Effects of magnetic field on plasma evolution in relativistic heavy-ion collisions

Shreyansh Shankar Dave

arXiv : 1703.08162v2

Collaborators: Arpan Das, P.S. Saumia, Ajit M Srivastava

Institute of Physics, Bhubaneswar, India

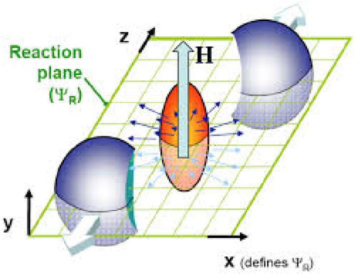
XQCD-2017, 26-28 June

Outline

Motivation

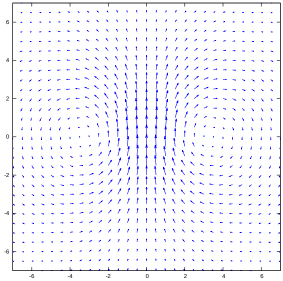
- Heavy-ion collision experiments leads to production of quark-gluon plasma.
- The main goal of this experiment to explore QCD phase diagram,phase transitions and investigate physical properties, e.g. **viscosity**, EoS, thermal conductivity, etc. of QGP.
- To extract the viscosity of QGP people fit the measured elliptic flow from experiment with viscous hydrodynamics simulation.
- In our work we found that magnetic field can change the elliptic flow.
- Motivation of the work is to show effects of magnetic field on elliptic flow.
- This is important for determination of viscosity of QGP. It also provides a way to determine initial stage magnetic field.

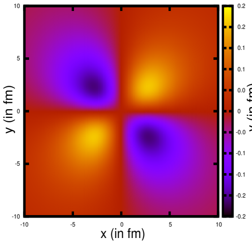
Production of Magnetic Field in HIC

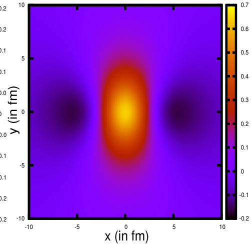


- No magnetic field in the case of central collisions. Fluctuations??
- Magnetic field generates in the non-central collisions along the y-axis at the center.
- Magnitude of magnetic field at the center can be $\sim 10^{15}$ Tesla (~ 0.1 GeV^2) (10^4 times stronger than mag. field of a magnetar).

Production of Magnetic Field in HIC







Survival of Magnetic Field due to conducting plasma

- Medium forms at thermalization time $\tau_0 < 1$ fm (uncertain) in the presence of time varying magnetic field.
- Quick thermalization and large conductivity of the plasma may protect magnetic field (of high magnitude) from decay.

Ref.: Kirill Tuchin, Phys. Rev. C, 88, 024911 (2013).

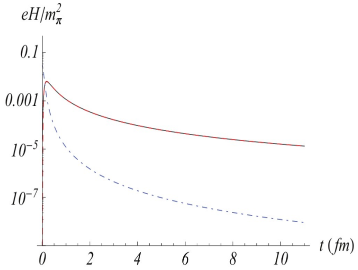


Figure: Time evolution of the magnetic field created by a point unit charge at the center in vacuum (blue) and in plasma of conductivity σ=5.8 MeV (red).

Fluid which we are considering

- In heavy-ion collisions there is spatial temperature profile vary from center (200-300 MeV) to zero in vacuum(outer).
- Conductivity also vary with the temperature. From lattice calculation conductivity of QGP is given by,

$$\sigma_{QGP} = 0.04T \tag{1}$$

- For simplicity we are considering ideal MHD fluid which has infinite conductivity everywhere.
- So we have electrically neutral, infinitely conducting fluid in the magnetic field.

