Influence of Skyrme-type Momentum Dependent Interaction on HICs Observables

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Outline

1, Symmetry energy and n/p effective mass splitting

2, Influence of Skyrme force on HIC observables

- isospin diffusion and DR(n/p) ratios

3, Covariance analysis on the HIC observables

- CI-R_2(n/p), CI-DR(n/p), CI-R_{21}(n/n),CI-R_{21}(p/p), R_{diff}

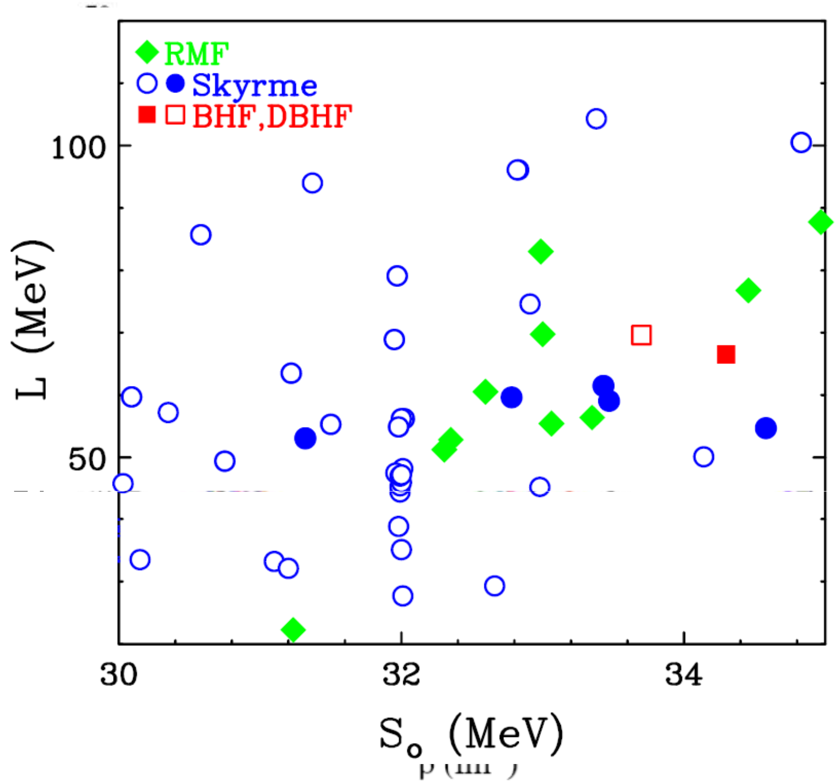
4, Summary and outlook

Isospin asymmetric Equation of State

$$E(\rho,\delta) = E(\rho,\delta=0) + S(\rho)\delta^2 + O(\delta^4)$$

It is a fundamental properties of nuclear matter, and is very important for understanding

- *properties of nuclear structure*
- *properties of neutron star*
- *properties of heavy ion reaction mechanism*



Density dependent of symmetry energy

$$S(\rho) = S_0 + \frac{L}{3}\left(\frac{\rho-\rho_o}{\rho_o}\right) + \frac{K_{\rm sym}}{18}\left(\frac{\rho-\rho_o}{\rho_o}\right)^2 + \ldots,$$

S_0: symemtry energy coefficient
L: slope of density dependent of symmetry energy
K_{sym}: curvature of density dependent of symmetry energy

S(ρ) is the density dependence of symmetry energy, it is a key ingredient of the isospin asymmetric *EOS. However, S(ρ)* uncertainty

Strategies for constraining the symmetry energy

- *Astrophysical measurements*
- *Nuclear structure*
- *Heavy Ion Collisions*

  large regions of ρ, T, δ,

measure the N/Z ratios of the emitted particles (n/p ratios, isospin diffusion, t/He3, N/Z ratios of IMFs, flow, pi-/pi+, ……)

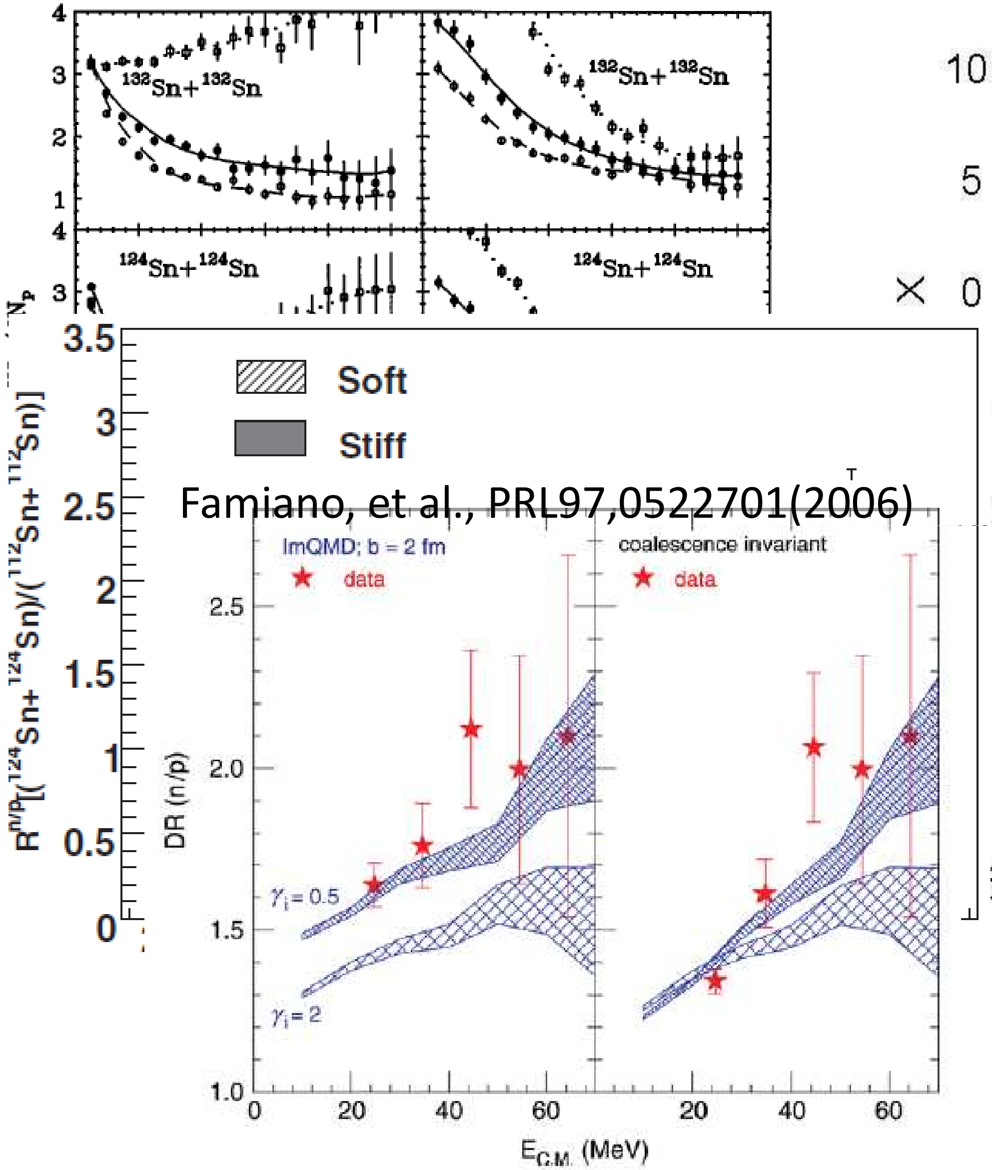
predictions from the transport model, in which the different symmetry potential can be used. QMD, BUU,AMD, SMF, ……

the symmetry energy information can be extracted.

