New dynamic critical phenomena in nuclear and quark superfluids

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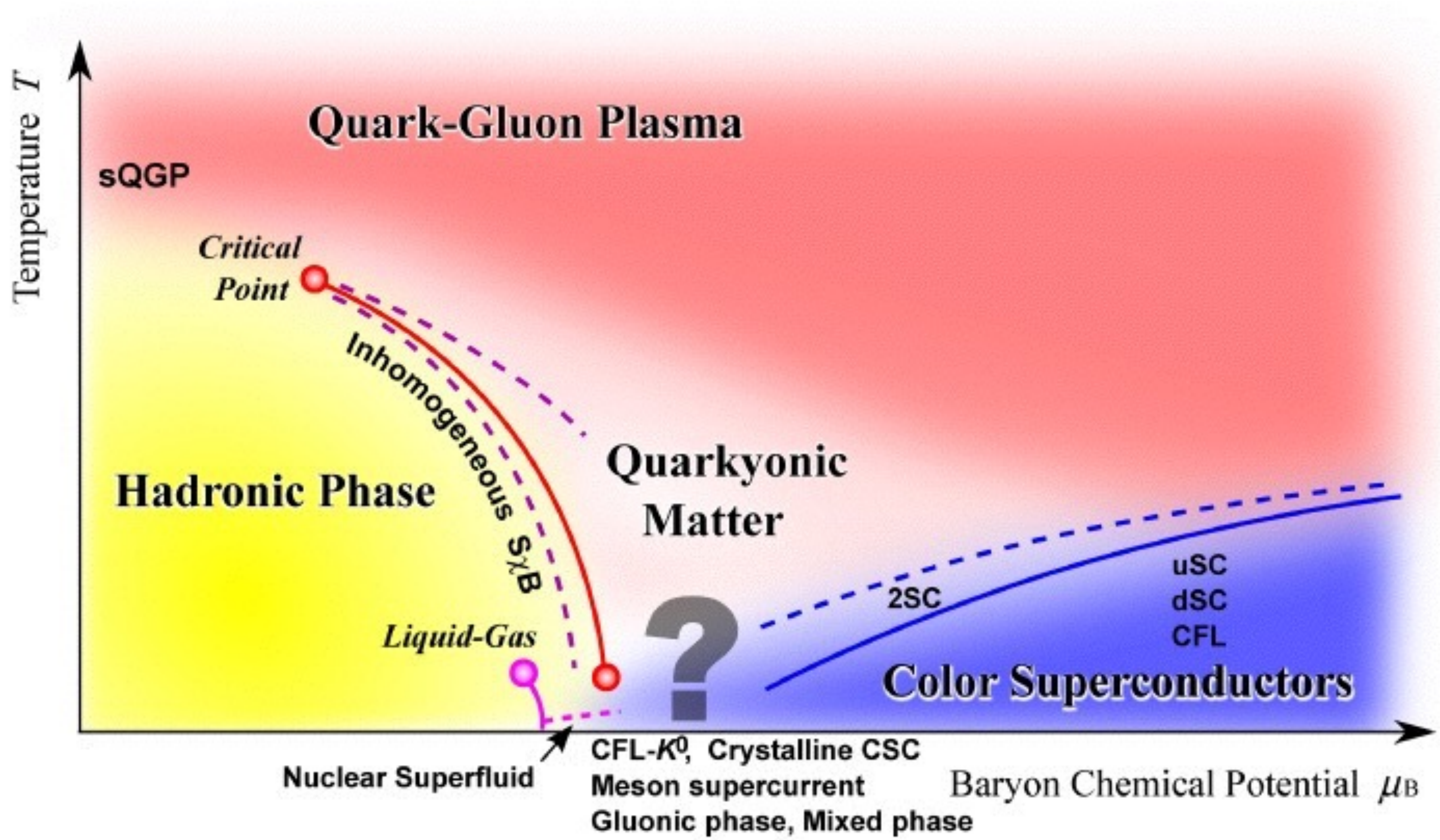
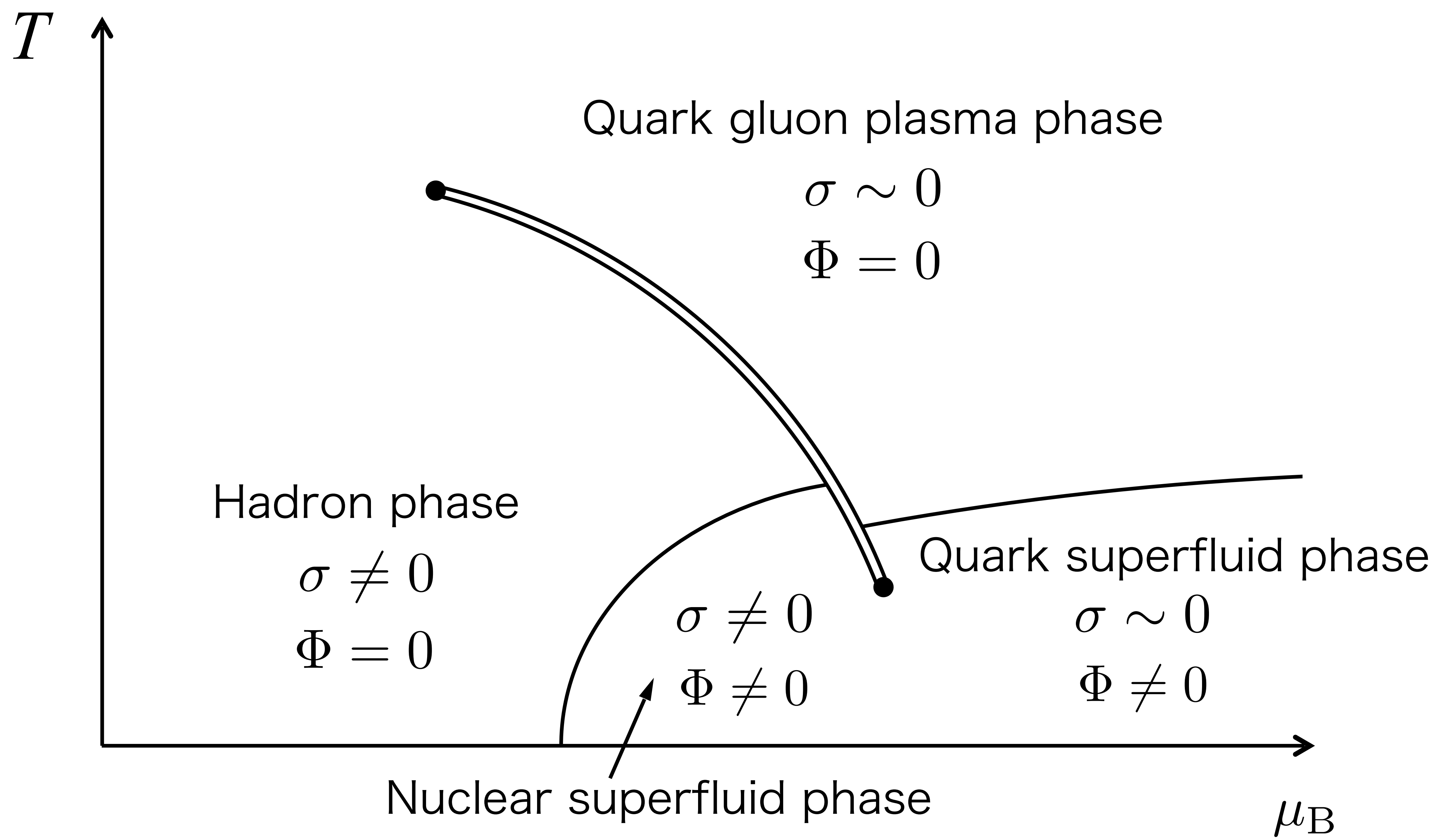
In collaboration with Naoki Yamamoto

