Measurement of $\pi\pi$ Scattering Lengths from K_{e4} and $K_{3\pi}$ Decays

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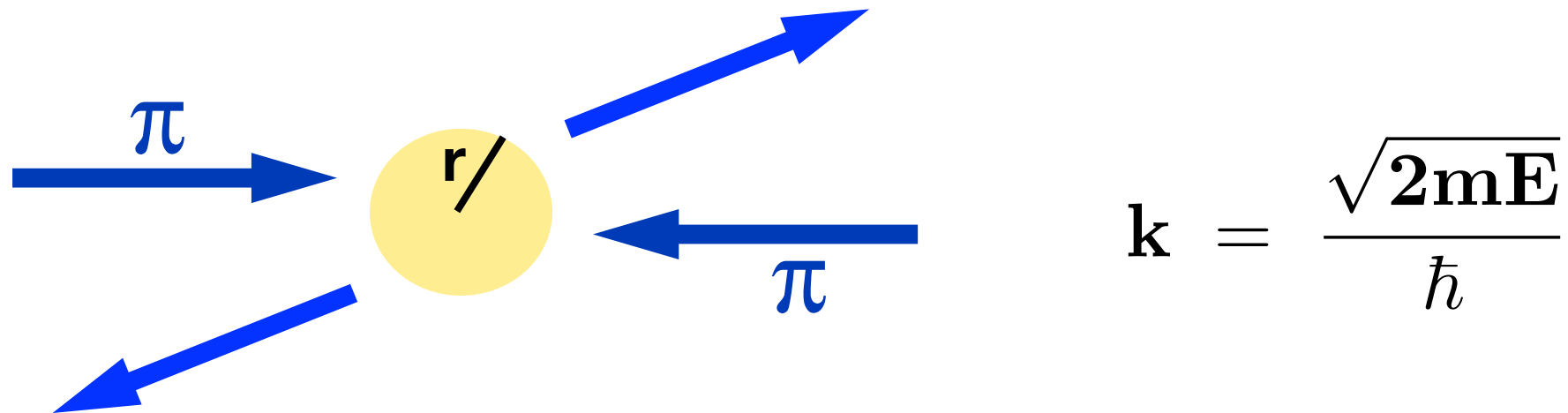JG|U

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$\pi\pi$ Scattering Lengths

$$k = \frac{\sqrt{2mE}}{\hbar}$$

At low energy _k r << 1_: **S-wave** dominates scattering amplitude.
Isospin _I = 0,2_ because of Bose statistics.

- Scattering matrix $\mathbf{S}|\pi\pi\rangle = \mathbf{e^{2i\delta}}|\pi\pi\rangle$ parametrized by two phases:
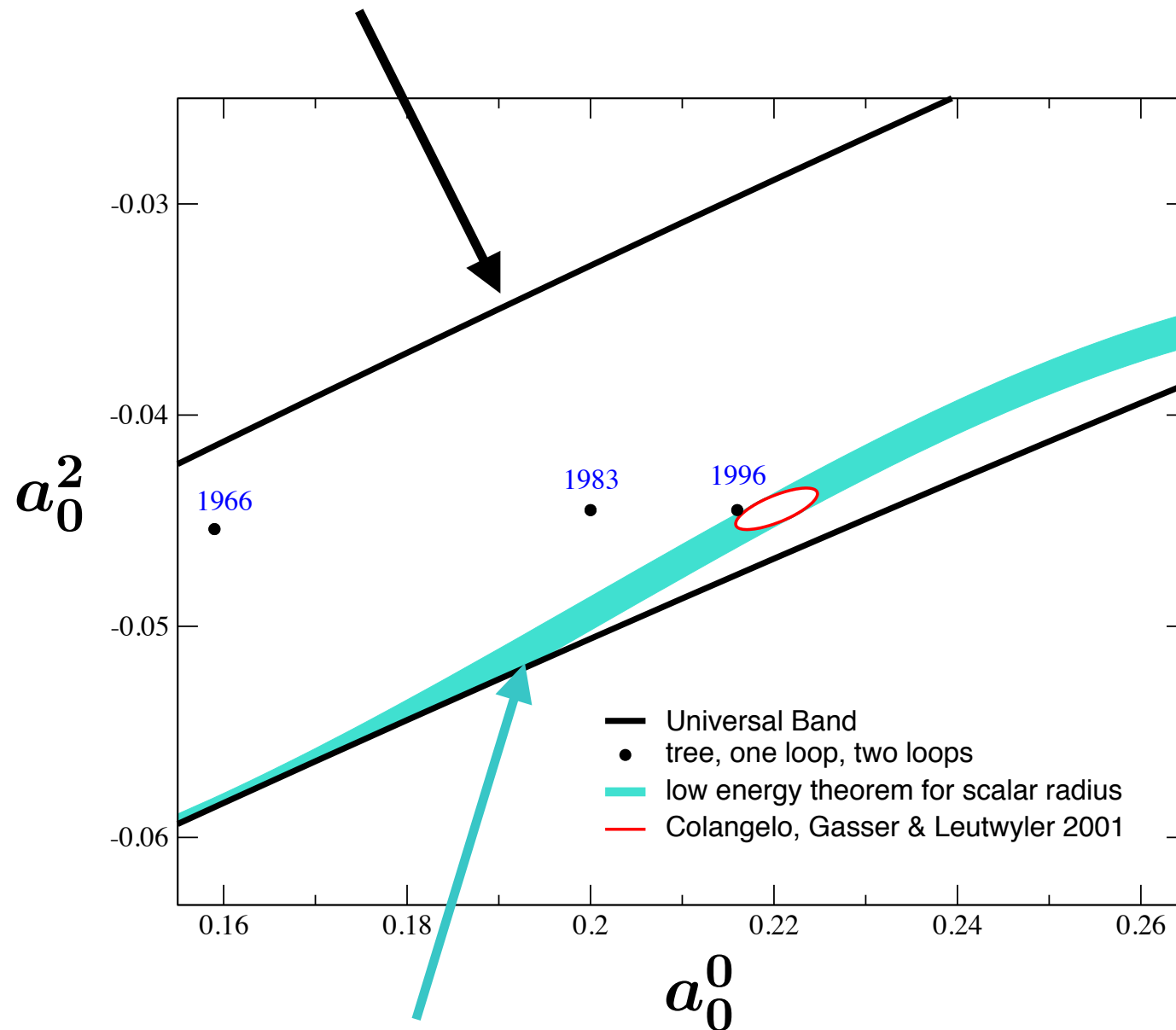
$$\delta_{0,2} = a_{0,2} \cdot k + \mathcal{O}(k^2)$$