Indirectly! (transport models)

HIC observables: n/p ratios and isospin diffusion, ……

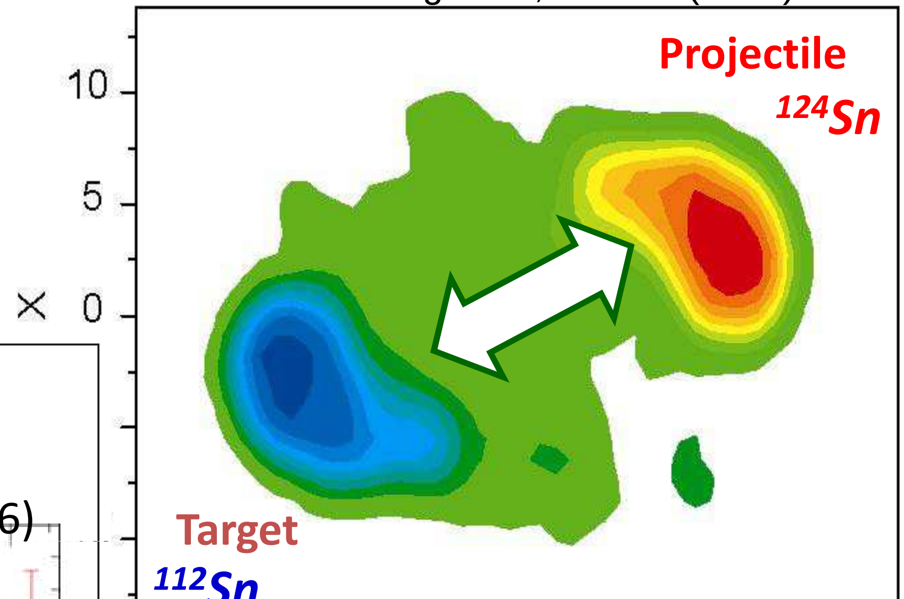
BALi, et al., PRL78(1997)1644

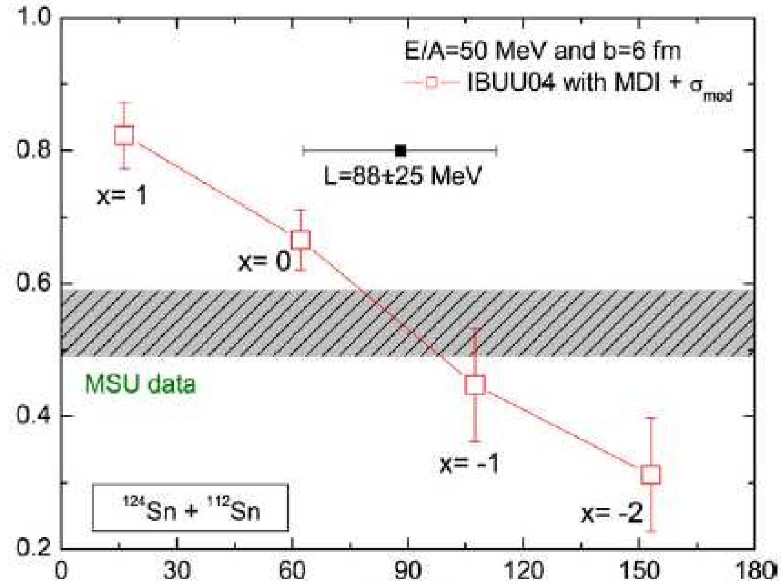


Famiano, et al., PRL97,0522701(2006)

Yingxun Zhang et al., PLB664(2008)145

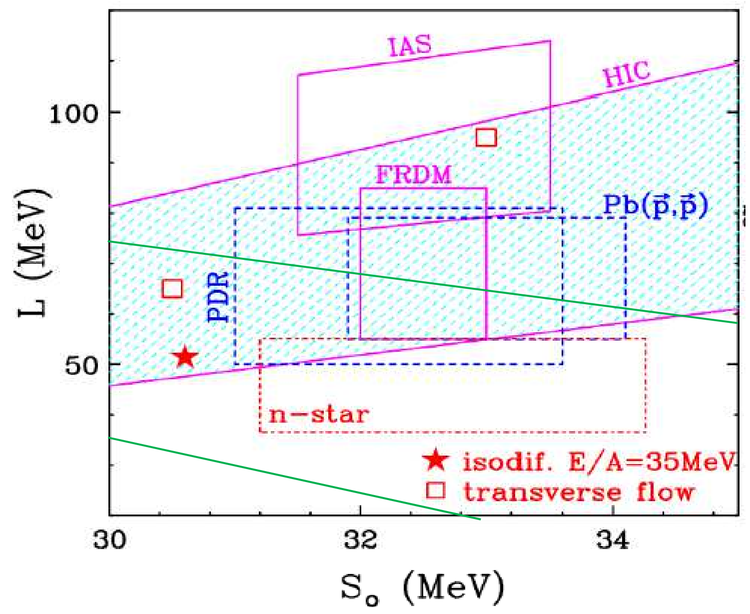
Tsang et al., PRL 92 (2004) 062701



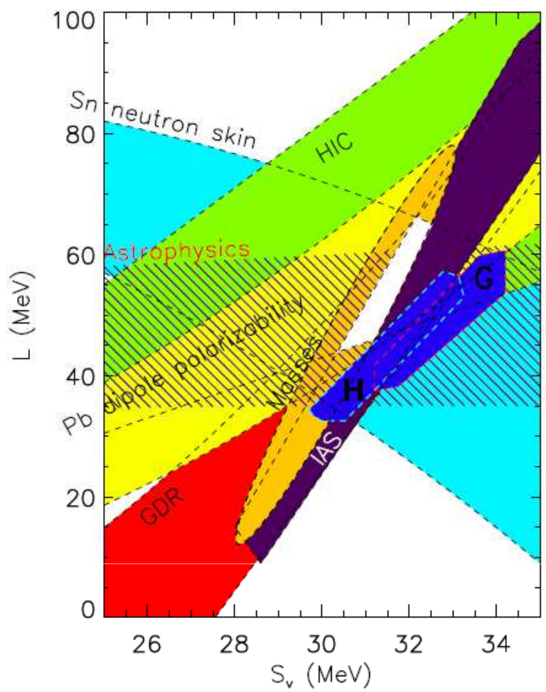


LWChen, et al., PRL72(2005)064309

M.B.Tsang, Yingxun Zhang, et.al., PRL2009



Lattimer, EPJA 50 (2014) 40



$$S(\rho) = S_0 + \frac{L}{3}\left(\frac{\rho-\rho_o}{\rho_o}\right) + \frac{K_{\rm sym}}{18}\left(\frac{\rho-\rho_o}{\rho_o}\right)^2 + \ldots,$$

Consensus on symmetry energy have been obtained at subsaturation density. Model uncertainties are left!

How to reduce the uncertainty of symmetry energy constraints?
density dependent, momentum dependent, tensor force, exchange term, …..

- Density dependent of symmetry energy from SHF

1) Density dependent of symmetry energy

$$S(\rho) = \frac{1}{3}\frac{\hbar^2}{2m}(\frac{3\pi^2}{2})^{2/3}\rho^{2/3} - \frac{1}{8}t_0(2x_0+1)\rho - \frac{1}{48}t_3(2x_3+1)\rho^{\sigma+1} - \frac{1}{24}(\frac{3\pi^2}{2})^{2/3}(3\Theta_v - 2\Theta_s)\rho^{5/3} \qquad \text{(C8)}$$

$$S(\rho) = \frac{1}{3}\epsilon_F\rho^{2/3} + A_{sym}\rho + B_{sym}\rho^{\sigma+1} + C_{sym}(m_s^*, m_v^*)\rho^{5/3} \qquad \text{(C11)}$$

Density dependent of symmetry depends not only on density, effective mass splitting but also isoscalar effective mass

$$f_I = \frac{1}{2\delta}(\frac{m}{m_n^*} - \frac{m}{m_p^*}) = \frac{m}{k}\frac{\partial((U_n - U_p)/2\delta)}{\partial k} = \frac{\partial U_{sym}}{\partial E_k}.$$

$$\begin{aligned} f_I &= \frac{m}{8\hbar^2}[t_2(2x_2+1) - t_1(2x_1+1)]\frac{\rho}{2} \\ &= m/8\hbar^2(\Theta_s - 2\Theta_v)\rho \\ &= (m/m_s^* - m/m_v^*) \end{aligned}$$

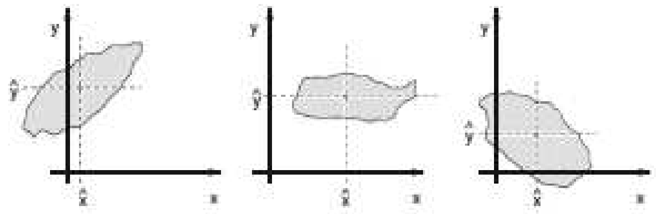
2) Correlation among the effective force parameters

$$C_{AB} = \frac{cov(A,B)}{\sigma(A)\sigma(B)} \quad (1)$$

$$cov(A,B) = \frac{1}{N-1}\sum_i (A_i - <A>)(B_i - <B>) \quad (2)$$

$$\sigma(X) = \sqrt{\frac{1}{N-1}\sum_i (X_i - <X>)^2}, X = A, B \quad (3)$$

$$<X> = \frac{1}{N}\sum_i X_i, i = 1, N. \quad (4)$$



C_AB=1 0 -1

Covariance between pairs of variables from 120 Skyrme sets

C_{AB}	K_0	S_0	L	Ms*	Mv*
K_0	1	0.003	0.161	0.131	0.295
S_0	0.003	1	**0.764**	0.397	0.228
L	0.161	**0.764**	1	**0.460**	0.212
Ms*	0.131	0.397	**0.460**	1	**0.715**
Mv*	0.295	0.228	0.212	**0.715**	1

- Momentum dependent potential and effective mass

Effective mass: slope of single particle potential as a function of k.

