Effect of breakup and transfer on complete and incomplete fusion in $^{6}Li+^{209}Bi$ reaction in multi-body Classical Molecular Dynamics calculation

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Plan of the Talk

- ❖ Motivation
- ❖ Multi-body classical molecular dynamics model details

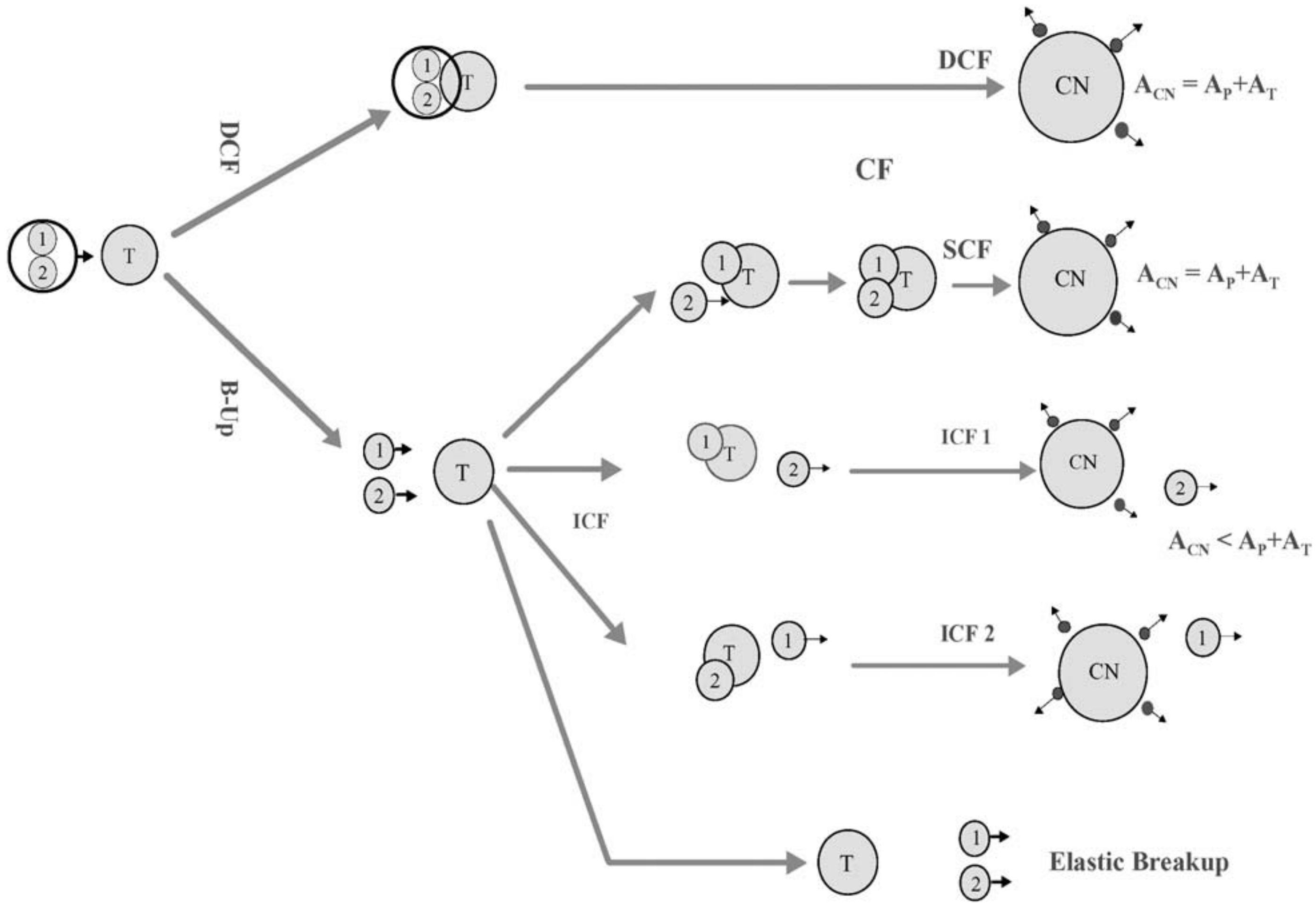
$^{6}Li+^{209}Bi$ reaction

- ❖ Various events (CF, ICF etc): simulation and probability calculations
- ❖ Complete, Incomplete/Total Fusion Cross Section calculation
- ❖ Conclusions

Breakup of stable weakly bound nuclei

- $^{6}Li \rightarrow {}^{4}He + d$ $S\alpha = 1.48$ MeV
- $^{7}Li \rightarrow {}^{4}He + t$ $S\alpha = 2.45$ MeV
-
-

Different processes in the reactions with weakly bound nuclei



Results from breakup measurement

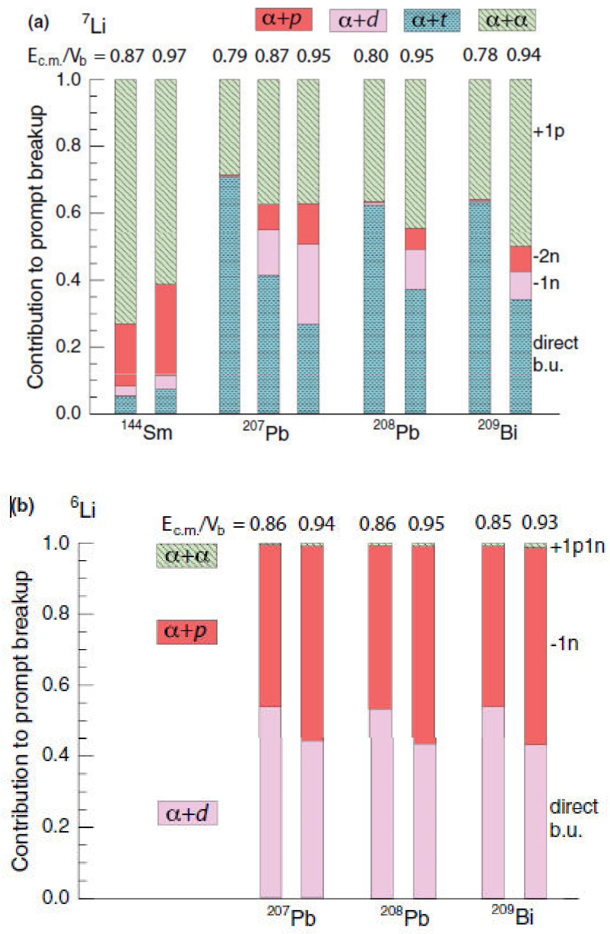
[D. H. Luong *et al,* Phys Rev C 88, 034609 (2013)]

- Breakup events in reactions of ^{6,7}Li arise either through direct excitation of ^{6,7}Li or through formation of intermediate nuclei via nucleon transfer which then undergo breakup.

- Coincidence measurements of breakup fragments by Luong *et al*, were carried out for ^{6,7}Li with various targets including ^{209}Bi at sub-barrier energies.

- For reactions involving ^{6}Li breakup was found to be predominantly triggered by *n* stripping leading to α+*p* for ^{6}Li ($^{6}\text{Li} \rightarrow {}^{5}\text{Li} \rightarrow \alpha + p$) more then direct cluster breakup ($^{6}\text{Li} \rightarrow \alpha + d$), and it is found to be target independent.

- In the case of ^{7}Li it was found to be predominantly triggered by *p* pickup leading to $\alpha+\alpha$ coincidence.



Break-up Reactions are usually studied using

- **Continuum Discretized Couple-Channel (CDCC)**

  [K. Hagino *et al.*, Phys. Rev. C **61**, 037602 (2000)]

- **Semi-classical couple channel approximation**

  [H. D. Marta, L. F. Canto, and R. Donangelo, Phy. Rev. C **78**, 034612 (2008)]
  [H D Marta *et al*, Phys. Rev. C **89**, 034625 (2014)]

- **Classical trajectory model**

  [Diaz-Torres, et al Phy. Rev.Lett. **98,** 152701 (2007)]

Continuum Discretized Couple-Channel (CDCC)

- More realistic than Couple-channel calculations such as CCFULL.
- Bound-continuum states couplings, with or without resonance states, discretized continuum states, continuum-continuum couplings.

Difficulties with CDCC

- It does not calculate sequential CF.
- Limited to 3-body (two fragments)

  (a poster yesterday: 4-body calculation)
- Very long computing time.

Semi-classical couple channel approximation

[H D Marta *et al*, Phys. Rev. C **89**, 034625 (2014)]

- Limited to 3-body (two fragments)
- Can distinguish between DCF, SCF, ICF etc and calculate corresponding cross sections.
- Direct reaction processes are not included.

Classical Trajectory Model

[Diaz-Torres, et al Phy. Rev.Lett. **98,** 152701 (2007)]

- Projectile: **2-body system**; assumed weak interaction potential; breakup initiated by a breakup probability function.
- **Three-body classical** system evolving under a given set of interaction potentials between each pair.

- **Deformation** and consequential **reorientation** of the entire projectile system is neglected.
- The interaction **potentials** are **not obtained self-consistently**.

➢ The **fusion probabilities are modified** when the **reorientation** of the approaching **deformed projectile** in the Coulomb field is taken into account. [Simenel *et al*, PRL 93, 102701 (2004); Desai & Godre, EPJA **47**, 146 (2011)]

➢ A new model is developed which is an **extension of the 3S-CMD model** [Godre, EPJ Web Conf, 86, 12 (2015)] **which can be used for three or many-body systems** [Morker & Godre, EPJ Web Conf, 86, 28 (2015)].

Model Calculation Details

(a) Construction of Nuclei
(b) Specification of Initial Conditions
(c) Dynamical Simulation
(d) Analysis

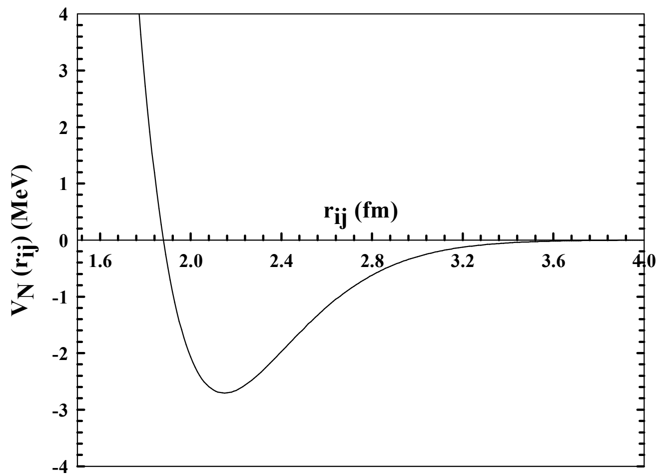
Construction of the nuclei

- Individual nuclei are constructed using a ***static potential energy minimization procedure***. The potential energy of nucleons is **cyclically minimized with respect to small displacements** of individual nucleon coordinates.
  [S. S. Godre and Y. R. Yaghmare, PRC 36, 1632 (1987)]

- **A soft-core Gaussian potential** (NN-potential) and **Coulomb potential** is chosen for interaction between nucleons.

$$V_{ij}(r_{ij}) = -V_0\left(1-\frac{C}{r_{ij}}\right)\exp\left(-\frac{r_{ij}^2}{r_0^2}\right)$$

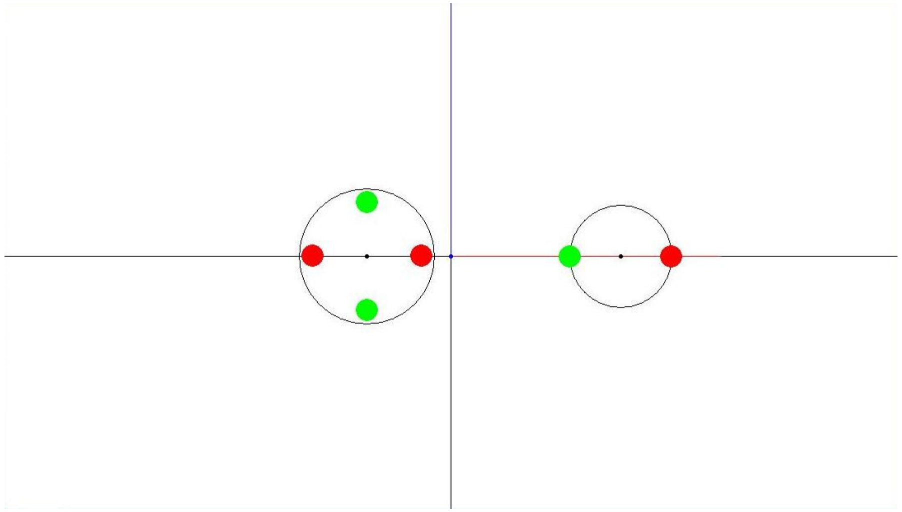
$$\mathrm{V_C}\left(\mathrm{r_{ij}}\right) = \frac{1.44}{\mathrm{r_{ij}}}(\mathrm{MeV})$$



- Parameters of the NN-potential (V_0, C, r_0) are chosen which reproduce ground-state properties of many nuclei.
- V_0= 710 MeV, C= 1.88 fm, r_0= 1.15 fm
  NN-potential between like particle pair - 20% weaker

  [W. D. Myers and W. J. Swiatecki, Ann. Phys.55, 395 (1969)]

