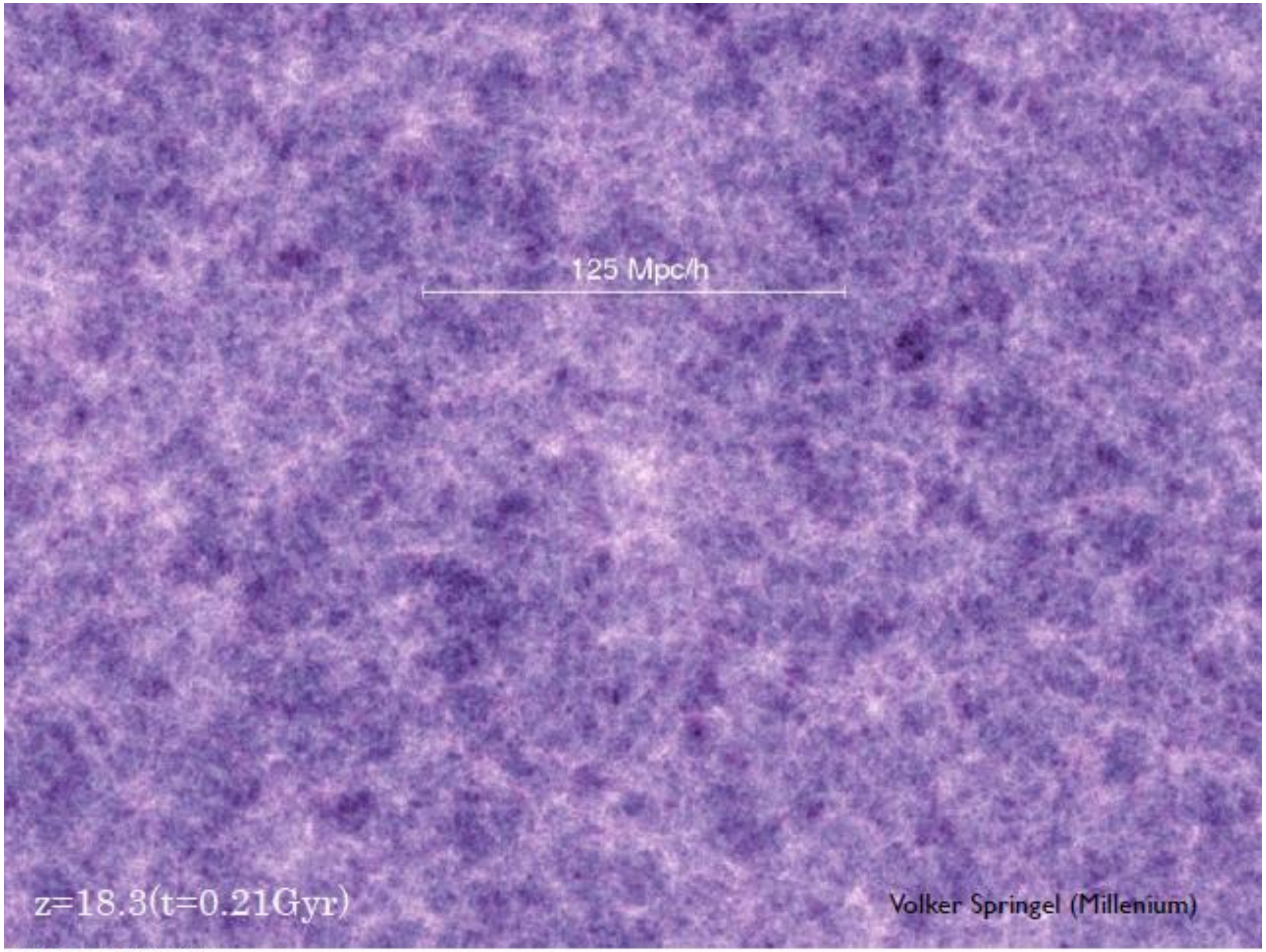
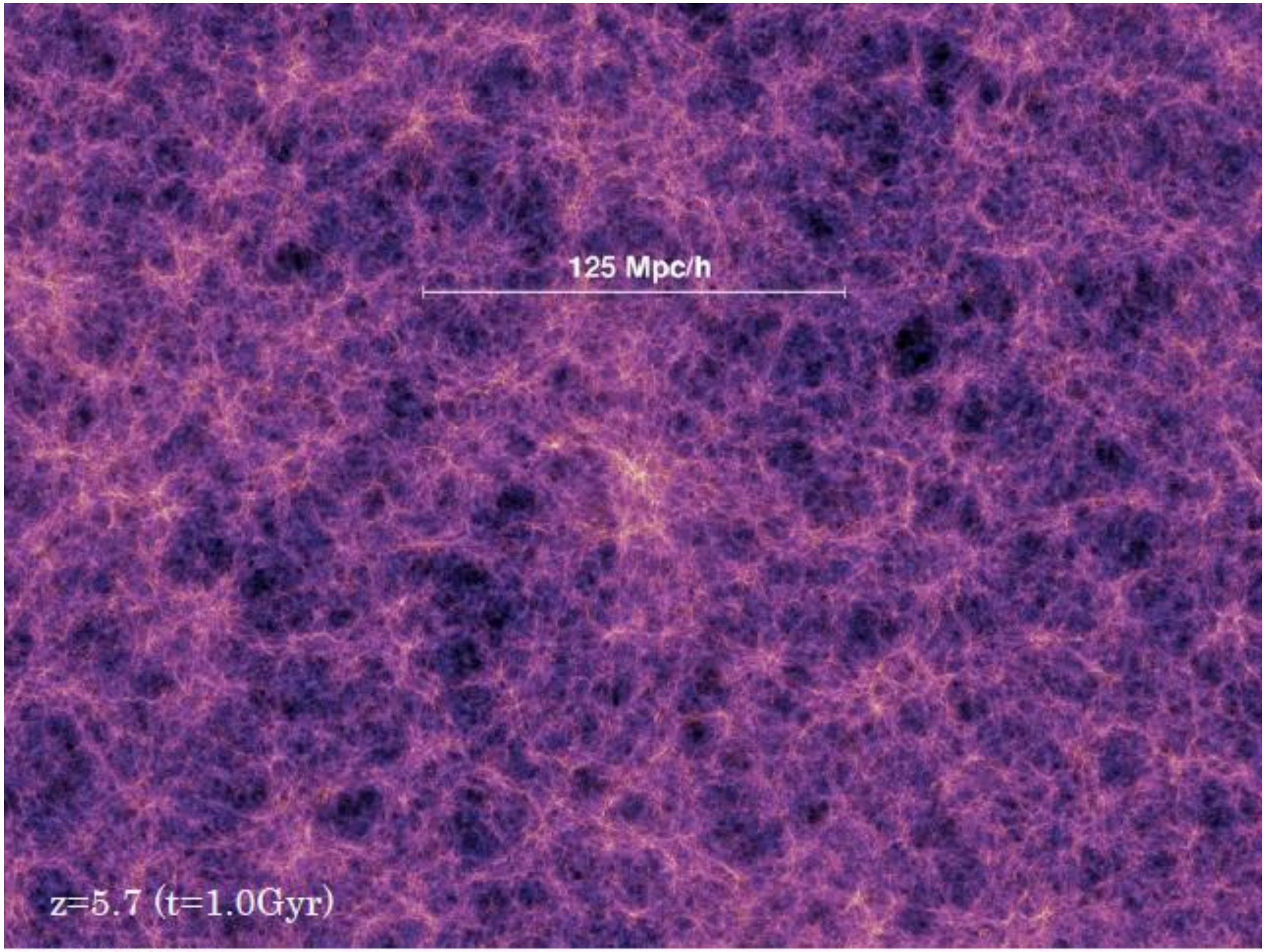
Phenomenology of dark matter structure formation