- At low energy **S-wave scattering lengths a_0, a_2** are essential parameters of **Chiral Perturbation Theory (ChPT).**

(In the following: $a_{0,2}$ quoted in units of m_π)

Theory predictions for a_0 and a_2

Universal band (from Roy equations)

■ Scattering lengths $\mathbf{a_0}$, $\mathbf{a_2}$ are directly connected to $\mathbf{m_\pi}$:

$$a_0 \sim \frac{7\,m_\pi^2}{32\pi\,F_\pi^2} = 0.16$$

$$a_2 \sim \frac{-m_\pi^2}{16\pi\,F_\pi^2} = -0.045$$

(Weinberg, PRL 17 (1996) 216)

■ Precise prediction within Chiral Perturbation Theory:

$$a_0 = 0.220 \pm 0.005$$

$$a_2 = -0.0444 \pm 0.0010$$

(Colangelo, Gasser, Leutwyler, PRL 86 (2001) 5008)

ChPT constraint:

$a_2 = -0.0444(8) + 0.236\,(a_0 - 0.22) - 0.61\,(a_0 - 0.22)^2 - 9.9\,(a_0 - 0.22)^3$

Measuring $\pi\pi$ Scattering Lengths

Three kinds of measurements have been performed:

- **Pionium lifetime $(\pi^+\pi^-)_{atom}$:** Measurement of $|a_2 - a_0|$

  → DIRAC experiment

- **Cusp in $K \rightarrow \pi\pi\pi$ ($K_{3\pi}$) decays:** Measurement of $a_2 - a_0$, a_2

  - $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ → **NA48/2:** ~60 million events
  - $K_L \rightarrow \pi^0 \pi^0 \pi^0$ → KTeV + NA48/2: (70+100) million events

- **$K^\pm \rightarrow \pi^+ \pi^- e^\pm \nu$ (K_{e4}) decays:** Measurement of a_0, a_2

