.11
c_2	1.30 ± 0.50	1.30 ± 0.76	1.36 ± 0.36	1.58 ± 0.42	1.07 ± 0.40	1.44 ± 0.24
c_3	-1.70 ± 0.11	-2.62 ± 0.10	-1.95 ± 0.12	-2.51 ± 0.16	-3.48 ± 0.15	-2.55 ± 0.12
c_4	1.81 ± 0.09	2.25 ± 0.07	2.12 ± 0.12	2.08 ± 0.08	2.41 ± 0.06	2.19 ± 0.10
d_1+d_2	1.29 ± 0.15	1.01 ± 0.14	1.21 ± 0.09	1.48 ± 0.16	1.27 ± 0.15	1.07 ± 0.07
d_3	-1.82 ± 0.23	-0.80 ± 0.21	-1.39 ± 0.08	-2.42 ± 0.32	-2.10 ± 0.28	-1.79 ± 0.13
d_4	-0.19 ± 3.65	2.54 ± 2.64	-0.41 ± 3.60	0.56 ± 2.11	-1.29 ± 2.20	0.24 ± 2.12
d_5	0.65 ± 0.17	-0.07 ± 0.16	0.18 ± 0.05	0.81 ± 0.19	0.44 ± 0.17	0.41 ± 0.06
d_{10}	-1.46 ± 2.27	-0.44 ± 2.41	-1.00 ± 2.22	-1.68 ± 2.27	-1.18 ± 2.23	-1.15 ± 2.26
d_{11}	-1.07 ± 2.19	-0.50 ± 2.24	-0.91 ± 2.18	-1.36 ± 2.20	0.38 ± 2.20	-0.95 ± 2.19
d_{12}	-0.19 ± 2.06	-1.73 ± 2.18	-0.61 ± 2.02	0.48 ± 2.06	-0.91 ± 2.04	-0.39 ± 2.02
d_{13}	-4.58 ± 2.51	-3.98 ± 2.82	-4.84 ± 2.38	-1.08 ± 2.30	-0.22 ± 2.09	-0.95 ± 2.16
$d_{14}-d_{15}$	-2.45 ± 0.27	-1.30 ± 0.25	-1.84 ± 0.13	-3.11 ± 0.28	-2.31 ± 0.26	-2.00 ± 0.13
d_{16}	5.76 ± 0.74	6.40 ± 0.80	6.06 ± 0.75	0.69 ± 0.72	-0.34 ± 0.75	0.54 ± 0.72
e_{10}	-0.32 ± 5.11	0.92 ± 4.90	-0.35 ± 5.10	0.98 ± 5.17	0.28 ± 5.02	0.97 ± 5.17
e_{11}	0.86 ± 5.12	-1.66 ± 5.06	0.75 ± 5.13	-0.64 ± 4.87	0.79 ± 4.54	-0.45 ± 4.82
e_{12}	1.02 ± 3.84	-3.54 ± 3.97	0.78 ± 3.84	-1.59 ± 3.88	-0.69 ± 3.82	-1.71 ± 3.87
e_{13}	2.49 ± 3.73	-3.47 ± 4.46	2.18 ± 3.73	-1.48 ± 3.65	-1.49 ± 3.45	-1.72 ± 3.58
e_{14}	0.58 ± 0.11	0.75 ± 0.10	0.52 ± 0.10	0.35 ± 0.15	1.30 ± 0.13	0.59 ± 0.12
e_{15}	-4.84 ± 0.71	0.41 ± 0.71	-3.05 ± 0.63	-1.60 ± 0.48	1.23 ± 0.48	-0.84 ± 0.37
e_{16}	2.48 ± 1.91	-1.32 ± 2.78	1.13 ± 1.38	-0.64 ± 0.82	-1.60 ± 0.83	-1.07 ± 0.51
e_{17}	-0.42 ± 0.11	-0.50 ± 0.11	-0.52 ± 0.11	-0.10 ± 0.09	-0.56 ± 0.09	-0.40 ± 0.10
e_{18}	1.37 ± 0.50	-1.22 ± 0.40	0.13 ± 0.64	-0.22 ± 0.28	-1.33 ± 0.22	-0.59 ± 0.36
e_{34}	-0.94 ± 4.82	1.51 ± 4.95	-0.85 ± 4.82	0.62 ± 4.83	0.75 ± 4.79	0.73 ± 4.83
$\chi^2_{\pi N}$	130 + 69	78 + 74	9 + 80	129 + 6	69 + 47	3 + 38
$\chi^2_{\pi\pi N}$	179	174	180	177	177	175