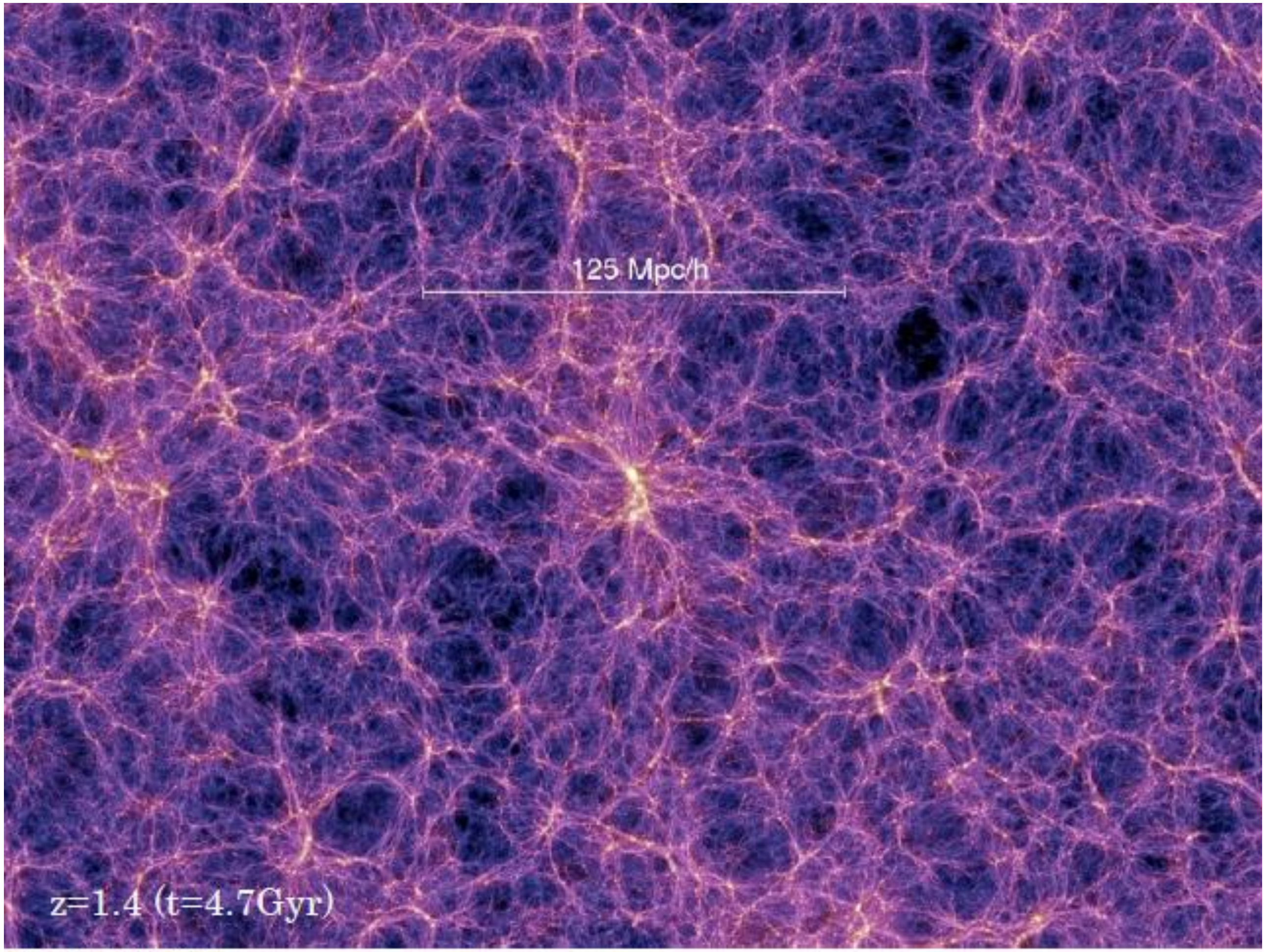
The halo model

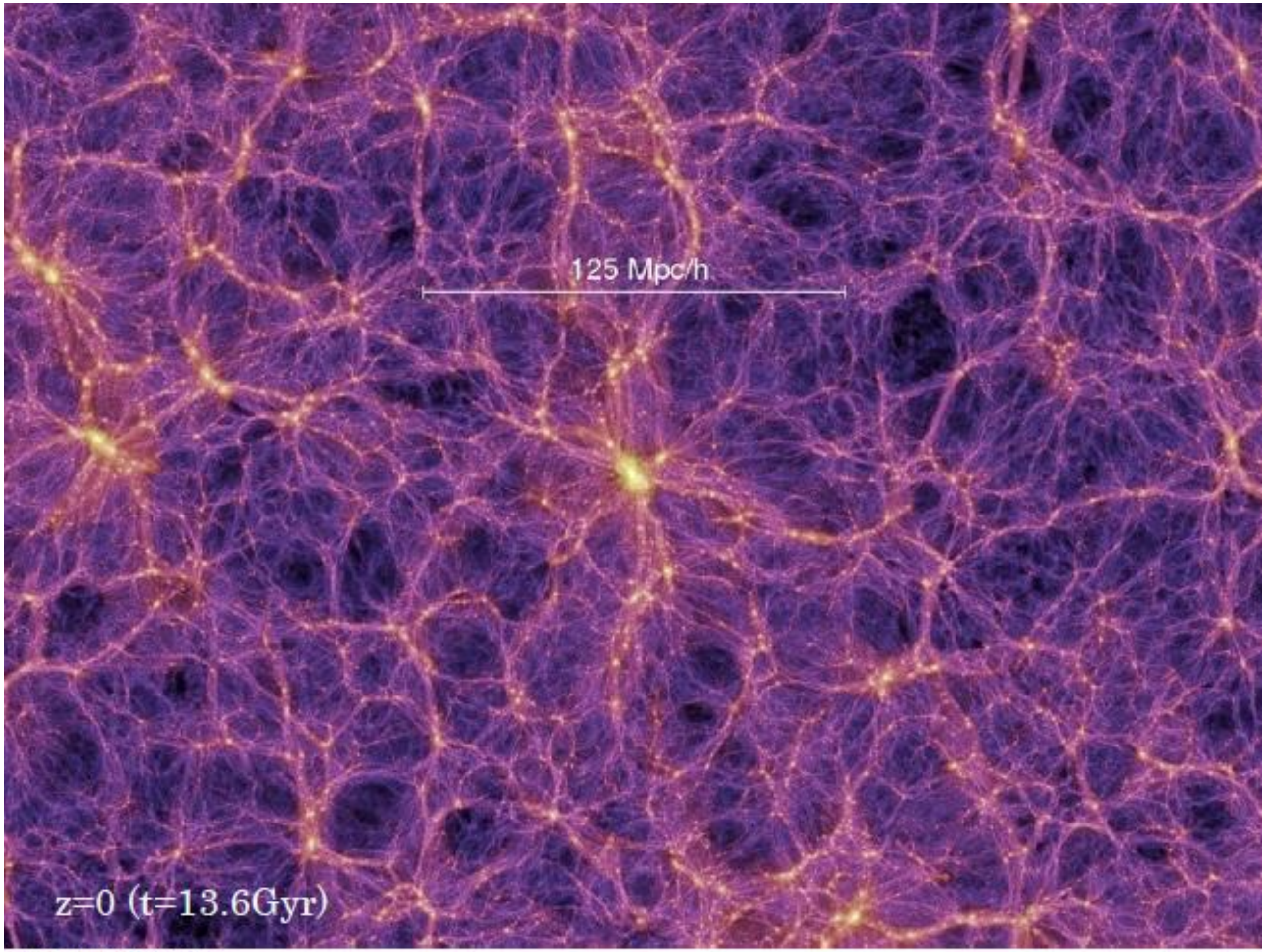
Halo abundances and clustering

Halo profiles


125 Mpc/h
z=18.3(t=0.21Gyr)
Volker Springel (Millenium)

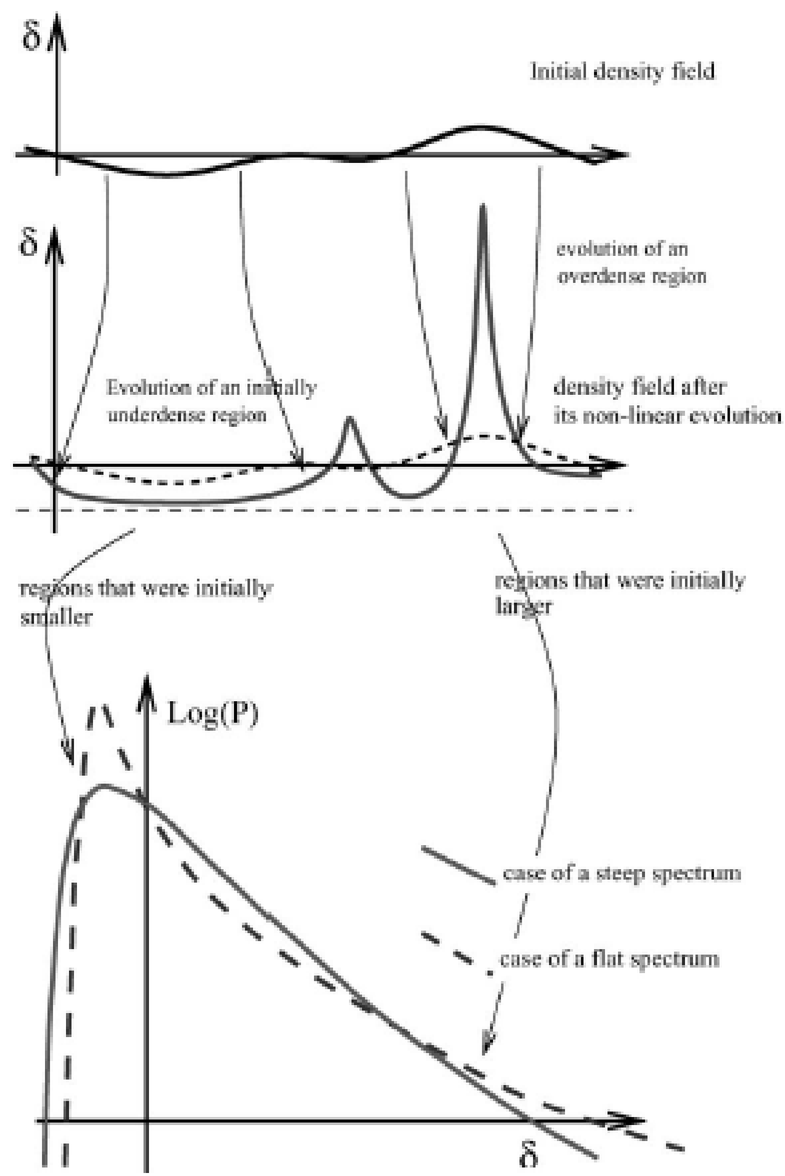

125 Mpc/h
z=5.7 (t=1.0Gyr)


125 Mpc/h
z=1.4 (t=4.7Gyr)

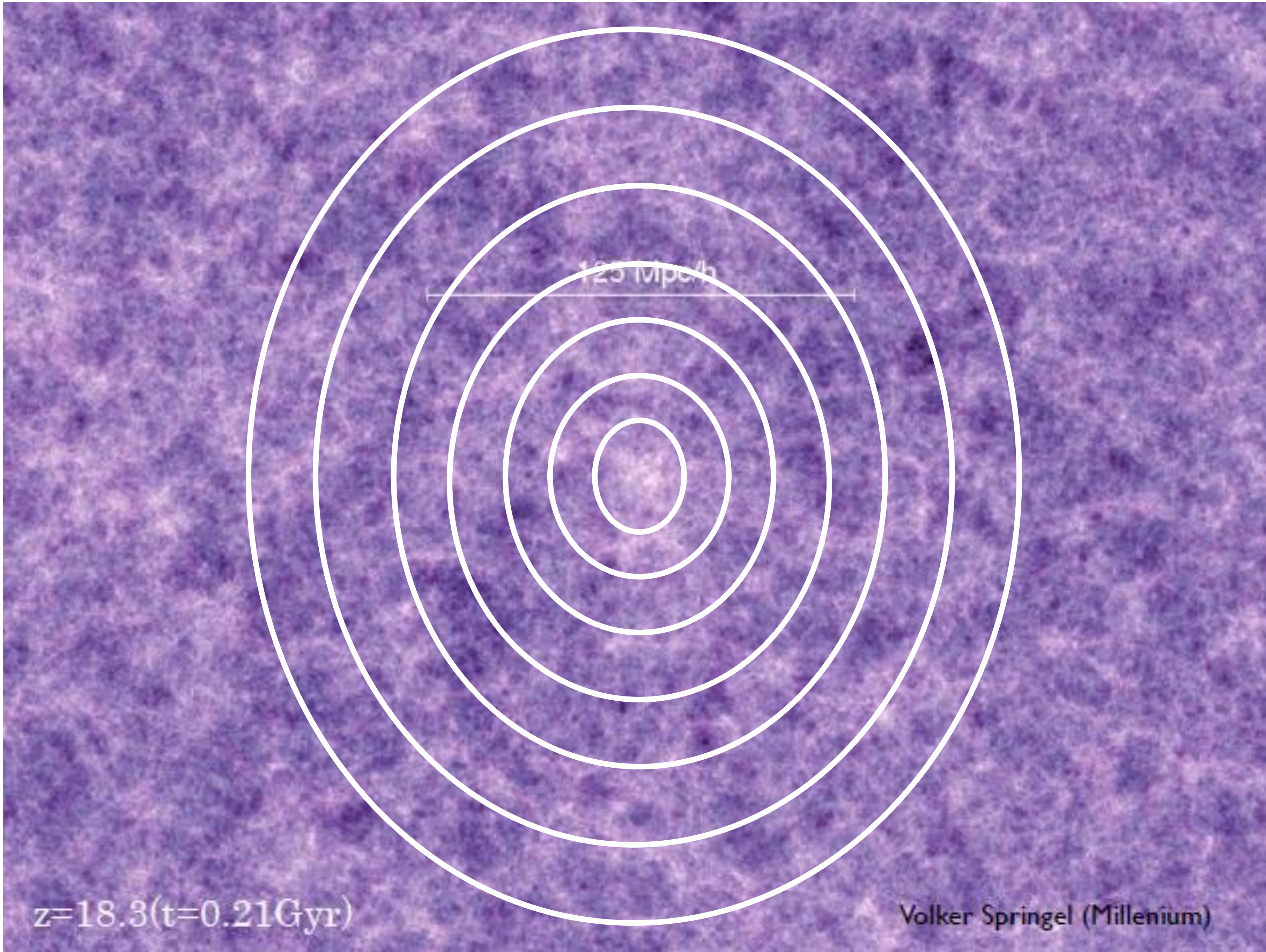

125 Mpc/h
z=0 (t=13.6Gyr)

Initially Gaussian fluctuation field becomes very non-Gaussian

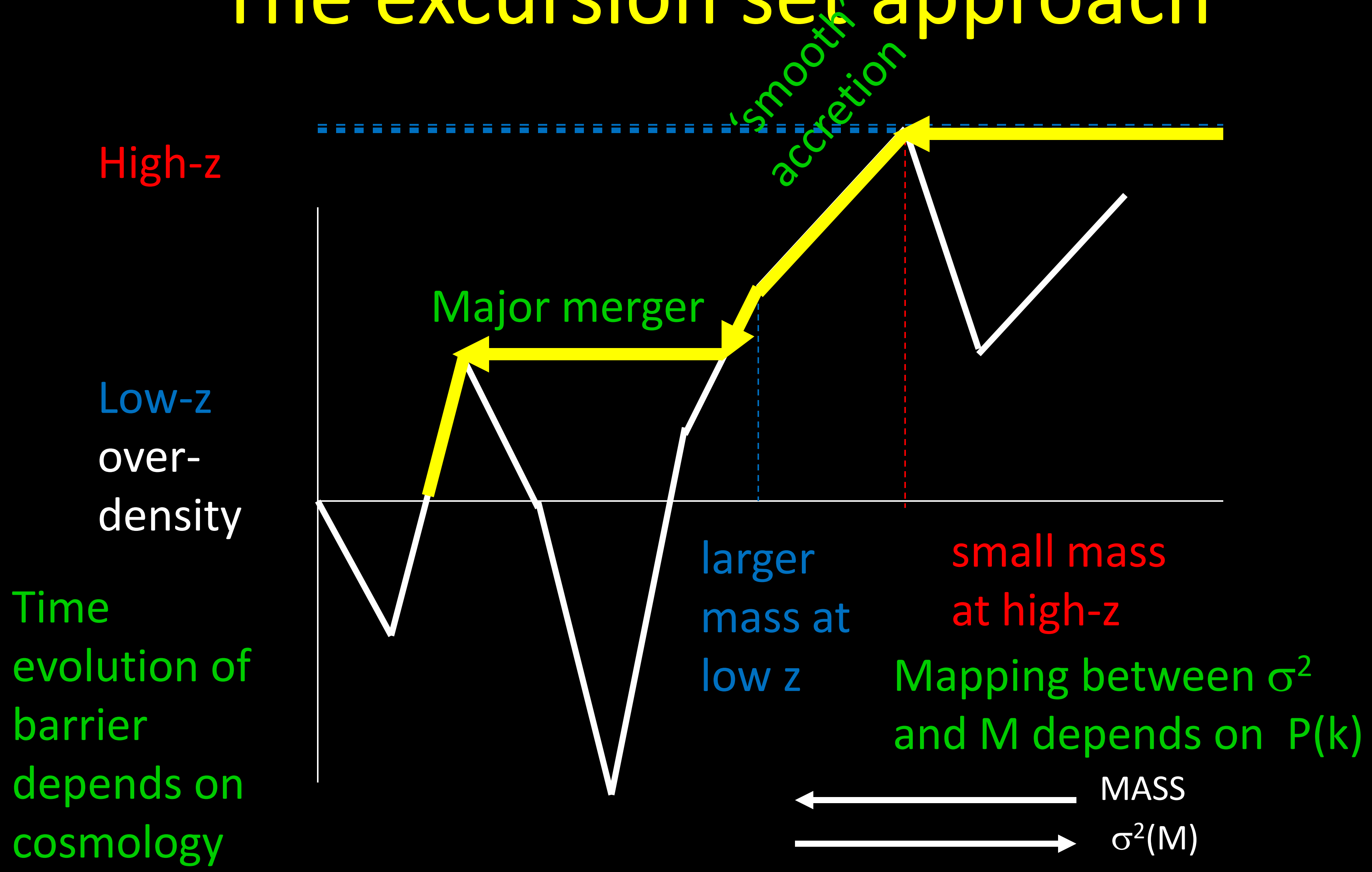
F. Bernardeau et al. / Physics Reports 367 (2002) 1–248



But wait …
We should be doing this in the INITIAL fluctuation field!


125 Mpc/h
z=18.3(t=0.21Gyr)
Volker Springel (Millenium)

The excursion set approach



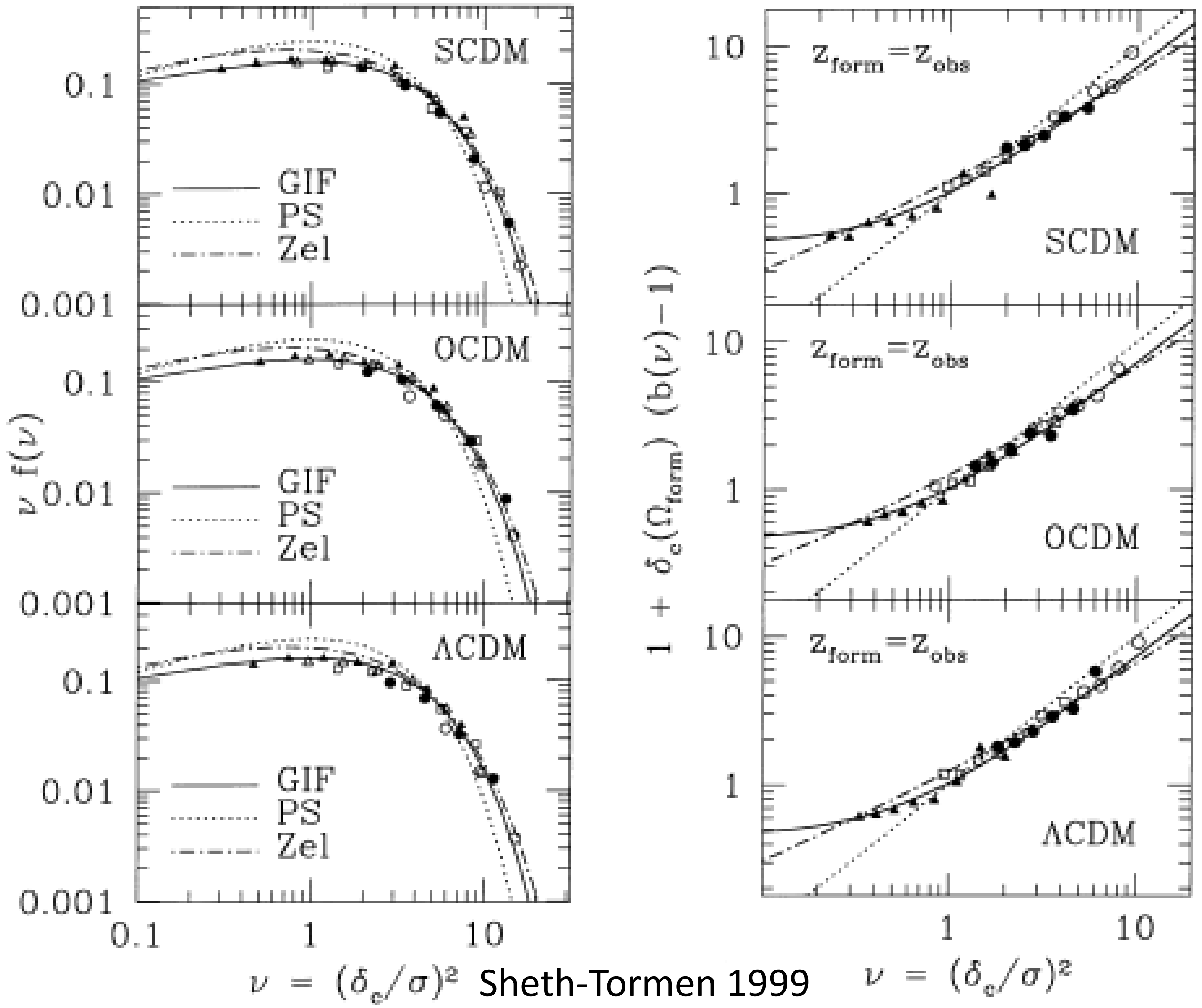
Simplification because…

- Everything local
- Evolution determined by cosmology (competition between gravity and expansion)
- Statistics determined by initial fluctuation field: for Gaussian, specified by initial power-spectrum $P(k)$
- Nearly universal in scaled units: $\delta_c(z)/\sigma(m)$ where $\sigma^2(m) = \langle\delta_m{}^2\rangle = \int dk/k \; k^3 P(k)/2\pi^2 \, W^2(kR_m)$ $m \propto R_m{}^3$
- Fact that only very fat cows are spherical is a detail (*crucial* for precision cosmology); in excursion set approach, mass-dependent barrier height increases with distance along walk

(Almost) universal mass function and halo bias

See Paranjape et al (2013) for recent progress in modeling this from first principles

See Castorina et al. (2014) for ν's



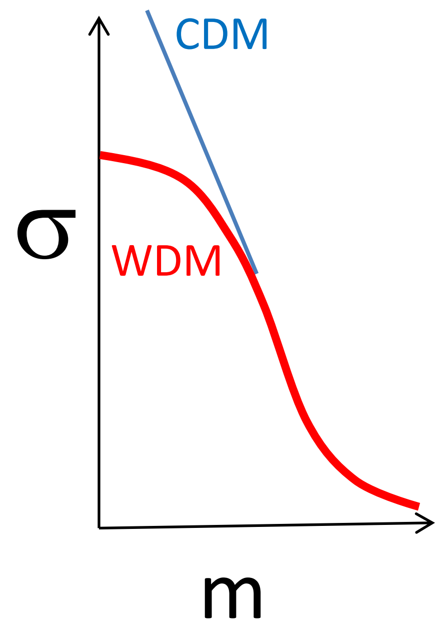
Sheth-Tormen 1999

Mass →

Mass →

For WDM …

- At small enough m, $\sigma(m)$ is flat
- Fraction of walks which didn't cross barrier prior to this σ = non-negligible smooth component which was never bound to anything
- f_{smooth} should be larger at high z
- Fewer halos (progenitors) at high z mean less concentrated halos at low z
- f_{smooth} should be larger in voids = voids are 'emptier' (even more so if $\delta_c(m)$ larger at small m)



Spherical evolution mapping …

$(R_{initial}/R)^3 = \text{Mass}/(\rho_{com}\text{Volume}) =$

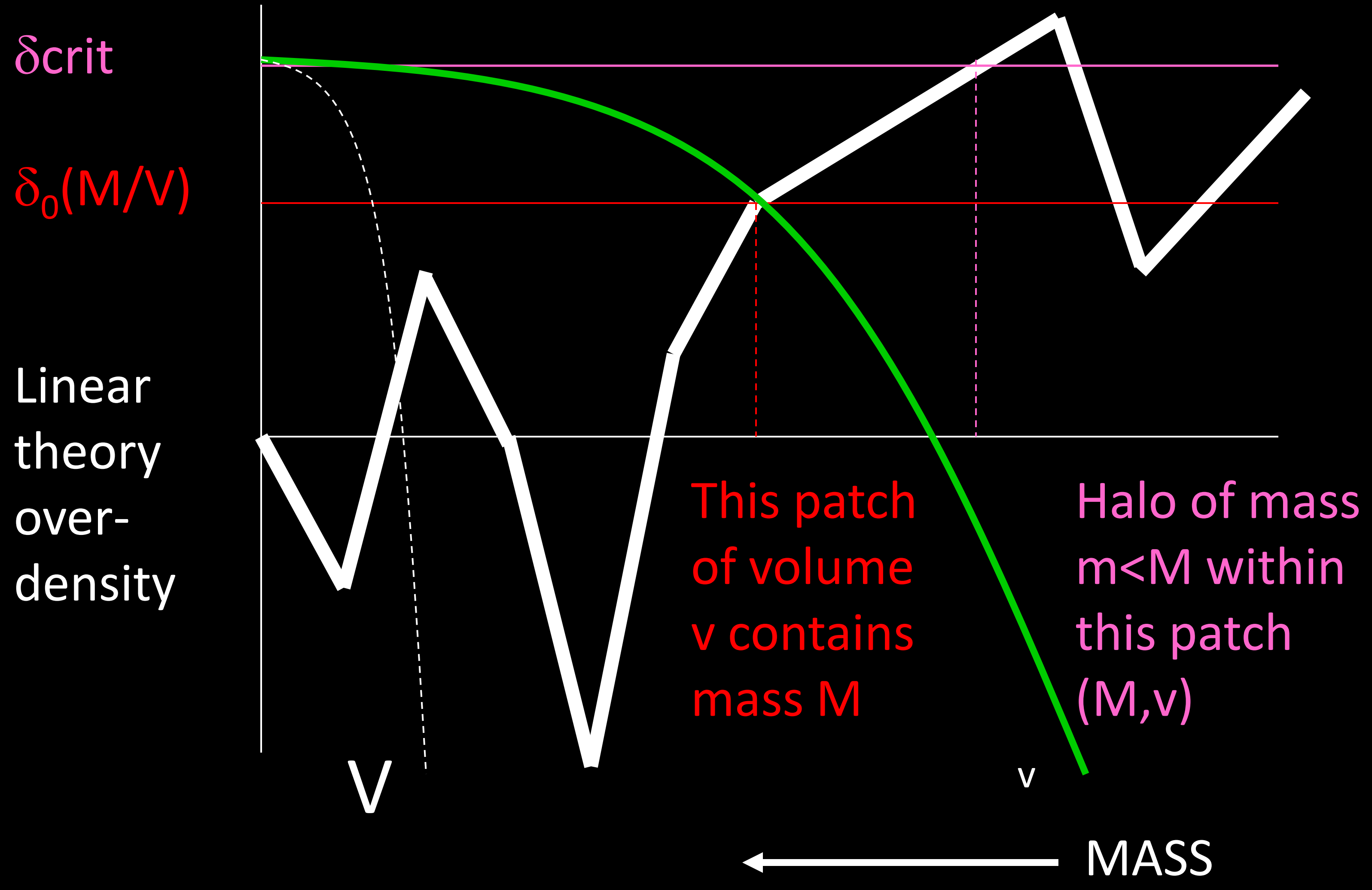
$1 + \delta \approx (1 - \delta_0/\delta_{sc})^{-\delta sc}$

… can be inverted:

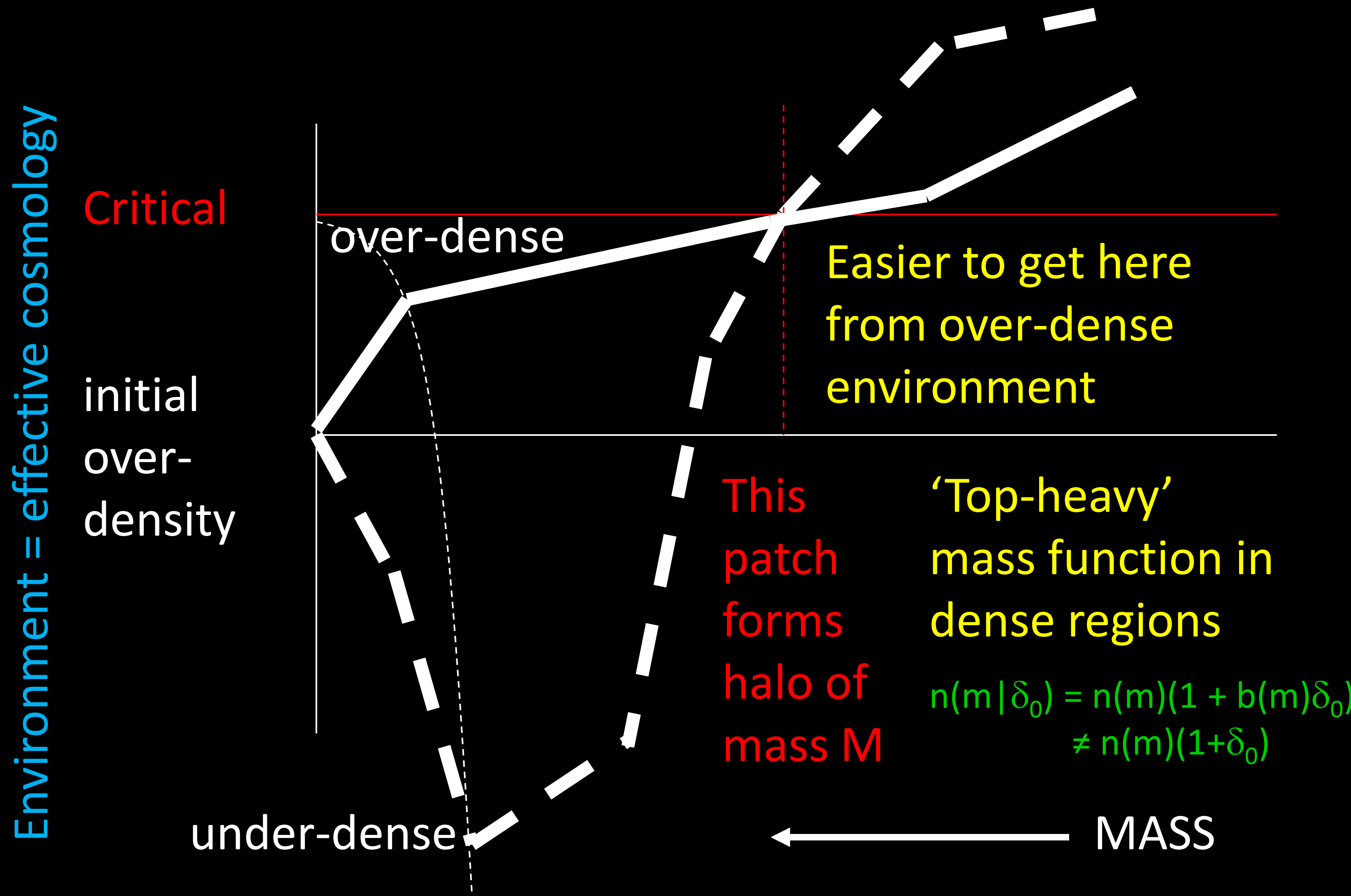
$(\delta_0/\delta_{sc}) \approx 1 - (M/\rho_{com}V)^{-1/\delta sc}$

N.B. For any V, there is a curve $\delta_0(M|V)$.

