Lectures & objectives

ISAPP 2014
(Belgirate)
21-30 July 2014

Transport of cosmic rays in the Galaxy and in the heliosphere (~ 4h30)

- What is GCR (Galactic Cosmic Ray) physics and transport
- Relevant time scales: ≠ species have ≠ phenomenology
- Main modelling ingredients: key parameters and uncertainties
- Tools to solve the transport equation

Charged signals: electrons/positrons, antibaryons (~1h30)

- What is astroparticle physics and DM (Dark Matter) indirect detection
- What are the astrophysical backgrounds + uncertainties [nuclear]
- Phenomenology of DM signals + uncertainties [transport and dark matter]
- Pros and Cons of DM indirect detection with charged GCRs

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Transport of cosmic rays (CR) in the Galaxy

I. Introduction; Galactic Cosmic Rays
1. Early history of CRs: discovery and disputes
2. GCR journey (from source to detector)
3. Timeline
4. Observables and questions

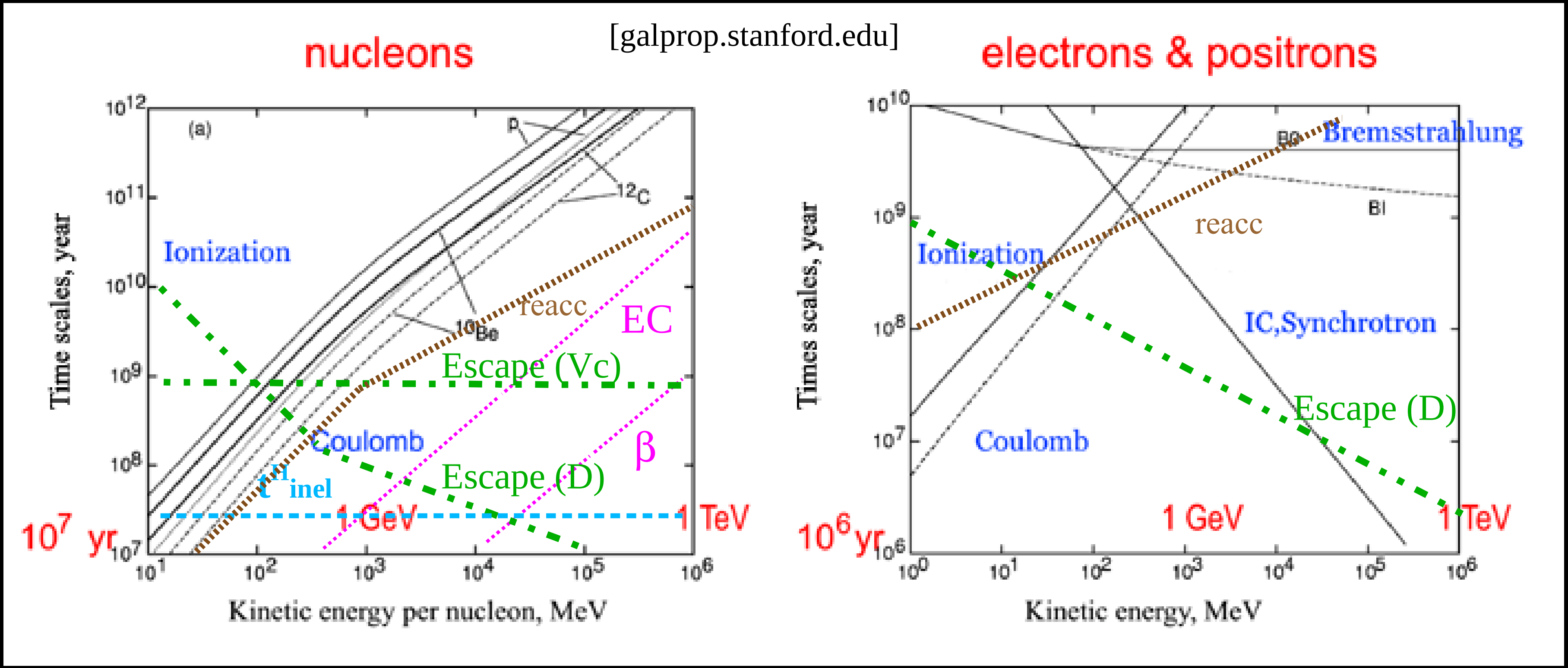
II. Processes, ingredients, characteristic times
1. Definitions
2. Diffusion (space and momentum)
3. Convection and adiabatic losses
4. Energy losses (continuous)
5. Catastrophic losses
6. All together

GCRs-II.pdf

III. Solving the equations: GCR phenomenology
1. From microphysics to effective models
2. Full set of equations (with source terms)
3 (Semi-)Analytical, numerical, & MC solutions
4. Stable species: degeneracy D_0 /L
5. Radioactive species and local ISM
6. Leptons and local sources

Time scales: all together

→ Numbers depend on MW model parameters (halo size, diffusion coefficient...)
→ Time scale for effects in the disc overestimated: CRs see density $n_{ISM} \geq \langle n \rangle \geq (h/H)\, n_{ISM}$



1. Dominant effects
   - Nuclei escape from the Galaxy
   - Leptons loose their energy

2. Local origin
   - Low energy radioactive nuclei
   - High energy electrons and positrons

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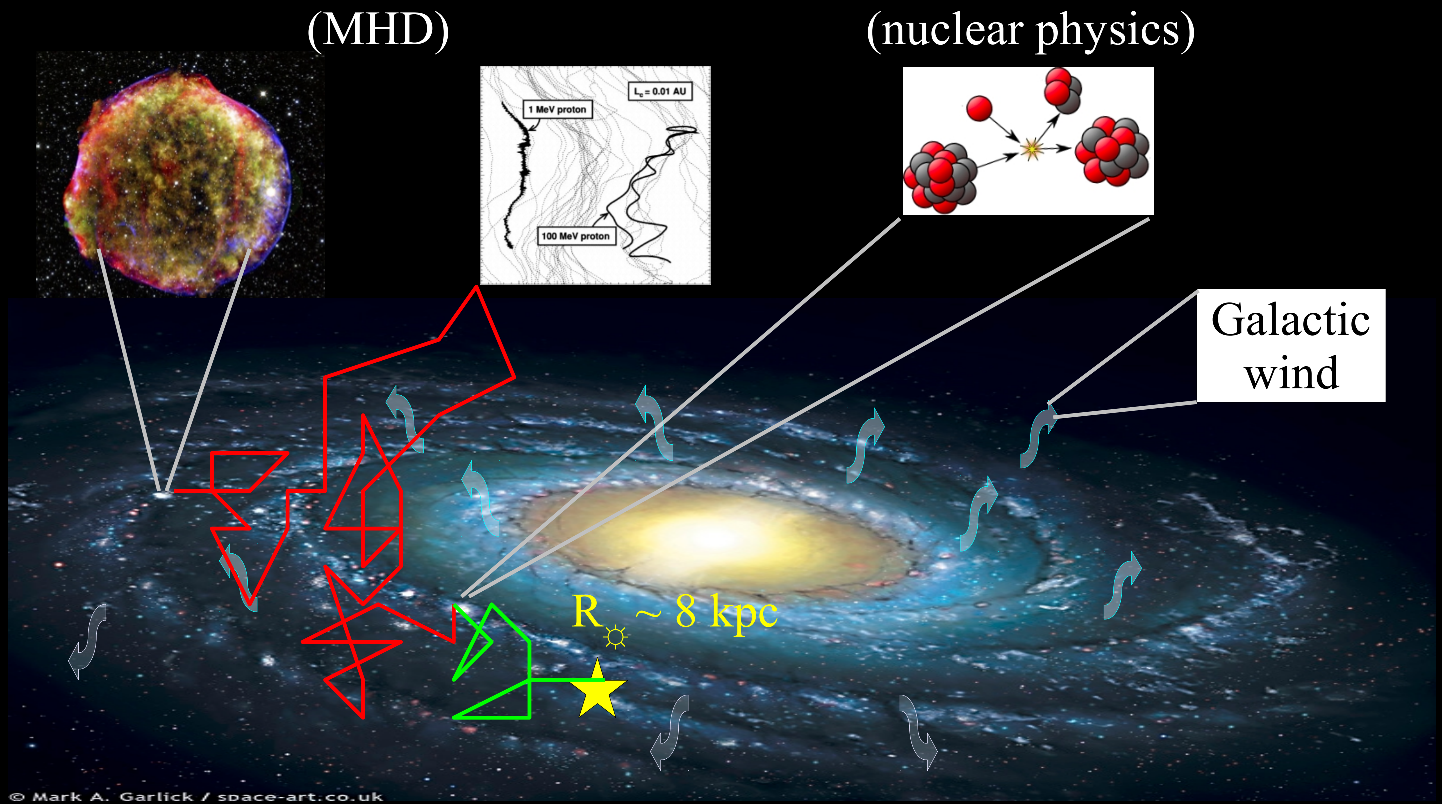
Charged cosmic rays in the Galaxy: reminder

1. Source injection
- spectrum ~ R^{-2}
- abundances

2. Transport in the Galaxy
- diffusion: $R^{-\delta}$
- convection
- energy gains/losses
- fragmentation/decay



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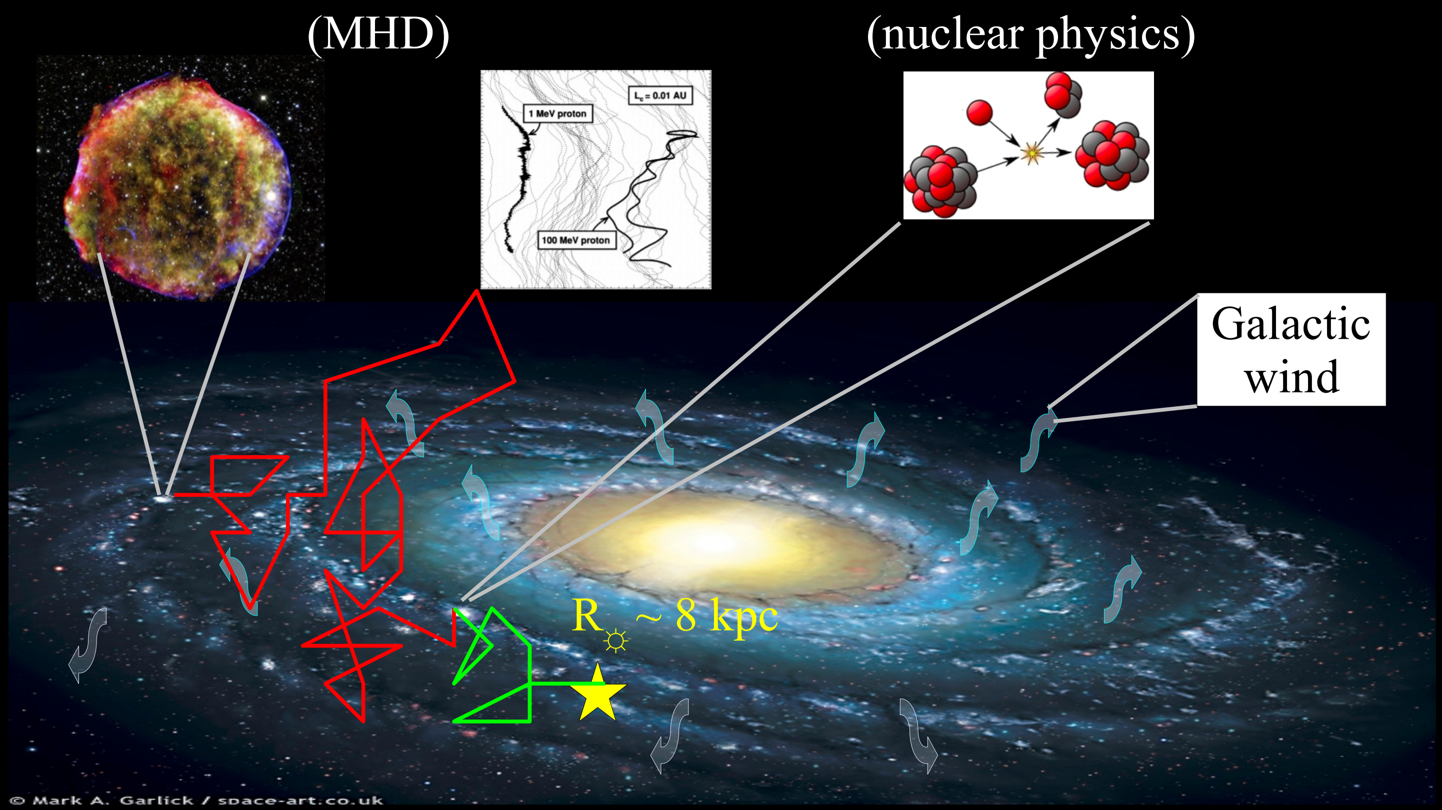

Do we understand the "standard" fluxes (everywhere and anytime)?
- Sources (SN, pulsars, ...)
- Nucleosynthesis (r and s-process for heavy nuclei)
- Acceleration mechanisms (injection, B amplification, E_{max})
- Propagation mechanisms (turbulence, spatial dependence, isotropy)
- Magneto-cosmico-gaseo properties of the Galaxy (MHD description)

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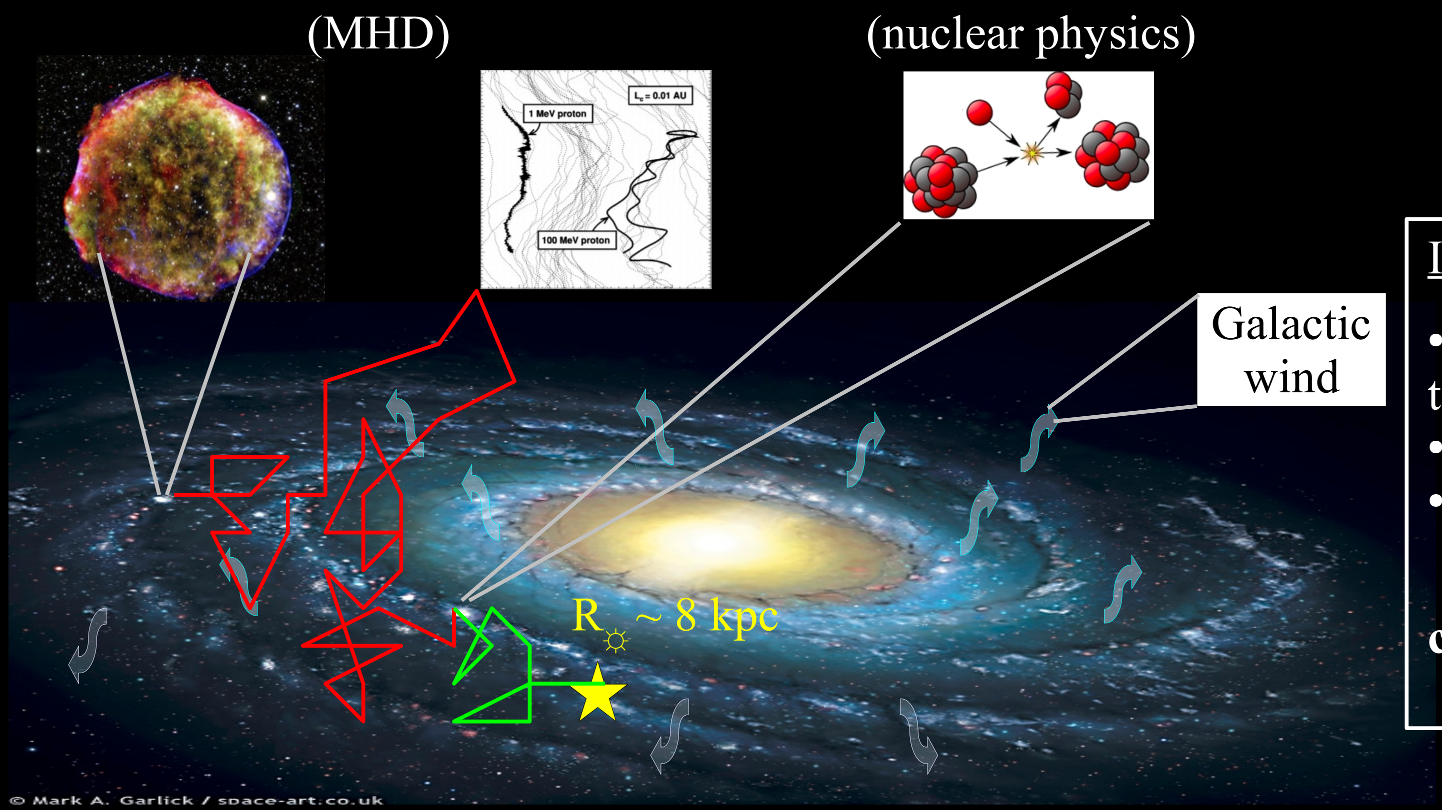

<u>In this lecture</u> (simpler task):

- Transport equations (all terms, all species)
- Set geometry/ingredients
- Solve the equations

→ Which GCR data constrain which parameters of the model?

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$\kappa_{||}$: Diffusion *along*[2] ***B***

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Analytical calculation

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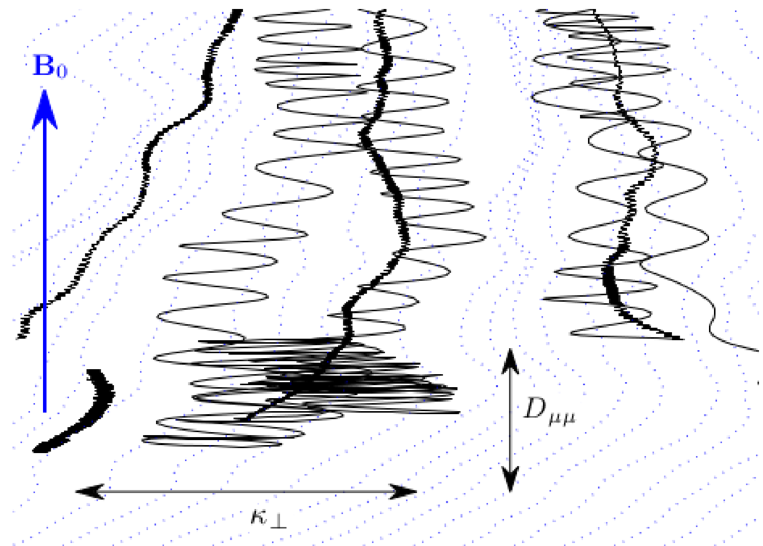

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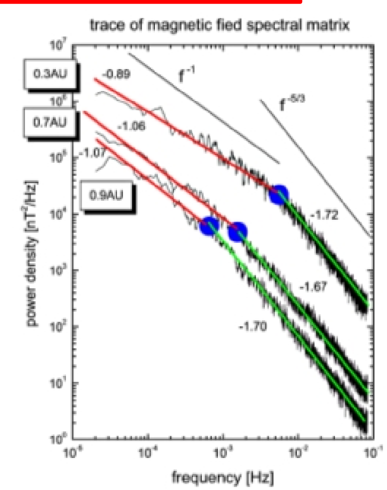
OR

Direct measurement

$$\langle \delta B_j(\boldsymbol{k}) \, \delta B_n(\boldsymbol{k}') \rangle$$

→ Turbulence is Kolmogorov for the Sun



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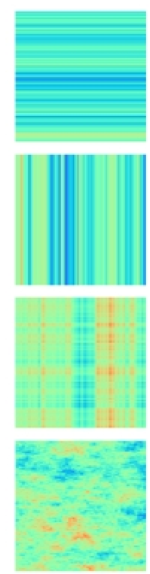

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(N.B.: also inputs of analytical calculations)

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[Adapted from R. Tautz (CRISM 2014)]

- **Physics problem:** motion in a turbulent field
- **Ansatz:** diffusion equation $\frac{\partial f}{\partial t} - S = \nabla \cdot \left(\kappa_{nj} \cdot \nabla f - \mathbf{v}\, f \right) + \frac{\partial}{\partial p} \left(p^2 D_p \frac{\partial}{\partial p} \frac{f}{p^2} - \dot{p}\, f \right) + \ldots$

$$\boldsymbol{\kappa} = \begin{pmatrix} \kappa_\perp & \kappa_A & 0 \\ -\kappa_A & \kappa_\perp & 0 \\ 0 & 0 & \kappa_{||} \end{pmatrix}$$

$\kappa_{||}$: Diffusion *along*[2] $\boldsymbol{B}$

$\kappa_\perp$: Diffusion *across*[3] $\boldsymbol{B}$

κ_A: Drift effects[4]

Analytical calculation

- Mean free path $\lambda_{||} \propto \kappa_{||} \propto \int_{-1}^{1} \mathrm{d}\mu \frac{(1-\mu^2)^2}{D_{\mu\mu}(\mu)}$

  Pitch angle $\mu = \cos(\widehat{\mathbf{v}, \mathbf{B}_0})$

- Fokker-Planck coefficient $D_{\mu\mu} = \int_0^\infty \mathrm{d}t\, \langle \dot{\mu}(t)\, \dot{\mu}^\star(0) \rangle$

  Taylor-Green-Kubo formula

- Equation of motion (Lorentz) $\dot{\mu} = \frac{\partial}{\partial t}\left(\frac{v_{||}}{v}\right) \overset{\text{static}}{=} \frac{\dot{v}_{||}}{v}$

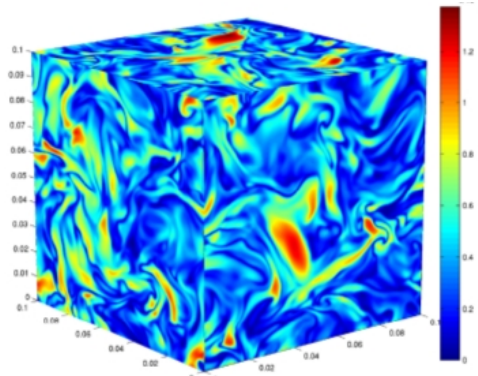
  Unknown $v_{x,y}$, unknown position in $\delta B_{x,y}$ $= \frac{\Omega}{v}\left(v_x \frac{\delta B_y}{B_0} - v_y \frac{\delta B_x}{B_0} \right)$

→ Can only be solved in ideal situations
- Quasi-Linear Theory ($\delta B \ll B$): QLT
- 2nd order QLT: SOQLT
- Non-linear guiding centre: NLGC

Numerical simulations

Reality: resonant wave-particle interaction with stochastic motion... turbulence model requires:

- Energy spectrum (diff.eq. for wave!): $W \propto k^{-s}$
- Geometry
- Dynamical behaviour
  - Instabilities
  - Damped waved
  - Intermittency



Diffusion in MHD turbulence

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- Energy spectrum (diff.eq. for wave!): $W \propto k^{-s}$
- Geometry
- Dynamical behaviour
  - Instabilities
  - Damped waved
  - Intermittency

→ Low resolution only (time-consuming)

→ Near future(?): full MHD treatment (CRs + gas + B)

Effective models: fit of δ (careful with the interpretation!)

$$\overbrace{\frac{\partial N^j}{\partial t}}^{\text{Variation}} + \overbrace{\left(-\vec{\nabla}\cdot\left(K(E,\vec{r})\vec{\nabla}\right)+\vec{\nabla}\cdot\vec{V}(\vec{r})\right)}^{\text{Spatial transport: diffusion+convection}} N^j + \overbrace{(\Gamma_{\rm rad}+\Gamma_{\rm inel})}^{\text{Catastrophic losses}} N^j + \overbrace{\frac{\partial}{\partial E}\left(b^j N^j - c^j\frac{\partial N^j}{\partial E}\right)}^{\text{E gains/losses}} = \overbrace{Q^j(E,\vec{r}) \quad + \sum_{m_i>m_j}\Gamma^{i\to j}N^i}^{\text{Source term: prim.+sec.}}$$

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Magnetic fields

- Everywhere: planetary → galaxy clusters
- Typical amplitudes: ~ μG – nT
- Two components (comparable strength):
  - ✓ Regular B0 (large scale)
  - ✓ Turbulent δB (small scale), i.e. <δB>=0

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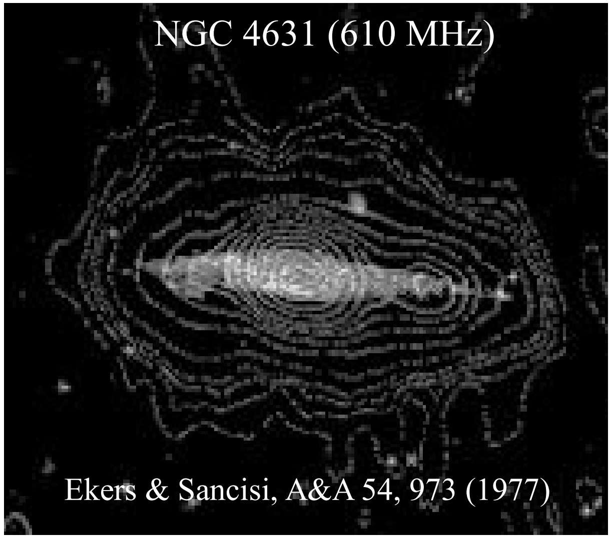

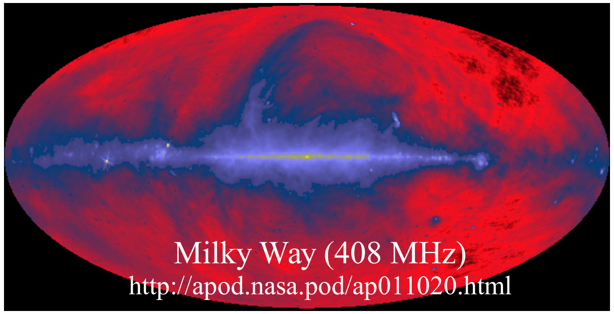

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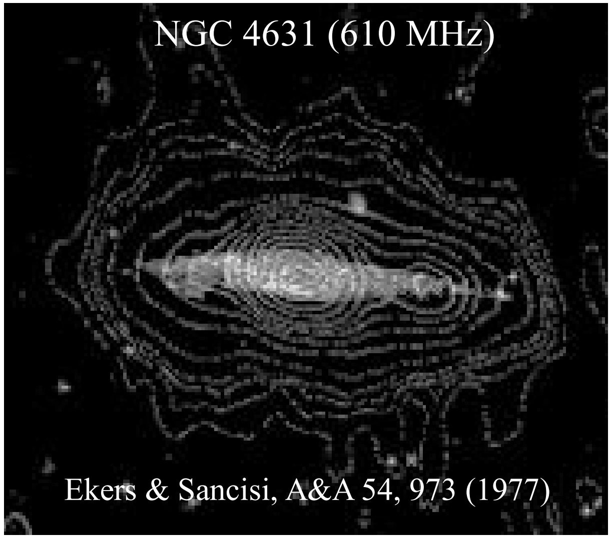