Effective mass:

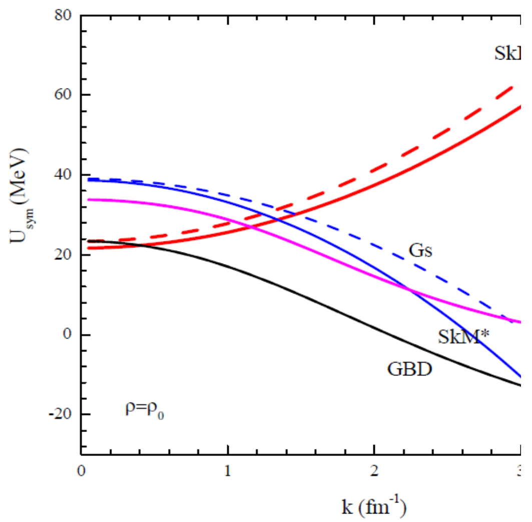
$$\frac{m_\tau^*}{m_\tau} = 1 - \frac{d\,U_\tau(k,\epsilon_\tau(k))}{d\epsilon_\tau} = \frac{k}{m_\tau}\frac{dk}{d\epsilon_\tau} = \left[1 + \frac{m_\tau}{k}\frac{dU_\tau(k,\epsilon_\tau(k))}{dk})\right]^{-1}$$

$$\frac{\widetilde{m_\tau}}{m_\tau} = \left[1 + \frac{m_\tau}{k}\frac{\partial U_\tau(k,\epsilon_\tau(k))}{\partial k})\right]^{-1}$$ K-mass

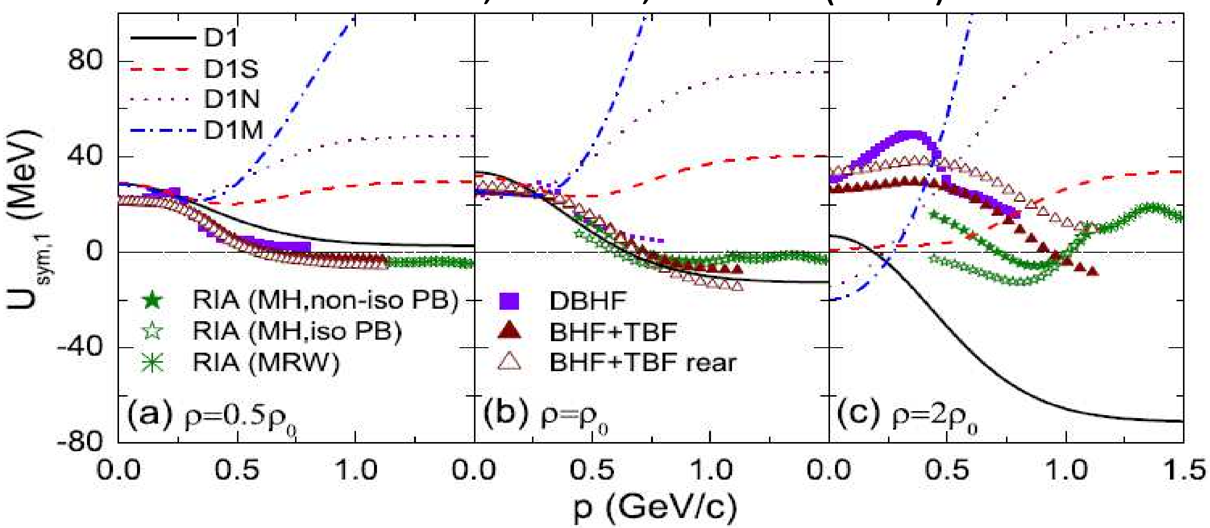
$$\frac{\overline{m_\tau}}{m_\tau} = 1 - \frac{\partial U_\tau(k,\epsilon_\tau(k))}{\partial \epsilon_\tau}$$ E-mass

Effective mass splitting: (mn*-mp*)/m

$$\frac{m}{m_n^*} - \frac{m}{m_p^*} = \frac{\partial(U_n - U_p)}{\partial E_k}$$



R. Chen et al., PRC 85, 024305 (2012).



• Uncertainties in Effective mass and effective mass splitting

ms*/m~0.65-0.9,

F.Chappert, PLB668(2008)402,

Table 2
Infinite and semi-infinite nuclear matter properties of the D1S and D1N interactions compared to empirical values

	D1S	D1N	Emp. values
ρ_0 (fm^{-3})	0.16	0.16	0.17 ± 0.02
E_0/A (MeV)	-15.9	-16.0	-16 ± 1
K_∞ (MeV)	210	230	220 ± 10
m^*/m	0.70	0.75	0.70 ± 0.05
E_{sym} (MeV)	32.0	29.3	30 ± 2
E_{surf} (MeV)	20.0	19.3	21 ± 2

P. Klupfel, et.al., PRC79, 0343310(2009)

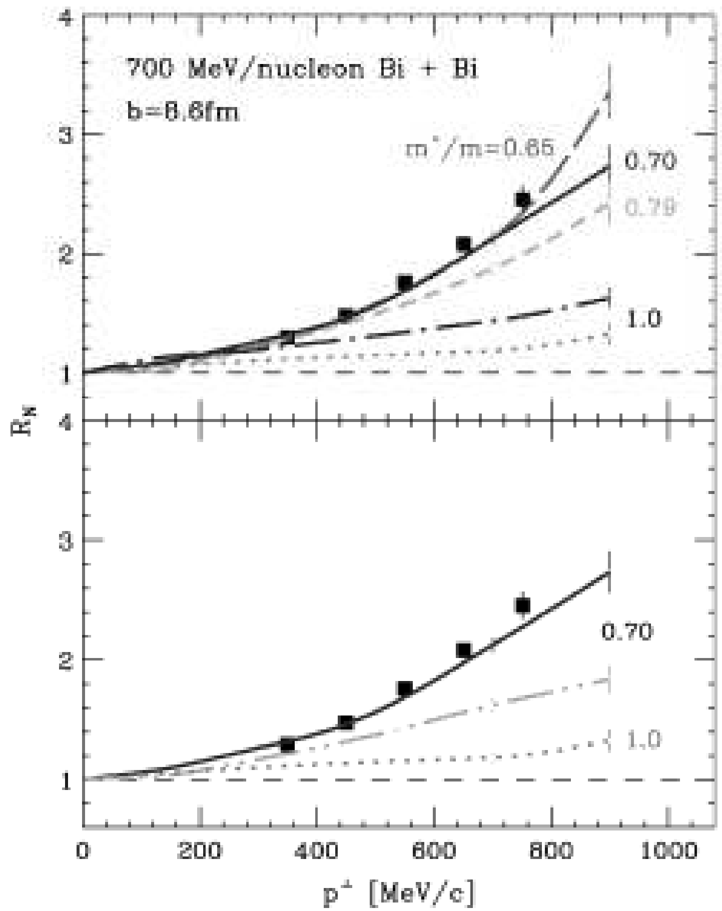
GQR Pb208, ms*/m=0.9

X.H.Li, et.al., PLB743(2015)408

Table 4
Nucleon isoscalar effective mass m_0^*/m and the neutron–ting m_{n-p}^* from the three cases studied in this work.

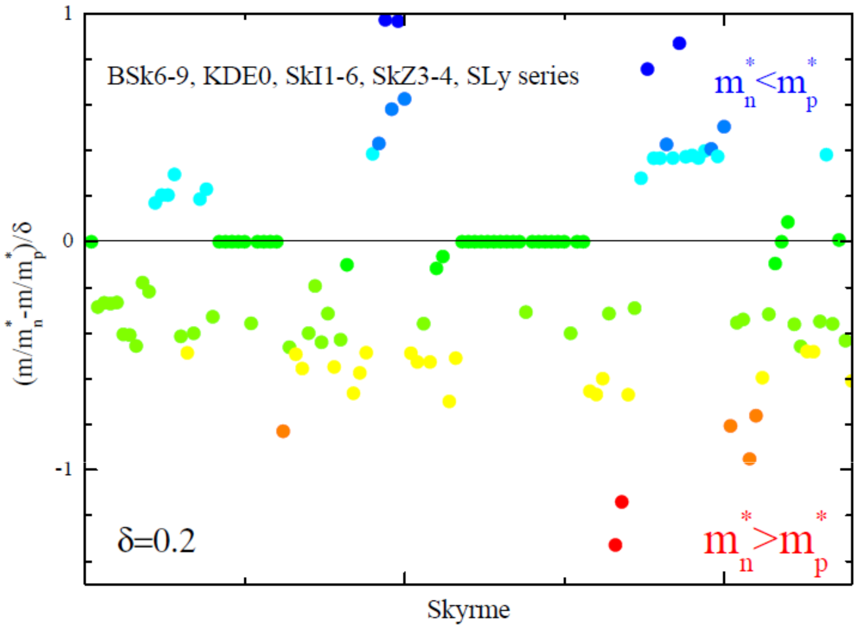
Case	m_0^*/m
I	0.65 ± 0.05
II	0.67 ± 0.06
III	0.65 ± 0.06

