

Lattice simulation of Chiral Magnetic Effect in Dirac Semimetals

D. L. Boyda, V. V. Braguta, M. I. Katsnelson,
A. Yu. Kotov, A. A. Nikolaev

arXiv:1608.07162, Phys.Rev. B94 (2016) no.20, 205147
arXiv:1704.07132
and 'in preparation'

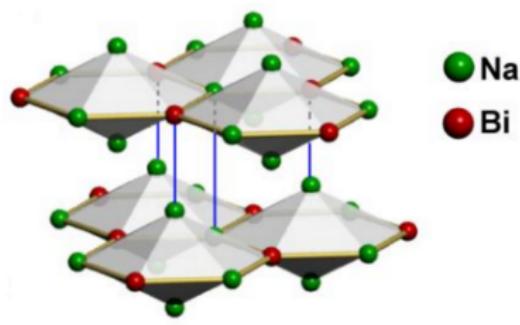
XQCD, 26-28 June 2017 Pisa

ITEP, Moscow

Dirac semimetals

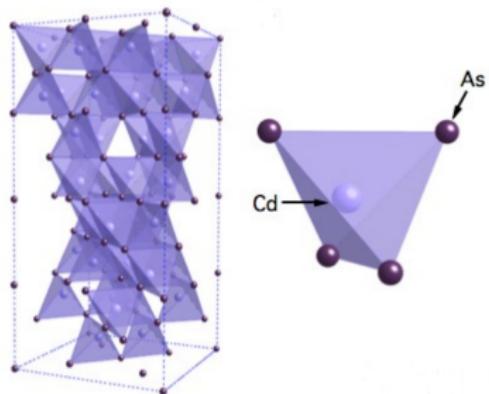
- ▶ Condensed matter analog of Relativistic QFT
 - ▶ massless fermions
 - ▶ strongly coupled
- ▶ 3D analog of graphene (2D)
- ▶ Perfect toolkit for tabletop study of phenomena, previously attributed to HEP (e.g. anomalous transport)

Dirac semimetals. Elementary structure



Na_3Bi

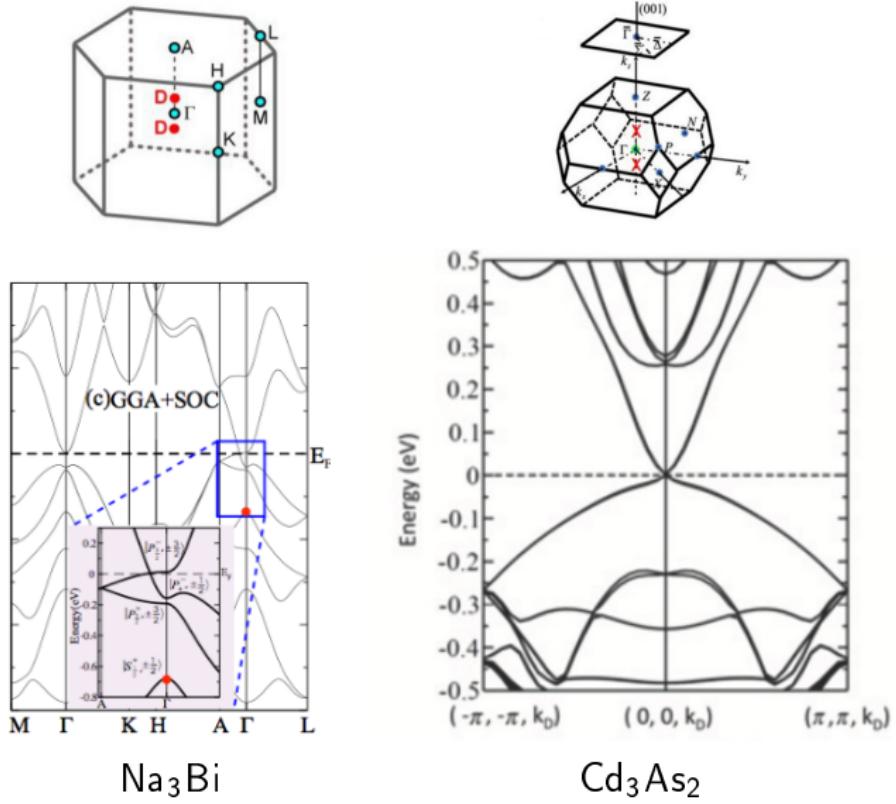
arXiv:1310.0391, Z. K. Liu et al.



Cd_3As_2

arXiv:1309.7892, M. Neupane et al.
arXiv:1309.7978, S. Borisenko et al.

Band structure



Na_3Bi

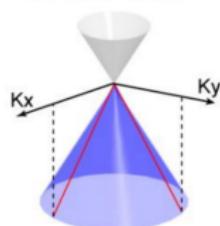
Cd_3As_2

How to study these materials?