  → S118 (Geneva-Saclay, 1977): ~30 000 events

  BNL E685 (2003): ~400 000 events

  NA48/2 (2009): ~1.1 million events

NA48/2 in 2003/2004

■ **Simultaneous K^+ and K^- beams** with $p_{K\pm} = (60 \pm 3)$ GeV/c.
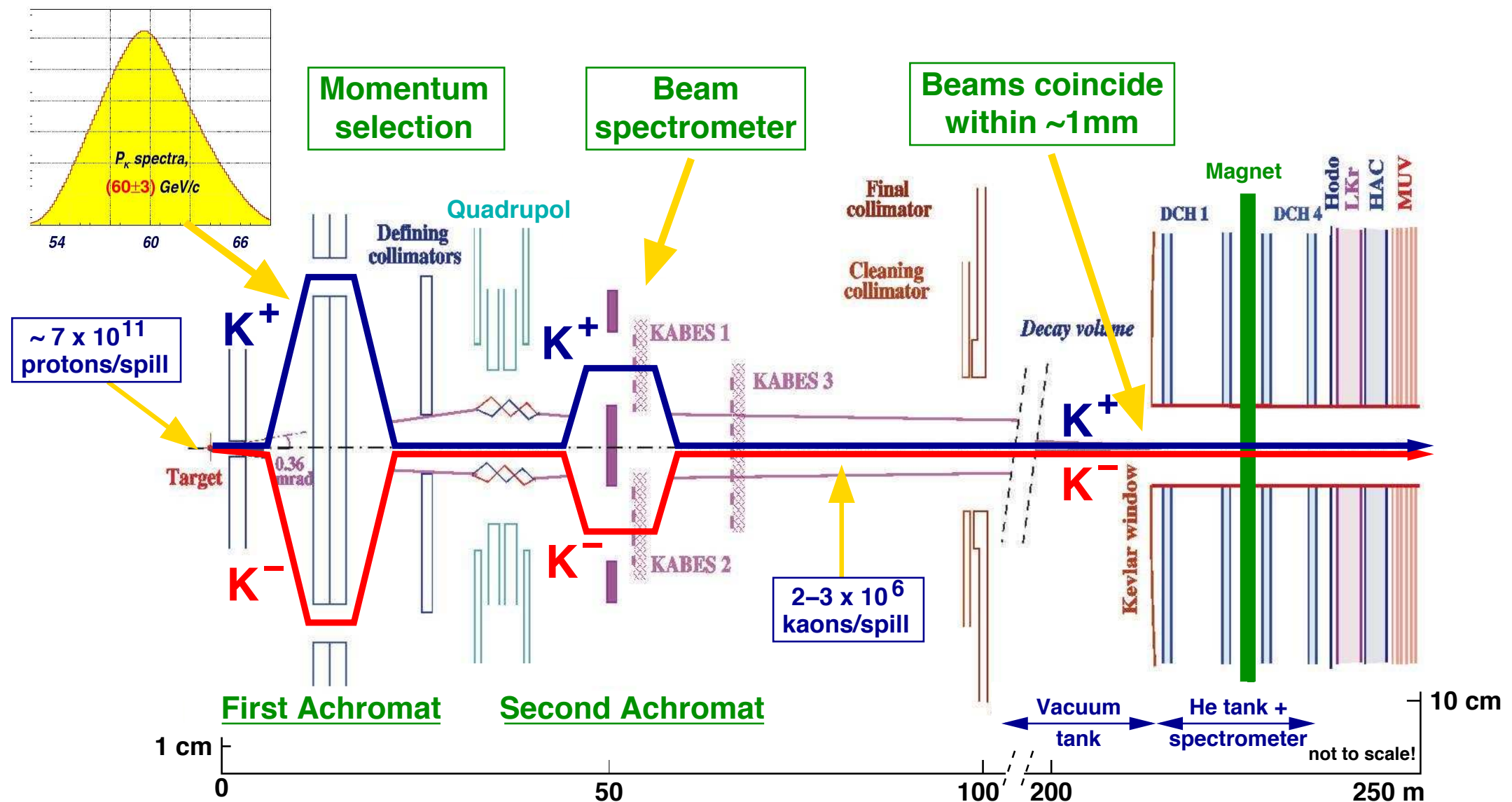
Trigger: ■ **3 charged tracks** or
■ **1 charged track + missing p_T**