P. Danielewicz / Nuclear Physics A 673 (2000) 375–410

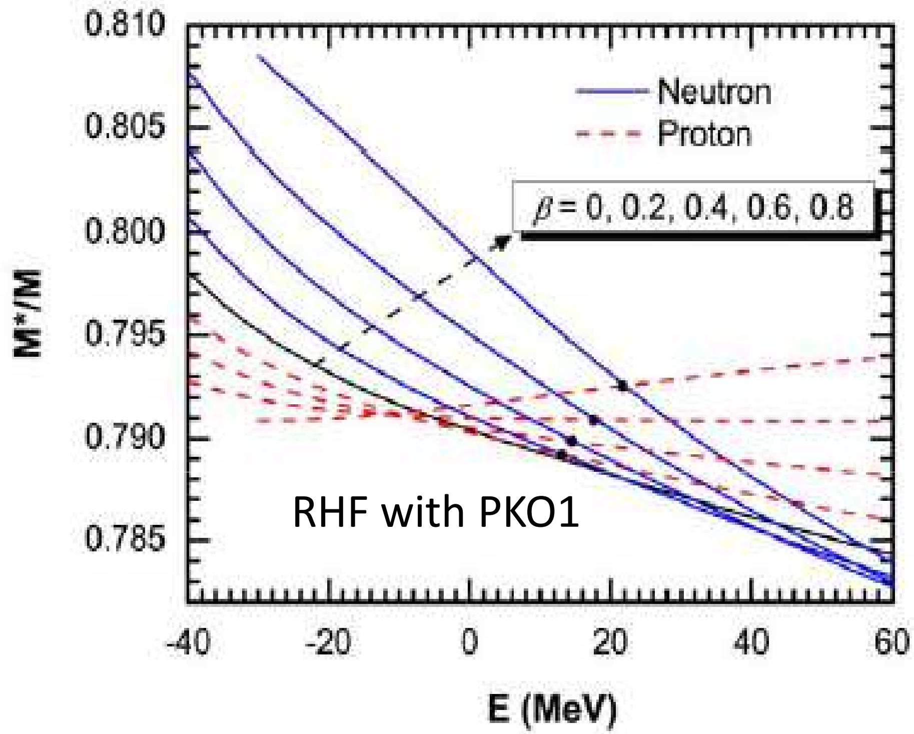


Effective mass splitting at normal density,

$$\frac{m}{m_n^*} - \frac{m}{m_p^*} = \frac{\partial (U_n - U_p)}{\partial E_k}$$



WHLong, N Van Giai, J.Meng, PLB640(2008)150



B.A.Li, C. Xu, et.al., PRC2006,2010,
X.H.Li, PLB743(2015)408)

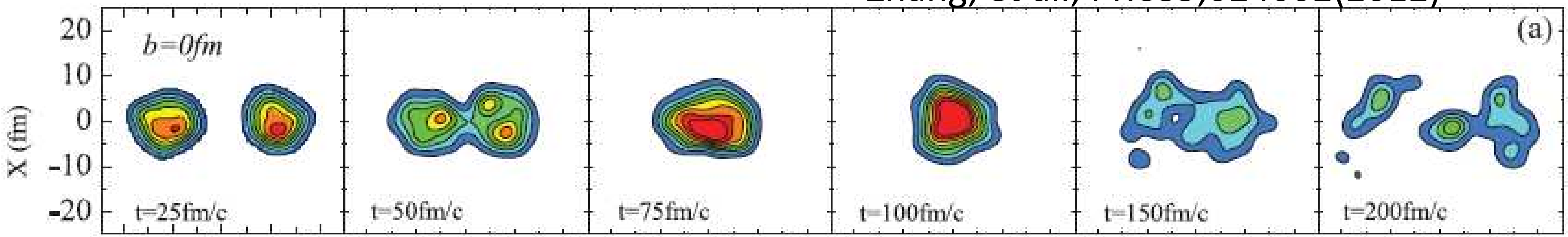
At normal density,

$(m_n^*-m_p^*)/m=0.32\delta$,
$(m_n^*-m_p^*)/m=(0.41+-0.15)\delta$

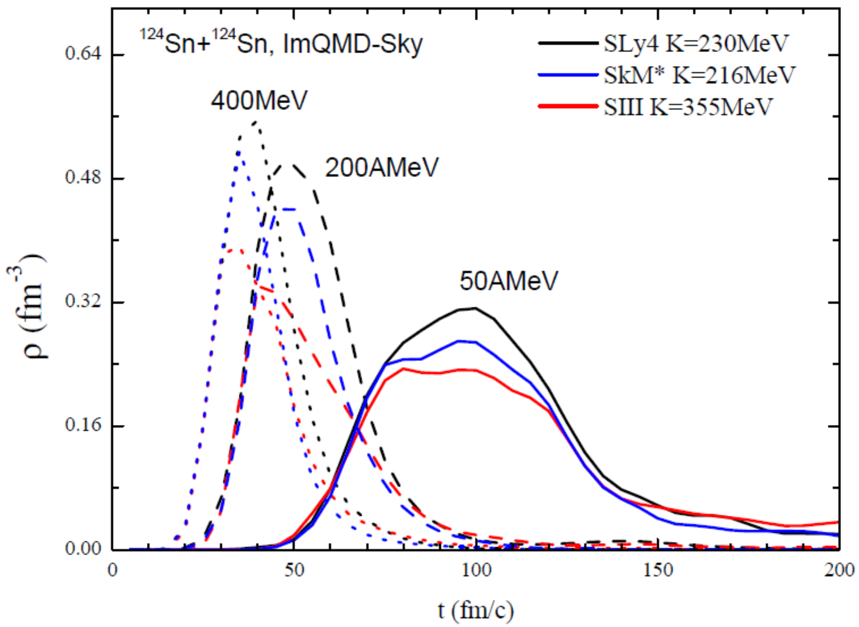
Effective mass splitting depends not only on the in-medium density but also on momentum or energy ??

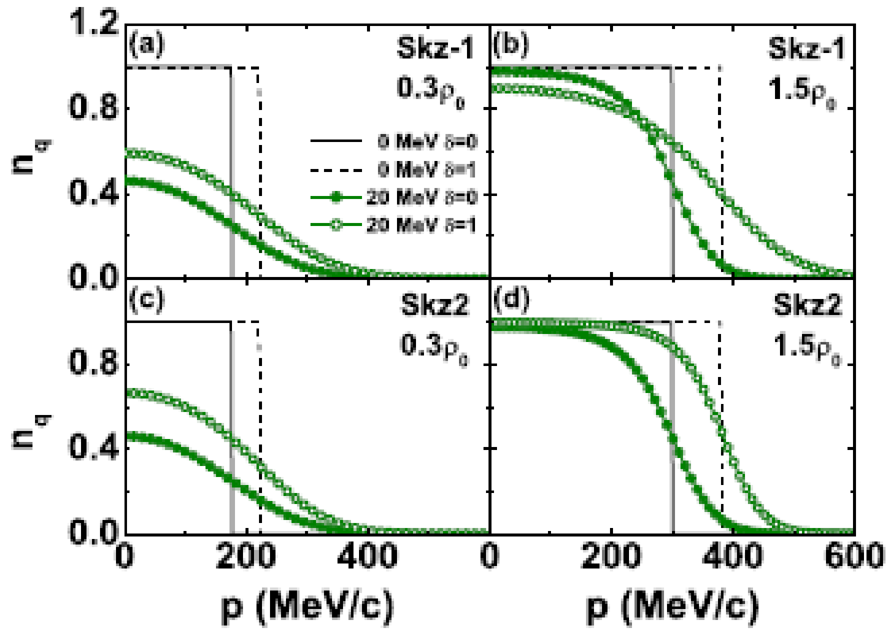
- What happens for n/p effective mass splitting away from the normal density and Fermi momentum ?
    - magnitude or sign of effective mass splitting can be changed with energy?

