Particle acceleration in twisted plasma waves with orbital angular momentum

J. Vieira¹
J.T. Mendonça¹, F. Quéré²

¹ GoLP / Instituto de Plasmas e Fusão Nuclear
Instituto Superior Técnico, Lisbon Portugal

² LIDYL, CEA, CNRS, Université Paris-Saclay, CEA Saclay, 91191 Gif-sur-Yvette France

web.ist.utl.pt/jorge.vieira || epp.tecnico.ulisboa.pt || golp.tecnico.ulisboa.pt
The topological freedom is an unique property of the plasma cannot be currently found in conventional devices.

**Plasma waves can have any shape**

Plasma waves are sustained by free electrons

![Plasma wave images](image)

Wakefields in conventional devices are sustained by metallic parts

**Implications of wake topology**

Plasma waves are ubiquitous

- Fundamental plasma processes wave-particle interactions
- Nonlinear optics
- Accelerators and light sources

- One-dimensional waves widely explored, require standard Gaussian drivers
- Plasma waves with non-trivial topologies unexplored, require non-Gaussian drivers

Changing the **topology of the plasma** can have deep ramifications and transform the outputs of **plasma accelerators and radiation sources**
The orbital angular momentum of light (OAM)

**Ultra-intense twisted lasers**

PW twisted laser for ion acceleration


High intensity twisted light from plasma mirrors

- Denoeud et al. PRL 118 033902 (2017)

**Orbital angular momentum**

\[ E_{\text{laser}} \propto \left( \frac{r}{w_0} \right)^{|\ell|} L_p^{|\ell|} \left( \frac{r}{w_0} \right) \exp \left( -\frac{r^2}{w_0^2} \right) \times \cos (\omega_0 t - k_0 z + \ell \phi) \]


**Goal:** to propose new configuration that allows transfer the angular momentum of the laser to a relativistic plasma wakefield carrying OAM
Conditions to excite a twisted wakefield

(Forward) Raman scattering

OAM transfer requires different photons (with different frequencies) carrying different OAM levels

Frequency and orbital angular momentum correlation

![Graph showing mode index vs. spectral intensity with OAM transfer](graph.png)

Plasma absorbs OAM $\Delta \ell$

Light spring with $\Delta \ell = 1$

Light spring with $\Delta \ell = 2$

Images adapted from G. Pariente, F. Quéré, Optics Letters 40 2037 (2015)
### Generation of an ultra-short light spring

#### Mathematical model for simulations and theory

**Beating two Laguerre-Gaussian modes**

\[
\Delta l \propto a_r^2 + a_{\ell+\Delta \ell}^2 + 2a_r a_{\ell+\Delta \ell} \cos \left[ \Delta k (ct - x) + \Delta \ell \theta + \Delta \phi (x) \right]
\]

\( \Delta l \) is the OAM difference between the two modes

\( \Delta k \) is the wavenumber difference

\( \Delta \phi \) is a phase difference

---

*G. Pariente, F. Quéré, Optics Letters 40 2037 (2015)*

---

### Experimental realisation*

**Spiral phase plate**

Light spring forms when thickness \( \approx \) laser duration

**Angular dependent group velocity dispersion**
osiris framework

- Massively Parallel, Fully Relativistic Particle-in-Cell (PIC) Code
- Visualization and Data Analysis Infrastructure
- Developed by the osiris.consortium ⇒ UCLA + IST

code features

- Scalability to ~ 1.6 M cores
- SIMD hardware optimized
- Parallel I/O
- Dynamic Load Balancing
- QED module
- Particle merging
- GPGPU support
- Xeon Phi support

Ricardo Fonseca
ricardo.fonseca@tecnico.ulisboa.pt

Frank Tsung
tsung@physics.ucla.edu

http://epp.tecnico.ulisboa.pt/
http://plasmasim.physics.ucla.edu/
Panofksy-Wenzel theorem

Transverse force acting on relativistic particle

$$\nabla_\perp E_x = \frac{\partial W_\perp}{\partial \xi}$$

Transverse wakefield

$$W_\perp = E_\perp + (e_x \times B)_\perp$$

OAM wakefield

Radial focusing (betatron motion)

$$\frac{\partial E_x}{\partial r} = \frac{\partial W_r}{\partial \xi} \propto \frac{\partial \phi}{\partial r}$$

Azimuthal force **new!**

$$\frac{1}{r} \frac{\partial E_x}{\partial \theta} = \frac{\partial W_\theta}{\partial \xi} \propto \frac{\ell_p}{r} \phi$$
Hamiltonian formulation

Hamiltonian of a charged particle
\[ \mathcal{H} = m_e c^2 \gamma + e \phi(r, \theta, \xi) \]