Ideal Relativistic Magneto-Hydrodynamics

- Dynamics of ideal relativistic magnetized fluid is governed by the equations,

Ideal RMHD Equations

- Energy-momentum conservation equation

$$\partial_\alpha\left((\epsilon + p_g + |b|^2)u^\alpha u^\beta - b^\alpha b^\beta + (p_g + \frac{|b|^2}{2})\eta^{\alpha\beta}\right) = 0 \quad (2)$$

- Maxwell's equations

$$\partial_\alpha(u^\alpha b^\beta - u^\beta b^\alpha) = 0 \quad (3)$$

Ideal Relativistic Magneto-Hydrodynamics

- EoS $p_g = \epsilon/3$.
- Metric: $\eta^{\alpha\beta} = diag(-1, 1, 1, 1)$.
- Fluid four velocity: $u^\alpha = \gamma(1, \vec{v})$; $u^\alpha u_\alpha = -1$.
- Four-vector b^α is related with the magnetic field and fluid velocity by,

$$b^\alpha = \gamma\left(\vec{v}.\vec{B}, \frac{\vec{B}}{\gamma^2} + \vec{v}(\vec{v}.\vec{B})\right) \tag{4}$$

- $u^\alpha b_\alpha = 0$

$$|b|^2 = b^\alpha b_\alpha = \frac{|\vec{B}|^2}{\gamma^2} + (\vec{v}.\vec{B})^2 \tag{5}$$

- Total pressure of the plasma is sum of thermal pressure p_g and magnetic pressure $\frac{|b|^2}{2}$; $p = p_g + \frac{|b|^2}{2}$.

$$p = p_g + \frac{|\vec{B}|^2}{\gamma^2} + (\vec{v}.\vec{B})^2 \tag{6}$$

Algorithm for solving RMHD Equations

- For computational purpose, the RMHD equations can be conveniently put in the following conservational form,[1]

$$\frac{\partial U}{\partial t} + \sum_{K} \frac{\partial F^k(U)}{\partial x^k} = 0, \tag{7}$$

where vector of conservative variables, $U = (m_x, m_y, m_z, B_x, B_y, B_z, E)$.

- Three components of momentum,

$$m_k = (\frac{4}{3}\epsilon\gamma^2 + B^2)v_k - (\vec{v}.\vec{B})B_k. \tag{8}$$

- The total energy density,

$$E = \frac{4}{3}\epsilon\gamma^2 - p_g + \frac{\vec{B}^2}{2} + \frac{v^2 B^2 - (\vec{v}.\vec{B})^2}{2} \tag{9}$$

[1] **A. Mignone and G. Bodo**, **Mon. Not. R. Astron. Soc. 368**, **1040 (2006)**.

Algorithm for solving RMHD Equations

- and F^k are the fluxes along the $x^k = (x, y, z)$ directions,

$$F^x(U) = \begin{bmatrix} m_x v_x - B_x \frac{b_x}{\gamma} + p \\ m_x v_x - B_x \frac{b_y}{\gamma} \\ m_x v_x - B_x \frac{b_z}{\gamma} \\ 0 \\ B_y v_x - B_x v_y \\ B_z v_x - B_x v_z \\ m_x \end{bmatrix}$$

- $F^{y,z}(U)$ are similarly defined by appropriate change of indices.

Algorithm for solving RMHD Equations

- U evolve with time following the conservation equation.
- Independent variables, $V = (\vec{v}, p_g, \vec{B})$, are required when computing the fluxes.
- To recover V from U, define : $W = \frac{4}{3}\epsilon\gamma^2$ and $S = \vec{m}.\vec{B}$,

$$E = W - p_g + \left(1 - \frac{1}{2\gamma^2}\right)|\vec{B}|^2 - \frac{S^2}{2W^2} \tag{10}$$

$$|\vec{m}|^2 = (W + |\vec{B}|^2)^2\left(1 - \frac{1}{\gamma^2}\right) - \frac{S^2}{W^2}(2W + |\vec{B}|^2) \tag{11}$$

- In the beginning of each time step, $\vec{m}$, $\vec{B}$ and S are known. γ in terms of W (only unknown) is,

$$\gamma = \left(1 - \frac{S^2(2W + |\vec{B}|^2) + |\vec{m}|^2 W^2}{(W + |\vec{B}|^2)^2 W^2}\right)^{-\frac{1}{2}} \tag{12}$$