➔ **Efficiencies > 99 %**

NA48 Detector

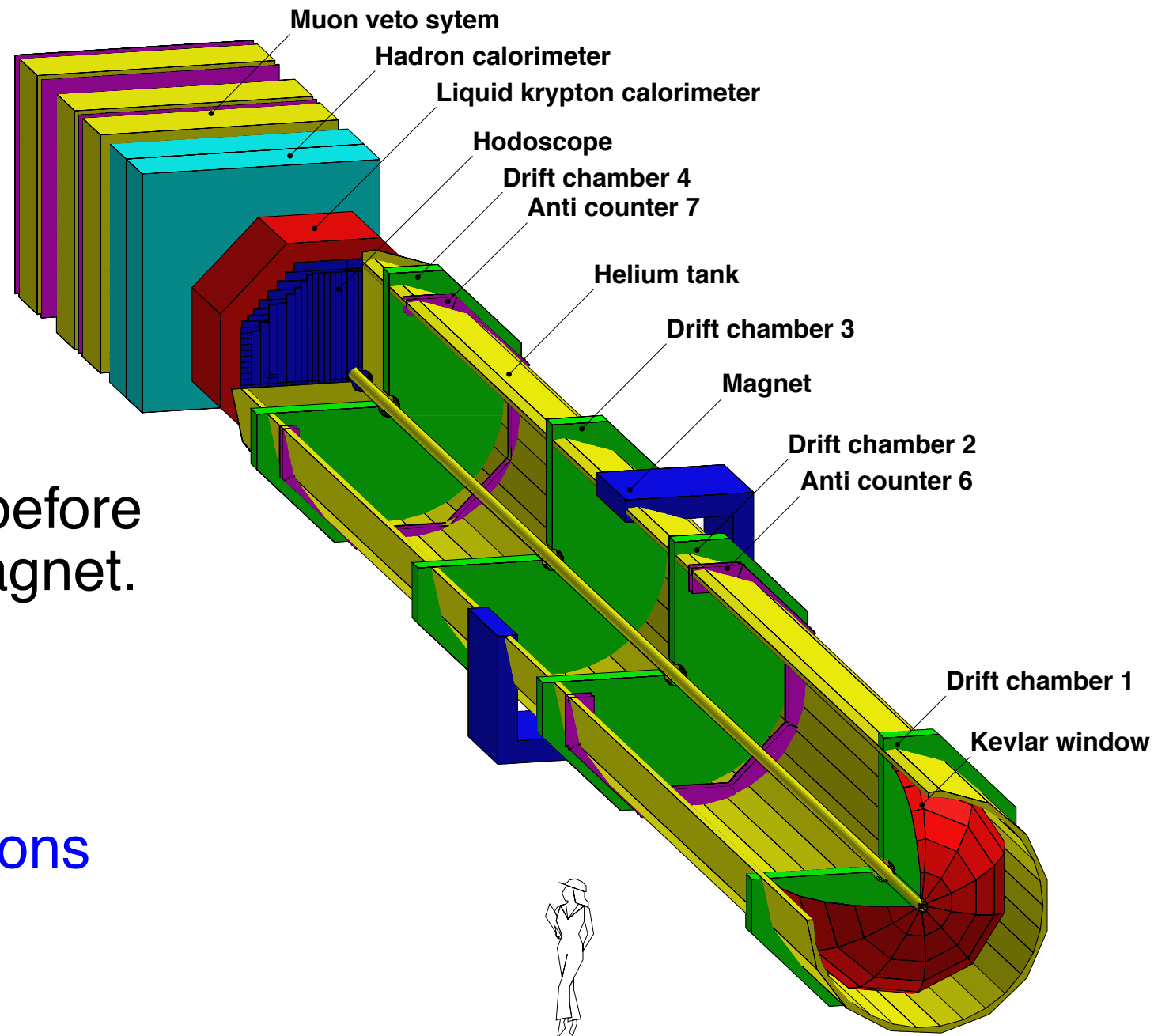**Main detector components:**

- **Magnet spectrometer**
  - Two drift chambers each before and after spectrometer magnet.
  - Momentum resolution:
    $\leq 1\%$ for 20 GeV/c tracks.

- Anti-counters for photons, muons

- **Liquid Krypton Calorimeter**

$$\frac{\Delta E}{E} = \frac{3.2\%}{\sqrt{E[\text{GeV}]}} \oplus \frac{90\,\text{MeV}}{E} \oplus 0.42\%$$

Muon veto sytem
Hadron calorimeter
Liquid krypton calorimeter
Hodoscope
Drift chamber 4
Anti counter 7
Helium tank
Drift chamber 3
Magnet
Drift chamber 2
Anti counter 6
Drift chamber 1
Kevlar window

Reconstruction of K → $\pi^\pm \pi^0 \pi^0$ Decays

$K \to \pi^\pm \pi^0 \pi^0$ selection:

- 1 charged track + 4 e.m. calorimeter clusters

- $\pi^0 \to \gamma\gamma$ selection:
  consider all 3 pairings and minimize vertex difference Δz

- **invariant $\pi^0 \pi^0$ mass $M(\pi^0\pi^0)$:** only calorimeter and vertex information used

➜ **100 million events**

   (mass resolution 1.3 MeV, negligible background)