Construction of *d* and α



	Calculated				Exprimental		
	B.E. (MeV)	**R_A (fm)**	**R_p (fm)**	**β_2**	**B.E. (MeV)**	**R (fm)**	**β_2**
^{2}H(*d*)	2.069	1.00	1.00		2.225	2.10	
^{4}He(α)	14.481	1.32	1.32	0.01	28.296	1.69	
^{6}Li(*d*+*α*)	18.017	1.93	2.00	1.11	31.990	2.54	.03
^{6}Li	29.150	1.52	1.52	0.01	31.990	2.54	.03
^{209}Bi	1606.16	5.55	5.69	-0.09	1640.00	5.52	0.0

Construction of ^{209}Bi

	Calculated				Exprimental		
	B.E. (MeV)	R_A (fm)	R_p (fm)	β_2	B.E. (MeV)	R (fm)	β_2
^{2}H(*d*)	2.069	1.00	1.00	7	2.225	2.10	
^{4}He(α)	14.481	1.32	1.32	0.01	28.296	1.69	
^{6}Li(*d+α)*	18.017	1.93	2.00	1.11	31.990	2.54	.03
^{6}Li	29.150	1.52	1.52	0.01	31.990	2.54	.03
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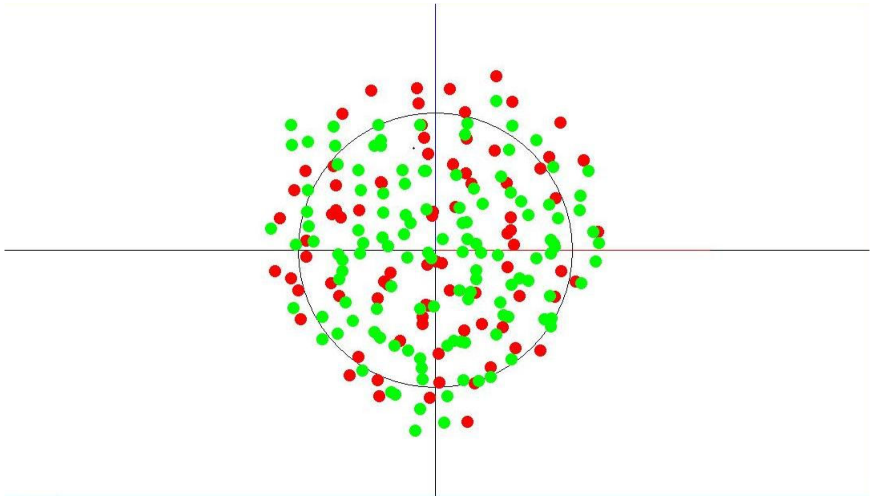
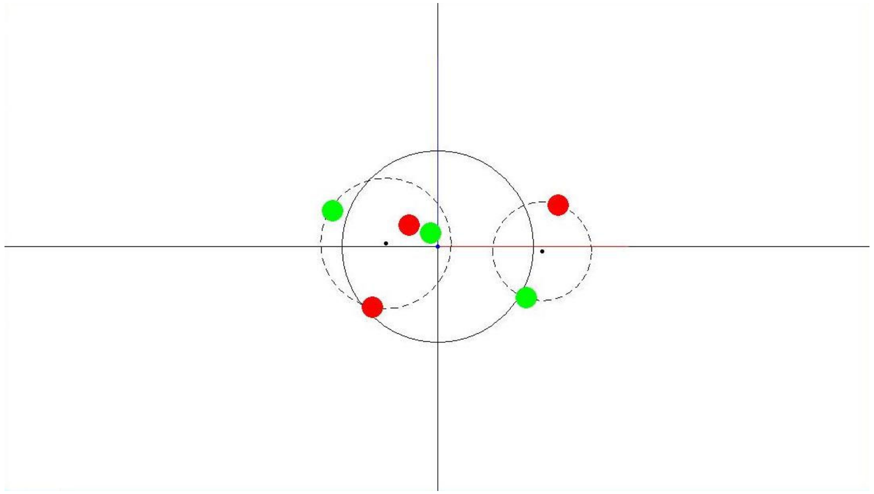
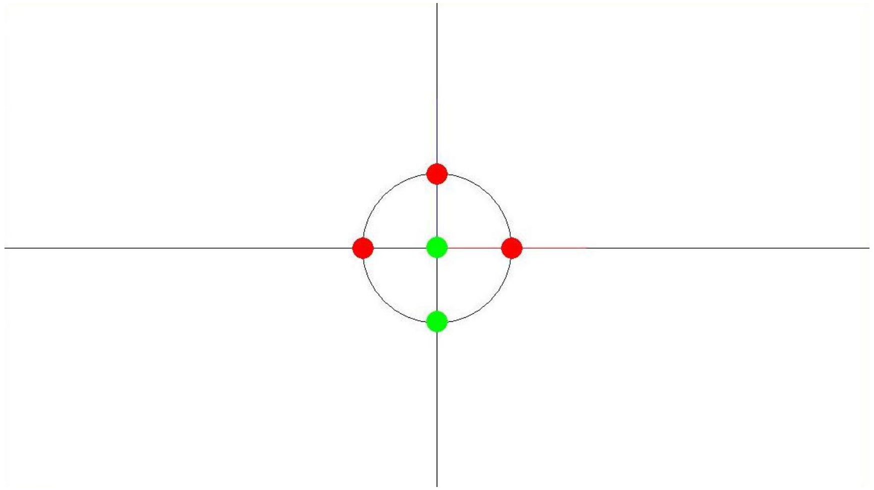


Construction of ^{6}Li as a weakly bound cluster of *d* and α





- The **potential energy (-1.446 MeV) between them** corresponds to the experimentally observed breakup threshold energy.
- The **interaction** between the fragments is generated **self-consistently** and the projectile and target both are **extended objects with desired size and shape deformation**.

	Calculated				Exprimental		
	B.E. (MeV)	R_A (fm)	R_p (fm)	β_2	B.E. (MeV)	R (fm)	β_2
^{2}H(*d*)	2.069	1.00	1.00		2.225	2.10	
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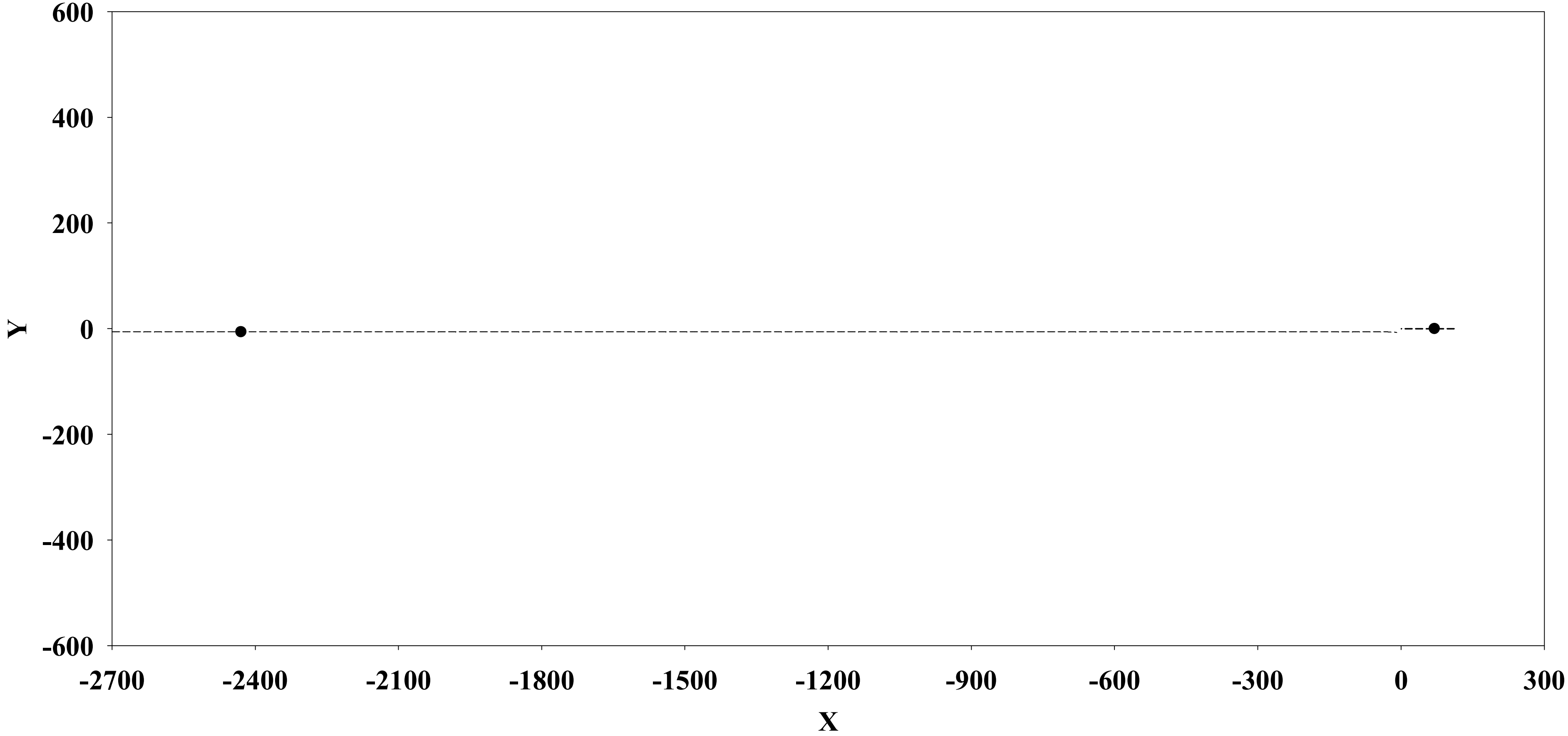
Dynamical Simulation in multi-body 3S-CMD

The dynamical collision simulation is carried out in the 3S-CMD in the following three stages

(1) Rutherford trajectory calculation

Rutherford trajectory calculation up to R_{in}= 2500 fm for given E_{cm} and *b*;



(2) Classical Rigid Body Dynamics (CRBD)

- The 3-body system of target and the projectile with its two constituent nuclei held rigidly, are placed on the Rutherford trajectories at R_{in}.

- This target projectile system as 2-body system, is then allowed to evolve further using the CRBD-model calculation. [Desai, & Godre, EPJA 47, 146 (2011)]

- The motion of the centre of masses and the orientation of the principal axes of the target and the projectile are obtained from the classical equations of motion for rigid bodies under the influence of the ion-ion potential and torques generated by the interaction between all the nucleons in the combined system.

Rigid-Body Equations of motion

In *fixed frame*,

- Total Linear Momentum

$$\dot{\vec{P}}_k = M_k \frac{d\vec{R}_k}{dt}$$

- Total Angular Momentum

  where

$$\dot{\vec{L}}_k = \vec{R}_k \times \vec{P}_k + A_k \overleftrightarrow{I}_k \vec{\Omega}_k$$

$\vec{\Omega}_k$ - body angular velocity

M_k - mass

I_k - moment of inertia tensor

$\vec{R}_k$ - position vector of the c.m. of the k^{th} nucleus.

- Total force on 1 due to 2

$$\vec{F}_{12} = \sum_{i=1}^{A_1} \sum_{j=1}^{A_2} \vec{F}_{ij}$$

- Total torque on1 due to 2

$$\vec{N}_{12} = \sum_{i=1}^{A_1} \sum_{j=1}^{A_2} \left(\vec{r}_{ij} \times \vec{F}_{ij} \right)$$