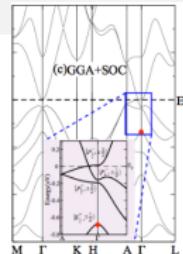
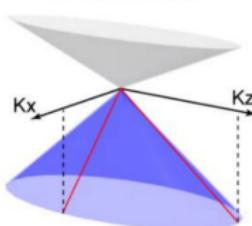
Effective theory approach

- If relevant energy is $E \sim E_f$

Kx-Ky
2D Dirac Cone



Kx-Kz
2D Dirac Cone



arXiv:1310.0391, Z. K. Liu et al.

$N = 2$ Dirac points with dispersion relation

$$E \sim \sqrt{v_{\perp}^2(k_x^2 + k_y^2) + v_{\parallel}^2 k_z^2}$$

Effective theory

- ▶ $N = 2$ Massless Dirac fermions
- ▶ Fermi velocity is $v_f \ll c$
- ▶ Anisotropy in Fermi velocity $v_{\parallel} \neq v_{\perp}$
effectively induced by strain (arXiv:1609.00615, S.Guan et al.)
- ▶ Electromagnetic interaction
($v_f \ll c \rightarrow$ neglect magnetic part)
- ▶ Effective charge is $\alpha_F = \frac{\alpha}{v_F} \sim 1$, $\alpha = \frac{1}{137}$

$$Z = \int D\bar{\psi} D\psi DA \exp \left(-\frac{1}{8\pi\alpha_F} \int d^4x [\partial_j A_4]^2 + i \int d^4x \bar{\psi} \left(\gamma^4 [\partial_4 + iA^4] + \xi_k \gamma^k \partial_k \right) \psi \right)$$

Lattice discretization

Staggered fermions ($N = 4 \rightarrow N = 2$ via rooting):

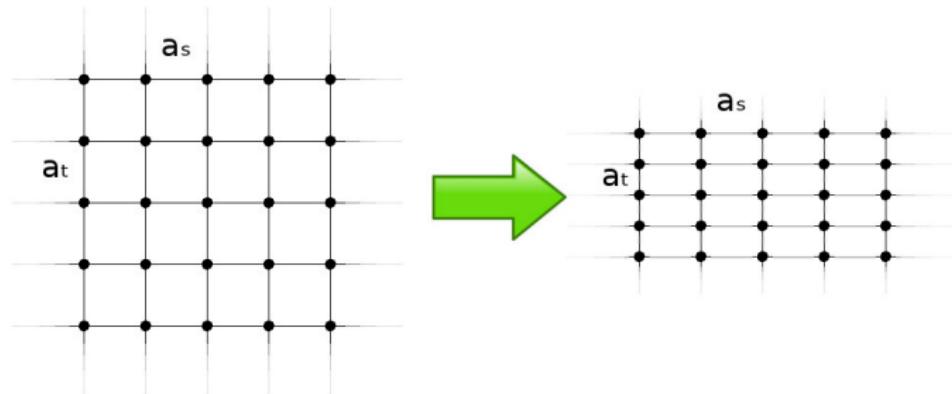
$$S_{\Psi \text{ int.}} = \sum_x m \bar{\Psi}_x \Psi_x + \frac{1}{2} \sum_{i=1,2,3} \xi_i (\bar{\Psi}_x \alpha_{x,i} \Psi_{x+\hat{i}} - \bar{\Psi}_x \alpha_{x,i} \Psi_{x-\hat{i}}) + \\ + \frac{1}{2} \bar{\Psi}_x \alpha_{x,4} \exp(i\theta_{I,4}(x)) \Psi_{x+\hat{4}} - \frac{1}{2} \bar{\Psi}_x \alpha_{x,4} \exp(-i\theta_{I,4}(x - \hat{4})) \Psi_{x-\hat{4}}$$

Noncompact gauge action:

$$S_g = \frac{\beta}{2} \sum_{i=1,2,3} \theta_{p,\hat{i}4}^2(x), \quad \beta = \frac{1}{4\pi\alpha_F} \\ \theta_{p,\hat{i}4} = \theta_{I,4}(x + \hat{i}) - \theta_{I,4}(x)$$

Lattice discretization

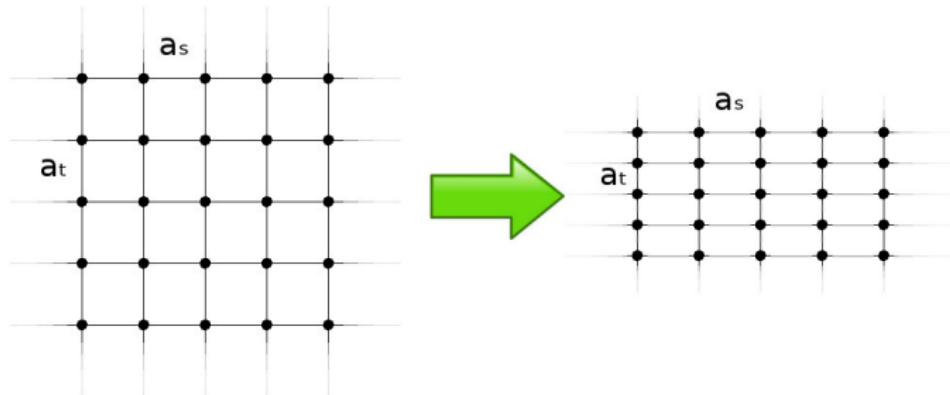
Different time- and space-like steps: $a_s \neq a_t$



The same idea was in PoS LATTICE2015, 046 (2016), Y. Araki.