$M_{3\pi} - M_K$

σ = 1.3 MeV/c^2

x 10^3

events/0.0001 GeV/c^2

ΔM (GeV/c^2)
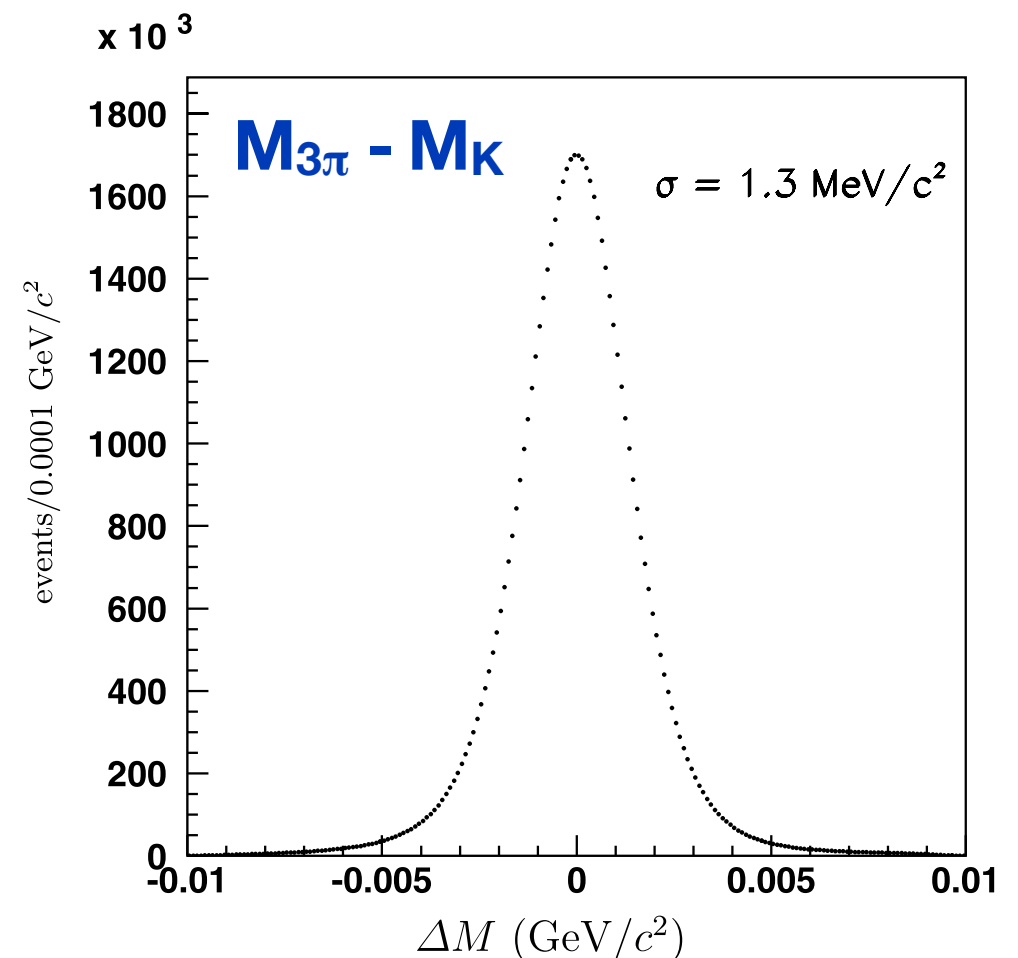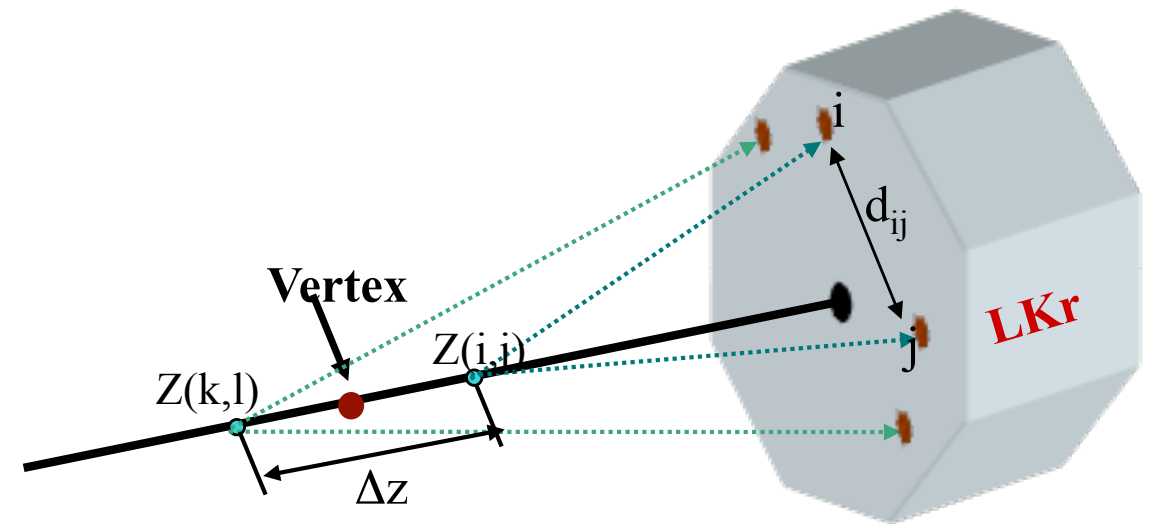
Cusp in K± → π± π⁰ π⁰ Decays
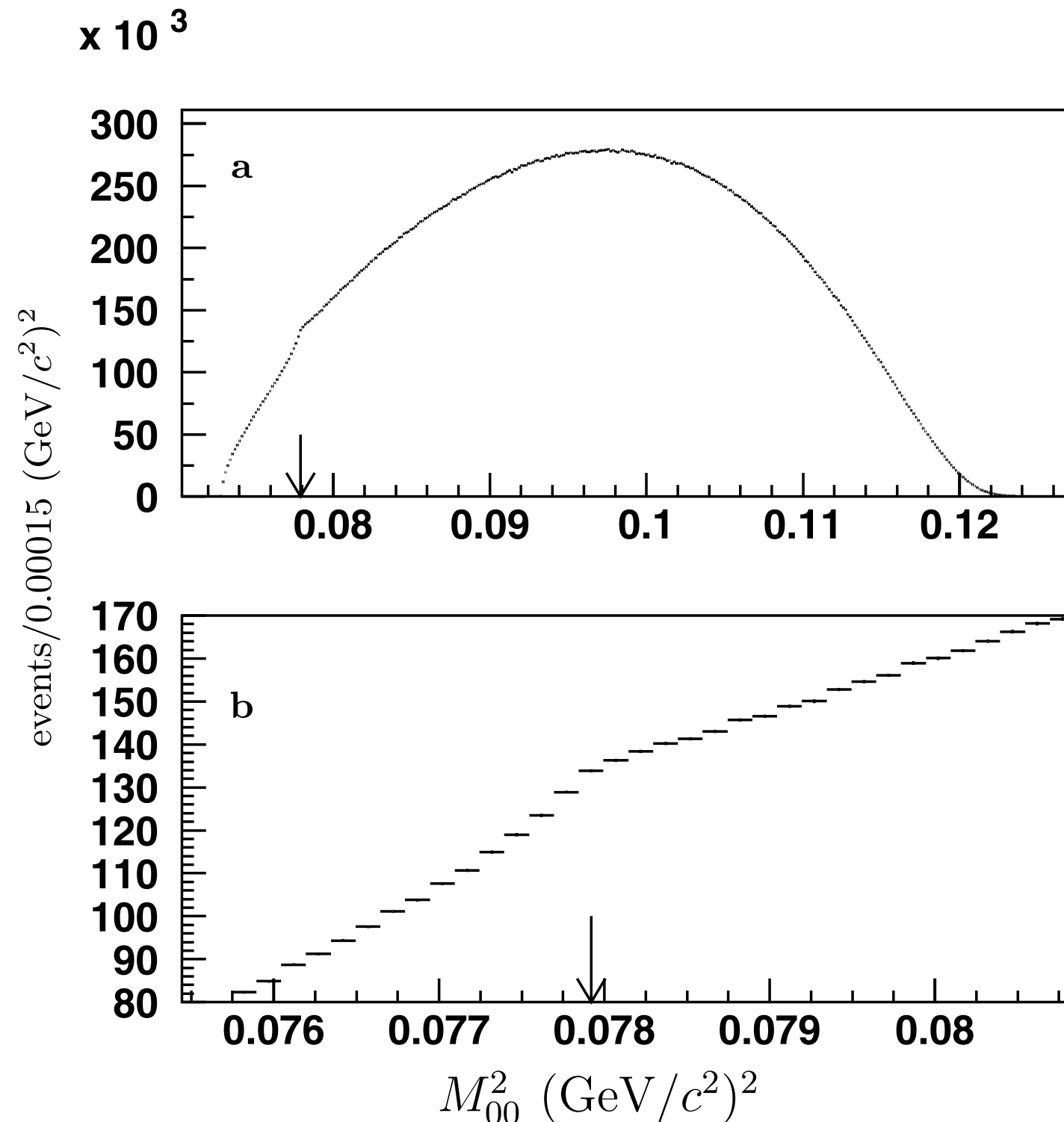
M($\pi^0\pi^0$) distribution:

