String Theory and Quantum Gravity

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Fundamental Problems in Quantum Physics - Erice

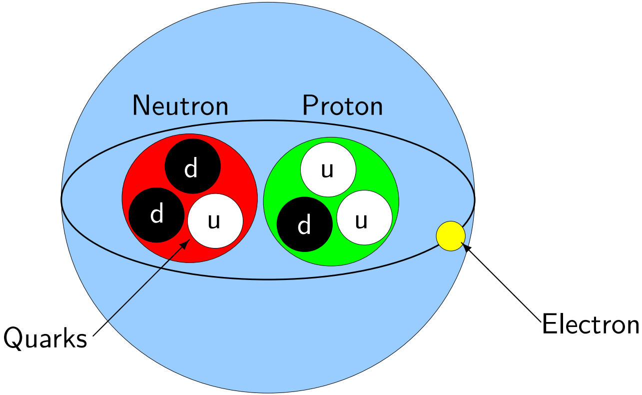
Plan for today

Give a non-technical account of String Theory, the present status and future perspective, elucidating the achievements and the open problems of the theory.

1. Motivation for String Theory.
2. What is String Theory?
3. Obstacles/open problems.

Fundamental particles

Q: What are the fundamental blocks of matter? Look at the atom!



Fundamental particles (fermions)

Fundamental particles

Leptons

e (electron)	ν_e(e-neutrino)
μ (muon)	ν_μ (μ-neutrino)
τ (tau)	ν_τ (τ-neutrino)

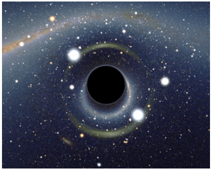
Quarks

u (up)	d (down)
c (charm)	s (strange)
t (top)	b (bottom)

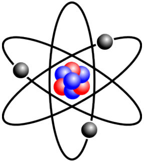
- All these particles form the matter, they are called fermions and have intrinsic spin $1/2$.

Fundamental forces (bosons)

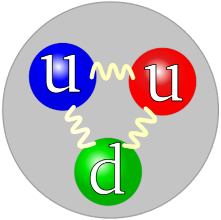
Gravity (graviton)



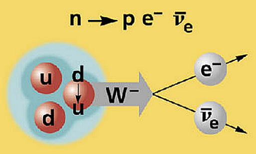
Electromagnetism (photon)



Nuclear Strong (gluons)



Nuclear Weak (W, Z bosons)



- Gravitons have spin 2. Gluons, W, Z bosons and photons have spin 1.

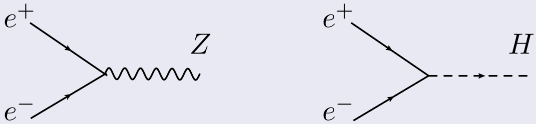
Standard Model

- At sub-atomic level $F_{grav} \sim 10^{-36} F_{weak}$, forget about gravity!

Standard Model: gauge QFT

$$\mathcal{L}_{SM} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi} \not{D} \psi + \psi_i y_{ij} \psi_j \phi + h.c. + |D_\mu \phi|^2 - V(\phi)$$

A set of basic rules



- Needed an extra boson of spin zero, which was found!
- Renormalizable theory (with dimensionless coupling constants) and shows remarkable agreement with the experiment!

Standard Model

The SM cannot exist by itself

- The particles of the standard model do couple to gravity.
- At some point gravity will become important and the SM will break down!

$$M_{pl} = \sqrt{\frac{\hbar c}{G_N}} \sim 1.22 \times 10^{19} GeV$$

- This corresponds to very small distances! ($\sim 1.61 \times 10^{-33}$ cm)

Pure Gravity

General Relativity: Einstein Hilbert Lagrangian

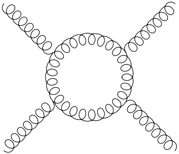
$$\mathcal{L}_{EH}[g] = \frac{1}{G_N}\sqrt{-g}\mathcal{R}$$

Consider a scattering process: $g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G_N}h_{\mu\nu}$

$$\mathcal{L}_{EH} = (\partial h)^2 + \sqrt{G_N}\, h(\partial h)^2 + \ldots$$

Mass dimension -1

The scattering of four gravitons is UV divergent at two loops:



$$\sim \quad E^2 G_N^2 \int^{\Lambda} d^4p \frac{p^6}{(p^2)^4} \sim E^2 G_N^2 \Lambda^2$$

Non-renormalizable: to avoid such divergence, we need to add a *new* term to the Lagrangian. And then another, and so on!

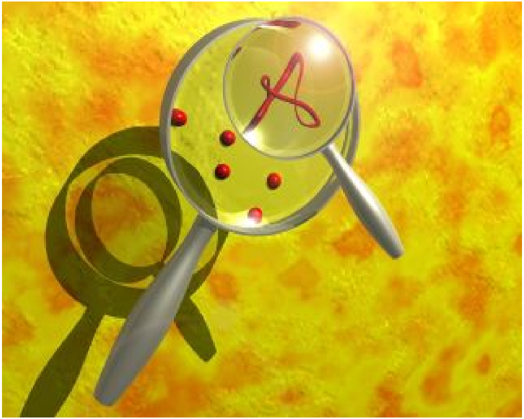
SM and GR as effective theories

Postulate

- The Standard model and GR are not the whole story, but the effective description, valid at energy scales small compared to M_{pl}, of a more fundamental theory.
- This fundamental theory provides a UV completion for the SM and GR.

