JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Results from the Dyson-Schwinger approach

Gernot Eichmann
University of Giessen, Germany

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Motivation

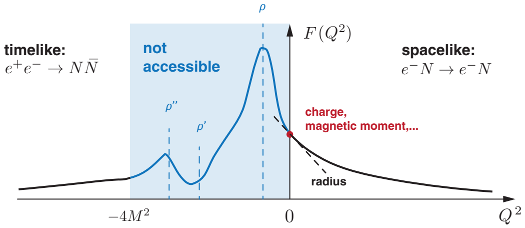
Goal: compute **hadron properties** (ground state & excitations, form factors, scattering amplitudes, etc.) from **quark-gluon substructure in QCD.**

QCD's Green functions $\leftrightarrow$ **"Dyson-Schwinger approach":**
Nonperturbative, covariant, all momentum scales, light and heavy quarks. But: **truncations!**

- **Baryon spectroscopy** from three-body Faddeev equation
  GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)
- **Elastic & transition form factors** for N and Δ
  GE, PRD 84 (2011); GE, Fischer, EPJ A48 (2012); GE, Nicmorus, PRD 85 (2012); ...
- **Tetraquark** interpretation for σ meson
  Heupel, GE, Fischer, PLB 718 (2012)
- Nucleon **Compton scattering**
  GE, Fischer, PRD 85 (2012) & PRD 87 (2013)
- **Muon g-2**
  GE, Fischer, Heupel, Williams, in preparation

Motivation

Sketch of a generic electromagnetic form factor:



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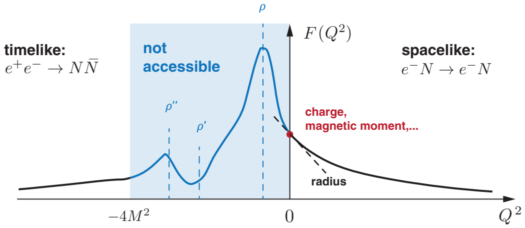
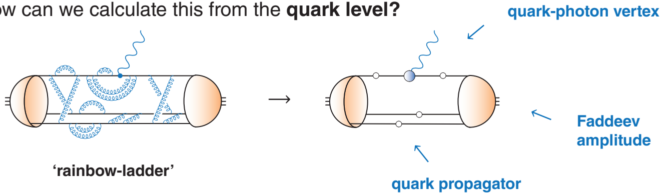


How can we calculate this from the **quark level?**



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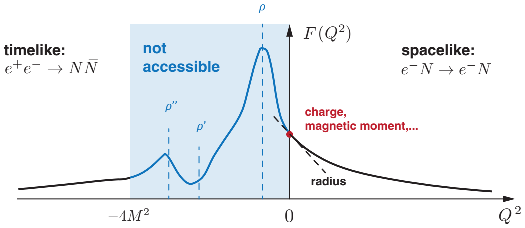

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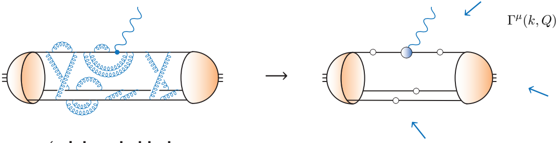


'rainbow-ladder'

quark-photon vertex:

$$\Gamma^\mu(k,Q) = \Big[i\gamma^\mu\,\Sigma_A + 2k^\mu(i\not{k}\,\Delta_A + \Delta_B)\Big] + \Big[i\sum_{j=1}^{8} f_j\,\tau_j^\mu(k,Q)\Big]$$

Faddeev amplitude:

...

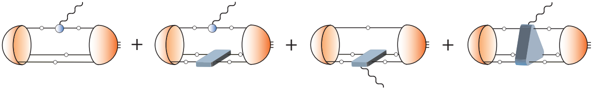
quark propagator:

$$S(p) = \frac{1}{A(p^2)}\,\frac{-i\not{p}+M(p^2)}{p^2+M^2(p^2)}$$

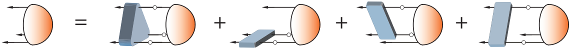
but not yet
em. gauge invariant!