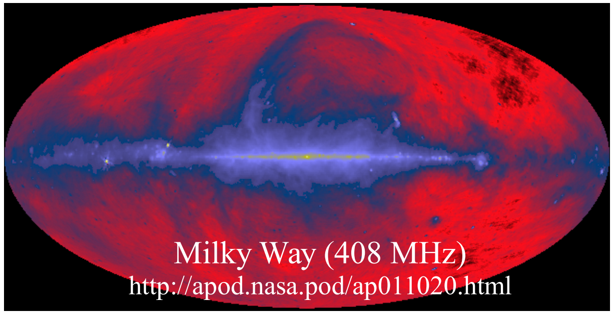

⇓

Geometry = camembert box
→ Diffusive halo half-height ~ L

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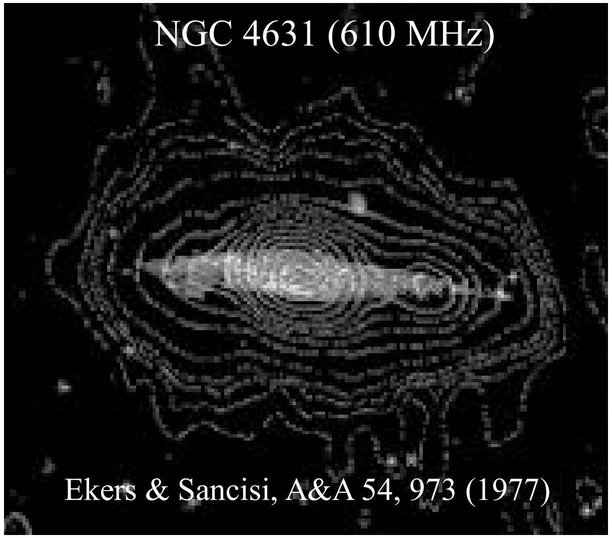
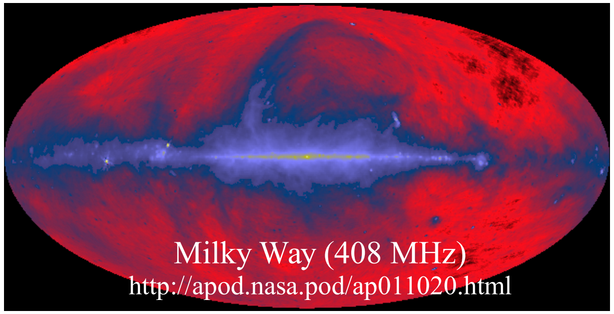
Diffusion coefficients

Diffusion+ confinement = geometry





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Usual simplifying assumptions

- Isotropic (no preferred diffusion direction)
- Standard (no sub-diffusion, Levy flights...)
- Spatial-independent diffusion coefficient
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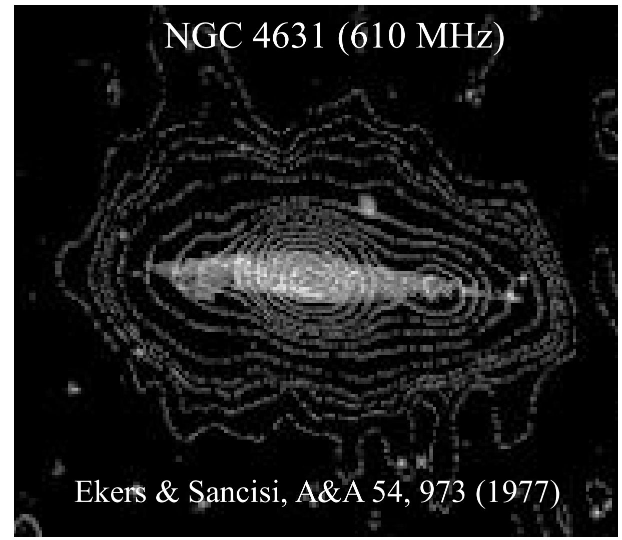
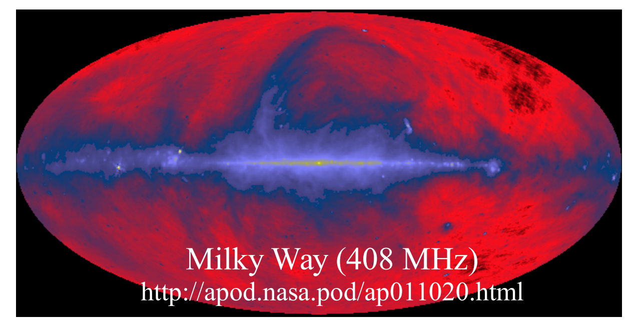
$\langle(\Delta x)^2\rangle \propto t^{\alpha+1}$
with α=0


NGC 4631 (610 MHz)
Ekers & Sancisi, A&A 54, 973 (1977)


Milky Way (408 MHz)
http://apod.nasa.pod/ap011020.html

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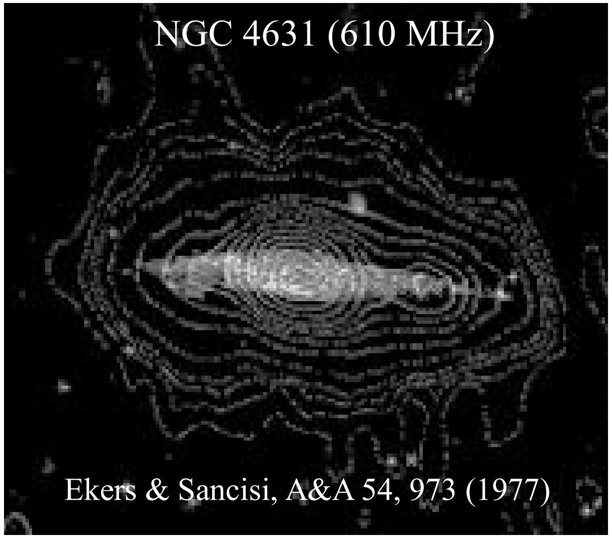

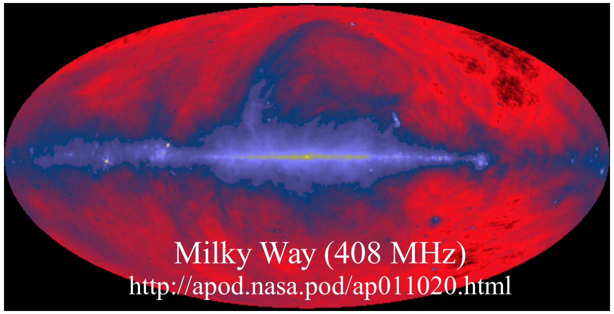

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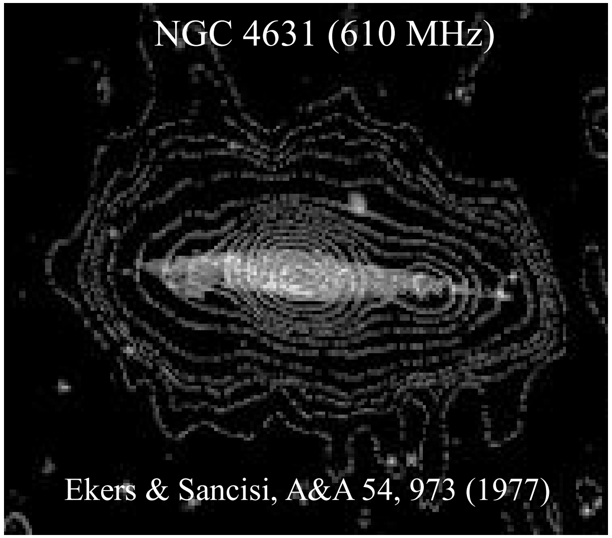

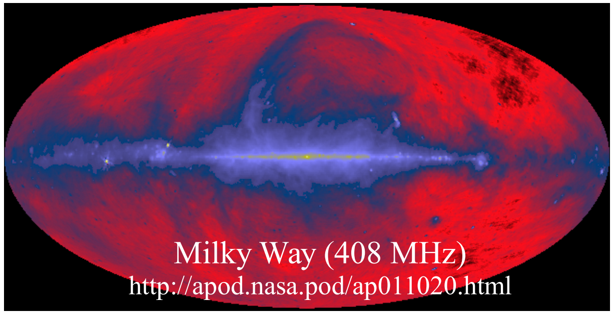

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→ $D = \beta\, D_0\, R^{\delta}$
→ D_{EE} μ $(pV_A)^2/D$

"Minimal" model
5 free parameters
D_0, δ, V_A, L, V_c

Geometry = camembert box
→ Diffusive halo half-height ~ L

Transport of cosmic rays (CR) in the Galaxy

III. Solving the transport equations: GCR phenomenology

1. Diffusion: from microphysics to effective models
2. Full set of equations (with source terms)
3 (Semi-)Analytical, numerical, & MC solutions
4. Stable species: degeneracy K_0 /L
5. Radioactive species and local ISM
6. Leptons and local sources

The full transport equation(s)

$$\overbrace{\frac{\partial N^j}{\partial t}}^{\text{Variation}} + \overbrace{\left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right) + \vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)}^{\text{Spatial transport: diffusion+convection}} N^j + \overbrace{\frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right)}^{\text{E losses and gains}} + \overbrace{(\Gamma_{\rm rad} + \Gamma_{\rm inel})}^{\text{Catastrophic losses}} N^j = \overbrace{Q^j(t,E,\vec{r})}^{\text{Source}}$$

$$b^j = \left\{ \left\langle \frac{dE}{dt} \right\rangle^{\text{(halo)+HE}}_{\text{sync, IC}} + \left\langle \frac{dE}{dt} \right\rangle^{\text{(disc)+LE}}_{\text{brem, ion, Coul}} - \left[\frac{(\vec{\nabla}\cdot\vec{V}_c)}{3}\frac{p^2}{E} \right]^{\text{(halo)+LE}} \right.$$

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$$\Gamma^{\text{(disc)+LE}}_{\text{inel}} + \Gamma^{\text{(halo)+LE}}_{\beta-\text{rad}} + \gamma^{\text{(disc)+LE}}_{\text{EC}-\text{rad}}$$

$$n_{\text{ISM}} v \sigma_{\text{inel}} + 1/(\gamma\tau^{\beta}) + n_{\text{ISM}} v \sigma_{\text{attach}}$$

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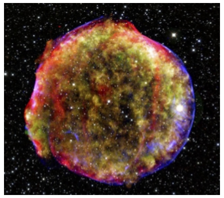
primary species
=
accelerated in sources



Primary source term: space-time granularity

$$Q(t,E,\vec{r}) = \sum_k Q_k\left(\theta(t-t_k), E, \delta(\vec{r}-\vec{r}_k)\right)$$

- Spatial distribution (positions)
- Age source / duration
- Spectrum (for all species)

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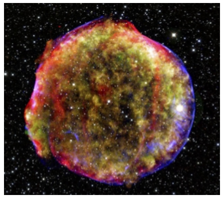

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→ 1[st] approximation:
- smooth spatial distribution
- steady-state

→ Refinement: add specific sources you believe may contribute to the total

The full transport equation(s)

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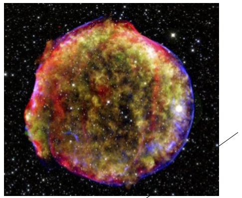
$$\left\langle\frac{dE}{dt}\right\rangle^{\rm (halo)+HE}_{\text{sync, IC}} + \left\langle\frac{dE}{dt}\right\rangle^{\rm (disc)+LE}_{\text{brem, ion, Coul}}$$

$$-\left[\frac{(\vec{\nabla}\cdot\vec{V}_c)}{3}\frac{p^2}{E}\right]^{\rm (halo)+LE}$$

$$\Gamma^{\rm (disc)+LE}_{\rm inel} + \Gamma^{\rm (halo)+LE}_{\beta-{\rm rad}} + \gamma^{\rm (disc)+LE}_{\rm EC-rad}$$

$$n_{\rm ISM} v \sigma_{\rm inel} + 1/(\gamma\tau^{\beta}) + n_{\rm ISM} v \sigma_{\rm attach}$$

primary species
=
accelerated in sources



$$Q^{\rm primary}_{\rm continuum}(r,E) \approx q^j R^{-\alpha^j} \times [f_{\rm SNR}(r)]^{\rm (disc)}$$

$$\left[Q^{\rm primary}_{\text{local sources}}(t,\delta(r-r_s),E)\right]$$

Primary source term: space-time granularity

$$Q(t,E,\vec{r}) = \sum_k Q_k\left(\theta(t-t_k),E,\delta(\vec{r}-\vec{r}_k)\right)$$

- Spatial distribution (positions)
- Age source / duration
- Spectrum (for all species)

→ 1[st] approximation:

- smooth spatial distribution
- steady-state

→ Refinement: add specific sources you believe may contribute to the total

The full transport equation(s)

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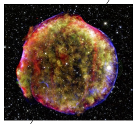

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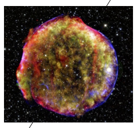

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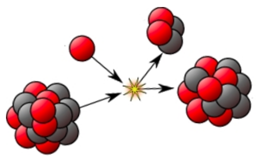

Secondary species
=
created in nuclear interaction of primary CR on ISM

Secondary source term: production

$$N^s(E_s,r,t) = \sum_{p={\rm CRs}} \sum_{i={\rm ISM}} \int_{E_{\rm th}}^{\infty} \frac{d\sigma_i^{ps}(E_p \to E_s)}{dE_s} n_i(r) v'_p N^p(E'_p)\, dE'_p$$

- ISM ~ 90% H, 10% He
- CRs: Mostly p and He (for e^+, pbar, dbar)
- Cross-sections: data + models

The full transport equation(s)

$$\overbrace{\frac{\partial N^j}{\partial t}}^{\text{Variation}} + \overbrace{\left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right)+\vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)}^{\text{Spatial transport: diffusion+convection}} N^j + \overbrace{\frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right)}^{\text{E losses and gains}} + \overbrace{\left(\Gamma_{\rm rad}+\Gamma_{\rm inel}\right)}^{\text{Catastrophic losses}} N^j = \overbrace{Q^j(t,E,\vec{r})}^{\text{Source}}$$

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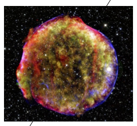

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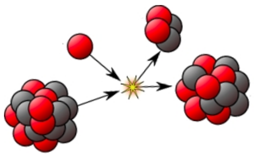

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For nuclei: $$\frac{d\sigma_i^{ps}(E_p\to E_s)}{dE_s} \simeq \sigma_i^{ps}\,\delta(E^p_{k/n}-E^s_{k/n})$$

→ models:
- Semi-empirical (Silberberg & Tsao)
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Δσ/σ ~ 10–20%

The full transport equation(s)

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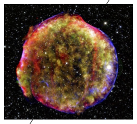

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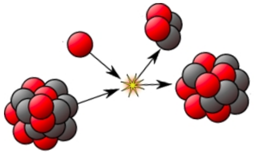

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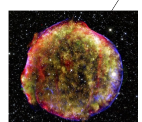

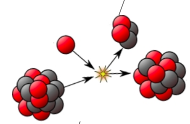

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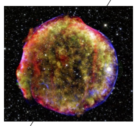

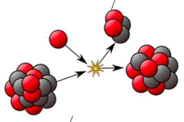

Remarks

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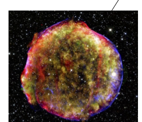

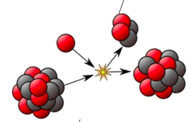

Remarks

→ 2nd order differential equation (space and energy) + time-derivative

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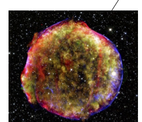

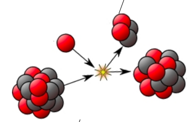

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Remarks

→ 2nd order differential equation (space and energy) + time-derivative

→ Linear equations: local sources or "DM" source term can be dealt with separately

The full transport equation(s)

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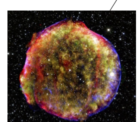

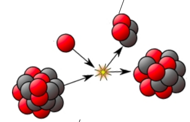

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Remarks

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→ Set of N coupled equations

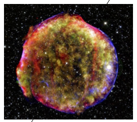
The full transport equation(s)

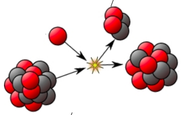
$$\overbrace{\frac{\partial N^j}{\partial t}}^{\text{Variation}} + \overbrace{\left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right) + \vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)}^{\text{Spatial transport: diffusion+convection}} N^j + \overbrace{\frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right)}^{\text{E losses and gains}} + \overbrace{\left(\Gamma_{\rm rad} + \Gamma_{\rm inel}\right)}^{\text{Catastrophic losses}} N^j = \boxed{\overbrace{Q^j(t,E,\vec{r})}^{\text{Source}}}$$

$$\left\{ \left\langle\frac{dE}{dt}\right\rangle^{\rm (halo)+HE}_{\rm sync,\,IC} + \left\langle\frac{dE}{dt}\right\rangle^{\rm (disc)+LE}_{\rm brem,\,ion,\,Coul} - \left[\frac{(\vec{\nabla}\cdot\vec{V}_c)}{3}\frac{p^2}{E}\right]^{\rm (halo)+LE} \right\}$$

$$\Gamma^{\rm (disc)+LE}_{\rm inel} + \Gamma^{\rm (halo)+LE}_{\beta-{\rm rad}} + \gamma^{\rm (disc)+LE}_{\rm EC-rad}$$

$$n_{\rm ISM} v\sigma_{\rm inel} + 1/(\gamma\tau^\beta) + n_{\rm ISM} v\sigma_{\rm attach}$$





$$Q^{\rm primary}_{\rm continuum}(r,E) \approx q^j R^{-\alpha^j} \times [f_{\rm SNR}(r)]^{\rm (disc)}$$

$$\left[Q^{\rm primary}_{\rm local\ sources}(t,\delta(r-r_s),E)\right]$$

$$Q^{\rm secondary}_{\bar{\rm p},\bar{\rm d},{\rm e}^\pm,\gamma,\nu}(r,E) = \sum_{(p,He)+(H,He)} \int n v' \frac{d\sigma}{dE} N^{p,He}(E')dE'$$

$$Q^{\rm secondary}_{\rm N}(r,E) = \sum_{m_p>m_s} n_{(H,He)} v\sigma^{ps}(E) N^p(r,E)$$

Remarks

→ $2^{\rm nd}$ order differential equation (space and energy) + time-derivative
→ Linear equations: local sources or "DM" source term can be dealt with separately
→ Set of N coupled equations

$(p, He) + \to \ldots (\%)$	^{11}B	^{10}Be	^{10}B	^{9}Be	^{7}Be
^{28}Si	-	1.7	-	1.9	1.9
^{24}Mg	2.3	2.7	2.5	3.0	3.1
^{20}Ne	2.1	2.6	2.2	2.8	3.0
^{16}O	20.3	20.0	23.0	21.7	22.7
^{15}N	3.2	5.5	1.1	4.2	2.9
^{14}N	5.4	5.0	5.6	5.5	5.7
^{13}C	4.2	1.2	1.2	2.9	1.9
^{12}C	56.9	22.8	44.8	26.5	27.9
^{11}B	-	30.6	16.1	15.6	9.5
^{10}Be	-	-	-	1.6	1.2
^{10}B	-	-	-	7.3	7.0
^{9}Be	-	-	-	-	6.9
Total	94.4%	92.1%	96.5%	93.0%	93.7%

The full transport equation(s)

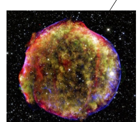
$$\overbrace{\frac{\partial N^j}{\partial t}}^{\text{Variation}} + \overbrace{\left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right)+\vec{\nabla}\cdot\vec{V_c}(\vec{r})\right)}^{\text{Spatial transport: diffusion+convection}} N^j + \overbrace{\frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right)}^{\text{E losses and gains}} + \overbrace{\left(\Gamma_{\rm rad}+\Gamma_{\rm inel}\right)}^{\text{Catastrophic losses}} N^j = \overbrace{Q^j(t,E,\vec{r})}^{\text{Source}}$$

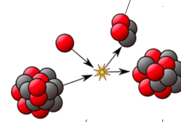
$$\left\langle\frac{dE}{dt}\right\rangle^{\rm (halo)+HE}_{\text{sync, IC}} + \left\langle\frac{dE}{dt}\right\rangle^{\rm (disc)+LE}_{\text{brem, ion, Coul}}$$

$$-\left[\frac{(\vec{\nabla}\cdot\vec{V_c})}{3}\frac{p^2}{E}\right]^{\rm (halo)+LE}$$

$$\Gamma^{\rm (disc)+LE}_{\rm inel} + \Gamma^{\rm (halo)+LE}_{\beta-{\rm rad}} + \gamma^{\rm (disc)+LE}_{\rm EC-rad}$$

$$n_{\rm ISM} v \sigma_{\rm inel} + 1/(\gamma\tau^\beta) + n_{\rm ISM} v \sigma_{\rm attach}$$





$$Q^{\rm primary}_{\rm continuum}(r,E) \approx q^j R^{-\alpha^j} \times [f_{\rm SNR}(r)]^{\rm (disc)}$$

$$\left[Q^{\rm primary}_{\text{local sources}}(t,\delta(r-r_s),E)\right]$$

$$Q^{\rm secondary}_{\bar{p},\bar{d},e^\pm,\gamma,\nu}(r,E) = \sum_{(p,He)+(H,He)} \int n v' \frac{d\sigma}{dE} N^{p,He}(E')dE'$$

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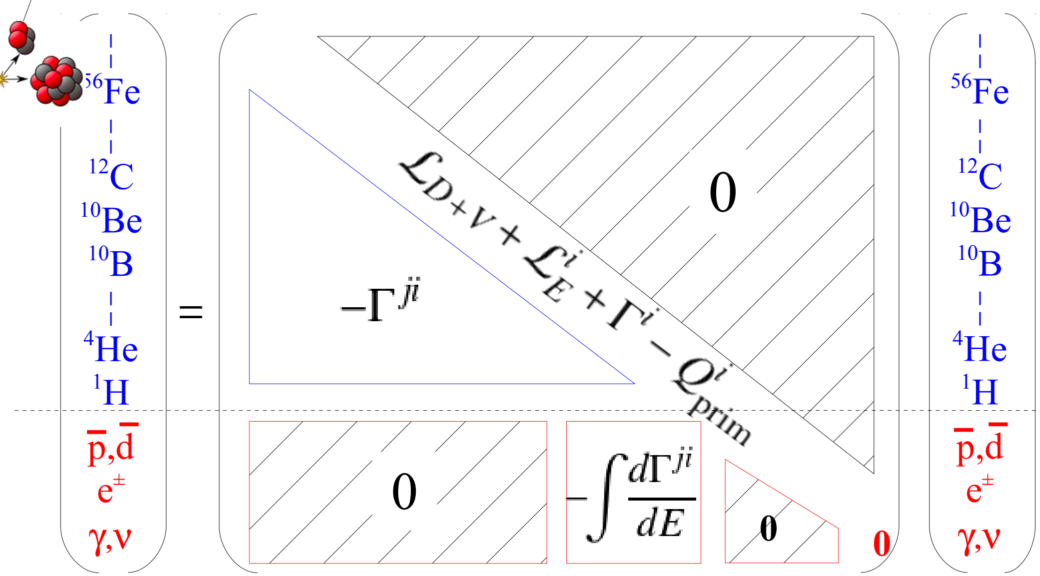


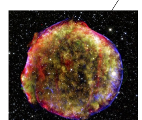
The full transport equation(s)

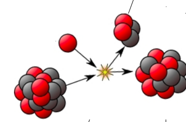
$$\overbrace{\frac{\partial N^j}{\partial t}}^{\text{Variation}} + \overbrace{\left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right)+\vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)}^{\text{Spatial transport: diffusion+convection}} N^j + \overbrace{\frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right)}^{\text{E losses and gains}} + \overbrace{\left(\Gamma_{\rm rad}+\Gamma_{\rm inel}\right)}^{\text{Catastrophic losses}} N^j = \overbrace{Q^j(t,E,\vec{r})}^{\text{Source}}$$

$$\left\{ \begin{array}{l} \left\langle \frac{dE}{dt} \right\rangle^{\rm (halo)+HE}_{\rm sync,\ IC} + \left\langle \frac{dE}{dt} \right\rangle^{\rm (disc)+LE}_{\rm brem,\ ion,\ Coul} \\ -\left[\frac{(\vec{\nabla}\cdot\vec{V}_c)}{3}\frac{p^2}{E}\right]^{\rm (halo)+LE} \end{array} \right.$$

$$\Gamma_{\rm inel}^{\rm (disc)+LE} + \Gamma_{\beta-{\rm rad}}^{\rm (halo)+LE} + \gamma_{\rm EC-rad}^{\rm (disc)+LE}$$

$$n_{\rm ISM} v \sigma_{\rm inel} + 1/(\gamma\tau^{\beta}) + n_{\rm ISM} v \sigma_{\rm attach}$$





$$\left\{ \begin{array}{l} Q^{\rm primary}_{\rm continuum}(r,E) \approx q^j R^{-\alpha^j} \times [f_{\rm SNR}(r)]^{\rm (disc)} \\ \left[Q^{\rm primary}_{\rm local\ sources}(t,\delta(r-r_s),E)\right] \end{array} \right.$$

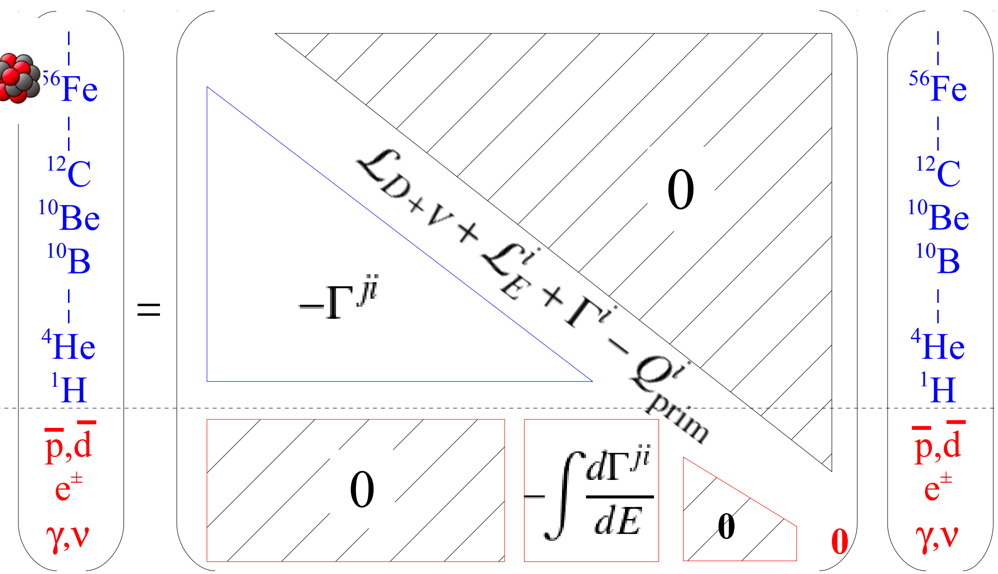
$$\left\{ \begin{array}{l} Q^{\rm secondary}_{\bar{p},\bar{d},e^{\pm},\gamma,\nu}(r,E) = \sum\limits_{(p,He)+(H,He)} \int n v' \frac{d\sigma}{dE} N^{p,He}(E')dE' \\ Q^{\rm secondary}_{\rm N}(r,E) = \sum\limits_{m_p>m_s} n_{(H,He)} v \sigma^{ps}(E) N^p(r,E) \end{array} \right.$$

