



**How much cooler would it be  
with some more neutrons?**

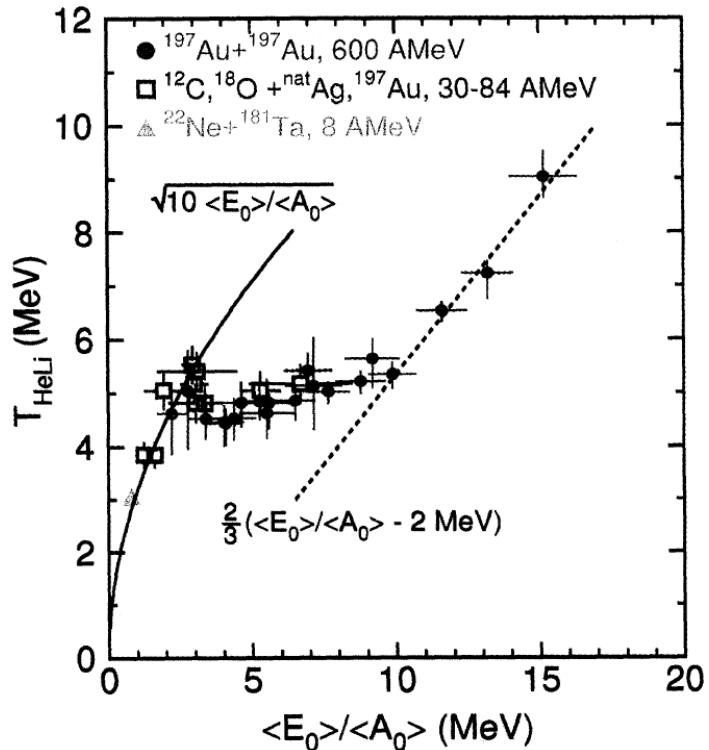
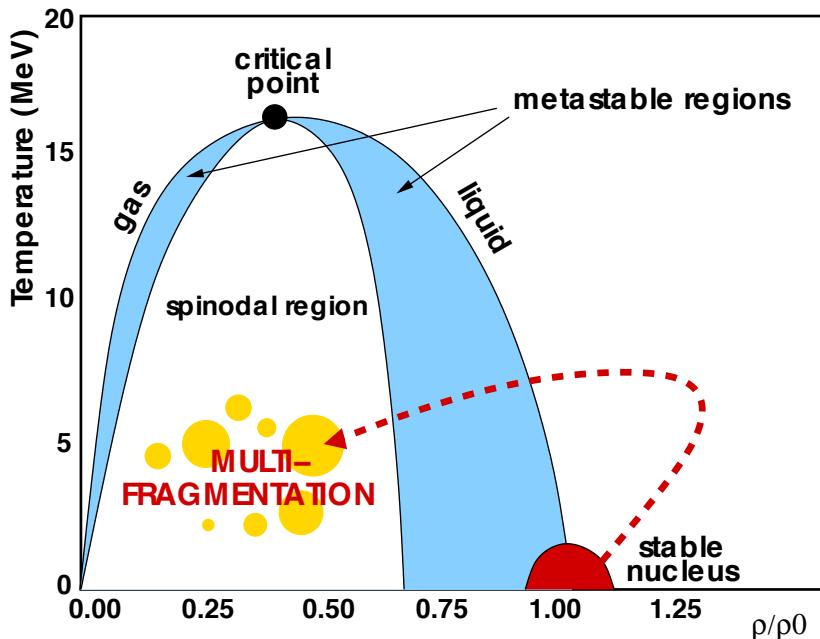
**Asymmetry Dependence of  
the Nuclear Caloric Curve**

**Alan McIntosh  
Texas A&M University**

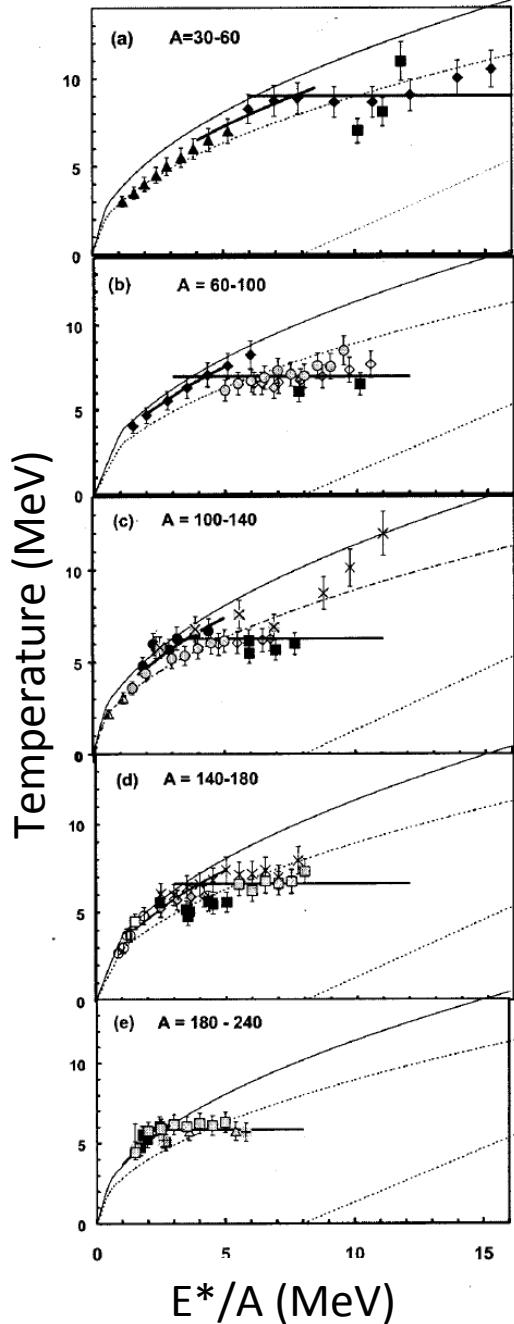
- Nuclear Equation of State:  
Background and Motivation
- The Measurement:  
Reconstruction Excited Nuclei and  
Extracting Their Temperatures
- Results:  
Temperature Decreases Linearly  
with Neutron Content
- Plans:  
Independent Experiment to  
Measure the Effect is Set to Run

# Nuclear Equation of State: T, $\rho$ , P, $E^*$ , I

- ❖ Heavy Ion Collisions at All Energies
- ❖ Nuclear Structure (e.g. Resonances)
- ❖ Supernovae (nucleosynthesis)
- ❖ Neutron Stars (Crust to Core)
- n-p Asymmetry Crucial



- Essential Piece of Nuclear Equation of State: T vs  $E^*/A$
- Search for & Study of Phase Transition
  - Liquid to Vapor
  - Evaporation to Multifragmentation



# Nuclear Caloric Curve

## MASS DEPENDENCE!

With increasing mass:

- Limiting Temperature decreases
- Onset of plateau at lower excitation

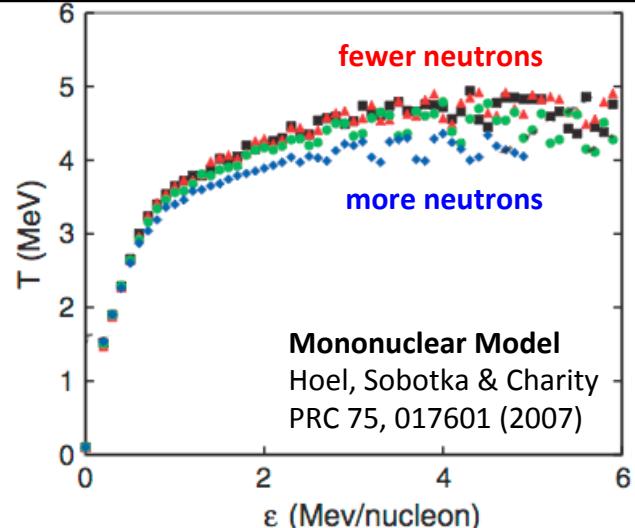
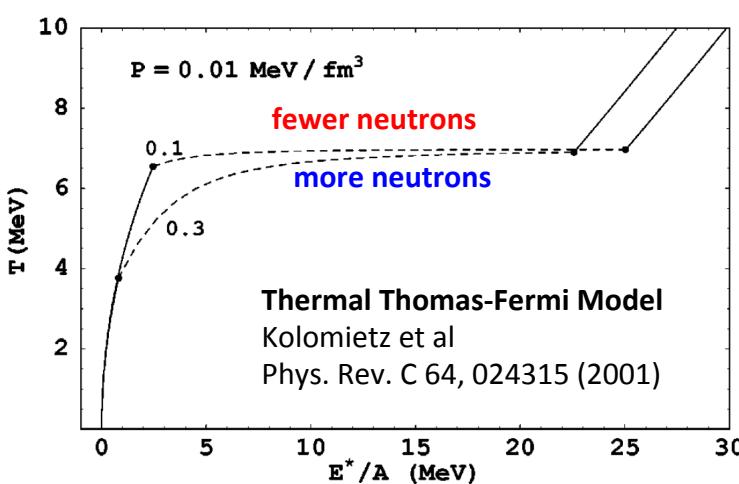
## ASYMMETRY DEPENDENCE?

- Does an n-p Asymmetry Dependence Exist?
- Which way does it go?
- How strong is it?

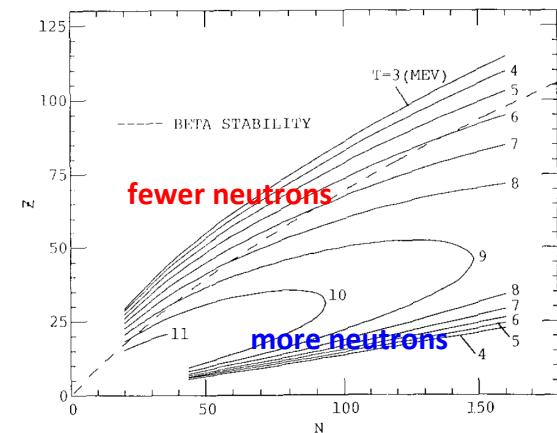
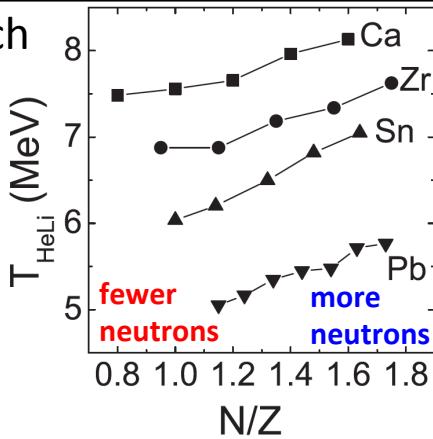
# Caloric Curve: Asymmetry Dependence?

## Theory

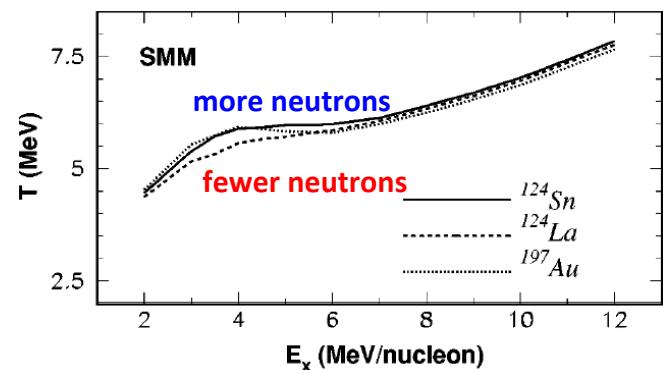
Neutron Rich  
→ Lower T



Neutron Rich  
→ Higher T



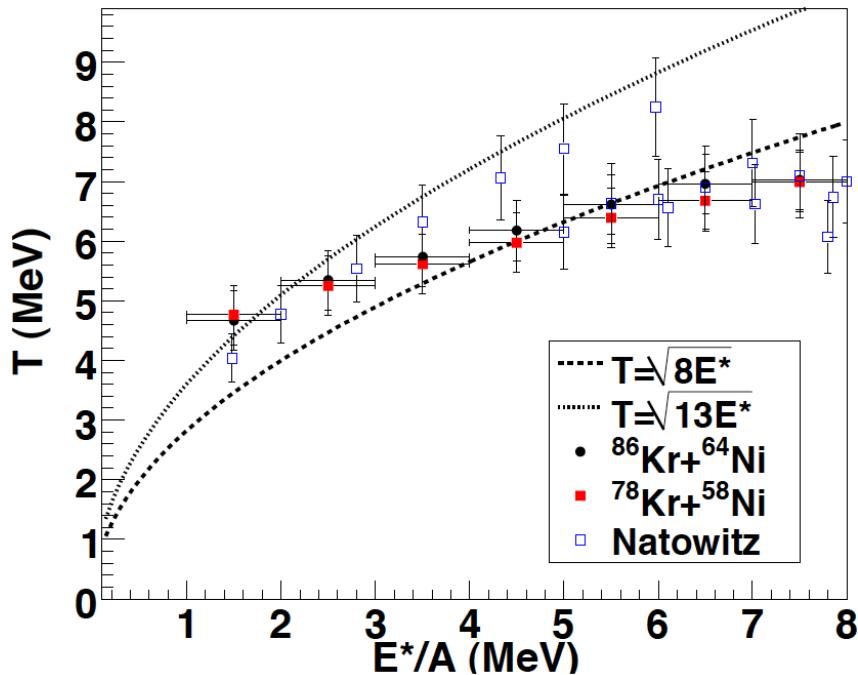
**Hot Liquid Drop Model**  
Besprosvany & Levit  
Phys. Lett B 217, 1 (1989)



**Statistical Multifragmentation Model**  
Ogul & Botvina  
Phys. Rev. C 66, 051601 (2002)

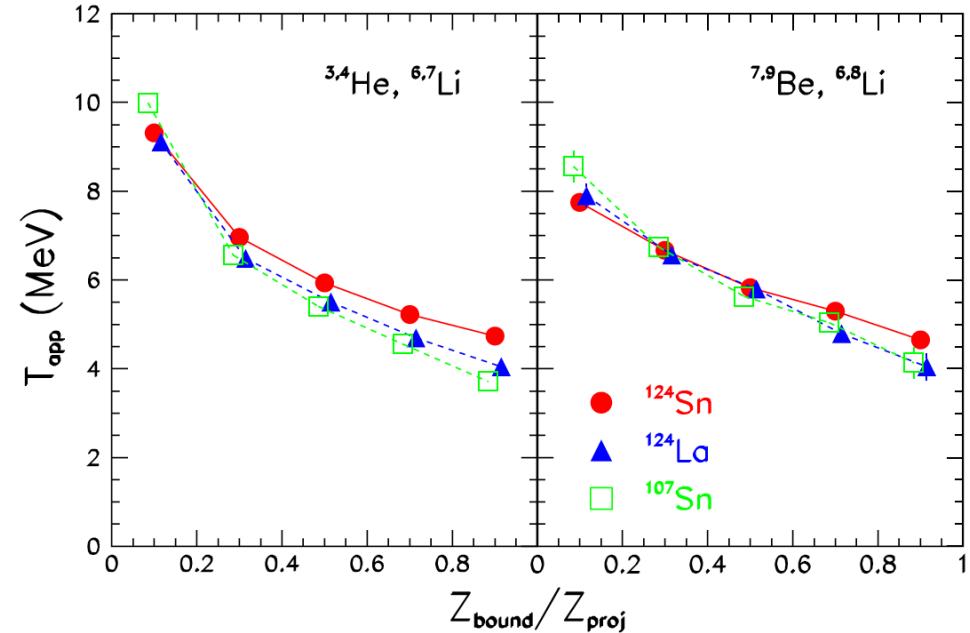
# Caloric Curve: Asymmetry Dependence?