Motivation

Em. gauge invariance requires



in consistency with **baryon's Faddeev equation** (2- and 3-body kernels):



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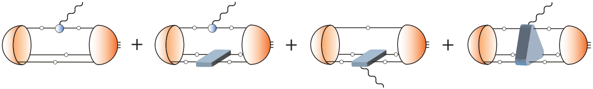

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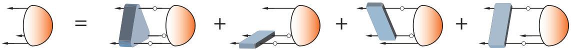
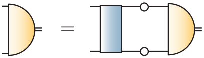
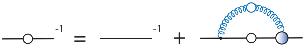


Meson Bethe-Salpeter equation:



Quark Dyson-Schwinger equation:



Kernels and quark propagator **not independent**
(Or we would lose **chiral symmetry!**)

- **PCAC:** massless pion in chiral limit, GMOR
- **vector current conservation,**
  em. gauge invariance (quark-photon vertex!)

⇒ **Symmetries** provide deep connection between quark-gluon and hadron level!

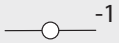
Dyson-Schwinger equations

QCD Lagrangian:
quarks, gluons (+ ghosts)

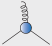
$$\mathcal{L} = \bar{\psi}(x)\,(i\partial\!\!\!/ + g A\!\!\!/ - M)\,\psi(x) - \frac{1}{4}\,F^a_{\mu\nu}\,F^{\mu\nu}_a$$

QCD & hadron properties are encoded in **QCD's Green functions.**
Their quantum equations of motion are the **Dyson-Schwinger equations (DSEs):**

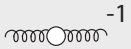
- **Quark propagator:**



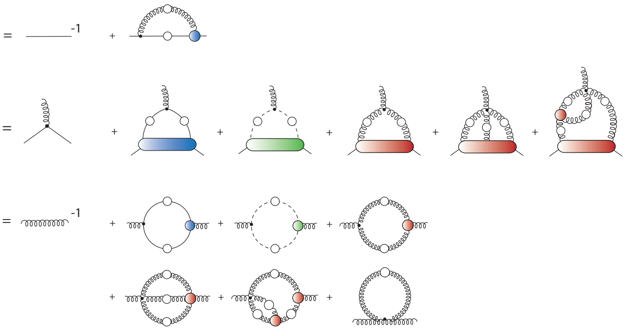
- **Quark-gluon vertex:**



- **Gluon propagator:**



- **Gluon self-interactions, ghosts, . . .**



Dyson-Schwinger equations

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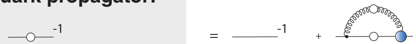

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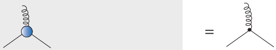

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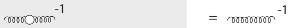


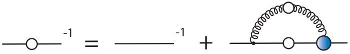
- **Gluon self-interactions, ghosts, . . .**

- **Truncation** ⇒ closed system, solveable.
  Ansätze for Green functions that are **not** solved (based on pQCD, lattice, FRG, ...)

- progress in determining **elementary propagators and vertices**
  Fischer, Maas, Pawlowski, Annals. Phys. 324 (2009)
  GE, Williams, Alkofer, Vujinovic, PRD 89 (2014), . . .

Dynamical quark mass

- **Dynamical chiral symmetry breaking:** generates “constituent-quark masses”
- Realized in **quark Dyson-Schwinger eq:**

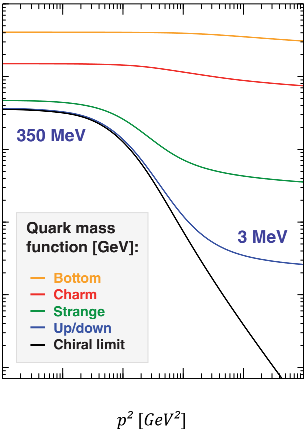


Dressed quark propagator has nonperturbatively enhanced **quark mass function** (DSE, Lattice, ...)

$$S_0(p) = \frac{-i\not{p}+m}{p^2+m^2} \longrightarrow S(p) = \frac{1}{A(p^2)}\,\frac{-i\not{p}+M(p^2)}{p^2+M^2(p^2)}$$

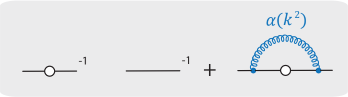
- Mass generation for **light hadrons!**

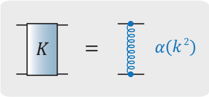
Fischer, J. Phys. G 32 (2006)

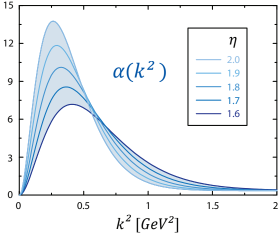


Structure of the kernel

Most hadron studies so far in **rainbow-ladder:**
tree-level vertex + effective coupling







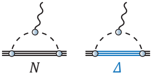
Ansatz for effective coupling:
Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

$$\alpha(k^2) = \alpha_{\mathrm{IR}}\left(k^2/\Lambda^2, \eta\right) + \alpha_{\mathrm{UV}}(k^2)$$

Adjust infrared scale Λ to physical observable,
keep width η as parameter

√ **DCSB, CVC, PCAC**
- ⇒ mass generation
- ⇒ Goldstone theorem, massless pion in χL
- ⇒ em. current conservation
- ⇒ Goldberger-Treiman

No pion cloud,
no flavor dependence,
no $U_A(1)$ anomaly, no
dynamical decay widths



, etc.