- The two nuclei are allowed to evolve through Classical rigid-body equations of motion:

$$\frac{d\vec{P}_k}{dt} = \vec{F}_{k\ell} \quad ; \quad \frac{d\vec{R}_k}{dt} = \frac{\vec{P}_k}{M_k}$$

$$\frac{d\vec{L}_k}{dt} = \vec{N}_{k\ell} \quad ; \quad \frac{dA_k}{dt} = \hat{\omega}_k A_k$$

for $\{k \neq \ell ; k = 1, 2\}$

$\vec{F}_{k\ell}$ - force exerted by the ℓ^{th} nucleus on the k^{th} nucleus,

$\vec{N}_{k\ell}$ - torque exerted by the ℓ^{th} nucleus on the k^{th} nucleus,

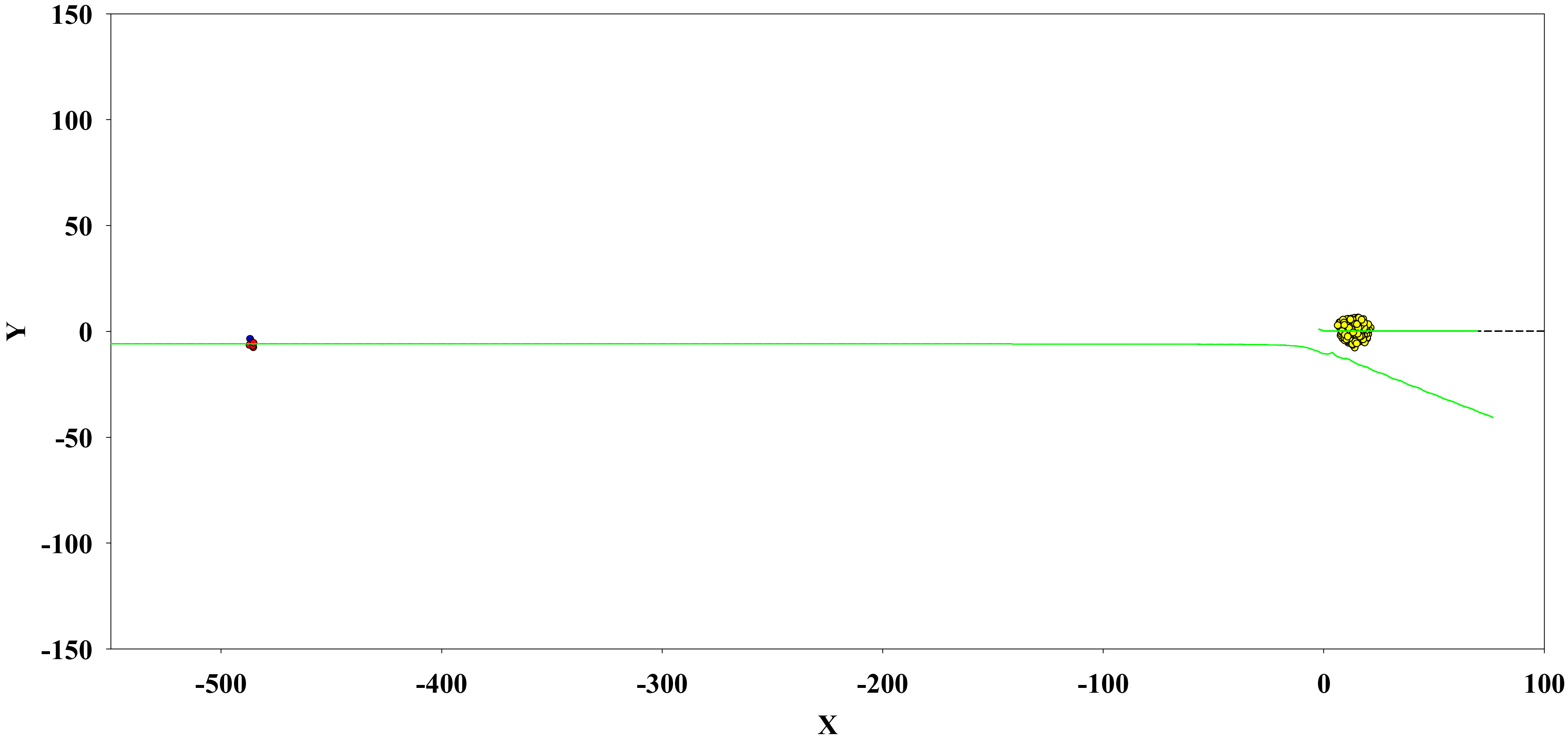
$\hat{\omega}_k$ - matrix defined by the angular velocity of the body in the fixed frame

$$\vec{\omega}_k = A_k \overleftrightarrow{I}_k^{-1} A_k^T (\vec{L}_k - \vec{R}_k \times \vec{P}_k) \quad ; \quad \{k = 1, 2\}$$

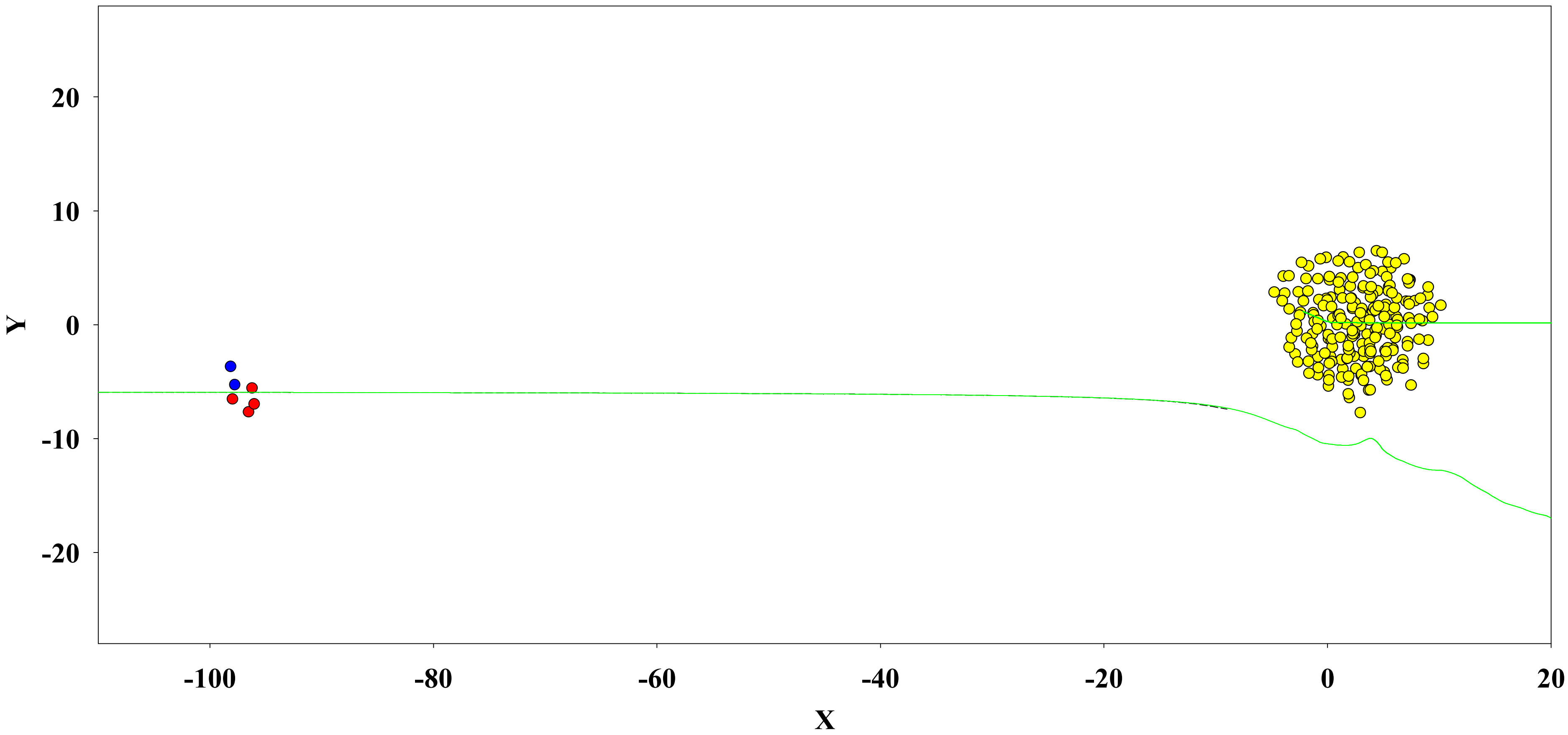
- Matrix $\hat{\omega}$ is given by

$$\hat{\omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$$

where $\omega_1, \omega_2, \omega_3$ are the three components of the angular velocity vector of the individual nuclei along their three principal axis in the *fixed frame*.



150
100
50
0
-50
-100
-150
Y
-500
-400
-300
-200
-100
0
100
X



20
10
0
-10
-20
Y
-100
-80
-60
-40
-20
0
20
X

(3) Classical Molecular Dynamics (CMD)

- Rigid-body constraints at about R_{cm}= 13 fm is relaxed and trajectories of all the nucleons in both the colliding nuclei obtained as in CMD calculation by solving coupled equations of motion:

$$m\frac{d^2 r_i}{dt^2} = -\nabla_i\left[\sum_{j\neq i} V_{ij}\right]$$

- If some fragments are further constrained to be rigid, it is dynamically evolved as in the CRBD-calculation.

- Relaxation of rigid-body constraints at appropriate stage takes care of excitations of the target and projectile fragments

$^{6}Li+^{209}Bi$ collision at Ecm = 42.7 MeV.

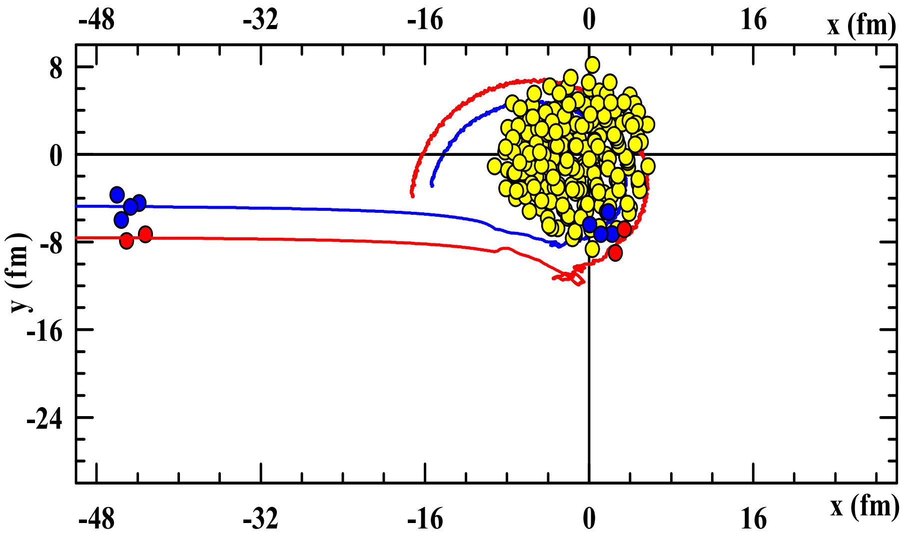


Fig (a). *b* = 5.6 fm (Complete Fusion)

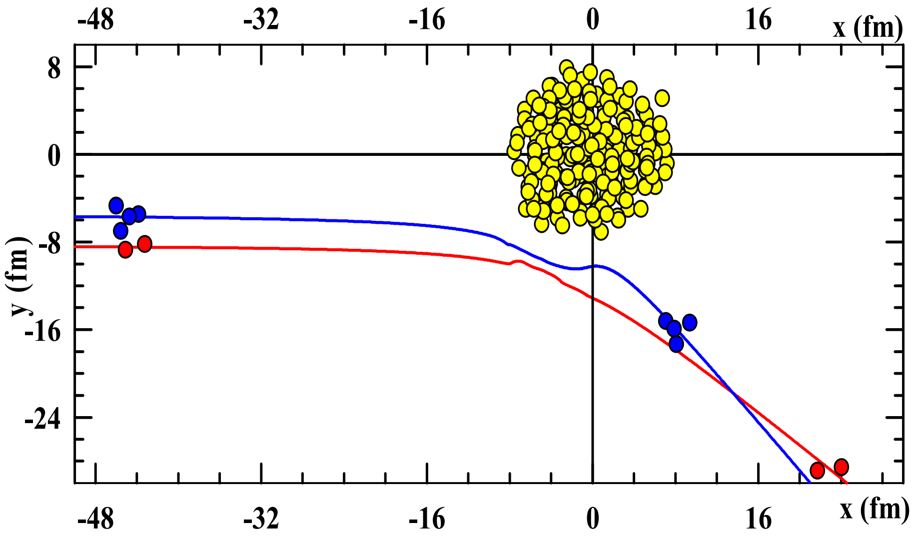


Fig (c). *b* = 6.5 fm (Breakup Scattering)

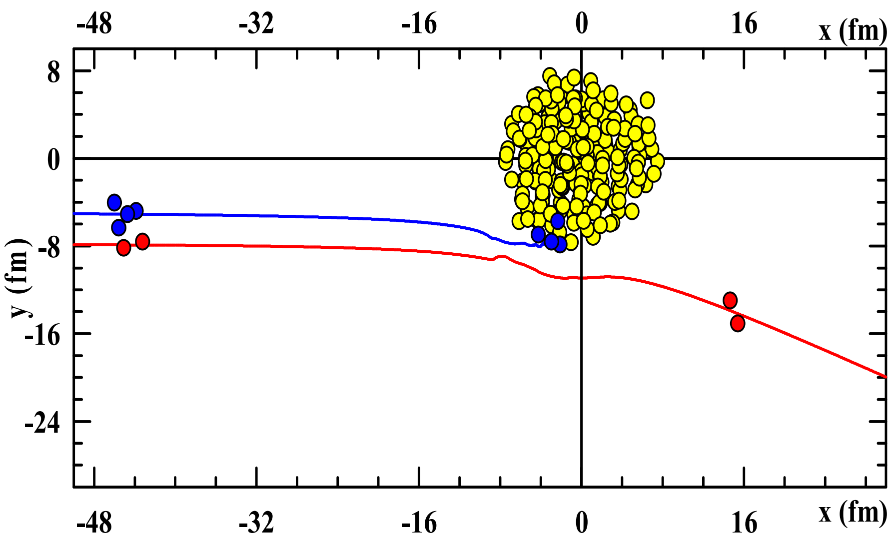


Fig (b). *b* = 5.9 fm (Incomplete Fusion)

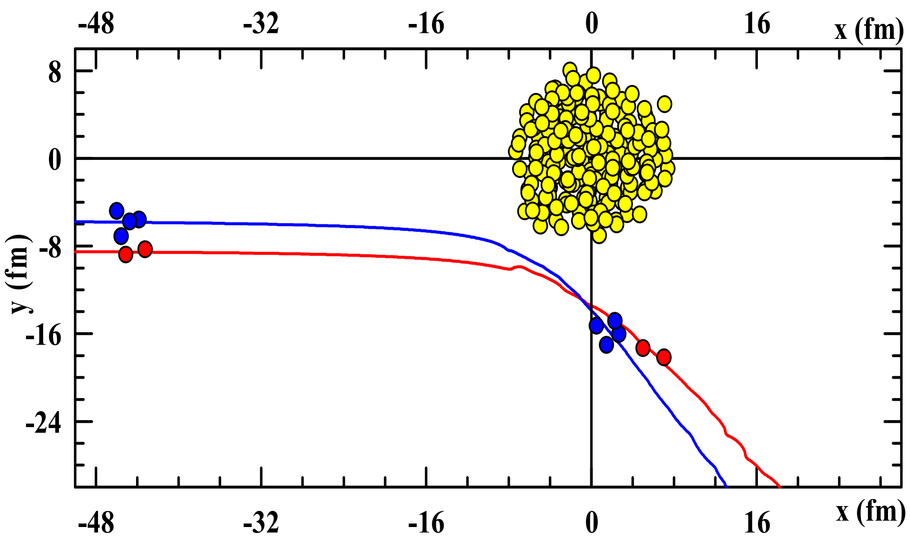


Fig (d). *b* = 6.6 fm (Scattering)

Event Probabilities

Calculated event probabilities (event-fractions),

$F(b)$ **= Nevent(*b*) / Ntotal(*b*)**

Ntotal: Total no of initially random orientations for given Ecm and *b*
500 at each value of *b* for Ecm =50 and 36 MeV
2000 for Ecm = 29 MeV;

Nevents: no of events analyzed DCF, SCF, ICF etc.