Twisted wakefield structure
\[ \phi = \phi(v_\phi t - x + \ell_p \theta) = \phi(u) \]

Hamilton’s equations
\[ \frac{dP_x}{dt} \approx \frac{dp_x}{dt} = -\frac{\partial \mathcal{H}}{\partial x} = \phi'(u) \]
\[ \frac{dP_\theta}{dt} \approx \frac{dL_x}{dt} = -\frac{\partial \mathcal{H}}{\partial \theta} = -\ell_p \phi' \]
\[ \frac{d\mathcal{H}}{dt} = \frac{\partial \mathcal{H}}{\partial t} = v_g \phi' \]

Constants of motion

Energy
\[ \gamma (1 - v_\phi v_x / c^2) = 1 + \Delta \phi / m_e c^2 \]

Ratio of angular momentum flux to energy
\[ \frac{\Delta L_x}{\Delta p_x} = \frac{\ell_p}{k_p} \Rightarrow \frac{\Delta L_x}{E} = \frac{\ell_p}{\omega_p} \]

Angular momentum is quantised.

Simulations confirm OAM quantisation

Simulations confirm OAM quantisation
Beams have a vortex density structure

**Vortex particle beam with \( \ell = 2 \)**

- Similar to twisted light
  - Ratio of angular momentum flux to energy flux for light
    \[
    \frac{J}{cP} = \frac{\ell}{\omega_0}
    \]
  - L. Allen et al., PRA 44 8185 (1992)

**Vortex particle beam with \( \ell = 4 \)**

- Ratio of angular momentum to energy for beam particle
  \[
  \frac{\Delta L_x}{E} = \frac{\ell}{\omega_p}
  \]

Vortex beam electrons move as a twisted ray of light
Energy gain of the vortex beam

**Lower energy gain**

Energy gain

\[ \Delta \gamma \propto \frac{1}{c^2(1 + \Delta \ell^2/r^2) - v^2_\phi} \leq 2\gamma^2_\phi \]

decreases with the plasma wave OAM

**Simulation confirmation**

Dephasing along the axial direction and also along the azimuthal direction \( e_\theta \)

Particles dragged along \( e_\theta \) due to the new azimuthal wakefield component \( E_\theta \)

**Reduced dephasing length**

Transverse slide of longitudinal wakefield

\[ x_5 \left[ c/\omega_p \right] \quad E_\theta \left[ m_e c/\omega_p e \right] \]

Simulation confirmation
Light springs spins in a plasma channel

Twisted wavefronts can spiral in a plasma channel causing the rotation of the light spring

Two key directions set wakefield response:
- rotation orientation: set by the OAM of light spring modes
- OAM orientation: set by the OAM difference between light spring modes

Wakefield phase velocity depends the relative orientation between these two rotational directions
Tuneable phase velocities

Wakefield phase velocity depends the relative sign between $\ell$ and $\Delta \ell$

$$
\frac{v_{\phi,\text{wake}}}{c} = \left[ 1 - \frac{k_p(0)^2}{2k_0^2} - \frac{2(1 + |\ell + \Delta \ell|)}{k_0^2 w_0^2} + \frac{2(|\ell + \Delta \ell| - |\ell|)}{k_0 w_0^2 \Delta k} \right]
$$

Phase velocity control

- $\Delta \ell > 0$ and $\ell > 0$
- $\Delta \ell < 0$ and $\ell > 0$
Energy gain control beyond planar wakefields

**Wakefield phase velocity depends the relative sign between \( \ell \) and \( \Delta \ell \)**

\[
\frac{v_{\phi, \text{wake}}}{c} = \left[ 1 - \frac{k_p(0)^2}{2k_0^2} - \frac{2(1 + |\ell + \Delta \ell|)}{k_0^2w_0^2} + \frac{2(|\ell + \Delta \ell| - |\ell|)}{k_0w_0^2\Delta k} \right]
\]

**Energy gain control**

Ordering of energy gain with \( \Delta \ell \) for constant \( \ell \)

- \( \Delta \ell = 1 \) \( \Delta E_{\text{max}} \sim 40 \)
- \( \Delta \ell = 0 \) \( \Delta E_{\text{max}} \sim 30 \)
- \( \Delta \ell = -1 \) \( \Delta E_{\text{max}} \sim 10 \)
Conclusions and future directions

Plasma wave topology is a new degree of freedom that impacts accelerators and light sources

- Light spring laser pulses as drivers for twisted plasma waves
- Generation of relativistic vortex bunches with quantised OAM levels
- Rotation of light spring enable all optical control of the wake phase velocity
- Additional questions motivated by this work
  - wakefield excitation by relativistic vortex bunches
  - twisted THz generation using vortex beams
  - vortex bunch magnetic moments and interaction with magnetic fields
  - ...
Thank you!