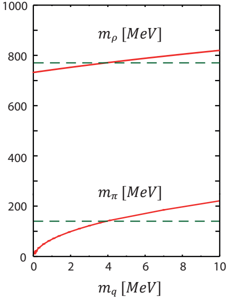
Pion cloud:
need infinite summation
of t-channel gluons

Mesons

- **Pseudoscalar & vector mesons:** rainbow-ladder is good. Masses, form factors, decays, $\pi\pi$ scattering, PDFs

  Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999); Bashir et al., Commun. Theor. Phys. 58 (2012)

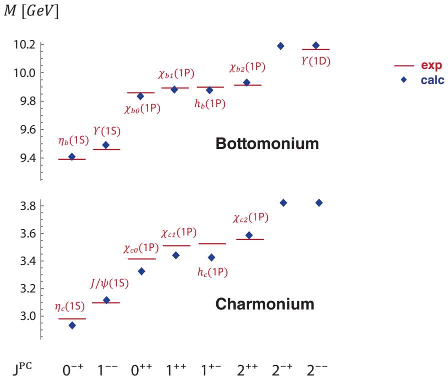
  Pion is Goldstone boson, satisfies GMOR: $m_\pi^2 \sim m_q$



- Need to go **beyond rainbow-ladder** for excited, scalar, axialvector mesons, η-η', etc.

  Fischer, Williams & Chang, Roberts, PRL 103 (2009)
  Alkofer et al., EPJ A38 (2008), Bhagwat et al., PRC 76 (2007)

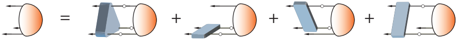
- **Heavy mesons** Blank, Krassnigg, PRD 84 (2011), Fischer et al., 1409.5076



Baryons

- **Baryons** from covariant Faddeev equation:
  full (64/128-dim.) tensor basis: s, p, d, f waves,
  same kernel as for mesons, scale set by f_π,

  GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010), GE, PRD 84 (2011),
  Sanchis-Alepuz, Fischer, 1408.5577



- **Diquark clustering** in baryons:
  similar results in quark-diquark approach

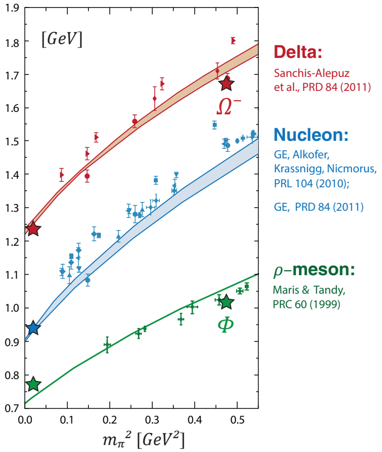
  Oettel, Alkofer, von Smekal, EPJ A8 (2000)
  GE, Cloet, Alkofer, Krassnigg, Roberts, PRC 79 (2009)

- **Excited baryons:** dominance of qq or qqq?

  $N^{+}_{1/2}$ (1440): 1.26 GeV
  $N^{-}_{1/2}$ (1535): 1.00 GeV

  Sanchis-Alepuz et al., preliminary

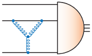
  Need to go beyond rainbow-ladder!



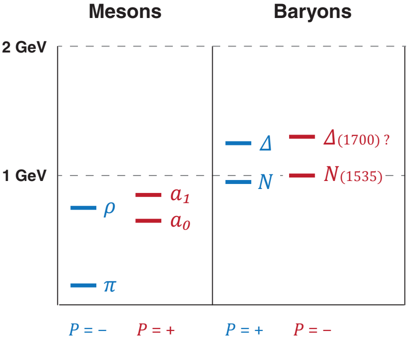
Beyond rainbow-ladder?