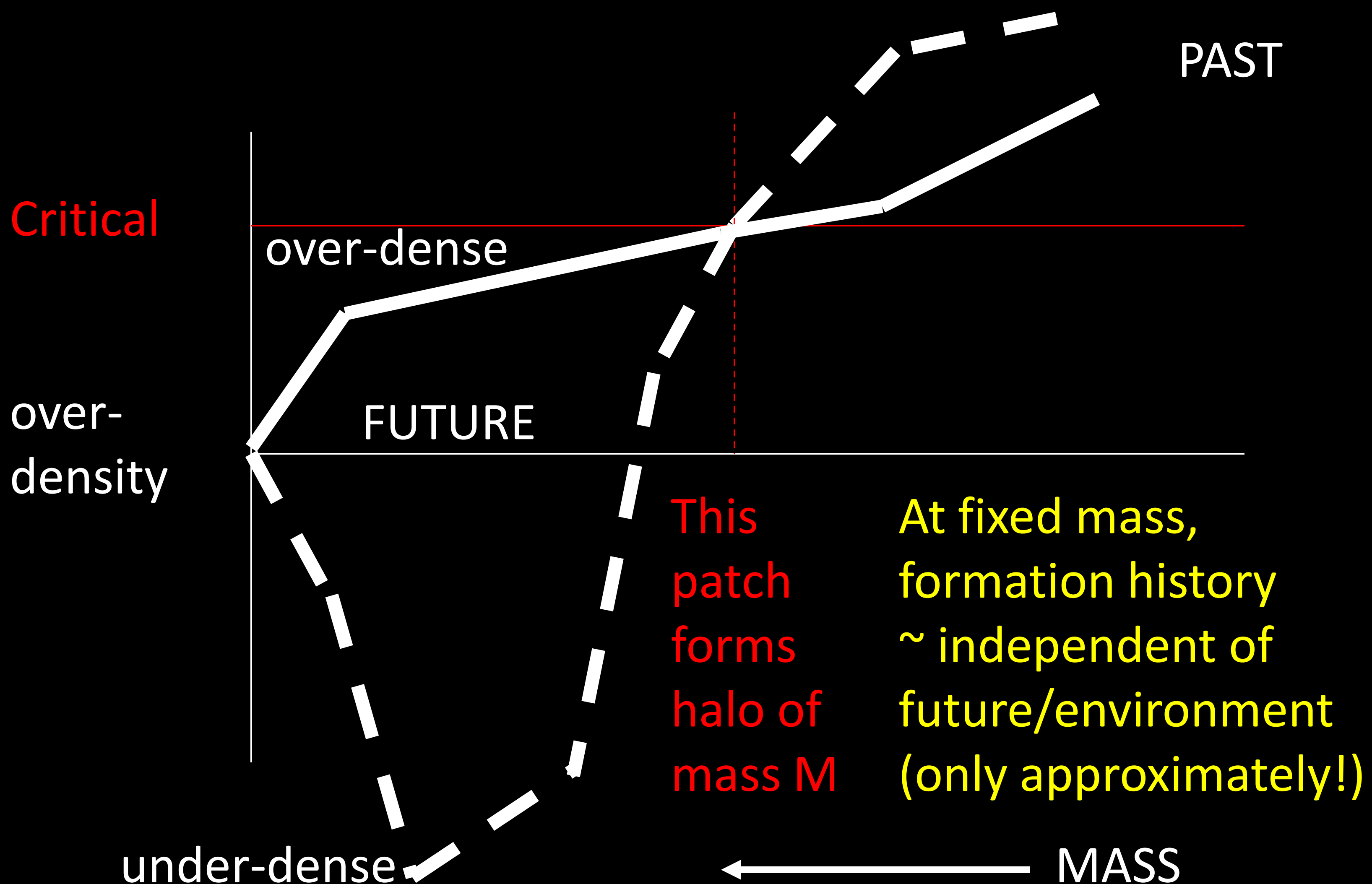
Moving barriers: The Nonlinear PDF



Correlations with environment



Correlations with environment



Large scale clustering/bias (from the peak-background split)

$$1 + \delta_h(\nu|\delta_0,S_0) = f(\nu|\delta_0,S_0)\ /f(\nu)$$
$$= 1 + b_1(\nu)\delta_0 + \ldots$$

- $b(\nu)$ directly from (derivatives of) $f(\nu)$ means halo abundances predict halo clustering
- $b(\nu)$ increases with ν

→ top-heavy mass function in dense regions:

$$n(m|\delta_0) = n(m)(1 + b(m)\delta_0 + \ldots) \neq n(m)(1+\delta_0)$$

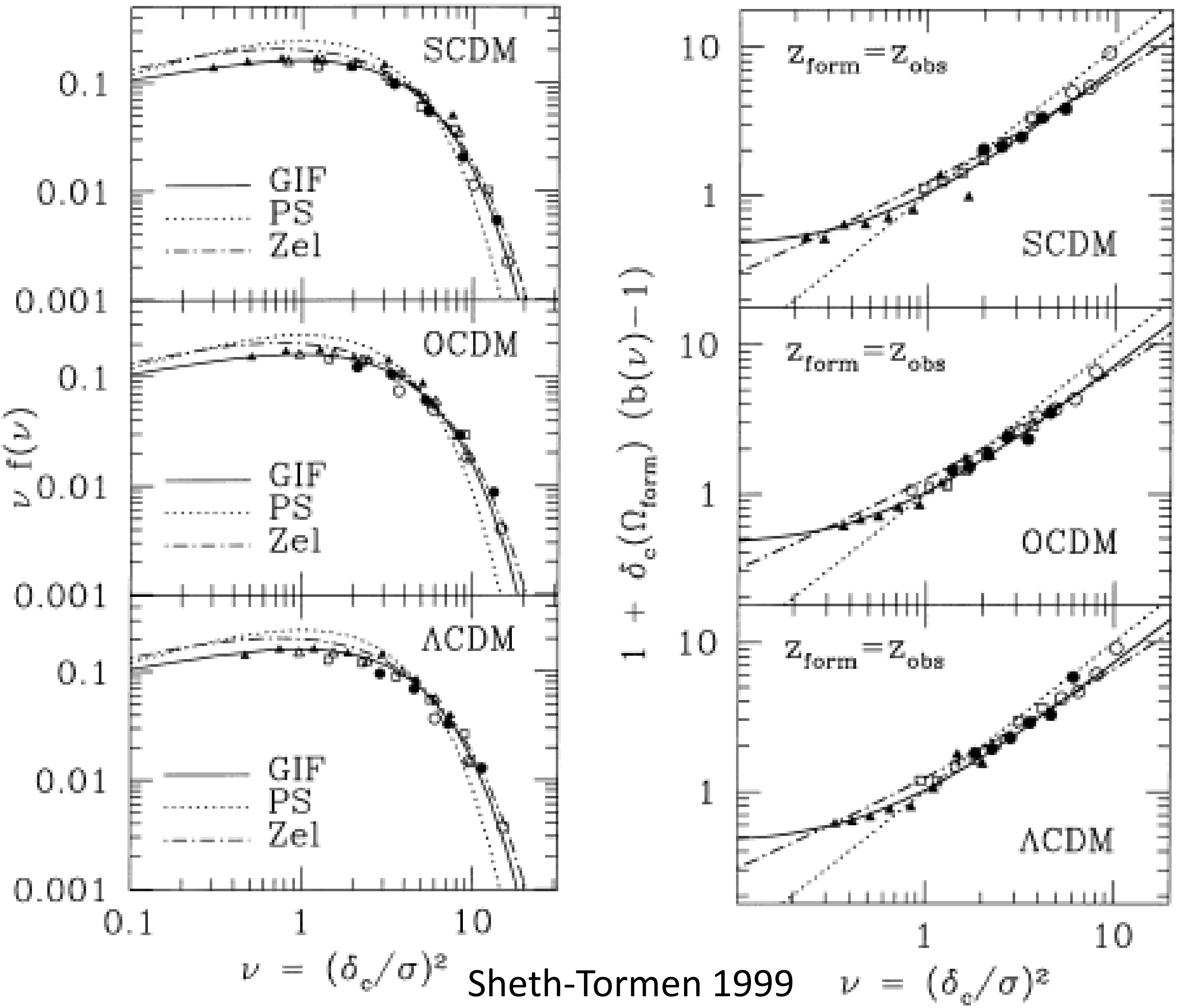
→ massive halos (i.e. larger ν) more clustered:

$$\langle\delta_h\delta_0\rangle = b_1(\nu)\ \langle\delta_0^2\rangle + \ldots$$

(Almost) universal mass function and halo bias

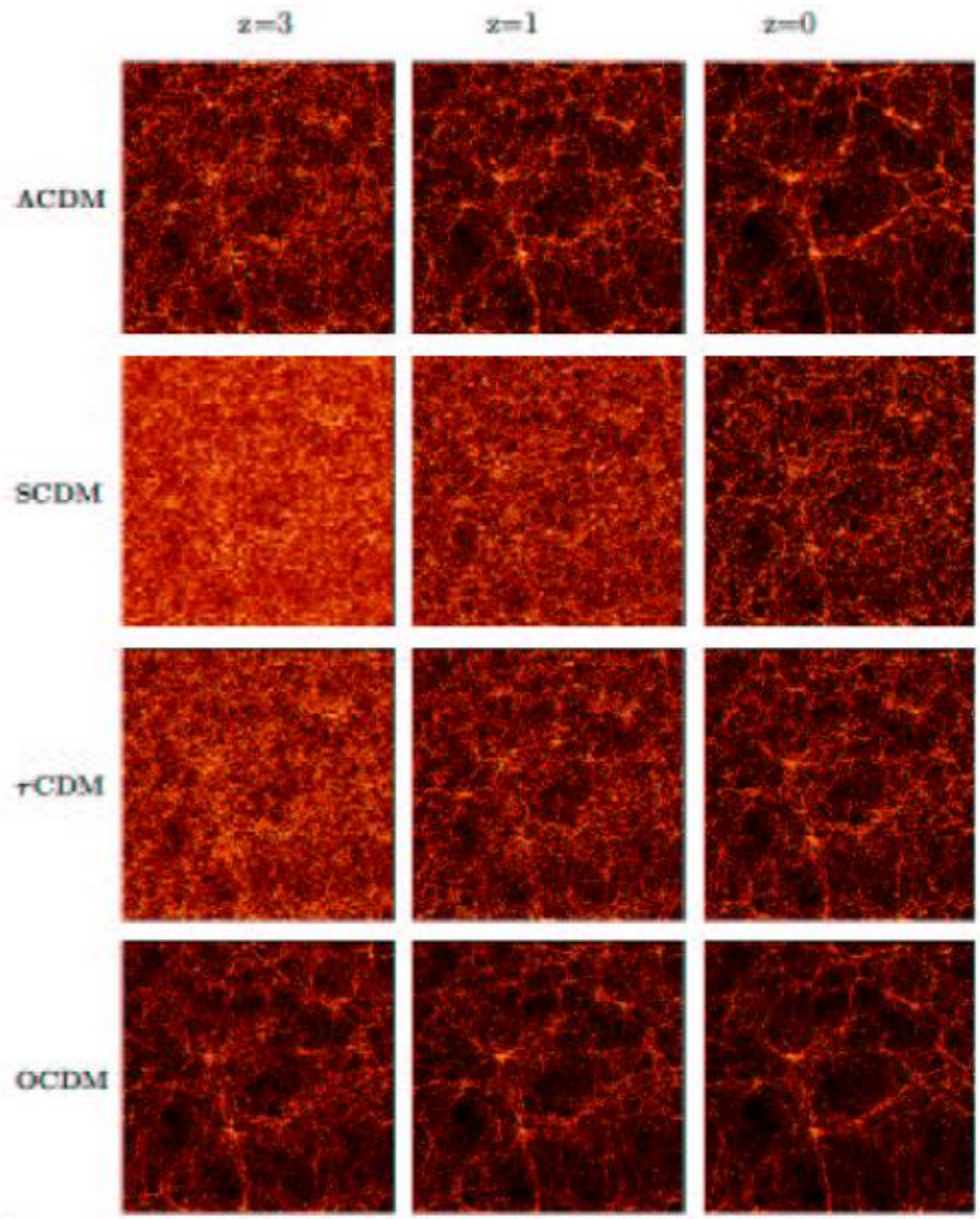
See Paranjape et al (2013) for recent progress in modeling this from first principles

See Castorina et al. (2014) for ν's

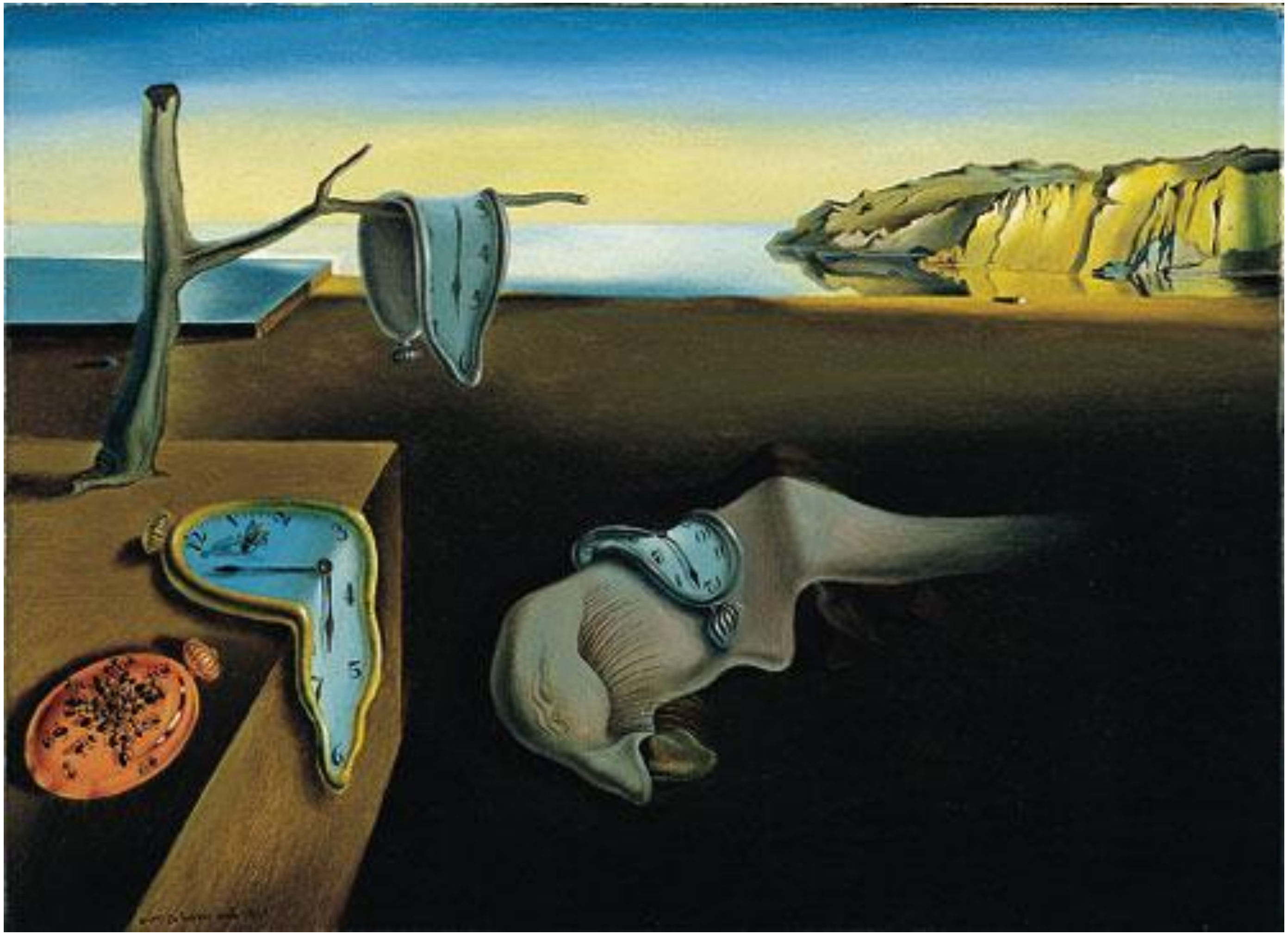


Sheth-Tormen 1999

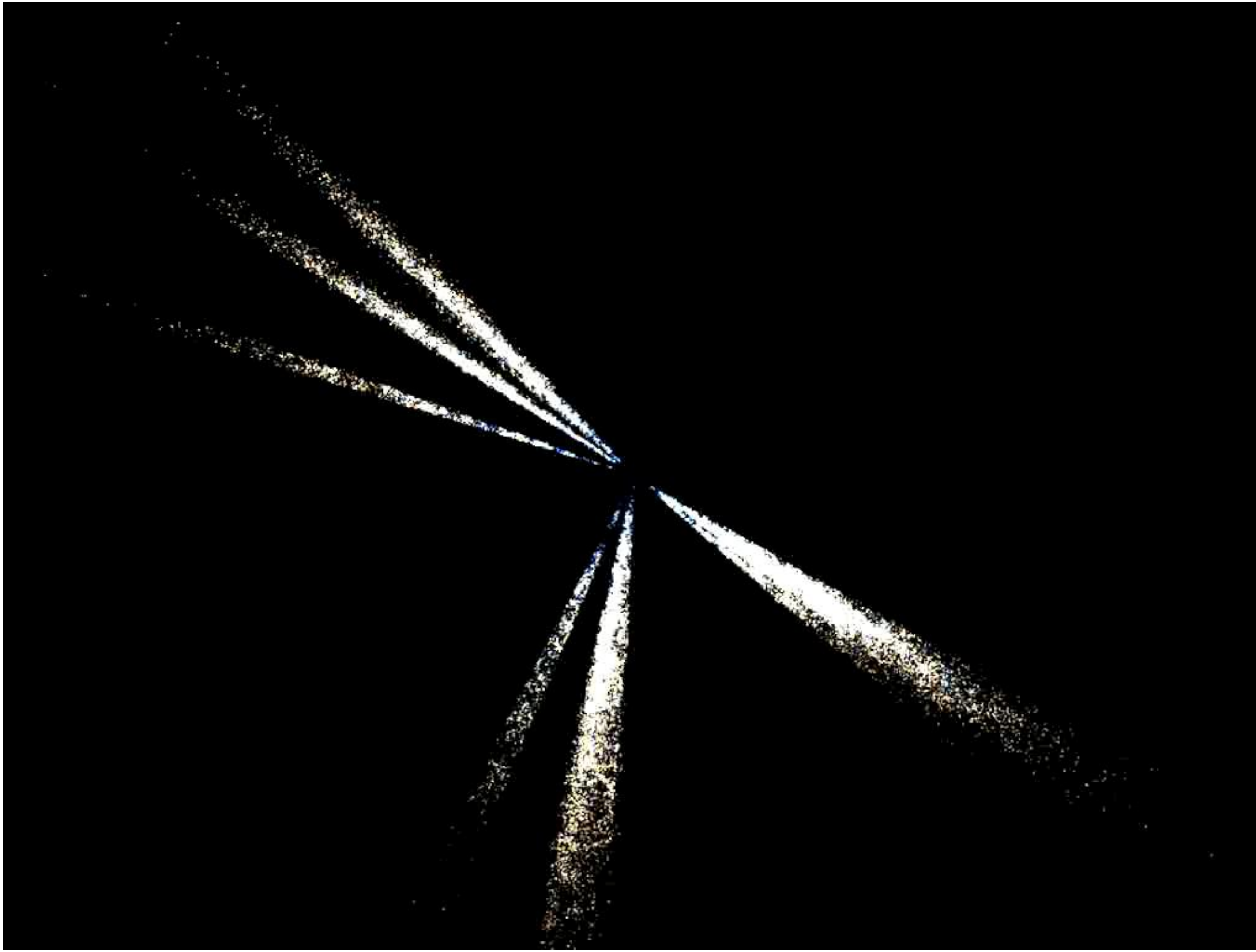
- Structure at a given time, and, more importantly, growth of structure, provides sharp constraints on models