Phase diagram of QCD



K. Fukushima, T. Hatsuda, Rept. Prog. Phys. (2010)

Phases of QCD



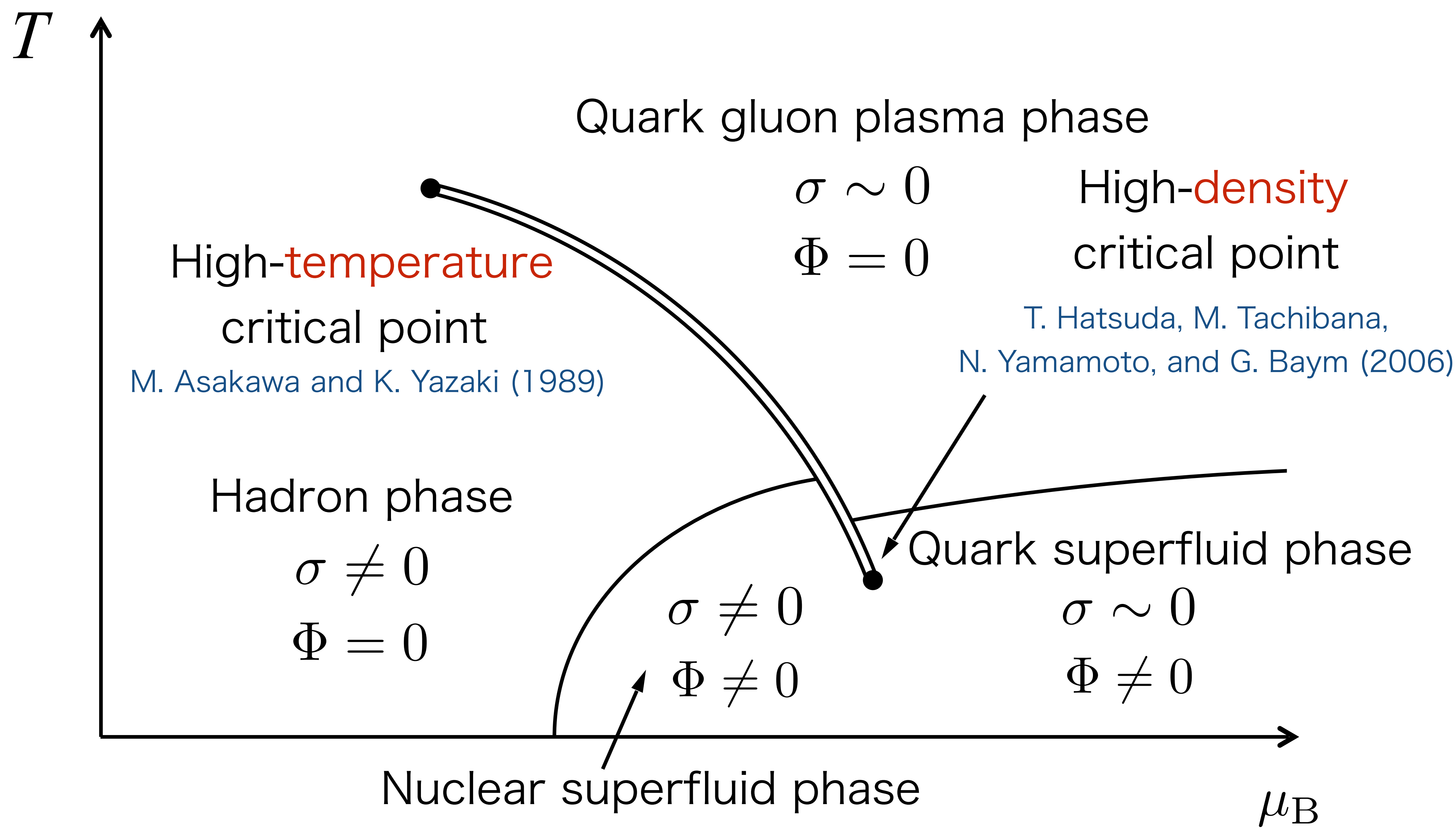
Chiral condensate: $\sigma \equiv \langle \bar{q} q \rangle$