Considered different cases with systematic relaxation of the RB-constraint on the projectile fragments and the bond between them
(keeping ^{209}Bi non-rigid and α rigid in all the cases in stage-3):

(a) ^{6}Li (as rigid-body);
(b) both α and ***d*** are rigid but free to move with respect to each other for Rcm<13 fm;
(c) same as in (b) but allowing ***d*** also to breakup.

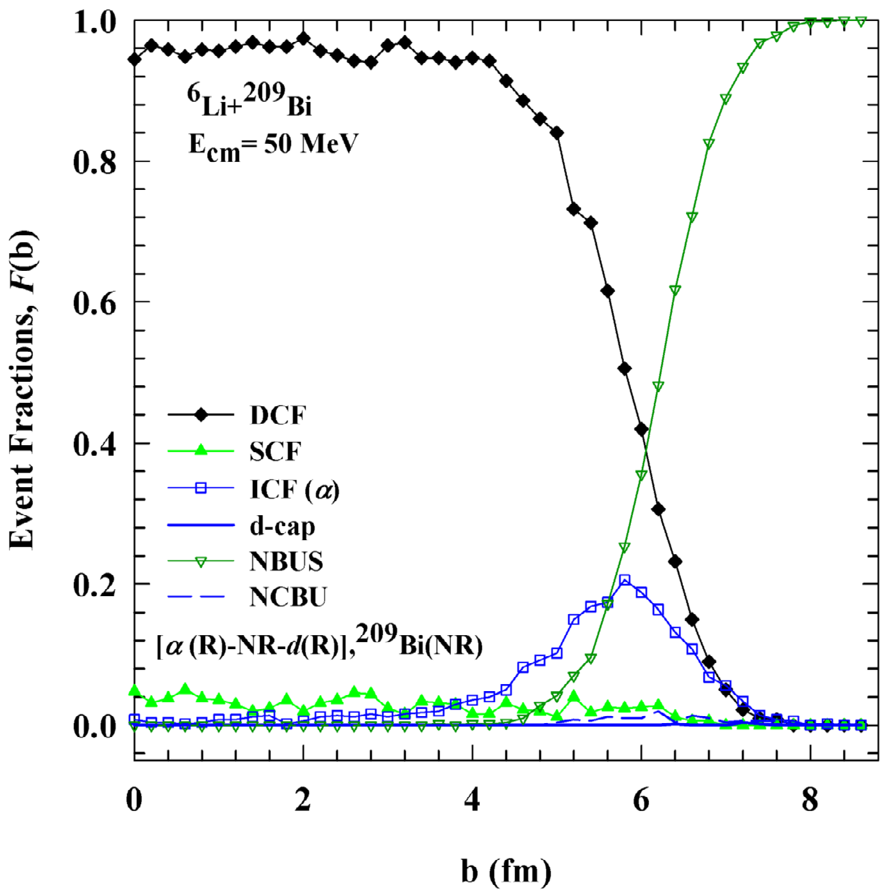
<u>Case-(b):</u> [α (R) -NR- *d* (R)]; ^{209}Bi (NR)

- As *b* increases, the relative angular momentum of the projectile fragments increases, resulting in more number of events following break up (ICF+NCBU).

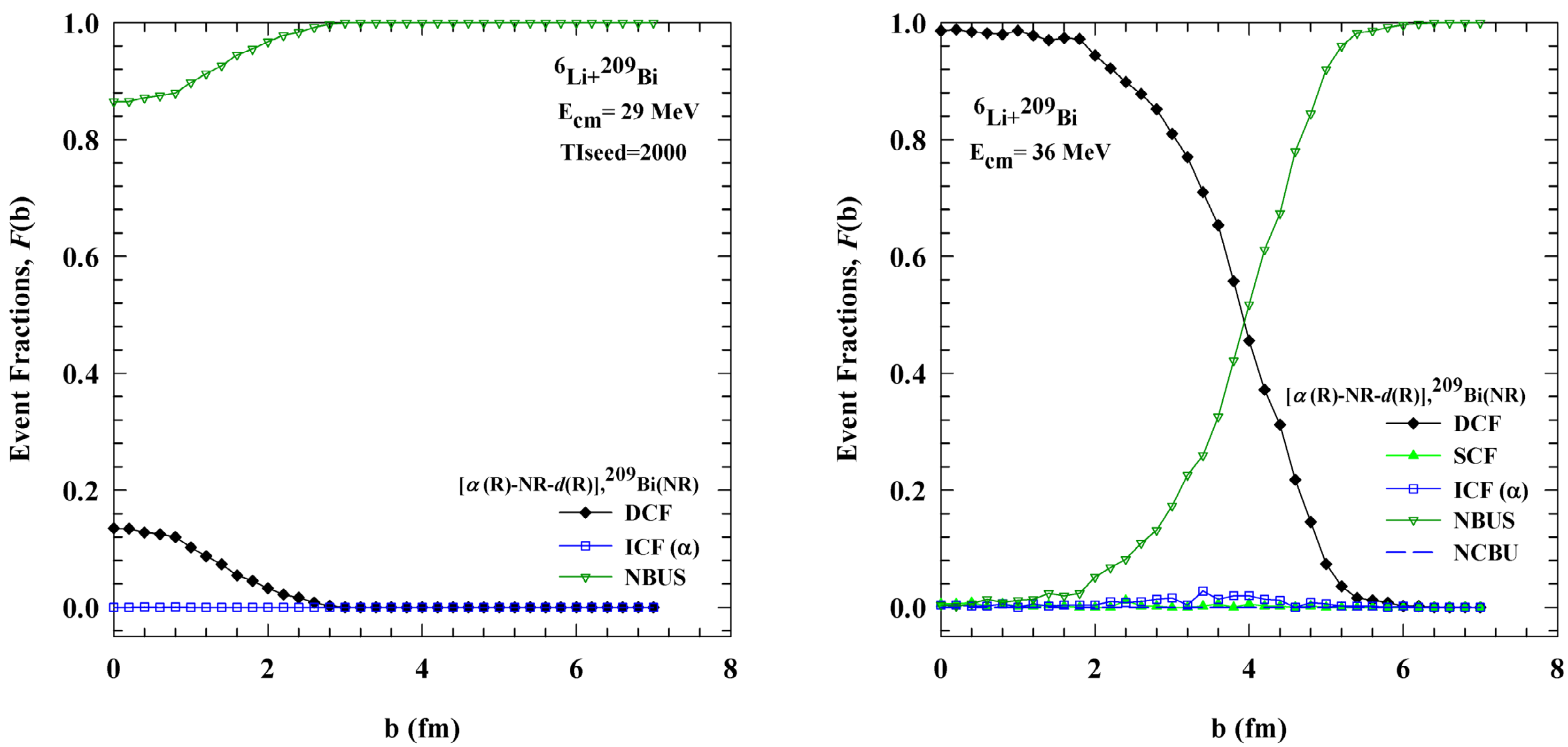
- At larger *b* the trajectories don't come very close to target and *F*(*b*) for events following break up again decreases.

- ICF(α) with α -capture, is negligible at low *b* and rises at higher *b*.

- ICF(***d***) with ***d***-capture and scattering following breakup (NCBU) are negligible. Thus where breakup occurs, either (SCF), or ICF(α) occurs.



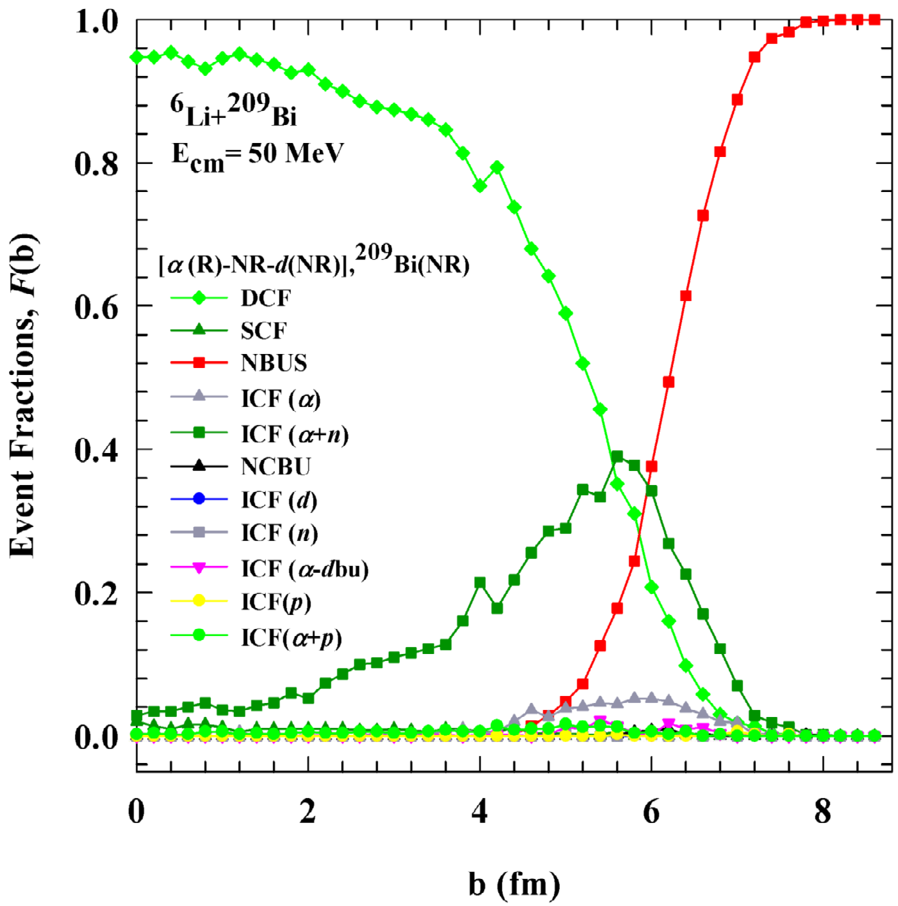
Case-(b): [α (R) -NR- *d* (R)]; ^{209}Bi (NR)



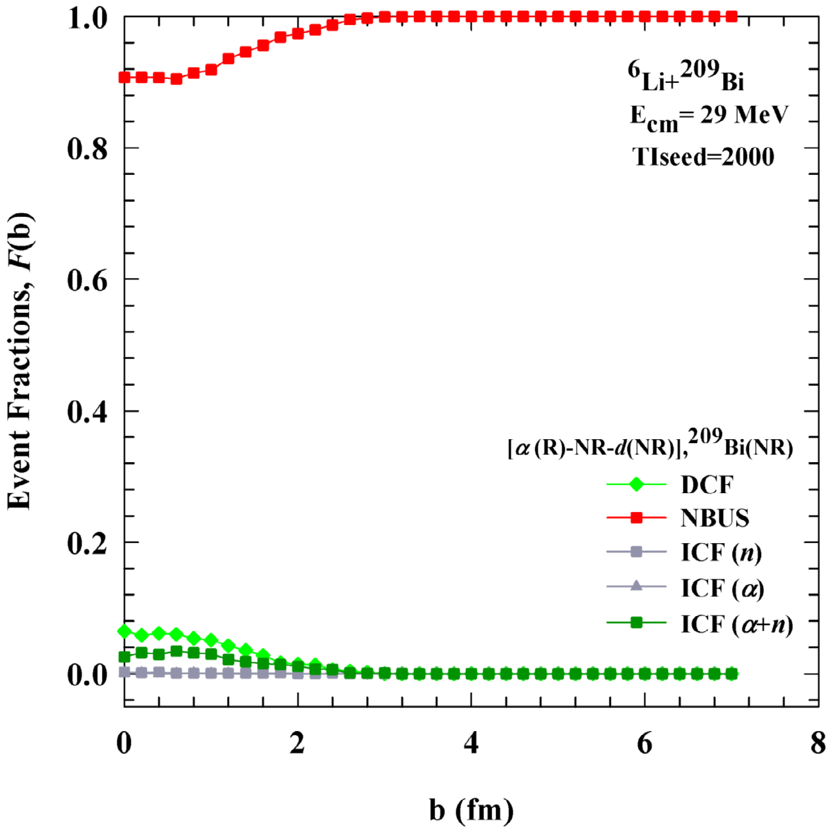
At lower energies,
DCF is the major component and ICF(α) and SCF are negligible.