Clear cusp at

$$M(\pi^0\pi^0) = 2\,m(\pi^\pm)$$

(were expecting peak
from pionium formation
$K \to \pi^\pm\,(\pi\pi)_{atom} \to \pi^\pm\,\pi^0\,\pi^0$)
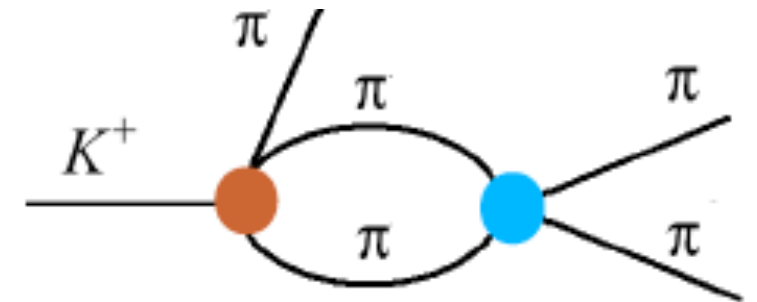
Theoretical Approach (CI)

Explanation:

- $\pi^+ \pi^- \rightarrow \pi^0 \pi^0$ rescattering amplitude
- depends on a_0 - a_2

  (Cabibbo, PRL 93 (2004) 121801,
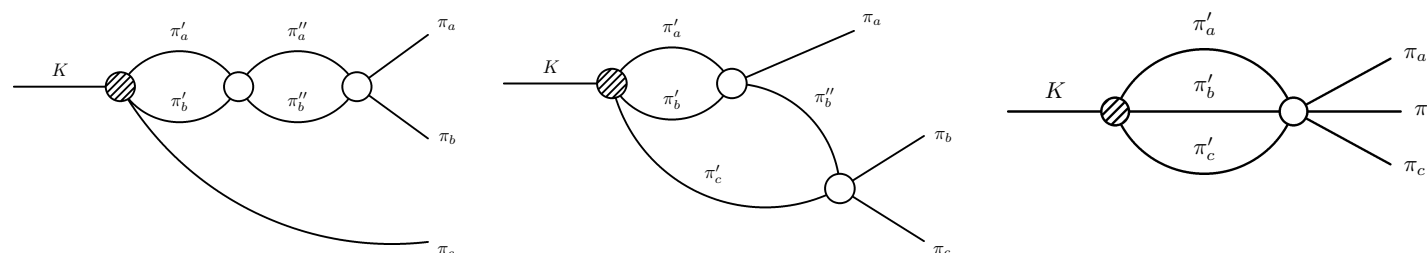 but predicted earlier: Budini, Fonda, PRL 6 (1961) 419)