~~Chiral symmetry~~

Diquark condensate: $\Phi \equiv \langle q q \rangle$

~~$U(1)_B$ symmetry~~

QCD Critical points



Chiral condensate: $\sigma \equiv \langle \bar{q} q \rangle$

~~Chiral symmetry~~

Diquark condensate: $\Phi \equiv \langle q q \rangle$

~~$U(1)_B$ symmetry~~

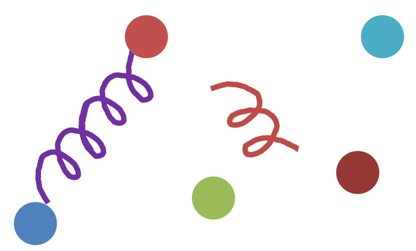
Universality classes of QCD critical points

Universality class	High-temperature	High-density
Static	3D Ising	?
Dynamic	Model H H. Fujii (2003), D. T. Son and M. A. Stephanov (2004)	?

What are the universality classes of the high-density critical point?

Dynamic universality class

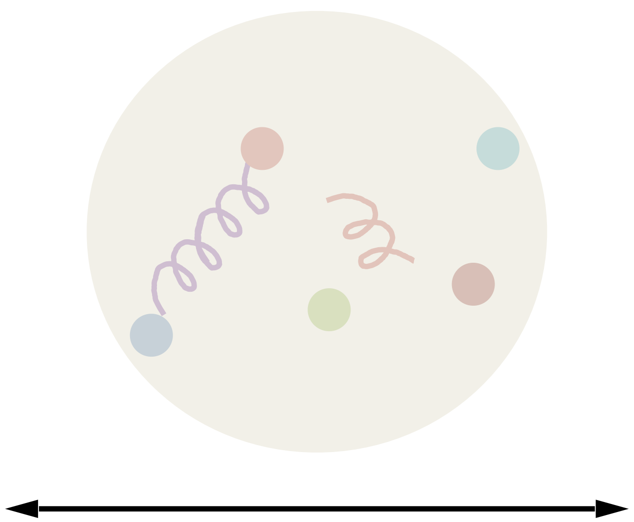
Microscopic theory



$$\xi \to \infty$$

Dynamic universality class

Microscopic theory



$\xi \to \infty$

Integrating out $\longrightarrow$

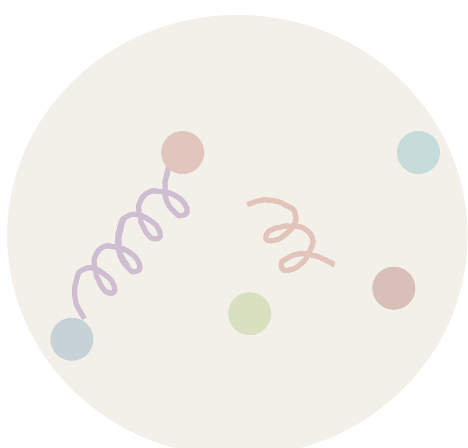
Effective theory

Hydrodynamic variables:

- Order parameters
- Conserved densities
- Nambu-Goldstone modes

Dynamic universality class

Microscopic theory



$\xi \to \infty$

Effective theory

Integrating out

Hydrodynamic variables:

- Order parameters
- Conserved densities
- Nambu-Goldstone modes

Same Symmetries

Classification based on hydrodynamic variables and symmetries

Conventional classification

Hohenberg and Halperin: Theory of dynamic critical phenomena

TABLE I. Some dynamical models treated by renormalization-group methods.