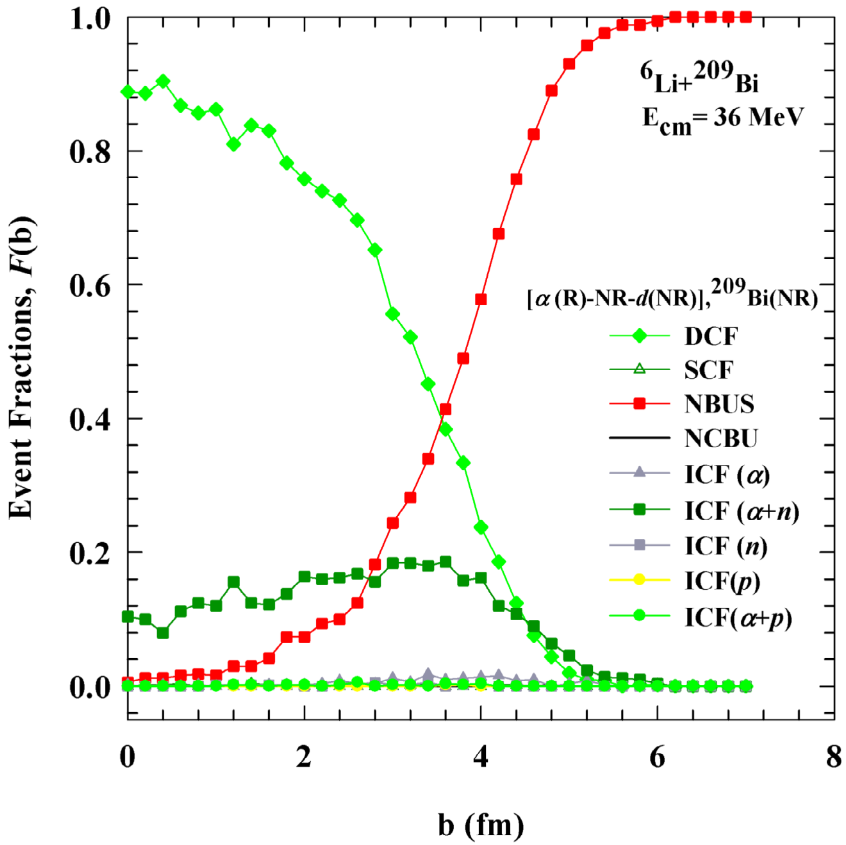
Case-(c): [α (R) -NR- *d* (NR)]; ^{209}Bi (NR)

- Events ICF(α+***n***) equivalent to ICF(^{5}**He**) or *n*-stripping followed by breakup of the resultant ^{5}Li→ α +***p*** with ***p*** scattered.
- ICF(α +***n***) distribution is much broader and larger as compared to ICF(α) in case(b).
- ICF(α +***n***) is substantially large compared to events ICF(α).
- in conformity with the recent expt. obs. of Luong *et al* which shows importance of direct reaction processes.

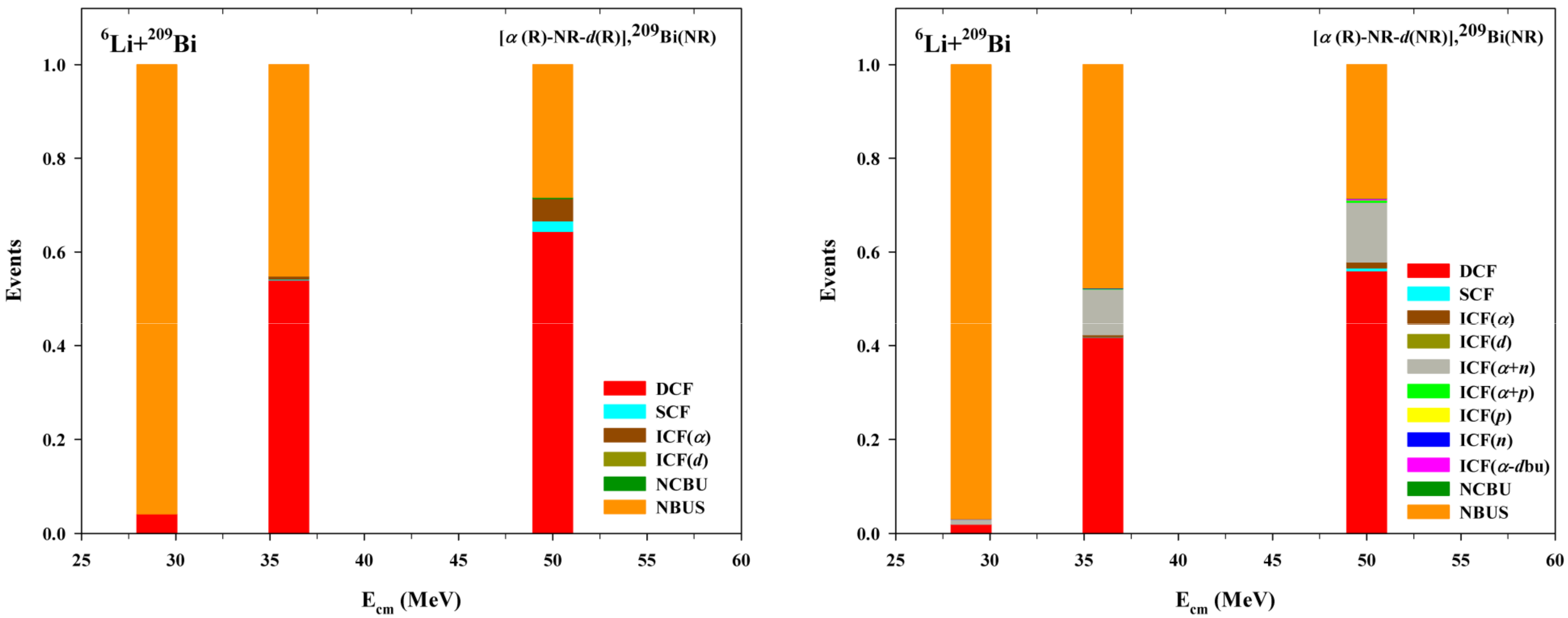


Case-(c): [α (R) -NR- *d* (NR)]; ^{209}Bi (NR)

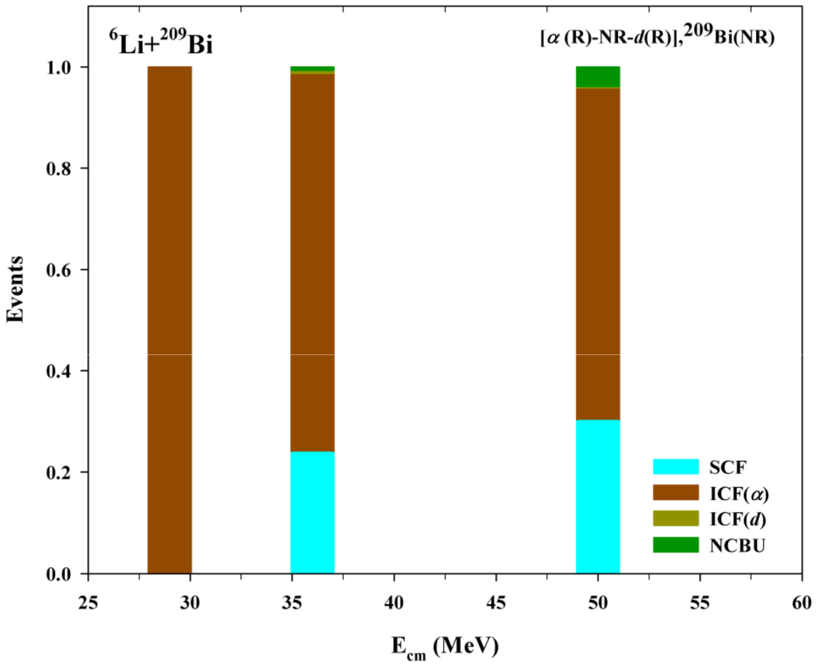


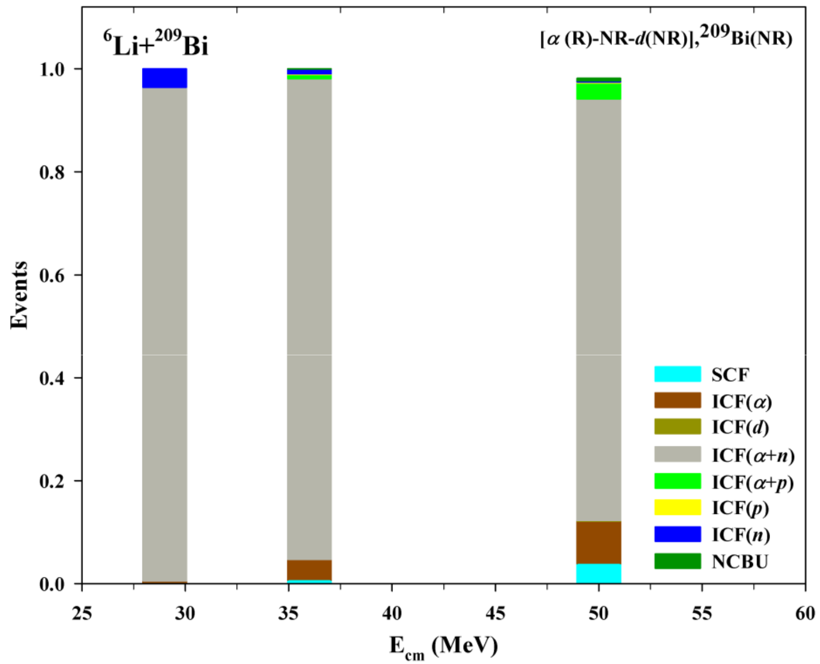


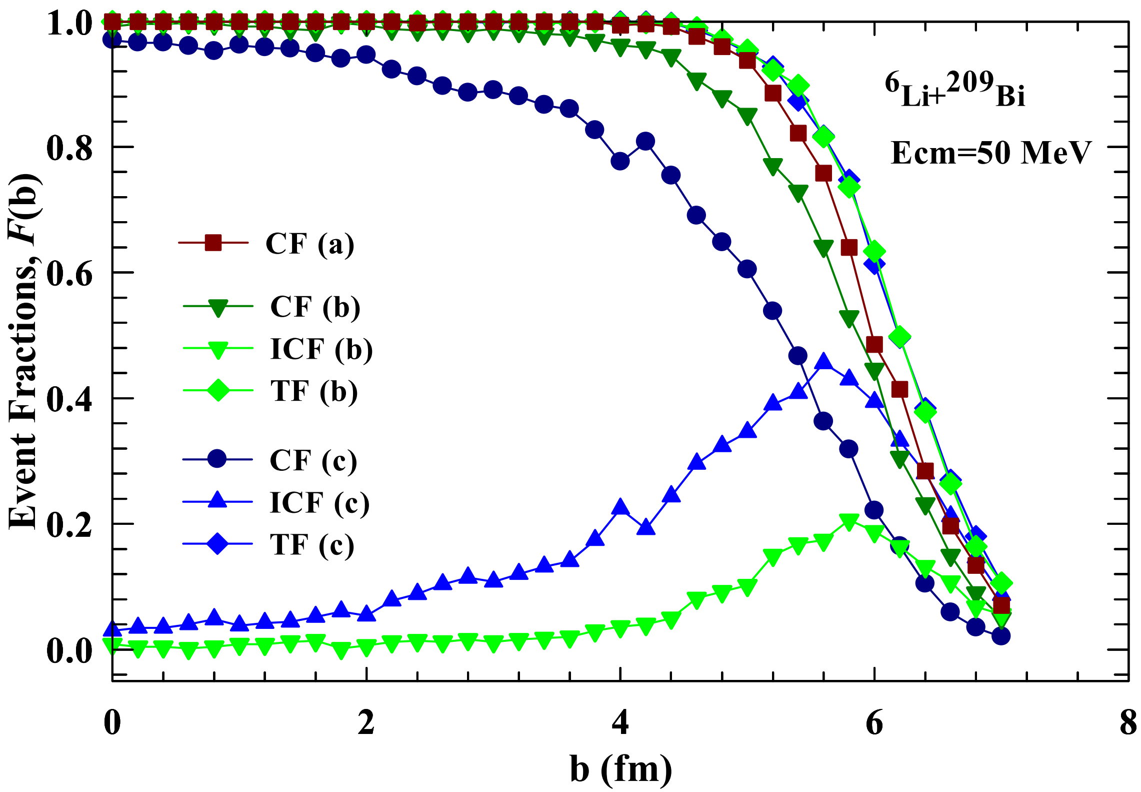
Events distribution as function of Energy