Lattice discretization

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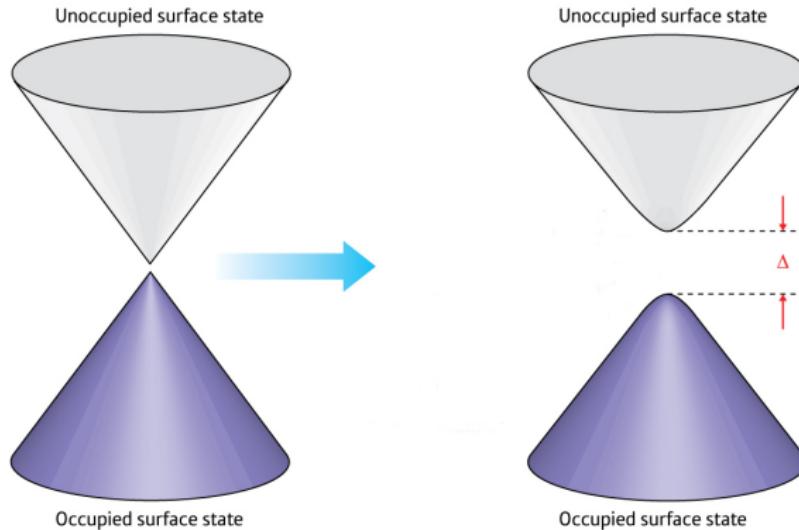
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Hamiltonian approach ($a_t \rightarrow 0$):

- ▶ Rooting $N_f : 4 \rightarrow 2$ (taste breaking $\sim a_t^2 (\gamma_5 \otimes \gamma_5 \gamma_4^T) \delta_4 \rightarrow 0$)
more details in arXiv:1704.07132
- ▶ Small FV effects
- ▶ More freedom in varying T, eB

Phase diagram. Semimetal - Insulator transition.

At large enough α gapped phase?



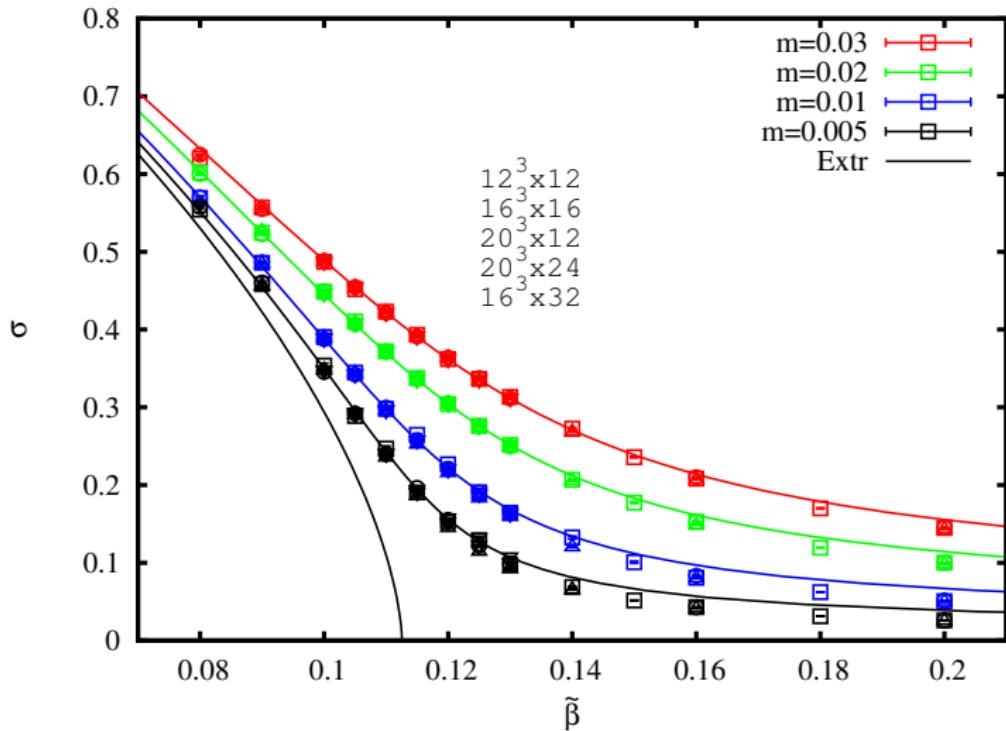
Is there a gap? What is the value of α_c ?