## Experiment



S. Wuenschel, Ph.D. Thesis, 2009

Slight offset of neutron-rich system,  
but not statistically significant

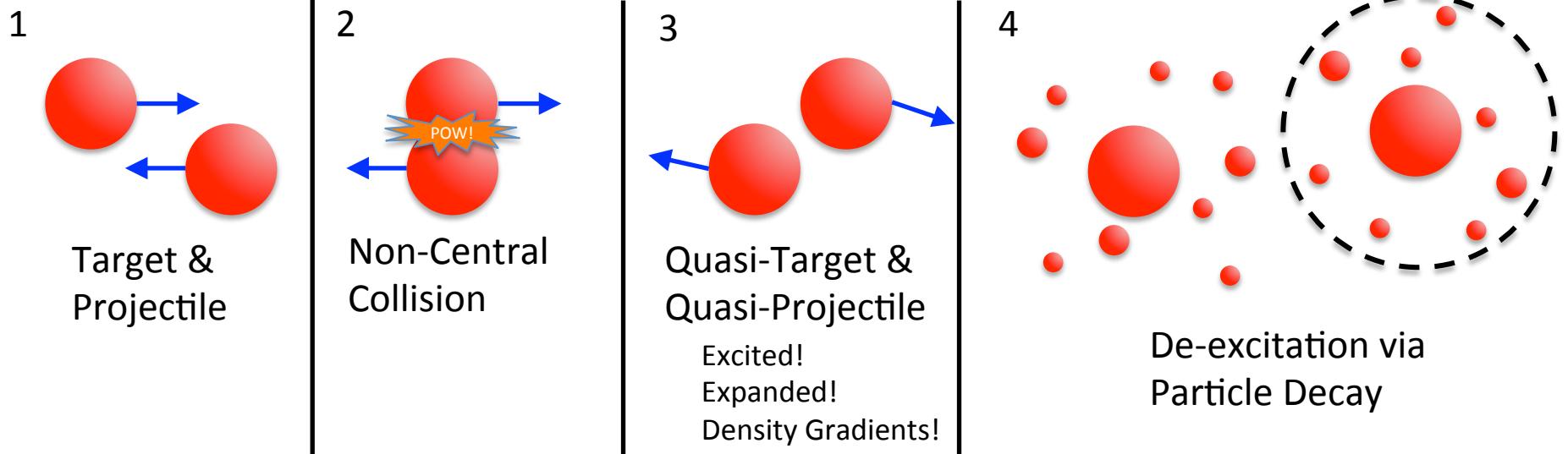


Sfienti et al., PRL 102, 152701 (2009)

Possible dependence on asymmetry,  
but not for all impact parameters.

Non-observation:  
Selection was on the system composition.  
Should use **reconstructed-source composition**

# Exciting Nuclear Matter!



The QP (quasi-projectile) is the primary excited fragment that exists momentarily after the nuclear collision

- We want to study the decay of the excited nuclear material (the QP)
- We use heavy ion collisions to create excited nuclear material
- From the reaction products, we reconstruct the QP

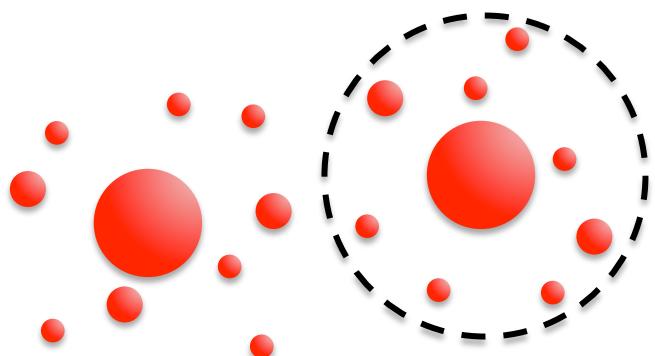
# Experiment

- NIMROD-ISiS Array
  - Full Silicon Coverage ( $4\pi$ )
  - Isotopic Resolution to  $Z=17$
  - Elemental Resolution to  $Z_{\text{projectile}}$
  - Neutron Ball ( $4\pi$ )

70Zn + 70Zn  
64Zn + 64Zn  
64Ni + 64Ni  
 $E = 35A \text{ MeV}$

## QP Reconstruction

Goal: select events with an equilibrated source



1. **Select particles** that may comprise the QP
  - Charged particles & free neutrons
  - Phase space selection using velocity cut
  - Calculate  $Z$ ,  $A$ ,  $p$ ,  $E^*$ , and asymmetry =  $m_s = (N-Z)/A$
2. **Select mass** (range) of the QP
3. **Select on-average spherical QP**

S. Wuenschel et al., Nucl. Inst. Meth. A 604 578 (2009)

Z. Kohley, Ph.D. Thesis, TAMU (2010)

# Thermometer: MQF

## Momentum Quadrupole Fluctuation Temperature

The quadrupole momentum distribution

$$Q_{xy} = p_x^2 - p_y^2$$

Contains information on the temperature through its fluctuations

$$\sigma_{xy}^2 = \int d^3p (p_x^2 - p_y^2)^2 f(p)$$

If  $f(p)$  is a Maxwell-Boltzmann distribution, then

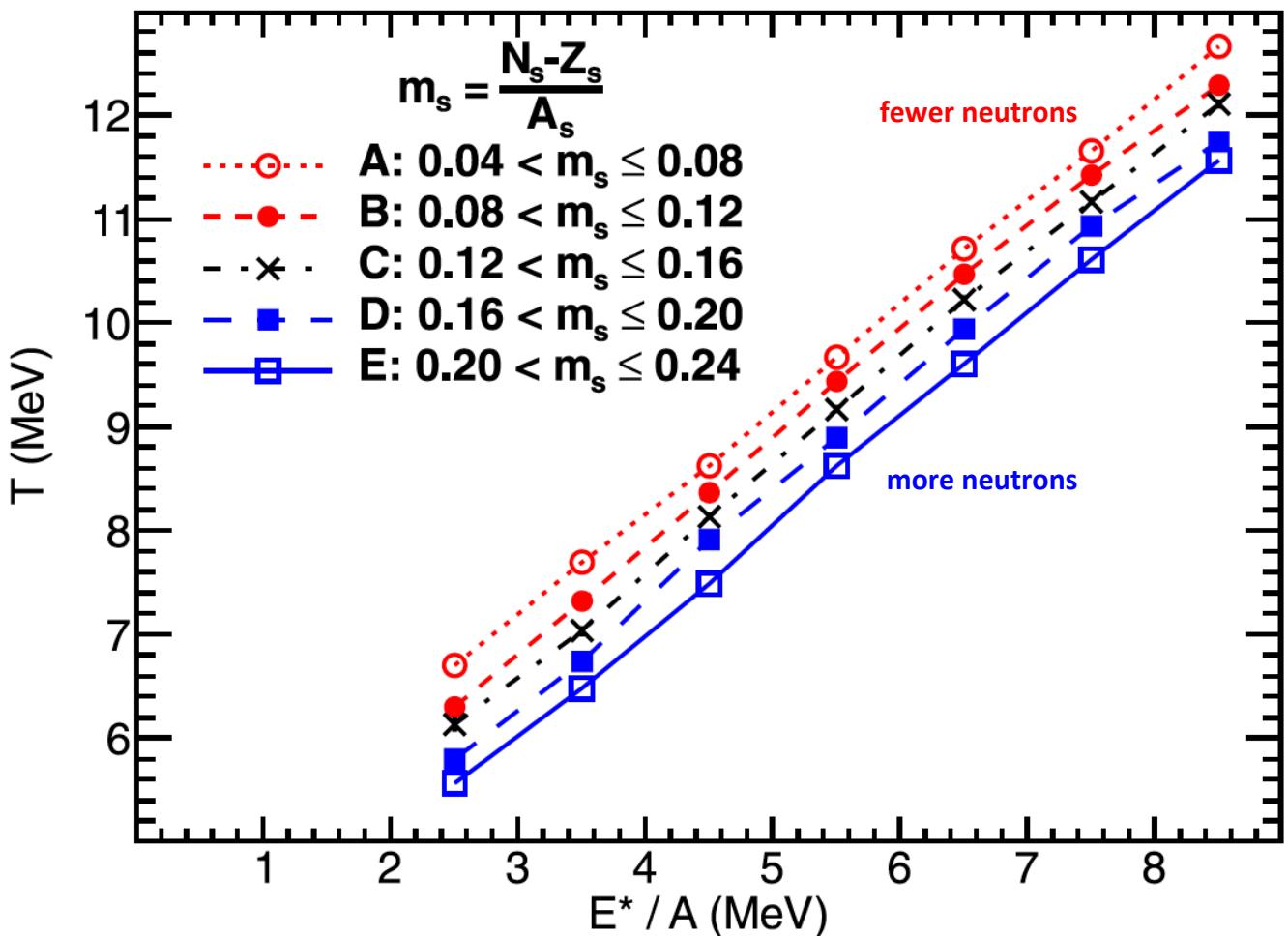
$$\sigma_{xy}^2 = 4m^2T^2$$

H. Zheng & A. Bonasera, PLB 696, 178 (2011)  
S. Wuenschel, NPA 843, 1 (2010)  
S. Wuenschel Ph.D. Thesis, TAMU (2009)

# Asymmetry Dependent Temperature

MQF Thermometer, Protons as Probe

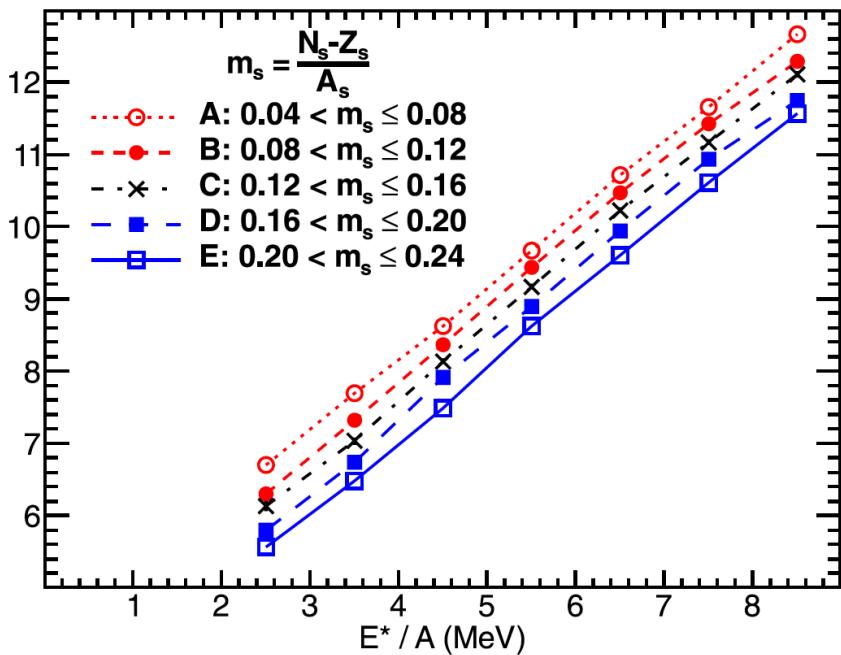
- $48 \leq A_{QP} \leq 52$
- 5 narrow asymmetry bins



Larger Asymmetry  
→ Lower Temperature  
 $> 1$  MeV shift!  
Evenly Spaced

# Importance of Reconstruction

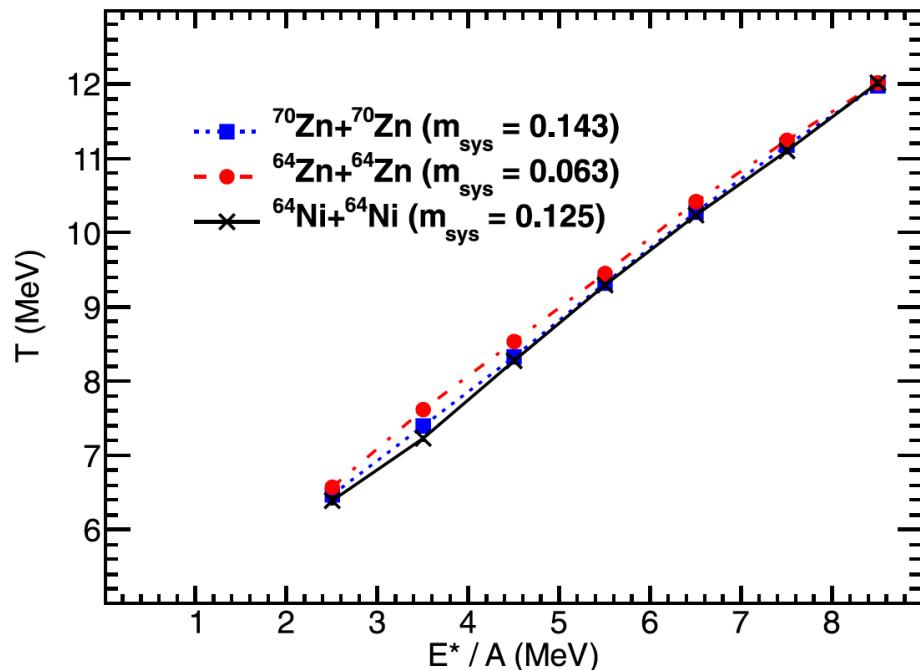
Asymmetry of Isotopically Reconstructed Source



Larger Asymmetry  
→ Lower Temperature  
Observed either way,  
but...

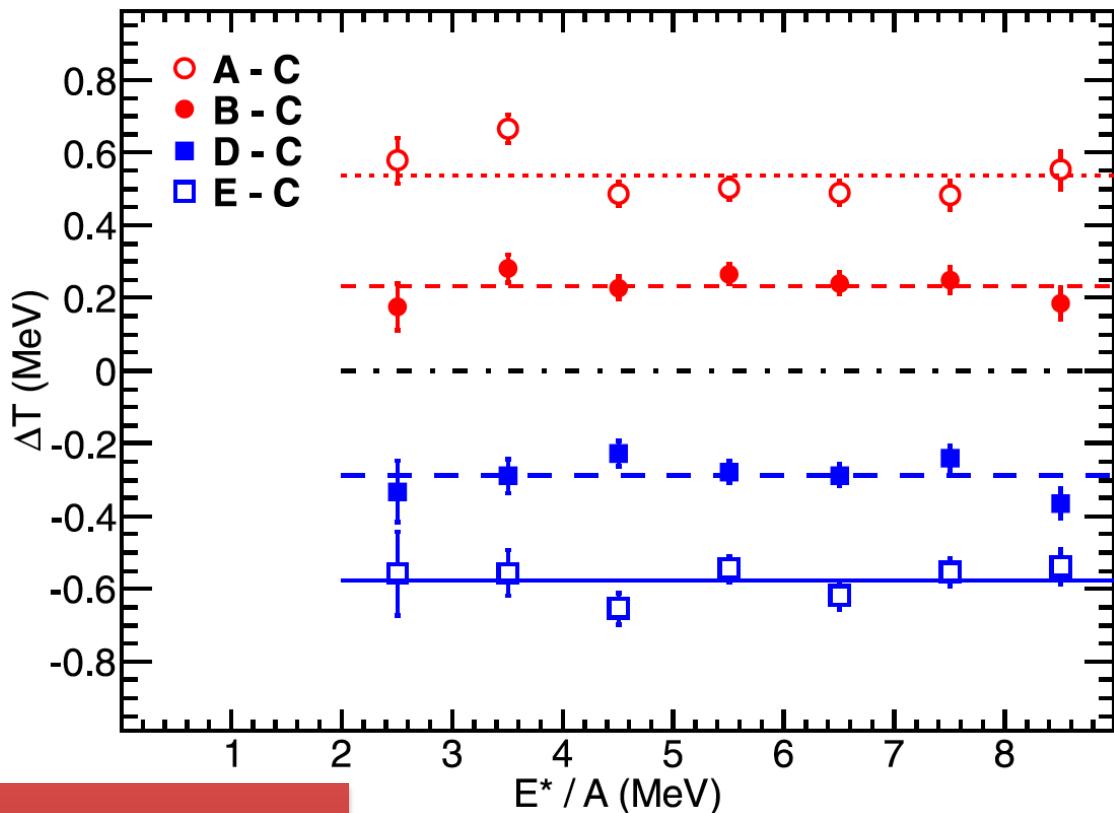
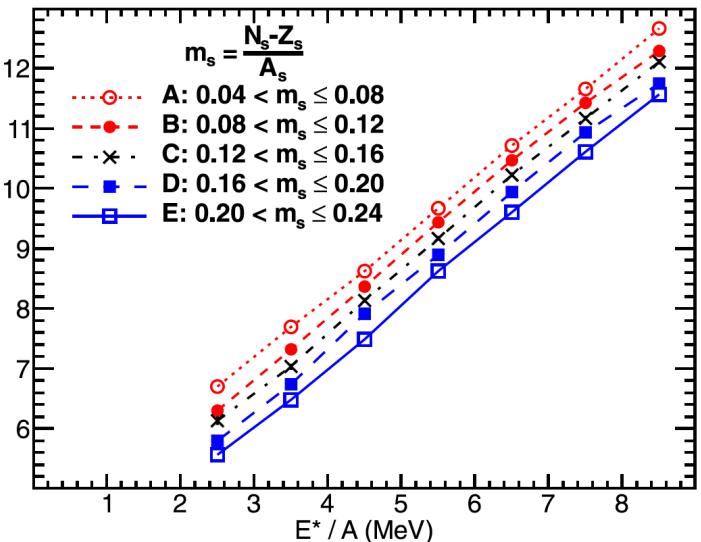
Asymmetry of Initial System