Zhang, et al., PRC85,024602(2012)

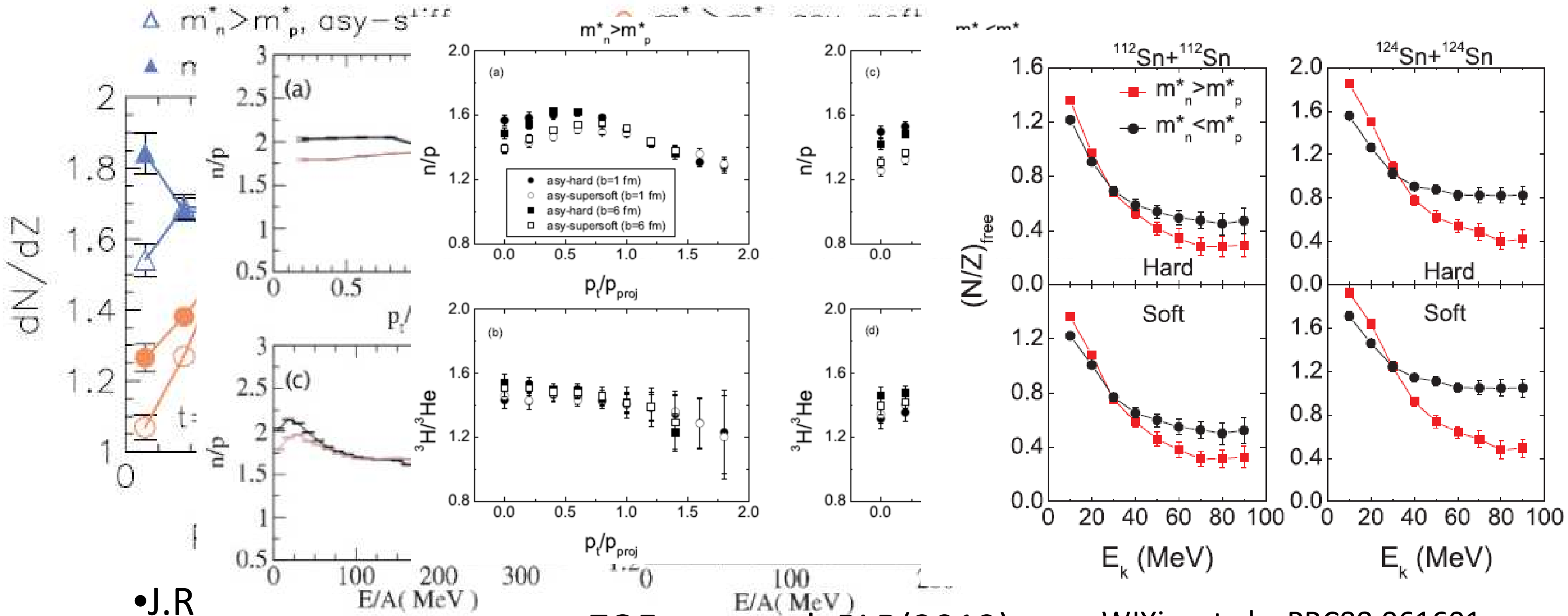


L.Ou, Z.X.Li, Y.X.Zhang, M.Liu, PLB697(2011)





• efforts on probing the momentum dependence of symmetry potential(or n/p effective mass splitting) by HICs



•J.R

ZQFeng, et al, PLB(2012)

WJXie, et al., PRC88,061601
MDI, Log form

V.Giordano, et al., PRC2010

H.H.Wolter

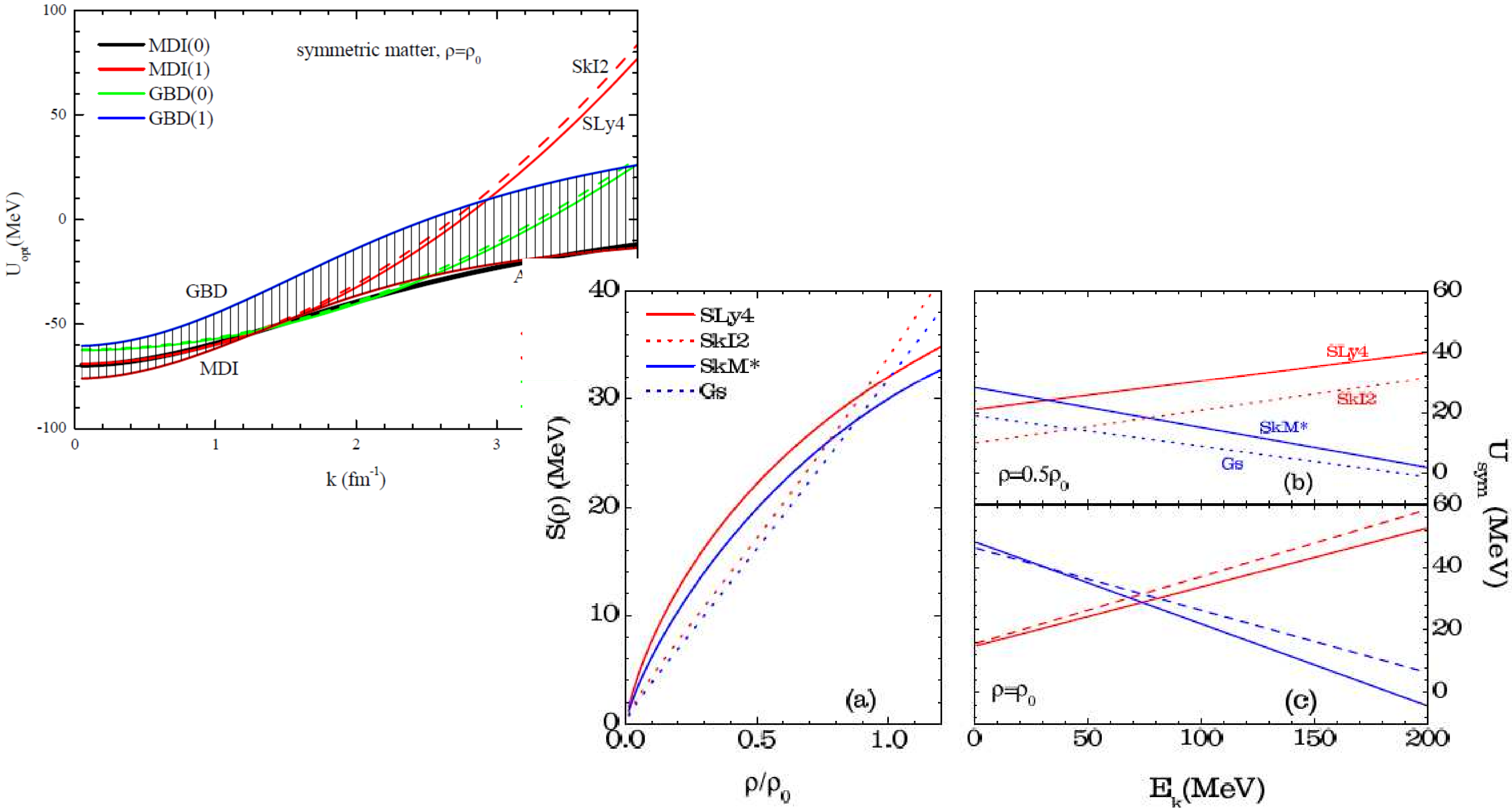
Here, we would like to investigate the influence of symmetry energy and n/p effective mass splitting on heavy ion collisions observables simultaneously by using the transport model calculations with Skyrme type interaction.

1. Simple and contain sufficient physics, widely used in nuclear structure, reaction and astrophysics.

2. In Skyrme EDF, one can easily choose different values L, m*_v for similar K_0 and S_0, m*_s from lot of sets.

3. One could get the constraints on Skyrme parameters, also on symmetry energy and n/p effective mass splitting from reaction data simultaneously.

Skyrme-type MDI



Below ~200MeV, optical potential for symmetric matter can be well approximated by Skyrme type interaction.

In the new version of ImQMD code, nucleons are represented by Gaussian wavepackets and the mean fields acting on these wavepackets are derived from an energy functional with the potential energy U that includes the full Skyrme potential energy without the spin-orbit term:

New version of ImQMD

$$U = U_\rho + U_{md} + U_{coul} \tag{2}$$

and U_{coul} is the Coulomb energy. The nuclear contributions are represented in a local form with

$$U_{\rho,md} = \int u_{\rho,md} d^3r$$

and

$$u_\rho = \frac{\alpha}{2}\frac{\rho^2}{\rho_0} + \frac{\beta}{\eta+1}\frac{\rho^{\eta+1}}{\rho_0^\eta} + \frac{g_{sur}}{2\rho_0}(\nabla\rho)^2 + \frac{g_{sur,iso}}{\rho_0}[\nabla(\rho_n - \rho_p)]^2 + A_{sym}\rho^2\delta^2 + B_{sym}\rho^{\eta+1}\delta^2$$

Y.X. Zhang, M.B.Tsang, Z.X. Li, HLiu, PLB(2014)

$$u_{\rm md} = \frac{1}{2\rho_0}\sum_{N_1,N_2={\rm n,p}}\frac{1}{16\pi^6}\int {\rm d}^3p_1{\rm d}^3p_2 f_{N_1}(\mathbf{p}_1)f_{N_2}(\mathbf{p}_2) \times 1.57\,[\ln(1.+5.\times10^{-4}(\Delta p)^2)]^2, \tag{4}$$

$$\delta(r_1 - r_2)(p_1 - p_2)^2$$

Changes:

$$\begin{aligned} u_{md} &= u_{md}(\rho\tau) + u_{md}(\rho_n\tau_n) + u_{md}(\rho_p\tau_p) \\ &= C_0\int d\vec{p}\, d\vec{p}'\, f(\vec{r},\vec{p})f(\vec{r},\vec{p}')(\vec{p}-\vec{p}')^2 \\ &\quad + D_0\left[\int d\vec{p}\, d\vec{p}'\, f_n(\vec{r},\vec{p})f_n(\vec{r},\vec{p}')(\vec{p}-\vec{p}')^2 + \int d\vec{p}\, d\vec{p}'\, f_p(\vec{r},\vec{p})f_p(\vec{r},\vec{p}')(\vec{p}-\vec{p}')^2\right] \end{aligned} \tag{5}$$

Select four parameter sets

*K_0 = 230 ± 20MeV, S_0 = 32 ± 2MeV, m*_s=0.7+/-0.1*
different L and n/p effective mass splitting.