Remarks

→ 2nd order differential equation (space and energy) + time-derivative

→ Linear equations: local sources or “DM” source term can be dealt with separately

→ Set of N coupled equations



→ Solve iteratively (or diagonalise) for nuclei

The full transport equation(s)

$$\overbrace{\frac{\partial N^j}{\partial t}}^{\text{Variation}} + \overbrace{\left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right)+\vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)}^{\text{Spatial transport: diffusion+convection}} N^j + \overbrace{\frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right)}^{\text{E losses and gains}} + \overbrace{\left(\Gamma_{\rm rad}+\Gamma_{\rm inel}\right)}^{\text{Catastrophic losses}} N^j = \overbrace{Q^j(t,E,\vec{r})}^{\text{Source}}$$

$$\left\langle\frac{dE}{dt}\right\rangle^{\rm (halo)+HE}_{\text{sync, IC}} + \left\langle\frac{dE}{dt}\right\rangle^{\rm (disc)+LE}_{\text{brem, ion, Coul}}$$

$$-\left[\frac{(\vec{\nabla}\cdot\vec{V}_c)}{3}\frac{p^2}{E}\right]^{\rm (halo)+LE}$$

$$\Gamma^{\rm (disc)+LE}_{\rm inel} + \Gamma^{\rm (halo)+LE}_{\beta-\rm rad} + \gamma^{\rm (disc)+LE}_{\rm EC-rad}$$

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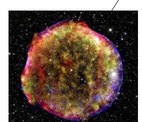

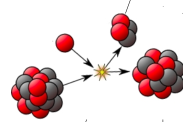

$$Q^{\rm primary}_{\rm continuum}(r,E) \approx q^j R^{-\alpha^j} \times [f_{\rm SNR}(r)]^{\rm (disc)}$$

$$\left[Q^{\rm primary}_{\text{local sources}}(t,\delta(r-r_s),E)\right]$$

$$Q^{\rm secondary}_{\bar{p},\bar{d},e^\pm,\gamma,\nu}(r,E) = \sum_{(p,He)+(H,He)} \int nv' \frac{d\sigma}{dE} N^{p,He}(E')dE'$$

$$Q^{\rm secondary}_{\rm N}(r,E) = \sum_{m_p>m_s} n_{(H,He)} v\sigma^{ps}(E) N^p(r,E)$$

Remarks

→ 2^{nd} order differential equation (space and energy) + time-derivative

→ Linear equations: local sources or "DM" source term can be dealt with separately

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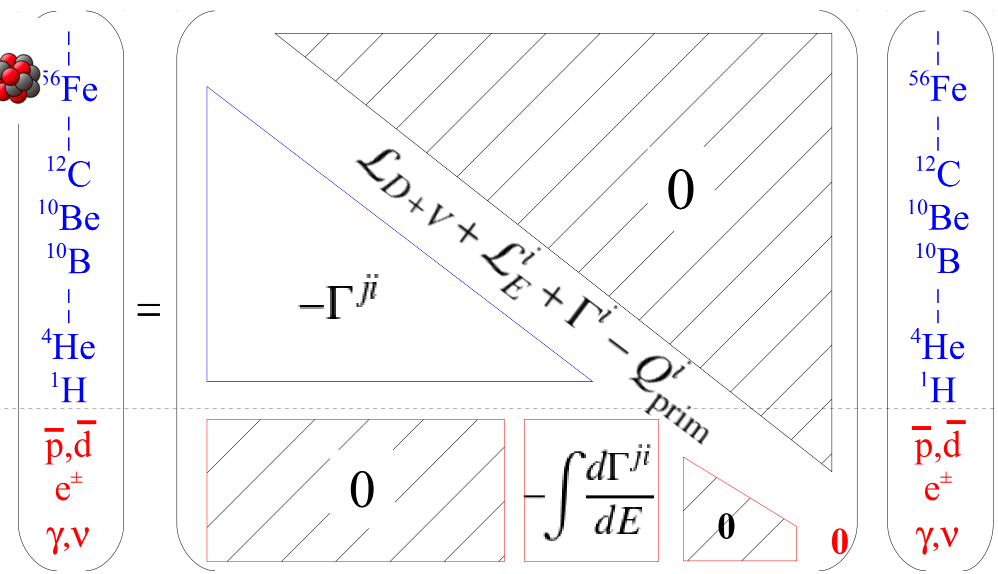

→ Solve iteratively (or diagonalise) for nuclei

→ Calculate p̄, e$^\pm$, γ, ν from p, He afterwards

Transport of cosmic rays (CR) in the Galaxy

III. Solving the transport equations: GCR phenomenology

1. Diffusion: from microphysics to effective models
2. Full set of equations (with source terms)
3 (Semi-)Analytical, numerical, & MC solutions
4. Stable species: degeneracy K_0 /L
5. Radioactive species and local ISM
6. Leptons and local sources

Solving the transport equation

$$\overbrace{\frac{\partial N^j}{\partial t}}^{\text{Variation}} + \overbrace{\left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right) + \vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)}^{\text{Spatial transport: diffusion+convection}} N^j + \overbrace{\frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right)}^{\text{E losses and gains}} + \overbrace{\left(\Gamma_{\text{rad}} + \Gamma_{\text{inel}}\right)}^{\text{Catastrophic losses}} N^j = \boxed{\overbrace{Q^j(t,E,\vec{r})}^{\text{Source}}}$$

	(Semi-)analytical	*Numerical*	*Monte Carlo*

Solving the transport equation

$$\overbrace{\frac{\partial N^j}{\partial t}}^{\text{Variation}} + \overbrace{\left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right)+\vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)}^{\text{Spatial transport: diffusion+convection}} N^j + \overbrace{\frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right)}^{\text{E losses and gains}} + \overbrace{(\Gamma_{\rm rad}+\Gamma_{\rm inel})}^{\text{Catastrophic losses}} N^j = \boxed{\overbrace{Q^j(t,E,\vec{r})}^{\text{Source}}}$$

	(Semi-)analytical	*Numerical*	*Monte Carlo*
Approach	<u>Simplify the problem</u>: • keep dominant effects only • simplify the geometry - spherical/cylindrical/1D halo - sources/gas in thin disc $\delta(z)$		

Solving the transport equation

$$\overbrace{\frac{\partial N^j}{\partial t}}^{\text{Variation}} + \overbrace{\left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right)+\vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)}^{\text{Spatial transport: diffusion+convection}} N^j + \overbrace{\frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right)}^{\text{E losses and gains}} + \overbrace{\left(\Gamma_{\rm rad}+\Gamma_{\rm inel}\right)}^{\text{Catastrophic losses}} N^j = \overbrace{Q^j(t,E,\vec{r})}^{\text{Source}}$$

	(Semi-)analytical	*Numerical*	*Monte Carlo*
Approach	Simplify the problem: • keep dominant effects only • simplify the geometry - spherical/cylindrical/1D halo - sources/gas in thin disc δ(z)		
Tools	• Green functions $\mathcal{L}N(x)=f(x)\begin{cases}\mathcal{L}G(x\vert x')=\delta(x-x')\\ N^{\rm part}(x)=\int G(x\vert x')f(x')dx'\end{cases}$		

Solving the transport equation

$$\overbrace{\frac{\partial N^j}{\partial t}}^{\text{Variation}} + \overbrace{\left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right)+\vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)}^{\text{Spatial transport: diffusion+convection}} N^j + \overbrace{\frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right)}^{\text{E losses and gains}} + \overbrace{(\Gamma_{\rm rad}+\Gamma_{\rm inel})}^{\text{Catastrophic losses}} N^j = \overbrace{Q^j(t,E,\vec{r})}^{\text{Source}}$$

	(Semi-)analytical	*Numerical*	*Monte Carlo*
Approach	Simplify the problem: • keep dominant effects only • simplify the geometry - spherical/cylindrical/1D halo - sources/gas in thin disc $\delta(z)$		
Tools	• Green functions • Fourier/Bessel expansion • Differential equations		

Solving the transport equation

$$\overbrace{\frac{\partial N^j}{\partial t}}^{\text{Variation}} + \overbrace{\left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right)+\vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)}^{\text{Spatial transport: diffusion+convection}} N^j + \overbrace{\frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right)}^{\text{E losses and gains}} + \overbrace{(\Gamma_{\rm rad}+\Gamma_{\rm inel})}^{\text{Catastrophic losses}} N^j = \boxed{\overbrace{Q^j(t,E,\vec{r})}^{\text{Source}}}$$

	(Semi-)analytical	*Numerical*	*Monte Carlo*
Approach	<u>Simplify the problem</u>: • keep dominant effects only • simplify the geometry - spherical/cylindrical/1D halo - sources/gas in thin disc δ(z)		
Tools	• Green functions • Fourier/Bessel expansion • Differential equations		
Pros and cons	• Useful to understand the physics • Fast and no instabilities • Only solve approximate model • New solution for new problem		

Solving the transport equation

$$\overbrace{\frac{\partial N^j}{\partial t}}^{\text{Variation}} + \overbrace{\left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right)+\vec{\nabla}\cdot\vec{V_c}(\vec{r})\right)}^{\text{Spatial transport: diffusion+convection}} N^j + \overbrace{\frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right)}^{\text{E losses and gains}} + \overbrace{(\Gamma_{\rm rad}+\Gamma_{\rm inel})}^{\text{Catastrophic losses}}\ N^j = \boxed{\overbrace{Q^j(t,E,\vec{r})}^{\text{Source}}}$$

	(Semi-)analytical	*Numerical*	*Monte Carlo*
Approach	<u>Simplify the problem</u>: • keep dominant effects only • simplify the geometry - spherical/cylindrical/1D halo - sources/gas in thin disc δ(z)	<u>Finite difference scheme</u>: • discretise the equation • scheme (e.g., Crank-Nicholson) • invert tridiagonal matrix	
Tools	• Green functions • Fourier/Bessel expansion • Differential equations		
Pros and cons	• Useful to understand the physics • Fast and no instabilities • Only solve approximate model • New solution for new problem		

Solving the transport equation

$$\overbrace{\frac{\partial N^j}{\partial t}}^{\text{Variation}} + \overbrace{\left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right)+\vec{\nabla}\cdot\vec{V_c}(\vec{r})\right)}^{\text{Spatial transport: diffusion+convection}} N^j + \overbrace{\frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right)}^{\text{E losses and gains}} + \overbrace{\left(\Gamma_{\rm rad}+\Gamma_{\rm inel}\right)}^{\text{Catastrophic losses}} N^j = \boxed{\overbrace{Q^j(t,E,\vec{r})}^{\text{Source}}}$$

	(Semi-)analytical	*Numerical*	*Monte Carlo*
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Tools	• Green functions • Fourier/Bessel expansion • Differential equations		
Pros and cons	• Useful to understand the physics • Fast and no instabilities • Only solve approximate model • New solution for new problem		

$$\begin{cases} \dfrac{\partial u}{\partial t} = a\dfrac{\partial^2 u}{\partial x^2} \\[2ex] \dfrac{u_i^{n+1}-u_i^n}{\Delta t} = \dfrac{a}{2(\Delta x)^2}\underbrace{\left(\overbrace{\left(u_{i+1}^{n+1}-2u_i^{n+1}+u_{i-1}^{n+1}\right)}^{\text{Implicit scheme}}+\overbrace{\left(u_{i+1}^n-2u_i^n+u_{i-1}^n\right)}^{\text{Explicit scheme}}\right)}_{\text{Crank-Nicholson scheme}} \end{cases}$$

Solving the transport equation

$$\overbrace{\frac{\partial N^j}{\partial t}}^{\text{Variation}} + \overbrace{\left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right)+\vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)}^{\text{Spatial transport: diffusion+convection}} N^j + \overbrace{\frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right)}^{\text{E losses and gains}} + \overbrace{\left(\Gamma_{\rm rad}+\Gamma_{\rm inel}\right)}^{\text{Catastrophic losses}} N^j = \boxed{\overbrace{Q^j(t,E,\vec{r})}^{\text{Source}}}$$

	(Semi-)analytical	*Numerical*	*Monte Carlo*
Approach	<u>Simplify the problem</u>: • keep dominant effects only • simplify the geometry - spherical/cylindrical/1D halo - sources/gas in thin disc δ(z)	<u>Finite difference scheme</u>: • discretise the equation • scheme (e.g., Crank-Nicholson) • invert tridiagonal matrix	
Tools	• Green functions • Fourier/Bessel expansion • Differential equations	• Numerical solvers • Numerical recipes (Press et al.) • NAG libraries, GSL (free)	
Pros and cons	• Useful to understand the physics • Fast and no instabilities • Only solve approximate model • New solution for new problem		

Solving the transport equation

$$\overbrace{\frac{\partial N^j}{\partial t}}^{\text{Variation}} + \overbrace{\left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right)+\vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)}^{\text{Spatial transport: diffusion+convection}} N^j + \overbrace{\frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right)}^{\text{E losses and gains}} + \overbrace{\left(\Gamma_{\rm rad}+\Gamma_{\rm inel}\right)}^{\text{Catastrophic losses}} N^j = \boxed{\overbrace{Q^j(t,E,\vec{r})}^{\text{Source}}}$$

	(Semi-)analytical	*Numerical*	*Monte Carlo*
Approach	<u>Simplify the problem</u>: • keep dominant effects only • simplify the geometry - spherical/cylindrical/1D halo - sources/gas in thin disc δ(z)	<u>Finite difference scheme</u>: • discretise the equation • scheme (e.g., Crank-Nicholson) • invert tridiagonal matrix	
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Pros and cons	• Useful to understand the physics • Fast and no instabilities • Only solve approximate model • New solution for new problem	• Very simple algebra • Any new input easily included • Slow, huge memory for high res. • Less insight in the physics	

Solving the transport equation

$$\overbrace{\frac{\partial N^j}{\partial t}}^{\text{Variation}} + \overbrace{\left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right)+\vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)}^{\text{Spatial transport: diffusion+convection}} N^j + \overbrace{\frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right)}^{\text{E losses and gains}} + \overbrace{\left(\Gamma_{\rm rad}+\Gamma_{\rm inel}\right)}^{\text{Catastrophic losses}} N^j = \overbrace{Q^j(t,E,\vec{r})}^{\text{Source}}$$

	(Semi-)analytical	*Numerical*	*Monte Carlo*
Approach	Simplify the problem: • keep dominant effects only • simplify the geometry - spherical/cylindrical/1D halo - sources/gas in thin disc δ(z)	Finite difference scheme: • discretise the equation • scheme (e.g., Crank-Nicholson) • invert tridiagonal matrix	Follow each particle: • N particles at t=0 • evolve each of them to t+1 • draw random numbers for D 1D : $\Delta z = \pm\sqrt{2D\Delta t}$
Tools	• Green functions • Fourier/Bessel expansion • Differential equations	• Numerical solvers • Numerical recipes (Press et al.) • NAG libraries, GSL (free)	
Pros and cons	• Useful to understand the physics • Fast and no instabilities • Only solve approximate model • New solution for new problem	• Very simple algebra • Any new input easily included • Slow, huge memory for high res. • Less insight in the physics	

Solving the transport equation

$$\overbrace{\frac{\partial N^j}{\partial t}}^{\text{Variation}} + \overbrace{\left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right) + \vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)}^{\text{Spatial transport: diffusion+convection}} N^j + \overbrace{\frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right)}^{\text{E losses and gains}} + \overbrace{\left(\Gamma_{\rm rad} + \Gamma_{\rm inel}\right)}^{\text{Catastrophic losses}} N^j = \boxed{\overbrace{Q^j(t,E,\vec{r})}^{\text{Source}}}$$

	(Semi-)analytical	*Numerical*	*Monte Carlo*
Approach	Simplify the problem: • keep dominant effects only • simplify the geometry - spherical/cylindrical/1D halo - sources/gas in thin disc δ(z)	Finite difference scheme: • discretise the equation • scheme (e.g., Crank-Nicholson) • invert tridiagonal matrix	Follow each particle: • N particles at t=0 • evolve each of them to t+1 • draw random numbers for D 1D : $\Delta z = \pm\sqrt{2D\Delta t}$
Tools	• Green functions • Fourier/Bessel expansion • Differential equations	• Numerical solvers • Numerical recipes (Press et al.) • NAG libraries, GSL (free)	• Stochastic differential equations • Markov process • MPI
Pros and cons	• Useful to understand the physics • Fast and no instabilities • Only solve approximate model • New solution for new problem	• Very simple algebra • Any new input easily included • Slow, huge memory for high res. • Less insight in the physics	

Solving the transport equation

$$\overbrace{\frac{\partial N^j}{\partial t}}^{\text{Variation}} + \overbrace{\left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right)+\vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)}^{\text{Spatial transport: diffusion+convection}} N^j + \overbrace{\frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right)}^{\text{E losses and gains}} + \overbrace{\left(\Gamma_{\text{rad}}+\Gamma_{\text{inel}}\right)}^{\text{Catastrophic losses}} N^j = \overbrace{Q^j(t,E,\vec{r})}^{\text{Source}}$$

	(Semi-)analytical	*Numerical*	*Monte Carlo*
Approach	Simplify the problem: • keep dominant effects only • simplify the geometry - spherical/cylindrical/1D halo - sources/gas in thin disc δ(z)	Finite difference scheme: • discretise the equation • scheme (e.g., Crank-Nicholson) • invert tridiagonal matrix	Follow each particle: • N particles at t=0 • evolve each of them to t+1 • draw random numbers for D 1D : $\Delta z = \pm\sqrt{2D\Delta t}$
Tools	• Green functions • Fourier/Bessel expansion • Differential equations	• Numerical solvers • Numerical recipes (Press et al.) • NAG libraries, GSL (free)	• Stochastic differential equations • Markov process • MPI
Pros and cons	• Useful to understand the physics • Fast and no instabilities • Only solve approximate model • New solution for new problem	• Very simple algebra • Any new input easily included • Slow, huge memory for high res. • Less insight in the physics	• Statistical properties (along path) • No grid (t step), (for/back)-ward • Very slow, statistical errors • Massively parallel problem

Solving the transport equation

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	(Semi-)analytical	*Numerical*	*Monte Carlo*
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Codes and/or references	Webber (1970+) Ptuskin (1980+) Schlickeiser (1990+) USINE (2000+)	<u>Semi-analytical</u> Analytical for spatial derivatives Numerical for energy derivatives *GALPROP* (Strong et al. 1998) *DRAGON* (Evoli et al. 2008) PICARD (Kissmann et al., 2013)	Webber & Rockstroh (1997) Farahat et al. (2008) *Kopp, Büshing et al. (2012)*

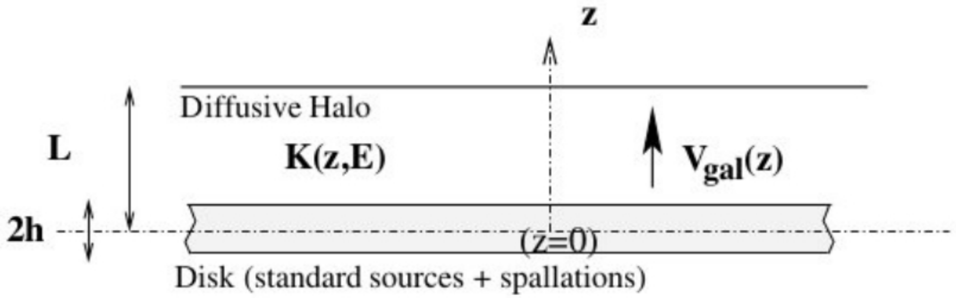
Transport of cosmic rays (CR) in the Galaxy

III. Solving the transport equations: GCR phenomenology

1. Diffusion: from microphysics to effective models
2. Full set of equations (with source terms)
3 (Semi-)Analytical, numerical, & MC solutions
4. Stable species: degeneracy D_0 /L
5. Radioactive species and local ISM
6. Leptons and local sources

Stable nuclei: simple 1D model

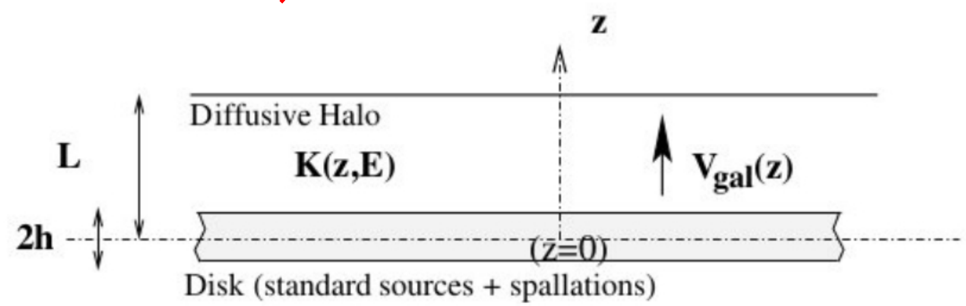
$$\frac{\partial N^j}{\partial t}+\left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right)+\vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)N^j+\frac{\partial}{\partial E}\left(b^j N^j-D_{EE}\frac{\partial N^j}{\partial E}\right)+(\Gamma_{\rm rad}+\Gamma_{\rm inel})N^j=Q^j(t,E,\vec{r})$$



Stable nuclei: simple 1D model

$$\frac{\partial N^j}{\partial t} + \left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right) + \vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)N^j + \frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right) + (\Gamma_{\rm rad} + \Gamma_{\rm inel})N^j = Q^j(t,E,\vec{r})$$

- Simplified 1D geometry (infinite plane)
- Neglect energy losses/gains
- D(E) isotropic, space-independent



Stable nuclei: simple 1D model

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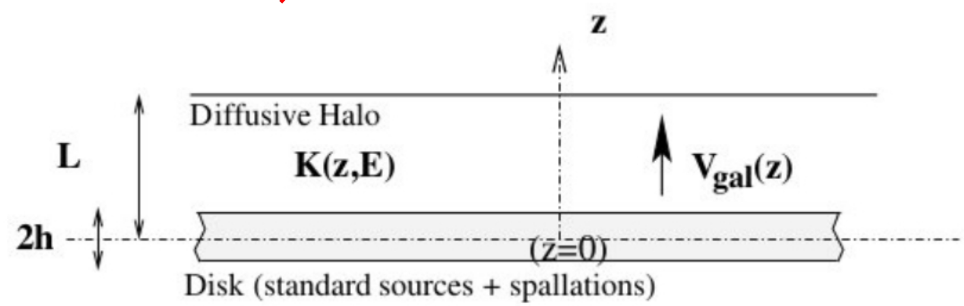

Solution for stable species

$$-DN'' + nv\sigma 2h\delta(z)\times N = 2h\delta(z)Q(E)$$

- Solve in the halo
- Ensure condition N(z=L)=0
- Integrate around 0

$$\int_{-\epsilon}^{+\epsilon}$$

Find solution N(0)

Stable nuclei: simple 1D model

$$\frac{\partial N^j}{\partial t}+\left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right)+\vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)N^j+\frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right)+(\Gamma_{\rm rad}+\Gamma_{\rm inel})N^j = Q^j(t,E,\vec{r})$$

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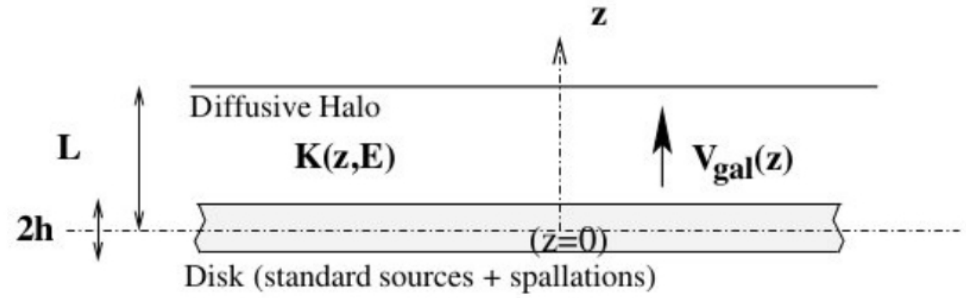

Solution for stable species

$$-DN''+nv\sigma 2h\delta(z)\times N = 2h\delta(z)Q(E)$$

- Solve in the halo
- Ensure condition N(z=L)=0
- Integrate around 0

$$N(0)=\frac{2hQ(E)}{2D/L+2hnv\sigma}$$

Stable nuclei: simple 1D model

$$\frac{\partial N^j}{\partial t}+\left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right)+\vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)N^j+\frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right)+\left(\Gamma_{\rm rad}+\Gamma_{\rm inel}\right)N^j = Q^j(t,E,\vec{r})$$

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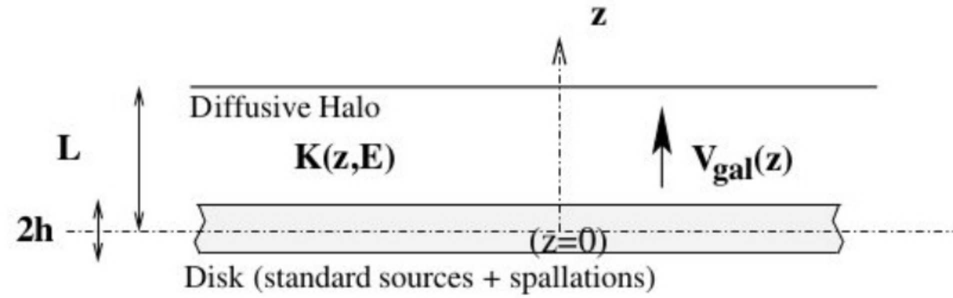


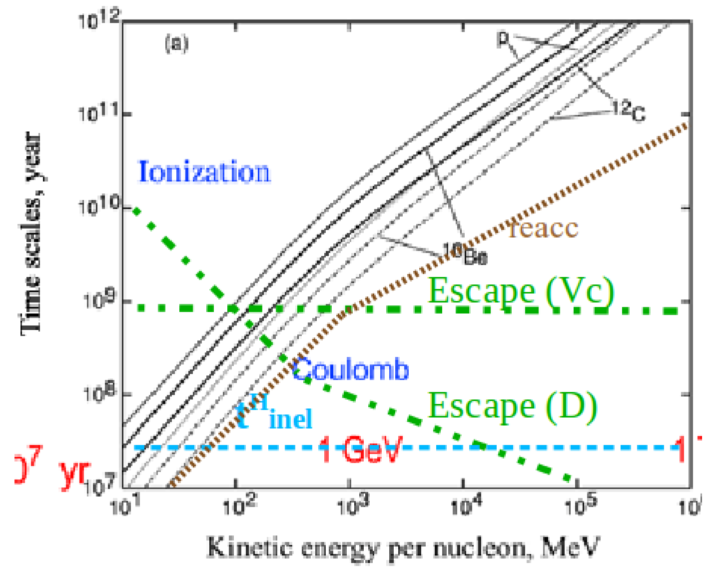
Solution for stable species

$$-DN'' + nv\sigma 2h\delta(z)\times N = 2h\delta(z)Q(E)$$

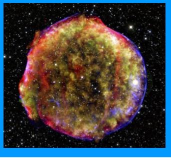
- Solve in the halo
- Ensure condition N(z=L)=0
- Integrate around 0

$$N(0)=\frac{2hQ(E)}{2D/L+2hnv\sigma}$$

Valid above ~10 GeV/n



Stable nuclei: ratio of primary species



$$Q^{\rm prim}(R) = q_0 R^{-\alpha}$$

$$\frac{\partial N^j}{\partial t} + \left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right) + \vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)N^j + \frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right) + (\Gamma_{\rm rad} + \Gamma_{\rm inel})N^j = Q^j(t,E,\vec{r})$$