Order parameter:

$$\sigma = \langle \bar{\psi} \psi \rangle = \frac{1}{V} \langle \text{tr } D^{-1} \rangle$$

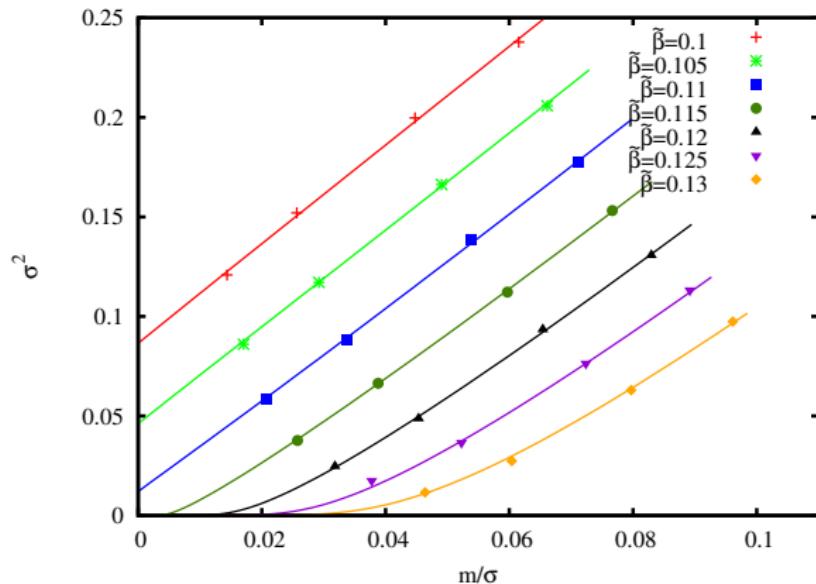
Isotropic case $v_{F,x} = v_{F,y} = v_{F,z} = v_F$.

$$\xi_i = v_F \frac{a_t}{a_s} = \frac{1}{2}$$



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$$\xi_i = v_F \frac{a_t}{a_s} = \frac{1}{2} \quad 16^3 \times 32$$



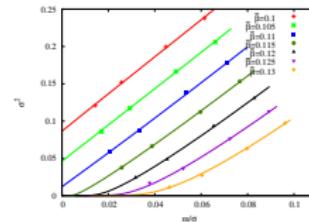
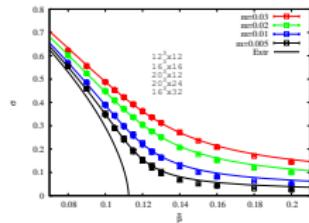
Equation of State (motivated by QED & graphene):

$$m(X_0 + X_1(1 - \beta/\beta_c)) = \sigma^\delta + Y_1(1 - \beta/\beta_c)\sigma^b.$$

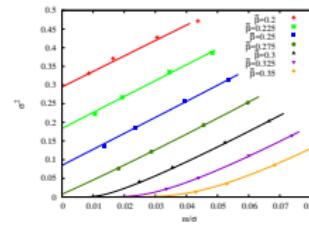
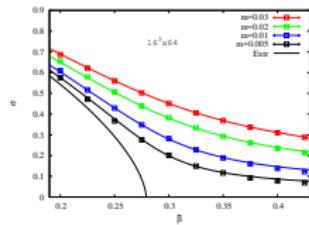
Classical: $\delta = 3$, $b = 1$.

Isotropic case $v_{F,x} = v_{F,y} = v_{F,z} = v_F$.

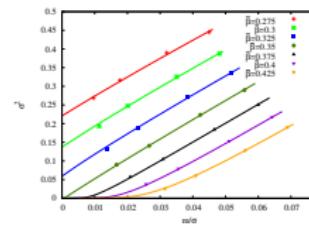
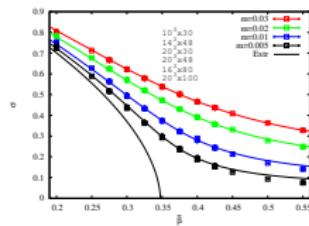
$$\xi_i = \frac{1}{2}$$