Para.	Rho_0	E_0	K_0	Q_0	J	L	K_{sym}	m*_s	m*_n/m*_p
SLy4	0.16	-15.97	229.91	363.11	32	46	-120	0.69	<1
SkI2	0.158	-15.78	240.93	339.70	33	104	71	0.68	<1
SkM*	0.16	-15.77	216.61	386.09	30	46	-156	0.79	>1
Gs	0.158	-15.59	237.29	348.79	31	93	14	0.78	>1

	Small L	**Large L**
m_n*<m_p*	SLy4 (L=46MeV)	SkI2 (L=104MeV)
m_n*>m_p*	SkM* (L=46MeV)	Gs (L=93MeV)

• Isospin diffusion and isospin transport ratios as a function of rapidity

A=124Sn, B=112Sn

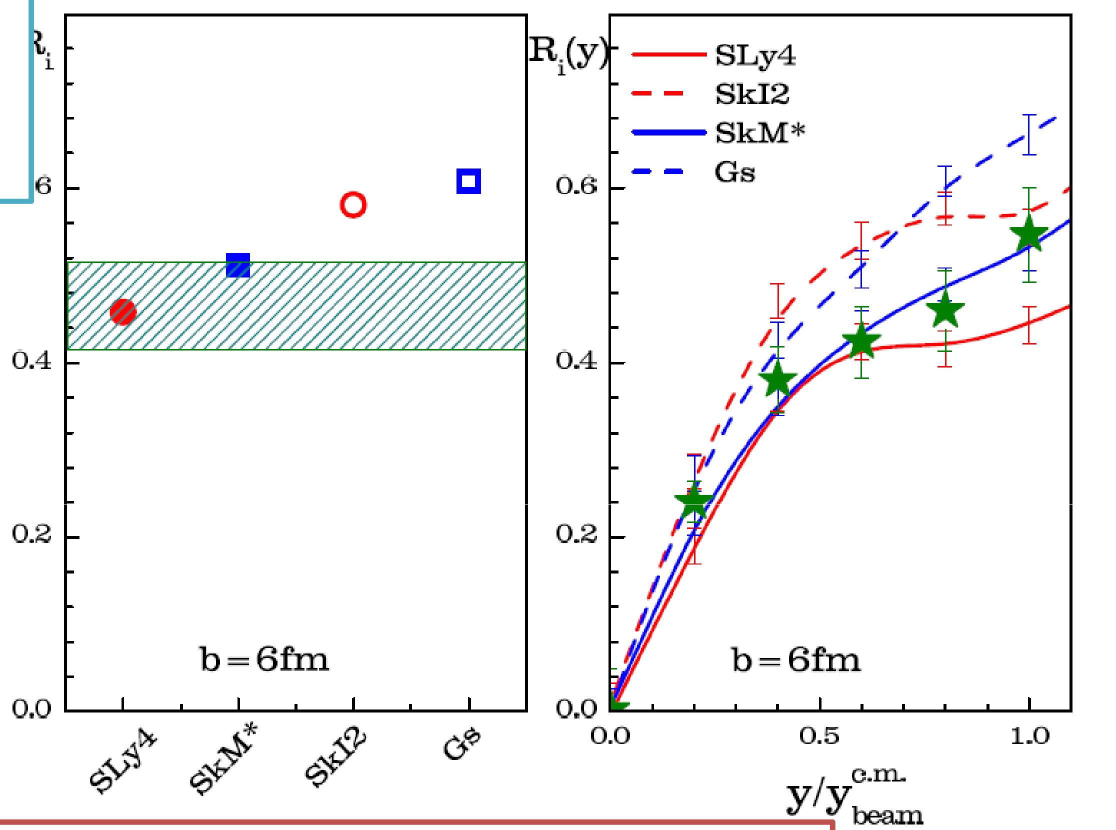
Isospin diffusion occurs only in asymmetric systems A+B, and diffusion ability depends on the symmetry energy and n/p effective mass splitting.

For $m^*_n<m^*_p$, the isospin diffusion process is accelerated due to larger Lane potential at subsaturation density.

$R_i=(2X-X_{AA}-X_{BB})/(X_{AA}-X_{BB})$

In absence of isospin diffusion R=1 or R=-1,
R~0 for isospin equilibrium

Zhang, Tsang, Li, Liu,, PLB732,186(2014)



Ri(SLy4, L=46MeV, m*_n<m*_p)<Ri(SkM*, L=46MeV, m*_n>m*_p) <
Ri(SkI2, L=104MeV, m*_n<m*_p)<Ri(Gs, L=93MeV, m*_n>m*_p)

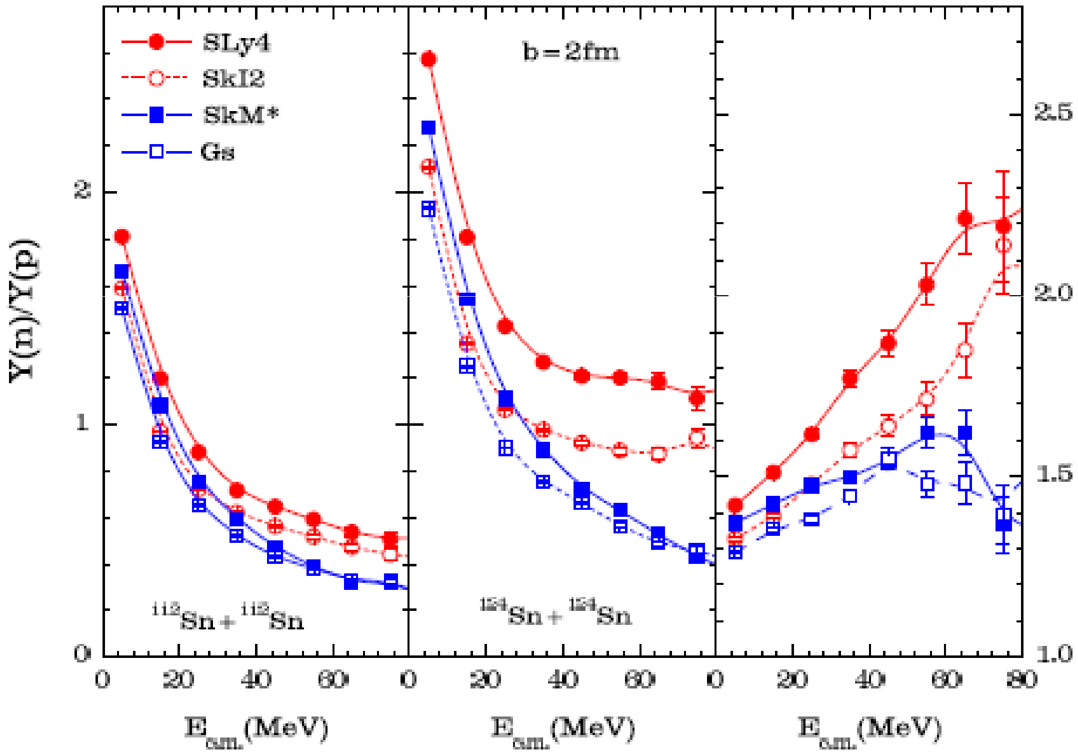
R_i is more sensitive to L than n/p effective mass splitting.

- n/p and DR(n/p) ratios as a function of kinetic energy

$$R_{n/p} = Y(n)/Y(p) = \frac{dM_n / dE_k}{dM_p / dE_k}$$

DR(n/p)=$R_{n/p}$(124)/$R_{n/p}$(112)

50AMeV, b=2fm



- The Larger the L is, the smaller the n/p ratio is.
- $m^*_n<m^*_p$ enhance the Y(n)/Y(p) ratios at higher kinetic energy region.
- DR(n/p) ratios are sensitive to the n/p effective mass splitting.