- Simplified 1D geometry (infinite plane)
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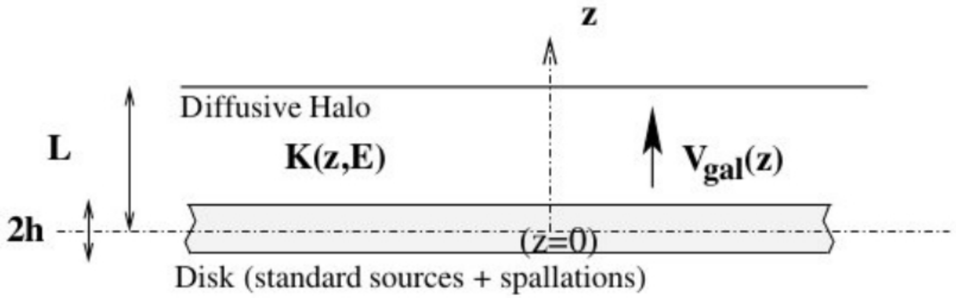


Solution for stable species

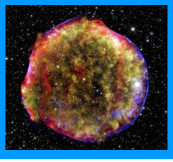
$$-DN'' + nv\sigma 2h\delta(z)\times N = 2h\delta(z)Q(E)$$

- Solve in the halo
- Ensure condition N(z=L)=0
- Integrate around 0

$$N(0) = \frac{2hQ(E)}{2D/L + 2hnv\sigma}$$

$$\frac{N^1(0)}{N^2(0)} \propto \frac{R^{-\alpha_1}}{R^{-\alpha_2}} \times \frac{2D/L + 2hnv\sigma_2}{2D/L + 2hnv\sigma_1}$$

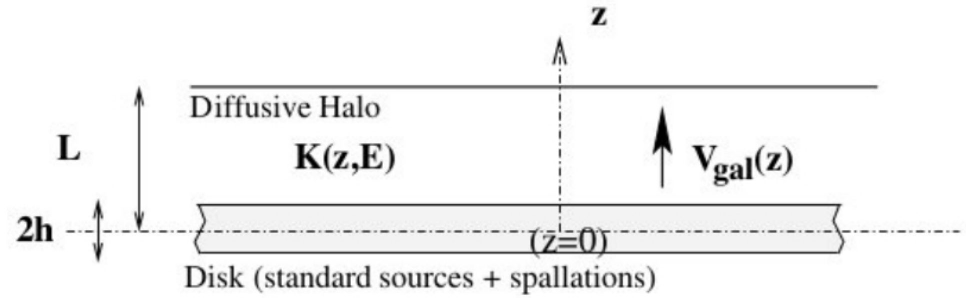
Stable nuclei: ratio of primary species (LE)



$$Q^{\rm prim}(R) = q_0 R^{-\alpha}$$

$$\frac{\partial N^j}{\partial t} + \left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right) + \vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)N^j + \frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right) + (\Gamma_{\rm rad} + \Gamma_{\rm inel})N^j = Q^j(t,E,\vec{r})$$

- Simplified 1D geometry (infinite plane)
- Neglect energy losses/gains
- D(E) isotropic, space-independent



Solution for stable species

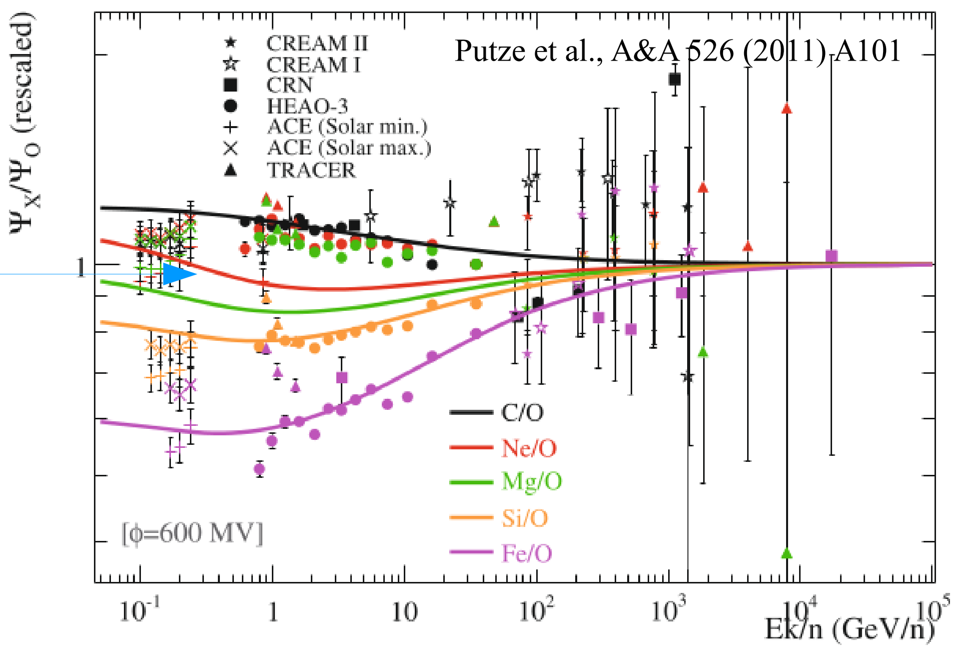
$$-DN'' + nv\sigma 2h\delta(z)\times N = 2h\delta(z)Q(E)$$

- Solve in the halo
- Ensure condition N(z=L)=0
- Integrate around 0

$$N(0) = \frac{2hQ(E)}{2D/L + 2hnv\sigma}$$

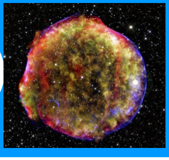
$$\frac{N^1(0)}{N^2(0)} \propto \frac{R^{-\alpha_1}}{R^{-\alpha_2}} \times \frac{2D/L + 2hnv\sigma_2}{2D/L + 2hnv\sigma_1}$$

$(\alpha_1=\alpha_2)$



The heavier the nucleus, the more destroyed it is at low energy

Stable nuclei: ratio of primary species (HE)



$$Q^{\rm prim}(R) = q_0 R^{-\alpha}$$

$$\frac{\partial N^j}{\partial t} + \left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right) + \vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)N^j + \frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right) + (\Gamma_{\rm rad} + \Gamma_{\rm inel})N^j = Q^j(t,E,\vec{r})$$

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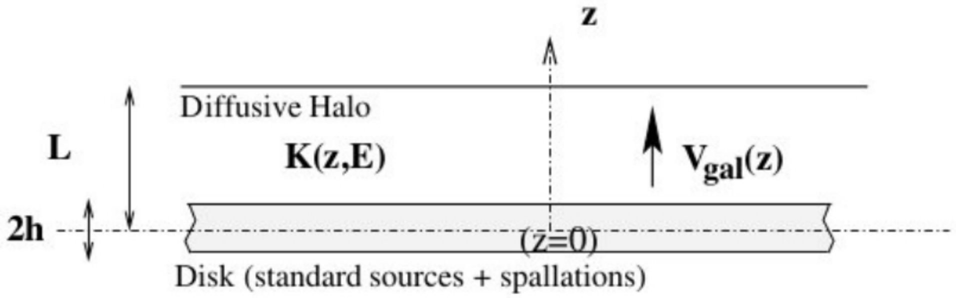


Solution for stable species

$$-DN'' + nv\sigma 2h\delta(z)\times N = 2h\delta(z)Q(E)$$

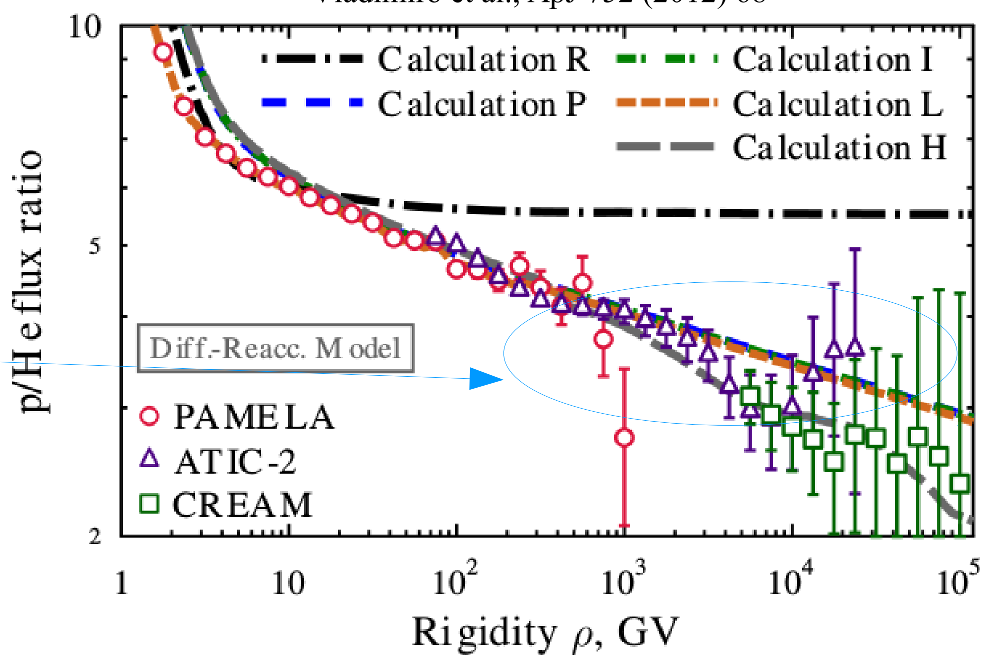
- Solve in the halo
- Ensure condition N(z=L)=0
- Integrate around 0

$$N(0) = \frac{2hQ(E)}{2D/L + 2hnv\sigma}$$

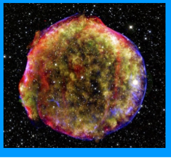
$$\frac{N^1(0)}{N^2(0)} \propto \frac{R^{-\alpha_1}}{R^{-\alpha_2}} \times \frac{2D/L + 2hnv\sigma_2}{2D/L + 2hnv\sigma_1}$$

Hint at $\alpha_p \neq \alpha_{He}$?
(collective effects, acceleration in WR winds)

Vladimiro et al., ApJ 752 (2012) 68



Stable nuclei: primary flux (HE)



$$Q^{\rm prim}(R) = q_0 R^{-\alpha}$$

$$\frac{\partial N^j}{\partial t} + \left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right) + \vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)N^j + \frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right) + (\Gamma_{\rm rad} + \Gamma_{\rm inel})N^j = Q^j(t,E,\vec{r})$$

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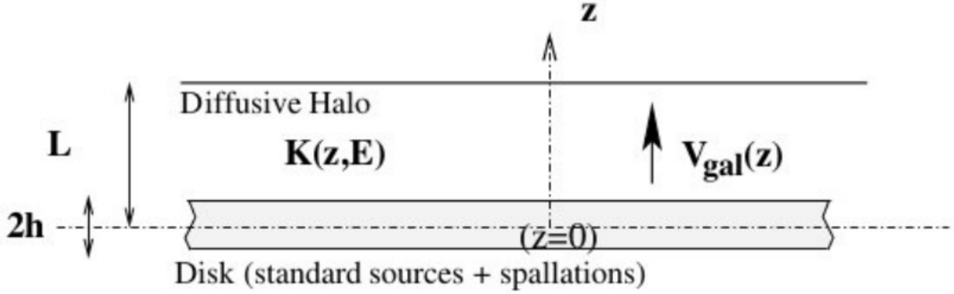


Solution for stable species

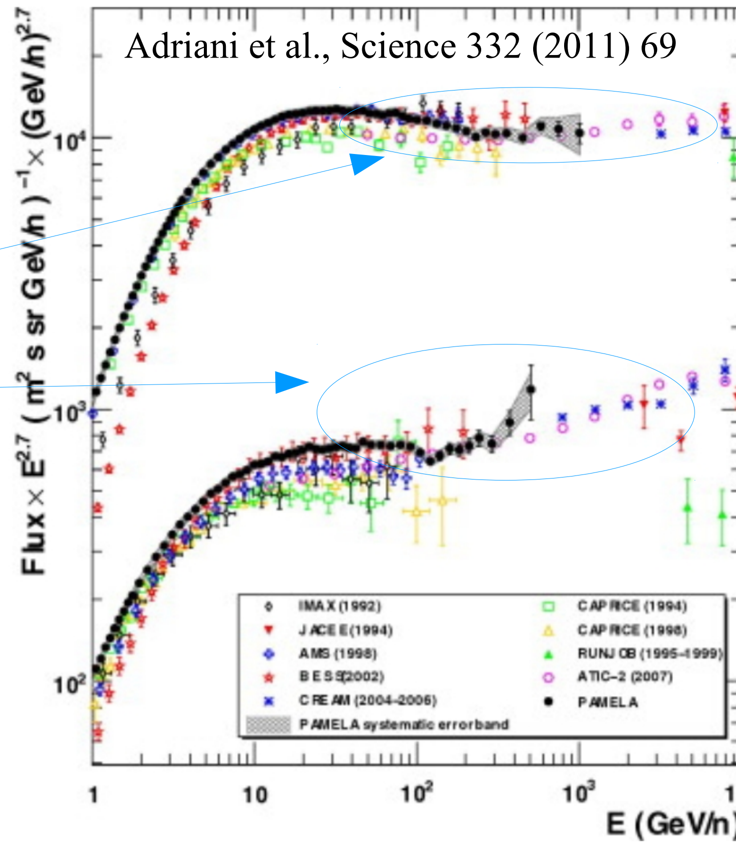
$$-DN'' + nv\sigma 2h\delta(z)\times N = 2h\delta(z)Q(E)$$

- Solve in the halo
- Ensure condition N(z=L)=0
- Integrate around 0

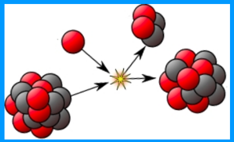
$$N(0) = \frac{2hQ(E)}{2D/L + 2hnv\sigma}$$

$$\rightarrow\ N^p(0) \propto \frac{Q(E)}{D(E)} = \frac{R^{-\alpha}}{R^{\delta}} = R^{-(\alpha+\delta)\approx -2.8}$$

Departure from a pure power law?
Concavity in the spectrum?



Stable nuclei: secondary/primary (HE)



$$Q^{sec}(R) = nv\sigma^{ps}N^{p}(0)$$

$$\frac{\partial N^j}{\partial t} + \left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right) + \vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)N^j + \frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right) + (\Gamma_{\rm rad} + \Gamma_{\rm inel})N^j = Q^j(t,E,\vec{r})$$

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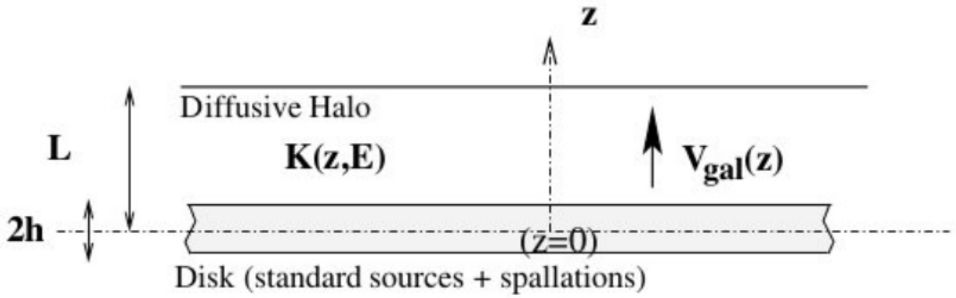


Solution for stable species

$$-DN'' + nv\sigma 2h\delta(z)\times N = 2h\delta(z)Q(E)$$

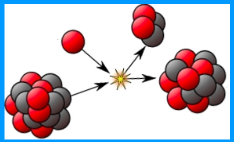
- Solve in the halo
- Ensure condition N(z=L)=0
- Integrate around 0

$$N(0) = \frac{2hQ(E)}{2D/L + 2hnv\sigma}$$

$$\rightarrow\ N^{sec}(0) = \frac{Q^{sec}(E)}{D(E)} \propto \frac{nv\sigma^{ps}N^p(0)}{2D/L}$$

$$\rightarrow\ \frac{N^s}{N^p}(0) \propto \frac{L}{D_0}R^{-\delta}$$

Stable nuclei: secondary/primary + δ



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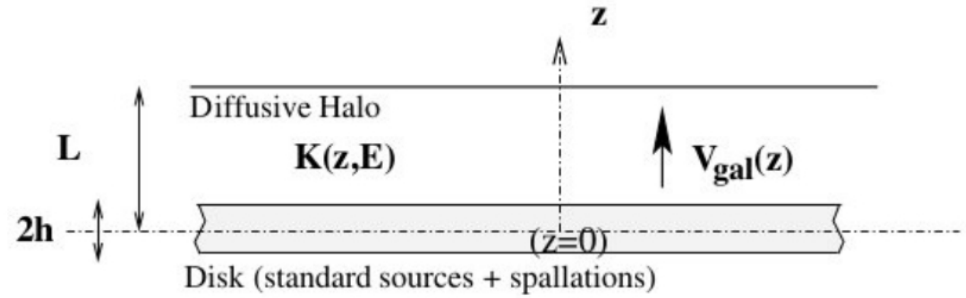

Solution for stable species

$$-DN'' + nv\sigma 2h\delta(z) \times N = 2h\delta(z)Q(E)$$

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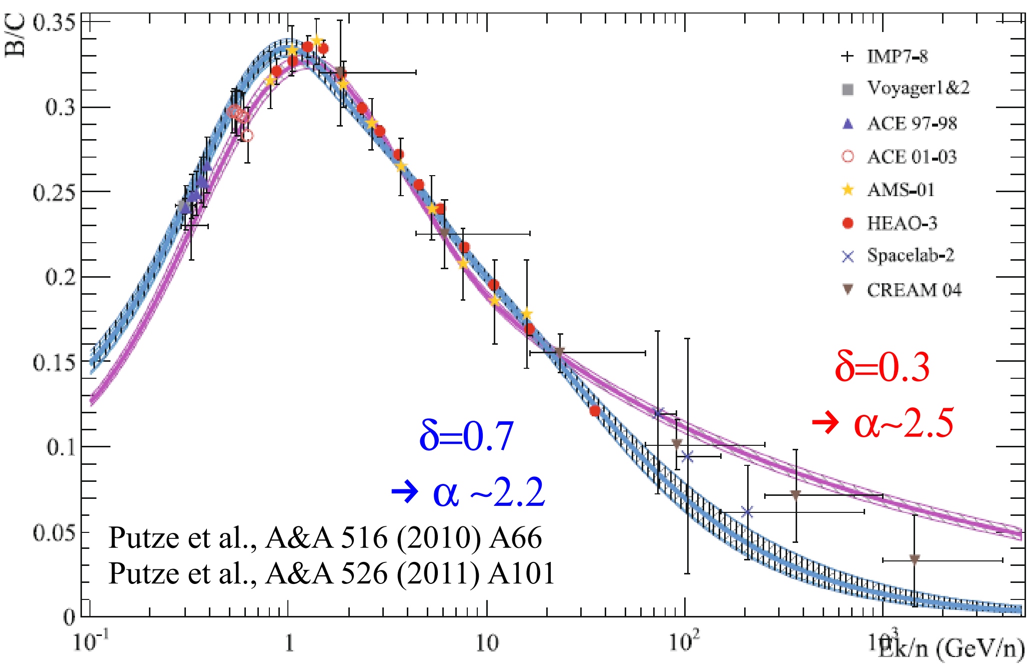
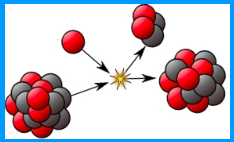
$$\rightarrow \frac{N^s}{N^p}(0) \propto \frac{L}{D_0}R^{-\delta}$$

→ Current data not at high energy enough to determine δ



Stable nuclei: secondary/primary + δ



$$Q^{sec}(R) = nv\sigma^{ps}N^{p}(0)$$

$$\frac{\partial N^j}{\partial t} + \left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right) + \vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)N^j + \frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right) + (\Gamma_{rad} + \Gamma_{inel})N^j = Q^j(t,E,\vec{r})$$

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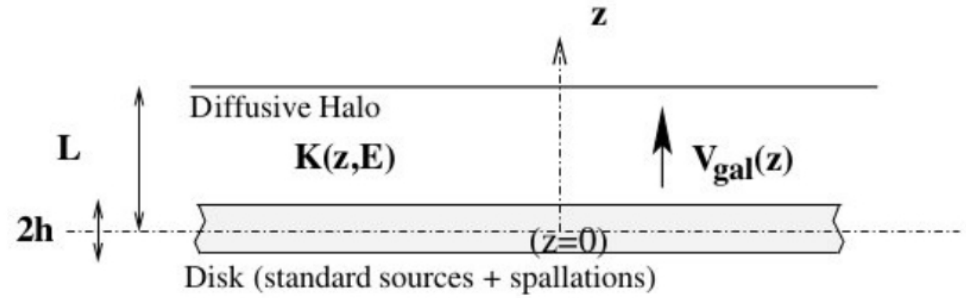


Solution for stable species

$$-DN'' + nv\sigma 2h\delta(z)\times N = 2h\delta(z)Q(E)$$

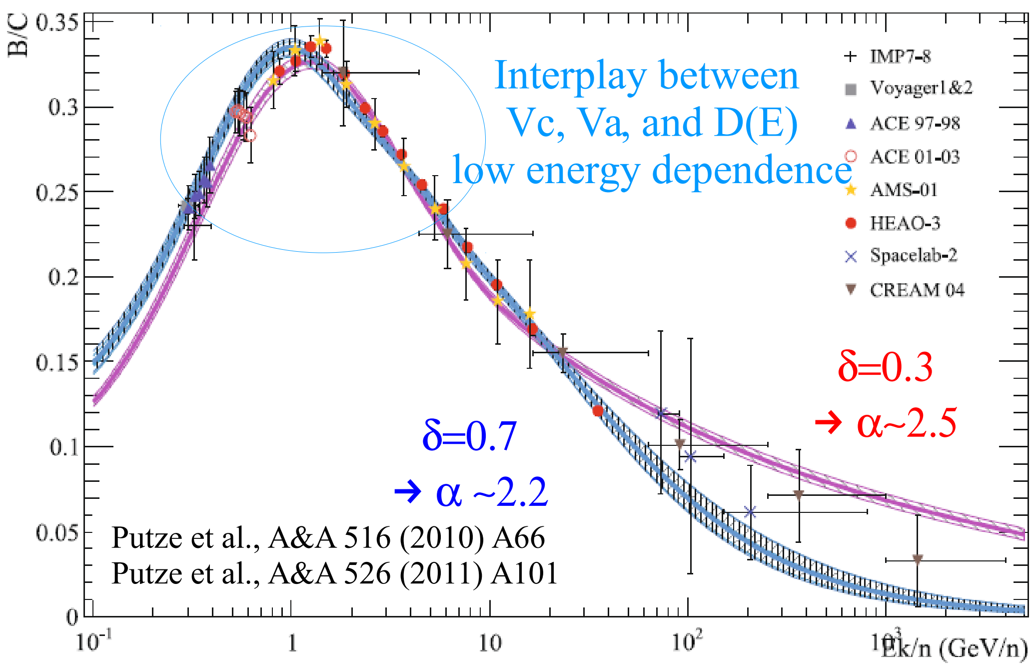
- Solve in the halo
- Ensure condition N(z=L)=0
- Integrate around 0

$$N(0) = \frac{2hQ(E)}{2D/L + 2hnv\sigma}$$

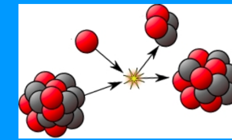
$$\rightarrow N^{sec}(0) = \frac{Q^{sec}(E)}{D(E)} \propto \frac{nv\sigma^{ps}N^{p}(0)}{2D/L}$$

$$\rightarrow \frac{N^s}{N^p}(0) \propto \frac{L}{D_0}R^{-\delta}$$

→ Current data not at high energy enough to determine δ



Stable nuclei: secondary/primary + δ



$Q^{\rm sec}(R) = nv\sigma^{ps}N^p(0)$

$$\frac{\partial N^j}{\partial t} + \left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right) + \vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)N^j + \frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right) + (\Gamma_{\rm rad} + \Gamma_{\rm inel})N^j = Q^j(t,E,\vec{r})$$

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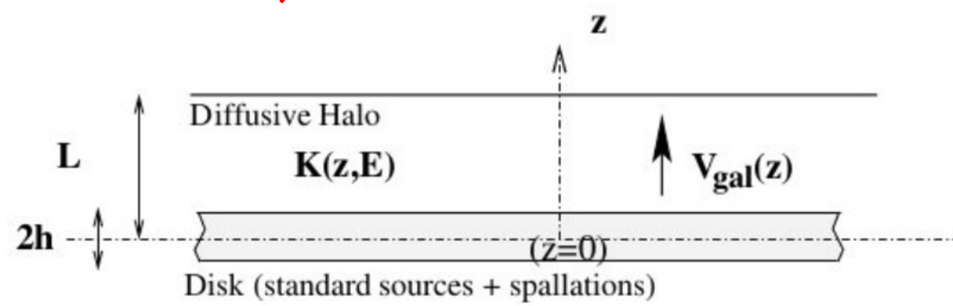


Solution for stable species

$$-DN'' + nv\sigma 2h\delta(z)\times N = 2h\delta(z)Q(E)$$

- Solve in the halo
- Ensure condition N(z=L)=0
- Integrate around 0

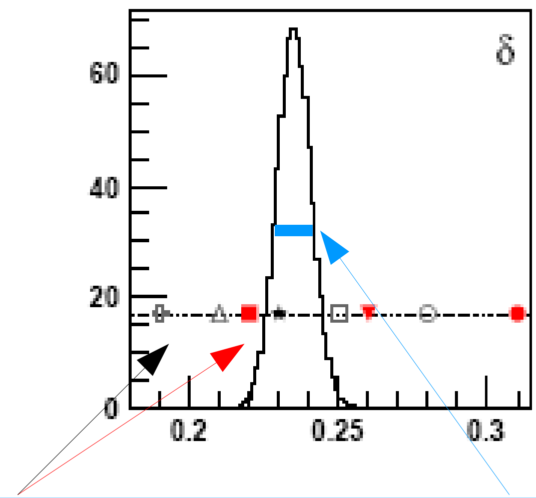
$$N(0) = \frac{2hQ(E)}{2D/L + 2hnv\sigma}$$

$$\rightarrow N^{\rm sec}(0) = \frac{Q^{\rm sec}(E)}{D(E)} \propto \frac{nv\sigma^{ps}N^p(0)}{2D/L}$$

$$\rightarrow \frac{N^s}{N^p}(0) \propto \frac{L}{D_0}R^{-\delta}$$

→ Current data not at high energy enough to determine δ

DM, Putze & Derome
A&A **516**, 67(2010)