- **Rainbow-ladder:**
  "s waves" ok, but "p waves" too low,
  excited states problematic

- **Gluonic corrections**
  (e.g. from 3g vertex):



  - ▹ large repulsive shifts for p-wave mesons
    Fischer, Williams & Chang, Roberts, PRL 103 (2009)
  - ▹ also for baryons? Could be understood from quark-diquark structure ⇒ N* level ordering!
    Chen et al., FBS 53 (2012)



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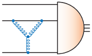

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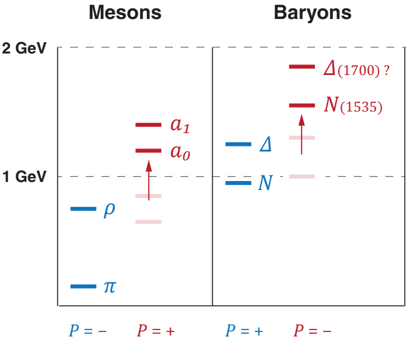

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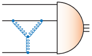

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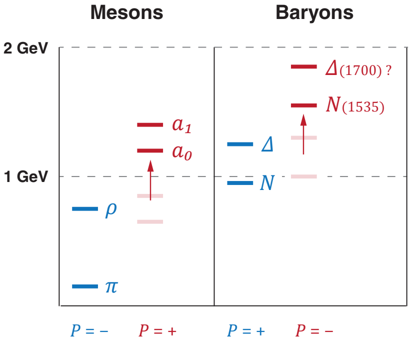
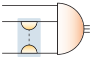
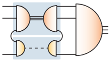
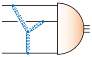


- **Pion cloud, coupled-channel effects?**
  Clearer signals expected for form factors
  Sanchis-Alepuz, Fischer, Kubrak, PLB 733C (2014)





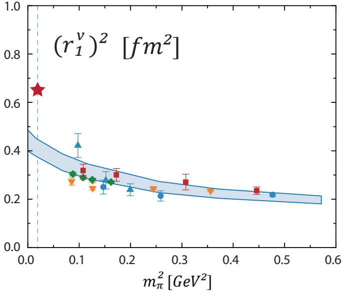
- **3-body forces?** 3-gluon vertex has zero crossing (DSE & lattice!)
  Huber, von Smekal, JHEP 1304 (2013)
  GE, Williams, Alkofer, Vujinovic, PRD 89 (2014)



Electromagnetic form factors

Nucleon charge radii:

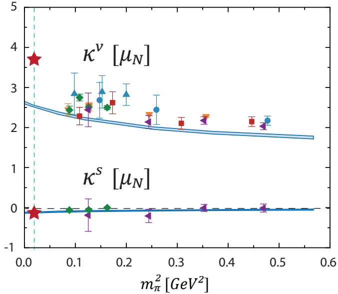
isovector (p-n) Dirac (F1) radius



- **Pion-cloud effects** missing in chiral region (⇒ divergence!), agreement with lattice at larger quark masses.

Nucleon magnetic moments:

isovector (p-n), isoscalar (p+n)



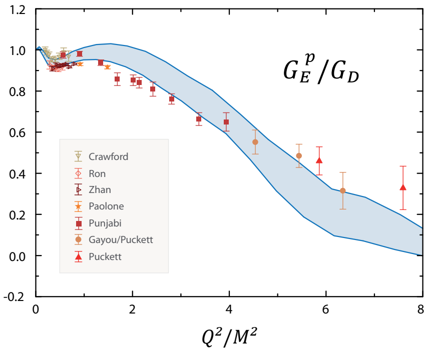
- **But:** pion-cloud **cancels** in κ^s ⇔ **quark core**

Exp: $\kappa^s = -0.12$

Calc: $\kappa^s = -0.12(1)$ **!!**

GE, PRD 84 (2011)

Large Q^2



- Faddeev result consistent with data: **OAM in nucleon amplitude**
- Underway: investigate **two-photon effects** via Compton scattering amplitude

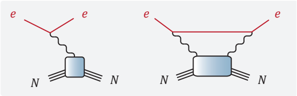
Electric proton form factor

at large momenta GE, PRD 84 (2011)

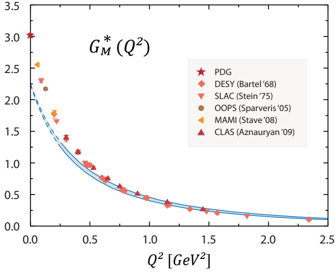
- **Rosenbluth method** suggested $G_E/G_M = \text{const.}$, in agreement with perturbative scaling