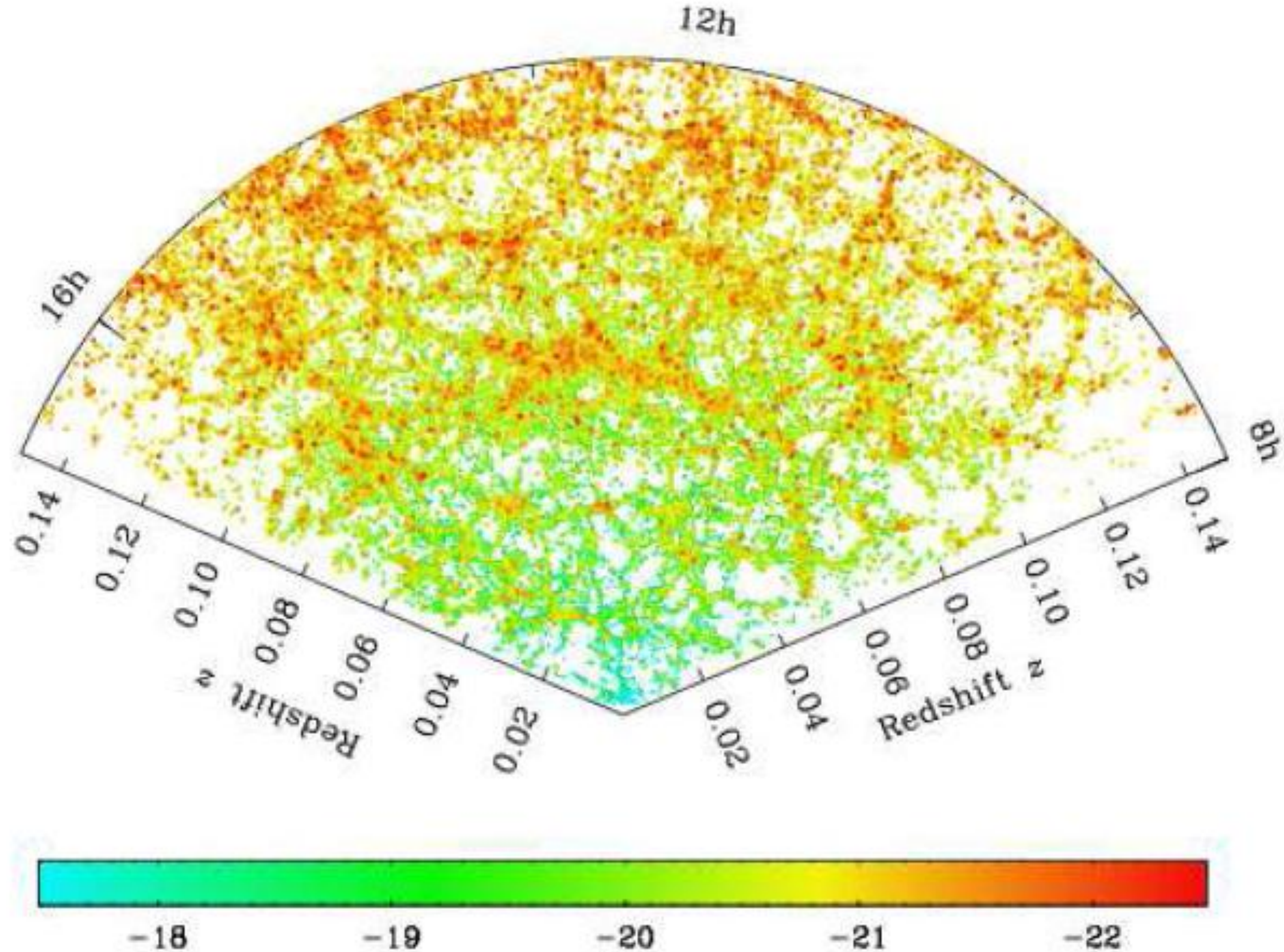


Hierarchical clustering in GR



= the persistence of memory

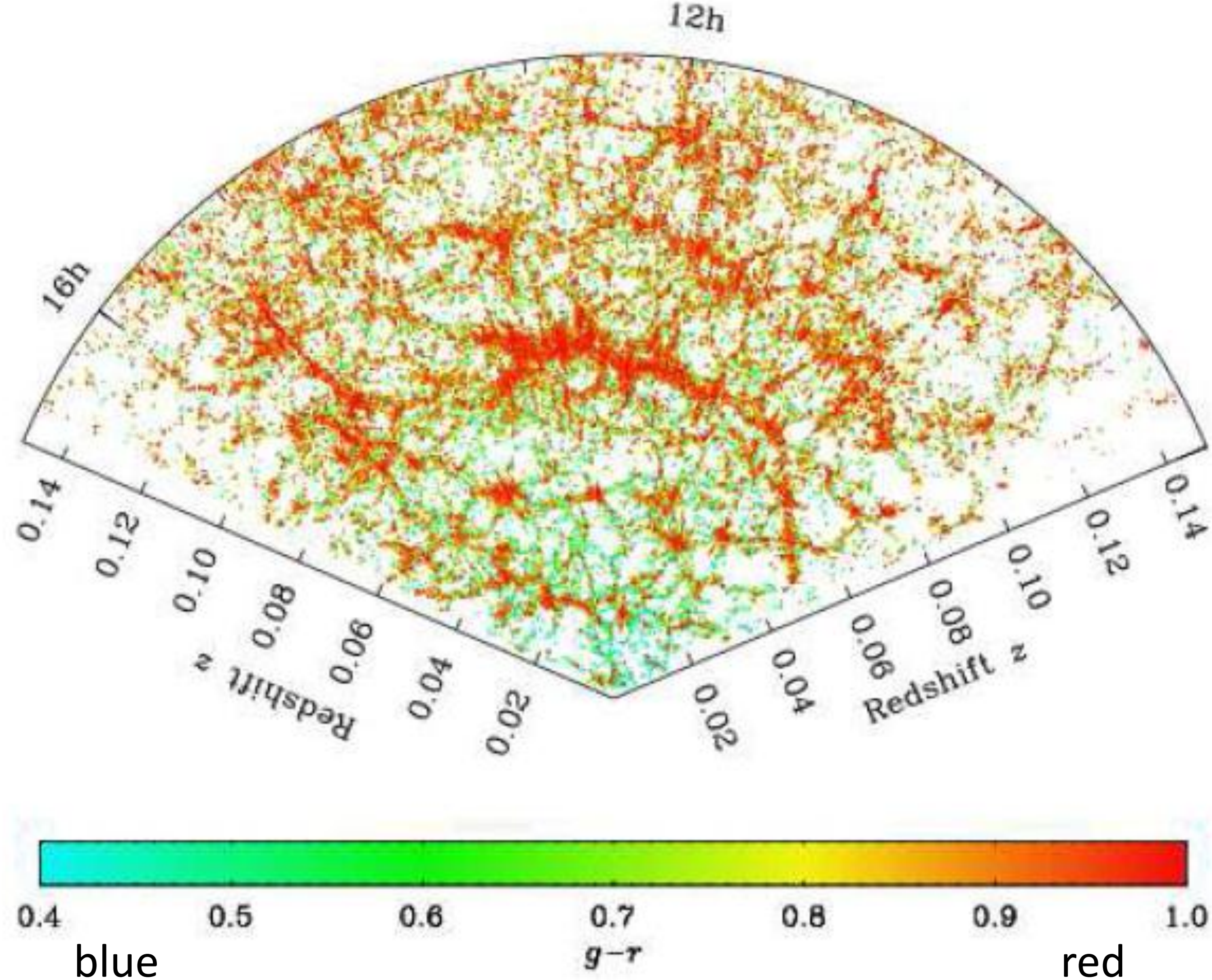




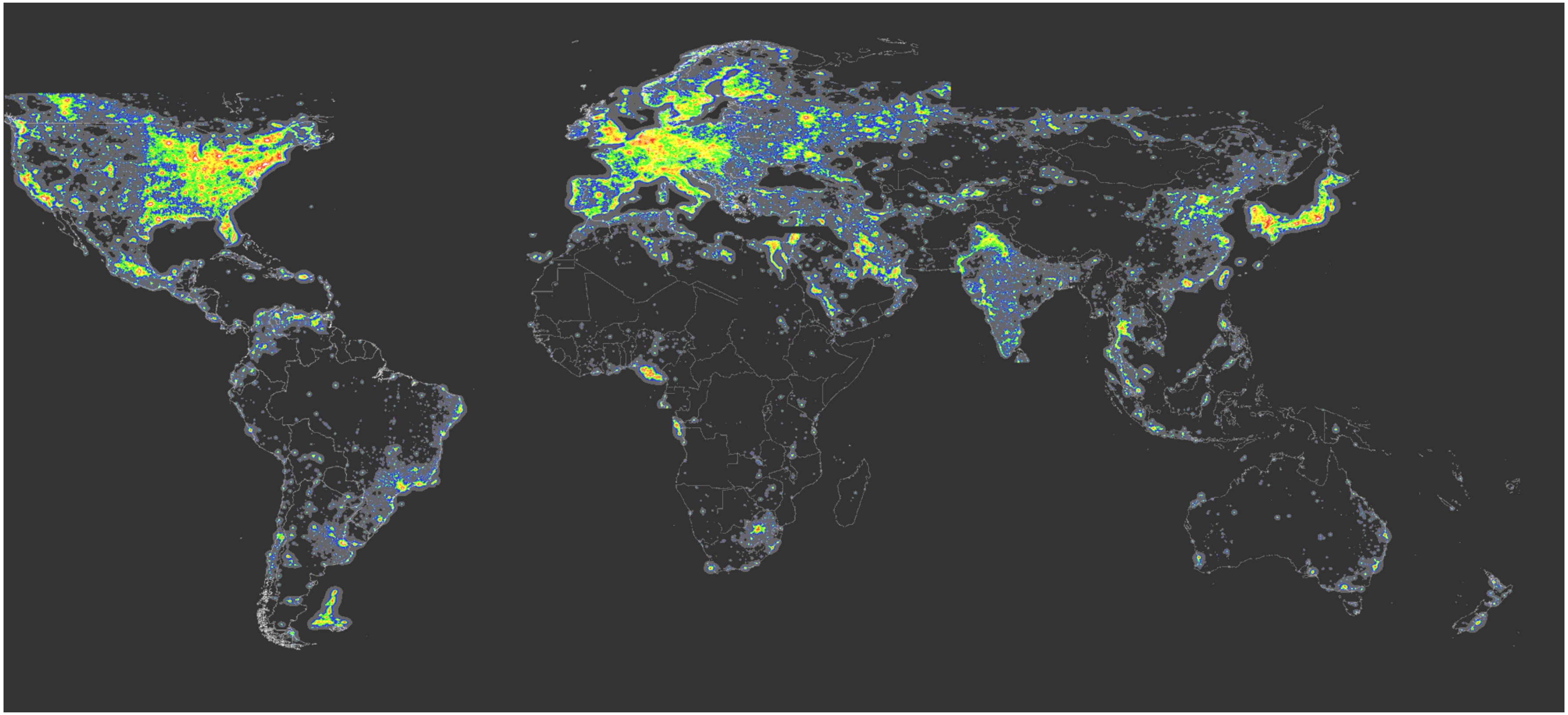
Not luminous

Luminous

Zehavi et al. 2010 (SDSS)


12h
16h
8h
0.14
0.12
0.10
0.08
0.06
0.04
0.02
Redshift z
0.02
0.04
0.06
0.08
0.10
0.12
0.14
Redshift z
0.4
0.5
0.6
0.7
0.8
0.9
1.0
g−r
blue
red

Complication: Light is a biased tracer



Not all galaxies are fair tracers of dark matter;
To use galaxies as probes of underlying dark matter distribution, must understand 'bias'

You can observe a lot just by watching

How to describe different point processes which are all built from the same underlying density field?

THE HALO MODEL

Review in Physics Reports (Cooray & Sheth 2002)

A THEORY OF THE SPATIAL DISTRIBUTION OF GALAXIES*

J. NEYMAN AND E. L. SCOTT
Statistical Laboratory, University of California
Received February 18, 1952

ABSTRACT

A theory of the spatial distribution of galaxies is built, based on the following four main assumptions: (i) galaxies occur only in clusters; (ii) the number of galaxies varies from cluster to cluster, subject to a probabilistic law; (iii) the distribution of galaxies within a cluster is also subject to a probabilistic law; and (iv) the distribution of cluster centers in space is subject to a probabilistic law described as quasi-uniform. The main result obtained is the joint probability generating function $G_{N_1, N_2}(t_1, t_2)$ of numbers N_1 and N_2 of galaxies visible on photographs from two arbitrarily placed regions ω_1 and ω_2, taken with fixed limiting magnitudes m_1 and m_2, respectively. The theory ignores the possibility of light-absorbing clouds. The function $G_{N_1, N_2}(t_1, t_2)$ is expressed in terms of four functions left unspecified, which govern the details of the structure contemplated. Methods are indicated whereby approximations to these functions can be obtained and whereby the general validity of the hypotheses can be tested.

Center-satellite process requires knowledge of how

1) halo abundance; 2) halo clustering; 3) halo profiles;

4) number of galaxies per halo; all depend on halo mass (+ ...)

(Revived, then discarded in 1970s by Peebles, McClelland & Silk)

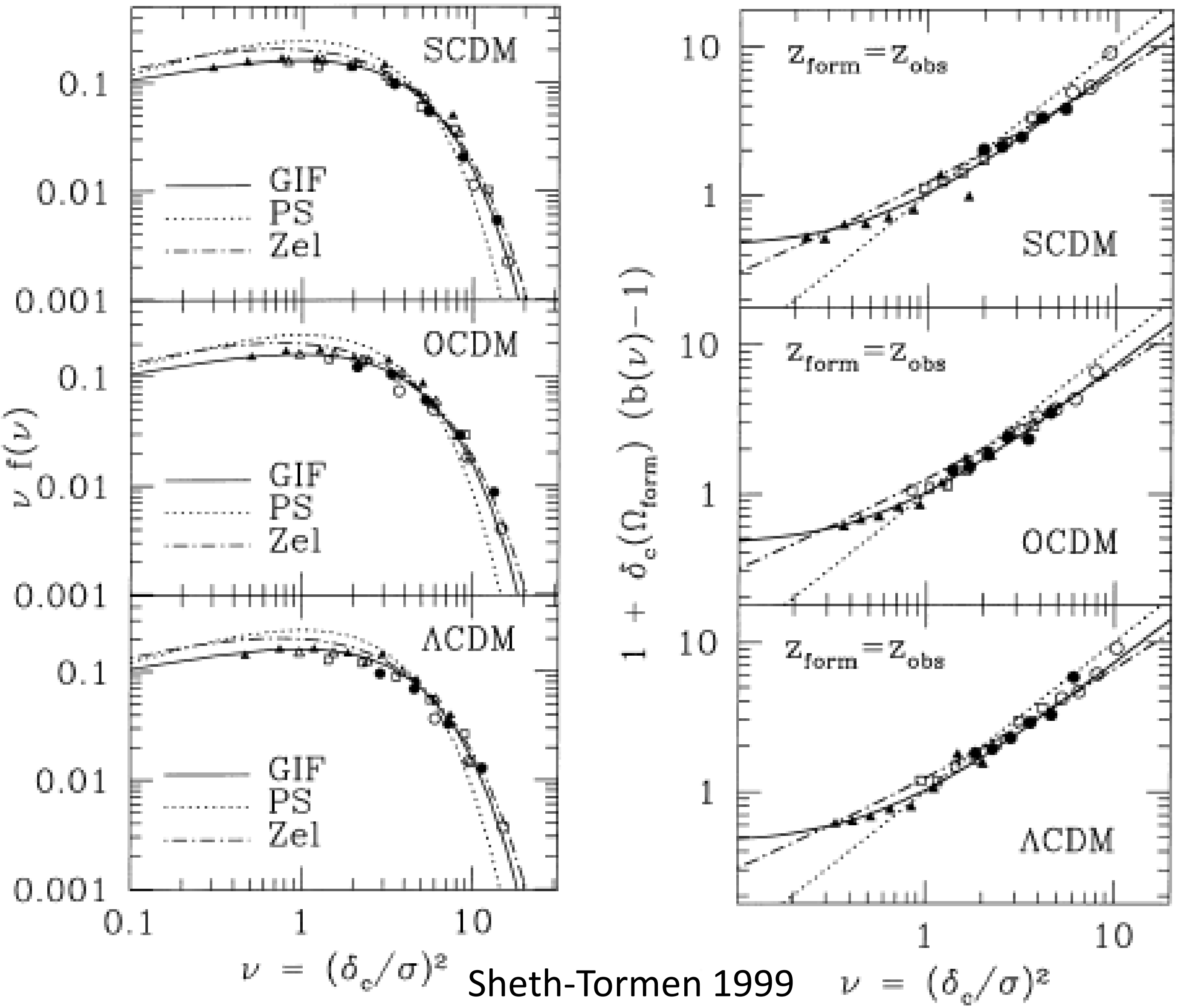
(Almost) universal mass function

$(m/\rho)\,(dn/d\ln\nu) = \nu f(\nu) = A\,[1 + (q\nu)^{-p}]\,\mathrm{sqrt}(q\nu/2\pi)\,\exp(-q\nu/2)$