Each system may populate a broad range of QP asymmetry



Much more pronounced for selection on **source composition** rather than system composition

# Excitation Independence

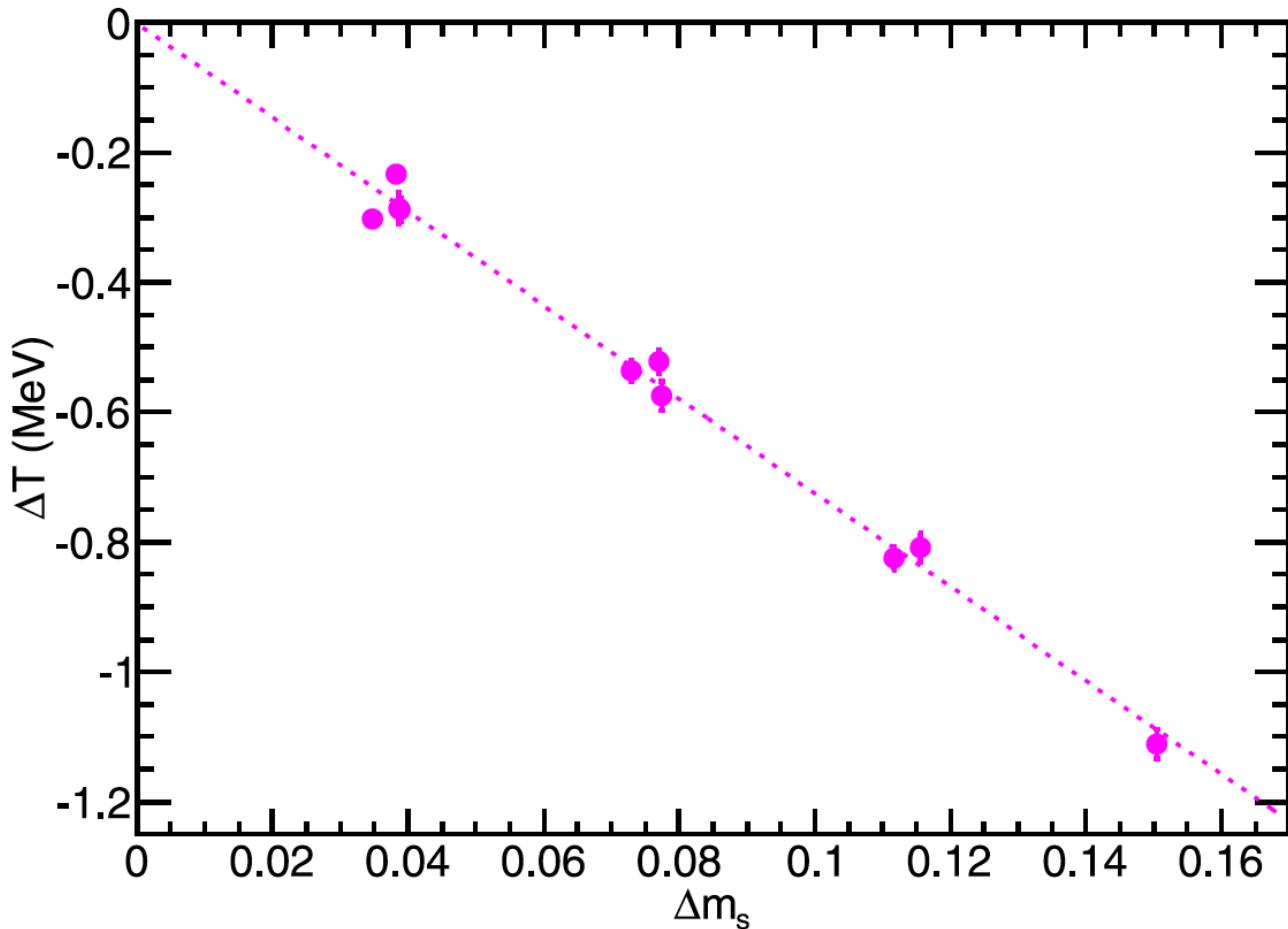
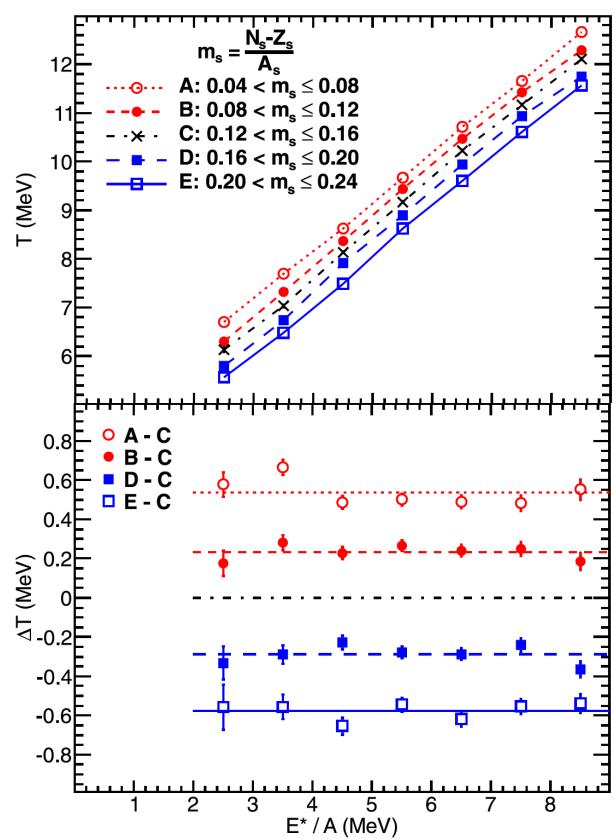


Larger Asymmetry  
→ Lower Temperature

Temperature shift does  
not show a trend with  
excitation.

Four of ten pairwise  
differences shown.

# Quantifying the Asymmetry Dependence



A change of 0.15 units of  $m_s = (N-Z)/A$  corresponds to a temperature decrease of 1.1 MeV

Linear relationship between change in asymmetry and change in temperature

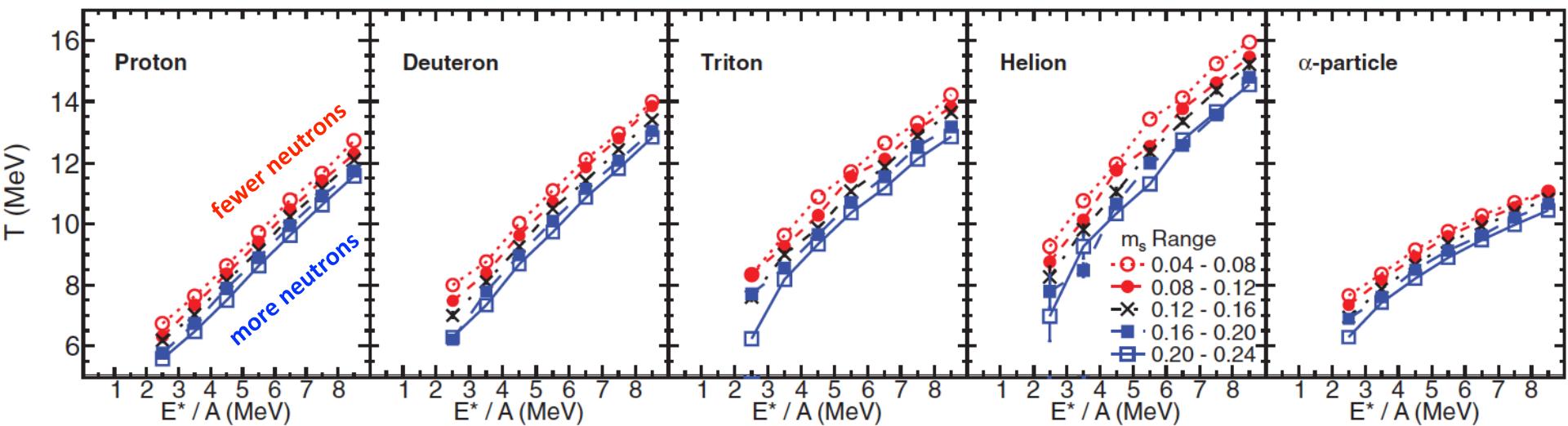


### Asymmetry Dependence

- ✓ MQF Protons

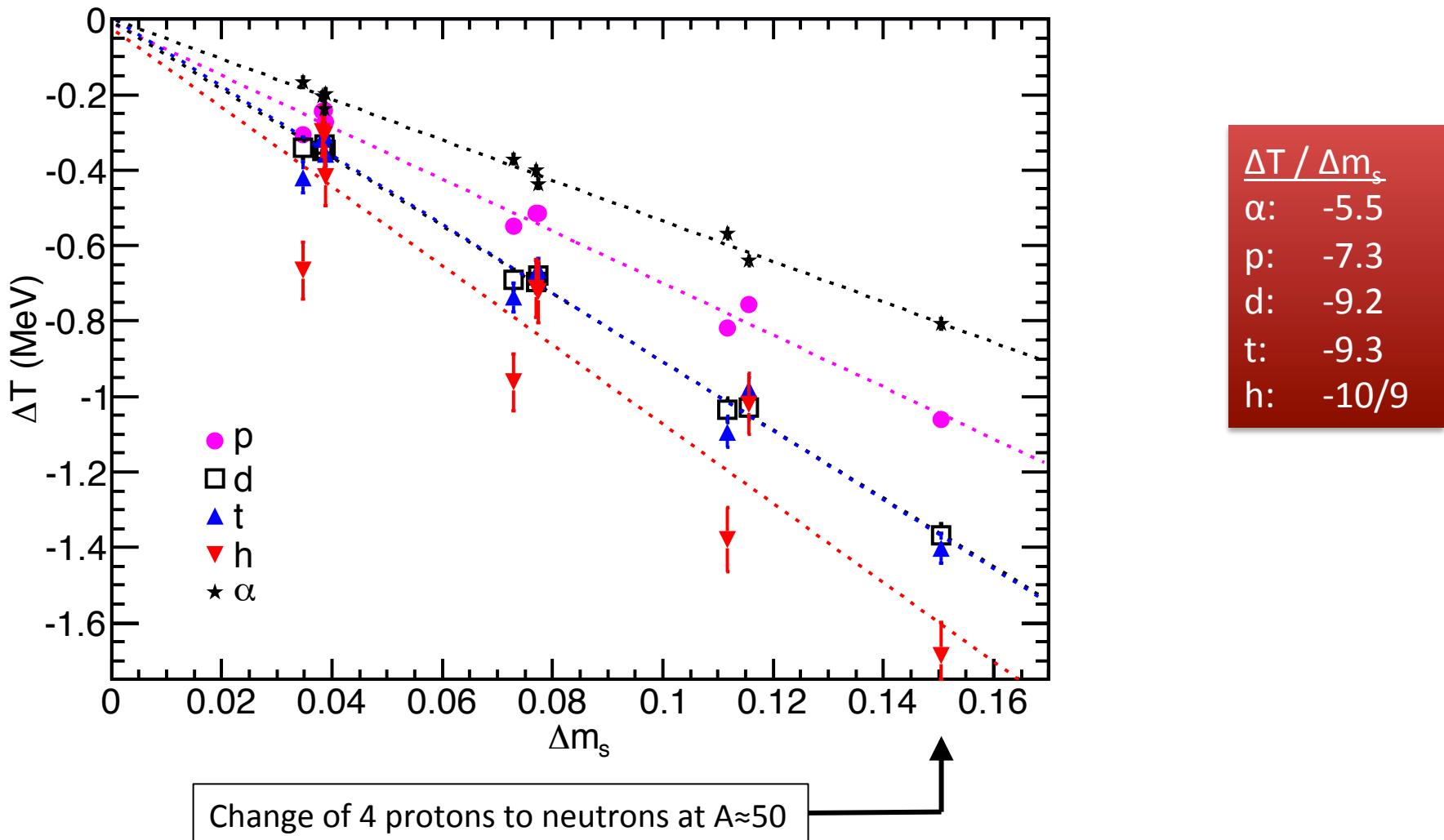
Do other probes of the temperature exhibit an asymmetry dependence?

# Caloric Curves for Light Charged Particles Using the MQF Thermometer

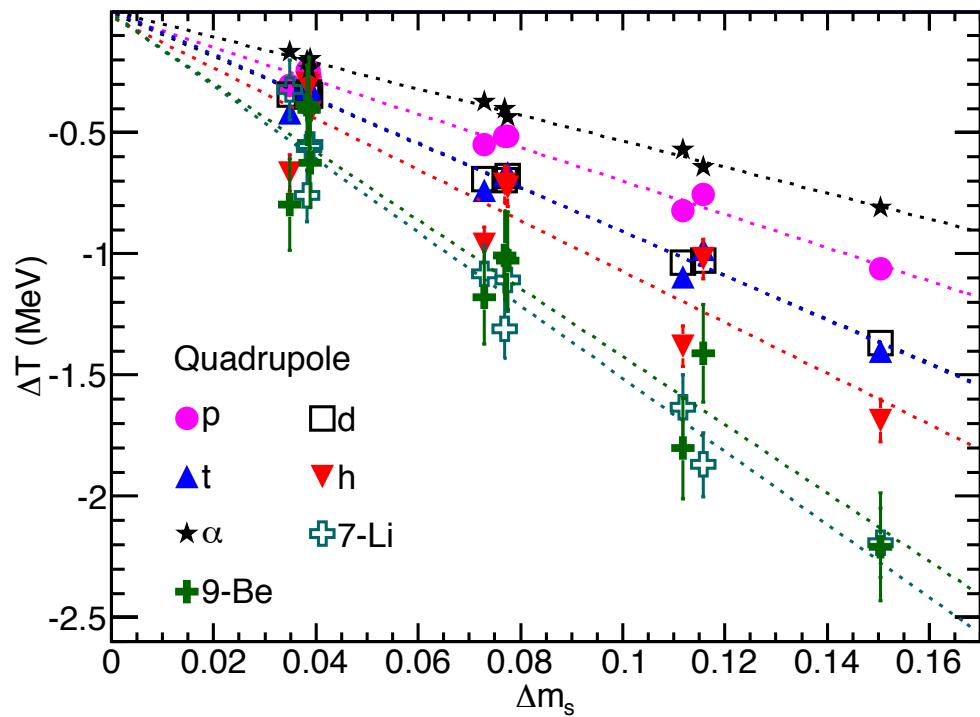
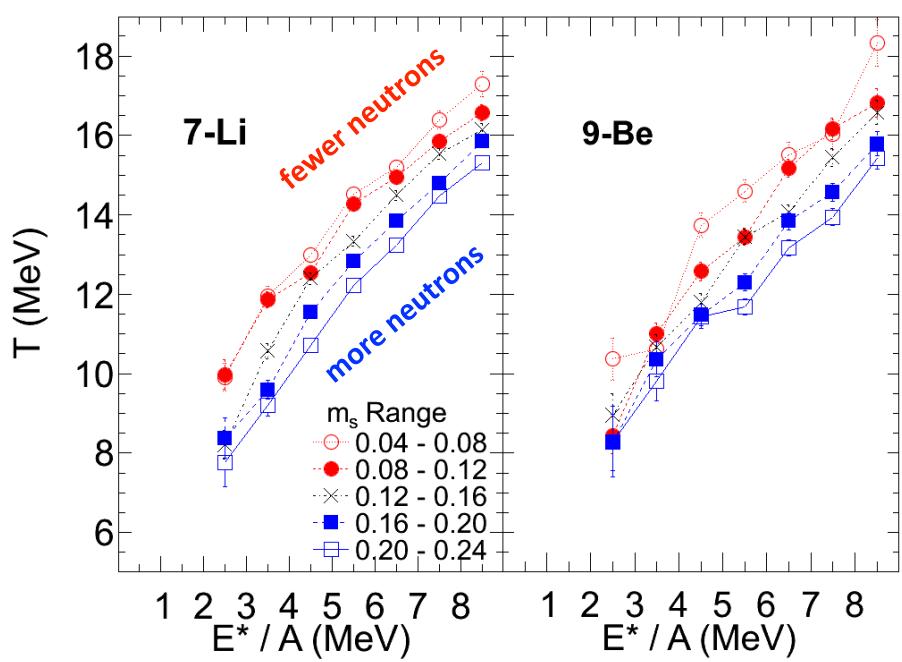


For All LCPs:  
Larger Asymmetry  
 $\rightarrow$  Lower Temperature

# Asymmetry Dependence of Temperature



# Temperatures Using Heavier Probes



Larger Asymmetry  
→ Lower Temperature



### Asymmetry Dependence

- ✓ MQF Protons
- ✓ MQF Deuterons
- ✓ MQF Tritons
- ✓ MQF Helions
- ✓ MQF Alphas
- ✓ MQF 7-Li
- ✓ MQF 9-Be

Do other probes of the temperature exhibit an asymmetry dependence?