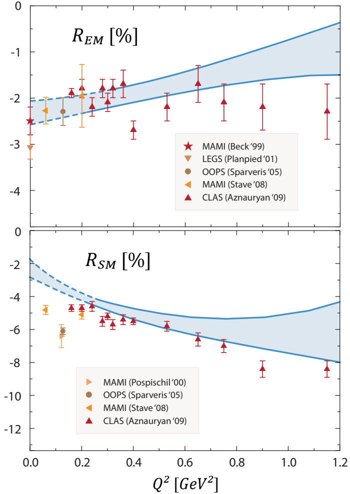
  Polarization experiments at JLAB showed **falloff** in G_E/G_M, with possible **zero crossing**

- Difference likely due to **two-photon corrections**
  Guichon, Vanderhaeghen, PRL 91 (2003)



Nucleon-Δ-γ transition

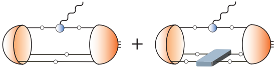


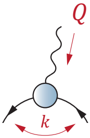


- **Magnetic dipole transition (G_M^*)** dominant: quark spin flip (s wave). "Core + 25% pion cloud"
- **Electric & Coulomb quadrupole transitions** small & negative, encode deformation.

  Ratios reproduced without pion cloud:
  OAM from **relativistic p waves** in the quark core!
  GE & Nicmorus, PRD 85 (2012)

Quark-photon vertex

Current matrix element: $\langle H|J^\mu|H\rangle =$ 



- Structure of **quark-photon vertex:**

$$\Gamma^\mu(k,Q) = \Big[i\gamma^\mu\, \Sigma_A + 2k^\mu (i\not{k}\, \Delta_A + \Delta_B) \Big] + \Big[i \sum_{j=1}^{8} f_j\, \tau_j^\mu(k,Q) \Big]$$

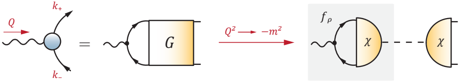
Ball-Chiu vertex,
depends only on
quark propagator
Ball, Chiu, PRD 22 (1980)

necessary for
electromagnetic
gauge invariance!

Transverse part:
vanishes at $Q \to 0$, no kin. singularities;
contains ρ-meson poles &
anomalous magnetic moment
Kizilersu et al, PRD 92 (1995);
GE, Fischer, PRD 87 (2013)

$$Q^\mu\, \Gamma^\mu(k,Q) = S^{-1}(k+\tfrac{Q}{2}) - S^{-1}(k-\tfrac{Q}{2})$$

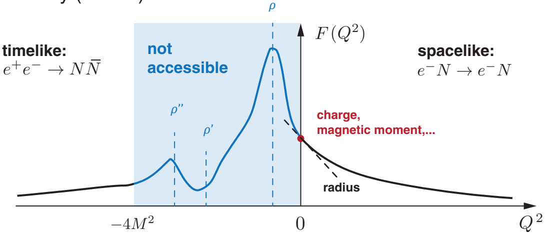
- Calculated from rainbow-ladder **Bethe-Salpeter equation**



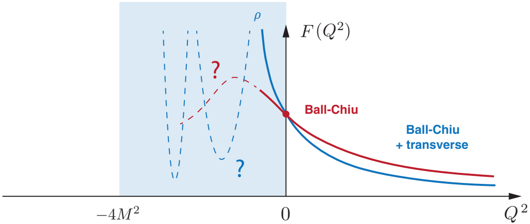
Quark-photon vertex
has ρ-meson poles:
'vector-meson dominance'

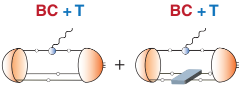
Quark-photon vertex

Structure of quark-photon vertex is reflected in form factors.
Experimentally (sketch):



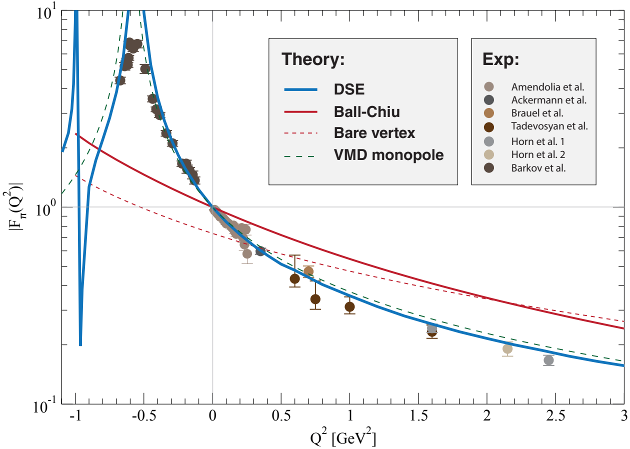
Calculated:
(Sketch)