where all $\nu = (\delta_c/\sigma)^2$ and A ensures integral over all ν is unity

and halo bias

$b(\nu) = 1 + d\ln f/d\delta_c$

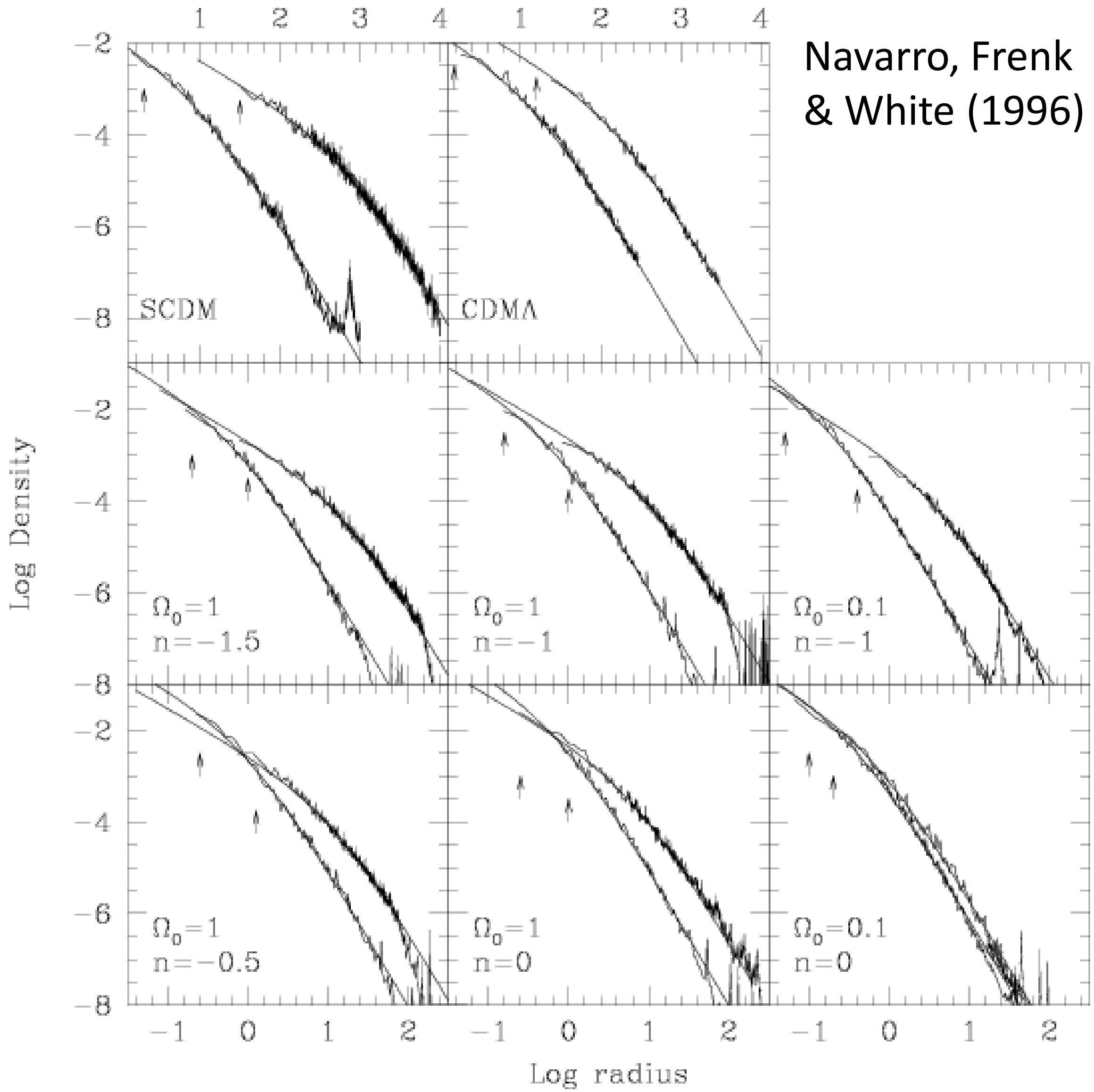


Sheth-Tormen 1999

Universal Halo Profiles

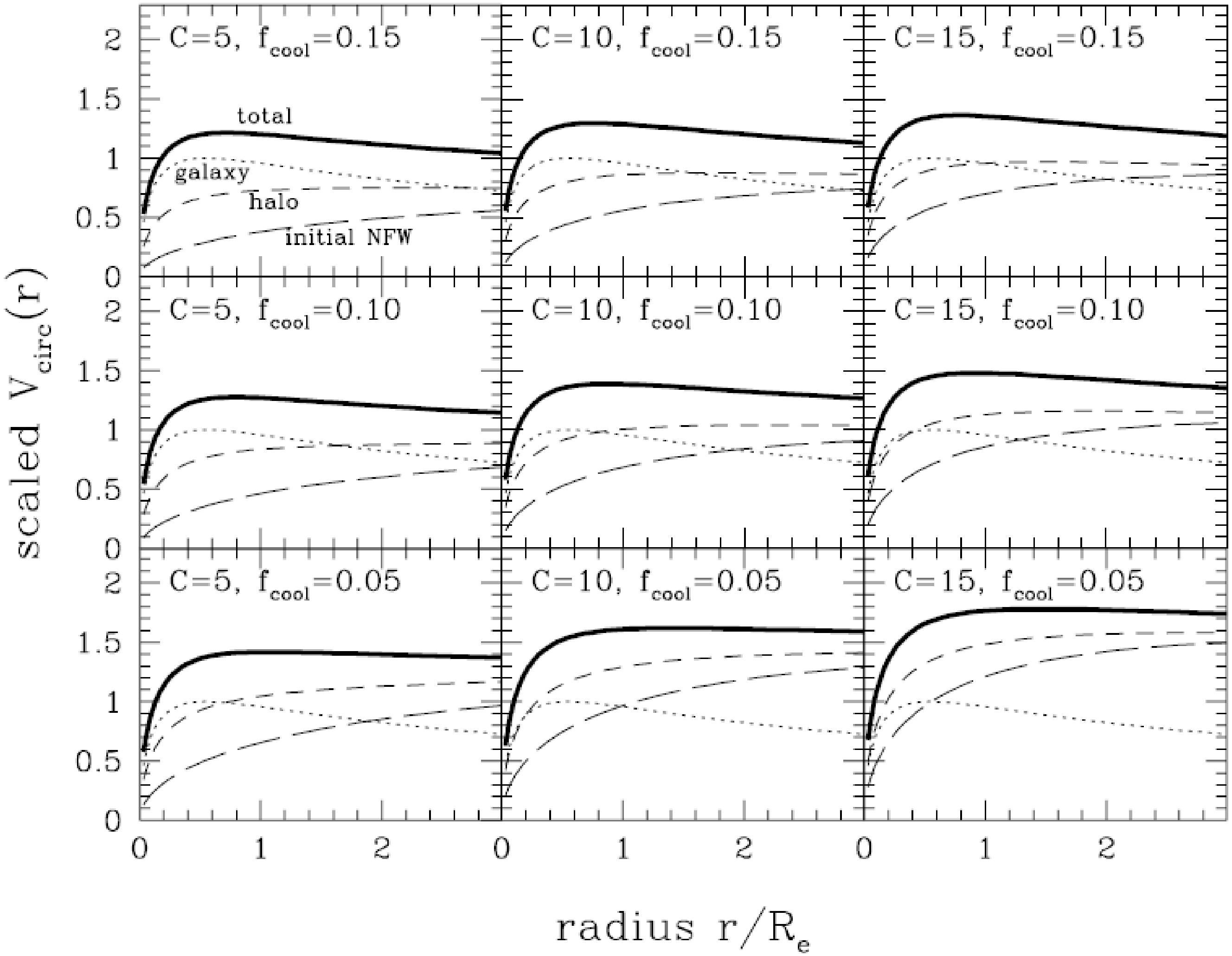
$\rho(r) = 4\rho_s/(r/r_s)/(1+ r/r_s)^2$

- Not quite isothermal
- Depend on halo mass, formation time
- Massive halos less concentrated (partially built-in from GRF initial conditions)
- Distribution of shapes (axis-ratios) known (Jing & Suto 2001)



Navarro, Frenk & White (1996)

Baryonic effects on the profile: Adiabatic contraction



Adiabatic contraction …

$$r\,[M_g(<r) + M_{dm}(<r)] = r_i\, M_{g+dm}(<r_i)$$

- Dark matter initially within r_i and now within r is

$$M_{dm}(<r) = (1 - f_g)\, M_{tot}(<r_i)$$

- Circular velocity from

$$V_{circ}^2(r) = GM(<r)/r = (r_i\,/r)^2\, GM_{g+dm}(<r_i)/r_i$$

$$V_{circ}(r) = (r_i\,/r)\, V_{circ}(r_i)$$

- In general, solve numerically. But, for (realistic) Hernquist galaxy $M_g(<r) = M_g\,(r/s_g)^2/(1+r/s_g)^2$ result is analytic:

$$f_g\, r^3 + (r+s_g)^2\,[(1-f_g)\, r - r_i]\, m_{g+dm}(r_i) = 0$$

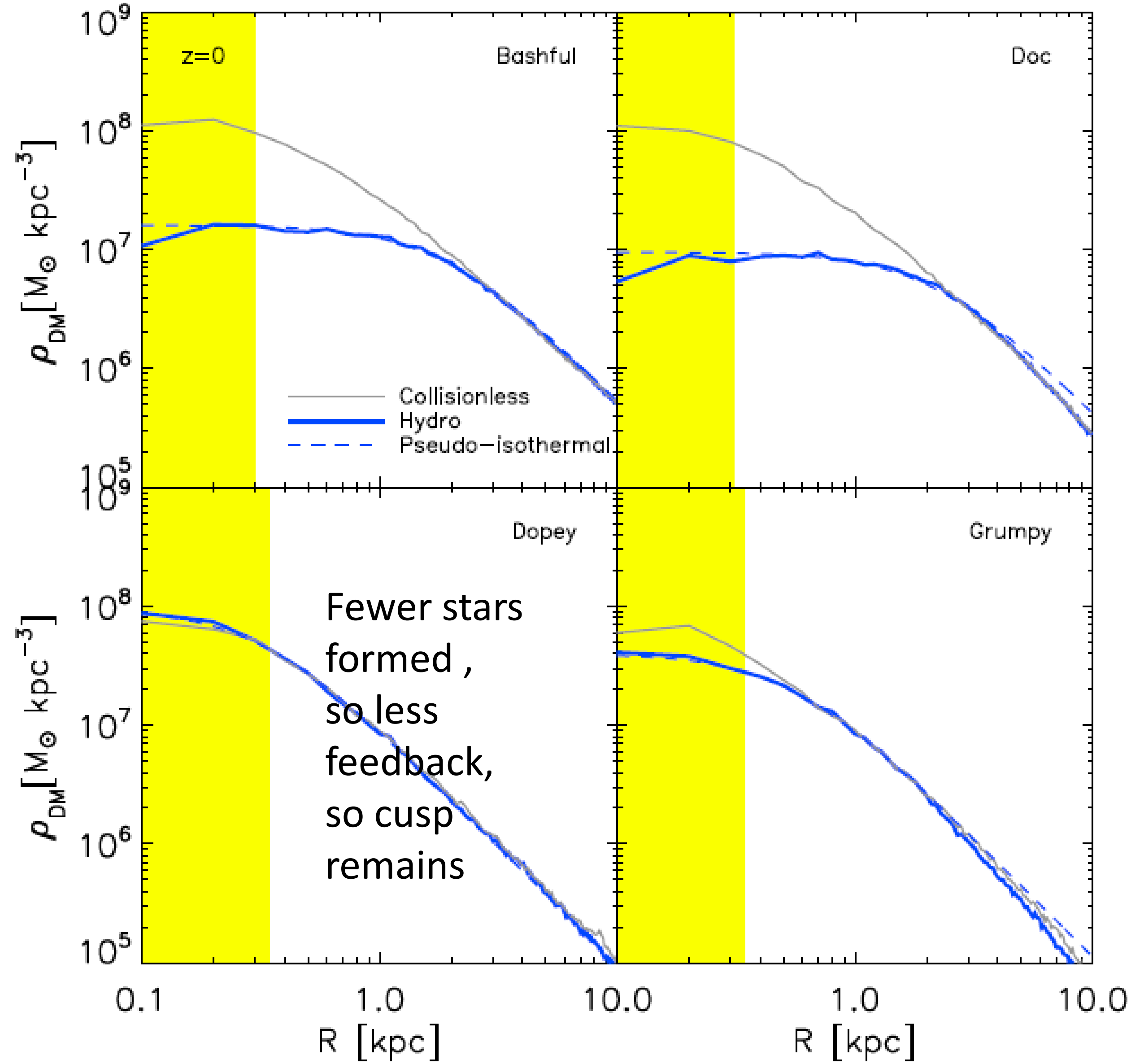
where $f_g = M_g/M_{tot}$. Get r by solving the cubic (Keeton 2001).

… increases circular velocities.

Inclusion of star formation feedback related effects can heat (expand) the gas, thus the dark matter as well: remove the cusp

Binding energy is $M(GM/R) \sim M^{5/3}$ so removal of cusp easier at low mass

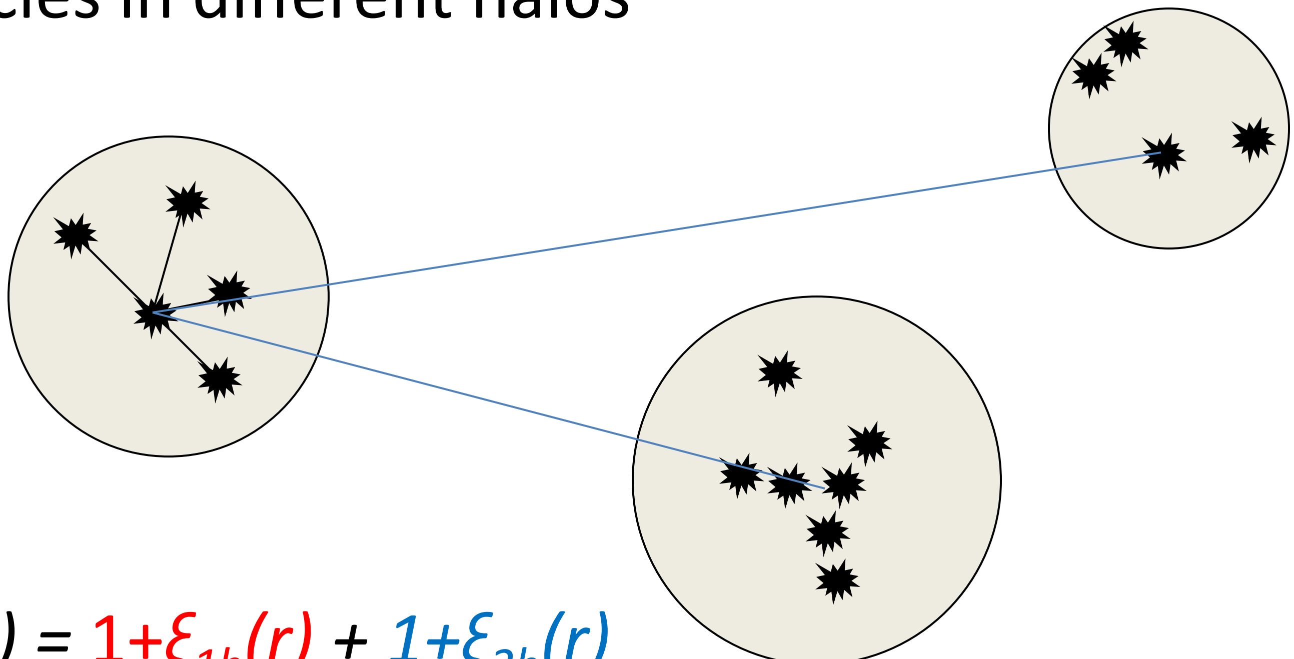
What remains has smaller V_{circ}, thus resolving the too-big-to-fail problem with <u>no</u> new physics



Madau et al. 2014

The halo-model of clustering

- Two types of pairs: both particles in same halo, or particles in different halos



- $1+\xi(r) = 1+\xi_{1h}(r) + 1+\xi_{2h}(r)$
- *All* physics can be decomposed similarly: 'nonlinear' effects from within halo, 'linear' from outside

The dark-matter correlation function

$$\xi_{dm}(r) = \xi_{1h}(r) + \xi_{2h}(r)$$

- $\xi_{1h}(r) \sim \int dm\, n(m)\, m^2\, \xi_{dm}(r|m)/\rho^2$
- $n(m)$: comoving number density of m-halos
- Comoving mass density: $\rho = \int dm\, n(m)\, m$
- $\xi_{dm}(r|m)$: fraction of total pairs, m^2, in an m-halo which have separation r; depends on (convolution of) density profile within m-halos
- This term only matters on scales smaller than the virial radius of a typical M_* halo (~ Mpc)
  - Need not know spatial distribution of halos!

$$\xi_{dm}(r) = \xi_{1h}(r) + \xi_{2h}(r)$$

- $\xi_{2h}(r) \approx \int dm_1 \, \frac{m_1 n(m_1)}{\rho} \int dm_2 \, \frac{m_2 n(m_2)}{\rho} \, \xi_{2h}(r|m_1,m_2)$
- Two-halo term dominates on large scales, where peak-background split estimate of halo clustering should be accurate: $\delta_h \sim b(m)\,\delta_{dm}$
- $\xi_{2h}(r|m_1,m_2) \sim \langle\delta_h^2\rangle \sim b(m_1)b(m_2)\,\langle\delta_{dm}^2\rangle$
- $\xi_{2h}(r) \approx [\int dm \; mn(m)\; b(m)/\rho]^2 \; \xi_{dm}(r)$
- On large scales, linear theory is accurate: $\xi_{dm}(r) \approx \xi_{Lin}(r)$ so $\xi_{2h}(r) \approx b_{eff}^2 \; \xi_{Lin}(r)$

Dark matter power spectrum

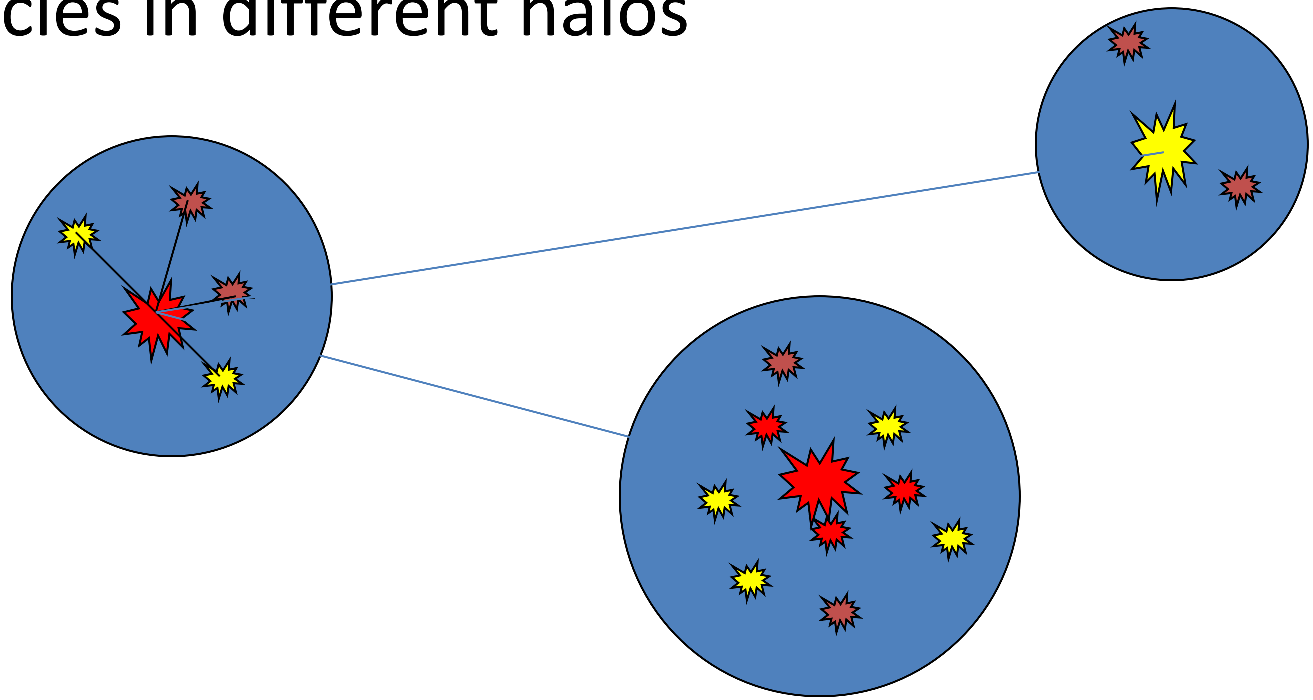
- Convolutions in real space are products in k-space, so $P(k)$ is easier than $\xi_{1h}(r)$

$$P(k) = P_{1h}(k) + P_{2h}(k)$$

- $P_{1h}(k) = \int dm\, n(m)\, m^2\, |u_{dm}(k|m)|^2/\rho^2$
- $P_{2h}(k) \approx [\int dm\, n(m)\, b(m)\, m\, u_{dm}(k|m)/\rho]^2\, P_{dm}(k)$

The halo-model of galaxy clustering

- Two types of particles: central + 'satellite'
- Two types of pairs: both particles in same halo, or particles in different halos



- $1+\xi_{obs}(r) = 1+\xi_{1h}(r) + 1+\xi_{2h}(r)$

$$1+\xi_{1h}(r) = 1+\xi_{cs}(r) + 1+\xi_{ss}(r)$$

The halo-model of galaxy clustering

- Write as sum of two components:
  - $1+\xi_{1gal}(r) = \int dm\, n(m)\, g_2(m)\, \xi_{dm}(m|r)/\rho_{gal}^2$
  - $\xi_{2gal}(r) \approx [\int dm\, n(m)\, g_1(m)\, b(m)/\rho_{gal}]^2\, \xi_{dm}(r)$
  - $\rho_{gal} = \int dm\, n(m)\, g_1(m)$: number density of galaxies
  - $\xi_{dm}(m|r)$: fraction of pairs in m-halos at separation *r*

- Think of mean number of galaxies, $g_1(m)$, as a weight applied to each dark matter halo
  - Galaxies 'biased' if $g_1(m)$ not proportional to m, …, $g_n(m)$ not proportional to m^n (Jing, Mo & Boerner 1998; Benson et al. 2000; Peacock & Smith 2000; Seljak 2000; Scoccimarro et al. 2001)
  - Central + Poisson satellites model works well
- Similarly, Y_{SZ} or T_X are just a weight applied to halos, so same formalism can model cluster clustering

The halo-model of galaxy clustering

- Write as sum of two components:
  - $1+\xi_{1gal}(r) = \int dm\, n(m)\, g_2(m)\, \xi_{dm}(m|r)/\rho_{gal}^2$
  - $\xi_{2gal}(r) \approx [\int dm\, n(m)\, g_1(m)\, b(m)/\rho_{gal}]^2\, \xi_{dm}(r)$
  - $\rho_{gal} = \int dm\, n(m)\, g_1(m)$: number density of galaxies
  - $\xi_{dm}(m|r)$: fraction of pairs in m-halos at separation *r*

- Handle 'assembly bias' easily by treating m as vector (m, c, spin, …)
  - See Musso et al. (2012, 2014), Dalal et al. (2008)
  - Statements that halo model cannot treat this bias are based on common but NOT essential assumption that m = halo mass only