Algorithm for solving RMHD Equations

- From EoS,

$$p_g(W) = \frac{W}{4\gamma^2} \tag{13}$$

- Unknown quantity W can be found out from,

$$f(W) = W - p_g + \left(1 - \frac{1}{2\gamma^2}\right)|\vec{B}|^2 - \frac{S^2}{2W^2} - E = 0 \tag{14}$$

- This equation is solved using Newton-Raphson method to get W.
- Once W has been computed, one can get back γ and p_g . Velocities can be found by expression of m_k,

$$v_k = \frac{1}{(W + |\vec{B}|^2)}\left(m_k + \frac{S}{W}B_k\right) \tag{15}$$

Simulation details

- We have performed (3+1)-d simulation on lattice $200 \times 200 \times 200$ with lattice sapcing of 0.1 fm.
- We perform low energy collisions with $\sqrt{s} = 20$ GeV and with Cu nuclei.
- Because of computational limitations we have taken radius of copper as 4.0 fm with skin 0.4 fm.
- Optical Glauber and Glauber Monte-Carlo like initial energy density are taken into account for the simulations.
- We have taken EOS of ideal relativistic gas $p_g = \rho/3$ and zero chemical potential for simplicity.
- Initial central temperature set to be ~ 180 MeV.

Simulation details

- Magnetic field produced by two oppositely moving, uniform charged spheres with appropriate Lorentz γ factor is taken as the initial magnetic field profile at time τ_0 after the collision.
- We use Leap-Frog 2nd order method to solve ideal RMHD equations numerically in (3+1)D, with system size 20 fm.
- Initial fluid velocity in the transverse plane taken to be zero.
- We have taken longitudinal velocity profile $\propto z$ with suitable maximum velocity at the edge of the plasma.
- We have done our calculation in central rapidity bin.

Simulation details

- We have Fourier analysed azimuthal distribution function,

$$r(\phi) = \frac{\delta P(\phi)}{\overline{P}} = \frac{P(\phi) - \overline{P}}{\overline{P}} = \sum_n \Big(a_n \cos(n\phi) + b_n \sin(n\phi) \Big) \tag{16}$$

where,

$$a_n = \frac{1}{\pi} \int_0^{2\pi} r(\phi) \cos(n\phi)\, d\phi, b_n = \frac{1}{\pi} \int_0^{2\pi} r(\phi) \sin(n\phi)\, d\phi \tag{17}$$

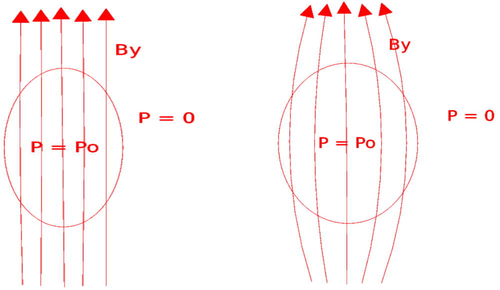
- Flow coefficients,

$$v_n^{rms} = \sqrt{a_n^2 + b_n^2} \tag{18}$$

- With fluctuations, elliptic flow $= v_2^{rms}$
- Without fluctuations, elliptic flow $= a_2$

Enhancement of elliptic flow due to magnetic field

- In MHD different kinds of wave motion are possible due to transverse deformation of magnetic lines.
- When magnetic field lines (frozen in the plasma) expand perpendicular to the direction of the manetic field, it cost energy, and feel tension, and try to become straight again.
- By this EoS becomes stiffer perpendicular to magnetic field cause larger sound speed.