# Albergo Thermometer

H/He

Li/He

---

Double Yield Ratio

$$R = \frac{Y(d)/Y(t)}{Y(h)/Y(\alpha)}$$

Account for binding  
energy differences and  
spin-degeneracies

$$T_{raw} = \frac{14.3 MeV}{\ln(1.59R)}$$

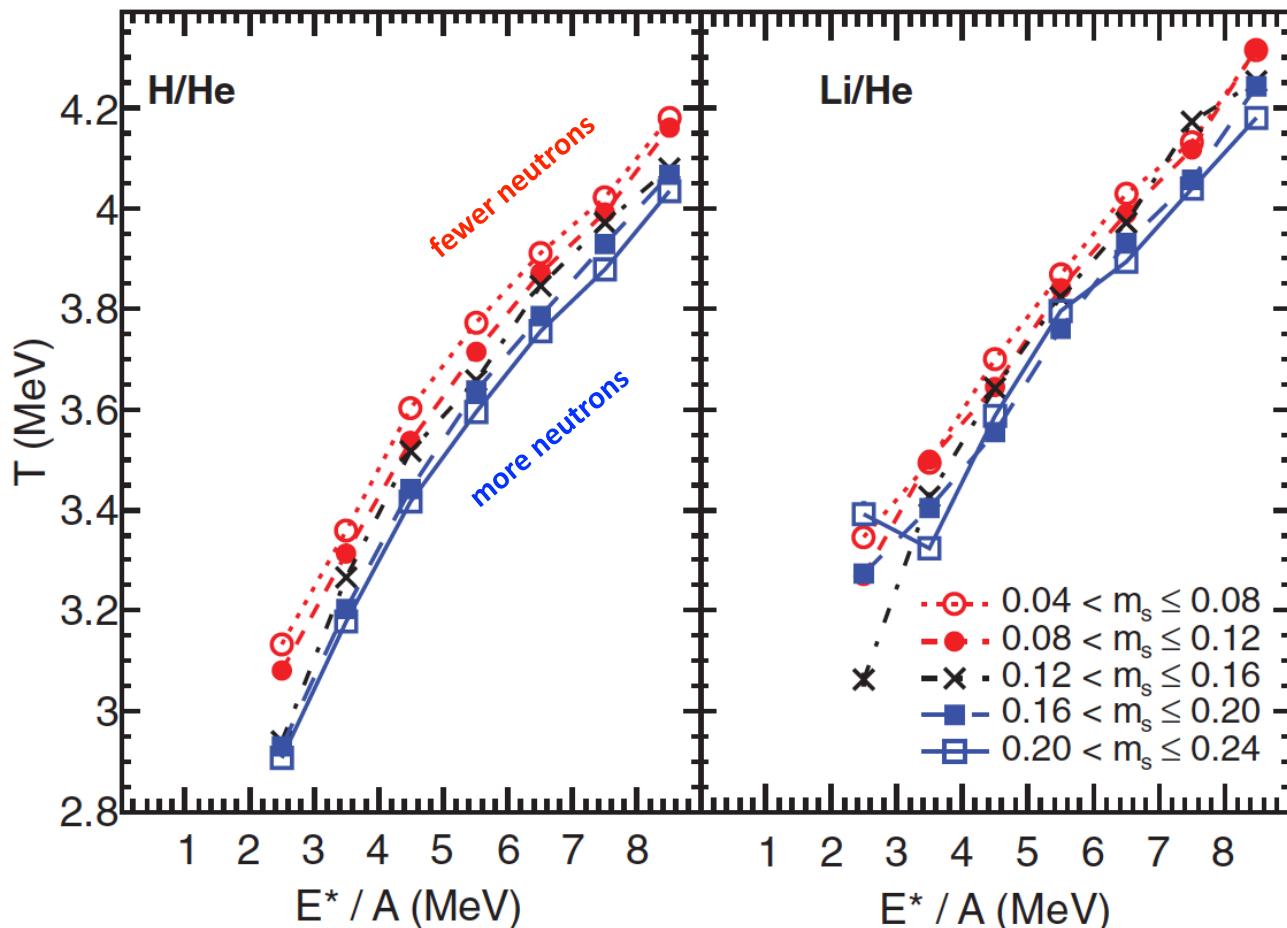
Correction for  
secondary decay ( $\approx 3\%$ )

$$\frac{1}{T} = \frac{1}{T_{raw}} - 0.0097$$

$$T_{raw} = \frac{13.3 MeV}{\ln(2.18R)}$$

$$\frac{1}{T} = \frac{1}{T_{raw}} + 0.0051$$

# Albergo Temperature: Asymmetry Dependent

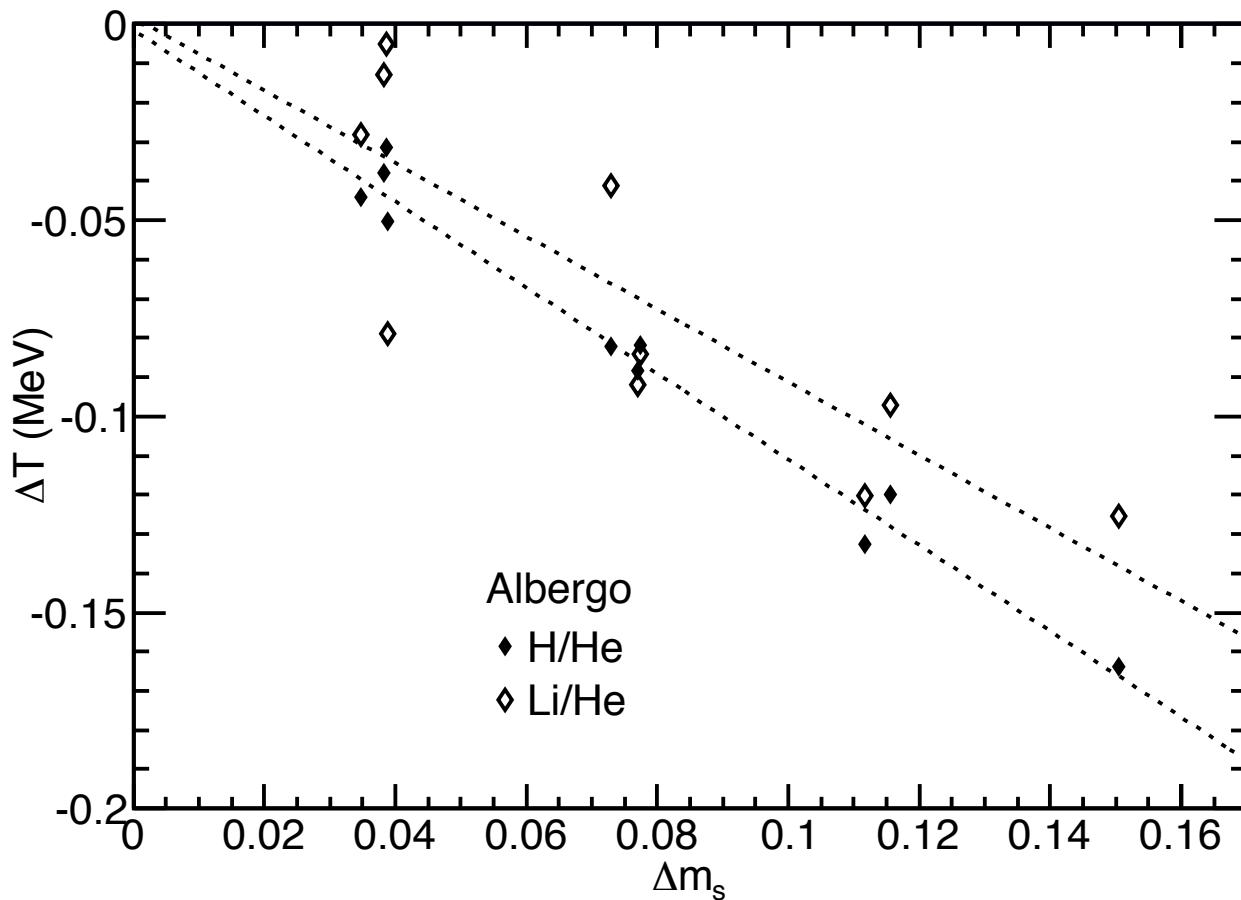


Temperature is smaller than for MQF  
(Chemical vs Kinetic)

Asymmetry dependence is smaller than MQF  
(Lower Temperature)

Larger Asymmetry  
→ Lower Temperature

# Albergo: Asymmetry Dependence of T



Larger Asymmetry  
→ Lower Temperature  
Linear Relationship

Stronger dependence for MQF than for Albergo

- Smaller value of temperature for Albergo than MQF
- Different methods (chemical vs kinetic)



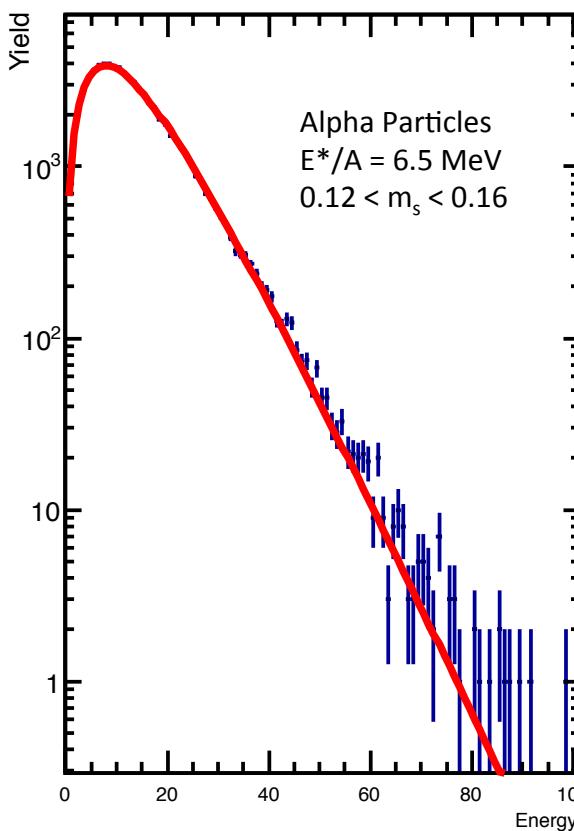
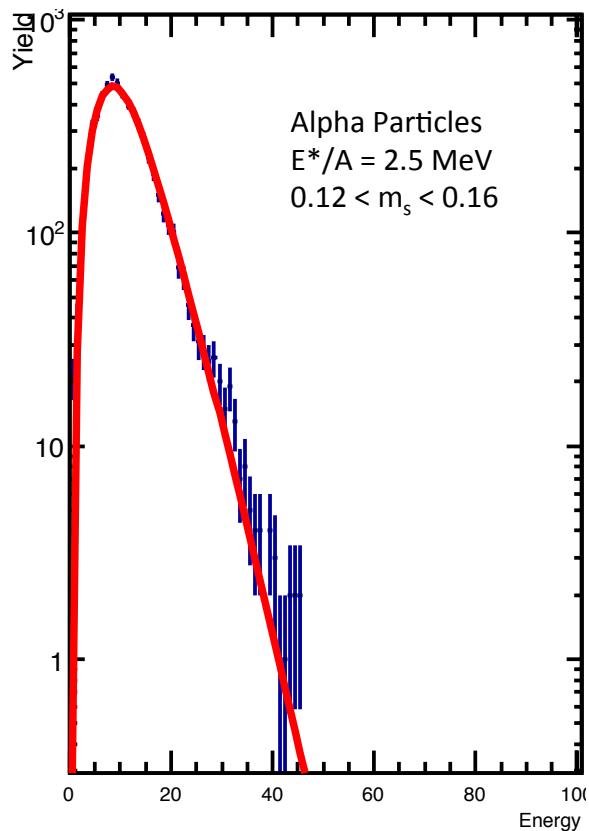
### Asymmetry Dependence

- ✓ MQF Protons
- ✓ MQF Deuterons
- ✓ MQF Tritons
- ✓ MQF Helions
- ✓ MQF Alphas
- ✓ MQF 7-Li
- ✓ MQF 9-Be
- ✓ Albergo H / He
- ✓ Albergo Li / He

Do other probes of the temperature exhibit an asymmetry dependence?

# Slope Temperatures

Kinetic Energies in the QP frame.  $\theta < 90^\circ$



Maxwell-Boltzmann

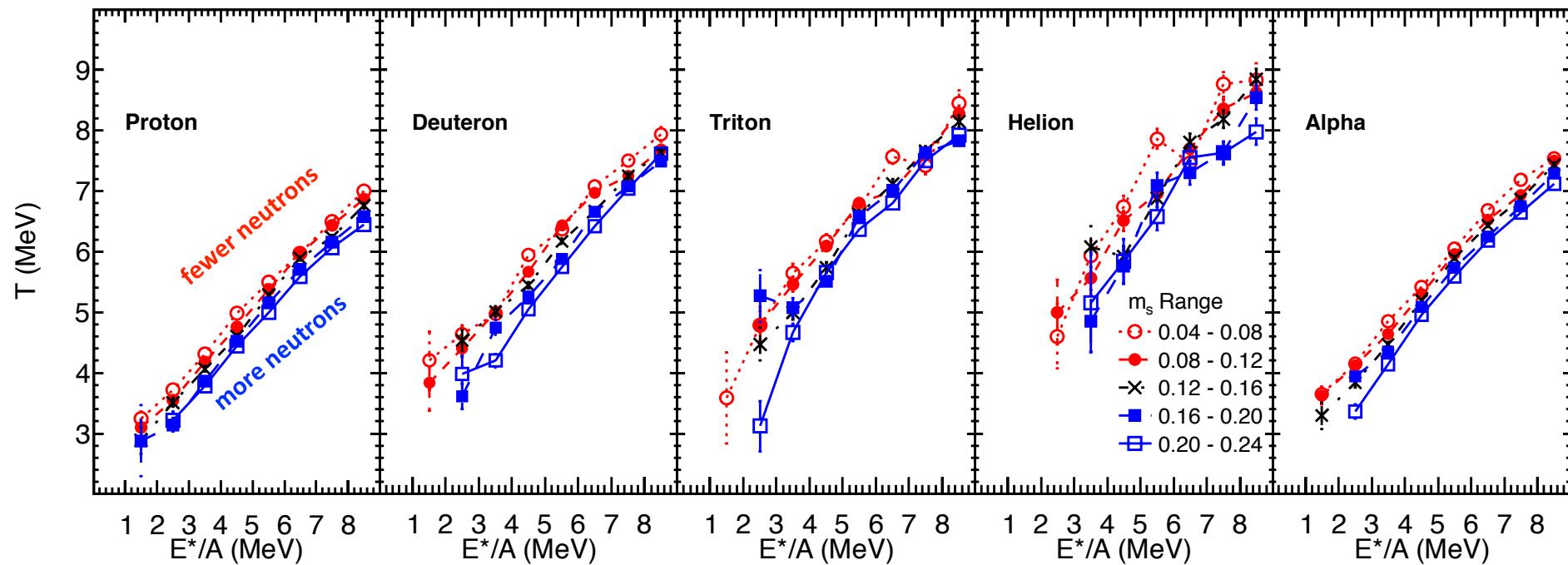
$$Y(E) \propto (E - B) \exp\left(-\frac{E}{T}\right)$$

for  $E \geq B + T$

With a modification for a diffuse barrier at lower energies.