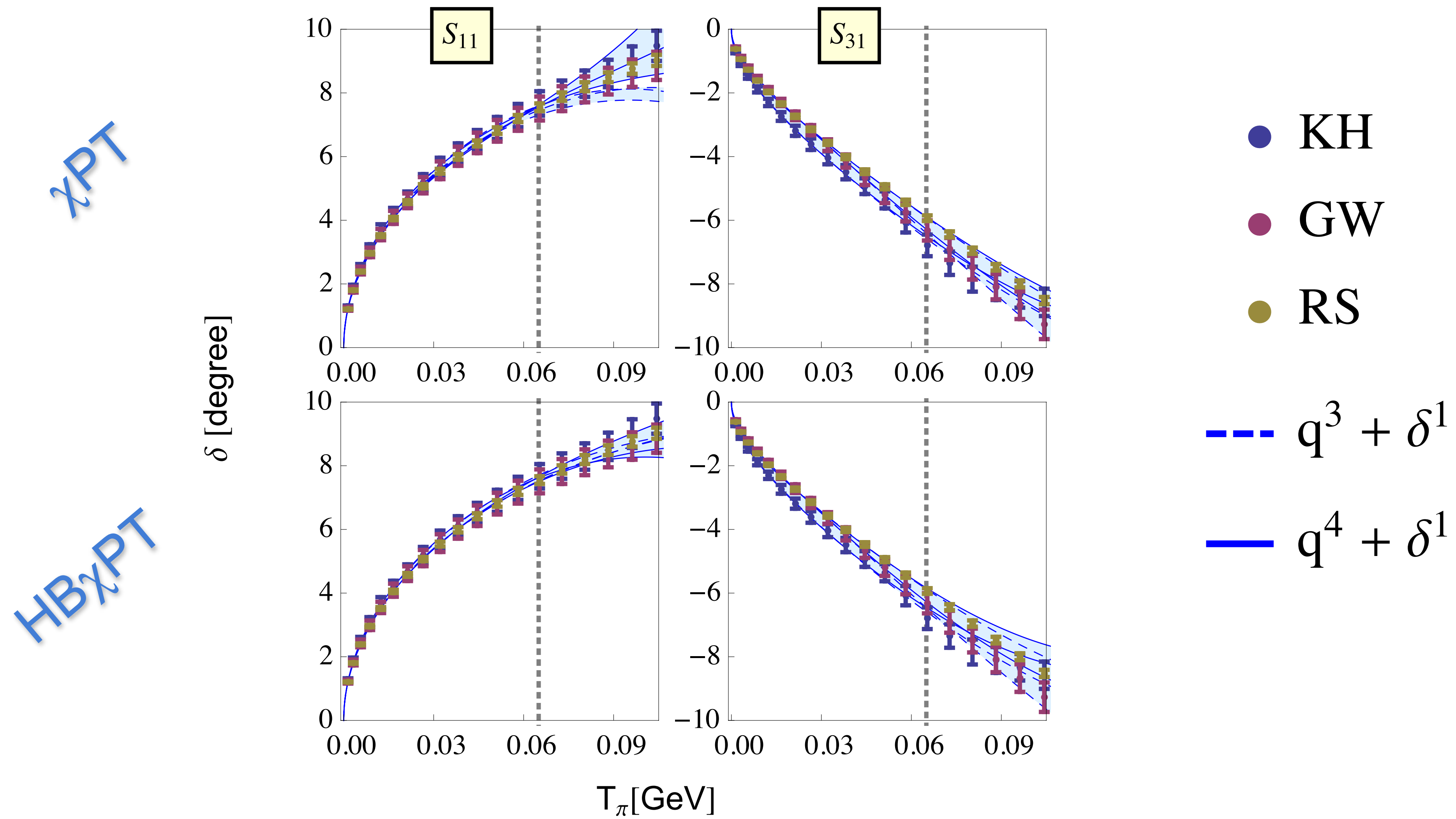
$|c_i| \sim 1.0 < 3.0 < 5.5$ $\quad$ $|d_i| \sim 1.5 < 4.0 < 7.0$ $\quad$ $|e_i| \sim 2.0 < 5.5 < 9.0$