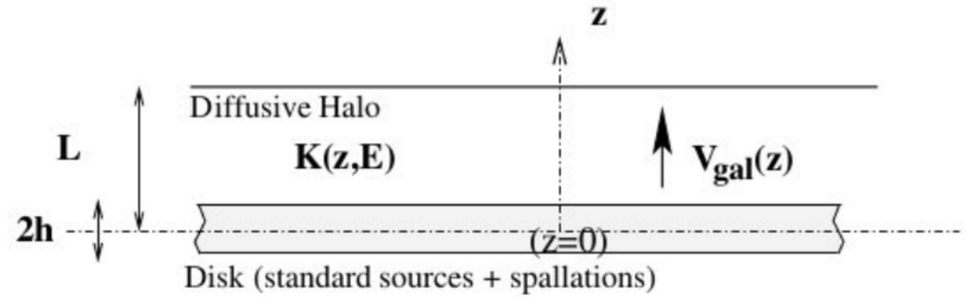


→ Nuclear uncertainties > B/C data uncertainties

Stable nuclei: secondary/primary and D₀/L degeneracy

$$\frac{\partial N^j}{\partial t}+\left(-\vec\nabla\cdot\left(D(E,\vec r)\vec\nabla\right)+\vec\nabla\cdot\vec V_c(\vec r)\right)N^j+\frac{\partial}{\partial E}\left(b^jN^j-D_{EE}\frac{\partial N^j}{\partial E}\right)+(\Gamma_{\rm rad}+\Gamma_{\rm inel})N^j=Q^j(t,E,\vec r)$$

- Simplified 1D geometry (infinite plane)
- Neglect energy losses/gains
- D(E) isotropic, space-independent



Solution for stable species

$$-DN''+nv\sigma 2h\delta(z)\times N=2h\delta(z)Q(E)$$

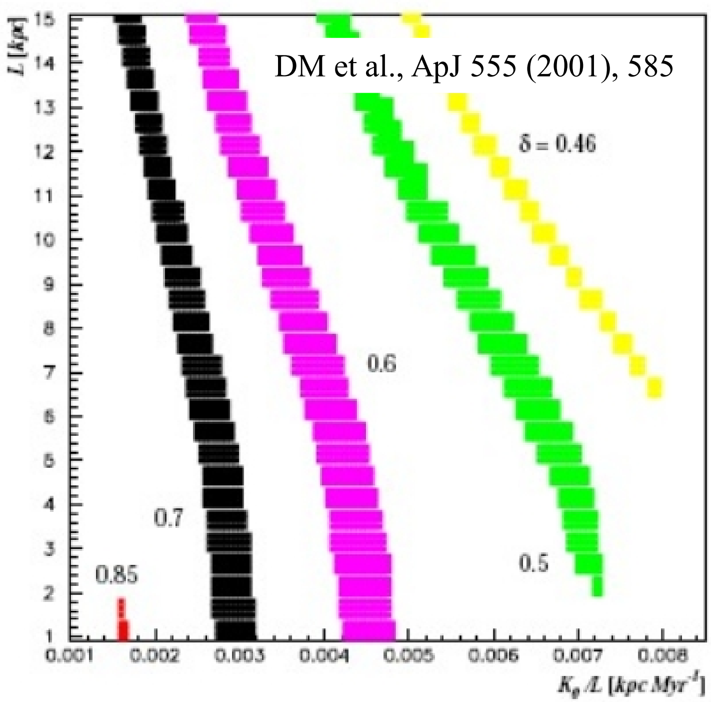
- Solve in the halo
- Ensure condition N(z=L)=0
- Integrate around 0

$$N(0)=\frac{2hQ(E)}{2D/L+2hnv\sigma}$$

$$\rightarrow\quad N^{\rm sec}(0)=\frac{Q^{\rm sec}(E)}{D(E)}\propto\frac{nv\sigma^{ps}N^p(0)}{2D/L}$$

$$\rightarrow\quad \frac{N^s}{N^p}(0)\propto\frac{L}{D_0}R^{-\delta}$$

→ Current data not at high energy enough to determine δ



→ Strong D₀/L degeneracy
≠ D₀ and L (but same D₀/L) gives same B/C

Stable nuclei: secondary/primary and D_0/L degeneracy

$$\frac{\partial N^j}{\partial t} + \left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right) + \vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)N^j + \frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right) + (\Gamma_{\rm rad} + \Gamma_{\rm inel})N^j = Q^j(t,E,\vec{r})$$

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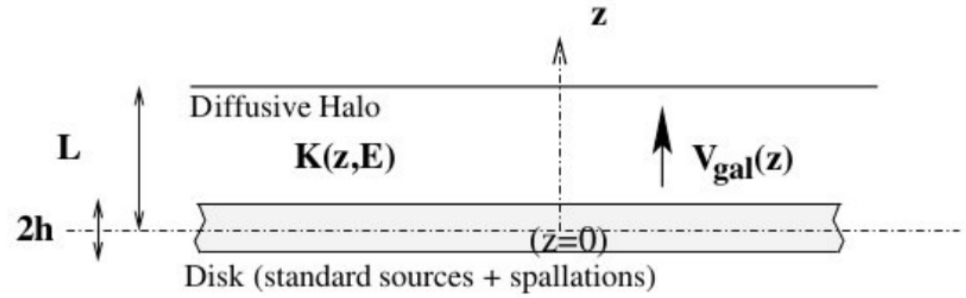

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Leaky-Box equation and grammage

$$\frac{N^s}{\tau_{\rm esc}} + nv\sigma N^s = nv\sigma^{ps}N^p$$

$$\frac{N^s}{N^p} = nv\sigma^{ps}\tau_{esc} \qquad \tau_{esc} = \lambda_{\rm esc} n\bar{m}v$$

$$\lambda_{\rm esc}(1\ {\rm GV}) = \frac{N^s}{N^p}\times\frac{\bar{m}}{\sigma^{ps}}$$

$$\lambda_{\rm esc}(1\ {\rm GV}) = 0.3\times\frac{2\cdot 10^{-24}}{20\cdot 10^{-27}} \sim 30\ {\rm g\ cm^{-2}}$$

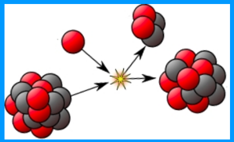
N.B.: correct answer ~ 10 g/cm^2

→ Strong D_0/L degeneracy
≠ D_0 and L (but same D_0/L) gives same B/C

III. Solving the transport equations: GCR phenomenology

1. Diffusion: from microphysics to effective models
2. Full set of equations (with source terms)
3 (Semi-)Analytical, numerical, & MC solutions
4. Stable species: degeneracy K_0 /L
5. Radioactive species and local ISM
6. Leptons and local sources

Secondary radioactive nuclei (LE)



$$Q^{\mathrm{sec}}(R) = nv\sigma^{ps}N^{p}(0)$$

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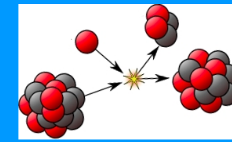

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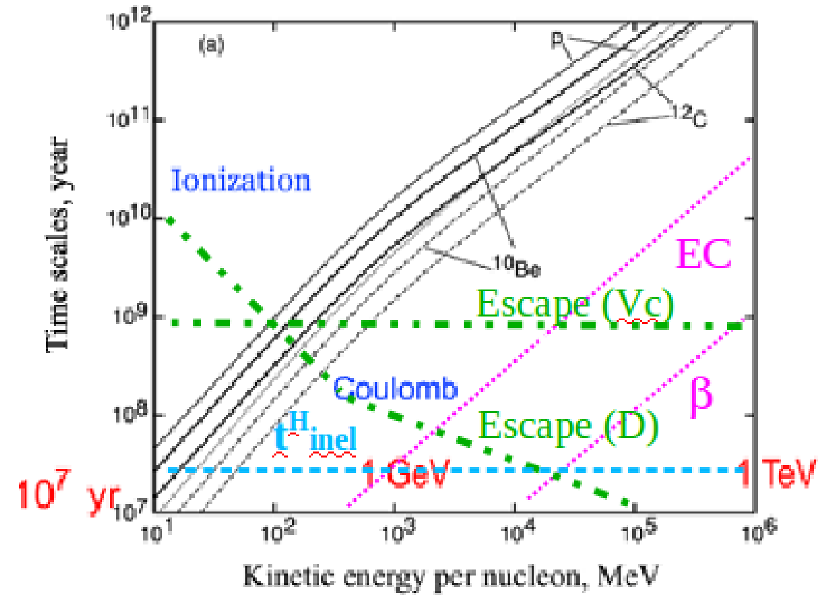

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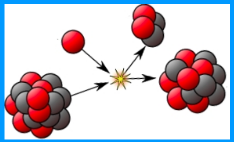

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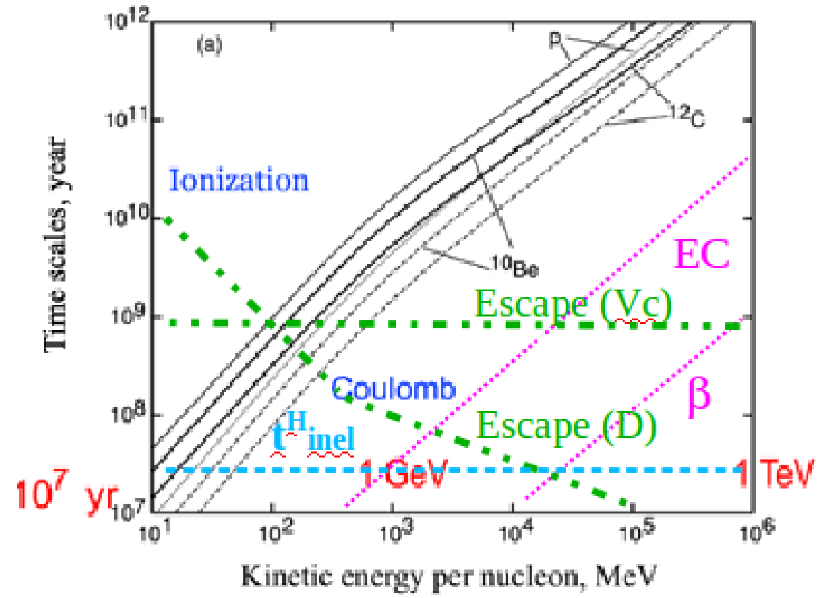
- Only decay and diffusion
- Unbounded diffusion (not sensitive to L)

$$-D\Delta_{\vec{r}}N^r + \frac{N^r}{\gamma\tau_0} = Q(r) \quad \Rightarrow \begin{cases} -\Delta_{\vec{r}}N^r + \lambda^2 N^r = \dfrac{Q(r)}{D} \\ \lambda = 1/\sqrt{D\gamma\tau_0} \end{cases}$$



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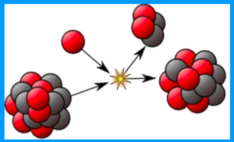

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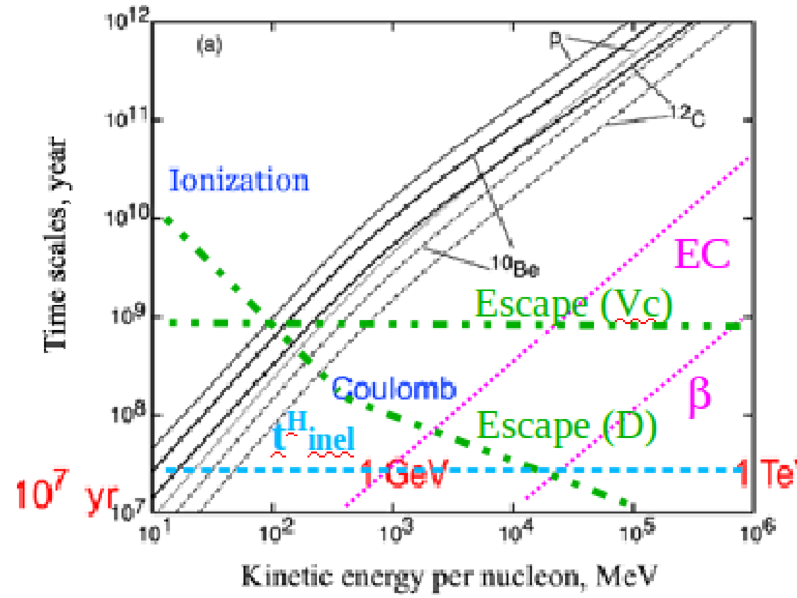
- Green function: solution for $Q(r)=\delta(r)$

$$\mathcal{L}N(x) = f(x) \begin{cases} \mathcal{L}G(x|x') = \delta(x-x') \\ N^{\rm part}(x) = \int G(x|x')f(x')dx' \end{cases}$$

$G(r) = \exp(-\lambda r)/(4\pi r)$

→ *only depends on the distance to the source*



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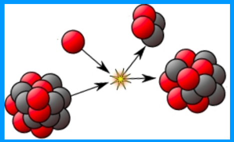

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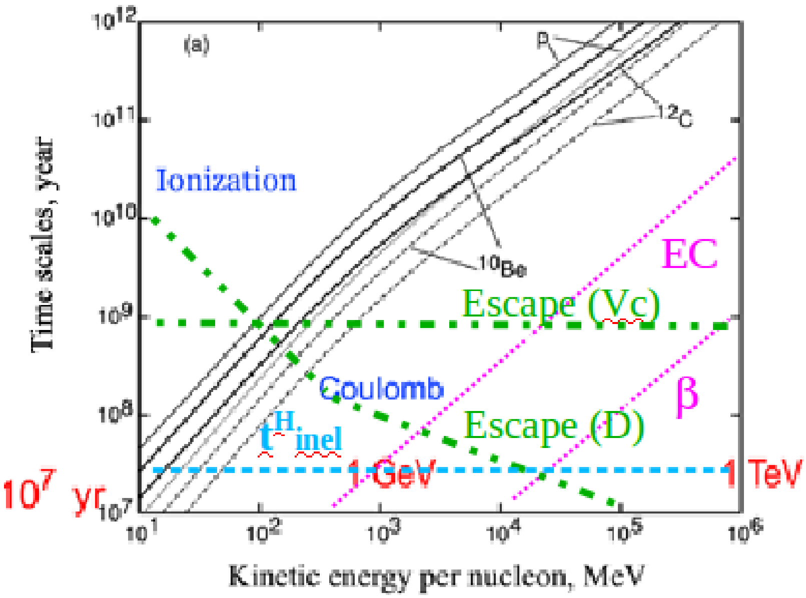

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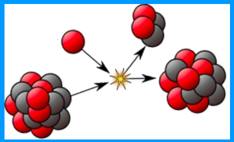

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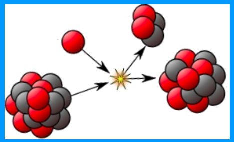

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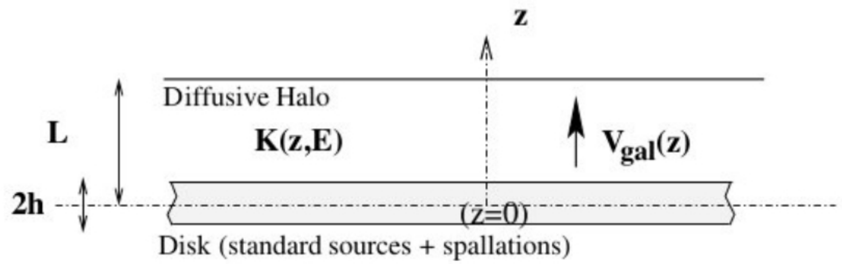

$$Q^r(z) = 2h\delta(z) n_{\mathrm{ISM}} v \sigma^{p\to r} N^p(z)$$

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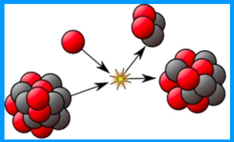

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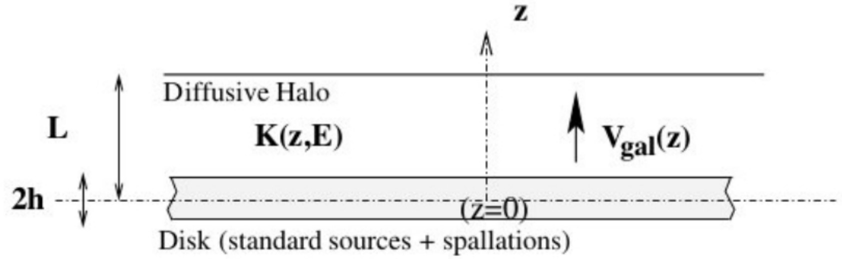

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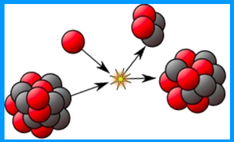
Find solution $N^r(0)$

N.B.: r is the distance, and we now change to (r,z) variables

$d^3r=4\pi r^2dr = 2\pi r dr dz$

Secondary radioactive nuclei (LE)



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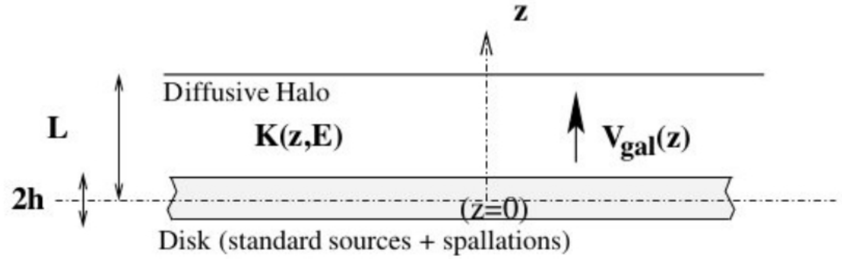

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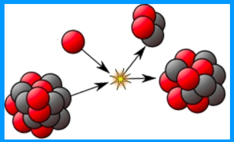
→ Direct measure of the diffusion coefficient

Secondary radioactive nuclei (LE)



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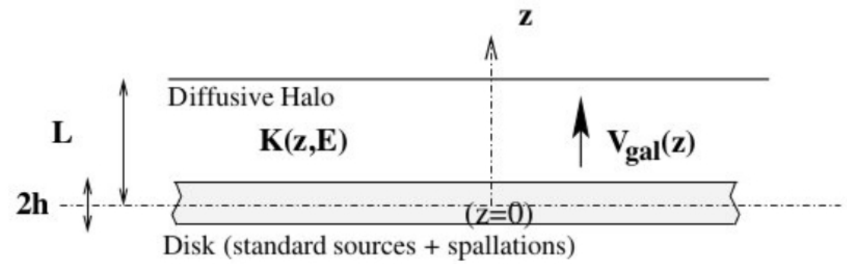

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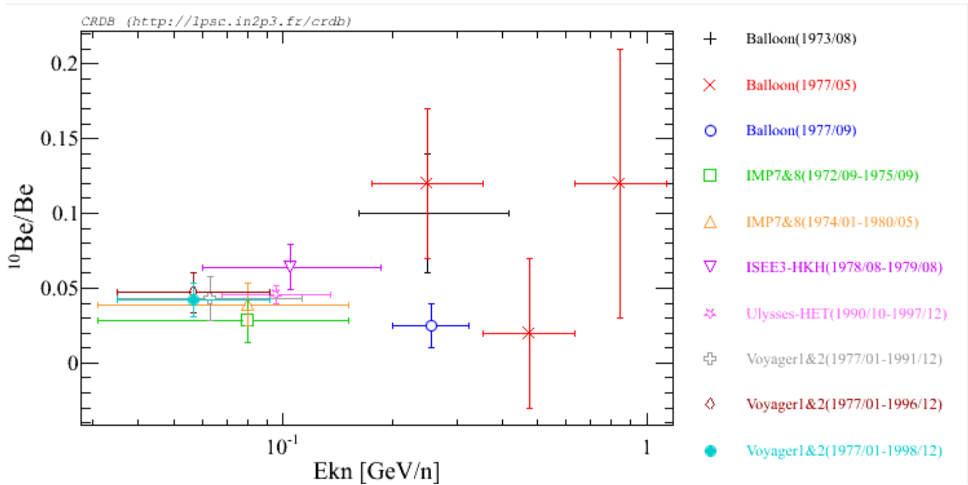
→ Direct measure of the diffusion coefficient

→ Indirect measure of L

But data are scarce...

- No direct measurement of ^{10}Be
- Only ~ one energy for ^{10}Be/Be or ^{10}Be/^{9}Be (also ^{26}Al, ^{36}Cl, ^{54}Mn)



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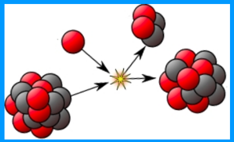

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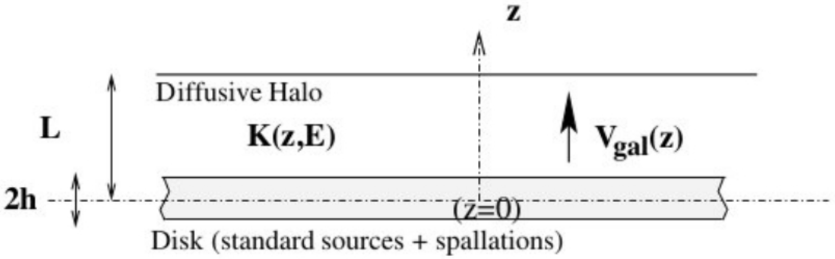

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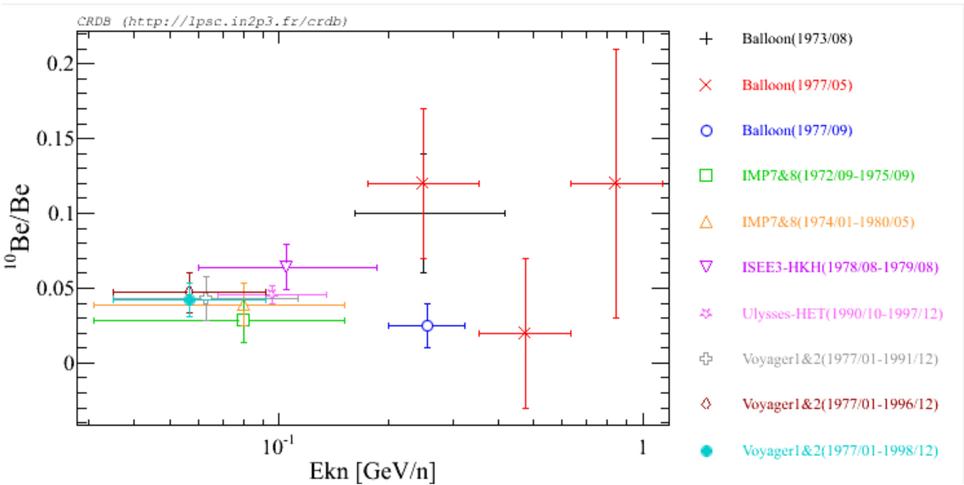
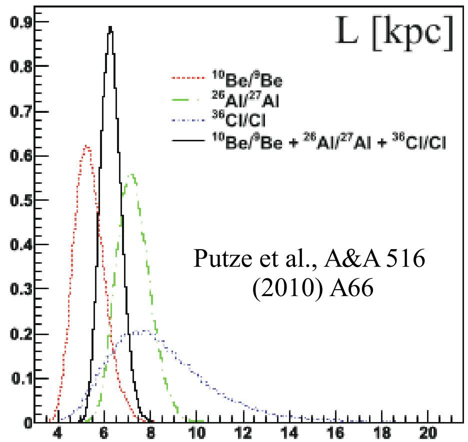




Secondary radioactive nuclei: the local ISM (LISM)

From how far away radioactive nuclei come?

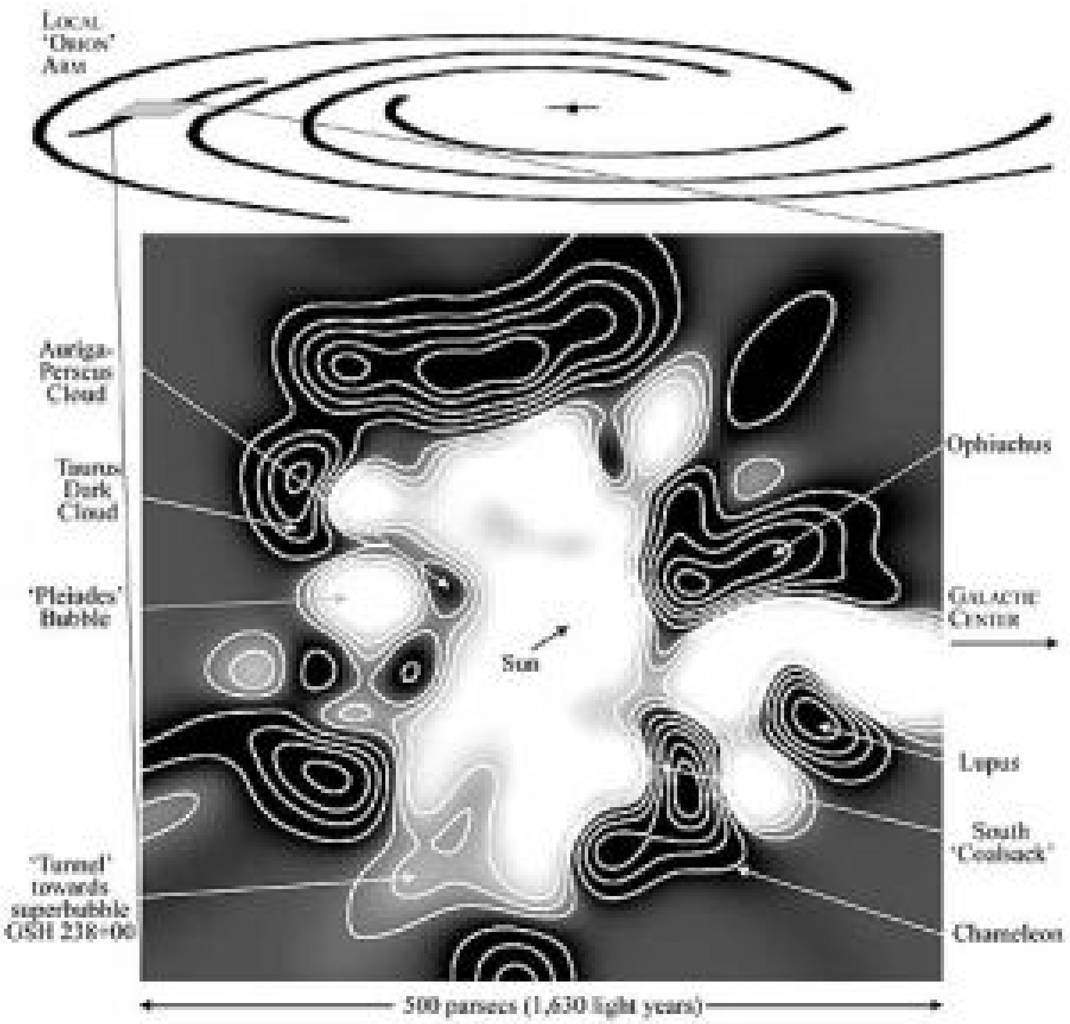
Calculate d (1 GeV)

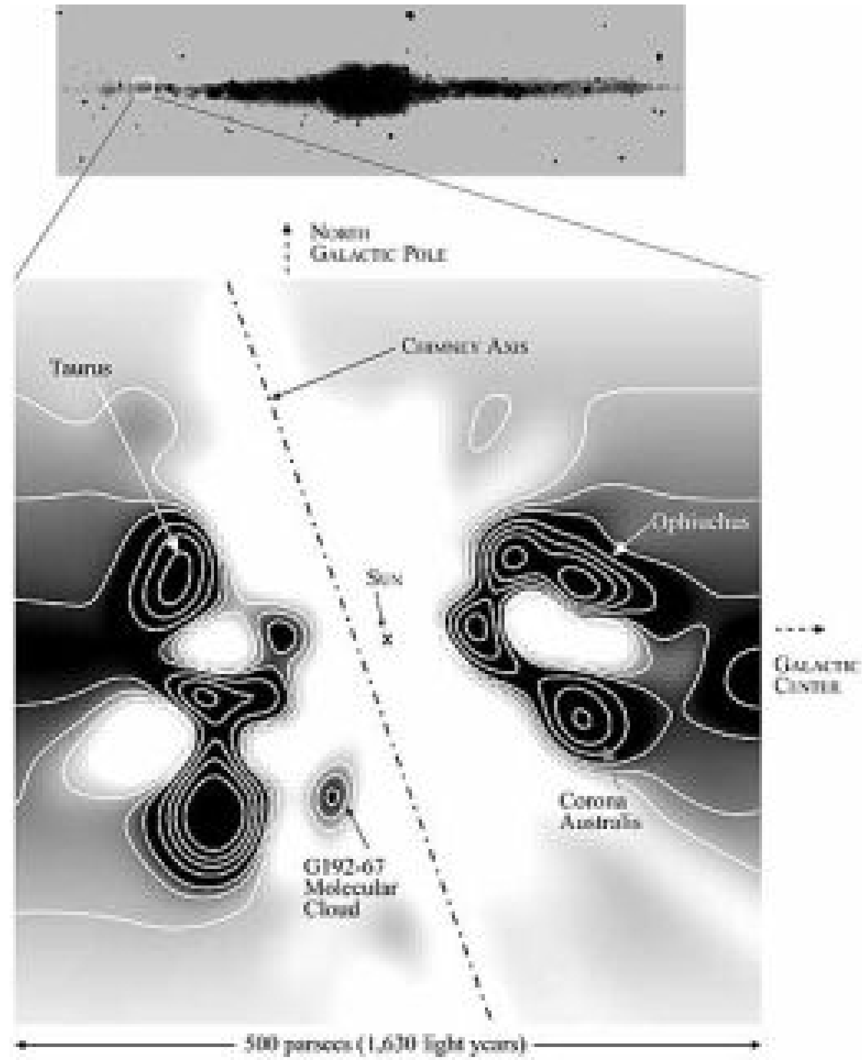
$D = 0.05\ \mathrm{kpc^2/Myr} \times (E/1\ \mathrm{GeV})^{0.5}$

$\tau\ (^{10}\mathrm{Be}) \sim 1.5\ \mathrm{Myr}$

Secondary radioactive nuclei: the local ISM (LISM)

Radioactive species are sensitive to small scales (~ 300 pc)
→ the local gas is not homogeneous on this scale!