Enhancement of elliptic flow due to magnetic field

- When we write MHD equations for the perturbations (from equilibrium value) of energy density, velocity and magnetic field , MHD equations provide three sound velocities for plane wave solution with wave vector $\vec{k}$,
  1. When $\vec{k}\perp\vec{B}$, MHD equations gives magnetosonic wave of velocity,
  $$c_{\perp}^2 = c_s^2 + v_A^2 \qquad (19)$$
  2. When $\vec{k}||\vec{B}$, MHD equations gives magnetosonic wave of velocity,
  $$c_{||}^2 = c_s^2 \qquad (20)$$
  3. When $\vec{k}||\vec{B}\perp\vec{v}$, then transverse wave called *Alfvén* wave moves with velocity v_A^2.

  where,
  $$c_s = \left(\frac{\partial p}{\partial \epsilon}\right)^{1/2}, v_A \sim \left(\frac{B_0^2}{8\pi\epsilon}\right)^{1/2} \qquad (21)$$

Enhancement of elliptic flow due to magnetic field

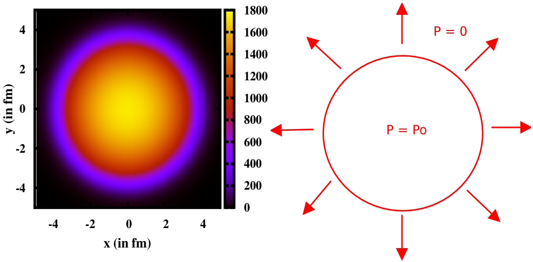
- The solution of hydrodynamics equations in small velocity approximation gives initial fluid velocity for gaussian energy density profile,[2]

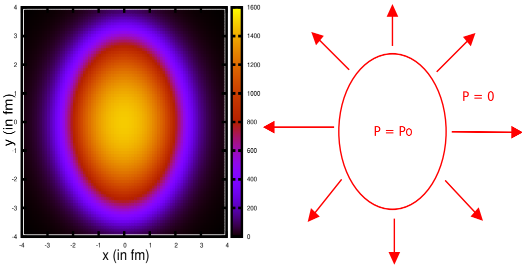
$$v_x = \frac{c_s^2 x}{\sigma_x^2} t, \tag{22}$$

$$v_y = \frac{c_s^2 y}{\sigma_y^2} t \tag{23}$$

σ_x and σ_y are the the widths of the transverse distribution.

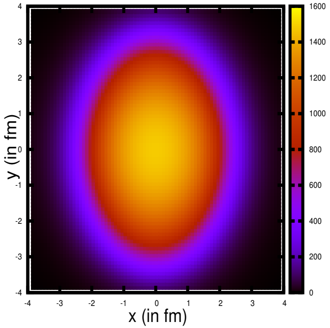
[2] **J-Y Ollitrault**, **Eur. J. Phys. 29 (2008) 275-302**.

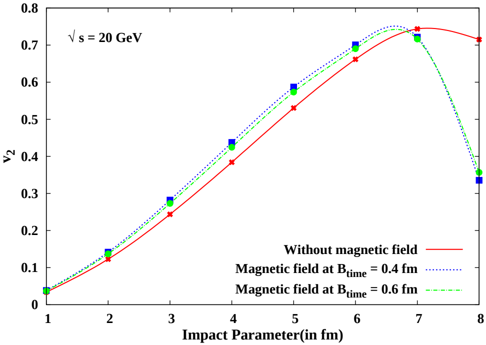

1800
1600
1400
1200
1000
800
600
400
200
0
x (in fm)
y (in fm)
P = 0
P = Po


1600
1400
1200
1000
800
600
400
200
0
x (in fm)
y (in fm)
P = 0
P = Po

Enhancement of elliptic flow due to magnetic field

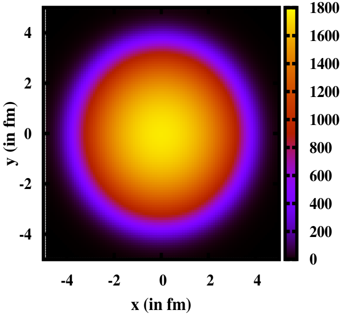
Ideal Relativistic Magnetohydrodynamics Simulation result :