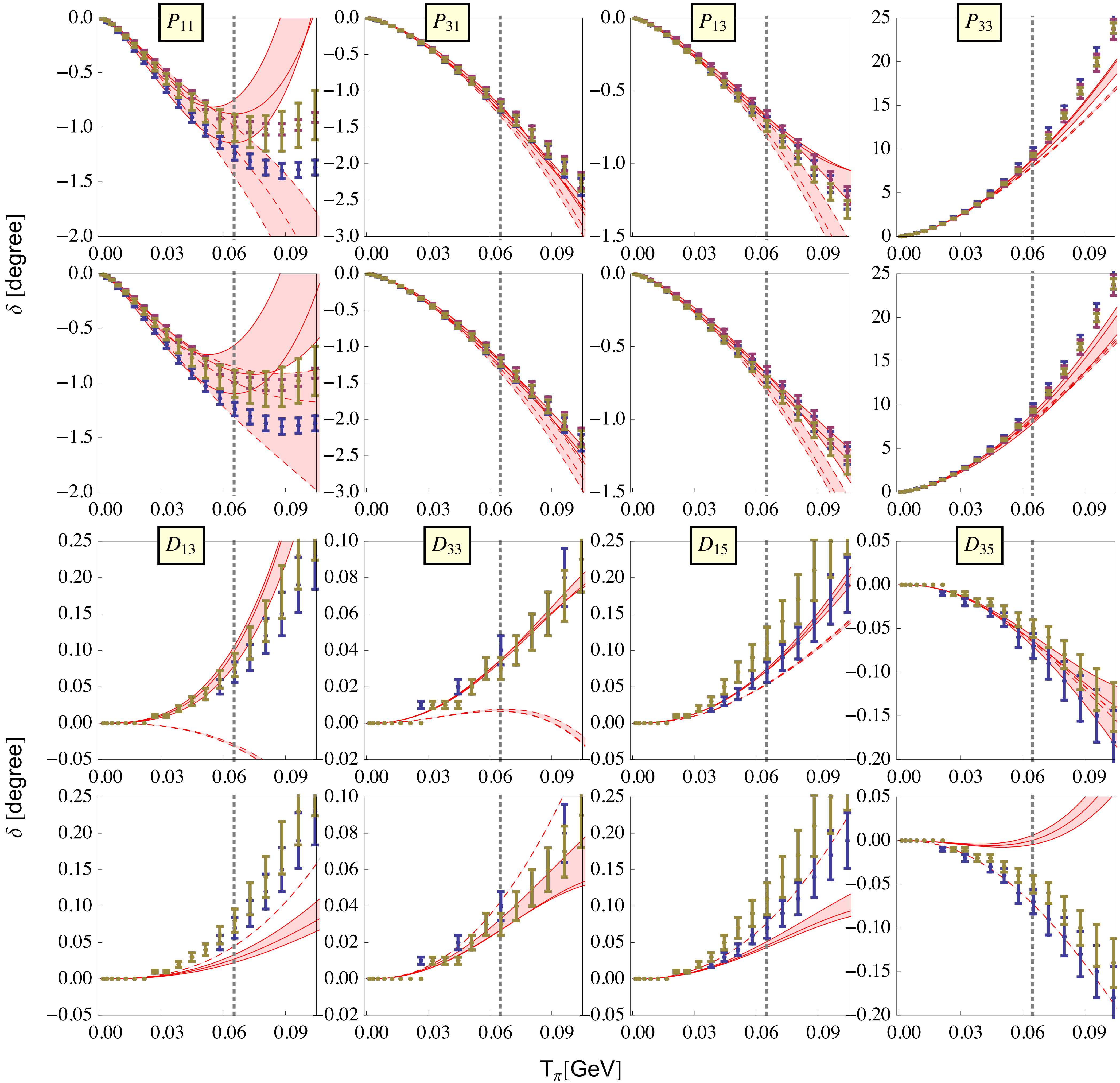
Predictions

Partial Waves