More complete computation: (Cabibbo, Isidori, JHEP03 (2005) 21)
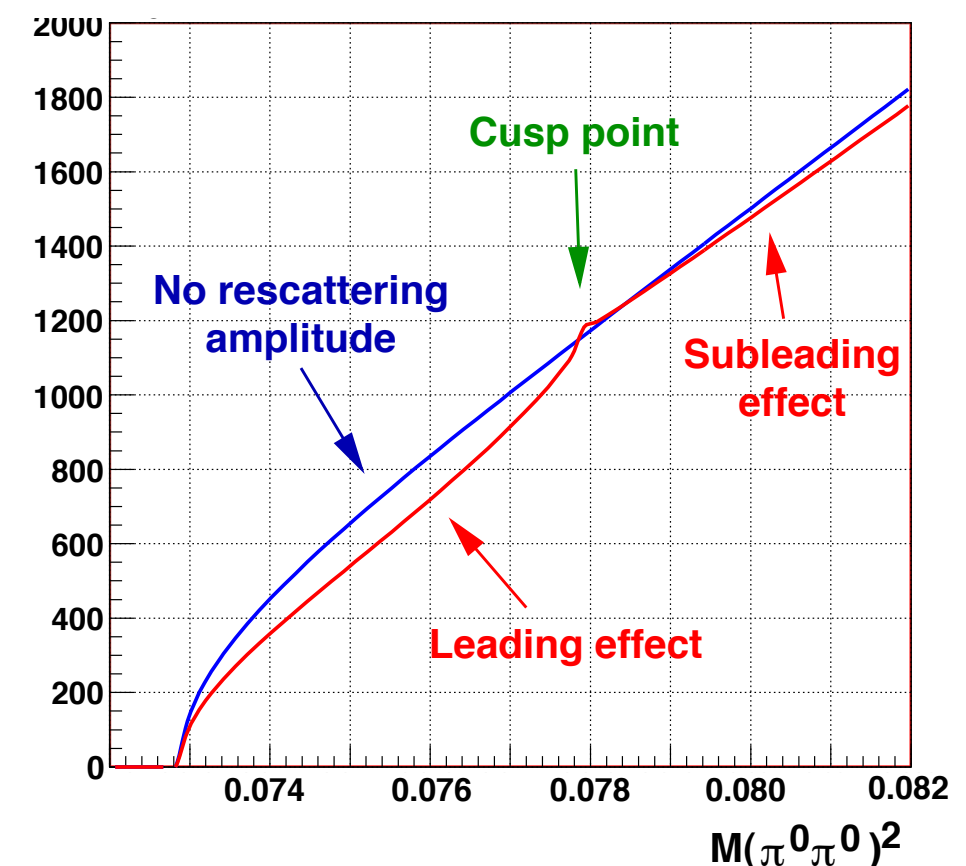
- rescattering corrections from
  $\pi^+ \pi^- \rightarrow \pi^0 \pi^0$, $\pi^+ \pi^0 \rightarrow \pi^+ \pi^0$, ...

- two-loop level $O(a_i^2)$ corrections

  $\Rightarrow$ sensitivity to a_2 alone

- no $O(a_i^3)$, no radiative corrections

Theoretical Approach (BB)

Approach by the Bern-Bonn group:

- based on an effective non-relativistic lagrangian

- different structure of the expansion (w.r.t. CI)

- simultaneous fitting of neutral and charged amplitudes to extract Dalitz plot slope parameters (modified w.r.t. PDG parametrization)

- electromagnetic effects and radiative corrections outside the cusp point are included