Power spectrum

- Convolutions in real space are products in k-space, so P(k) is easier than ξ(r):

$$P(k) = P_{1h}(k) + P_{2h}(k)$$

- $P_{1h}(k) = \int dm\, n(m)\, g_2(m)\, |u_{dm}(k|m)|^2/\rho^2$
- $P_{2h}(k) \approx [\int dm\, n(m)\, b(m)\, g_1(m)\, u_{dm}(k|m)/\rho]^2\, P_{dm}(k)$

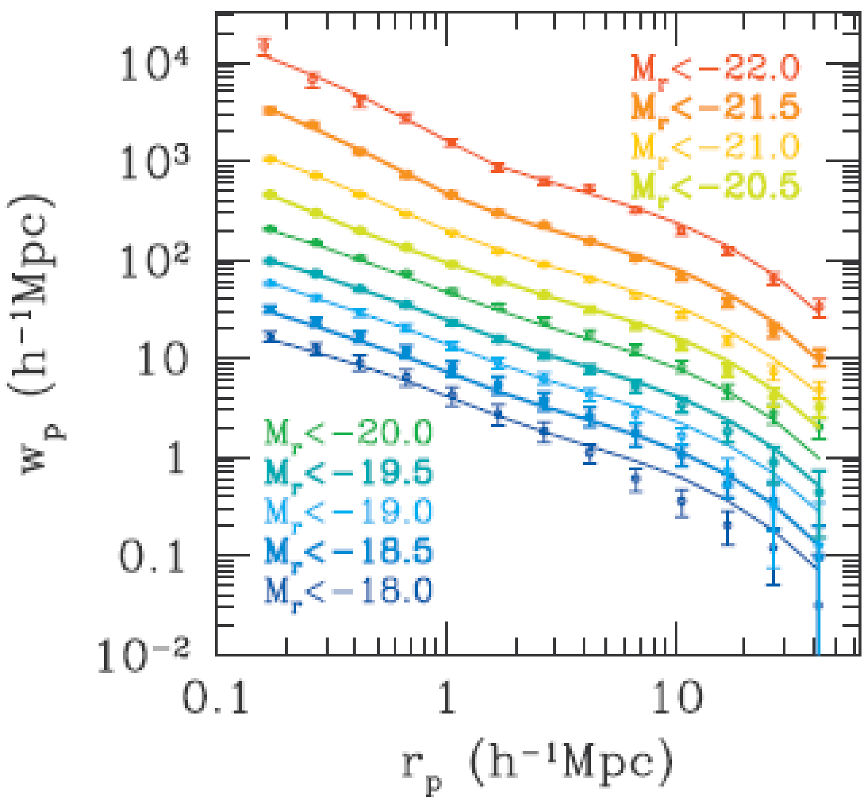
Bells and whistles (which matter for CDM→WDM)

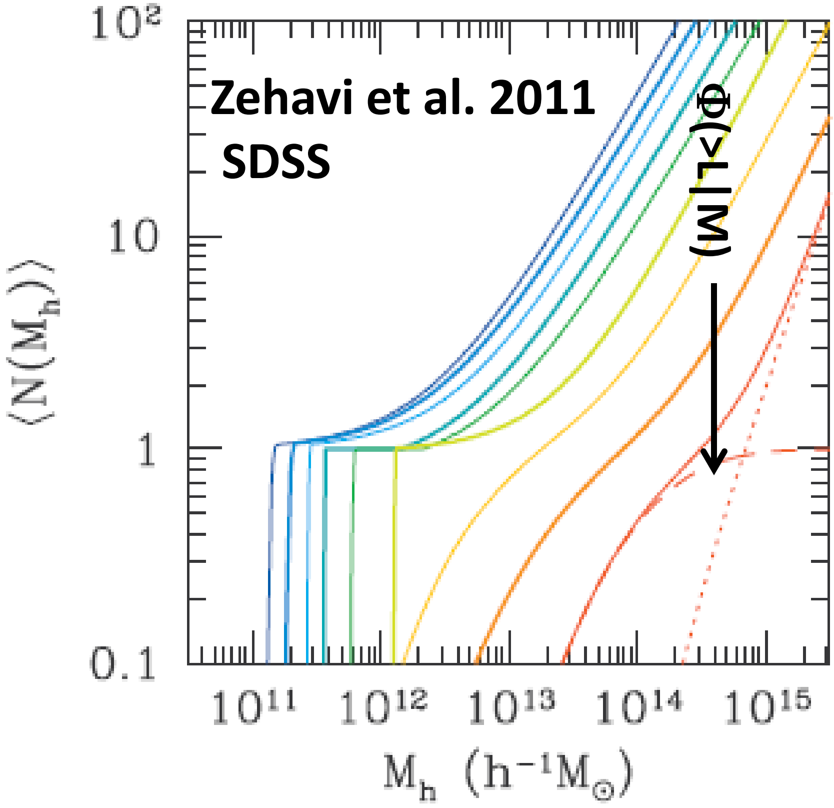
- Mass-concentration and scatter
  - Different profiles for red vs blue
- Distribution of halo shapes
  - Correlation of shapes with surrounding large scale structure
  - Projection effects matter for conc-m relation!
- Substructure = galaxies? Correlations with concentration/formation, time/environment
  - Correlation of substructure with large scale structure

Halo Model: HOD, CLF, SHAM

- Goal is to infer p(N|m) from measurements of abundance and clustering
  - Abundance constrains $\langle N|m\rangle = g_1(m)$
  - 1-halo term of n-pt clustering constrains $g_n(m)$
- HOD uses abundance and 2pt statistics to constrain p(N|m) from different samples (Zehavi et al. 2011; Skibba et al. 2014)
- CLF now does too, to constrain $\phi(L|m)$ (Lu et al. 2014)
- Since $\langle N(>L)|m\rangle = \phi(>L|m)$, HOD~CLF but with different systematics
- SHAM uses abundance only, but gets 2pt stats quite well anyway (Moster et al. 2013)
  - Problematic for color selected samples

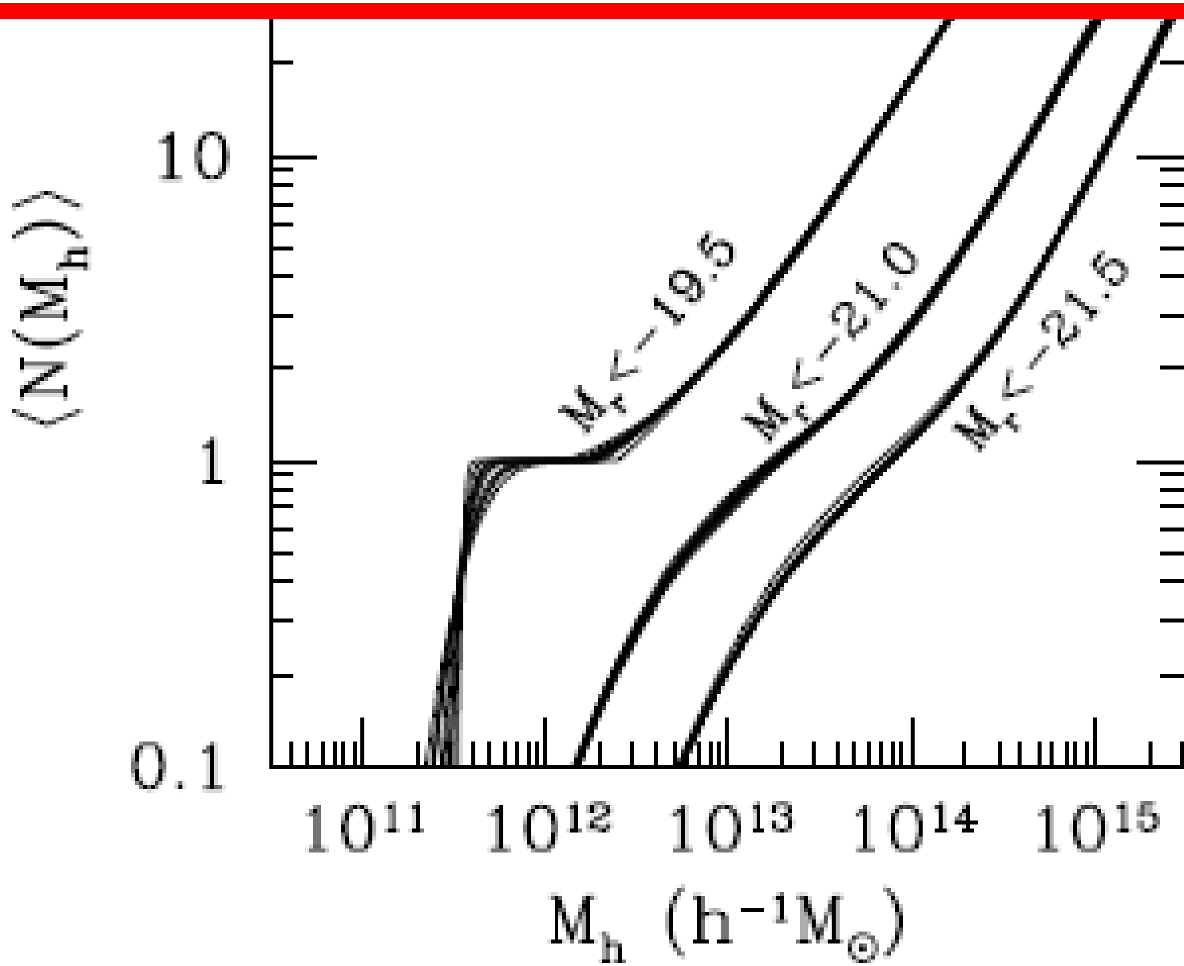
Luminosity dependence of clustering



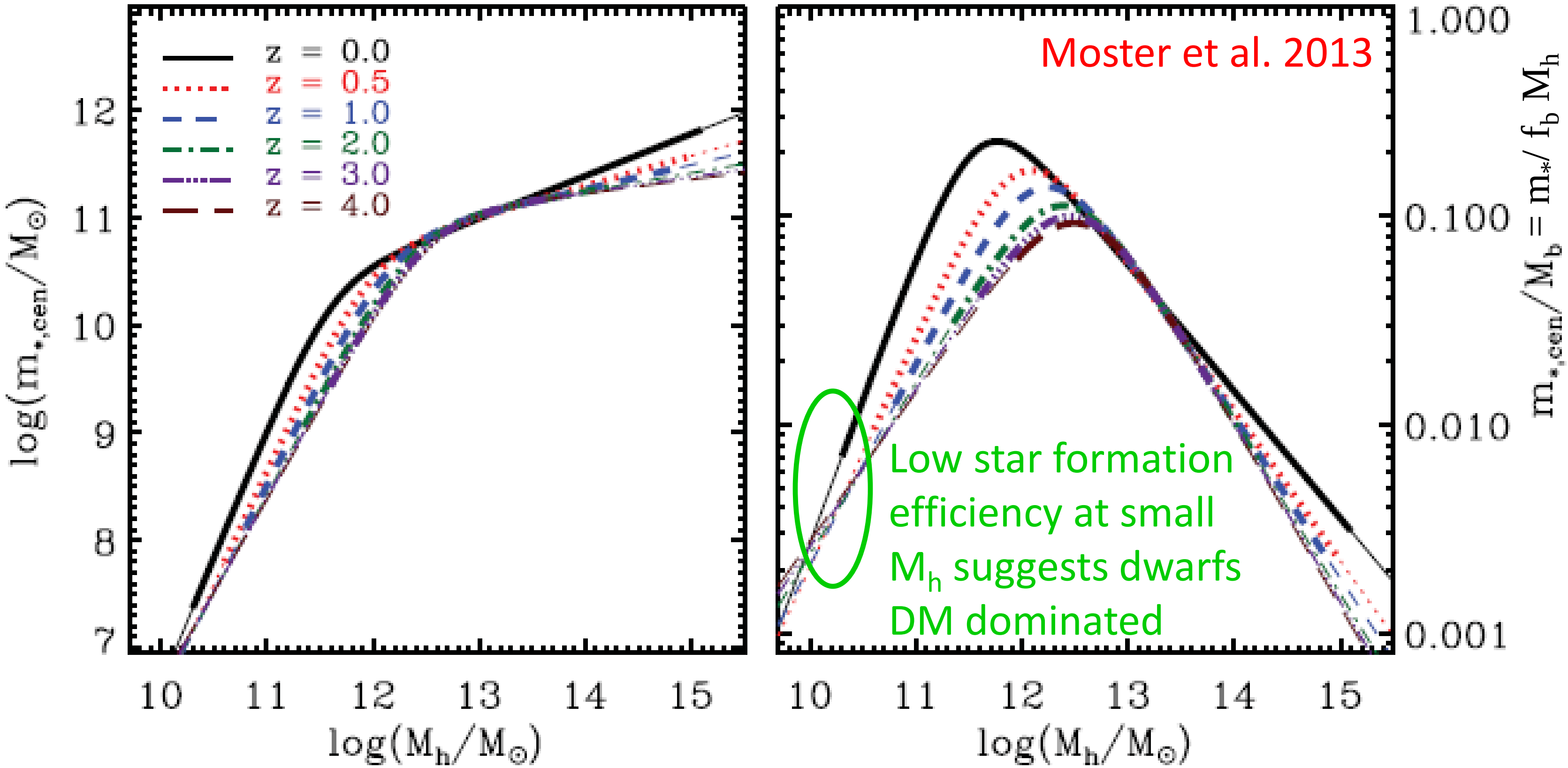


$$<N_{gal}|m> = f_{cen}(m)\ [1 + <N_{sat}|m>]$$

$$\langle N(M_h)\rangle = \frac{1}{2}\left[1+\mathrm{erf}\left(\frac{\log M_h - \log M_{\rm min}}{\sigma_{\log M}}\right)\right]\left[1+\left(\frac{M_h - M_0}{M_1'}\right)^{\alpha}\right], \tag{6}$$

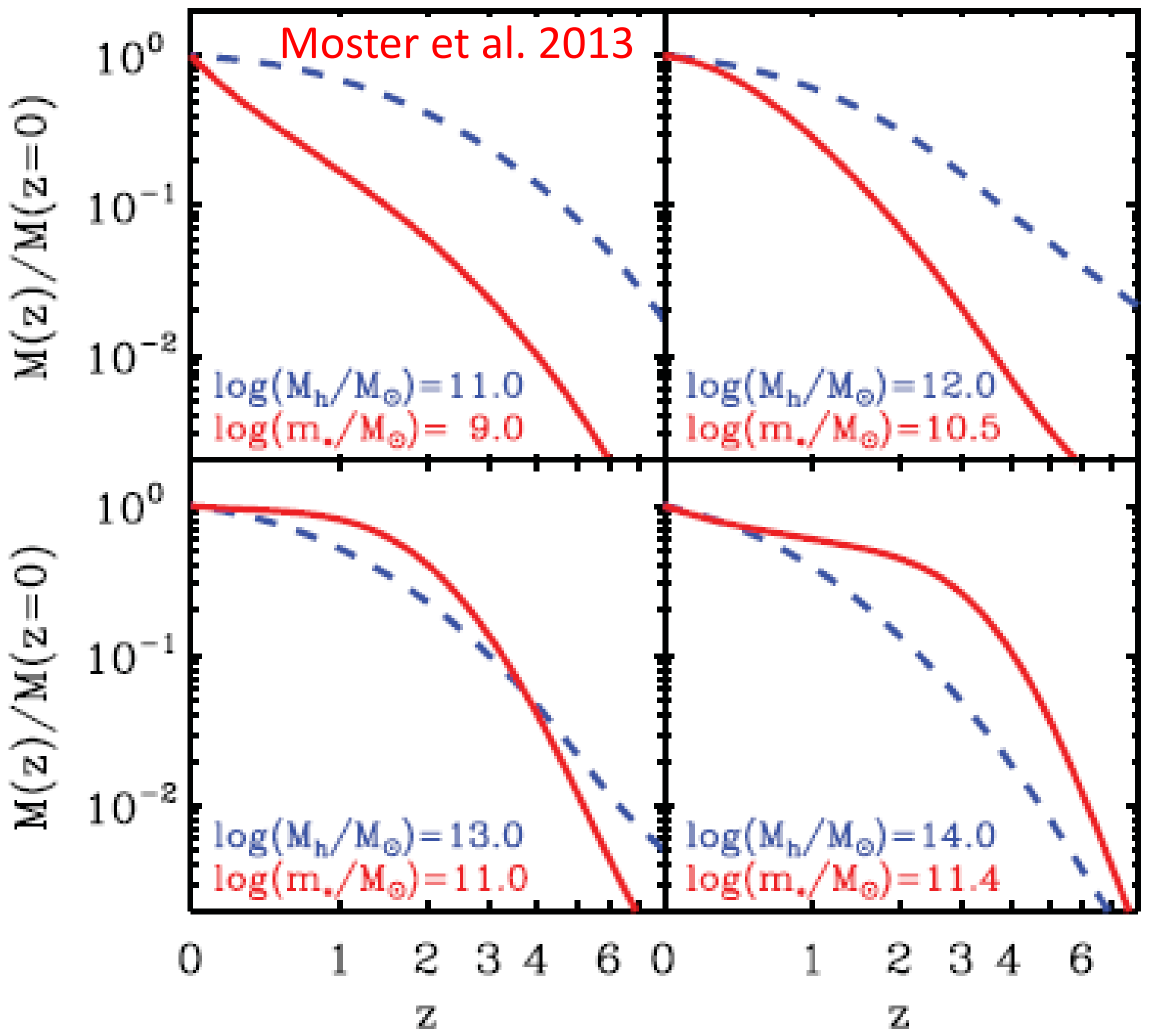


From ϕ(L|M) or ϕ(M*|M) can determine <M*|M >; i.e. star formation efficiency as function of halo mass



Knowing $\langle M_* | M_h \rangle$ at each z yields estimates of $SFR(M_h,z)$ for the population (i.e., not object by object)

- Knowing M_*-M_h at each z yields $M_*(z)$ given $M_*(0)$ and $M_h(0)$
- Since $M_h(z)$ also known, can compare growth in situ vs mergers
- Hence, can deduce SFR(M_h,z) for the population (but not object by object)
- Clustering also predicted - OK



This is a very active field:

This is a very active field:

Nobody goes there anymore – it’s too crowded

The other half of phase-space: Velocities

Just as statistics can be split into two regimes, so too can the physics: linear + nonlinear

'Infall' velocities from spherical model

$$(R_{initial}/R_t)^3 = \text{Mass}/(\rho_{com}\text{Volume})$$

$$= 1 + \delta \approx (1 - \delta_t/\delta_{sc})^{-\delta sc}$$

$$R(t)/R_{initial} \approx (1 - D(t)\, \delta_{initial}/\delta_{sc})^{\delta sc/3}$$

Now use $v(t) = dR(t)/dt$ so

$$v(t)/HR = (d\ln R/dt)/(d\ln a/dt)$$

$$\approx (d\ln D/d\ln a)\, D(t)\delta_{initial}/3$$

$$\approx \Omega^{0.56}\, D(t)\delta_{initial}/3$$

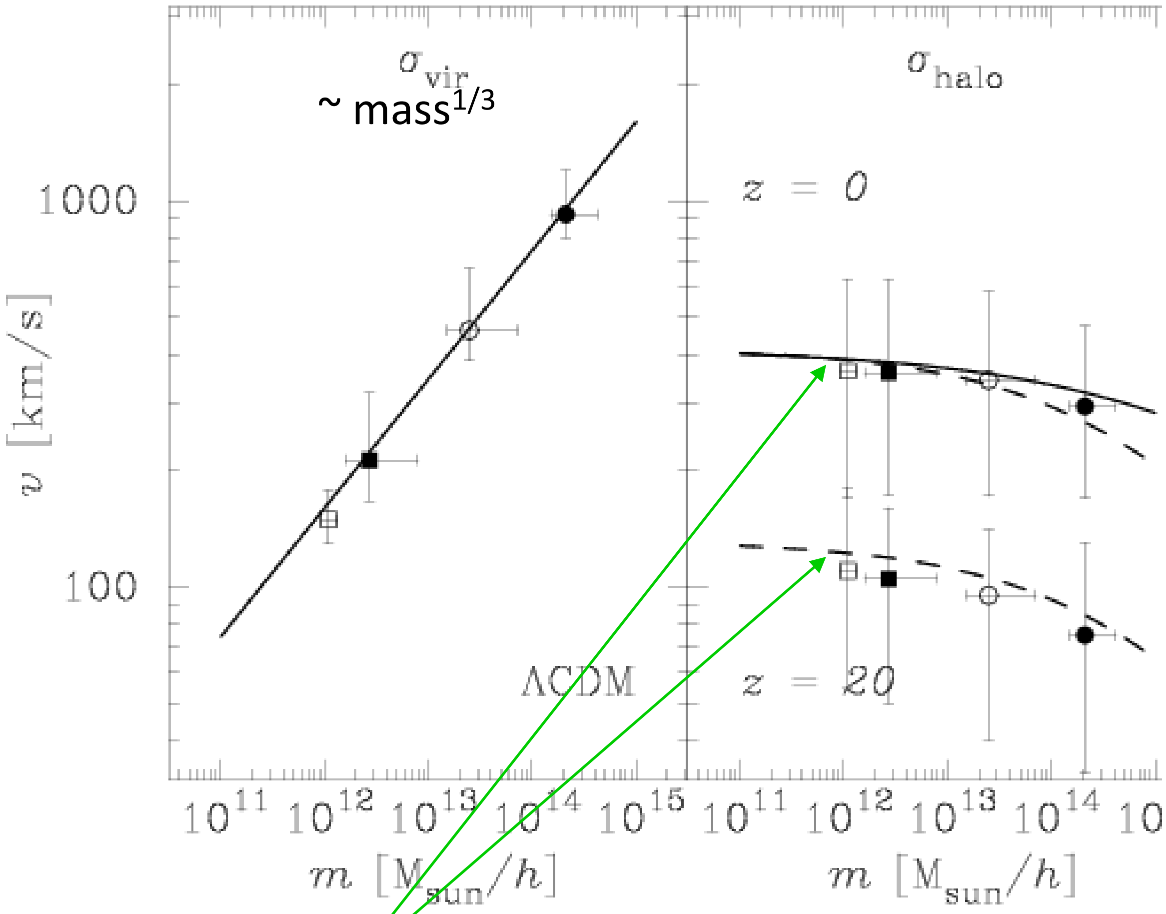
Non-Maxwellian Velocities?