Breakup events distribution as function of Energy





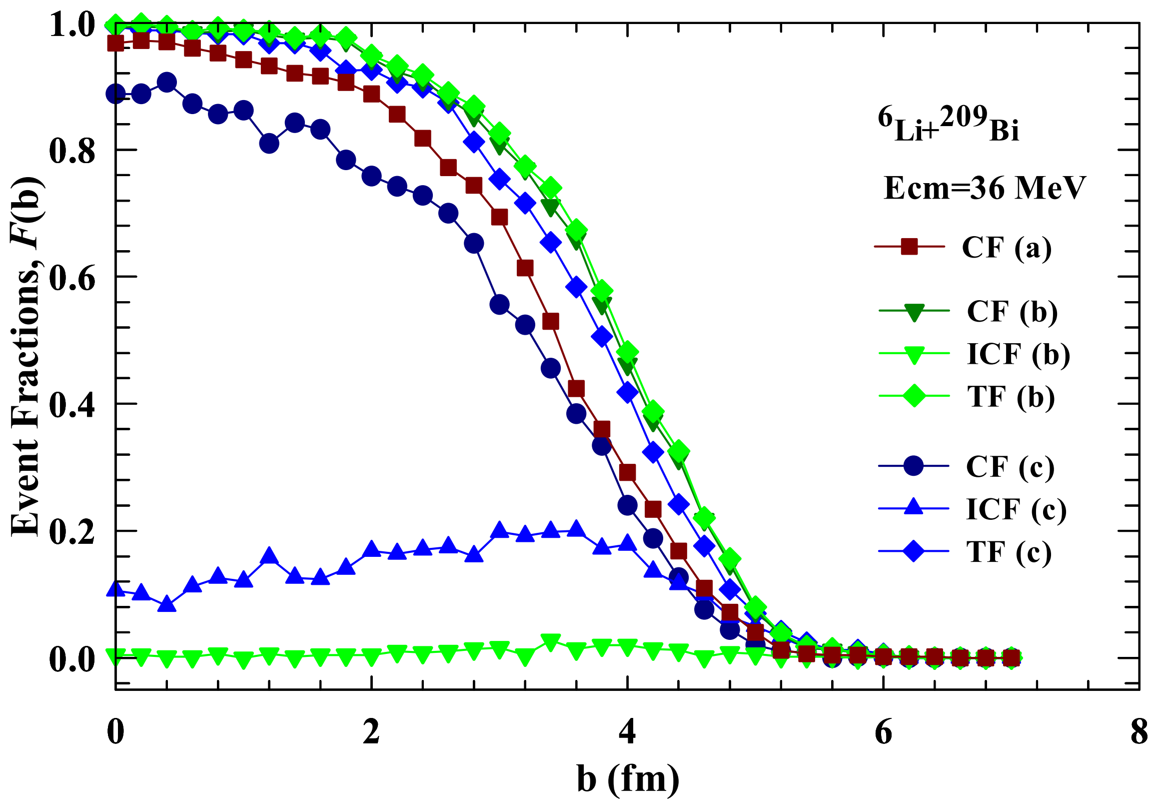


- TF in cases (b) and (c) are almost identical which implies that the differences in ICF are responsible for the CF values to be different in the two cases.

- Since there is no break-up in case-(a), CF in this case is close to TF in the other cases.

- A small difference may be attributed to complete lack of internal excitations which tends to lower the fusion probability.

- This conforms to the experimental observation of suppression of complete fusion at higher energies as compared to the case where there is no breakup.



- ❑ suppression of CF diminishes as energy is lowered
- ❑ internal excitations in both the cases-(b&c) leads to enhancement of CF cross sections at lower energies compared to higher energies.

Complete Fusion (CF) Cross Sections

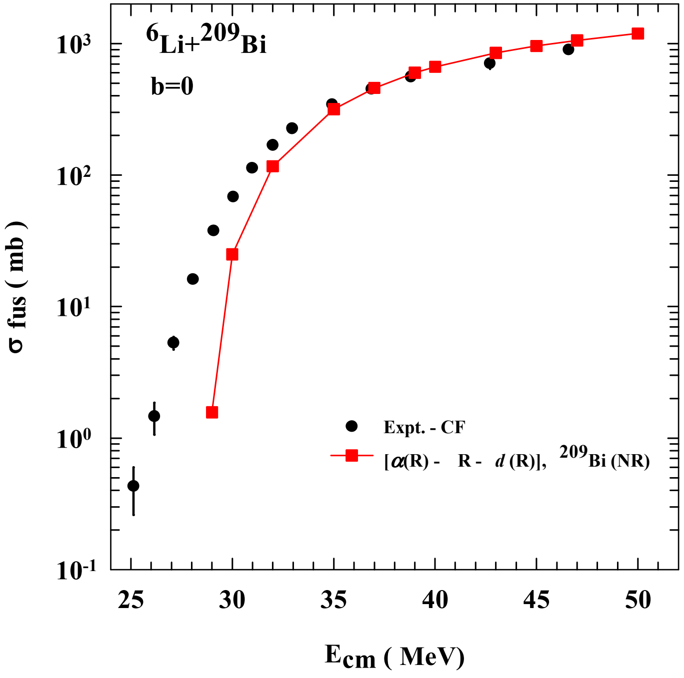
CF (Def): both the projectile fragments are captured by the target for long time.

- $\sigma_{CF} = \sigma_{DCF} + \sigma_{SCF}$
- Ion-ion potential is obtained as a function of the separation between the centre of masses of the target and the projectile. Barrier parameters (V_B, R_B, ω_B) for *b*=0, obtained at every E_{CM} by the dynamically generated ion-ion potential.
- Fusion cross section is calculated using Wong's formula,
  [C. Y. Wong, Phys. Rev. Lett., 31, 766 (1973)].

$$\sigma(E_{CM}) = \left[\frac{R_B^2 \hbar\omega_0}{2E_{CM}}\right] \ln\left\{1 + \exp\left(2\pi \frac{E_{CM} - V_B}{\hbar\omega_0}\right)\right\}$$

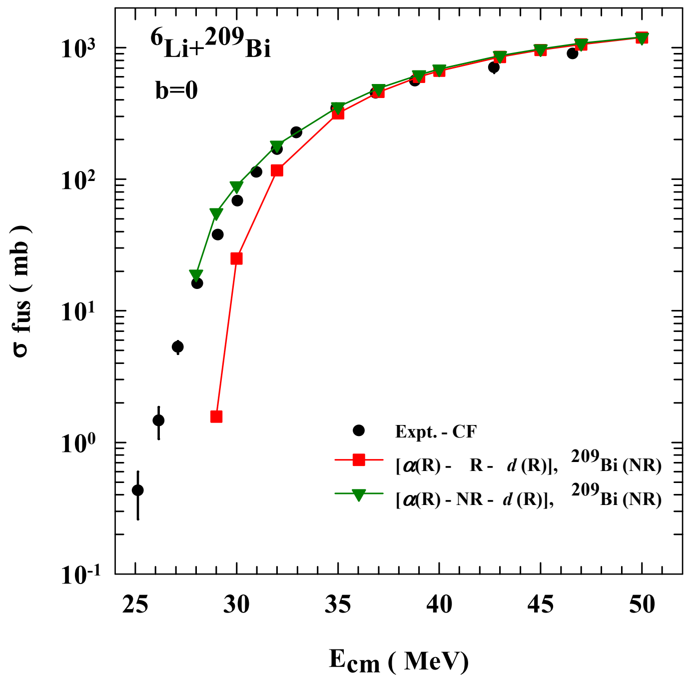
- For given E_{CM} a large number of randomly chosen initial orientations are considered and orientation-averaged fusion cross section is calculated.

➢ Trajectories and the barrier parameters are obtained classically while quantum tunneling taken into account by using Wong's formula in a semi-classical manner.



Case(a)

- No internal excitations in the rigid projectile which lowers fusion probability resulting in highly underestimated CF cross sections at lower energies compared to the experimental data **[Dasgupta *et al*, PRC 70, 024606 (2004)]** and that for the case-(b).



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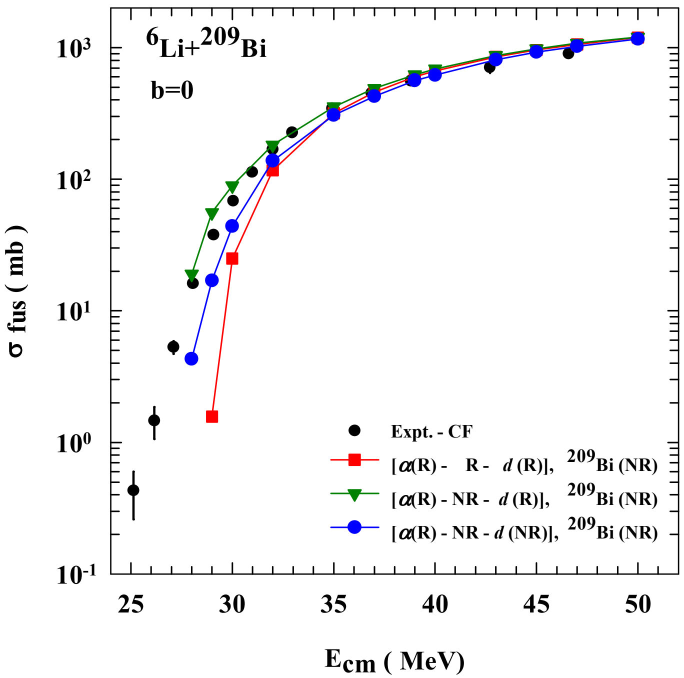
<u>Case(b)</u>

- Two fragments are rigid but the projectile is allowed to get excited leading to significant enhancement in CF cross section at all energies as compared to case-(a).
- Since the excited projectile can also possibly break up, resulting in loss of flux for CF and contributing to ICF events
- Thus CF cross sections for case-(b) are less then the TF cross section for this case.



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Case(b)

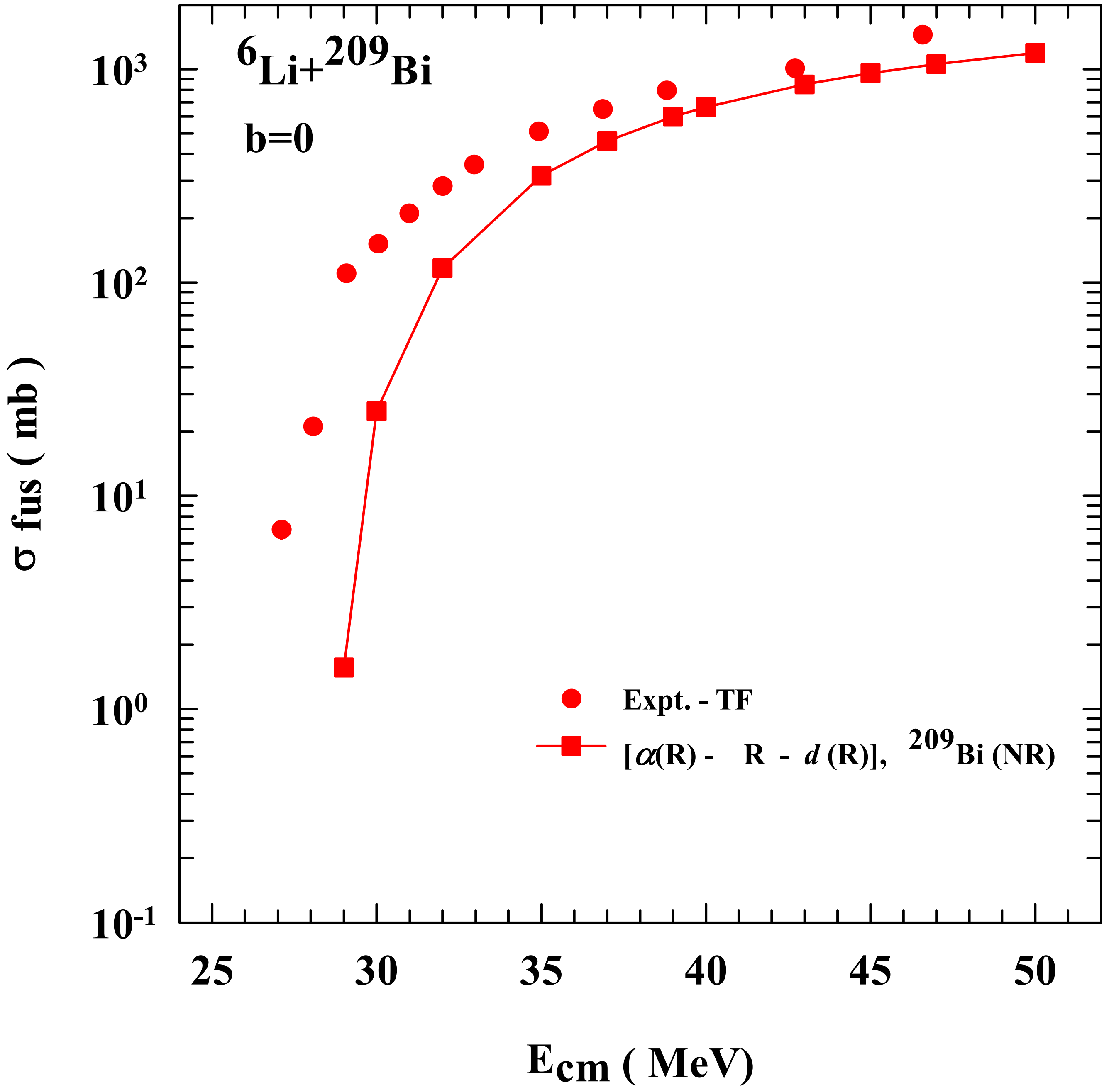
- Two fragments are rigid but the projectile is allowed to get excited leading to significant enhancement in CF cross section at all energies as compared to case-(a).
- Since the excited projectile can also possibly break up, resulting in loss of flux for CF and contributing to ICF events
- Thus CF cross sections for case-(b) are less then the TF cross section for this case.