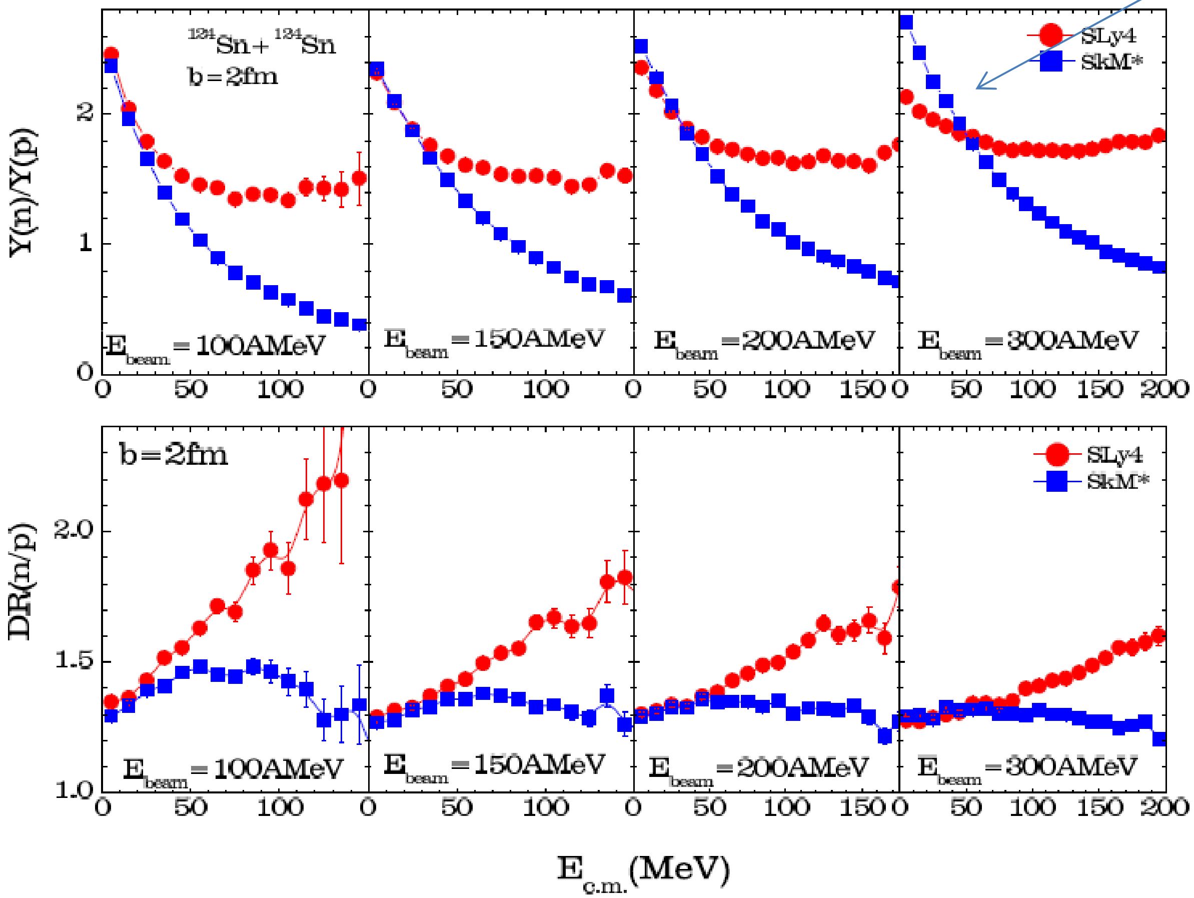
Y(n)/Y(p) and DR(n/p) are more sensitive to n/p effective mass splitting than L

n/p and DR(n/p) at different beam energy

SLy4 (S_0=32MeV, L=46MeV, $m^*_n < m^*_p$)

SkM*(S_0=30MeV, L=46MeV, $m^*_n > m^*_p$)

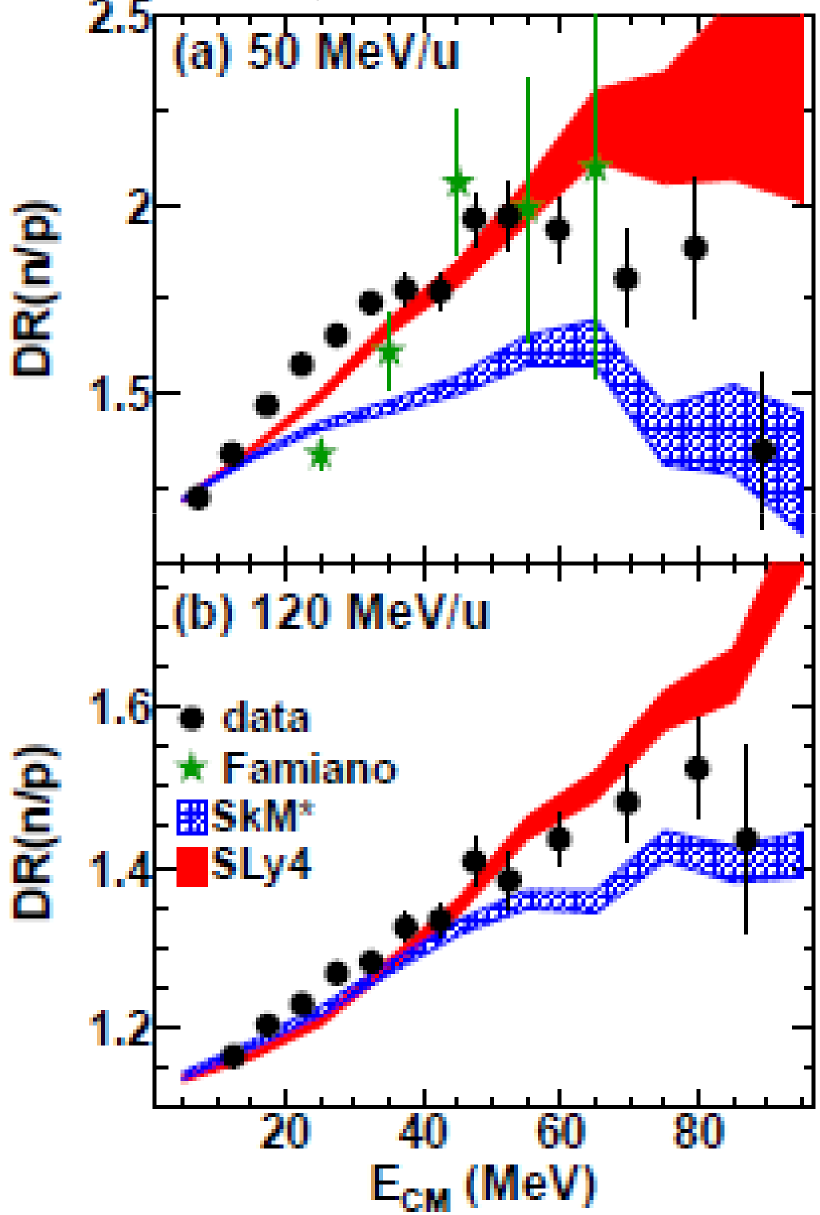
Cross over



- the Y(n)/Y(p) obtained with $m^*_n > m^*_p$ are greater than that with $m^*_n < m^*_p$ cases at lower beam energy for higher beam energy.

Theoretical predictions and New data on coalescence invariant DR(n/p)