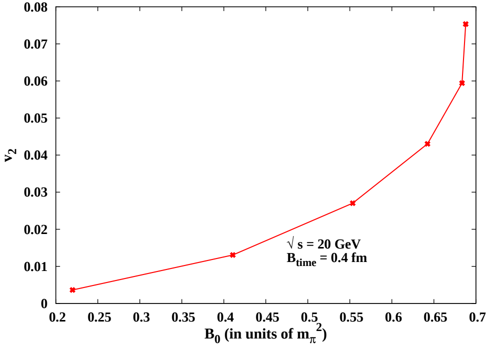




Enhancement of elliptic flow due to magnetic field

Ideal Relativistic Magnetohydrodynamics Simulation result :

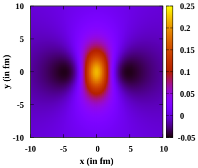


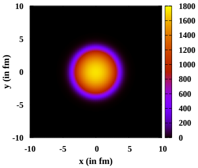


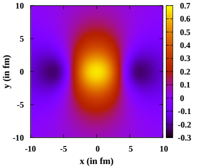
Such situation can arise in the collision of deformed nucleus case.

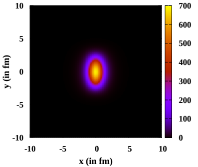
Low impact parameter magnetic field well inside the plasma region, hence argument of sound speed holds true.

High impact parameter extension of magnetic field much outside the plasma region. Lenz's law opposes the expansion of the conducting fluid in x-direction.

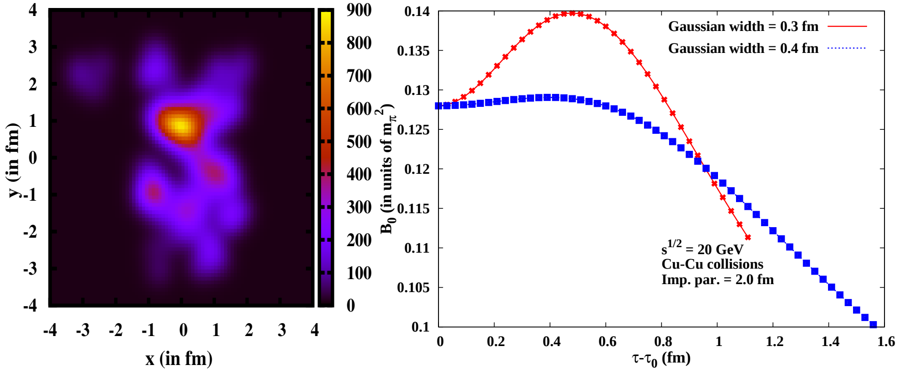






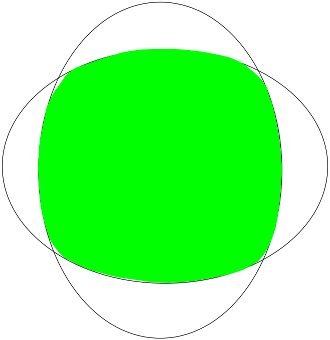


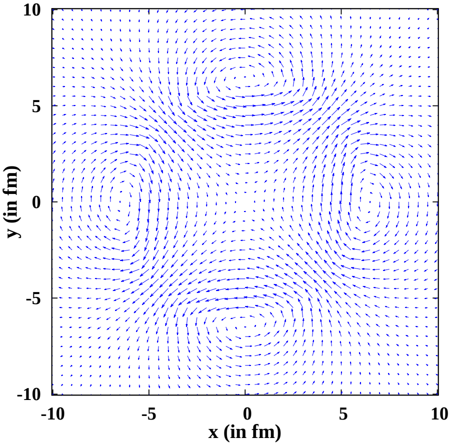
Enhancement of magnetic field due to fluctuation



- When fluctuations evolve, magnetic flux can get reorganized. This can lead to local and temporary enhancement of magnetic field.