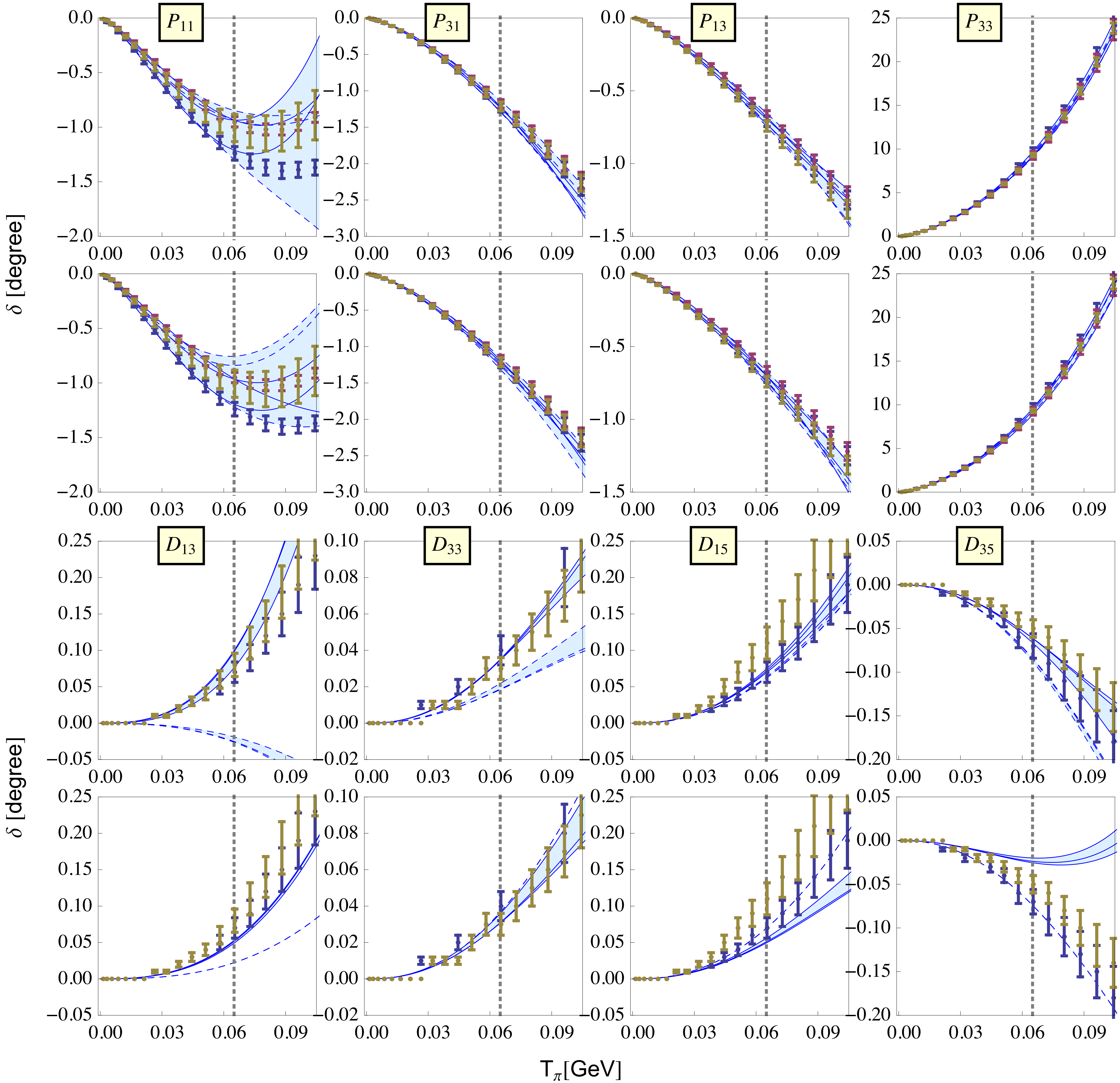


Partial Waves



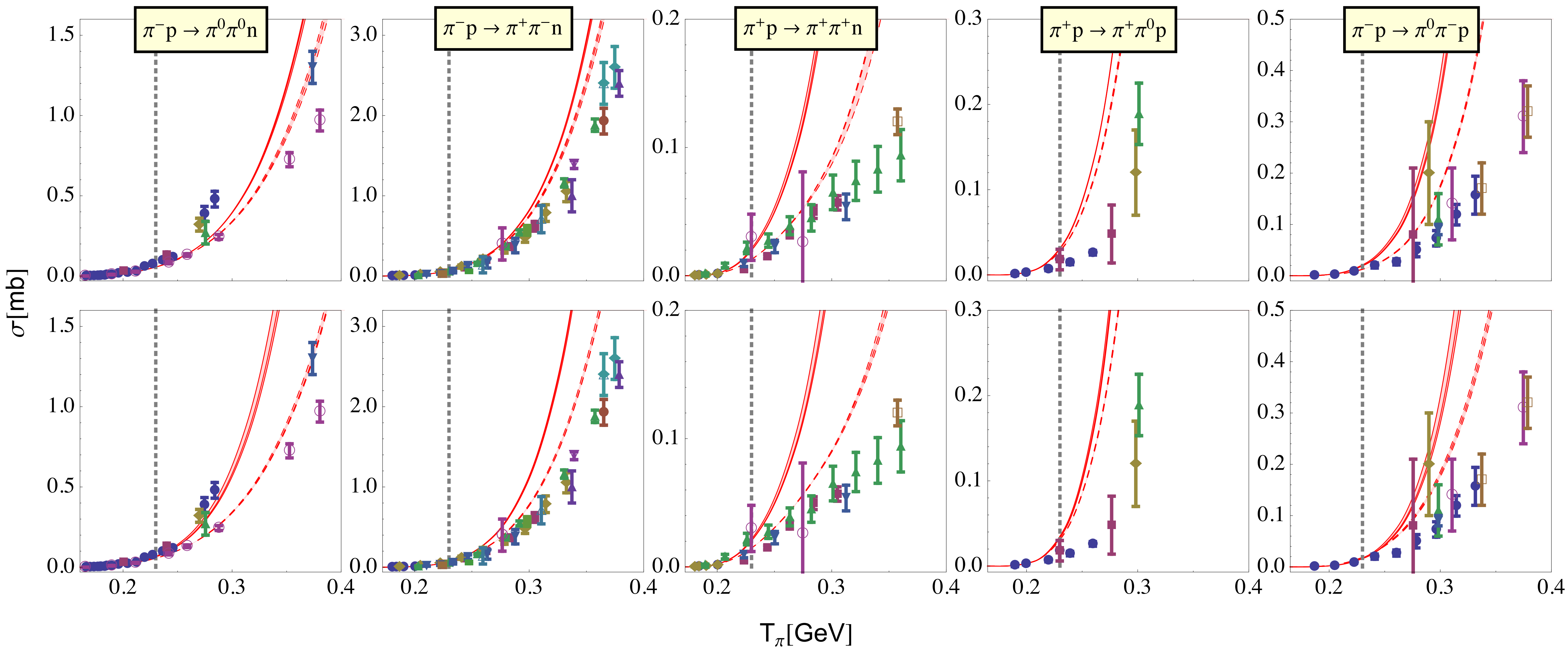


P_{11}
P_{31}
P_{13}
P_{33}
D_{13}
D_{33}
D_{15}
D_{35}
δ [degree]
δ [degree]
T_π[GeV]