Model	Designation	System	Dimension order of parameter	Non-conserved fields	Conserved fields	Non-vanishing Poisson bracket
Relaxational	A	Kinetic Ising anisotropic magnets	n	ψ	None	None
	B	Kinetic Ising uniaxial ferromagnet	n	None	ψ	None
	C	Anisotropic magnets structural transition	n	ψ	m	None
Fluid	H	Gas–liquid binary fluid	1	None	ψ $\mathbf{j}$	$\{\psi, \mathbf{j}\}$
Symmetric planar magnet	E	Easy-plane magnet, $h_z = 0$	2	ψ	m	$\{\psi, m\}$
Asymmetric planar magnet	F	Easy-plane magnet, $h_z \neq 0$ superfluid helium	2	ψ	m	$\{\psi, m\}$
Isotropic antiferromagnet	G	Heisenberg antiferromagnet	3	ψ	$\mathbf{m}$	$\{\psi, \mathbf{m}\}$
Isotropic ferromagnet	J	Heisenberg ferromagnet	3	None	ψ	$\{\psi, \psi\}$

P. C. Hohenberg and B. I. Halperin (1977)

Universality classes of QCD critical points

Universality class	High-temperature	High-density
Static	3D Ising	3D Ising
Dynamic	Model H H. Fujii (2003), D. T. Son and M. A. Stephanov (2004)	New class

New dynamic universality class beyond the conventional classification

Critical phenomena of high-density QCD critical point

1 Hydrodynamic variables

2 Static critical phenomena

Ginzburg-Landau theory

3 Dynamic critical phenomena

Langevin theory

Hydrodynamic variables

- Chiral condensate: $\sigma \equiv \bar{q}q - \langle \bar{q}q \rangle$
- Baryon number density: $n \equiv \bar{q}\gamma^0 q - \left\langle \bar{q}\gamma^0 q \right\rangle$
- Superfluid phonon: θ defined by $\langle qq \rangle \sim e^{i\theta}$
- Energy density: $\varepsilon \equiv T^{00} - \left\langle T^{00} \right\rangle$
- Momentum density: $\pi^i \equiv T^{0i}$

Ginzburg-Landau theory

$$F[\sigma, n, \theta] = \int \mathrm{d}\boldsymbol{r} \left[\frac{a}{2}(\boldsymbol{\nabla}\sigma)^2 + b\boldsymbol{\nabla}\sigma \cdot \boldsymbol{\nabla}n + \frac{c}{2}(\boldsymbol{\nabla}n)^2 + \frac{d}{2}(\boldsymbol{\nabla}\theta)^2 + V(\sigma, n) + \cdots \right]$$

$$V(\sigma, n) = \frac{A}{2}\sigma^2 + B\sigma n + \frac{C}{2}n^2$$

- Near critical point $\longrightarrow$ Small order parameter
- Interested in long-range behavior $\longrightarrow$ Derivative expansion
- QCD symmetries $\longrightarrow$ constrains on the expansion

chiral symmetry, baryon number symmetry, CPT symmetries

Ginzburg-Landau theory

$$F[\sigma, n, \theta] = \int \mathrm{d}\boldsymbol{r} \left[\frac{a}{2}(\boldsymbol{\nabla}\sigma)^2 + b\boldsymbol{\nabla}\sigma \cdot \boldsymbol{\nabla} n + \frac{c}{2}(\boldsymbol{\nabla} n)^2 + \frac{d}{2}(\boldsymbol{\nabla}\theta)^2 + V(\sigma, n) + \cdots \right]$$

$$V(\sigma, n) = \frac{A}{2}\sigma^2 + B\sigma n + \frac{C}{2}n^2$$

- Superfluid phonon θ is irrelevant to the statics.

$$F[\sigma, n, \theta] = F_{\mathrm{MF}}[\sigma, n] + F_{\mathrm{MF}}[\theta] + \gamma\sigma(\boldsymbol{\nabla}\theta)^2 + \cdots$$

decoupled from
the time reversal symmetry

derivative coupling
due to U(1) symmetry

Ginzburg-Landau theory

$$F[\sigma, n, \theta] = \int \mathrm{d}\boldsymbol{r} \left[\frac{a}{2}(\boldsymbol{\nabla}\sigma)^2 + b\boldsymbol{\nabla}\sigma \cdot \boldsymbol{\nabla} n + \frac{c}{2}(\boldsymbol{\nabla} n)^2 + \frac{d}{2}(\boldsymbol{\nabla}\theta)^2 + V(\sigma, n) + \cdots \right]$$