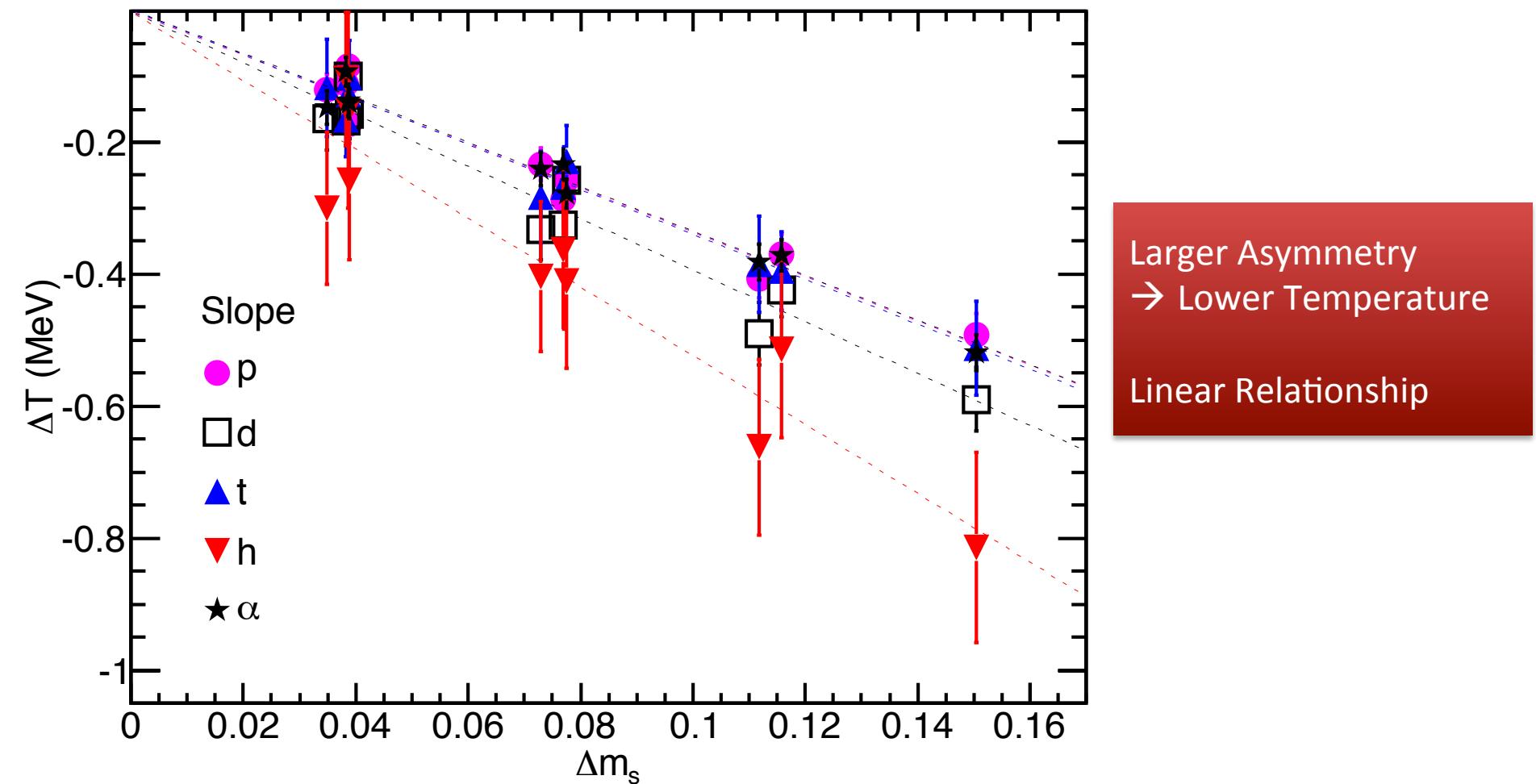
Yanez, Phys. Rev. C 68, 011602(R) (2003)

# Slope Temperature: Asymmetry Dependent

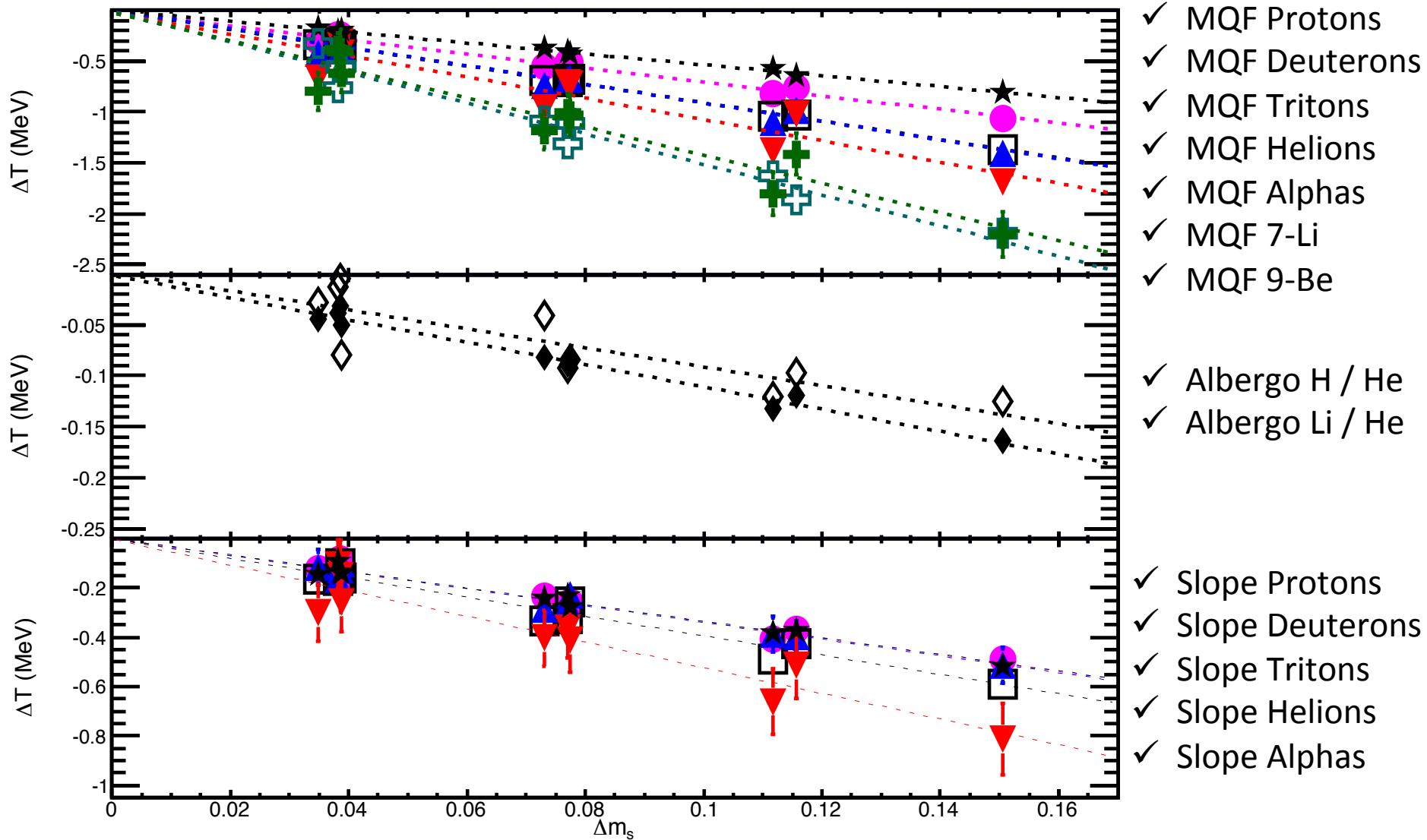


Larger Asymmetry  
→ Lower Temperature

# Asymmetry Dependence of Slope Temperature



Q: How much cooler would it be with some more neutrons?



A: It depends on the thermometer, but it sure would be cooler.

Q: How much cooler would it be if we didn't need to measure neutrons?

Q: How much cooler would it be if we didn't need to measure neutrons?

To confirm our observation of an asymmetry dependence of the caloric curve, we will conduct a new experiment.

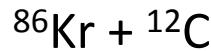
Fusion reactions produce excited nuclei with known n-p asymmetry and known excitation.

Free neutrons are not needed!

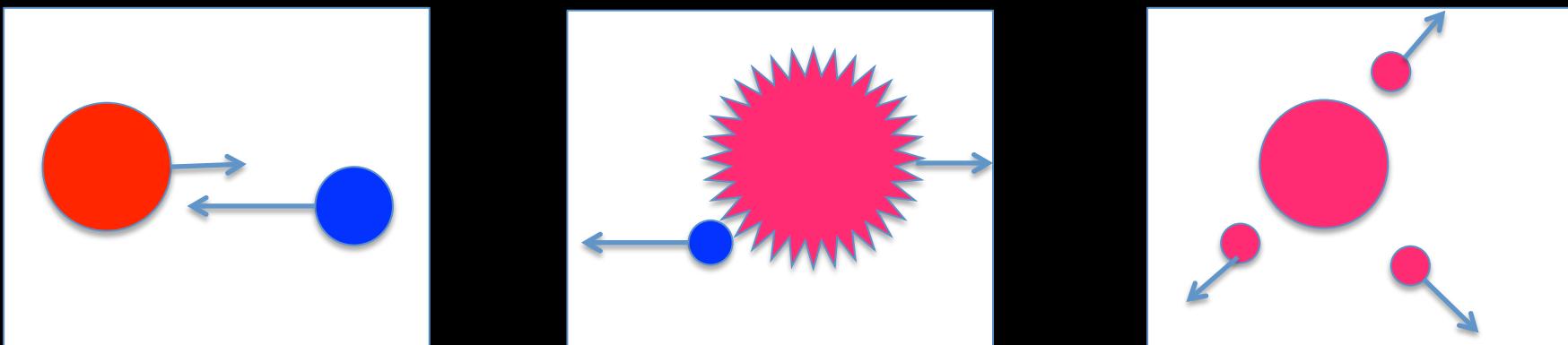
Fusion-evaporation residues will provide event characterization.

Light charged particles that are evaporated from the compound nucleus will provide the temperature in multiple ways.

# The Fusion Reactions



$$E/A = 15, 25, 35 \text{ MeV}$$



$E^*/A = 1.3, 2.0 \text{ MeV, and } 2.8 \text{ MeV for } ^{86}\text{Kr}.$   
 $\text{For } ^{78}\text{Kr, } \sim 10\% \text{ higher.}$

$$(N-Z)/A = 0.070 \text{ and } 0.163$$

# Experimental Configuration

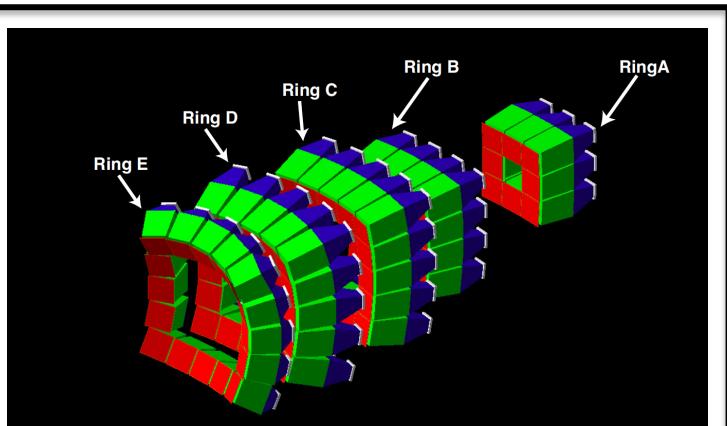
## Quadrupole Triplet Spectrometer

Measure Fusion-Evaporation Residues

Time-Of-Flight,  $\Delta E$ ,  $E \rightarrow$  Velocity, Energy, Z, A

$0.9^\circ \leq \theta \leq 2.3^\circ$

P. Cammarata et al., NIMA 792, 61 (2015)  
L. Heilborn et al., article in preparation



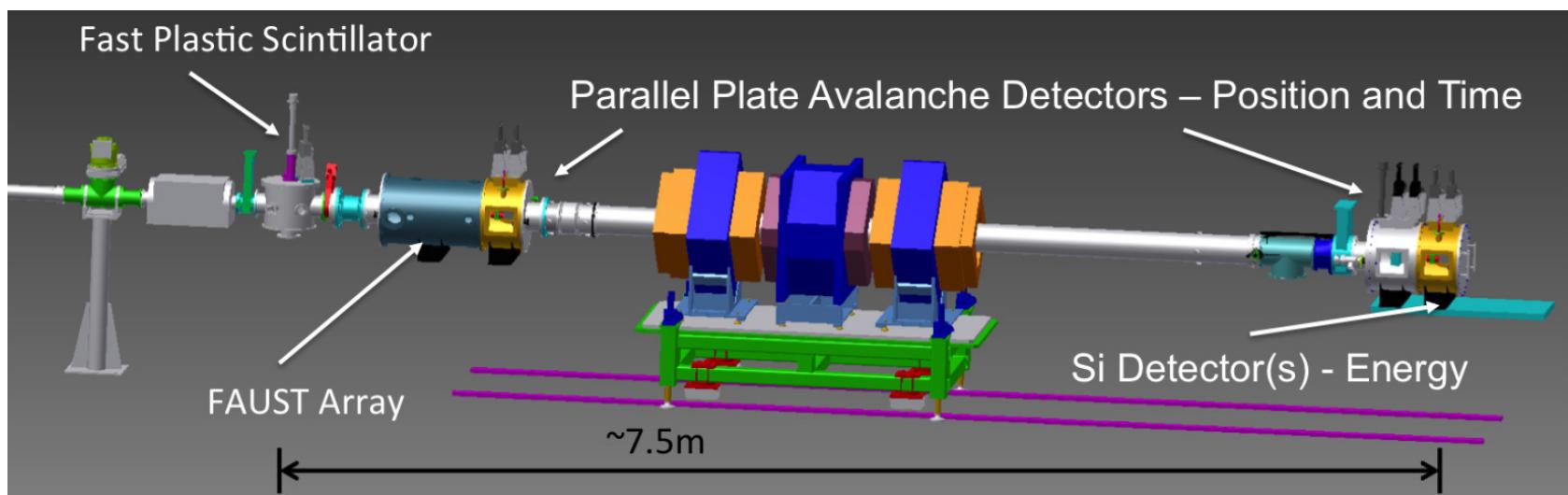
## FAUSTUPS

Measure Light Charged Particles

Position-Sensitive

$\Delta E$ ,  $E \rightarrow Z$ , A, Energy

$1.6^\circ \leq \theta \leq 45^\circ$



# Acknowledgements



Cyclotron Institute  
Texas A&M University



## Yennello Research Group & Collaborators

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M. Youngs, S. Wuenschel, A. Zarrella, H. Zheng, S.J. Yennello

Department of Energy  
DE-FG03-93ER40773



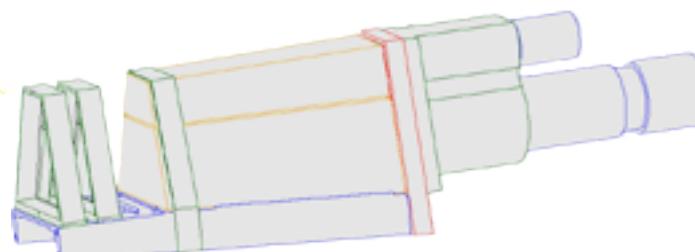
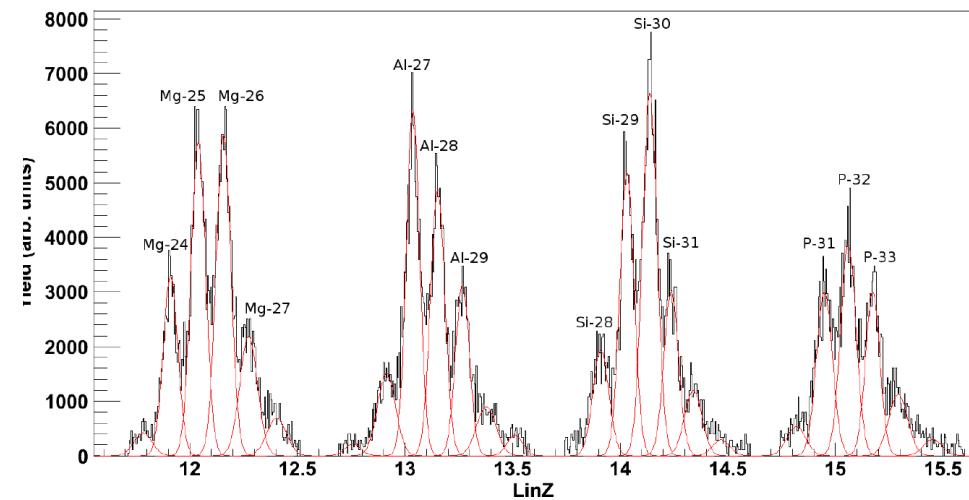
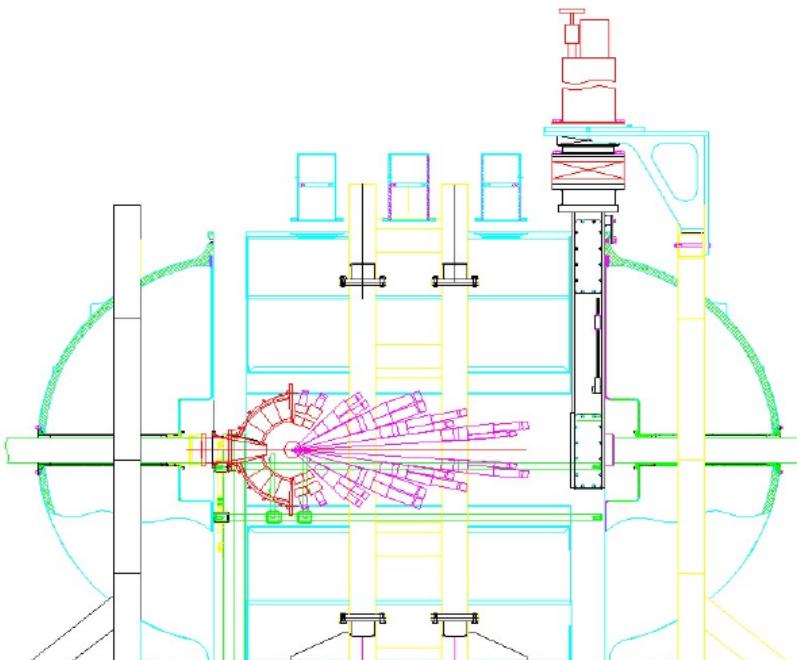
Welch Foundation  
A-1266

## Supplementary Material

# NIMROD-ISIS Array

- Full Silicon Coverage ( $4\pi$ )
- Isotopic Resolution to  $Z=17$
- Elemental Resolution to  $Z_{\text{projectile}}$
- Neutron Ball ( $4\pi$ )

70Zn + 70Zn  
64Zn + 64Zn  
64Ni + 64Ni  
 $E = 35\text{A MeV}$



S. Wuenschel et al., Nucl. Instrum. Methods. A604, 578–583 (2009)

Z. Kohley, Ph.D Thesis, TAMU (2010)

# QP Reconstruction

Cut 1/3:  
Velocity

Remove particles that do not belong (on average) to a statistically emitting projectile-like source.