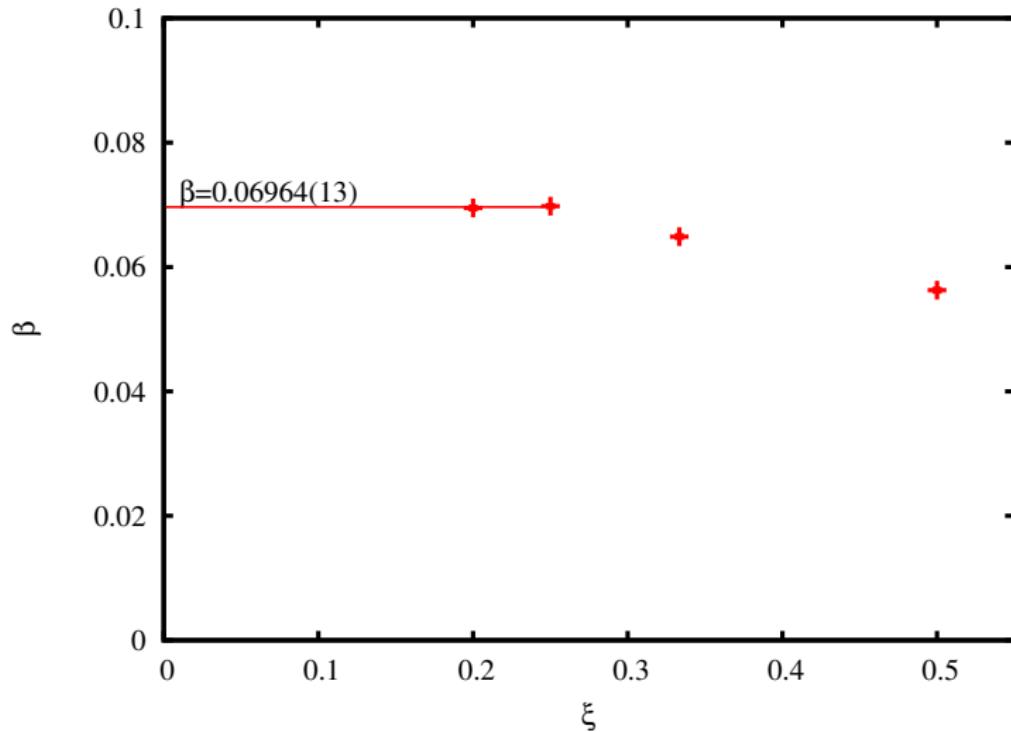
$$\xi_i = \frac{1}{4}$$



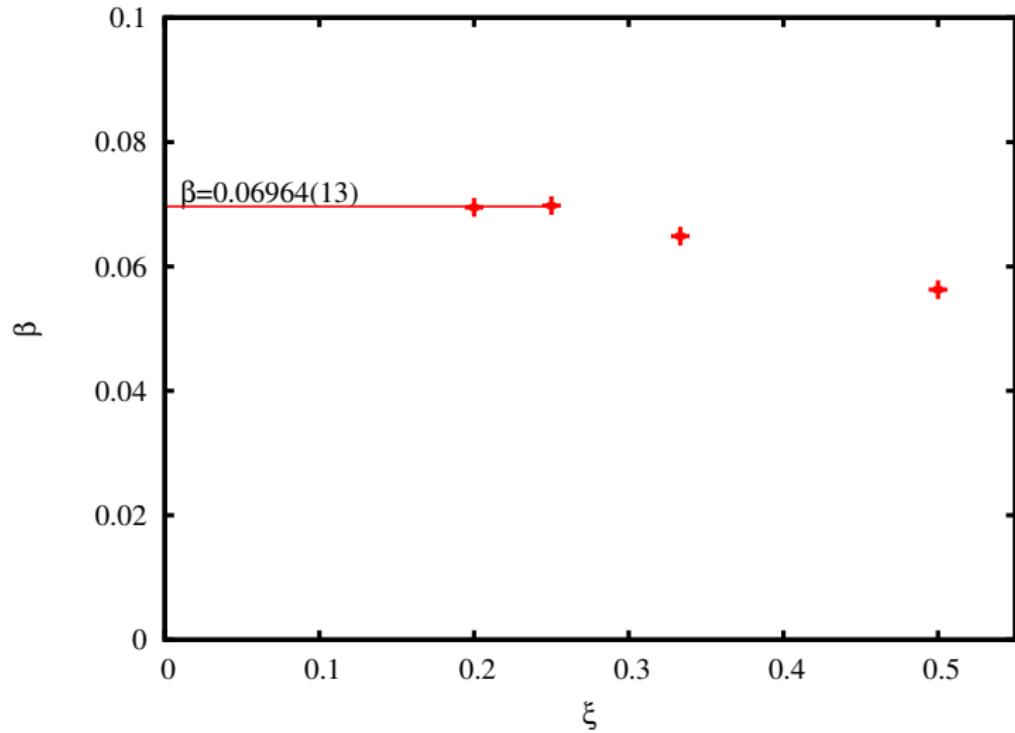
$$\xi_i = \frac{1}{5}$$



Isotropic case $v_{F,x} = v_{F,y} = v_{F,z} = v_F$.



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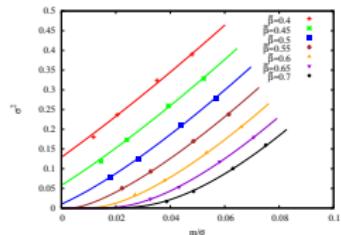
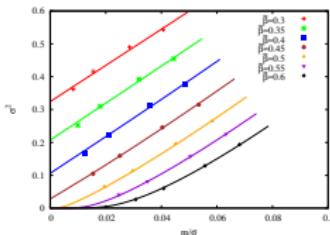
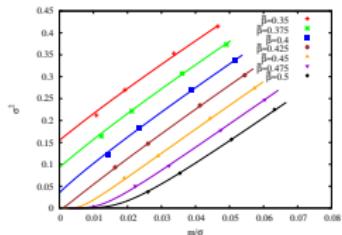
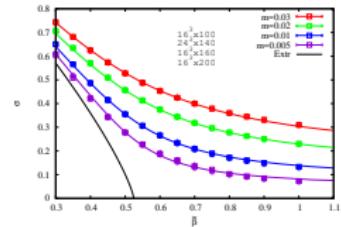
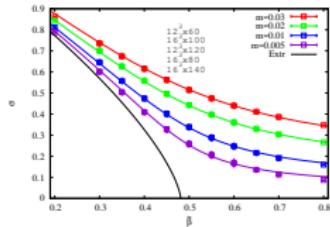
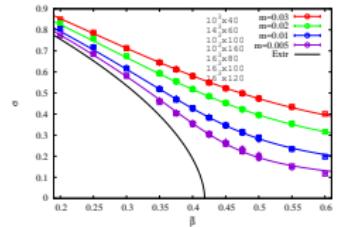
$$\beta_c \sim 0.069 \rightarrow \alpha_c \sim 1.14$$