$$V(\sigma, n) = \frac{A}{2}\sigma^2 + B\sigma n + \frac{C}{2}n^2$$

- Statics

$$\langle \sigma(\boldsymbol{r})\sigma(0) \rangle = \frac{1}{4\pi r} e^{-r/\xi} \qquad \xi \sim \frac{1}{\sqrt{AC - B^2}} \to \infty$$

$$\chi_{\mathrm{B}} \equiv \frac{\partial n}{\partial \mu} = T \left\langle n^2 \right\rangle_{\boldsymbol{q} \to \boldsymbol{0}} \sim \xi^{2-\eta} \qquad \eta = 0.04$$

Same static universality class as that of high-temperature critical point

Langevin equation

$$\dot{x}_i(\boldsymbol{r},t) = -\gamma_{ij}\frac{\delta F}{\delta x_j} - \int \mathrm{d}\boldsymbol{r}'\,[x_i(\boldsymbol{r}), x_j(\boldsymbol{r}')]\,\frac{\delta F}{\delta x_j(\boldsymbol{r}')} + \text{noise term}$$

dissipative term reversible term

$$(x_i \equiv \sigma, n, \theta)$$

- Time reversal symmetry of correlation functions $\longrightarrow$ $\gamma_{ij} = \gamma_{ji}$
- Poisson brackets

$$[\theta(\boldsymbol{r}), n(\boldsymbol{r}')] = \delta(\boldsymbol{r} - \boldsymbol{r}')$$

$$[x_i(\boldsymbol{r}), x_j(\boldsymbol{r}')] = 0 \qquad \text{otherwise}$$

Langevin equation

$$\dot{x}_i(\boldsymbol{r},t) = -\gamma_{ij}\frac{\delta F}{\delta x_j} - \int \mathrm{d}\boldsymbol{r}'\,[x_i(\boldsymbol{r}), x_j(\boldsymbol{r}')]\,\frac{\delta F}{\delta x_j(\boldsymbol{r}')} + \text{noise term}$$

derivative expansion

$$\dot{\sigma}(\boldsymbol{r}) = -\Gamma_{\sigma\sigma}\frac{\delta F}{\delta\sigma(\boldsymbol{r})} + \Gamma_{\sigma n}\boldsymbol{\nabla}^2\frac{\delta F}{\delta n(\boldsymbol{r})}$$

$$\dot{n}(\boldsymbol{r}) = \Gamma_{\sigma n}\boldsymbol{\nabla}^2\frac{\delta F}{\delta\sigma(\boldsymbol{r})} + \Gamma_{nn}\boldsymbol{\nabla}^2\frac{\delta F}{\delta n(\boldsymbol{r})} - \int \mathrm{d}\boldsymbol{r}'\,[n(\boldsymbol{r}), \theta(\boldsymbol{r}')]\,\frac{\delta F}{\delta\theta(\boldsymbol{r}')}$$

$$\dot{\theta}(\boldsymbol{r}) = -\Gamma_{\theta\theta}\frac{\delta F}{\delta\theta(\boldsymbol{r})} - \int \mathrm{d}\boldsymbol{r}'\,[\theta(\boldsymbol{r}), n(\boldsymbol{r}')]\,\frac{\delta F}{\delta n(\boldsymbol{r}')}$$

Hydrodynamic modes

$$\begin{pmatrix} i\omega - \Gamma_{\sigma\sigma}A - (\Gamma_{\sigma\sigma}a + \Gamma_{\sigma n}B)\boldsymbol{q}^2 & -\Gamma_{\sigma\sigma}B - (\Gamma_{\sigma\sigma}b + \Gamma_{\sigma n}C)\boldsymbol{q}^2 & 0 \\ -(\Gamma_{\sigma n}A + \Gamma_{nn}B)\boldsymbol{q}^2 & i\omega - (\Gamma_{\sigma n}B + \Gamma_{nn}C)\boldsymbol{q}^2 & d\boldsymbol{q}^2 \\ -B - b\boldsymbol{q}^2 & -C - c\boldsymbol{q}^2 & i\omega - \Gamma_{\theta\theta}d\boldsymbol{q}^2 \end{pmatrix} \begin{pmatrix} \sigma \\ n \\ \theta \end{pmatrix} = \mathbf{0}$$