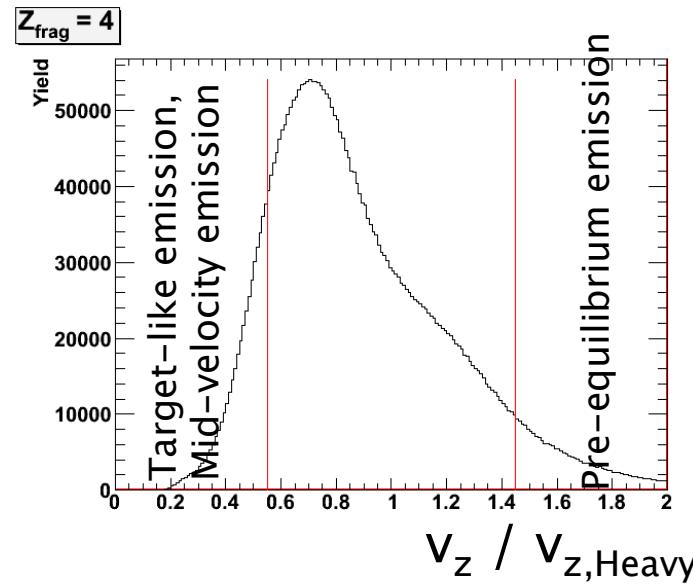
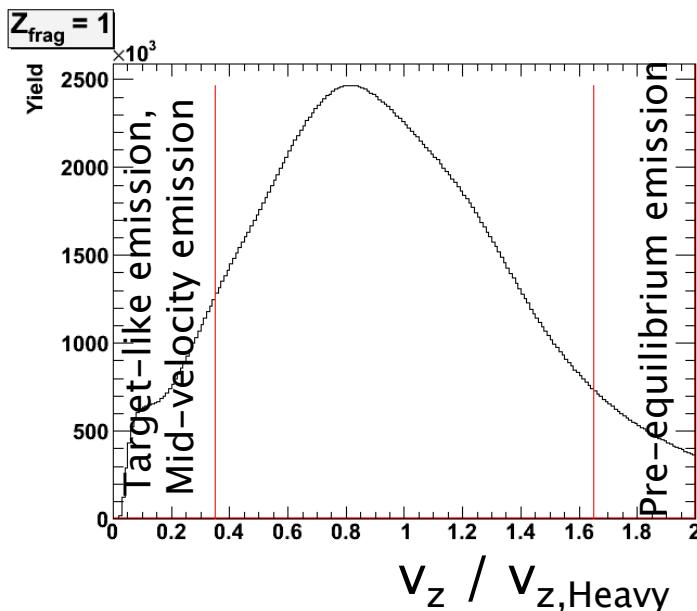
Compare laboratory parallel velocity of each particle to that of the heaviest charged particle measured in the event.

$$Z = 1 : \frac{v_z}{v_{z,PLF}} \leq 1.65$$

$$Z = 2 : \frac{v_z}{v_{z,PLF}} \leq 1.60$$

$$Z \geq 3 : \frac{v_z}{v_{z,PLF}} \leq 1.45$$

Steckmeyer et al., NPA 686, 537 (2001)



# QP Reconstruction

- **Mass Selection Considerations**
- Mass close to beam - well defined system
- Not too close to beam: significant  $E^*$ , overlap of target and projectile
- Sufficient statistics

$$48 \leq A_{QP} \leq 52$$

$$m_{\text{source}} = \frac{N_{QP} - Z_{QP}}{A_{QP}}$$

Largest uncertainty in  $A_{QP}$ : free neutron multiplicity

- Uncertainty in excitation
  - ▶ relatively small (compared to results)
- Uncertainty in asymmetry  $(N-Z)/A$ 
  - ▶ relatively small (compared to results)

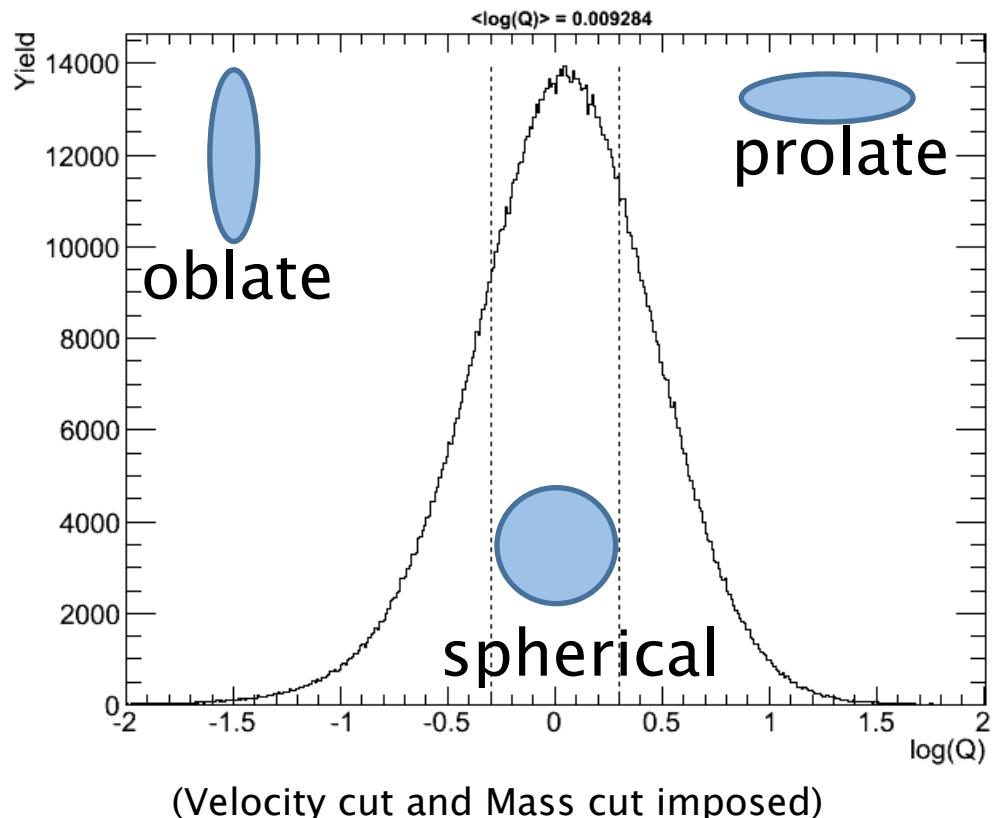
Marini et al., NIMA 707, 80 (2013)

# QP Reconstruction

Cut 3/3: Sphericity

$$Q = \frac{\sum p_{z,i}^2}{\frac{1}{2} \sum p_{T,i}^2}$$
$$-0.3 \leq \log Q \leq 0.3$$

Select events with  
near-zero average  
momentum  
quadrupole.

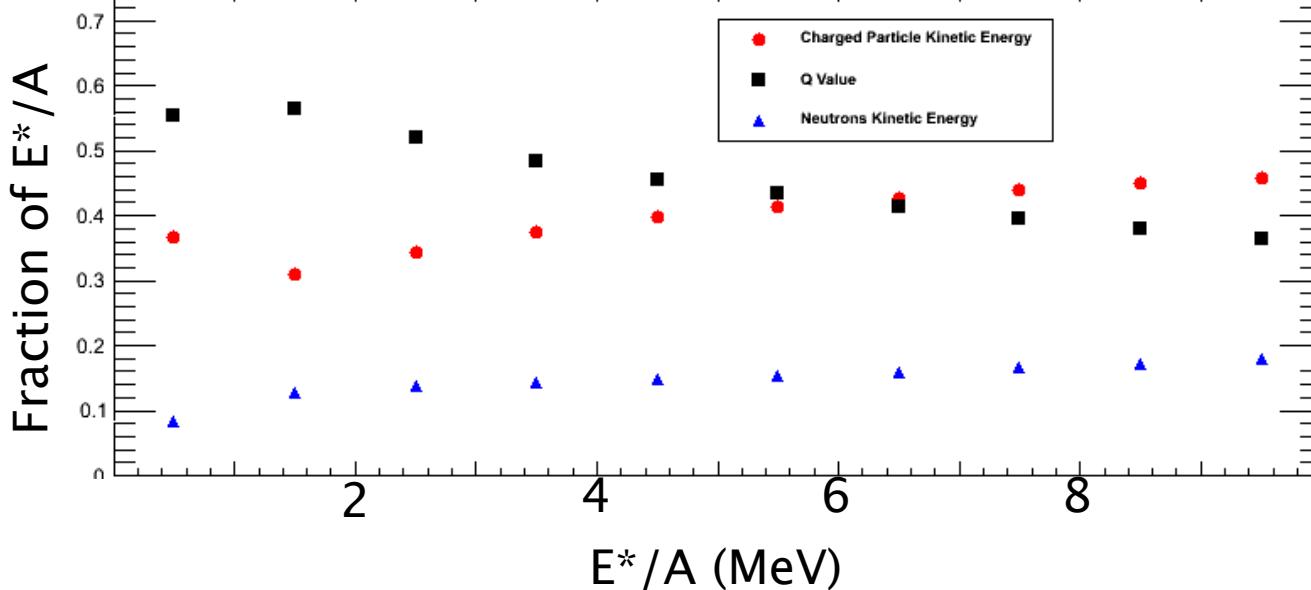


Concept to select thermally equilibrated events:  
Shape equilibration is slow relative to thermal equilibration.

S. Wuenschel, NPA 843, 1 (2010)  
S. Wuenschel, Ph.D. Thesis, Texas A&M University, (2009)

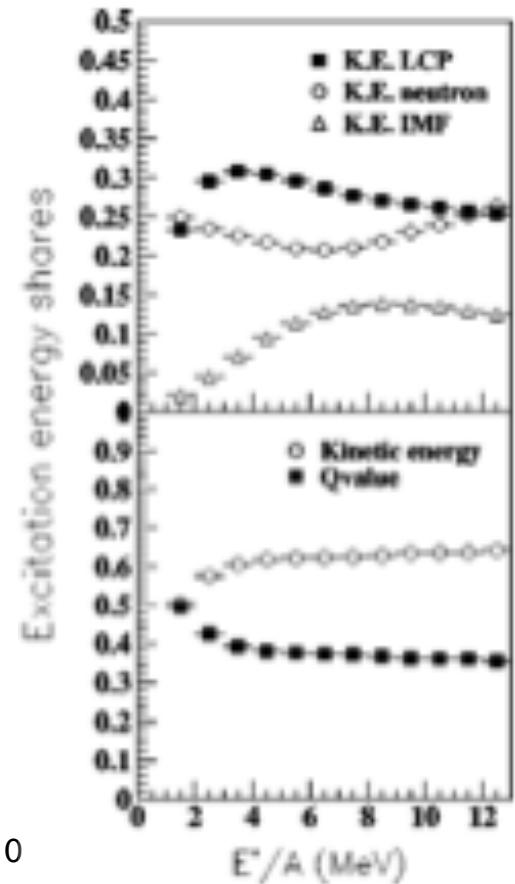
# QP Identity

$$Z_{QP} = \sum_i^{CP} Z_i \quad A_{QP} = \sum_i^{CP} A_i + M_n \quad \vec{v}_{QP} m_{QP} = \sum_i^{CP} \vec{v}_i m_i \quad E_{QP}^* = \sum_i^{CP} \frac{3}{2} K_{\perp,i} + M_n \langle K_n \rangle - Q$$



Excitation energy sharing is in reasonable agreement with previously published data:  
~ 40% Charged particle KE  
~ 40% Q value  
~ 20% Neutrons

Lefort et al. PRC 64, 064603 (20  
5–15 GeV/c Hadrons on Au



# Neutron Measurement

$$M_{\text{meas}} = (\epsilon_{QP} M_{QP} + \epsilon_{QT} M_{QT}) \left( \frac{\epsilon_{\text{lab}}}{\epsilon_{\text{sim}}} \right) + M_{\text{bkg}}$$

Efficiency  $\epsilon_{\text{lab}}$  measured with a calibrated Cf source.

**Simulations** to determine efficiency  $\epsilon_{QP}$ ,  $\epsilon_{QT}$ ,  $\epsilon_{\text{sim}}$ .

Efficiencies are model-independent (CoMD, HIPSE-SIMON).

Efficiencies are system-independent.

$$M_n = \frac{M_{\text{meas}} - M_{\text{bkg}}}{\left( \epsilon_{QP} + \frac{N_T}{N_P} \epsilon_{QT} \right) \left( \frac{\epsilon_{\text{lab}}}{\epsilon_{\text{sim}}} \right)}$$

Marini et al., NIMA 707, 80 (2013)  
Wada et al., PRC 69, 044610 (2004)

# Neutron Uncertainty

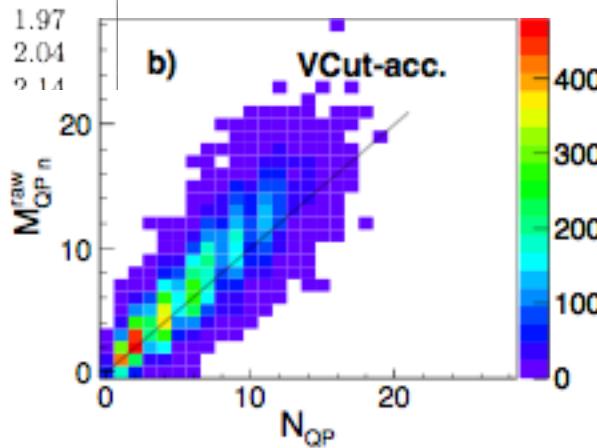
$$\sigma_{\text{raw}}^2 = \sigma_{\text{true}}^2 + \sigma_{\text{eff}}^2 + \sigma_{\text{bkg}}^2$$

$N_{QP}$	$(M_{QP}^{\text{raw}})$	$\sigma(M_{QP}^{\text{raw}})$
HIPSE		
0	1.46	1.10
1	2.13	1.21
2	3.02	1.64
3	3.67	1.68
4	4.06	1.75
5	4.72	1.88
6	5.28	1.97
7	5.94	2.04
8	6.50	2.14

raw width: 5.36

width due to efficiency: < 2.1 (worst case)

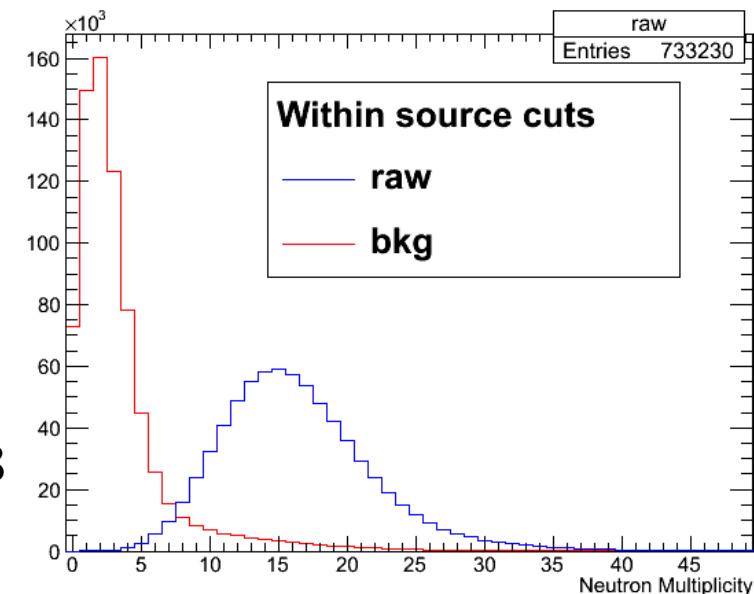
efficiency: 9% effect



raw width: 5.36

width due to background: 1.8

efficiency: 6% effect



Marini et al., NIMA 707, 80 (2013)

**Net effect: we know the QP neutron multiplicity to within 11% ( $1\sigma$ ).**

# Calculation of Neutron Uncertainty

We know the QP neutron multiplicity to within 11% ( $1\sigma$ ).  
How big is this?