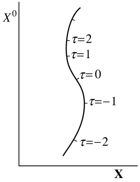
String Theory

- The divergences above arise because the particles can get very close to each other.
- What is the "form" of the elementary particles?
- Propose that they are little (really tiny!) strings!

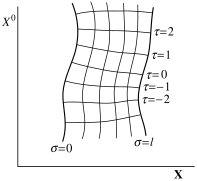


String Theory

Relativistic theory of strings consistent with quantum mechanics!



$$S_{part} = -m \int d\tau \sqrt{\left(-g_{\mu\nu} \frac{dX^\mu}{d\tau} \frac{dX^\nu}{d\tau}\right)}$$



$$S_{string} = -T \int d^2\sigma \sqrt{\left(-\det g_{\mu\nu} \frac{\partial X^\mu}{\partial \sigma^a} \frac{\partial X^\nu}{\partial \sigma^b}\right)}$$

$T = \frac{1}{2\pi\alpha'}$: tension of the string.

String theory

- Introduce a new field, h_{ab}, the world-sheet metric:

Polyakov action

$$S_p = -\frac{T}{2}\int d^2\sigma\sqrt{-\det h}h^{ab}g_{\mu\nu}\frac{\partial X^\mu}{\partial\sigma^a}\frac{\partial X^\nu}{\partial\sigma^b}$$

Bosonic string: A theory of D bosonic fields X^μ in $2d$.

- Weyl invariance

$$h^{ab}(\sigma) \to \Omega^2(\sigma)h^{ab}(\sigma)$$

- Consistency of the theory requires this to be a symmetry at the quantum level, but

$$\text{Anomaly} = -\frac{D-26}{12}\mathcal{R}^{2d}$$

- Quantum consistency fixes $D = 26$!!

Superstring theory

- The bosonic string spectrum contains an state with mass:

$$M^2 = -\frac{1}{\alpha'}$$

- This is terrible! a tachyon leads to instabilities.
- Solution: Introduce fermionic modes!

Superstring action

$$S_{super} = -\frac{T}{2}\int d^2\sigma \left(\eta^{ab}\partial_a X^\mu \partial_b X_\mu - i\bar{\Psi}^\mu \gamma^a \partial_a \Psi_\mu\right)$$

Majorana spinor in $2d$

- Super-symmetry:

$$\delta_\epsilon X^\mu = \bar{\epsilon}\,\Psi^\mu, \quad \delta_\epsilon \Psi^\mu = -i\gamma^a \partial_a \Psi_\mu \epsilon$$

Superstring theory

Features of Super-String Theory

- Now $D = 10$, which is better!
- The tachyon is removed!
- There is a natural symmetry between bosons and fermions.
- You have no choice! the theory is pretty unique and has a single continuous parameter, the string tension T, or α'.
- Discrete choices:

Type I

Type IIA

Type IIB

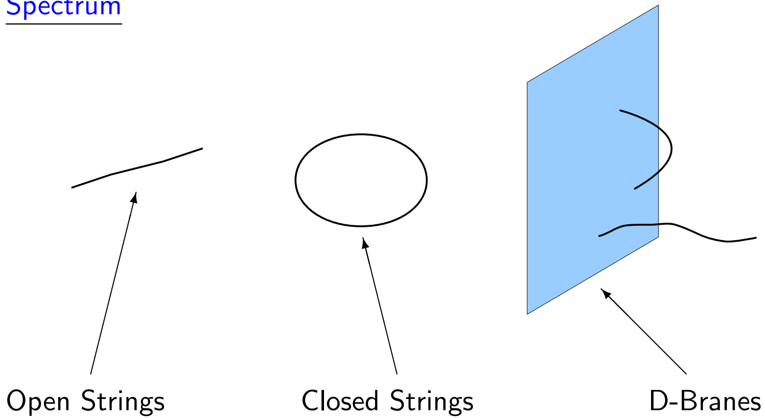
$SO(32)$

$E_8 \times E_8$

- There is a web of dualities relating them, and they may be different regions of the same theory (M-theory).

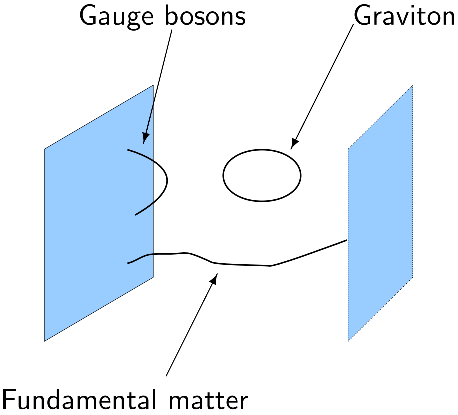
String theory spectrum

Spectrum



- The lowest lying states are massless, with the same quantum numbers as the particles we see in nature!!