$\alpha_c \approx 1.8660$, J.Gonzalez, arXiv:1509.00210, Ladder approximation

Anisotropy in Fermi velocity

- ▶ $v_{F,1} = v_{F,2} = v_F$
- ▶ $v_{F,3} = \zeta v_F$, ζ is varied
- ▶ $\beta = \frac{v_F}{4\pi e^2}$

Anisotropy in Fermi velocity

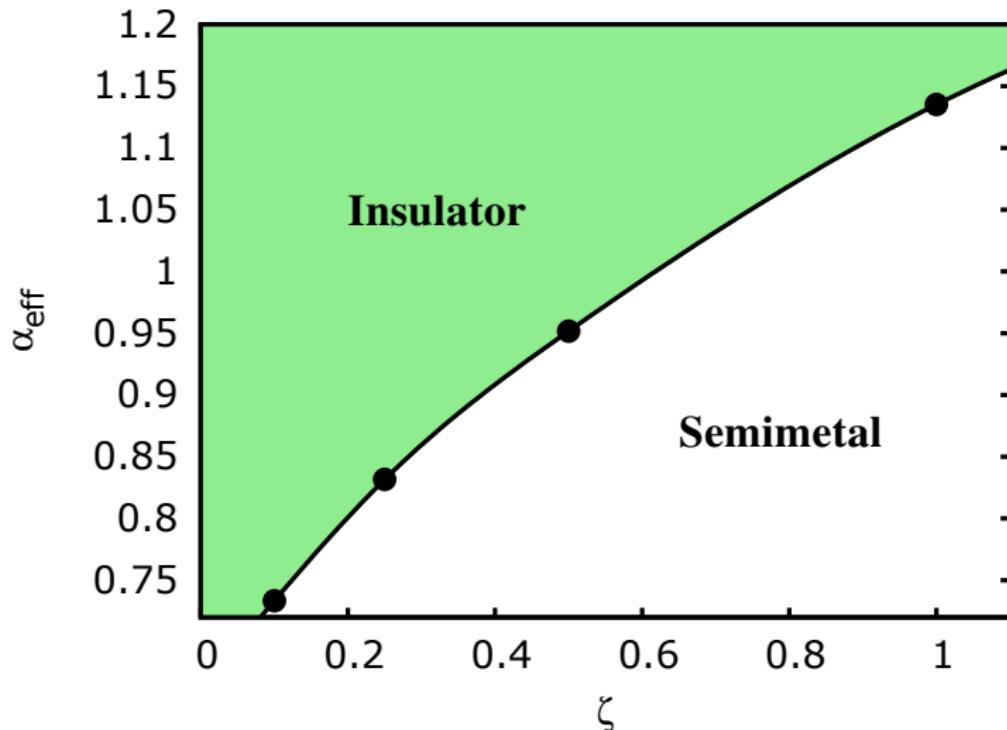


$$\zeta = \frac{1}{2}$$

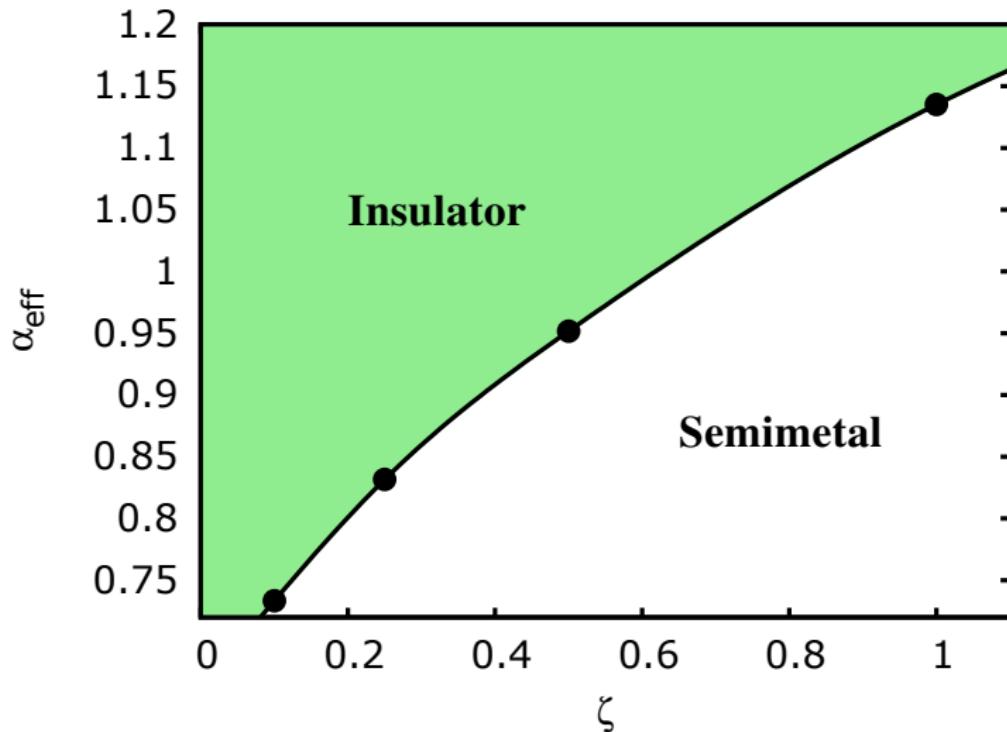
$$\zeta = \frac{1}{4}$$

$$\zeta = \frac{1}{10}$$

Phase diagram

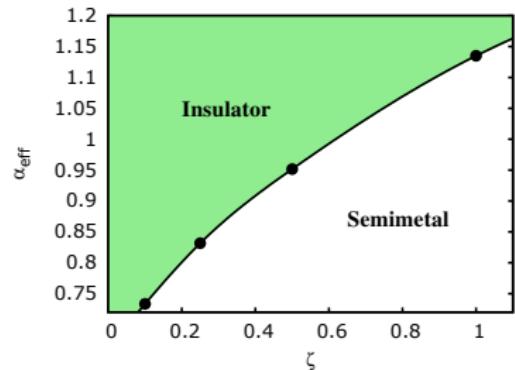


Phase diagram



$$\zeta : 1 \rightarrow 0 \implies 3D \rightarrow 2D$$

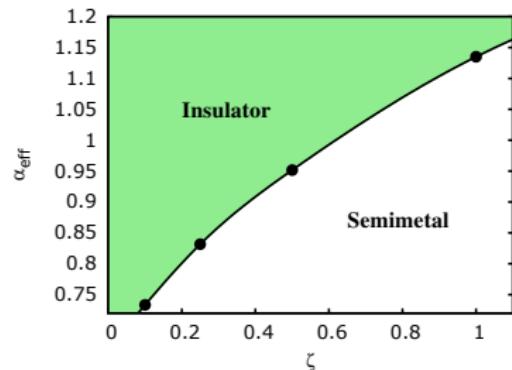
Phase diagram



- ▶ Na_3Bi : $\alpha_{\text{eff}} \approx 7$
- ▶ Cd_3As_2 : $\alpha_{\text{eff}} \approx 2$

Both should be in insulator phase!

Phase diagram



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Possible solutions:

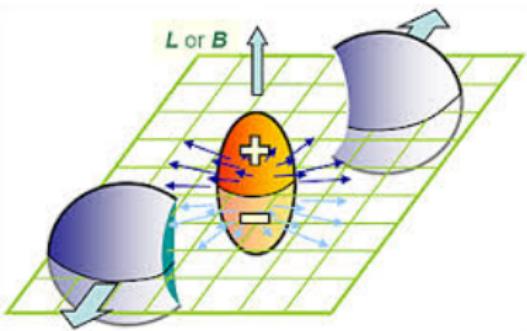
- ▶ Screening by bound electrons(ϵ)
- ▶ Renormalization

Chiral Magnetic Effect

Chiral Magnetic Effect

"A system with a nonzero chirality responds to a magnetic field by inducing a current along the magnetic field. This is the Chiral Magnetic Effect."