For a source of 50 nucleons where 5 become free neutrons, the free neutrons contribute 0.97 MeV/nucleon to the excitation energy.

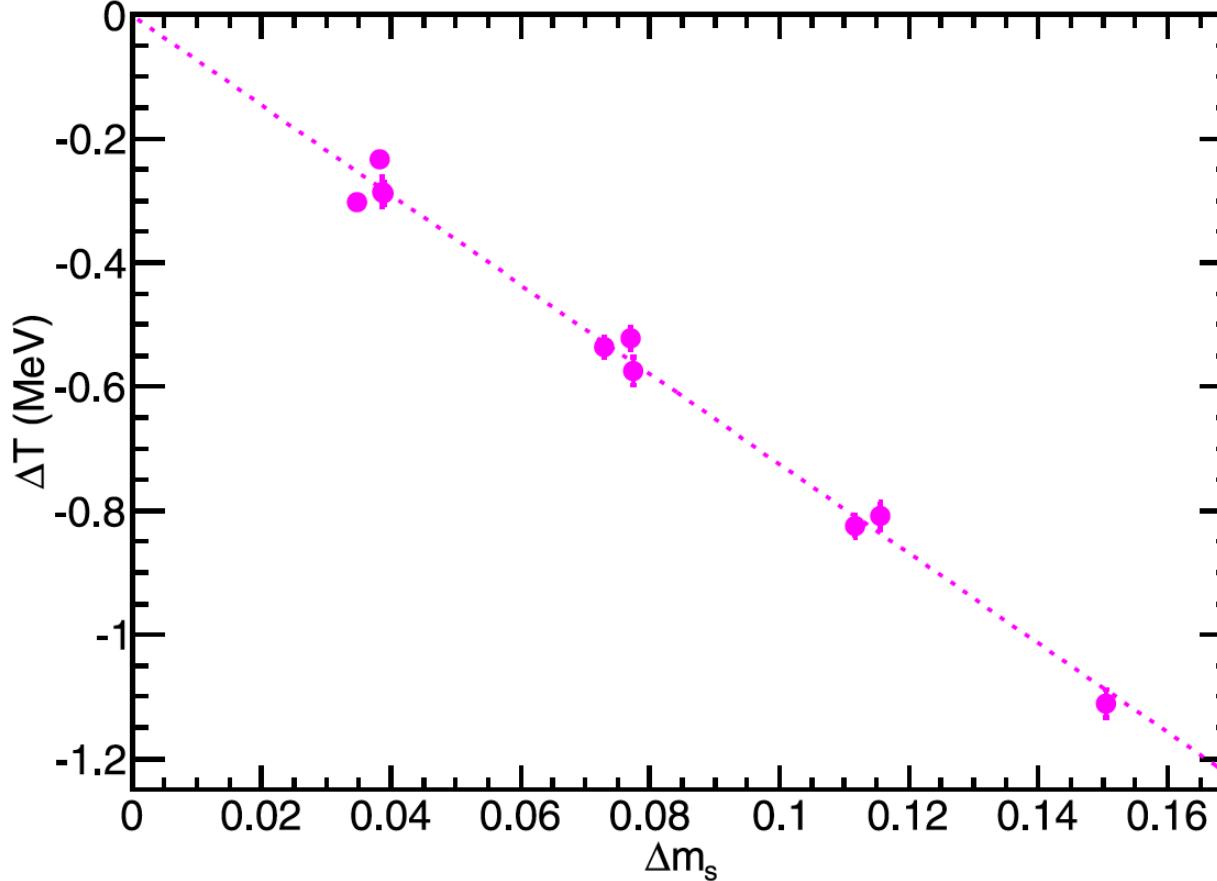
An uncertainty of 11% on the free neutron multiplicity corresponds to an uncertainty of 0.11 MeV/nucleon.

This uncertainty of 0.11 MeV/nucleon is significantly smaller than the spacing between even the closest caloric curves.

asymmetry

For a source of 50 nucleons where 5 become free neutrons, an error of 1 neutron corresponds to a  $2\sigma$  variation. It would require an error of  $4\sigma$  to shift from one asymmetry bin to another.

# Robust Asymmetry Dependence



We vary the neutron kinetic energy to physically unrealistic extremes:

- Neutron KE to 50%: slope  $\Delta T/\Delta m_s$  decreases only to 75%
- Neutron KE to 150%: slope  $\Delta T/\Delta m_s$  increases only to 125%

→ Some uncertainty in magnitude of the correlation, but not in its existence

## TARGET:

If target is too small, excitation energy is very low

e.g. Kr+p gives  $E^*/A$  below 1MeV, even at  $E_{beam}=35A$  MeV

If target is too large, incomplete fusion is very incomplete

e.g. in Kr+Al, the zinc fuses with less of the aluminum target than Kr+C

Target of  $^{12}C$  is good compromise

Kr+C at 15,25,35A MeV fuses with more than half of the target

Reaches nearly  $E^*/A = 3$ MeV

## PROJECTILE:

If projectile is too heavy,  $E^*/A$  will be low, cyclotron limitation

e.g. Sn+C: Sn max energy around 15A MeV  $\rightarrow E^*/A = 1.2$ MeV

If projectile is too light, compound nucleus lighter than I'd like

e.g. Ar+C  $\rightarrow$  Cr ( $A=48$ ) is not terrible, but larger A is better to study caloric curve

## ASYMMETRY:

Choose combination with large range in  $(N-Z)/A$

$^{78}Kr + ^{8}Be \rightarrow ^{86}Zr$  (8Be is 2/3 of  $^{12}C$  target)

$^{86}Kr + ^{8}Be \rightarrow ^{94}Zr$

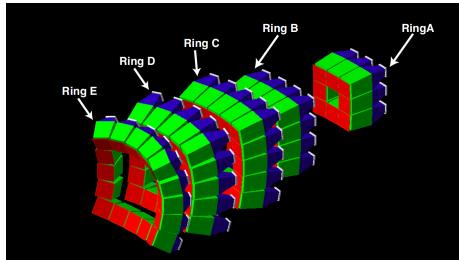
$\rightarrow (N-Z)/A = 0.070$  and  $0.163$

## BEAM ENERGY:

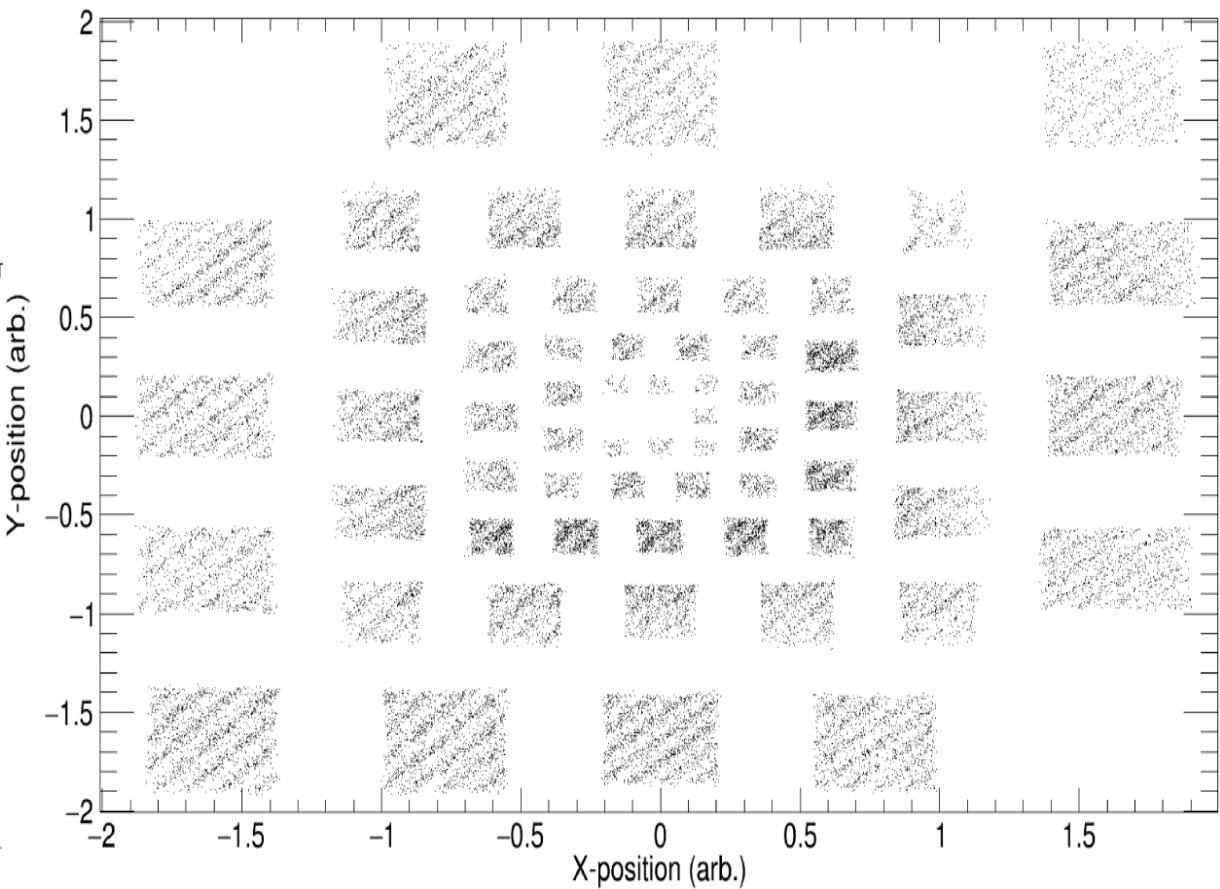
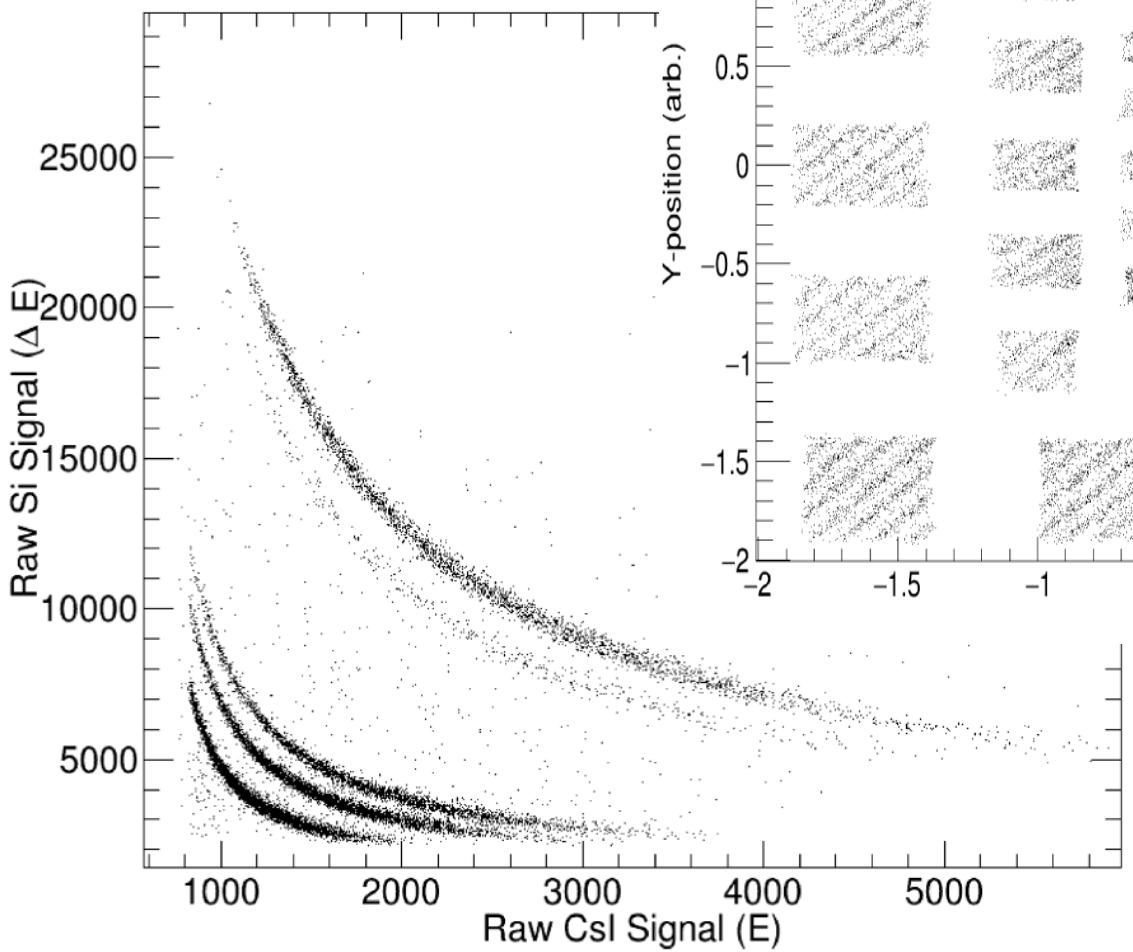
@ 15, 25, 35 A MeV

$\rightarrow E^*/A = 1.3$  MeV, 2.0 MeV, and 2.8 MeV for  $^{86}Kr$  beam. For  $^{78}Kr$ ,  $\sim 10\%$  higher.