- $v = v_{vir} + v_{halo}$
- Maxwellian/Gaussian velocity within halo (dispersion depends on parent halo mass) + Gaussian velocity of parent halo (from linear theory ≈ independent of *m*)
- Hence, at fixed *m*, distribution of v is convolution of two Gaussians, i.e.,

p(v|*m*) is Gaussian, with dispersion

$$\sigma_{\mathrm{vir}}^2(m) + \sigma_{\mathrm{Lin}}^2 = (m/m_*)^{2/3}\sigma_{\mathrm{vir}}^2(m_*) + \sigma_{\mathrm{Lin}}^2$$

Two contributions to velocities



- Virial motions (i.e., nonlinear theory terms) dominate for particles in massive halos
- Halo motions (linear theory) dominate for particles in low mass halos

Growth rate of halo motions ~ consistent with linear theory

Exponential tails are generic

- $p(\mathrm{v}) = \int dm\; mn(m)\; \mathrm{G}(\mathrm{v}|m)$

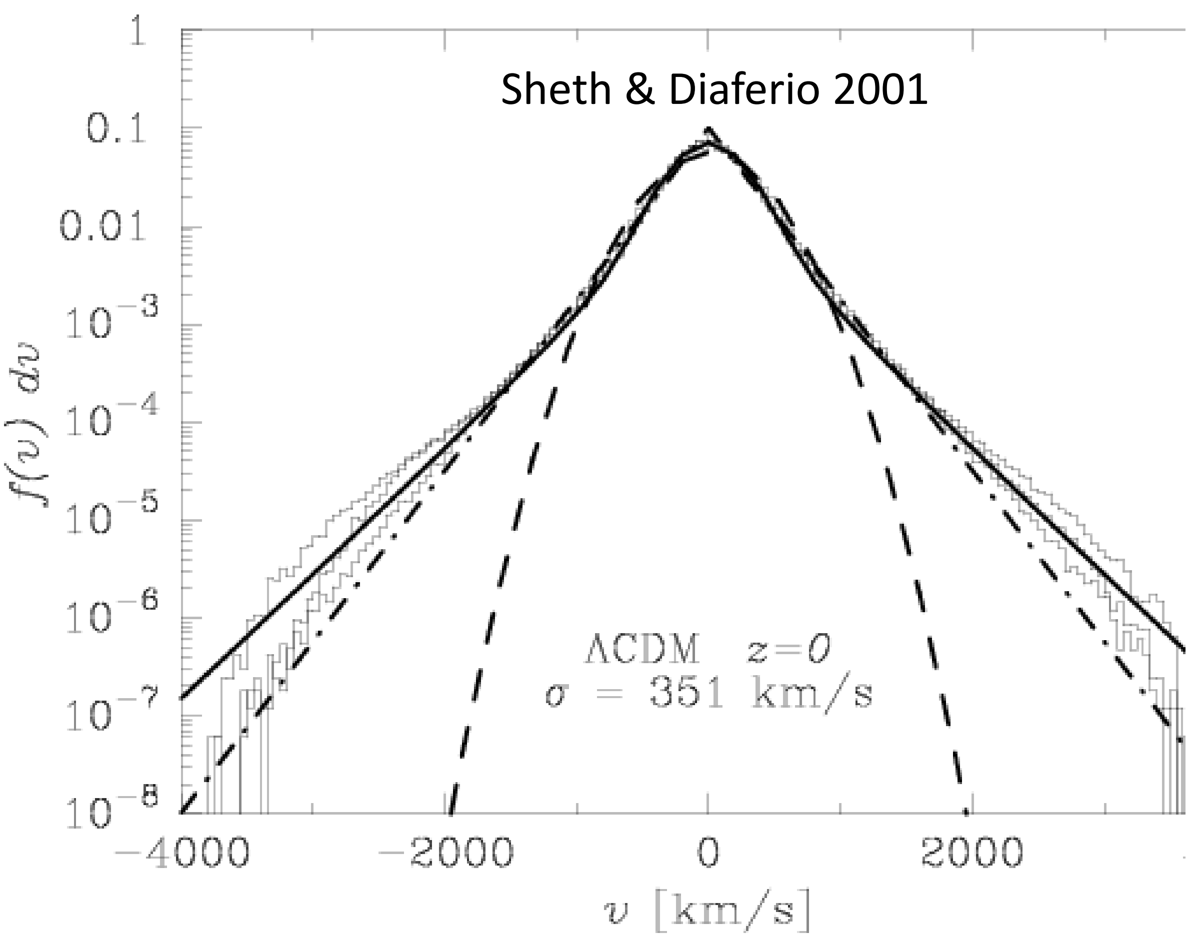
  $\mathcal{F}(t) = \int d\mathrm{v}\; e^{i\mathrm{v}t}\; p(\mathrm{v}) = \int dm\; n(m)m\; \mathrm{e}^{-t^2\sigma_{\mathrm{vir}}^2(m)/2}\; \mathrm{e}^{-t^2\sigma_{\mathrm{Lin}}^2/2}$

- For $P(k) \sim k^{-1}$, mass function $n(m) \sim$ power-law times $\exp[-(m/m_*)^{2/3}/2]$, so integral is:

  $\mathcal{F}(t) = \mathrm{e}^{-t^2\sigma_{\mathrm{Lin}}^2/2}\; [1 + t^2\sigma_{\mathrm{vir}}^2(m_*)]^{-1/2}$

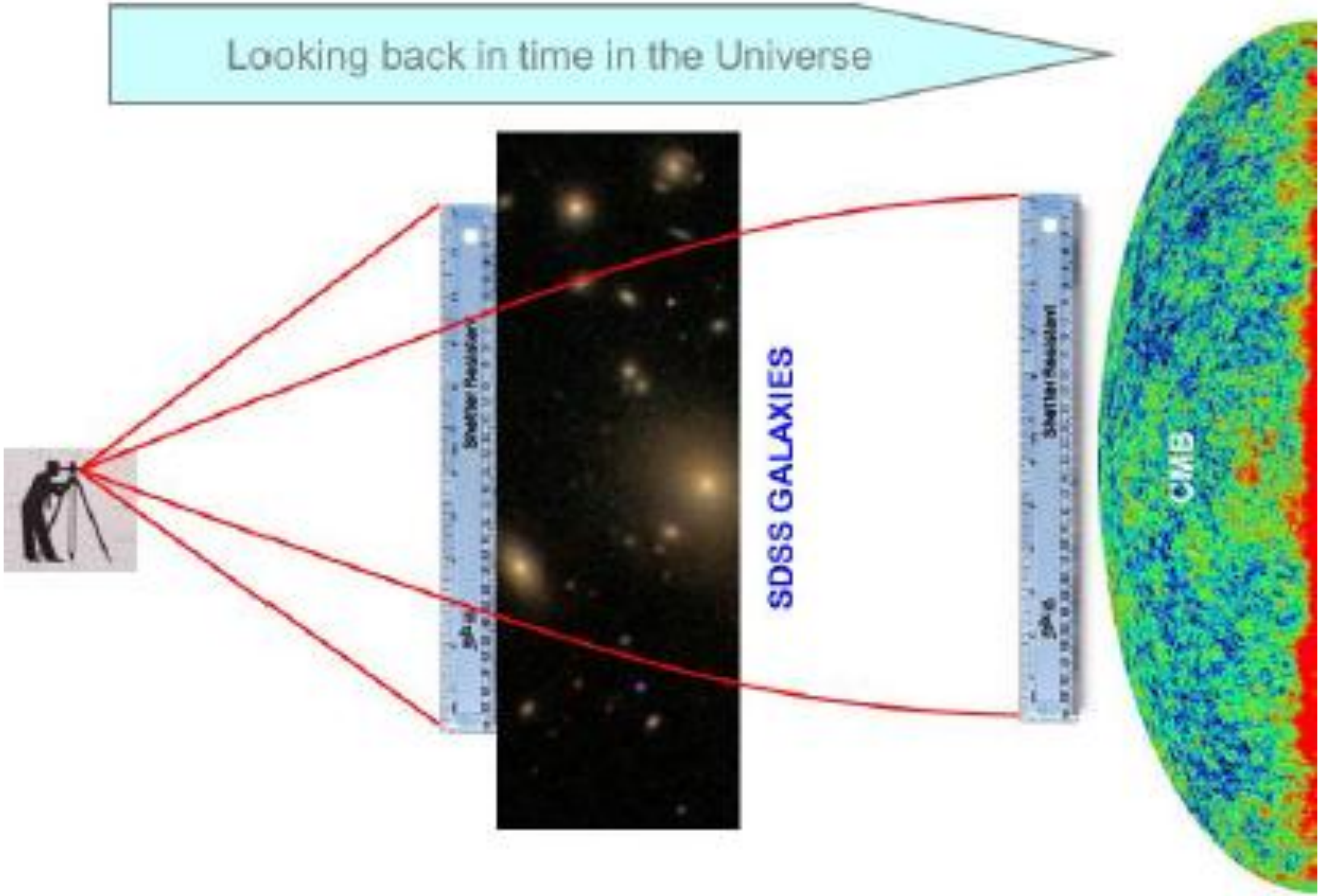
- Fourier transform is product of Gaussian and FT of K_0 Bessel function, so $p(\mathrm{v})$ is convolution of $\mathrm{G}(\mathrm{v})$ with $K_0(\mathrm{v})$
- Since $\sigma_{\mathrm{vir}}(m_*) \sim \sigma_{\mathrm{Lin}}$, $p(\mathrm{v}) \sim$ Gaussian at $|\mathrm{v}|<\sigma_{\mathrm{Lin}}$ but exponential-like tails extend to large v

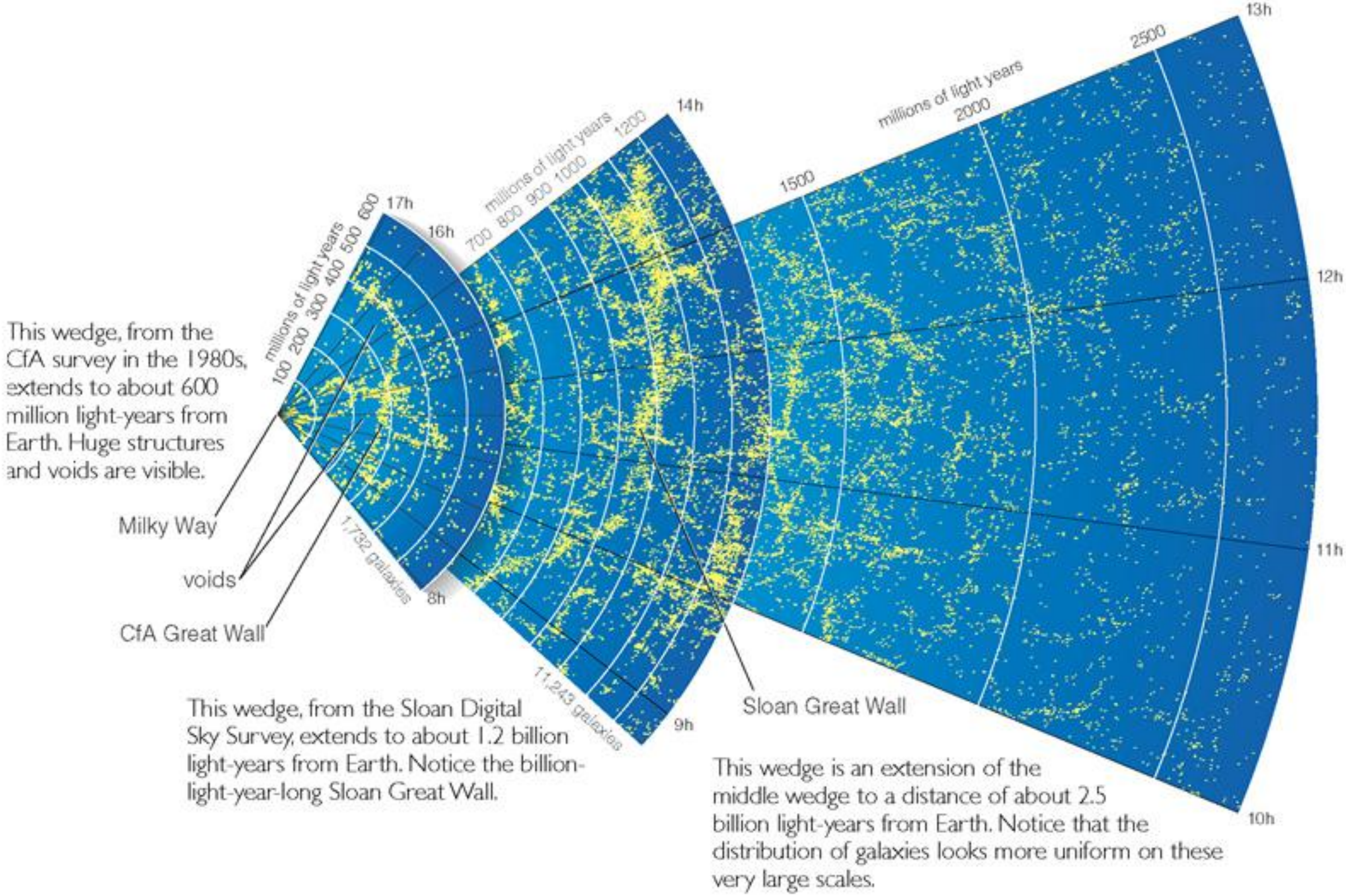
Comparison with simulations



Gaussian core with exponential tails as expected

Baryon Oscillations in the Galaxy Distribution



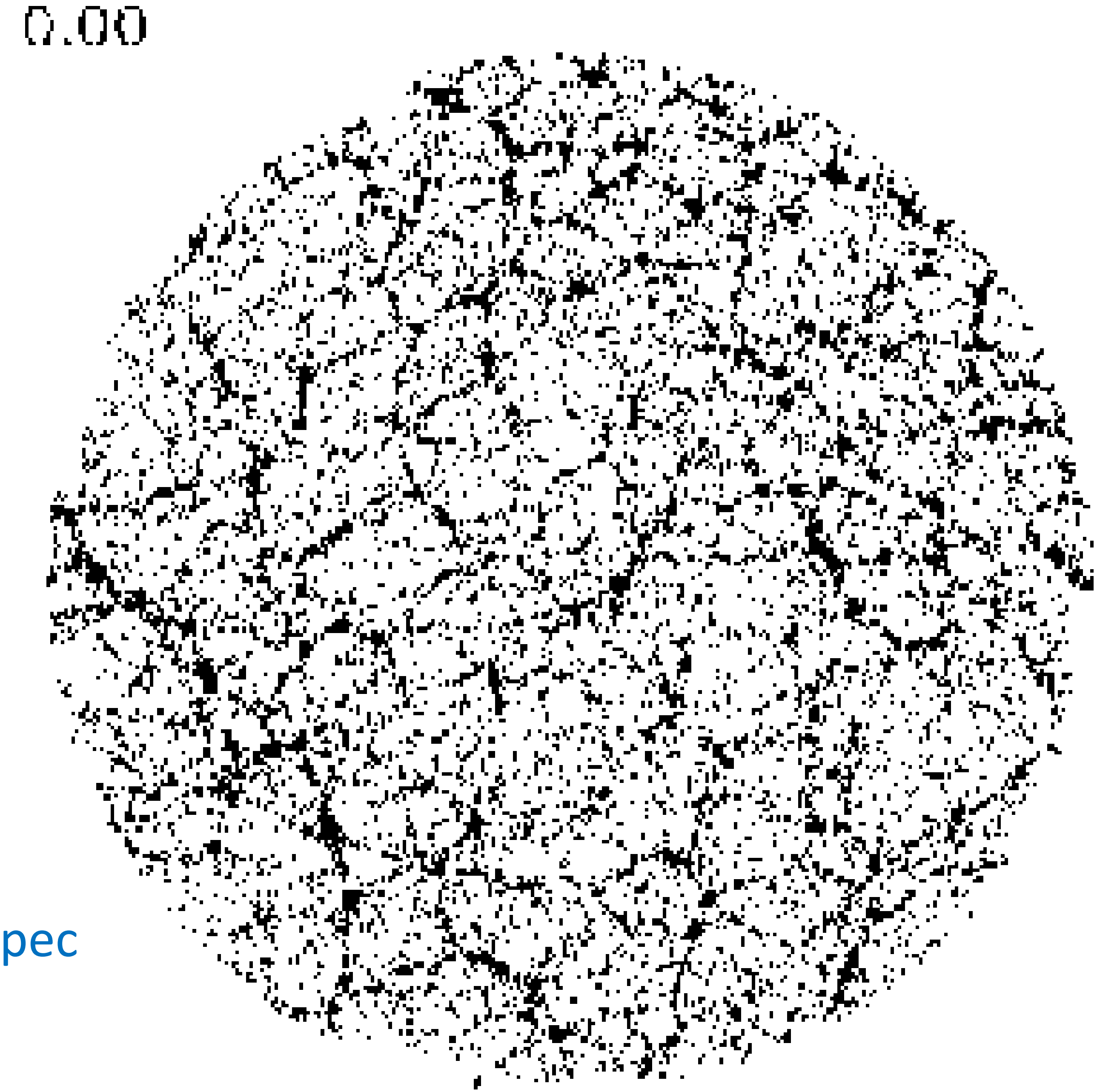


Structures in galaxy maps look very similar to the ones found in models in which dark matter is WIMPs

Redshift space distortions:

peculiar velocities driven by gravity

$cz_{obs} = Hd + v_{pec}$



Halos and Fingers-of-God

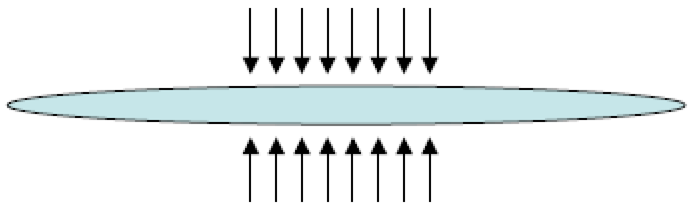
- Virial equilibrium:
- $V^2 = GM/r = GM/(3M/4\pi 200\rho)^{1/3}$
- Since halos have same density, massive halos have larger random internal velocities: $V^2 \sim M^{2/3}$
- $V^2 = GM/r = (G/H^2)\ (M/r^3)\ (Hr)^2$