- Ball-Chiu part is dominant **(em. gauge invariance):** charge, magnetic moments
- Transverse part changes slope and charge radii. No pion cloud in RL ⇒ timelike ρ-**meson poles**

Pion form factor



Spacelike and timelike region:

A. Krassnigg (Schladming 2010)

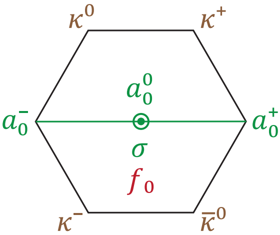
extension of Maris & Tandy, Nucl. Phys. Proc. Suppl. 161 (2006)

Include **pion cloud:**

Kubrak et al., in preparation

Tetraquarks

Light scalar mesons (0^{++}) don't fit into the conventional meson spectrum:



a_0 (980 MeV) } $u\bar{u}, d\bar{d}, u\bar{d}$
σ (500 MeV) }
κ (680 MeV) $u\bar{s}, d\bar{s}$
f_0 (980 MeV) $s\bar{s}$

- Why are σ, κ so **light** compared to a_0, f_0?
  Why are the masses of a_0, f_0 degenerate?

- Why do they have so different **decay widths**?

  $\Gamma(\sigma, \kappa) \approx$ 550 MeV
  $\Gamma(a_0, f_0) \approx$ 50–100 MeV

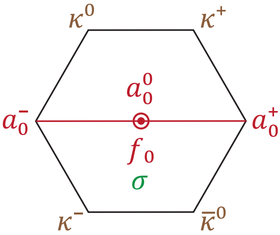
- Why do both f_0 and a_0 couple to $K\bar{K}$?
  (hidden strange-quark content of a_0?)

- Scalar mesons ~ **p-waves,** should have masses similar to axial-vectors: a_1, $f_1 \sim$ 1.3 GeV

- Increasing evidence for non-$q\bar{q}$ nature from dispersive analyses, linear σ model, ...
  Pelaez, Mod. Phys. Lett. A19 (2004) & PoS CD12 (2013),
  Parganlija et al., PRD 87 (2013), . . .

Tetraquarks

Could these be light **tetraquark** (diquark-antidiquark) states? Jaffe '77

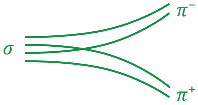


a_0 (980 MeV) $\}$ $us\bar{u}\bar{s}$, ...
f_0 (980 MeV)

κ (680 MeV) $us\bar{u}\bar{d}$, ...

σ (500 MeV) $ud\bar{u}\bar{d}$

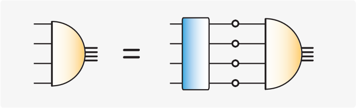
- Explains **mass ordering**:
  f_0 and a_0 have same strangeness content
- Explains **decay widths**:
  f_0 and a_0 decay into K$\overline{\text{K}}$;
  "OZI-superallowed" mechanism
  leads to large widths for σ, κ :



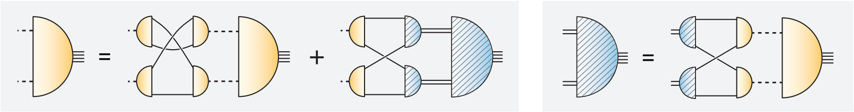
- Actual **scalar** $q\bar{q}$ **ground states** would be
  "1st radially excited" nonet $\sim$ 1.3–1.5 GeV

Tetraquarks

Start from **four-quark bound-state equation:**



Keep only qq, $q\bar{q}$ interactions with separable T-matrix.
Obtain coupled **diquark-antidiquark / meson-meson** equations:
Heupel, GE, Fischer, PLB 718 (2012)



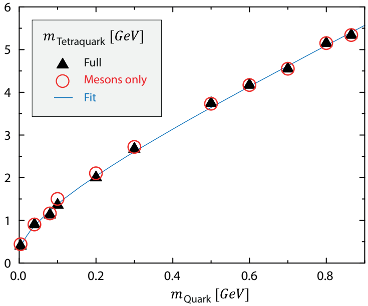
So far:
- 0^{++}, isoscalar, 4 identical quarks: $nn\bar{n}\bar{n}$, $ss\bar{s}\bar{s}$, $cc\bar{c}\bar{c}$, ...
- keep only pseudoscalar meson and scalar diquark