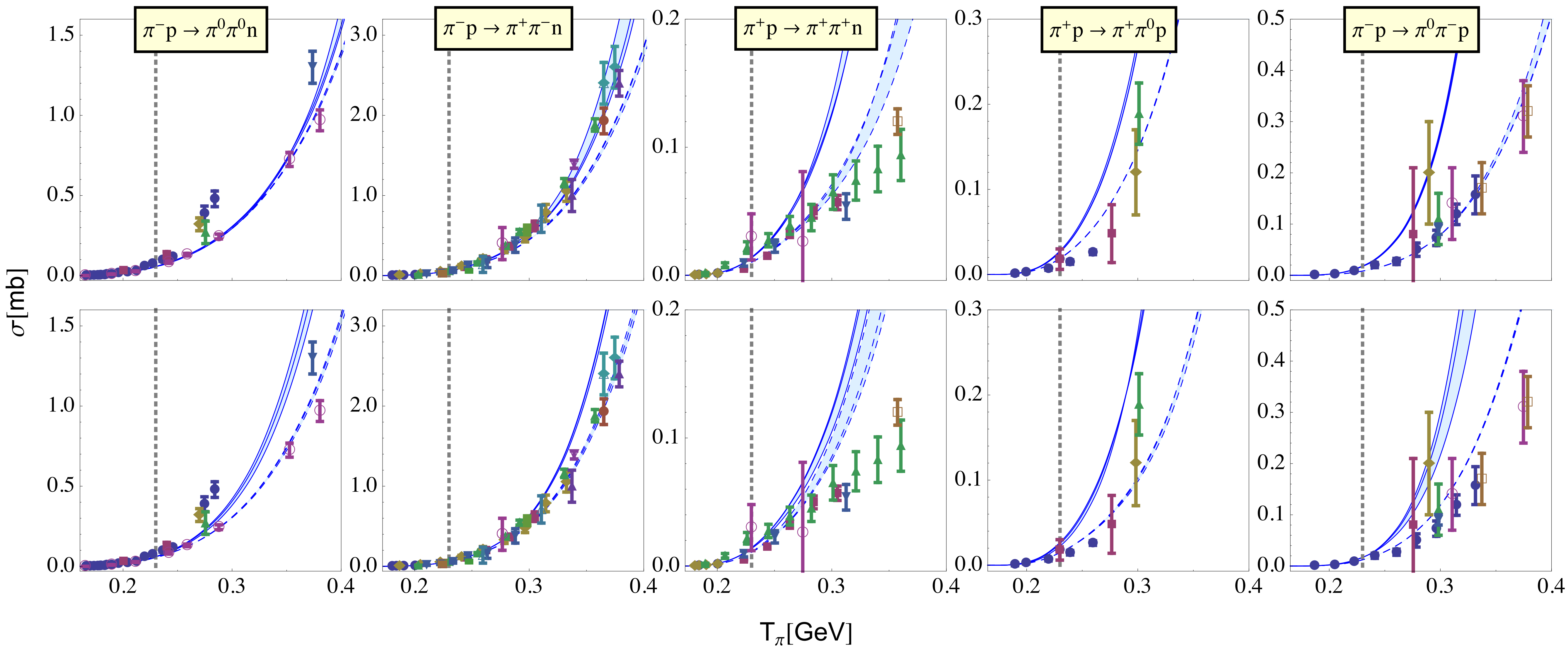


P11
P31
P13
P33
D13
D33
D15
D35
δ [degree]
δ [degree]
T_π[GeV]

Cross Sections



Cross Sections



Summary

Good description of the phase shifts in $\pi N \to \pi N$

- Fits in q^3 & q^4 comparable $\Longrightarrow$ convergency
- χPT ~ HBχPT $\Longrightarrow$ $1/m_N$ contributions not that important
- higher energy predictions for P_{11} (R) and P_{33} (Δ) problematic

Fair description of the cross sections in $\pi N \to \pi\pi N$

- $q^3 > q^4$ $\Longrightarrow$ bad convergency (too large LECs from $\pi N \to \pi N$)
- χPT $\gtrsim$ HBχPT $\Longrightarrow$ $1/m_N$ contributions important
- role of Δ and R underestimated?