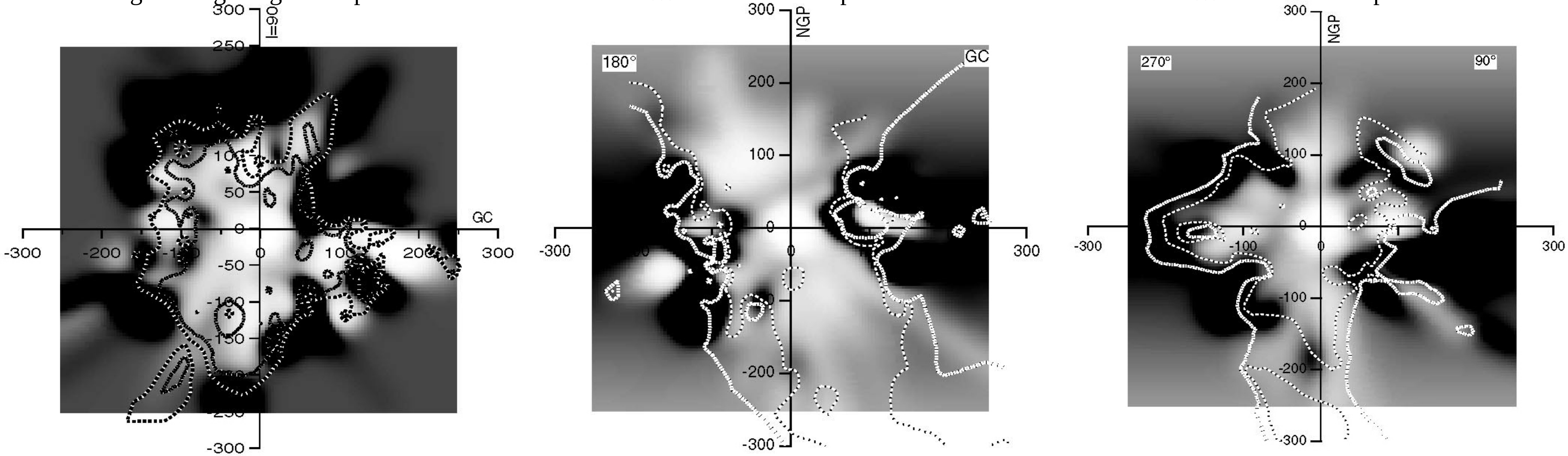




Secondary radioactive nuclei: LISM and local bubble

NaI absorption measurements (5890 Å): 1005 sight lines

Dense gas along the galactic plane … in the meridian plane … in the rotation plane



Lallement et al., A&A **411**, 447 (2003)

→ 20 SN explosions during the past 10-20 Myr (age of the local bubble)

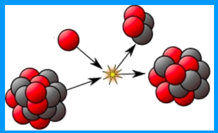
(local bubble linked to the formation of the Gould Belt 30-60 Myr ago?)

→ 1 more SN about 1 Myr ago

(The SN could be as close as ~ 40 pc from SS: contribution to the Pliocene-Pleistocen extinction?)

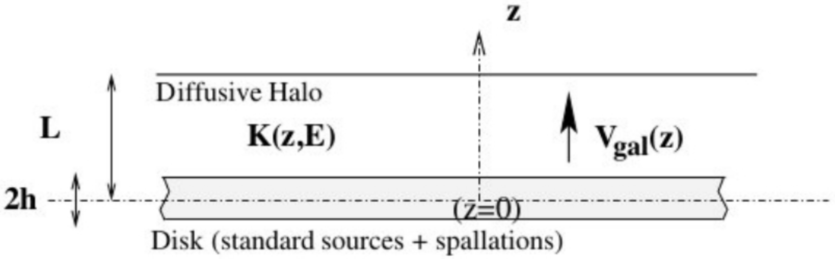
Maíz-Apellániz, ApJ **560**, L83 (2001)
Berghöfer & Breitschwerdt, A&A **390**, 299 (2002)
Benítez, Maíz-Apellániz & Canelles, Phys. Rev. Lett. **88**, 081101 (2002)

Secondary radioactive nuclei: toy model



$$Q^{sec}(R) = nv\sigma^{ps}N^p(0)$$

$$\frac{\partial N^j}{\partial t} + \left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right) + \vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)N^j + \frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right) + (\Gamma_{\rm rad} + \Gamma_{\rm inel})N^j = Q^j(t,E,\vec{r})$$



Solution for radioactive species

- Only decay and diffusion
- Unbounded diffusion (not sensitive to L)

$$-D\Delta_{\vec{r}}N^r + \frac{N^r}{\gamma\tau_0} = Q(r) \quad \Rightarrow \begin{cases} -\Delta_{\vec{r}}N^r + \lambda^2 N^r = \dfrac{Q(r)}{D} \\ \lambda = 1/\sqrt{D\gamma\tau_0} \end{cases}$$

$$Q^r(z) = 2h\delta(z)n_{\rm ISM}v\sigma^{p\to r}N^p(z)$$

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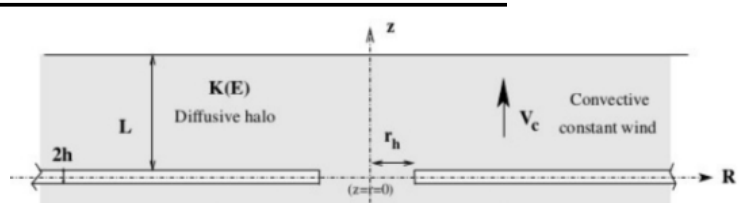
N.B.: r is the distance, and we now change to (r,z) variables

$$N^r(0) = hn_{\rm ISM}v\sigma^{p\to r}N^p(0)\sqrt{\frac{\gamma\tau_0}{D}}$$

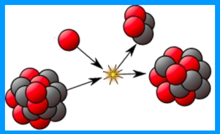
Extra parameter: size of the underdense region

→ Hole in the thin disc



$$\kappa \equiv \frac{N_{r_h}}{N_{r_h=0}}$$

Secondary radioactive nuclei: toy model



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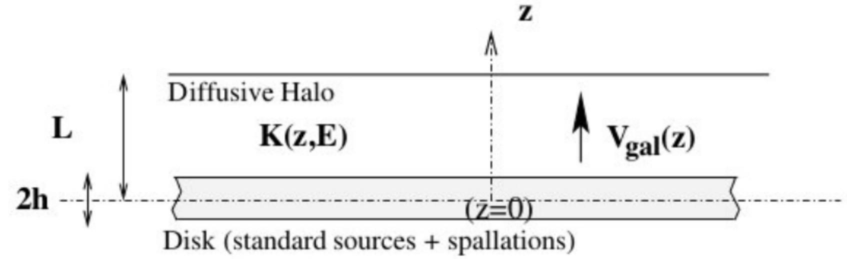

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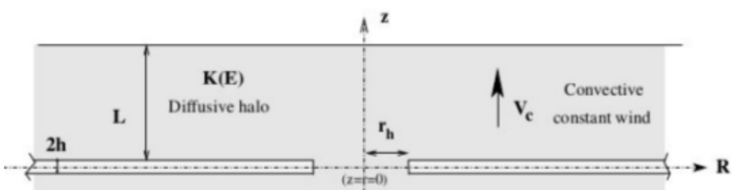
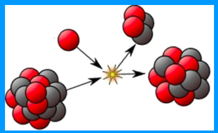


$$\kappa \equiv \frac{N_{r_h}}{N_{r_h=0}} = \exp\left(\frac{-r_h}{\sqrt{K\gamma\tau_0}}\right)$$

Donato et al, A&A 381 (2002) 539

Secondary radioactive nuclei: toy model



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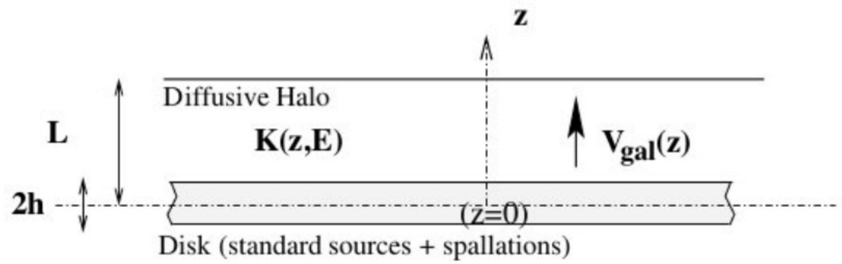

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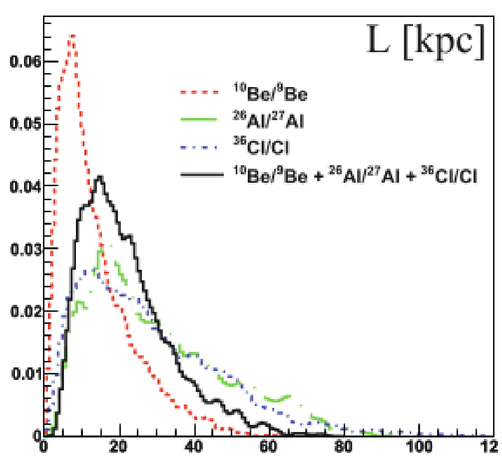
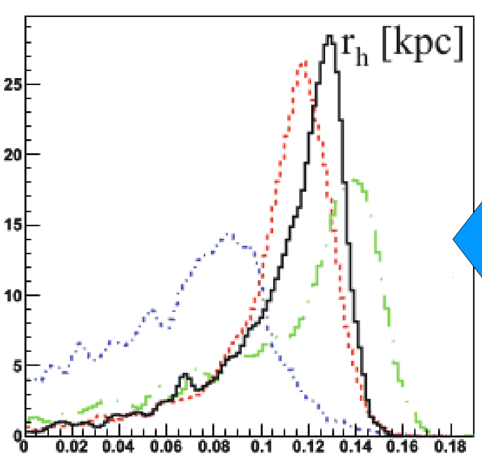
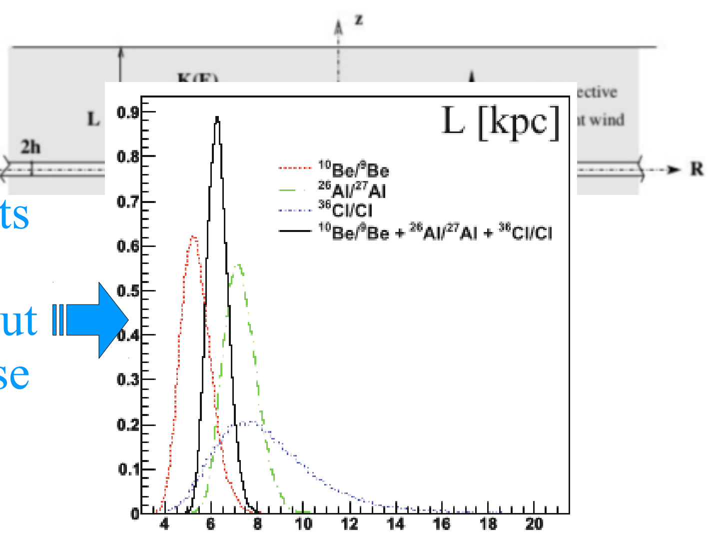


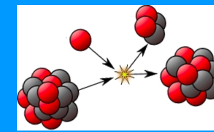


Constraints on L with/without underdense region



Putze et al., A&A 516 (2010) A66

Secondary radioactive nuclei: toy model



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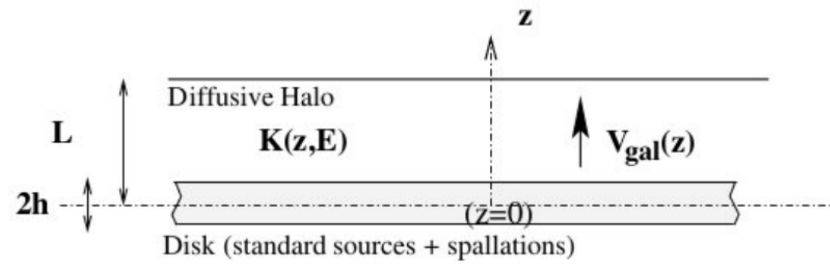

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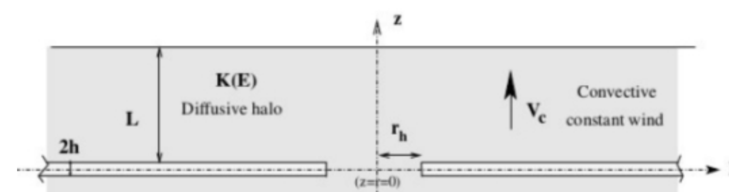


→ CR data point towards r_h~ 80 pc
→ extra parameter r_h: prevent lifting D_0/L degeneracy

→ D in the underdense region?
+ Bad fit Be/B data (nuclear uncertainties?)

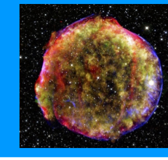
$$\kappa \equiv \frac{N_{r_h}}{N_{r_h=0}} = \exp\left(\frac{-r_h}{\sqrt{K\gamma\tau_0}}\right)$$

Transport of cosmic rays (CR) in the Galaxy

III. Solving the transport equations: GCR phenomenology

1. Diffusion: from microphysics to effective models
2. Full set of equations (with source terms)
3 (Semi-)Analytical, numerical, & MC solutions
4. Stable species: degeneracy K_0 /L
5. Radioactive species and local ISM
6. Leptons and local sources

Solution for primary electrons



$$Q^{\rm prim}(R) = q_0 R^{-\alpha}$$

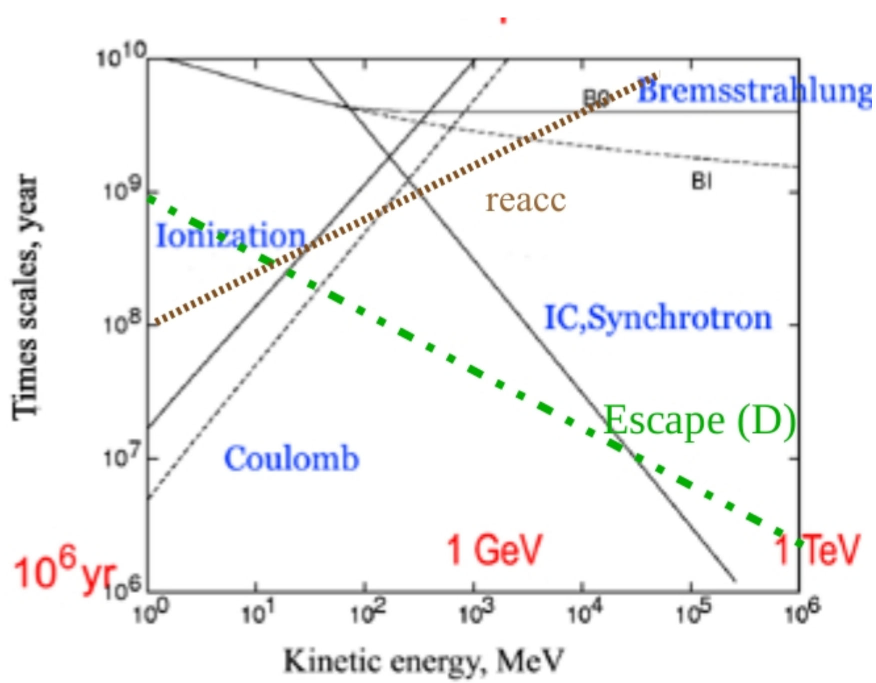
$$\frac{\partial N^j}{\partial t} + \left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right) + \vec{\nabla}\cdot\vec{V}_c(\vec{r})\right)N^j + \frac{\partial}{\partial E}\left(b^j N^j - D_{EE}\frac{\partial N^j}{\partial E}\right) + (\Gamma_{\rm rad} + \Gamma_{\rm inel})N^j = Q^j(t,E,\vec{r})$$

General time-dependent solution

Syrovatskii, Soviet Astronomy 3 (1959) 22

THE DISTRIBUTION OF RELATIVISTIC ELECTRONS IN THE GALAXY AND THE SPECTRUM OF SYNCHROTRON RADIO EMISSION

The problem of the diffusion of particles is solved, taking into account the regular changes of the particle energy during this process. The spatial dis-



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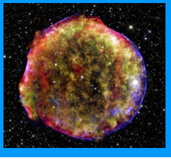

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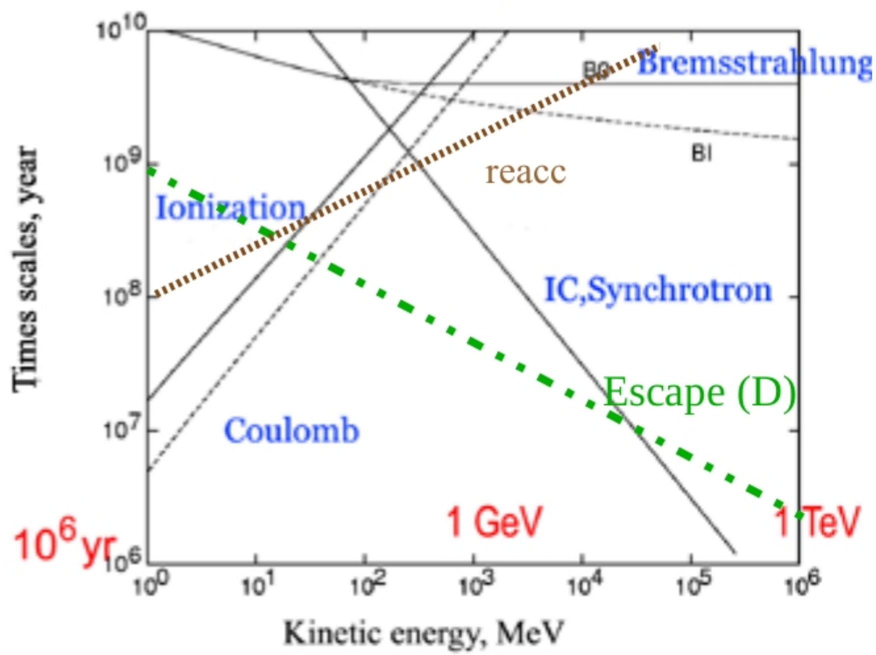
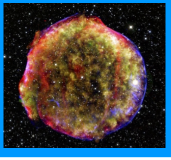
But no data to support time-dependent behaviour

For the problem of diffusion in the galaxy and the determination of the electron spectrum we can restrict ourselves to the investigation of the stationary conditions since there are no reasons for considering that the number of relativistic electrons supplied by the sources is appreciably time-dependent. Therefore, we shall use the source function (11) for the stationary case.



Solution for primary electrons



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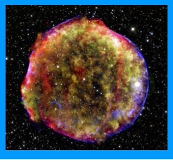
Synchrotron: B ~ 6 μG
IC: negligible

Green function solution (Ansatz: Syrovatskii variable)
→ similar variable used to solve propagation of UHECRs in the expanding Universe
Berezinsky & Gazizov, ApJ 643 (2006) 8
Alves Batista & Sigl (arXiv:1407.6150)

High energy electrons : local origin



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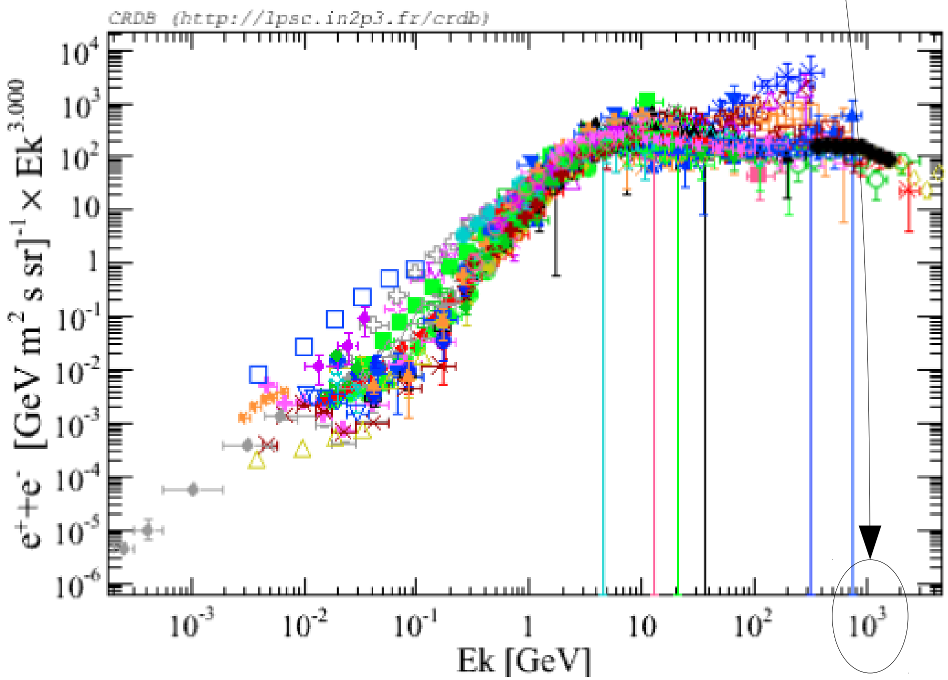
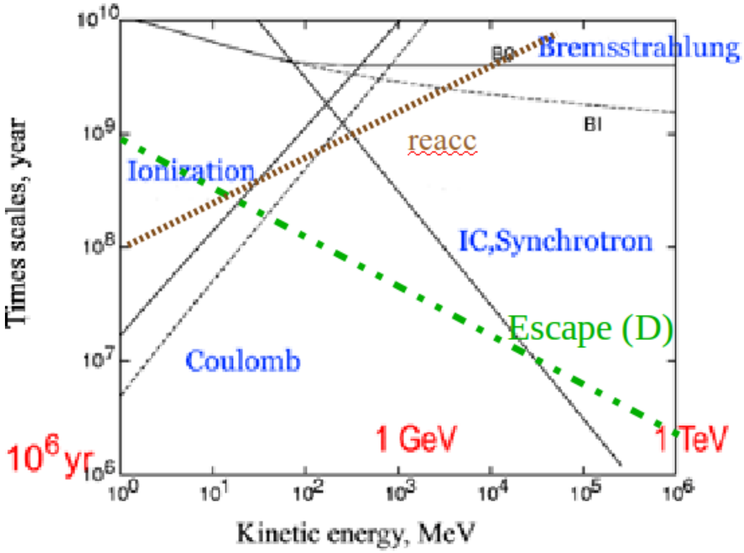
Origin of high energy electrons (TeV)

D = 0.05 kpc^2/Myr × (E/1 GeV)$^{0.5}$

Calculate d_{max} (1 TeV)





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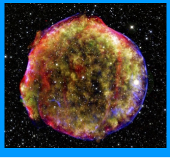

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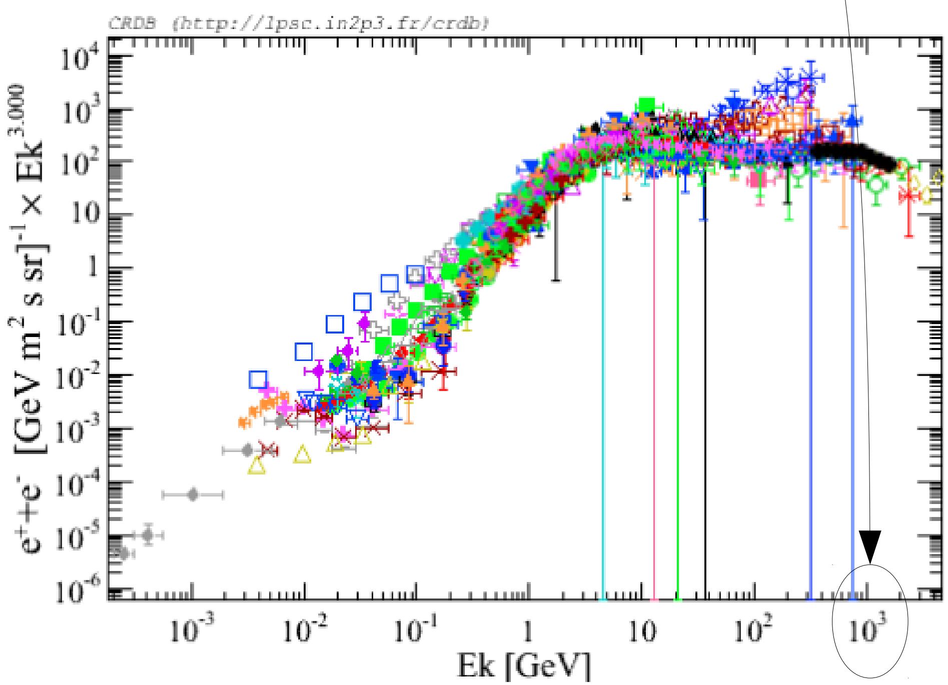
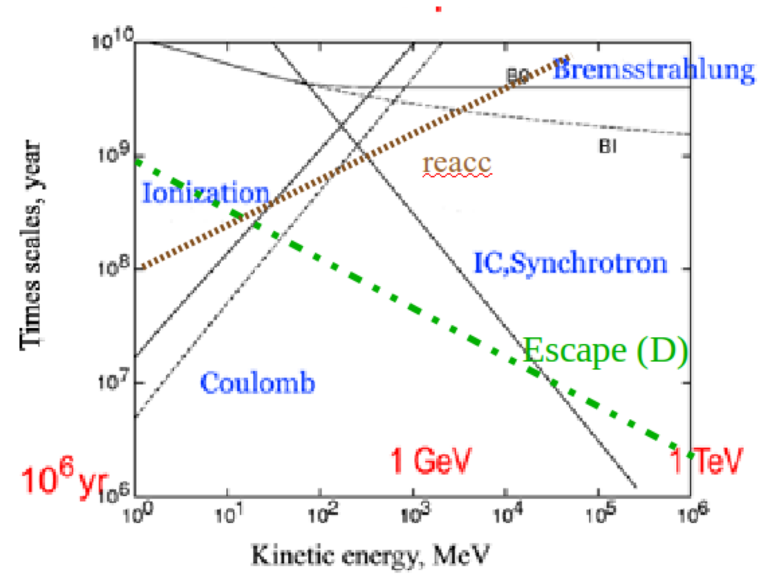
Origin of high energy electrons (TeV)

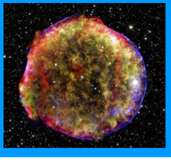
- $t_{IC} \sim 0.3$ Myr
- $d_{max} \sim (2Dt)^{1/2}$

$\rightarrow d_{max} \sim 1$ kpc





High energy electrons: single source?