$$\omega = -i\Gamma_{\sigma\sigma}A \qquad\qquad \omega^2 = c_{\mathrm{s}}^2\boldsymbol{q}^2$$

relaxation mode of σ — superfluid phonon

Dynamic critical phenomena

- Speed of superfluid phonon

$$c_\mathrm{s} \equiv \sqrt{\frac{d}{\chi_\mathrm{B}}} \to 0$$

$$\left(\begin{array}{c} F_\mathrm{MF}[\theta] = \int \mathrm{d}\boldsymbol{r} \frac{d}{2} (\boldsymbol{\nabla}\theta)^2 \\ \chi_\mathrm{B} \equiv \frac{\partial n}{\partial \mu} \sim \xi^{2-\eta} \end{array} \right)$$

"critical slowing down"

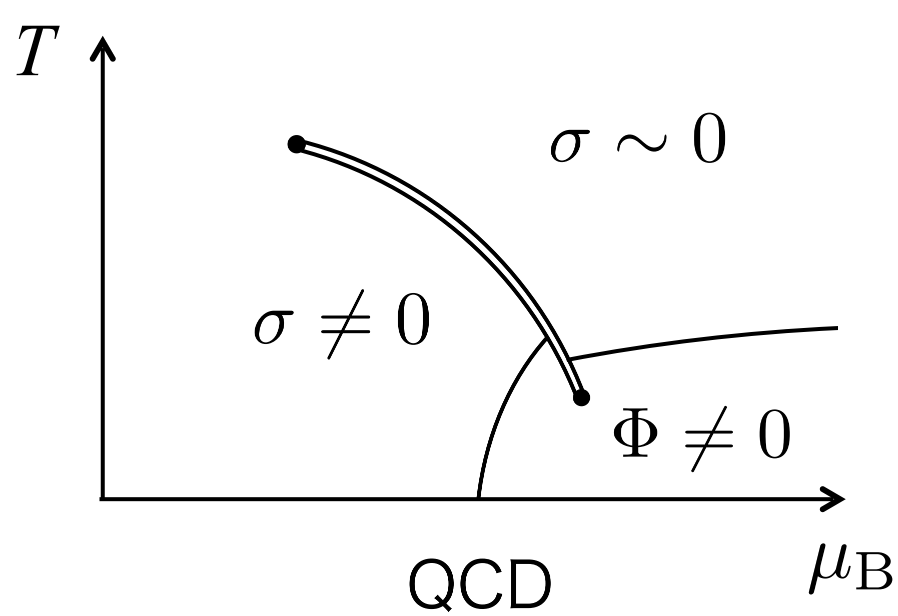
- Dynamic critical exponent $\xi^{-z} \sim c_\mathrm{s} |\boldsymbol{q}|$