Future extensions of the combined fit

- q^3 & q^4 + ΔNLO + RNLO
- ε^3 + RNLO

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Future extensions of the combined fit

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Thanks !!

Backup

Q^4

LECs	HB		Cov	
	no D waves	with D waves	no D waves	with D waves
c_1	-0.93 ± 0.08	-0.94 ± 0.08	-1.00 ± 0.10	-1.02 ± 0.09
c_2	2.93 ± 0.27	2.84 ± 0.27	3.28 ± 0.32	3.35 ± 0.23
c_3	-4.25 ± 0.11	-4.06 ± 0.11	-5.17 ± 0.16	-5.23 ± 0.11
c_4	3.08 ± 0.12	2.90 ± 0.12	3.53 ± 0.12	3.47 ± 0.10
$d_1 + d_2$	4.94 ± 0.08	4.76 ± 0.08	5.08 ± 0.08	5.09 ± 0.07
d_3	-3.93 ± 0.08	-3.82 ± 0.08	-5.01 ± 0.12	-5.01 ± 0.12
d_4	0.32 ± 1.81	0.61 ± 1.88	-2.33 ± 1.88	-2.32 ± 1.88
d_5	-0.42 ± 0.05	-0.37 ± 0.05	0.08 ± 0.06	0.07 ± 0.06
d_{10}	-6.36 ± 1.64	-6.08 ± 1.66	-6.11 ± 1.81	-6.22 ± 1.79
d_{11}	-2.46 ± 1.93	-2.43 ± 1.95	-2.13 ± 2.00	-2.14 ± 1.99
d_{12}	6.67 ± 1.62	6.32 ± 1.64	7.50 ± 1.74	7.75 ± 1.70
d_{13}	-2.23 ± 2.00	-2.32 ± 2.02	-1.19 ± 1.93	-1.30 ± 1.92
$d_{14} - d_{15}$	-8.50 ± 0.13	-8.23 ± 0.12	-8.86 ± 0.13	-8.93 ± 0.12
d_{16}	6.71 ± 0.69	6.45 ± 0.69	-0.78 ± 0.65	-0.72 ± 0.64
e_{10}	-4.91 ± 4.48	-4.21 ± 4.52	-3.69 ± 4.43	-3.69 ± 4.42
e_{11}	1.10 ± 4.57	0.68 ± 4.65	2.66 ± 4.08	2.65 ± 4.09
e_{12}	2.04 ± 3.60	1.85 ± 3.66	1.69 ± 3.52	1.70 ± 3.51
e_{13}	-1.78 ± 3.70	-1.43 ± 3.75	-2.54 ± 3.34	-2.50 ± 3.34
e_{14}	-3.26 ± 1.97	1.18 ± 0.10	-2.30 ± 2.25	0.40 ± 0.12
e_{15}	-3.88 ± 3.88	-13.55 ± 0.61	-0.58 ± 3.80	-5.50 ± 0.34
e_{16}	3.63 ± 1.82	8.29 ± 1.10	-0.62 ± 1.22	1.28 ± 0.47
e_{17}	2.34 ± 3.50	-0.46 ± 0.11	1.09 ± 2.31	0.32 ± 0.10
e_{18}	2.44 ± 3.50	6.10 ± 0.61	0.61 ± 1.93	1.57 ± 0.35
e_{34}	0.62 ± 4.81	0.51 ± 4.82	0.96 ± 4.77	0.95 ± 4.77
$\chi^2_{\pi N}$	24	41+200	31	31+34
$\chi^2_{\pi\pi N}$	270	257	227	228