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Singe source and cut-off in HE spectrum → very sensitive to D

Shen, ApJ 162 (1970) 181

PULSARS AND VERY HIGH-ENERGY COSMIC-RAY ELECTRONS

In the study of the propagation of cosmic-ray electrons, the use of a continuous source distribution is not valid in the range of very high energies. The electron spectrum in that energy range depends on the age and distance of a few local sources. It is shown that if the far-infrared background discovered recently exists in the Galaxy, the very high-energy electrons observed at Earth probably all come from the source Vela X, and a cutoff energy at about 2×10^3 BeV is predicted. Implications on the propagation of cosmic rays in the Galaxy are discussed.

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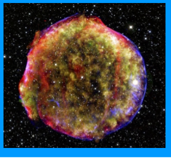

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Time-dependent solution for constant D

$$N_i(E) = \int_{(bE)^{-1}}^{0} \frac{K_i(t) f_i[E/(1-bEt)] \exp(-r_i^2/4Dt)}{(4\pi Dt)^{3/2}(1-bEt)^2}\, dt$$

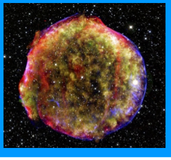
$Q_i = K_i(t) f_i(E)$ at a distance r_i :

$$N_i(E) = Q_i(E).(4\pi D t_i)^{-3/2}(1-bEt_i)^{\alpha-2}\exp(-r_i^2/4Dt_i) \quad \text{for } E < E_i = (bt_i)^{-1}$$

$$= 0 \quad \text{for } E > E_i = (bt_i)^{-1}$$

[α = injection spectral index]

High energy electrons: single source?



$Q^{\rm prim}(R) = q_0 R^{-\alpha}$

$$\cancel{\frac{\partial N^j}{\partial t}} + \left(-\vec{\nabla}\cdot\left(D(E,\vec{r})\vec{\nabla}\right) + \cancel{\vec{\nabla}\cdot\vec{V}_c(\vec{r})}\right)N^j + \frac{\partial}{\partial E}\left(b^j N^j - \cancel{D_{EE}\frac{\partial N^j}{\partial E}}\right) + \cancel{(\Gamma_{\rm rad} + \Gamma_{\rm inel})N^j} = Q^j(t,E,\vec{r})$$

Origin of high energy electrons (TeV)

- $t_{IC} \sim 0.3$ Myr
- $d_{max} \sim (2Dt)^{1/2}$

$\rightarrow d_{max} \sim 1$ kpc

Singe source and cut-off in HE spectrum → very sensitive to D

Shen, ApJ 162 (1970) 181

PULSARS AND VERY HIGH-ENERGY COSMIC-RAY ELECTRONS

In the study of the propagation of cosmic-ray electrons, the use of a continuous source distribution is not valid in the range of very high energies. The electron spectrum in that energy range depends on the age and distance of a few local sources. It is shown that if the far-infrared background discovered recently exists in the Galaxy, the very high-energy electrons observed at Earth probably all come from the source Vela X, and a cutoff energy at about 2×10^3 BeV is predicted. Implications on the propagation of cosmic rays in the Galaxy are discussed.

Time-dependent solution for constant D

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$Q_i = K_i(t) f_i(E)$ at a distance r_i

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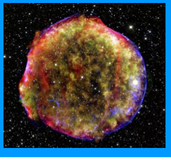
$$= 0 \quad \text{for } E > E_i = (bt_i)^{-1}$$

[α = injection spectral index]

Procedure (use of 50 pulsars)

→ sources @ r>1kpc: continuous space-time distribution
→ sources @ r<1kpc

High energy electrons: single source?



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Origin of high energy electrons (TeV)

Atoyan, Aharonian &Völk, PRD 52 (1995) 3265
Electrons and positrons in the galactic cosmic rays

→ *Apply procedure of Shen (1970)*
→ *More general solutions and analysis*

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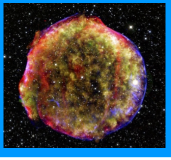

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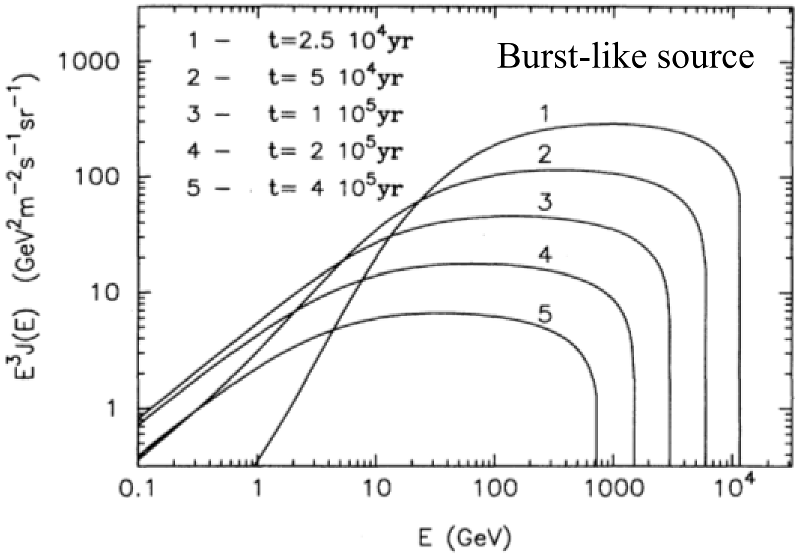
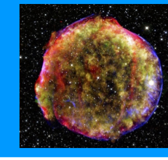


Injection slope α=2.2
Diffusion slope δ=0.6

High energy electrons: single source?



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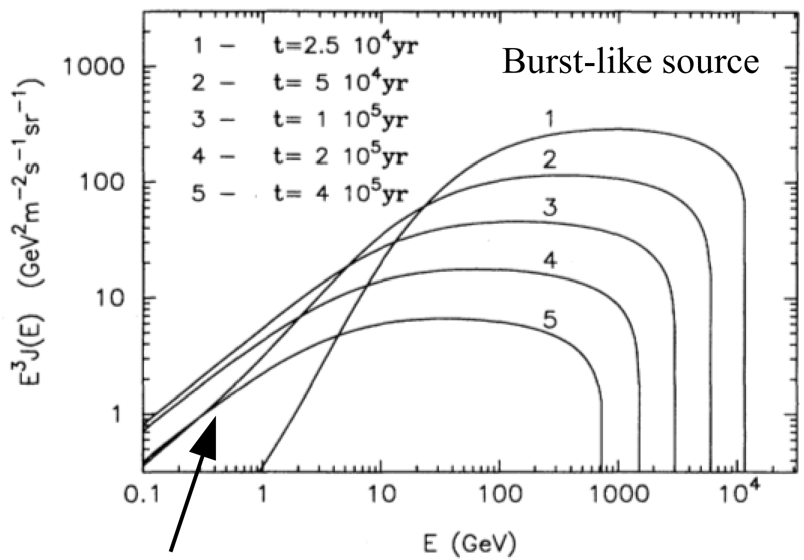

Low energy e± have not reached yet the observer

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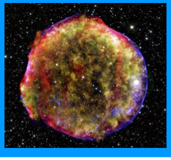

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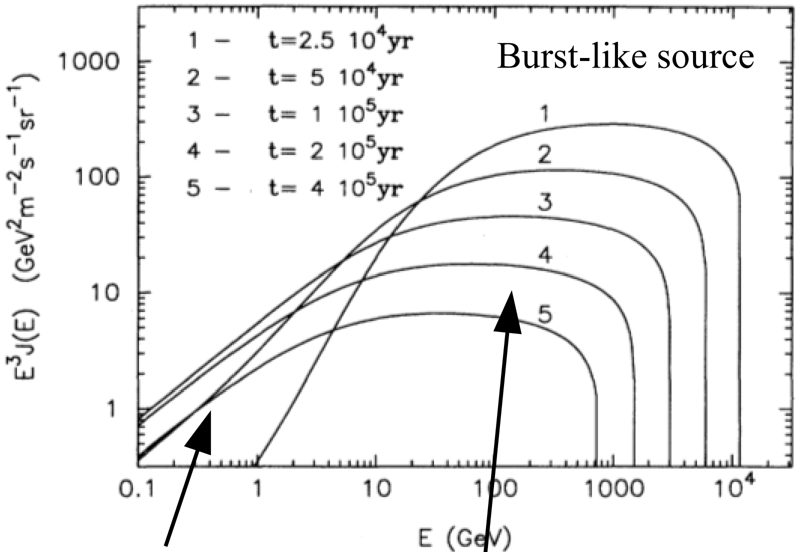

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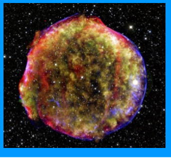
Balance transport/E losses give slope α+(3/2)δ~3.1

High energy electrons: single source?



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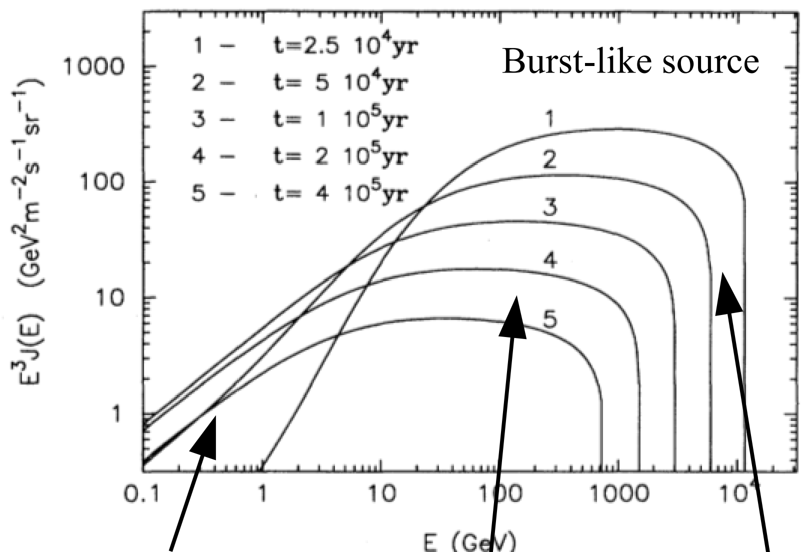

Injection slope α=2.2
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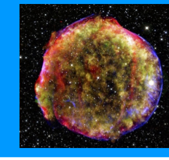
Cut-off from energy losses

Balance transport/E losses give slope α+(3/2)δ~3.1

High energy electrons: single source?



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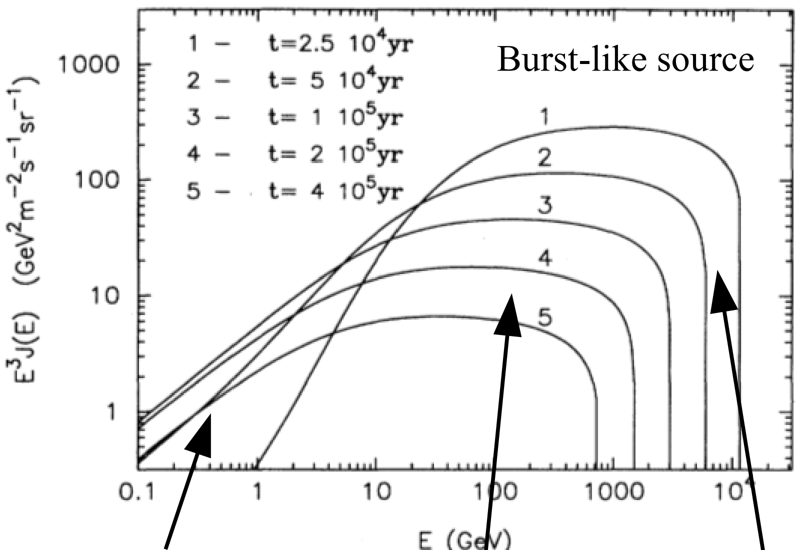

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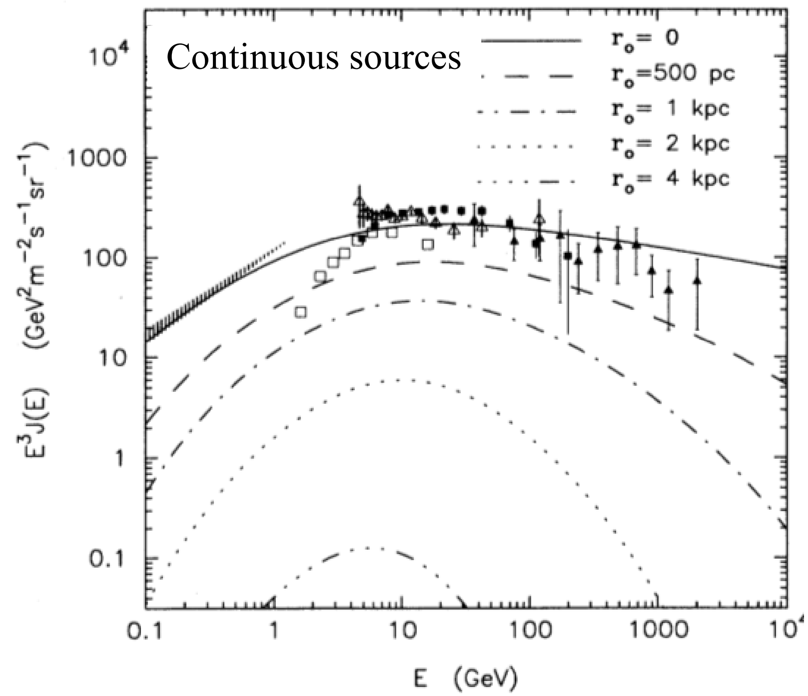

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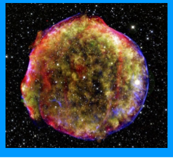

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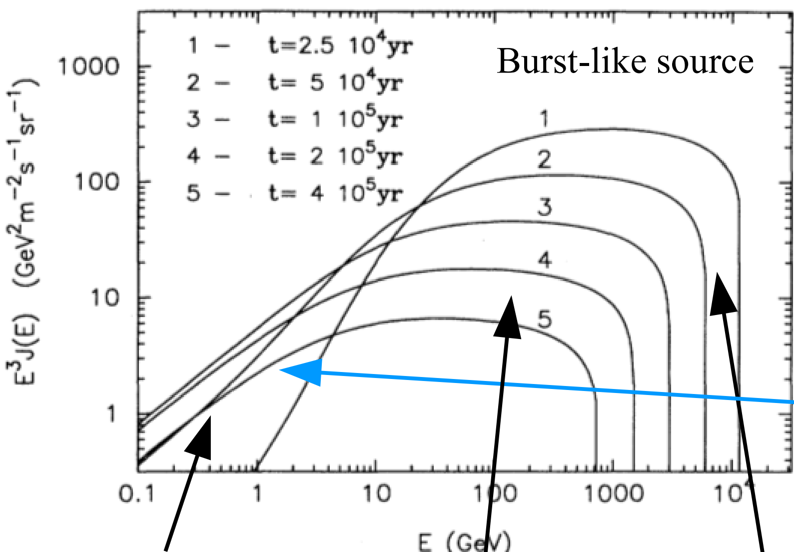
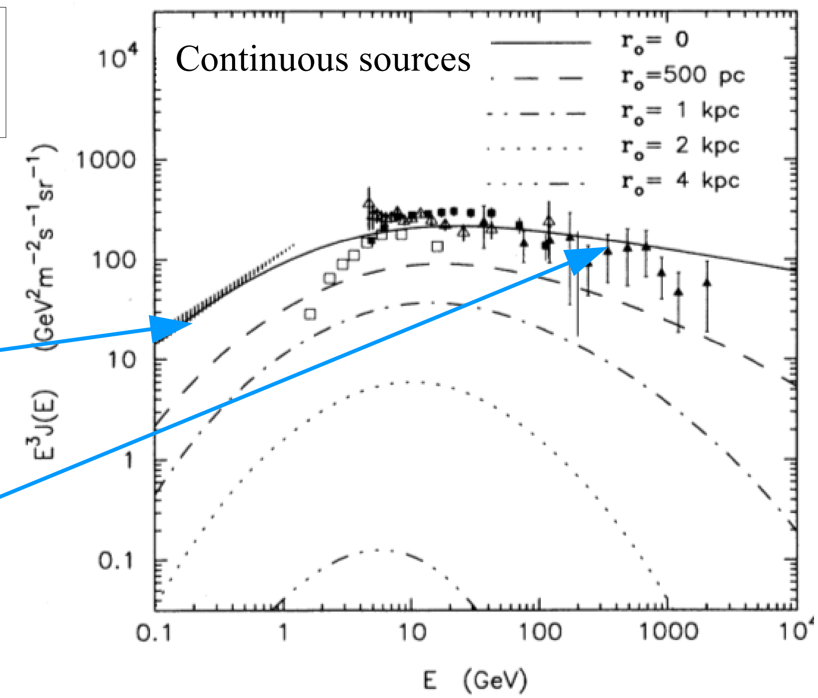


Injection slope α=2.2
Diffusion slope δ=0.6

slope ~ α ~ 2.2

slope ~ α+δ ~ 2.8



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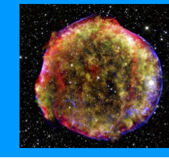

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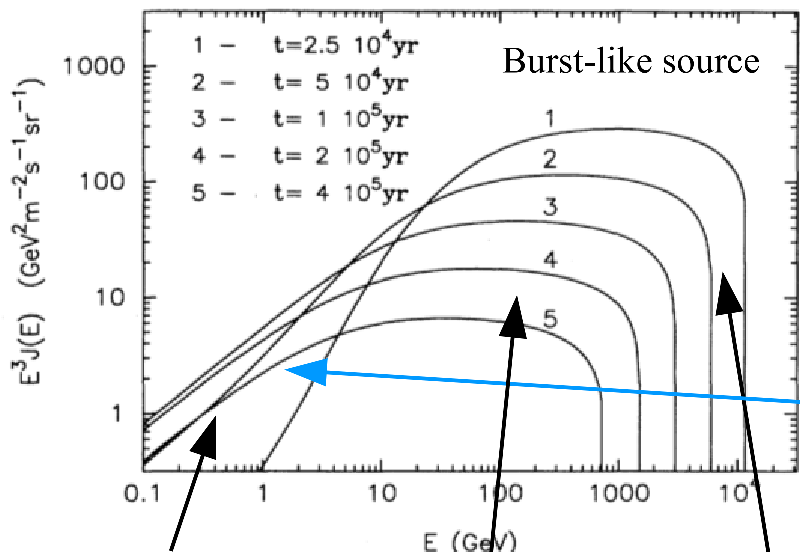

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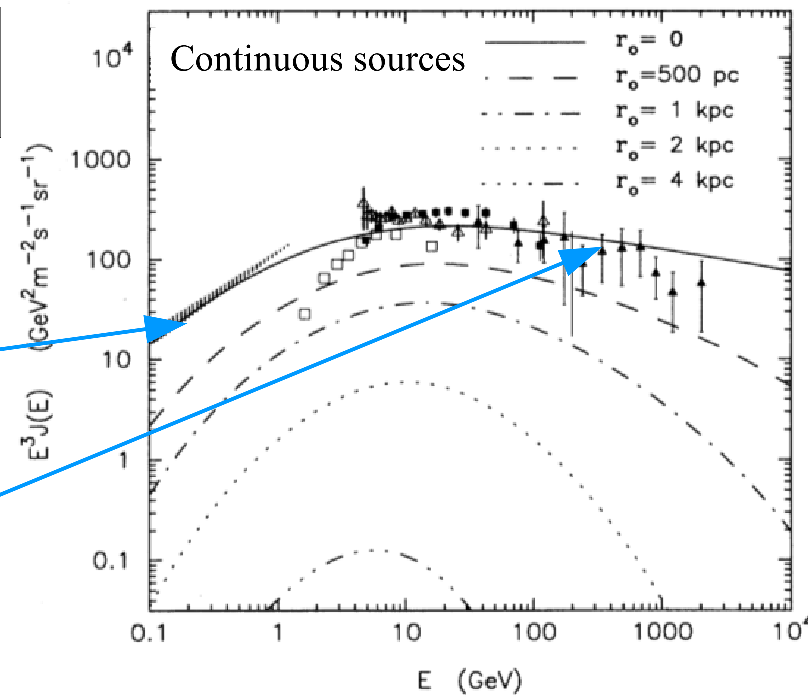


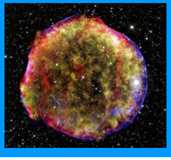
Low energy e± have not reached yet the observer

Cut-off from energy losses

Balance transport/E losses give slope α+(3/2)δ~3.1

→ Main difficulty is to get the numbers (sources, diffusion) correct

HE e± spectrum from a single local source



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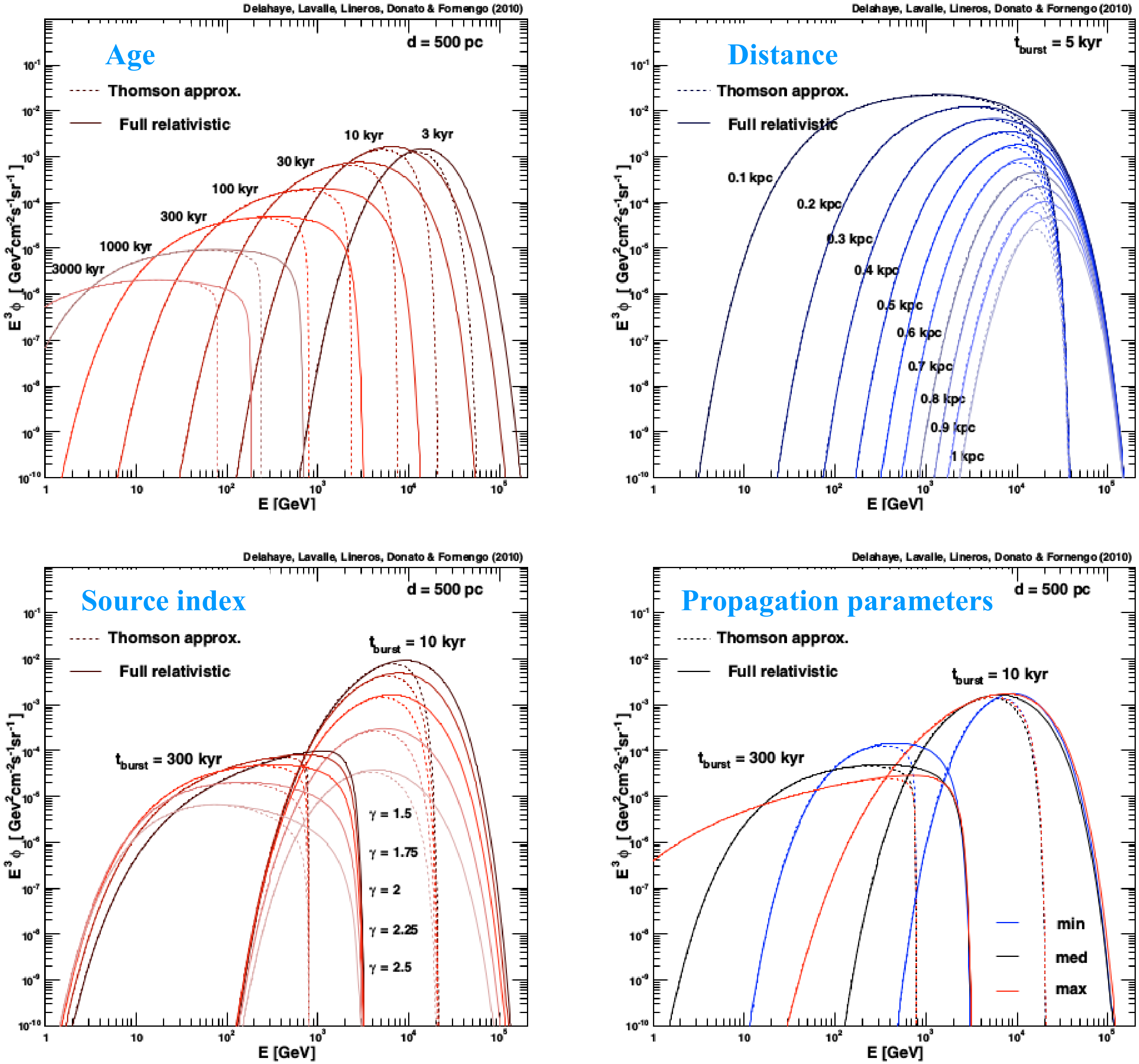


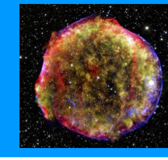
Delahaye et al., A&A 524 (2010) A51

Catalog of known nearby SNRs (and pulsars)