arXiv:0808.3382, Phys. Rev. D78 (2008) 074033
K. Fukushima, D. Kharzeev, H.J. Warringa.



Conductivity in external magnetic field

Chiral Magnetic Effect(CME) in Dirac semimetals:

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Q. Li et al., Nature Phys. 12 (2016) 550-554

H. Li et al., Nat. Comm. 7, 10301 (2016)

Conductivity in external magnetic field

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$\rho_5(\mu_5)?$

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$\rho_5(\mu_5)$?

- ▶ Small B : $\rho_5 \sim \mu_5 T^2 \implies \sigma_{||} \sim B^2$
- ▶ Large B : $\rho_5 \sim \mu_5 B \implies \sigma_{||} \sim B$

Conductivity on the lattice

Kubo relation ($K(\omega, t) = \frac{2\omega \cosh(w(\frac{1}{2T} - t))}{\sinh(\frac{\omega}{2T})}$):

$$\int d^3\bar{x} \langle j_i(\bar{x}, t)j_i(0) \rangle = \int_0^\infty \frac{d\omega}{2\pi} K(\omega, t) \sigma_i(\omega),$$

In the middle point:

$$C_i\left(\frac{1}{2T}\right) = \int_0^\infty \frac{d\omega}{2\pi} \frac{2\omega}{\sinh \frac{\omega}{2T}} \sigma_i(\omega)$$