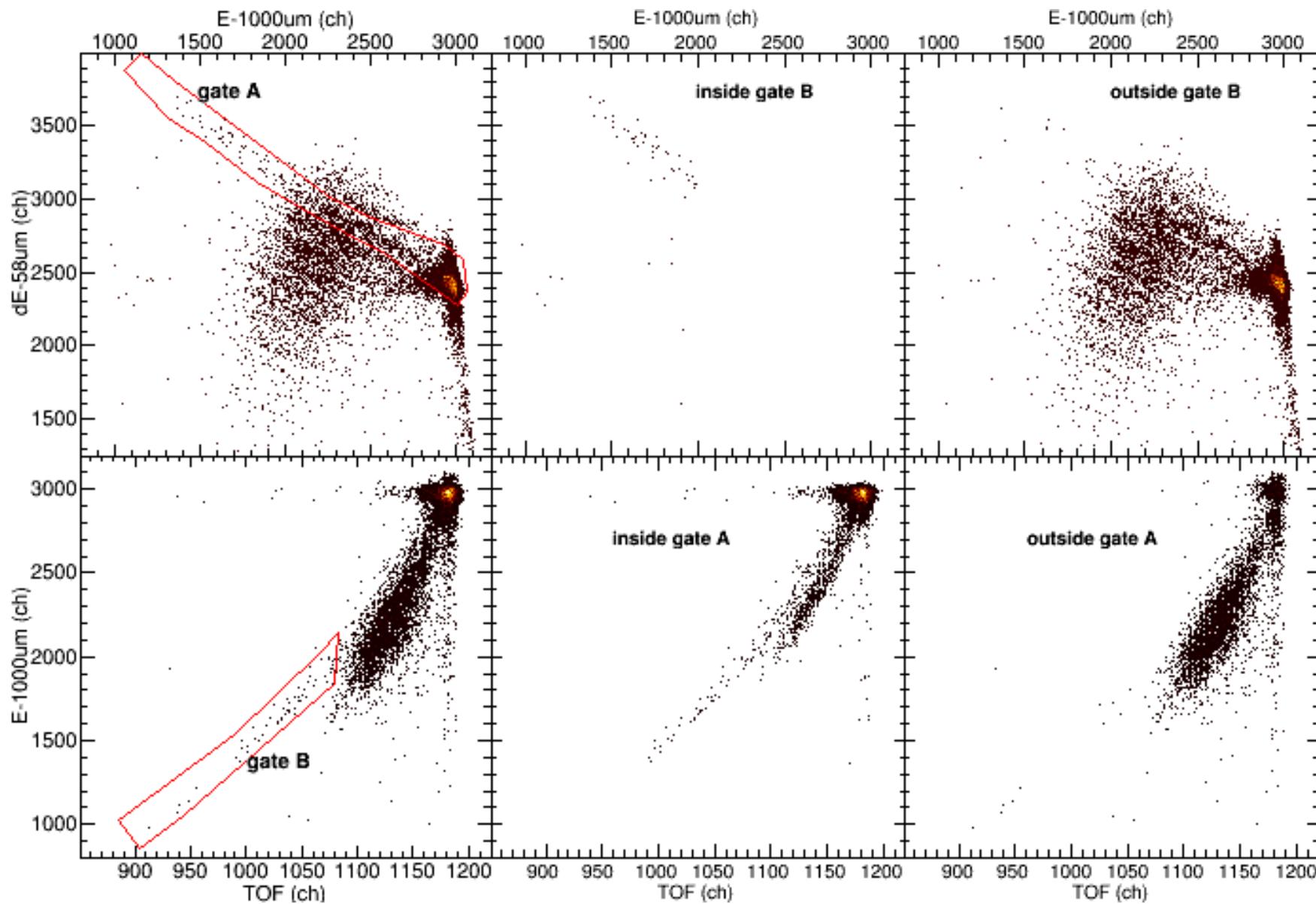
# FAUSTUPS



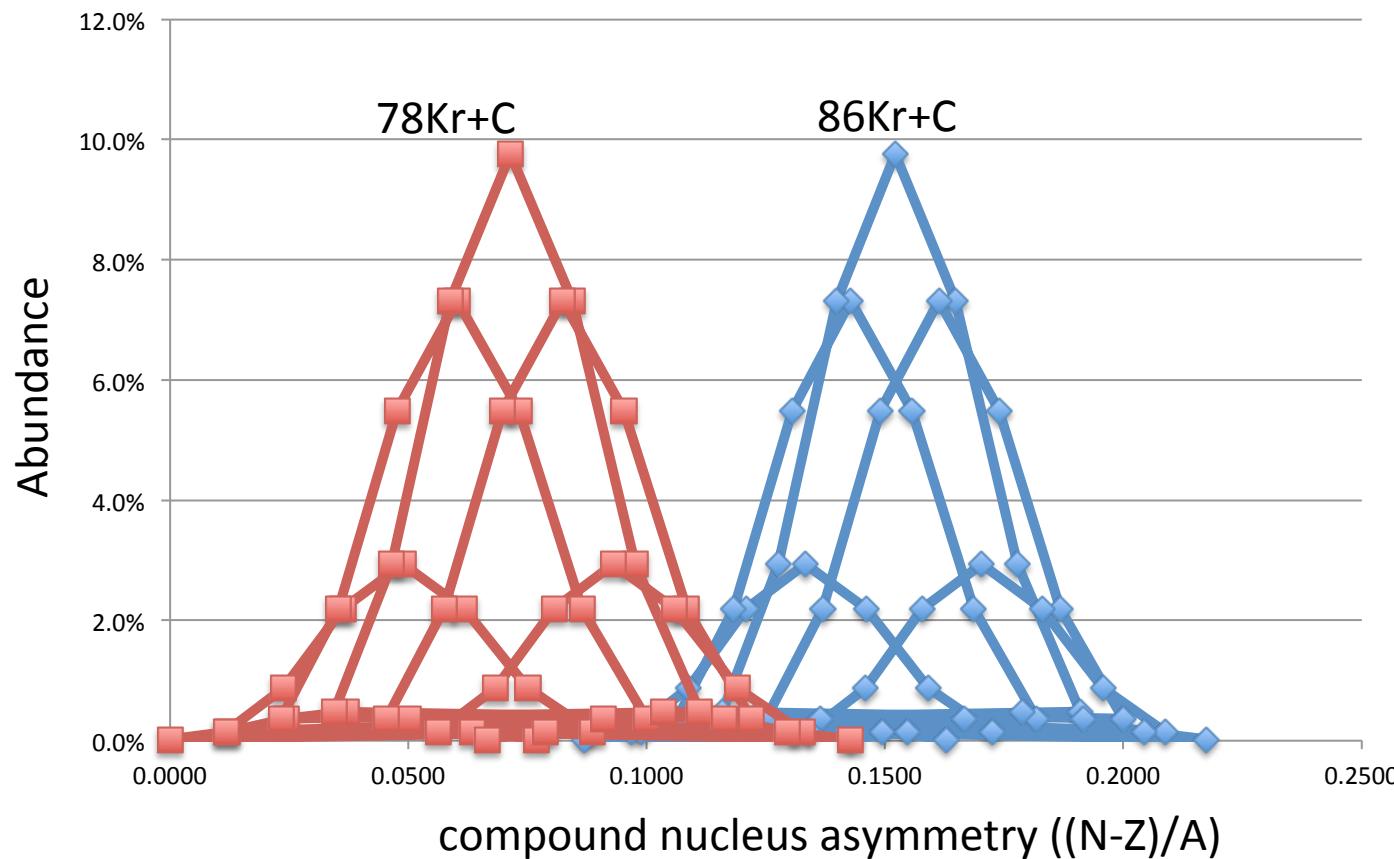
Upgrade to excellent position resolution.  
Retain excellent energy and isotopic resolution



# Quadrupole Triplet Spectrometer Provides {Z, A, E, V} using, {TOF, $\Delta E$ , E}



Even if you pick up every possible number of neutrons and protons with equal probability, the asymmetries of the compound nuclei for the 86Kr and 78Kr systems are still well separated.



This is combinatorics only. Physics may narrow the distributions since many of these cases don't occur and some others will be filtered out by the triplet. The distributions may shift toward each other if the  $^{78}\text{Kr}$  prefers to pick up neutrons and the  $^{86}\text{Kr}$  prefers to pick up protons.

## Preferential nucleon pickup

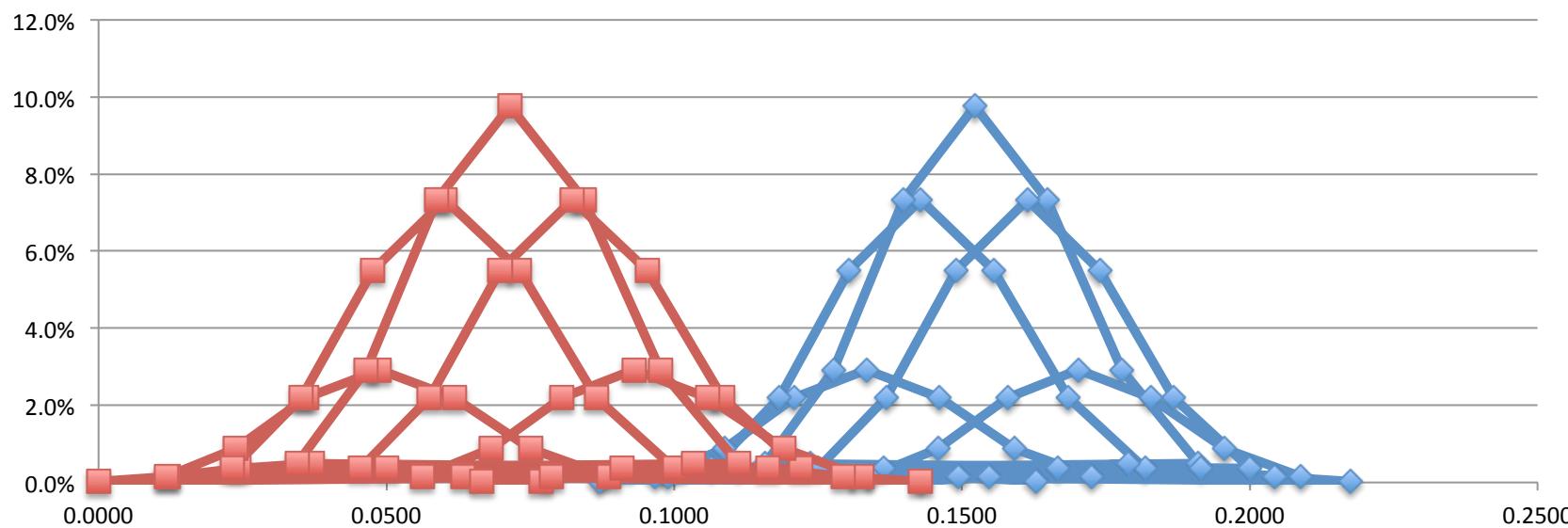
If the  $^{78}\text{Kr}$  prefers 5 neutron + 3 proton pickup and the  $^{86}\text{Kr}$  prefers 3 neutron + 5 proton pickup, then the central asymmetry of the two systems will still be separated in asymmetry.

A difference of 6/2 vs 2/6 will actually switch the asymmetries of the systems.

beam				pickup		compound nucleus			<b>delta</b>
	Z	A	N	$\Delta Z$	$\Delta N$	Z	N	A	
78Kr	36	78	42	2	6	38	48	86	<b>0.116</b>
78Kr	36	78	42	3	5	39	47	86	<b>0.093</b>
78Kr	36	78	42	4	4	40	46	86	<b>0.070</b>
78Kr	36	78	42	5	3	41	45	86	<b>0.047</b>
78Kr	36	78	42	6	2	42	44	86	<b>0.023</b>
86Kr	36	86	50	2	6	38	56	94	<b>0.191</b>
86Kr	36	86	50	3	5	39	55	94	<b>0.170</b>
86Kr	36	86	50	4	4	40	54	94	<b>0.149</b>
86Kr	36	86	50	5	3	41	53	94	<b>0.128</b>
86Kr	36	86	50	6	2	42	52	94	<b>0.106</b>

Is it true that  $(\Delta Z - \Delta N)_{^{86}\text{Kr}} > -2$ ?  
 Is it also true that  $(\Delta Z - \Delta N)_{^{78}\text{Kr}} < 2$ ?

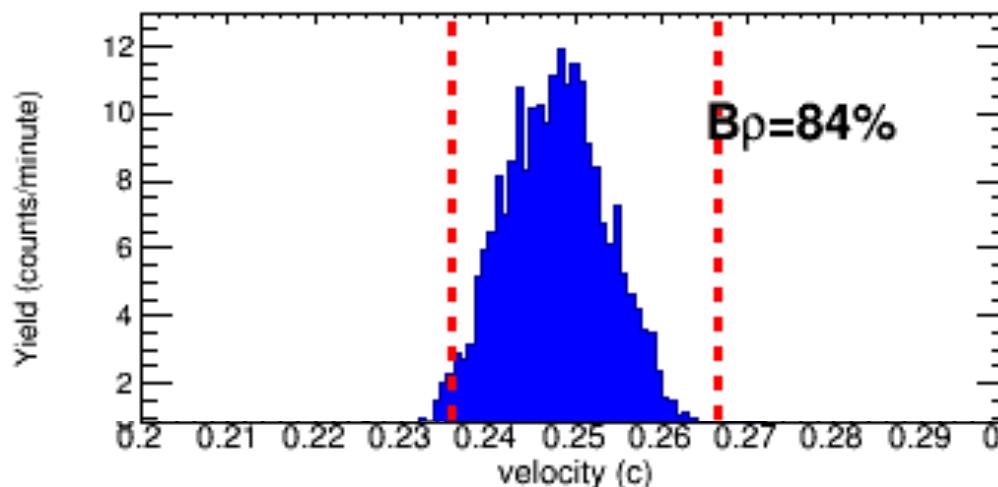
***Cluster structure of  $^{12}\text{C}$  to the rescue!... →***

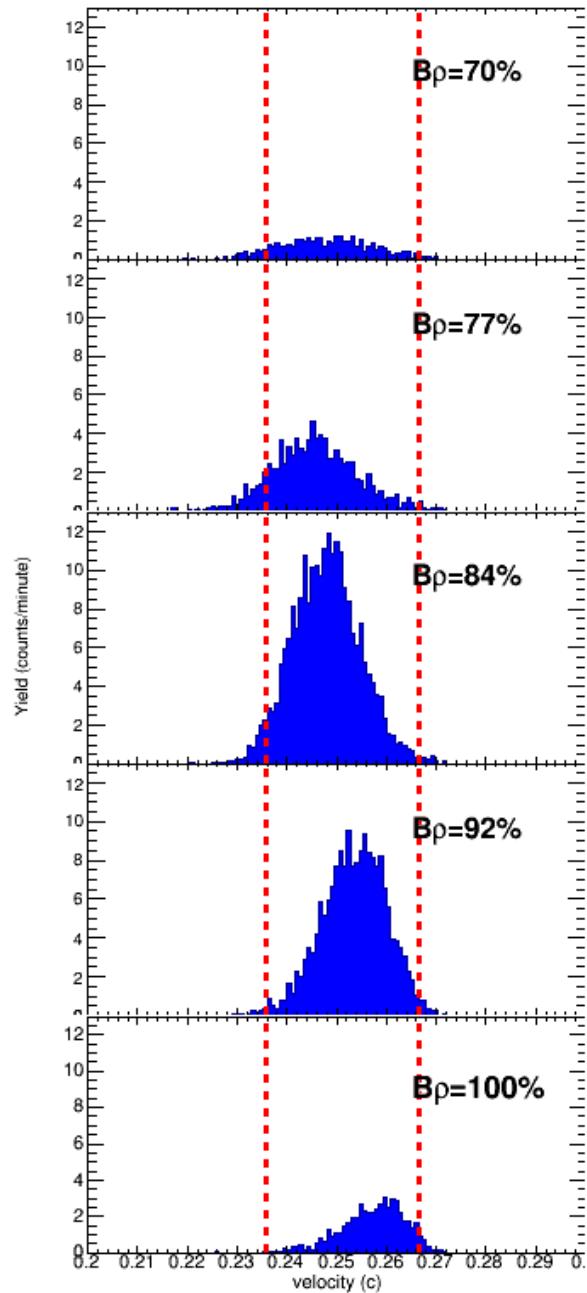


## $^{86}\text{Kr} + \text{C}$ @ 35A MeV

Velocity distribution of fusion-evaporation residues measured.

The velocity distributions indicate that on average about 8 of the 12 nucleons of the target fuse with the heavy projectile.





## 86Kr + C @ 35A MeV

Velocity distribution of fusion-evaporation residues measured.

The velocity distributions indicate that on average about 8 of the 12 nucleons of the target fuse with the heavy projectile.

**What if the un-fused part of the target gets dragged along some?**

**Bohne et al PRC 41 R5 (1990)**

$$\epsilon^* = \frac{1}{2} (\cos\theta_R v_P - v_R) v_R + Q_{gg}/m_R \\ + \frac{1}{2} \Delta m_P/m_R (\cos\theta_P v_P - v_{P'}) v_{P'} \\ + \frac{1}{2} \Delta m_T/m_R (\cos\theta_T v_P - v_{T'}) v_{T'},$$

using  $\cos = 1$ , and having 8 nucleon pickup from the carbon, and neglecting  $Q$ :

$$\epsilon^* = (1/2)(v_P - v_R)v_R + (1/2)(4/94)(v_P - v_{T'})v_{T'}$$

if  $v_{T'} = 0$ :

$$\epsilon^* = (1/2)(v_P - v_R)v_R = 0.5 * (0.2666 - 0.2453)(0.2453) = 0.00401 \text{ c}^2$$

if  $v_{T'} = v_R$ :

$$\epsilon^* = (1/2)(v_P - v_R)v_R * (1 + (4/94)) = 0.00418 \text{ c}^2$$

That's (obviously) a relative difference of  $4/94 = 4.3\%$  in the excitation energy for the two extreme cases of pre-equilibrium at rest and pre-equilibrium at residue velocity.

BUT  $\cos=1$  is not good.  $\cos\theta t$  can be much smaller than 1. Extreme is if  $v_P \cos\theta t = v_R$ .

This also makes the term zero. There will be a maximum around  $v_{T'} = 0.5 \cos\theta t v_P$ .

then if  $v_{T'} = 0$ :

$$\epsilon^* = (1/2)(v_P - v_R)v_R + (1/2)(4/94)(\cos\theta t v_P - v_{T'})v_{T'} = 0.5 * (0.2666 - 0.2453)(0.2453) = 0.00401 \text{ c}^2$$

or if  $v_{T'} = 0.5 \cos\theta t v_P$ :

$$\epsilon^* = (1/2)(v_P - v_R)v_R + (1/2)(4/94)(\cos\theta t v_P - v_{T'})v_{T'} = 0.00401 + (1/2)*(4/94) v_{T'}^2$$

$$\epsilon^* = 0.00401 + 0.000378 = 0.00439. \text{ This is a } 9\% \text{ effect.}$$