Delahaye et al., A&A 524 (2010) A51

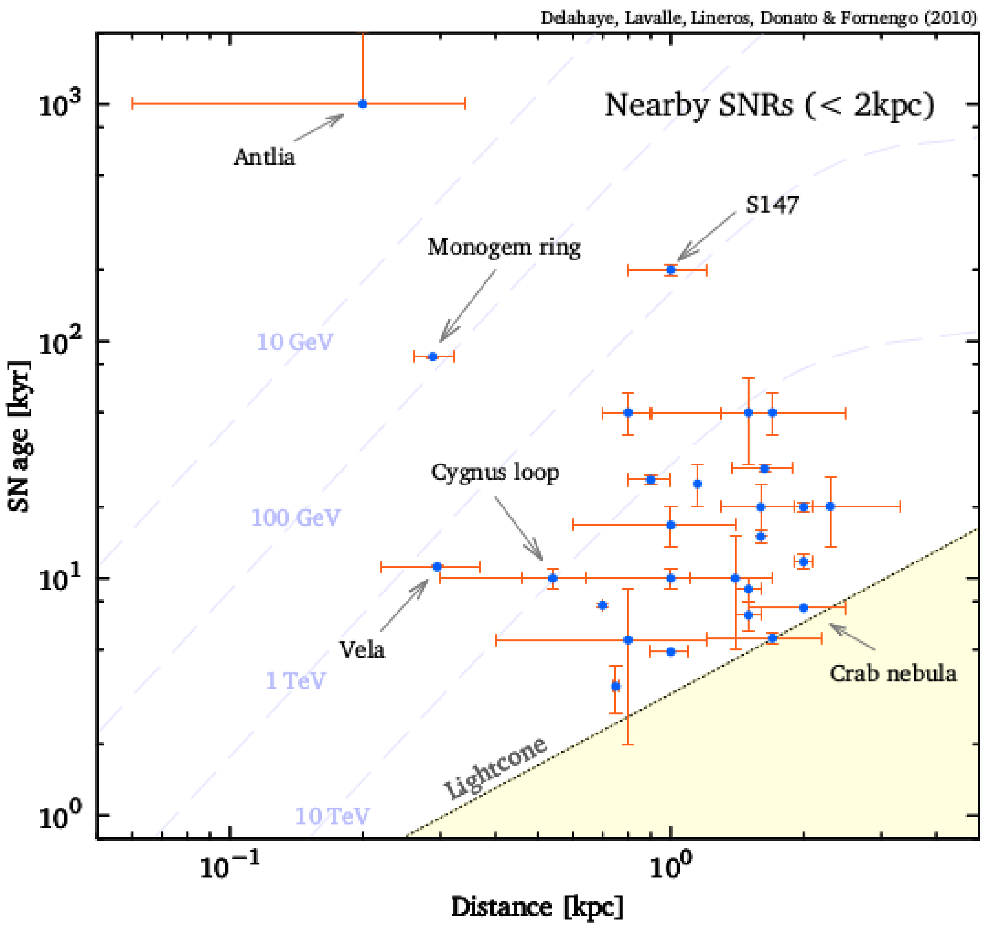
#	SNR G+long+lat	other name	distance [kpc]	radio index	Brightness [Jy]	age [kyr]	Pulsar
1	18.95 − 1.1		2. ± 0.1	0.28	40	11.75 ± 0.85	?
2	65.3 + 5.7		0.9 ± 0.1	0.58 ± 0.07	52	26 ± 1	–
3	65.7 + 1.2	DA 495	1.0 ± 0.4	0.45 ± 0.1	5	16.75 ± 3.25	unknown
4	69.0 + 2.7	CTB 80	2.0 ± 0.1	0.20 ± 0.10	60 ± 10	20 ± 1	J1952+3252
5	74.0 − 8.5	Cygnus Loop	$0.54^{+0.10}_{-0.08}$	0.4 ± 0.06	175 ± 30	10 ± 1	–
6	78.2 + 2.1	γ Cygni	1.5 ± 0.1	0.75 ± 0.03	275 ± 25	7 ± 1	–
7	82.2 + 5.3	W63	2.3 ± 1.0	0.36 ± 0.08	105 ± 10	20.1 ± 6.6	–
8	89.0 + 4.7	HB 21	1.7 ± 0.5	0.27 ± 0.07	200 ± 15	5.60 ± 0.28	–
9	93.7 − 0.2	CTB 104A or DA 551	1.5 ± 0.2	0.52 ± 0.12	42 ± 7	50 ± 20	–
10	114.3 + 0.3		0.7	0.49 ± 0.25	6.4 ± 1.4	7.7 ± 0.1	–
11	116.5 + 1.1		1.6	0.16 ± 0.11	10.9 ± 1.2	20 ± 5	B2334+61 ?
12	116.9 + 0.2	CTB 1	1.6	0.33 ± 0.13	6.4 ± 1.4	20 ± 5	B2334+61 ?
13	119.5 + 10.2	CTA 1	1.4 ± 0.3	0.57 ± 0.06	42.5 ± 2.5	10 ± 5	J0010+7309
14	127.1 + 0.5	R5	1.±0.1	0.43 ± 0.1	12 ± 1	25 ± 5	–
15	156.2 + 5.7		0.8 ± 0.5	$2.0^{+1.1}_{-0.7}$	4.2 ± 0.1	10 ± 1	B0450+55 ?
16	160.9 + 2.6	HB 9	0.8 ± 0.4	0.48 ± 0.03	~ 75	5.5 ± 1.5	B0458+46
17	180.0 − 1.7	S147	1.2 ± 0.4	0.75	74 ± 12	600 ± 10	J0538+2817
18	184.6-5.8	Crab nebula or 3C144 or SN1054	2.0 ± 0.5	0.3	1,040	7.5 ⋆	B0521+31
19	189.1 + 3.0	IC 443	1.5 ± 0.1	0.36 ± 0.04	160 ± 5	30 or 4	–
20	203.0 + 12.0	Monogem ring	$0.288^{+0.033}_{-0.027}$			86 ± 1	B0656+14
21	205.5 + 0.5	Monoceros Nebula	1.63 ± 0.25	0.66 ± 0.2	156.1 ± 19.9	29 ± 1	–
22	263.9 − 3.3	Vela(XYZ)	0.295 ± 0.075	variable	2,000 ± 700	11.2 ± 0.1	B0833-45
23	266.2-1.2	RX J0852.0-4622 or Vela Jr or SN1300	0.75 ± 0.01			3.5 ± 0.8 ⋆?	J0855-4644 ?
24	276.5 + 19.0	Antlia	0.2 ± 0.14			≥1000	B0950+08
25	315.1 + 2.7		1.7 ± 0.8	0.7		50 ± 10	J1423-56
26	330.0 + 15.0	Lupus Loop	1.2 ± 0.3			50 ± 10	B1507-44 ?
27	347.3 − 0.5	SN393	1.± 0.3			4.9 ⋆	–

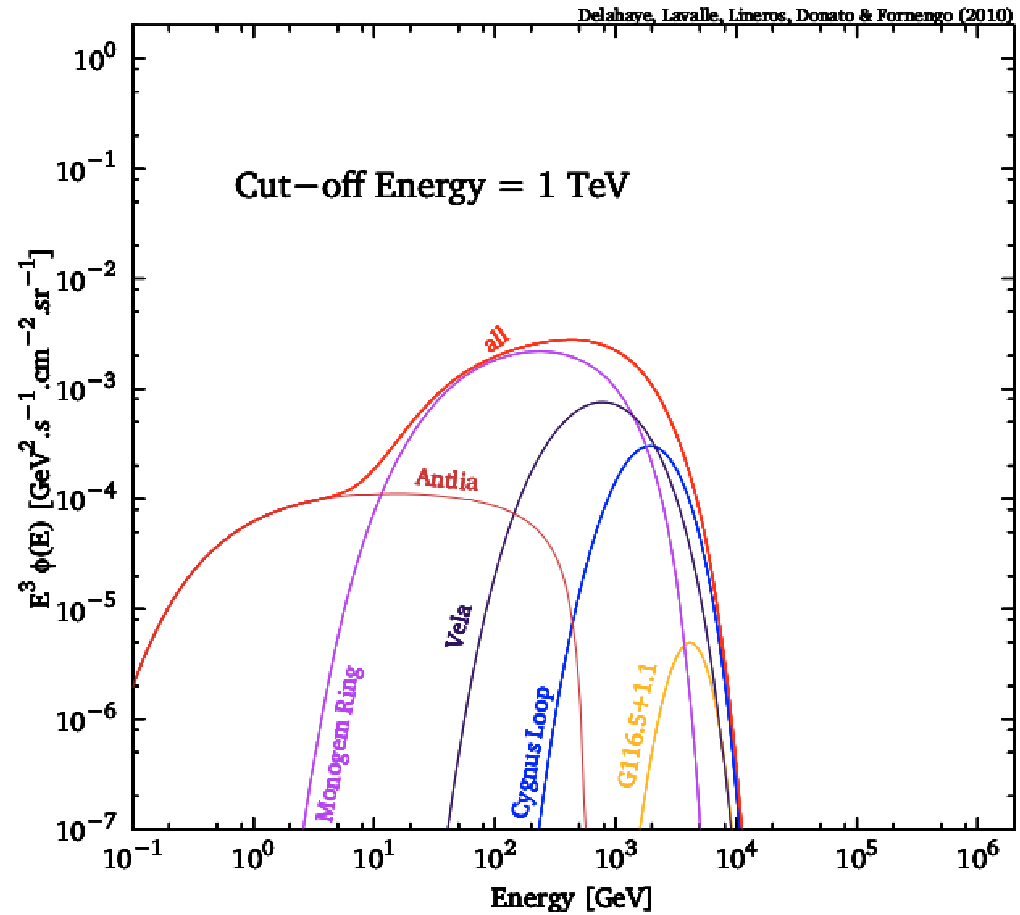
Electron flux from SNRs (<2 kpc)



$Q^{\mathrm{prim}}(R) = q_0 R^{-\alpha}$

Delahaye et al., A&A 524 (2010) A51





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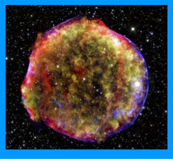

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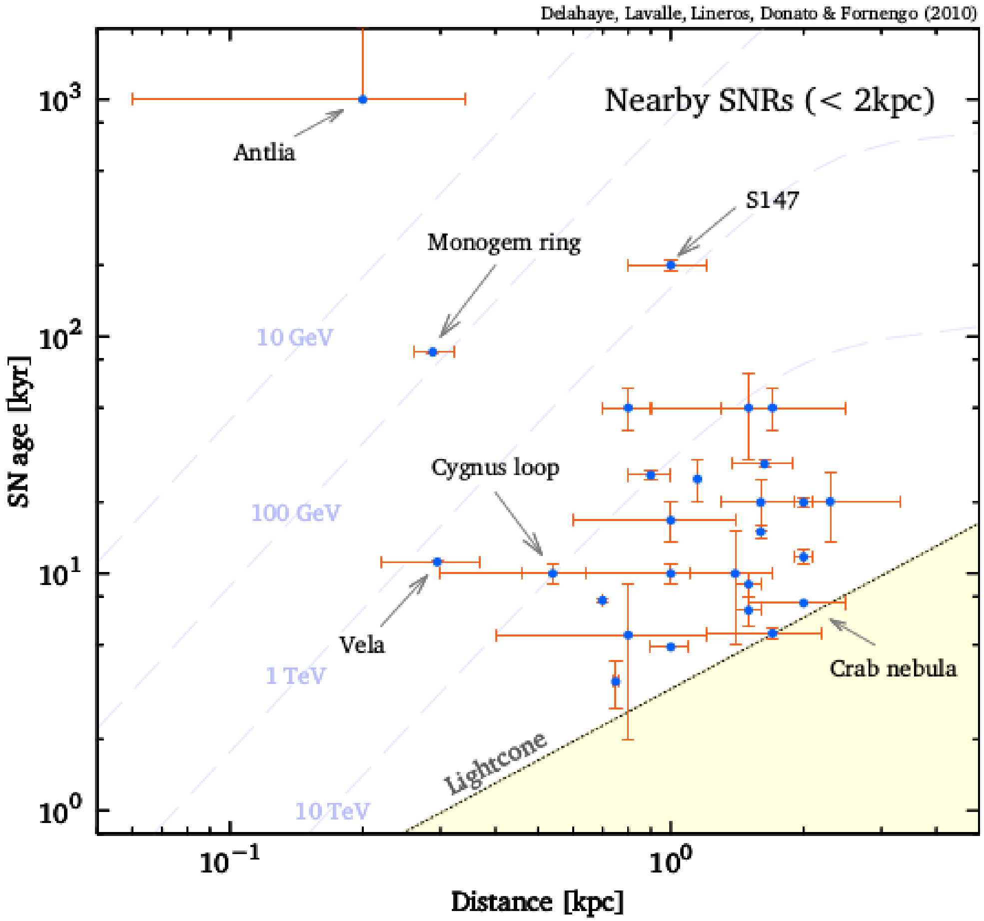

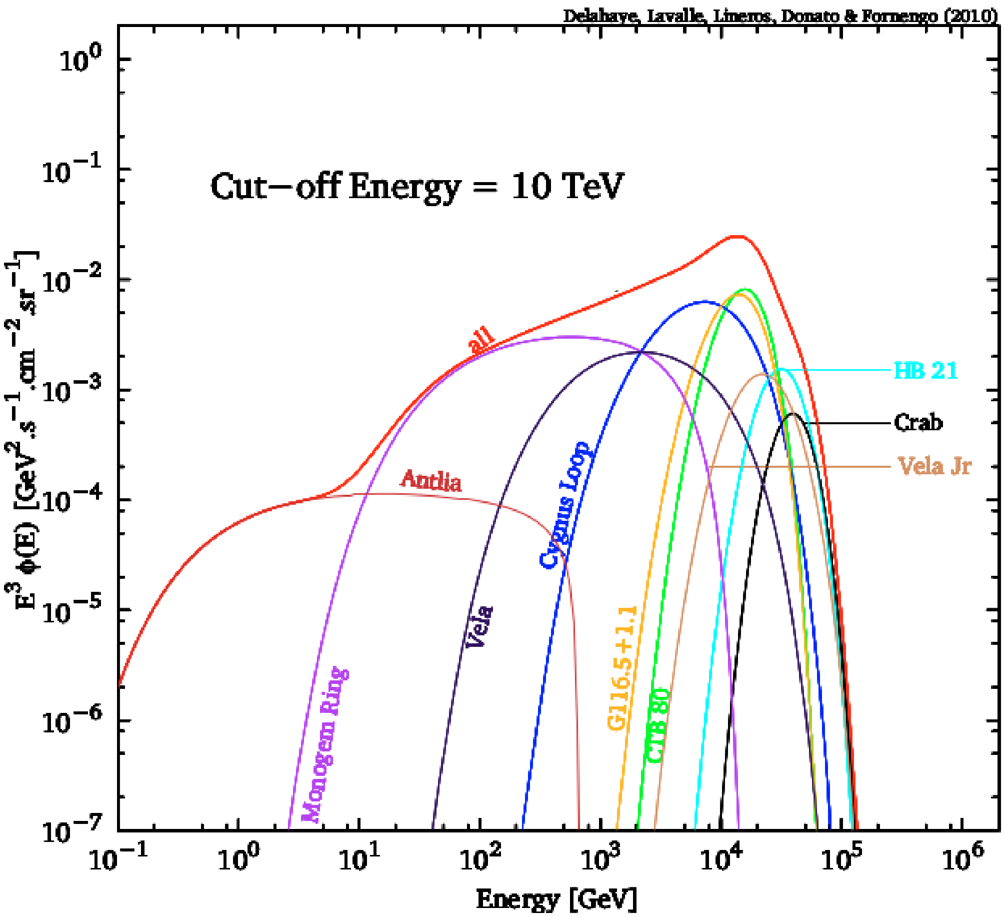

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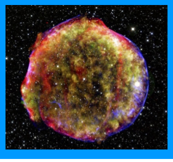

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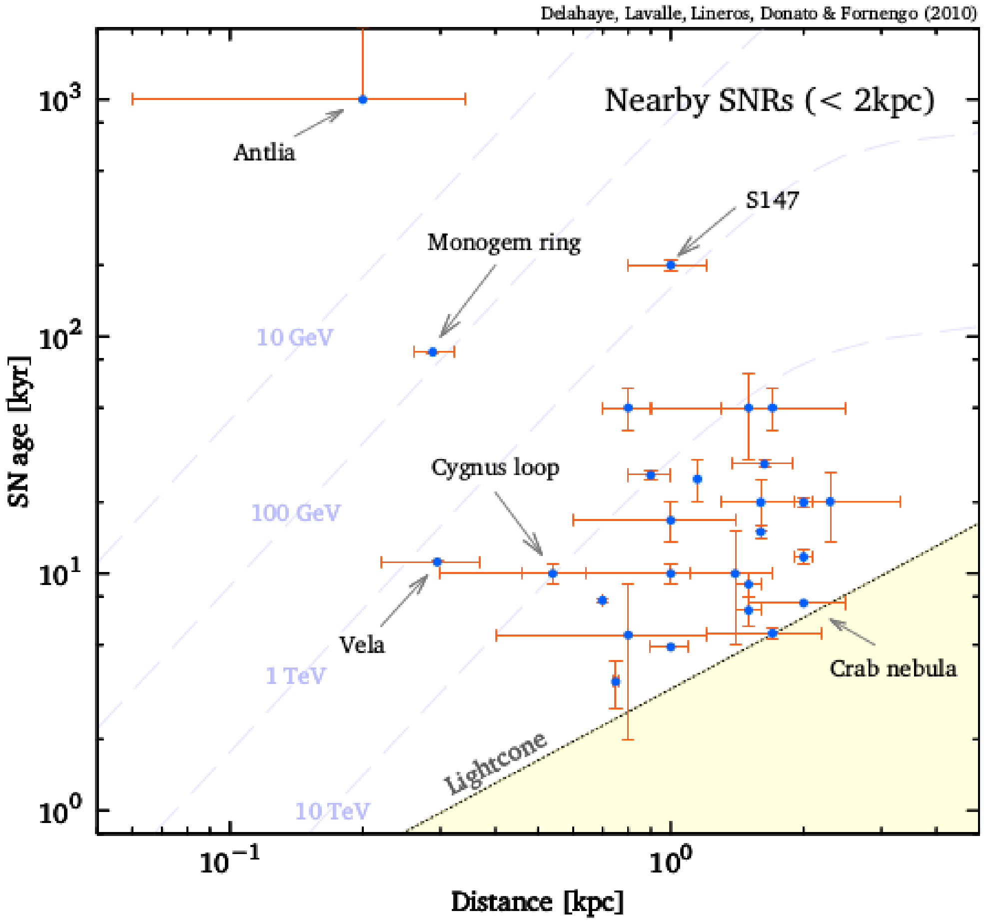
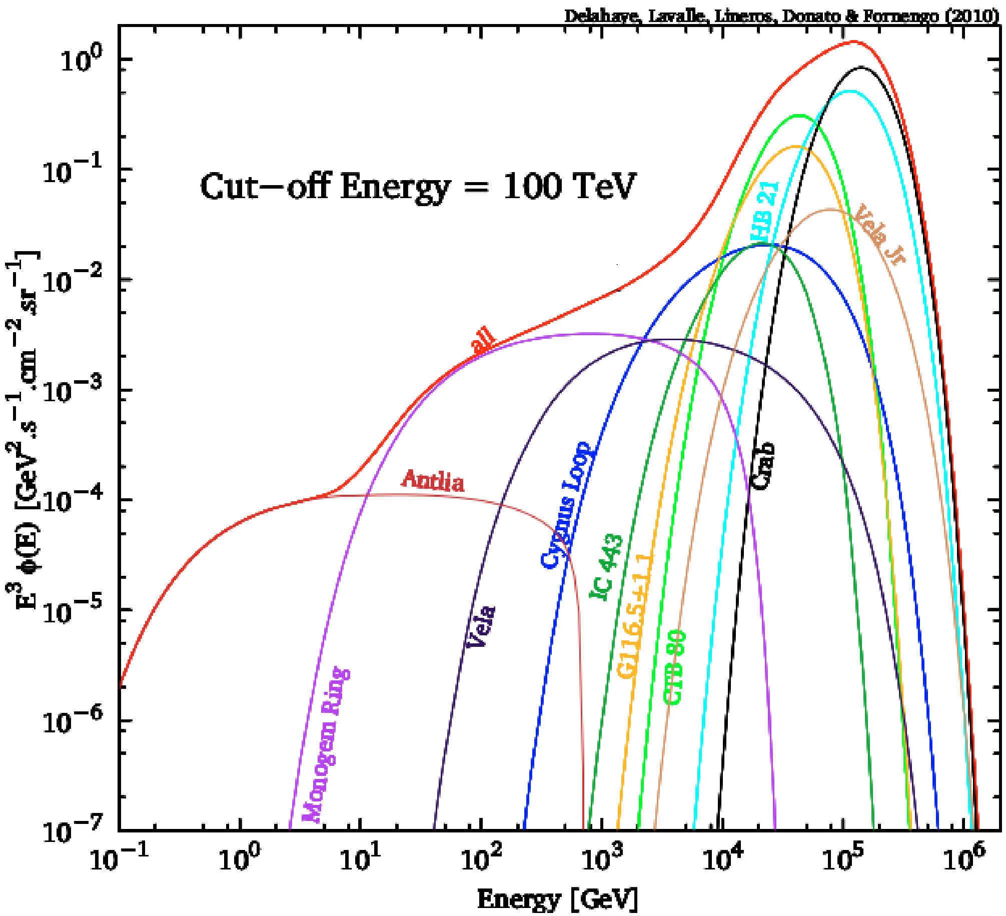
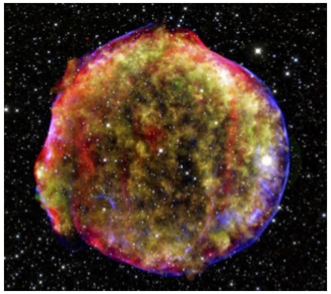




Full calculation for e±: distant+local contributions

Delahaye et al., A&A 524 (2010) A51



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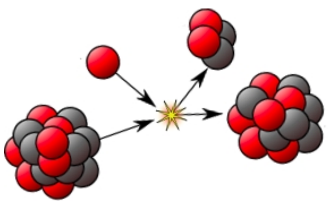
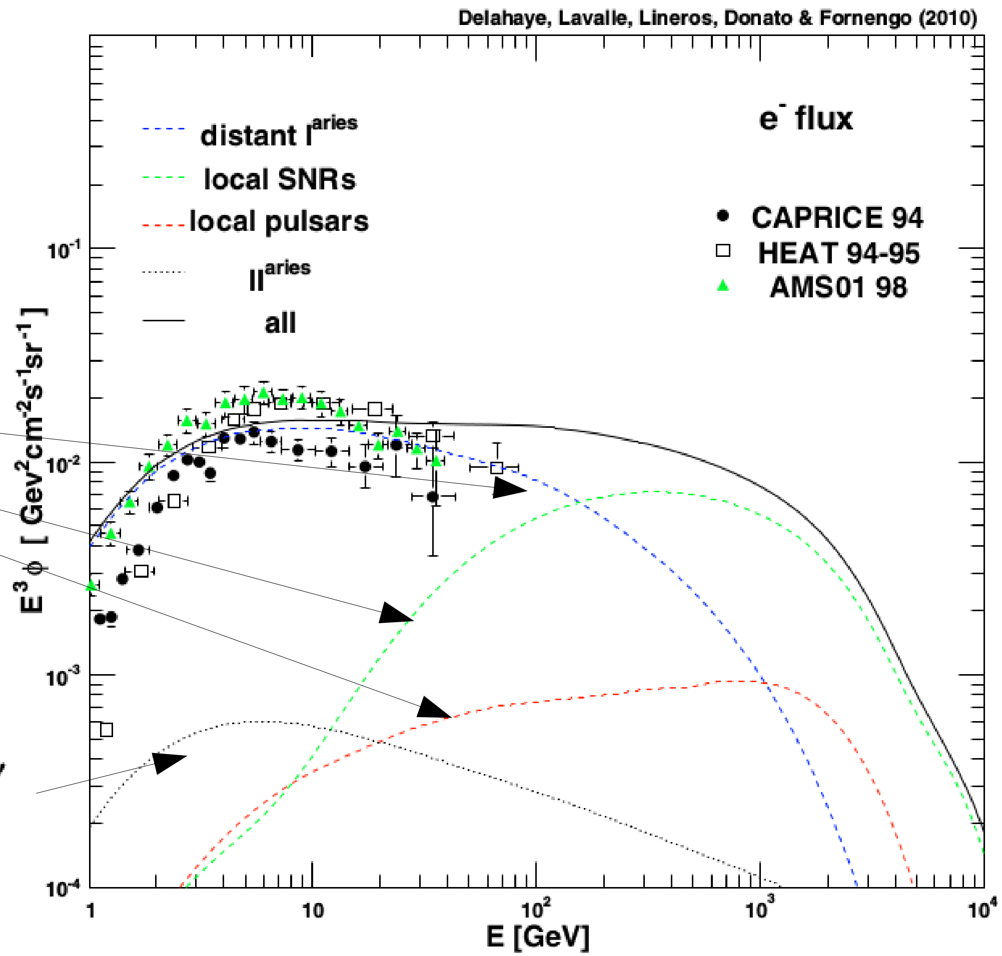


$$Q^{\text{secondary}}_{\bar{p},\bar{d},e^{\pm},\gamma,\nu}(r,E) = \sum_{(p,He)+(H,He)} \int n v' \frac{d\sigma}{dE} N^{p,He}(E')dE'$$



Exact flux difficult to predict because of uncertainties on

- Source position/age/spectrum
- Transport parameters (D value)
- Energy losses (B value/geometry)

« Standard » GCRs : summary and perspectives

Lecture II: processes, ingredients, characteristic times

→ **Different time scales for nuclei and leptons**

Nuclei
- Diffusive escape at high energy (>10 GeV/n)
- All effects compete @ GeV/n (convection, losses, reacceleration)

Leptons
- E losses dominate at high energy (>10 GeV)
- E losses dominate below 100 MeV (ion and coulomb)

Lecture III: solving the transport equations and phenomenology

→ **Diffusion coefficient (microphysics) to effective models, GCR phenomenology**

- secondary stable nuclei: *slope* δ of the diffusion coefficient
- secondary radioactive nuclei: local *value of* D_0 (but sensitive to LISM)
- high energy electrons and positrons: sensitive to *local source(s)*

→ Homogeneous 2-zone diffusion models successfully explain most of the existing data up to the knee (~PeV)

Ongoing/future developments/improvements in the modelling

- Phenomenology: space-time granularity, spatial-dependence (D and V)
- Improvement on source description (radio/X/γ-ray observations)
  - Spectra (not power-law): time-dependent, source dependent
  - In the source: secondary production, reacceleration
- Diffusion: use of $D_{//}$, $D_{\perp}$, anisotropic diffusion with more realistic B

→ self-consistent description (MHD) of B, CRs, and gas