$|c_i| \sim 1.0 < 3.0 < 5.5$ $\quad$ $|d_i| \sim 1.5 < 4.0 < 7.0$ $\quad$ $|e_i| \sim 2.0 < 5.5 < 9.0$

$Q^4+\delta^1$

LECs	HB		Cov	
	no D waves	with D waves	no D waves	with D waves
c_1	-1.33 ± 0.13	-1.28 ± 0.11	-1.10 ± 0.12	-1.14 ± 0.11
c_2	1.22 ± 0.42	1.36 ± 0.36	1.58 ± 0.34	1.44 ± 0.24
c_3	-2.05 ± 0.12	-1.95 ± 0.12	-2.58 ± 0.17	-2.55 ± 0.12
c_4	2.21 ± 0.12	2.12 ± 0.12	2.31 ± 0.13	2.19 ± 0.10
$d_1 + d_2$	1.32 ± 0.09	1.21 ± 0.09	1.04 ± 0.08	1.07 ± 0.07
d_3	-1.45 ± 0.08	-1.39 ± 0.08	-1.77 ± 0.13	-1.79 ± 0.13
d_4	-0.18 ± 4.10	-0.41 ± 3.60	0.05 ± 2.11	0.24 ± 2.12
d_5	0.16 ± 0.05	0.18 ± 0.05	0.41 ± 0.06	0.41 ± 0.06
d_{10}	-1.28 ± 2.24	-1.00 ± 2.22	-1.06 ± 2.26	-1.15 ± 2.26
d_{11}	-0.79 ± 2.19	-0.91 ± 2.18	-1.03 ± 2.18	-0.95 ± 2.19
d_{12}	-0.80 ± 2.04	-0.61 ± 2.02	-0.27 ± 2.02	-0.39 ± 2.02
d_{13}	-4.33 ± 2.48	-4.84 ± 2.38	-1.10 ± 2.17	-0.95 ± 2.16
$d_{14} - d_{15}$	-2.00 ± 0.13	-1.84 ± 0.13	-1.99 ± 0.14	-2.00 ± 0.13
d_{16}	6.12 ± 0.77	6.06 ± 0.75	0.44 ± 0.72	0.54 ± 0.72
e_{10}	-0.44 ± 5.13	-0.35 ± 5.10	0.91 ± 5.15	0.97 ± 5.17
e_{11}	0.54 ± 5.22	0.75 ± 5.13	-0.35 ± 4.79	-0.45 ± 4.82
e_{12}	0.39 ± 3.95	0.78 ± 3.84	-1.75 ± 3.86	-1.71 ± 3.87
e_{13}	1.99 ± 3.80	2.18 ± 3.73	-1.84 ± 3.57	-1.72 ± 3.58
e_{14}	-1.83 ± 2.12	0.52 ± 0.10	0.55 ± 2.33	0.59 ± 0.12
e_{15}	1.91 ± 4.12	-3.05 ± 0.63	-0.57 ± 3.96	-0.84 ± 0.37
e_{16}	-0.63 ± 1.88	1.13 ± 1.38	-1.54 ± 1.23	-1.07 ± 0.51
e_{17}	-0.43 ± 3.56	-0.52 ± 0.11	-1.50 ± 2.51	-0.40 ± 0.10
e_{18}	-0.42 ± 3.56	0.13 ± 0.64	-0.05 ± 2.07	-0.59 ± 0.36
e_{34}	-0.78 ± 4.83	-0.85 ± 4.82	0.78 ± 4.82	0.73 ± 4.83
$\chi^2_{\pi N}$	7	9+80	1	3+38
$\chi^2_{\pi\pi N}$	179	180	175	175

$|c_i| \sim 1.0 < 3.0 < 5.5$ $\quad$ $|d_i| \sim 1.5 < 4.0 < 7.0$ $\quad$ $|e_i| \sim 2.0 < 5.5 < 9.0$

D-waves