Case(c)

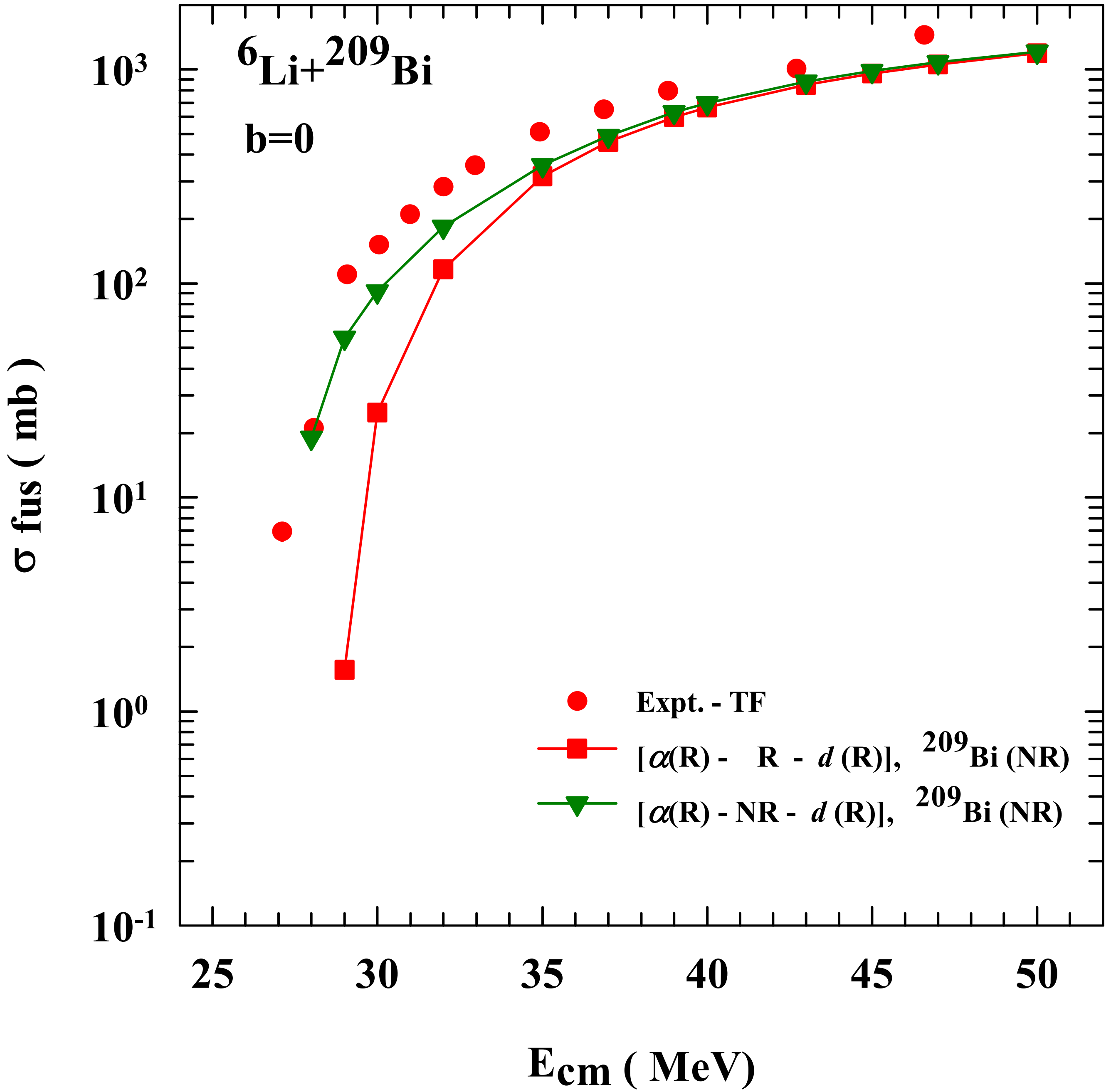
- Possible breakup of weakly bound ***d*** itself results in direct reaction process, n-stripping followed by breakup of the resulting unstable **^{5}Li→*α*+*p*** with ***p*** scattered leading to ICF(α+*n*) equivalent to ICF(^{5}He) events.
- It leads to significant reduction in CF cross sections compared to rigid ***d*** as in case-(b).

Total Fusion (TF) Cross Sections

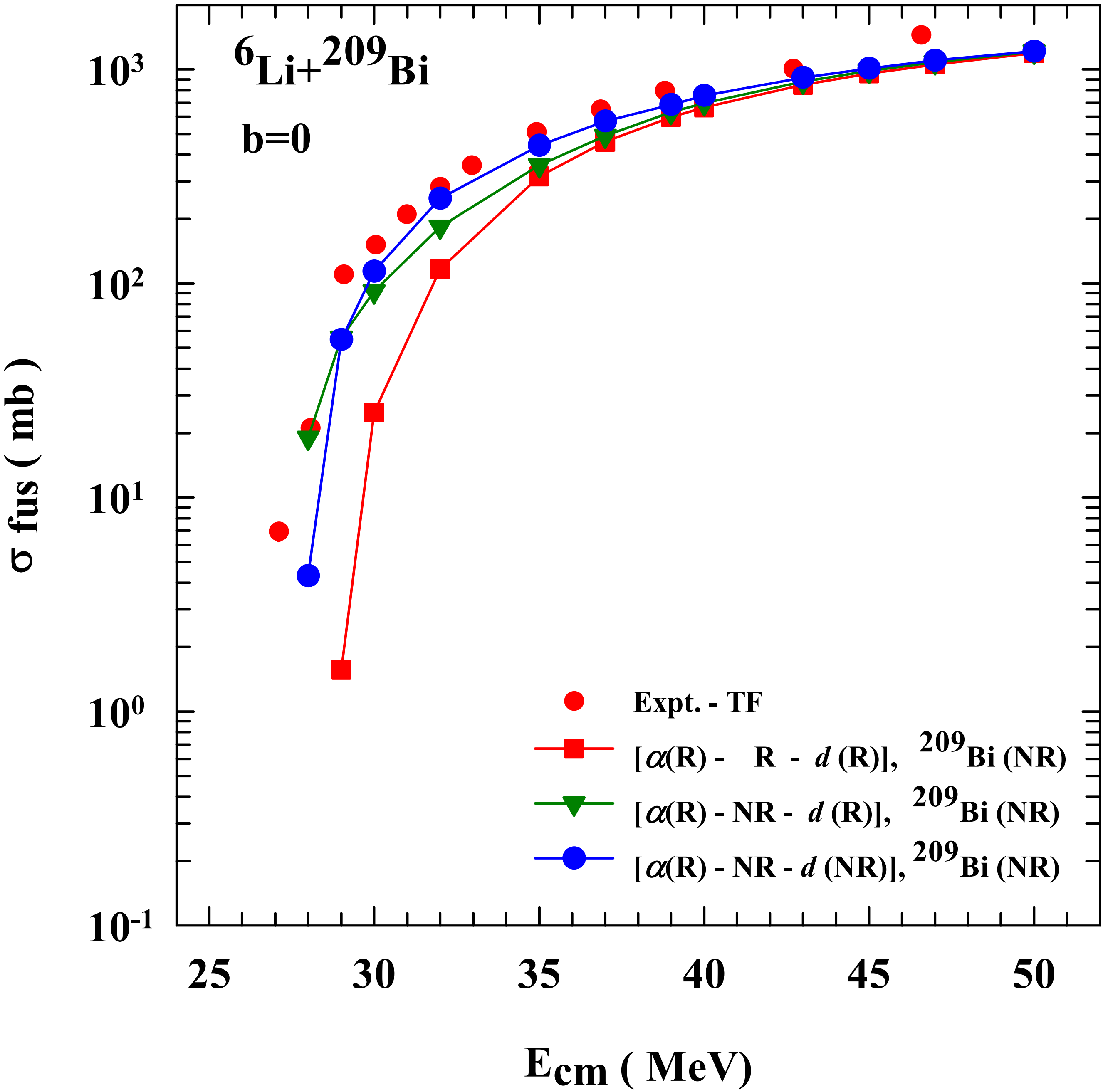
- **Incomplete Fusion (ICF):** only one of the projectile fragment or a part of the projectile is captured.
- Ion-ion potential is obtained as a function of the separation between the cm of the target and the projectile-fragment that is captured.
- Barrier parameters determined from this ion-ion potential (b=0) are used in the Wong's formula to get incomplete fusion cross section $\boldsymbol{\sigma_{ICF}}$.
- Total fusion cross section $\boldsymbol{\sigma_{TF}} = \boldsymbol{\sigma_{CF}} + \boldsymbol{\sigma_{ICF}}$



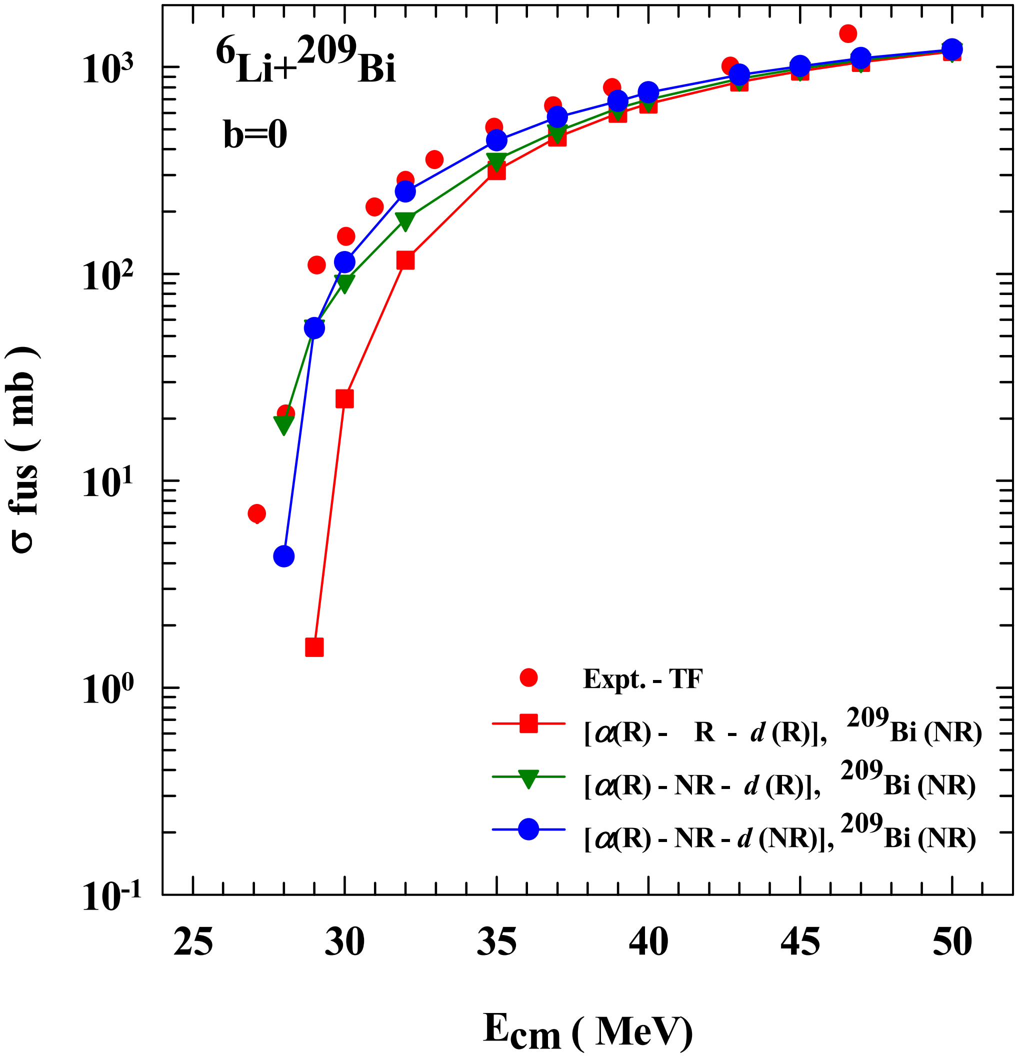
^{6}Li+^{209}Bi
b=0
Expt. - TF
[α(R) - R - d (R)], ^{209}Bi (NR)
σ fus (mb)
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
25
30
35
40
45
50
E_{cm} (MeV)



$^{6}Li+^{209}Bi$
b=0
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
σ fus (mb)
25
30
35
40
45
50
E_{cm} (MeV)
Expt. - TF
[α(R) - R - d (R)], ^{209}Bi (NR)
[α(R) - NR - d (R)], ^{209}Bi (NR)