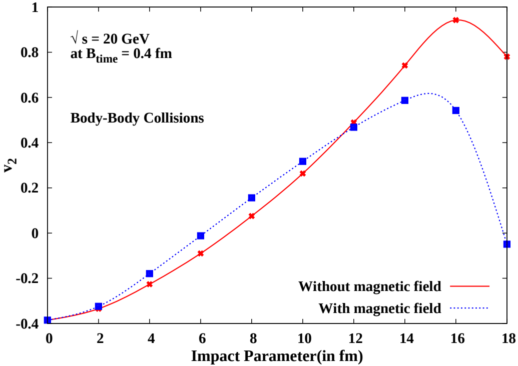
Deformed Nuclei collisions

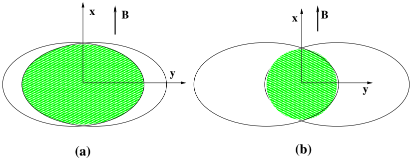




- Quardrupolar Magnetic field profile.

Deformed Nuclei collisions



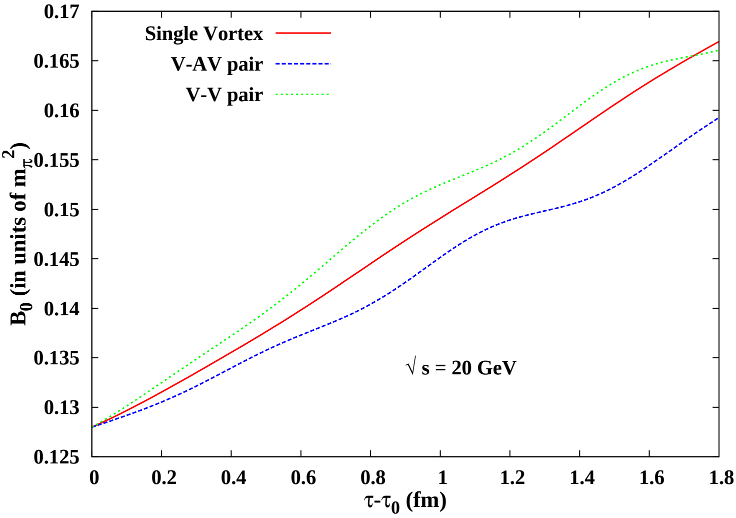


Dynamo Like effects in Heavy-ion collisions

- Superfluid vortices may be possible at FAIR and NICA[3]
- Symmetry breaking pattern from QGP to CFL phase, $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{c+L+R} \times Z_2$.
- This allows Superfluidity and superfluid vortex.
- Vortices (turbulence) are known to strongly increase magnetic field, the so-called Dynamo effect.
- We studied evolution of magnetic field in the presence of vortex configuration.
- We see strong increase in the magnetic field in the presence of vortices.
- Dynamo effect for ideal MHD may be possible in strong flux-folding regime as expected in the presence of vortices.[4]

[3] **A. Das, S.S. Dave, S. De, and A.M. Srivastava, arXiv: 1607.00480**.
[4] **S. I. Vainshtein & Ya. B. Zel'dovich, 1972 Sov. Phys. Usp. 15 159**.

Dynamo Like effects in Heavy-ion collisions



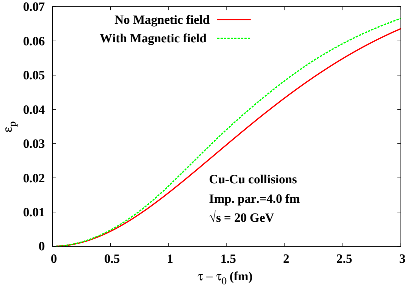
Conclusion

- Magnetic field can change ellipic flow with dependence on the impact parameter of the collisions. It can be very important in the study of viscosity of QGP and provides signal of presence of magnetic field.
- We found that magnetic field can get enhanced in the presence of fluctuations.
- Deformed nuclei can give very interesting possibility in the magnitude and profile of magnetic field like qudrupolar field configuration which can give beam focussing.
- Dynamo like effects are possible in low energy heavy-ion collisions in the presence of superfluid phase (Superfluid vortex) in ideal MHD limit.

Thank You !!