  $= (8\pi G/3H^2)\ (3M/4\pi r^3)\ (Hr)^2/2$

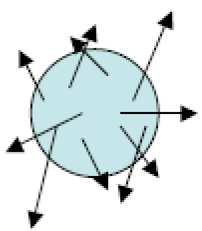
  $= 200\ \rho/\rho_c\ (Hr)^2/2 = \Omega\,(10\ Hr)^2$
- Halos should appear ~ten times longer along line of sight than perpendicular to it: 'Fingers-of-God'
- Think of V^2 as Temperature; then Pressure ~ $V^2\rho$

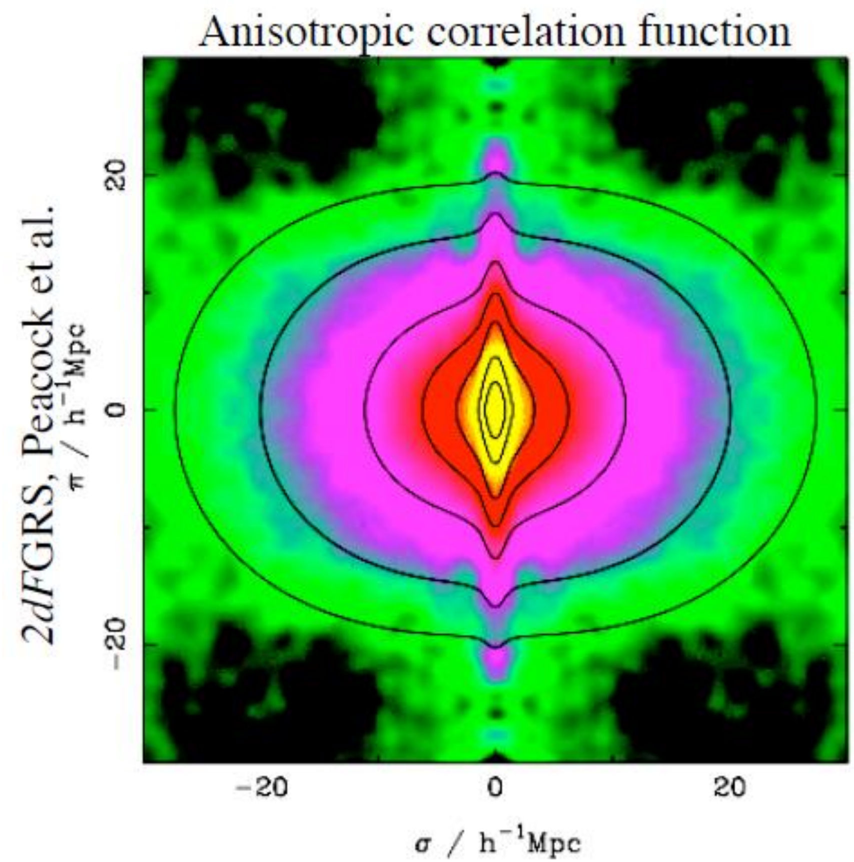
Redshift space distortions

Coherent infall



Random (thermal) motion



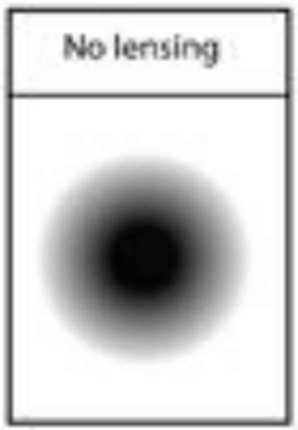


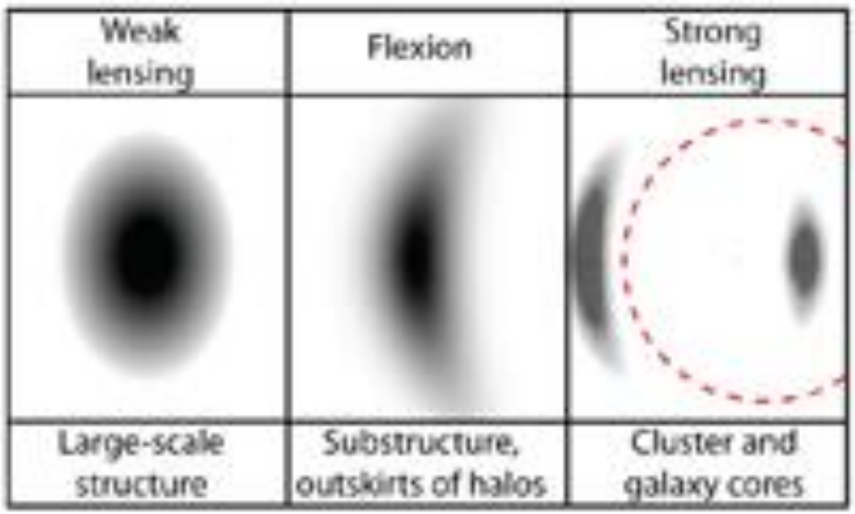
$$1 + \xi_s(s_\parallel, s_\perp) = \int_{-\infty}^{\infty} dr_\parallel \; [1 + \xi(r)] \; \mathcal{P}(\underbrace{r_\parallel - s_\parallel}_{\mathbf{v}_p}, \mathbf{r})$$

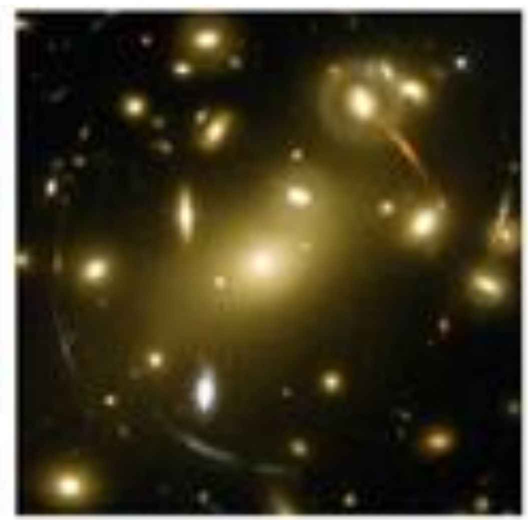
Cosmology from Gravitational Lensing

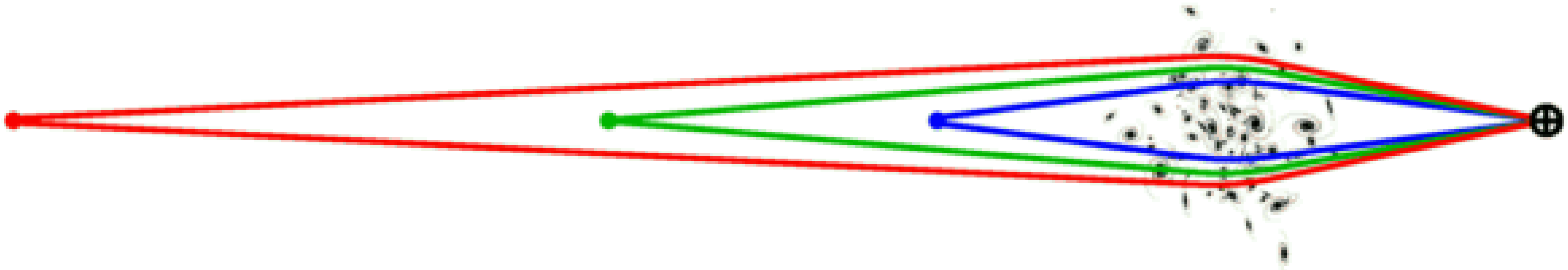
Volume as function of redshift

Growth of fluctuations with time

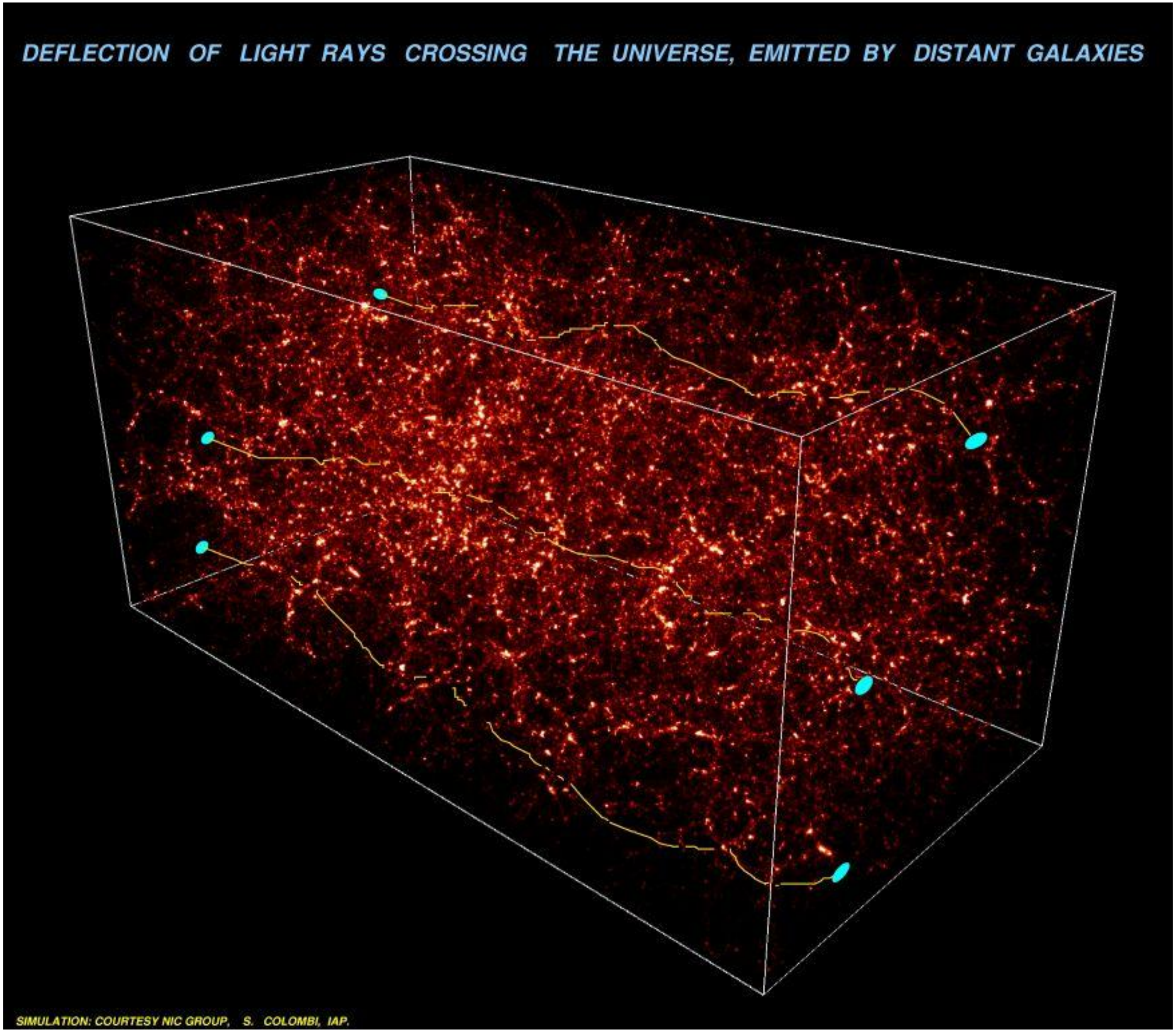








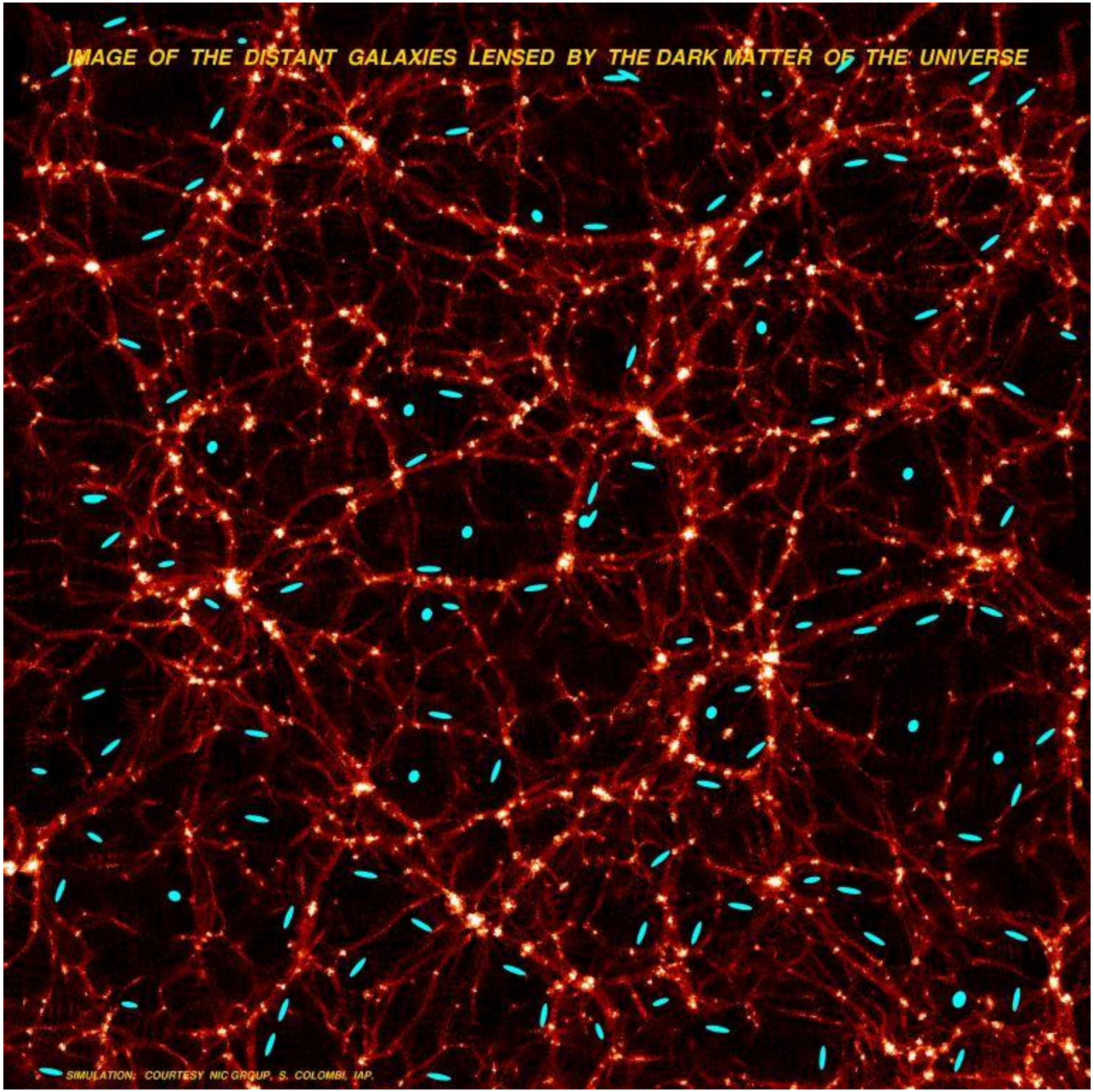
- Focal length strong function of cluster-centric distance; highly distorted images possible
- Strong lensing if source lies close to lens-observer axis; weaker effects if impact parameter large
- Strong lensing: Cosmology from distribution of image separations, magnification ratios, time delays; but these are rare events, so require large dataset
- Weak lensing: Cosmology from correlations (shapes or magnifications); small signal requires large dataset



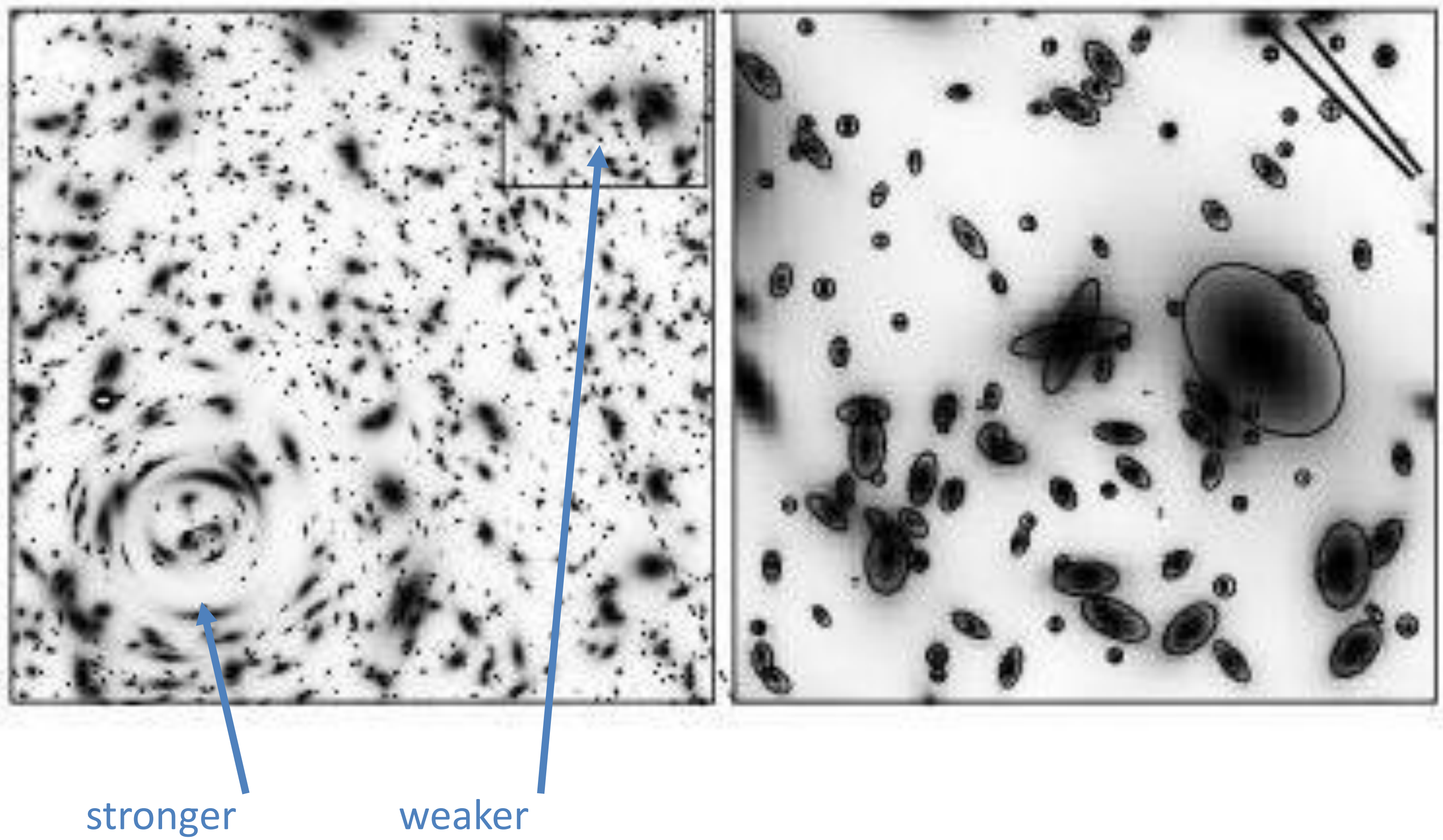
Lensing provides a measure of dark matter along line of sight

Weak lensing: Image distortions correlated with dark matter distribution

E.g., lensed image ellipticities aligned parallel to filaments, tangential to knots (clusters)



The shear power of lensing



Cosmology from measurements of correlated shapes; better constraints if finer bins in source or lens positions possible

Halo Model is simplistic …

- Nonlinear physics on small scales from virial theorem
- Linear perturbation theory on scales larger than virial radius (exploits 20 years of hard work between 1970-1990)
- Halo mass is more efficient language (than e.g., dark matter density) for describing nonlinear field

…but quite accurate!

Halo-model

≈

Circles in circles

Cosmology from Large Scale Structure Sky Surveys

- Baryon Acoustic Oscillations
- Cluster counts and clustering
- Redshift space distortions
- Weak gravitational lensing
- Supernovae IA
- Your name here!