⇒ **meson molecule** with **diquark-antidiquark admixture**

Tetraquarks

Tetraquark masses:
Heupel, GE, Fischer, PLB 718 (2012)

- up/down: $m \sim 400$ MeV
  Light tetraquark because
  it carries traces of the **pion**!
  $\Leftrightarrow$ exp. σ / f_0 (500)?
- strange: $m \sim 1.2$ GeV
- charm: $m \sim 5.3$ GeV



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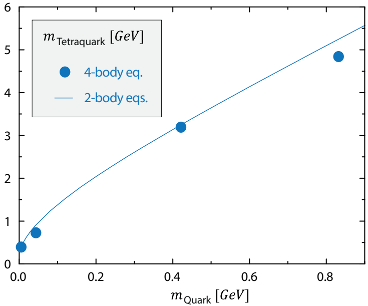
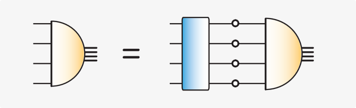


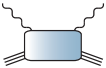
First results from genuine **four-body equation**
(rainbow-ladder & s waves only) Heupel, GE, Fischer, in preparation



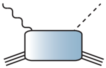
- No explicit pions and diquarks here,
  but **results almost identical!**
- No mixing with $q\bar{q}$ yet: "pure" tetraquark
- σ: 400 MeV, κ: 601 MeV, a_0, f_0: 785 MeV

Hadron scattering

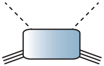
Can we extend this to **four-body scattering** processes?
GE, Fischer, PRD 85 (2012)



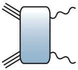
Compton scattering, DVCS, 2γ physics



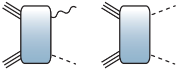
Meson photo- and electroproduction



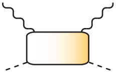
Nucleon-pion scattering



$\bar{p}p \to \gamma\gamma^*$
annihilation



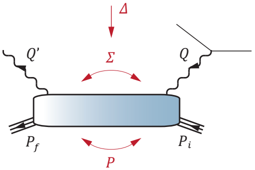
Meson production



Pion Compton scattering

$\Rightarrow$ Nonperturbative description of hadron-photon and hadron-meson scattering

Compton scattering

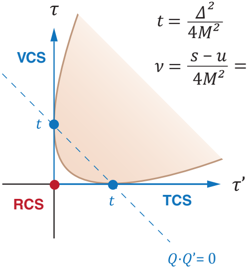


$$\tau = \frac{Q^2}{4M^2}$$

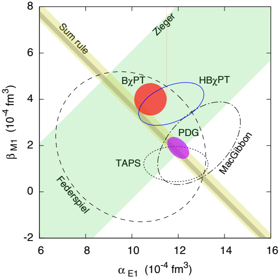
$$\tau' = \frac{Q'^2}{4M^2}$$

$$t = \frac{\Delta^2}{4M^2}$$

$$\nu = \frac{s-u}{4M^2} = -\frac{\Sigma\cdot P}{M^2}$$



- **RCS, VCS:** nucleon polarizabilities

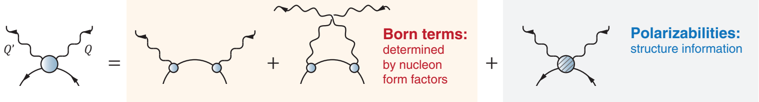


Krupina & Pascalutsa, PRL 110 (2013)

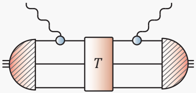
- **DVCS:** handbag dominance, GPDs
- **Forward limit:** structure functions in DIS
- **Timelike region:** $p\bar{p}$ annhihilation at PANDA
- **Spacelike region:** two-photon corrections to nucleon form factors, proton radius puzzle?

Compton scattering

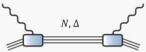
Nucleon Compton scattering amplitude at **hadron level:**

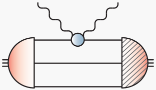


and at **quark level** (rainbow-ladder, modulo crossing & permutation): GE, Fischer, PRD 85 (2012)



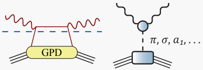
s- and u-channel
nucleon resonances:

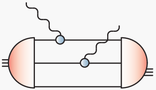




Quark Compton vertex:

handbag diagrams +
t-channel **meson poles**





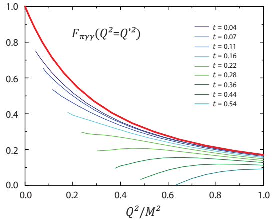
cat's ears diagrams

- √ crossing symmetry
- √ em. gauge invariance
- √ perturbative processes included
- √ s, t, u channel poles generated in QCD