ϵ_p Plot

$$\epsilon_p = \frac{T^{xx}_{pl} - T^{yy}_{pl}}{T^{xx}_{pl} + T^{yy}_{pl}} = \frac{v_x^2 - v_y^2}{v_x^2 + v_y^2 + \frac{1}{2\gamma^2}} \tag{24}$$

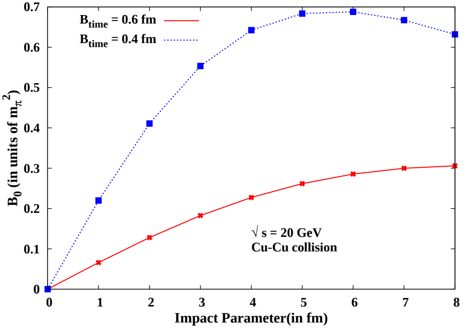


$$v_2 = \frac{T^{0x}_{pl} - T^{0y}_{pl}}{T^{0x}_{pl} + T^{0y}_{pl}} = \frac{v_x - v_y}{v_x + v_y} \tag{25}$$

Return

Central magnetic field vs impact parameter

- In heavy-ion collisions initial magnetic field decreases with time very fast in vacuum.
- It is not very clear at what time we will have thermalized plasma in which magnetic lines get frozen.
- We have calculated initial magnetic field profile at two different times and show results for both the field values.



Survival of Magnetic Field due to conducting plasma

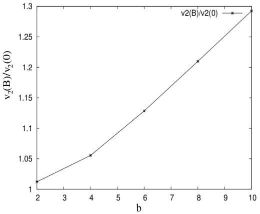
- Magnetic field genrated by a moving charge particle in vacuum, with velocity v along z-axis is given by,

$$\vec{H} = \frac{e\gamma}{4\pi} \frac{v|\vec{b} - \vec{b'}|\hat{\phi}}{(|\vec{b} - \vec{b'}| + \gamma^2(vt - z)^2)^{\frac{3}{2}}} \quad (26)$$

  where $\vec{b}$ is an observation point and $\vec{b'}$ is charge point.
- Magnitude of Magnetic field ($\propto \gamma$) is very high at time $t = 0$.
- But it decays very quickly at $t \neq 0$, because of presence of γ^2 in the denominator also.
- So in vacuum magnetic field decays very quickly.

Enhancement of elliptic flow due to magnetic field

- Since flow velocity is directly proportion to square of sound speed, asymmetry in the sound velocity can cause asymmetry in the flow.[5]

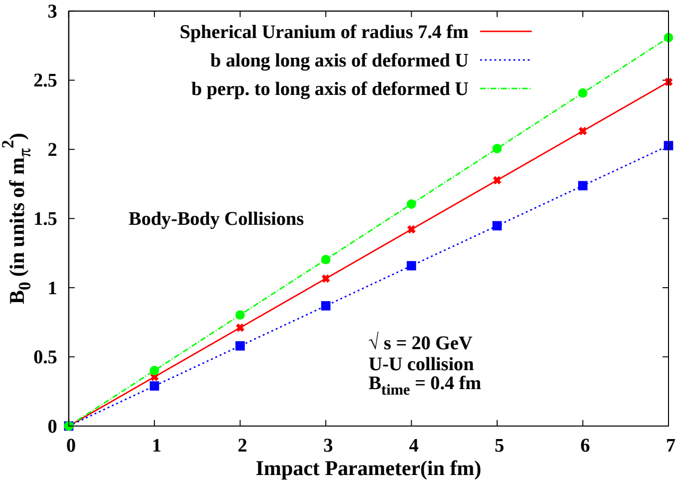


- So elliptic flow can arise becasue of magnetic field even in the azimuthally symmetric plasma region.
- Other calculation support this result.[6]

[5] **R.K. Mohapatra**, **P.S. Saumia**, **Ajit M Srivastava**, **Mod.Phys.Lett. A**, **26**, **2477-2486 (2011)**.

[6] **Kirill Tuchin**, **J. Phys. G: Nucl. Part. Phys. 39 025010 (2012)**.

Deformed Nuclei collisions



- Collision orientation dependence of magnetic field.