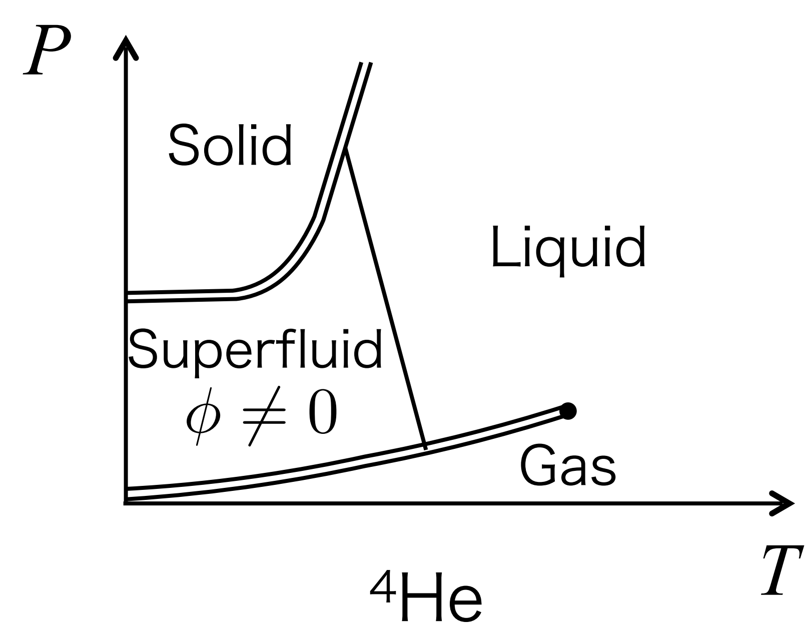
$$z = 2 - \frac{\eta}{2}$$

New dynamic universality class beyond Hohenberg and Halperin's classification

Why the universality class is new?

	Superfluidity	Interplay between chiral condensate and superfluid phonon
High-μ_B QCD critical point	✔	✔
High-T QCD critical point (Model H)		
Superfluid transition of ^{4}He (Model F)	✔	





ϕ: Superfluid gap

Future heavy-ion collisions

- Dynamic critical phenomena **can** distinguish the high-temperature and high-density critical points in QCD. (static critical phenomena **can not**.)
- Observation of the high-density critical point would provide the indirect evidence of the superfluidity in QCD.

Conclusion

- We found the new dynamic universality class beyond the conventional Hohenberg and Halperin's classification.

Universality class	High-temperature	High-density
Static	3D Ising	3D Ising NS, N. Yamamoto (2017)
Dynamic	Model H H. Fujii (2003), D. T. Son and M. A. Stephanov (2004)	New class NS, N. Yamamoto (2017)

Back up slides

Consequence of gauge symmetry

- QCD at finite density

$$\mathcal{L} = \mathcal{L}_0 + \frac{\mu}{3}\bar{q}\gamma^0 q$$
$$= \mathcal{L}_0 + \frac{1}{3}A_\mu \bar{q}\gamma^\mu q \qquad A^\mu \equiv (\mu, \mathbf{0})$$

$A_\mu \to A_\mu - \partial_\mu \alpha \quad \theta \to \theta + \alpha$ gauge symmetry

- Low-energy effective theory

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_{\mathrm{eff}}(\dot{\theta} + \mu, \boldsymbol{\nabla}\theta)$$

$$\boxed{n = \frac{\delta \mathcal{L}}{\delta \mu} = \frac{\delta \mathcal{L}}{\delta \dot{\theta}}}$$

Nonlinear dynamics

- Renormalization of kinetic coefficients

e.g. Model H   P. C. Hohenberg and B. I. Halperin (1977)

Diffusion rate: $D \propto \lambda$

Kinetic coefficient: $\lambda \sim \xi^{x_\lambda}$

Dynamic critical exponent: $\xi^{-z} \sim D\boldsymbol{q}^2$

$$z = 4 - \eta - x_\lambda \quad (x_\lambda = 0.946)$$

- In high-density QCD critical point, c_s does NOT depend on any kinetic coefficients even if the energy and momentum densities are included.
- Nonlinear dynamics is irrelevant.

Superfluid phonon ^{4}He

$$c_s = \sqrt{\frac{\rho_{\rm s}}{c_{\rm p}}}$$

stiffness constant: $\rho_{\rm s} \sim \xi^{-1}$

Note:

$$\begin{aligned} F[\phi] &= \int \mathrm{d}\boldsymbol{r}\ \frac{1}{2}|\boldsymbol{\nabla}\phi|^2 \\ &= \int \mathrm{d}\boldsymbol{r} \left[\frac{1}{2}(\boldsymbol{\nabla}h)^2 + \frac{\rho_{\rm s}}{2}(\boldsymbol{\nabla}\theta)^2\right] \end{aligned}$$

Here

$$\phi \equiv he^{i\theta} \qquad\qquad \rho_{\rm s} \equiv h^2$$