Compton scattering

Quark Compton vertex:

⇒ **handbag diagrams** + all **t-channel poles** (scalar, pion, ...).
$\pi\gamma\gamma$ **transition form factor** from residue at pion pole:
GE & Fischer, PRD 87 (2013) - Direct rainbow-ladder calculation: Maris, Tandy, PRC 65 (2002)

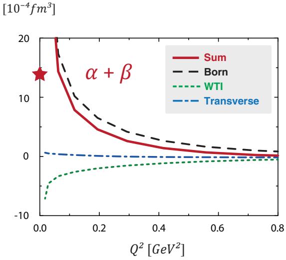


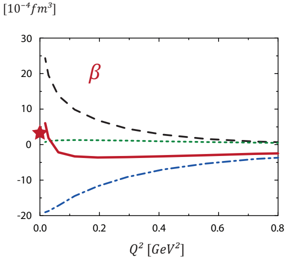
⇒ **not em. gauge invariant**, but comparable to ‚structure part‘ at nucleon level? **Need tensor basis** free of kin. singularities!
Tarrach, Nuovo Cim. 28 (1975), GE & Fischer, PRD 87 (2013) & PoS. Conf. X (2012)

⇒ **nucleon polarizabilities** (‘naive‘ extraction):

$\alpha + \beta$: dominated by **handbag**

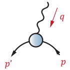
β: cancellation between **handbag** and **t-channel poles?**





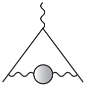
Muon g-2

- **Muon anomalous magnetic moment:**
  total SM prediction deviates from exp. by ~3σ

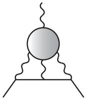


$$= ie\,\bar{u}(p')\left[F_1(q^2)\,\gamma^\mu - F_2(q^2)\,\frac{\sigma^{\mu\nu}q_\nu}{2m}\right]u(p)$$

- Theory uncertainty dominated by **QCD:**
  Is QCD contribution under control?



Hadronic vacuum polarization



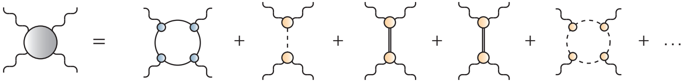
Hadronic light-by-light scattering

$a_\mu\ [10^{-10}]$

Jegerlehner, Nyffeler, Phys. Rept. 477 (2009)

Exp:	11 659 208.9	(6.3)	
QED:	11 658 471.9	(0.0)	
EW:	15.3	(0.2)	
Hadronic:			
• VP (LO+HO)	685.1	(4.3)	
• **LBL**	**10.5**	**(2.6)**	**?**
SM:	11 659 182.8	(4.9)	
Diff:	26.1	(8.0)	

- **LbL amplitude:** ENJL & MD model results
  Bijnens 1995, Hakayawa 1995, Knecht 2002, Melnikov 2004, Prades 2009, Jegerlehner 2009, Pauk 2014



Quark loop	pseudoscalar exchange	scalar exchange	axialvector exchange	π, K loop	
2	8 ... 11	-1	2	-2	$(\times 10^{-10})$

Muon g-2

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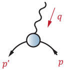
  total SM prediction deviates from exp. by ~3σ



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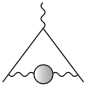

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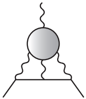

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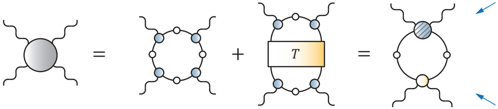
- **LbL amplitude** at quark level, derived from **gauge invariance:**
  GE, Fischer, PRD 85 (2012), Goecke, Fischer, Williams, PRD 87 (2013)



quark Compton vertex

Born terms

- **no double-counting, gauge invariant!**
- need to understand **structure of amplitude**
  GE et al., in preparation

Summary

- Progress in calculating hadron properties from the **quark-gluon level**

- **Rainbow-ladder** ok for meson (0-, 1-, ...) and baryon (1/2+, 3/2+) **ground states:** masses, electromagnetic properties

- Interesting physics encoded in **higher n-point functions:** Compton scattering, light-by-light amplitude, ...

- Progress in going **beyond rainbow-ladder** (necessary for 'p waves', excited states, etc.)

Interplay between experiment and theory:

- Hadron **masses, wave functions, form factors and scattering amplitudes from QCD**

- Refined tools for understanding fundamental **properties of QCD from experiment**