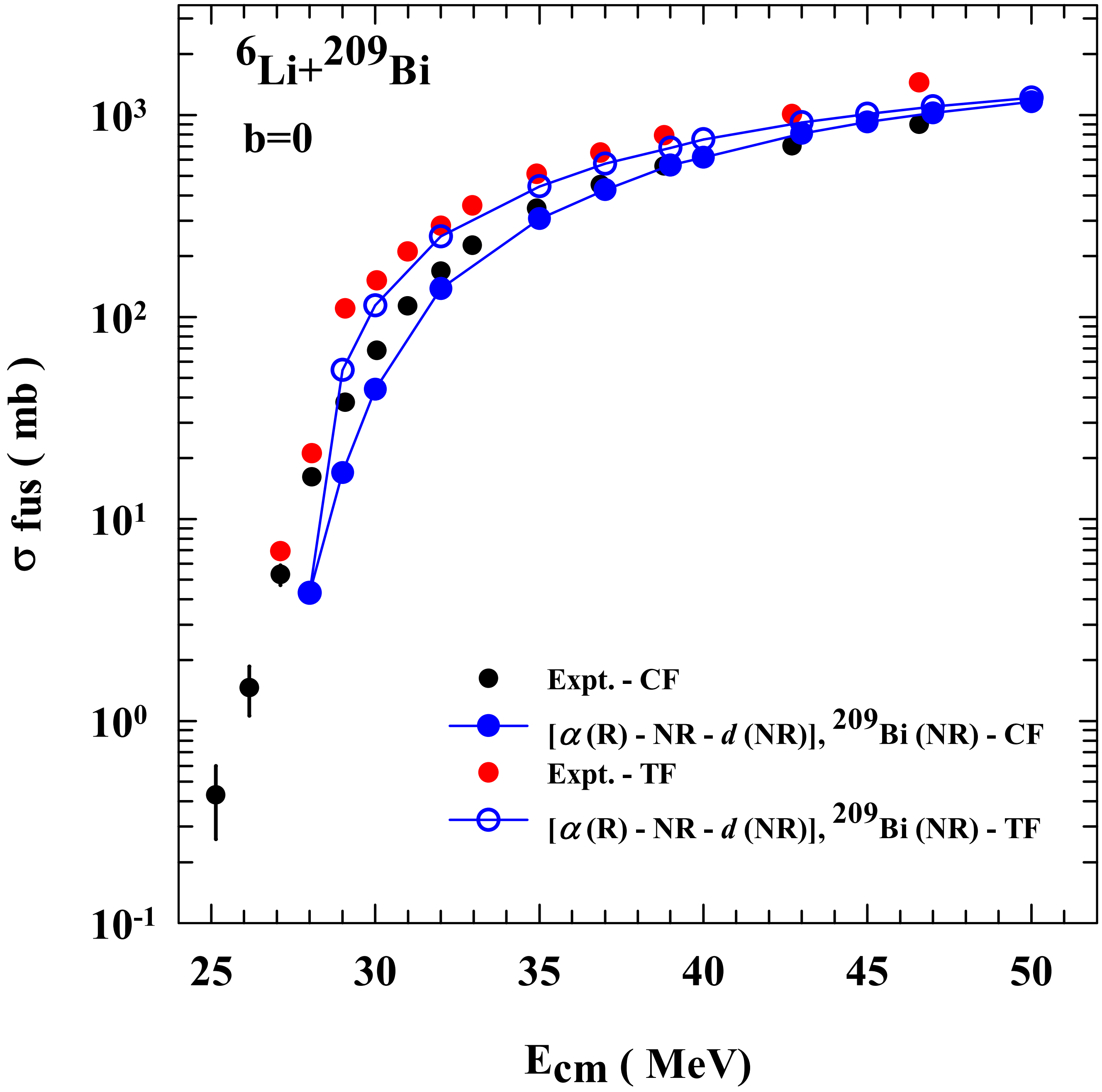
$^{6}Li+^{209}Bi$
b=0
σ fus (mb)
E_{cm} (MeV)
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
25
30
35
40
45
50
Expt. - TF
[α(R) - R - d (R)], ^{209}Bi (NR)
[α(R) - NR - d (R)], ^{209}Bi (NR)
[α(R) - NR - d (NR)], ^{209}Bi (NR)



TF cross sections for case-(b) & (c) have almost same values at all the energies which shows difference in CF in case (b) and (c) is due to ICF or ICF(α+n).

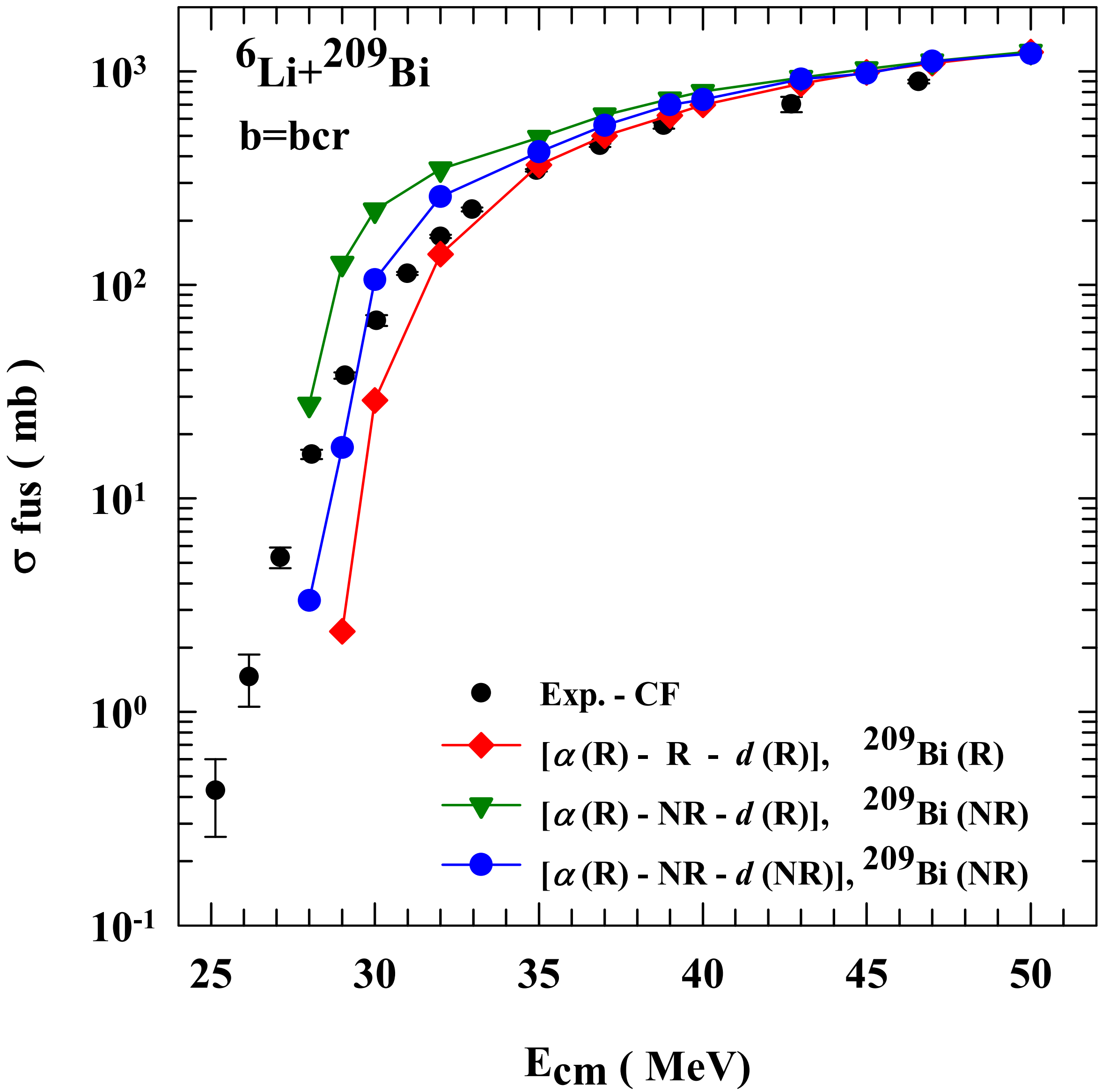
TF cross section data obtained from [Dasgupta *et al*, PRC 70, 024606 (2004)] . which, however, can not distinguish between ICF and direct reaction products.

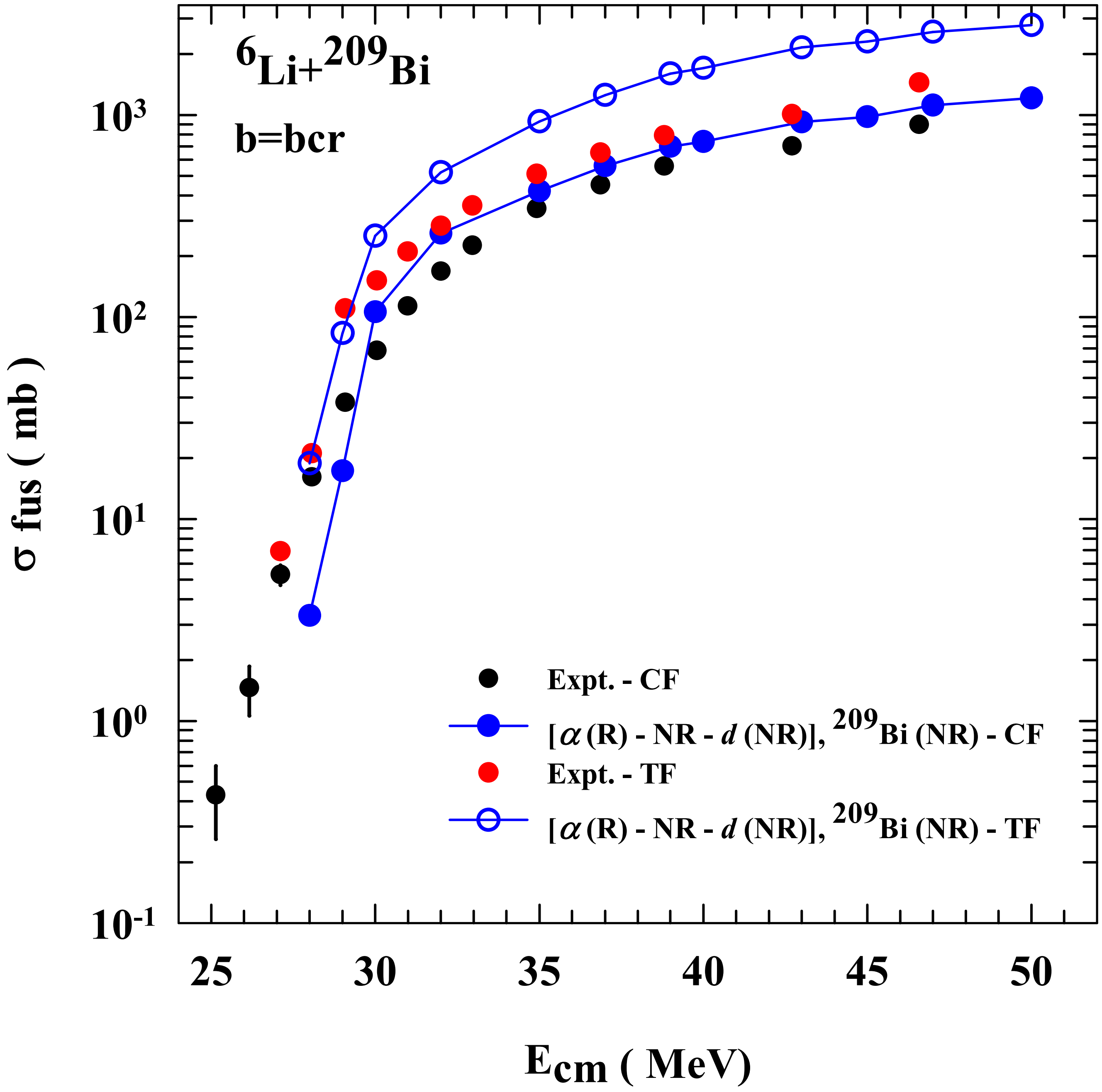
Calculated CF and TF cross sections in case-(c) gives good agreement with the expt.



6Li+209Bi
b=0
σ fus (mb)
Ecm (MeV)
Expt. - CF
[α (R) - NR - d (NR)], 209Bi (NR) - CF
Expt. - TF
[α (R) - NR - d (NR)], 209Bi (NR) - TF

[Morker & Godre, EPJ Web Conf, 86, 28 (201 5)]; [Rowley & Hagino, PRC 91, 044617 (2015)]





^{6}Li+^{209}Bi
b=bcr
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
σ fus (mb)
25
30
35
40
45
50
E$_{cm}$ (MeV)
Expt. - CF
[α (R) - NR - d (NR)], ^{209}Bi (NR) - CF
Expt. - TF
[α (R) - NR - d (NR)], ^{209}Bi (NR) - TF

Conclusions

- The comparative study of calculated CF and TF cross sections for $^{6}Li+^{209}Bi$ reactions with systematic relaxations of the rigid-body constraints on the target, projectile fragments and the bond between the projectile fragments exhibits the importance of constituent's excitations and breakup.

- As a result of allowing for internal excitations, CF cross sections are comparatively enhanced, but breakup of the constituents takes away the flux from CF, resulting in its comparative suppression.

- Therefore, the otherwise possible enhancement of CF cross section in the case-(b) due to the projectile excitations is suppressed at higher energies resulting in similar values as in the case-(a).

- The CF cross section calculation which allows for the breakup of 6Li into ***α*** and ***d,*** as well as breakup of ***d*** itself (case-c), gives reasonable agreement with the experiment.

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Thank You for Your Attention