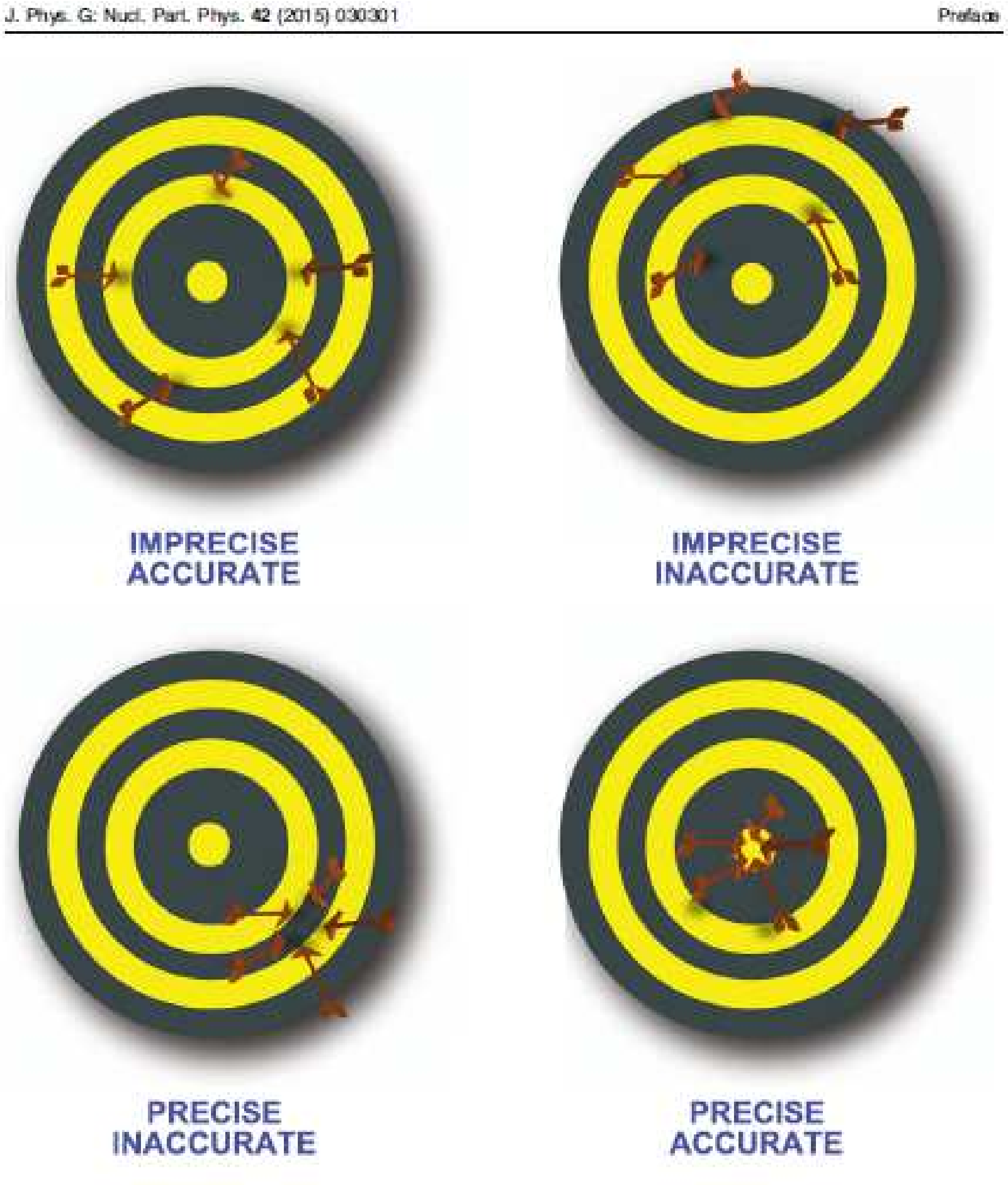
D.D.S.Coupland, et al., arXiv:1406.4546



- New data seems to favor small effective mass splitting at high momentum

- need more calculations with different effective mass splitting to understand this difference.

• Covariance analysis



Precise:

how near the model is to reality?

Accurate:

how well known are model para?

Precise-accurate:

1. What observables are best for constraining the interested physical quantities?
2. How can we estimate the uncertainties from statistical and systematic?
3. How to verify the model extrapolations ?
4. ……

D.G.Ireland, W.Nazarewicz, JPG:Nucl.Part.Phys.42(2015)030301..

input variables for ImQMD, similar relation as in MSL

L.W.Chen, et al., PRC82, 024321(2010)

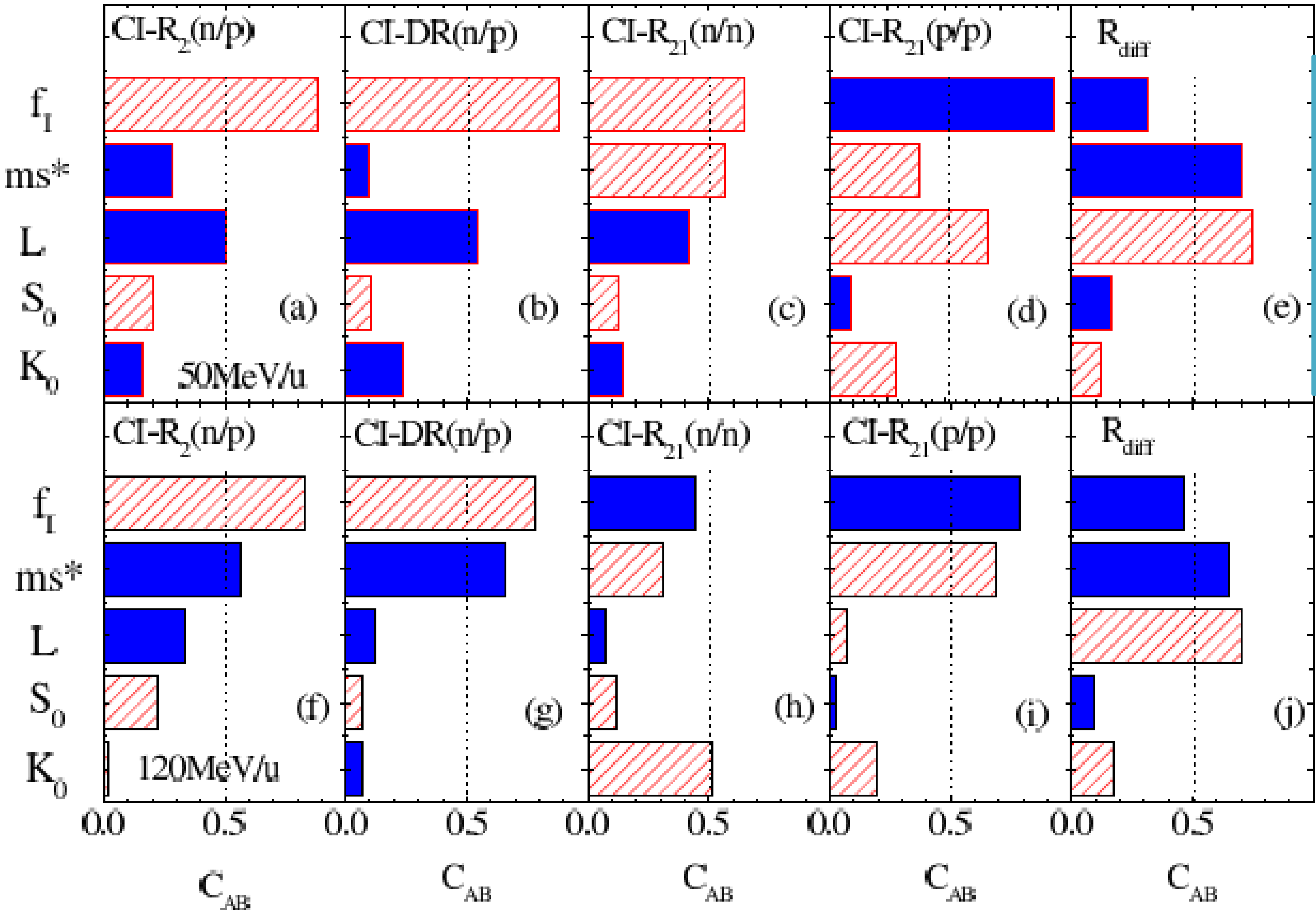
$$\{\alpha,\ \beta,\ \eta,\ A_{sym},\ B_{sym},\ C_0,\ D_0\} \Longleftarrow \{\rho_0,\ E_0,\ K_0,\ S_0,\ L,\ m_s^*,\ m_v^*\}$$

Table 1: List of twelve parameters used in the ImQMD calculations. $\rho_0 = 0.16 fm^{-3}$, $E_0 = -16MeV$, and $g_{sur} = 24.5MeV fm^2$, $g_{sur,iso} = -4.99MeV fm^2$

Para.	K_0 (MeV)	S_0(MeV)	L (MeV)	m_s^*/m	f_I
1	**230**	**32**	**46**	**0.7**	**-0.238**
2	**280**	32	46	0.7	-0.238
3	**330**	32	46	0.7	-0.238
4	230	**30**	46	0.7	-0.238
5	230	**34**	46	0.7	-0.238
6	230	32	**60**	0.7	-0.238
7	230	32	**80**	0.7	-0.238
8	230	32	**100**	0.7	-0.238
9	230	32	46	**0.85**	-0.238
10	230	32	46	**1.00**	-0.238
11	230	32	46	0.7	**0.0**
12(SLy4)	230	32	46	0.7	**0.178**

Covariance analysis from 12 Parameter sets

Y.X.Zhang,M.B.Tsang, Z.X.Li, submitted



Blue: negative correlation

Red: Positive correlation

- **Ms* also play important roles for isospin diffusion, and neutron to proton yield ratio observables at 120MeV/u. We need to pay attention again when we constrain the symmetry energy**

4, Summary and outlook

1, Developed a new version of ImQMD which can accommodate the Standard Skyrme interaction in parameters. It can bridge the reaction and structure study by using same EDF.

2, The Ri and Ri(y) support the SLy4 and SkM* interactions, they have L=46MeV.

3, high energy n/p yield ratio is sensitive to the effective mass splitting (momentum dependent of symmetry potential)

4, Covariance analysis suggest that the influence of isoscalar effective mass should also be seriously considered for further improving the constraints of symmetry energy.

5, combination analysis is required for constraining the symmetry energy by using HICs observables.

Thanks for your attention!