String theory spectrum

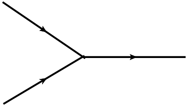


- All particles and forces under a unified framework!
- In addition, an infinite tower of (very) massive string states:

$$M^2 = \frac{n}{\alpha'}, \quad n = 1, 2, \ldots$$

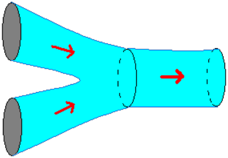
String theory and UV divergences

Point particles



VS

Strings



- Roughly, ST provides a soft cut-off: $\frac{1}{p^2} \rightarrow \frac{e^{-\alpha' p^2}}{p^2}$
- At low energies, $p^2 \ll 1/\alpha'$, we can forget about the string massive states, and we recover GR (or SUGRA).
- At higher and higher energies we need to consider the massive states and get a UV completion of GR!

$$\frac{1}{\alpha'} \sim M_{pl}^2$$

String theory, achievements and problems

Achievements

- Unifies all known fundamental particles and forces under one framework.
- Cures the divergence problems of gravity, providing a quantum theory of gravity.
- Other achievements:
  - Explanation of black hole entropy.
  - Precise implementation of the holographic principle (see later!).
  - Dualities that led to new results in gauge theories, CFT's, etc

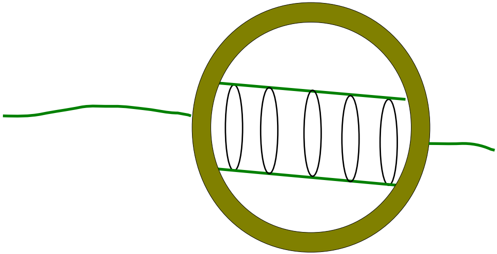
Problems

- Supersymmetry has not been yet observed in nature.
- Extra dimensions.
- Abundance of solutions and predictability.

Extra dimensions

Q: String Theory predicts 10 dimensions, why do we see only 4??

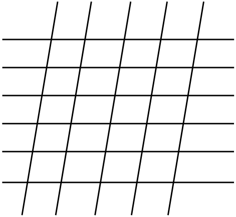
A: Not all of them have to be large!



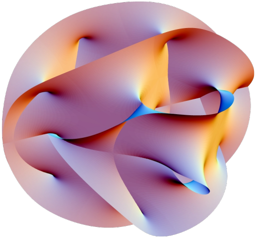
$$\text{2d Pipe} = \underset{\overset{\uparrow}{\text{Large dimension}}}{\mathbb{R}} \times \underset{\overset{\uparrow}{\text{Compact dimension}}}{S^1}$$

String compactification

- String Theory on 10D space-times of the form $M^{3,1} \times M_6$



$\times$



$M^{3,1}$: (3+1)d flat space-time

M_6: 6d Compact Manifold

4d Physics vs Geometry

- The 4d-physical theory depends on the compact manifold M_6!

$$\text{Physical theory on } M^{3,1} \quad \leftrightarrow \quad M_6$$

e.g. A SUSY gauge theory in 4d corresponds to a CY manifold.

Problems

- $6d$ manifolds are extremely rich and hard to study!
- *e.g.* Hundreds of thousands (but finite!) CY manifolds, and the explicit metric is not know even for a single one!
- It is hard to get models that a low energies look exactly as the Standard model (but too easy to get similar models!)

A landscape of string vacua

Predictability in String Theory

- String theory has a single adjustable parameter: α'.
- Originally it was hoped that string theory would allow (in principle!) to predict all fundamental constants, masses, etc.

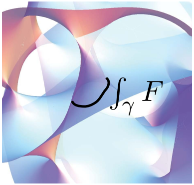
However

- Many classical solutions with space of the form $M^{3,1} \times CY_6$!!
- The metric is one of the fields of string theory, but we have others and we have an abundance of vacua!

Q: How many?

A landscape of string vacua

- String theory has field strengths F.



Quantization conditions along cycles of CY_6:

$$\int_{\gamma_i} F = n_i$$

- Solutions are labelled by $(n_1, n_2, \cdots)$
- There is an upper limit on n_i, roughly $n_i \lesssim 10$.
- The number of cycles in a CY_6 is roughly 500.

This results in 10^{500} vacua!

- We need to understand whether there are further constraints!

Conclusions

Achievements

- String theory unifies all known fundamental particles and forces under a single framework (*e.g.* it predicts gauge theories!)
- Cures the divergence problems of gravity, providing a quantum theory of gravity.
- Many other achievements I didn't have time to explain (see next talk!)

Conclusions

Future perspective

- It predicts supersymmetry, not yet observed in nature.
- At some scale, we should see extra dimensions, but this scale could be very high.
- The precise scale at which these should be observed depend on the model, and some models are already being ruled out at LHC!
- String theory has an abundance of vacua, but this is a finite value (most other theories have infinite families of vacua!).
- Still, we need to understand better the constraints imposed by String theory.
- Finally, most studies concern pertubative ST (much as perturbative QFT), but a rigorous definition of non-perturbative ST is missing.