Estimation:

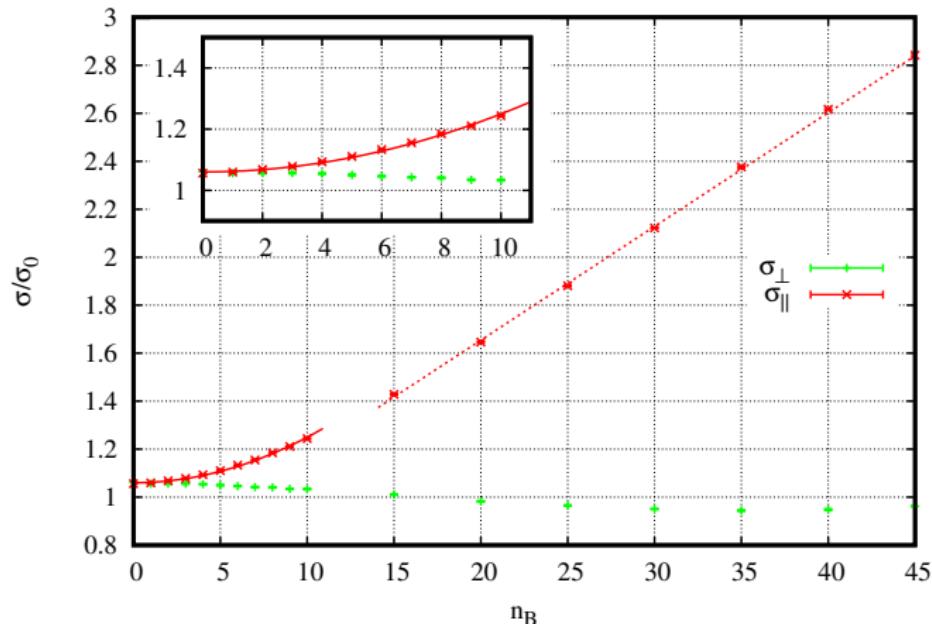
$$\sigma_i^{(0)} = \int_0^\infty \frac{d\omega}{2\pi} \frac{2\omega}{\sinh \frac{\omega}{2T}} \sigma_i(\omega) / \int_0^\infty \frac{d\omega}{2\pi} \frac{2\omega}{\sinh \frac{\omega}{2T}} = \frac{1}{\pi T^2} C_i\left(\frac{1}{2T}\right)$$

Conductivity in external magnetic field

$\beta = 0.25$ (Semimetal phase)

Lattice 20^4

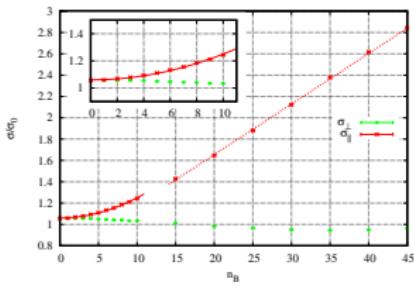
$a_t/a_s = 1/4$



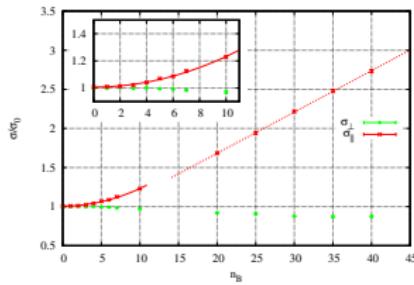
- ▶ Small B : $\rho_5 \sim \mu_5 T^2 \implies \sigma_{\parallel} \sim B^2$
- ▶ Large B : $\rho_5 \sim \mu_5 B \implies \sigma_{\parallel} \sim B$

Conductivity(different phases)

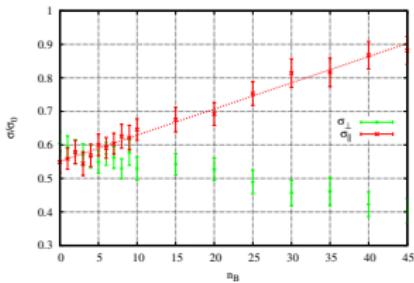
SM
 $\beta = 0.25$



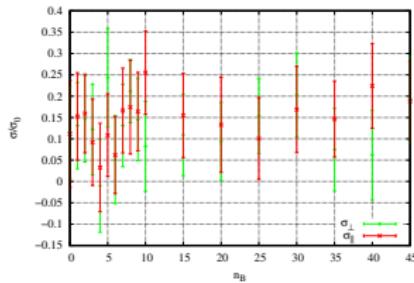
SM
 $\beta = 0.125$



Transition
 $\beta = 0.0625$



Insulator
 $\beta = 0.03125$



Conclusions

- ▶ Lattice Formulation and Study of DSM
- ▶ $\frac{a_t}{a_s} \rightarrow 0$ is important
- ▶ Semimetal-insulator transition in DSM
- ▶ Observed DSM should be insulating
- ▶ Screening? Renormalization?
- ▶ Conductivity of DSM in external Magnetic Field is measured
- ▶ Our results confirm CME in DSM