➜ **provides so far most complete description of rescattering effect**
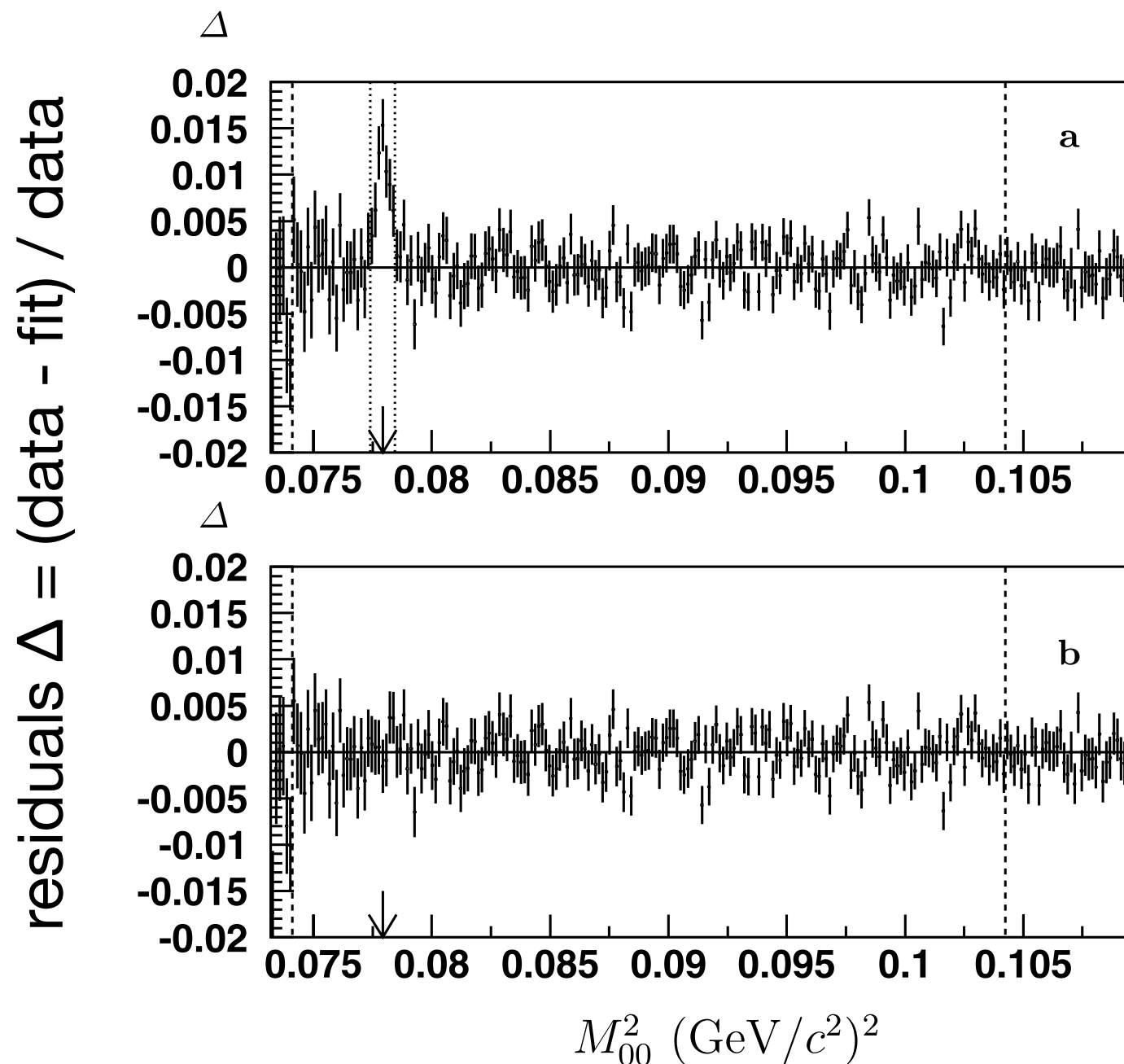
(Colangelo, Gasser, Kubis, Rusetsky, PLB 638 (2006) 187;

Bissinger, Fuhrer, Gasser, Kubis, Rusetsky, PLB 659 (2008) 576; NPH B806 (2009) 178)

Fit to the M($\pi^0\pi^0$) Spectrum (BB Model)

Free fit parameters: a₀-a₂, a₂, Dalitz plot parameters, normalizations

(fit also includes K → $\pi^\pm$ π^+ π^- decays)

7 bins around
cusp/pionium excluded

pionium fraction f$_{atom}$
left free in the fit

Fit Results on $a_0 - a_2$ and a_2

fit	χ^2/ndf	$a_0 - a_2$	a_2	f_{atom}
------	------	------	------	------
CI	206.3/195	0.2727(46)	-0.0392(80)	0.0533(91)
CI (a)	201.6/189	0.2689(50)	-0.0344(86)	0.0533
CI (c)	210.6/196	0.2749(21)	-0.0413	0.0441(76)
CI (a,c)	207.6/190	0.2741(21)	-0.0415	0.0441
------	------	------	------	------
BB	462.9/452	0.2815(43)	-0.0693(136)	0.0530(95)
BB (a)	458.5/446	0.2775(48)	-0.0593(142)	0.0542
BB (c)	467.3/453	0.2737(26)	-0.0417	0.0647(76)
BB (a,c)	459.8/447	0.2722(27)	-0.0421	0.0647

Rad. corr. off

a: pionium f_{atom} fixed

c: with ChPT constraint

a,c: both

(...): statistical error

fit	χ^2/ndf	$a_0 - a_2$	a_2	f_{atom}
CI	205.6/195	0.2483(45)	-0.0092(91)	0.0625(92)
CI (a)	202.9/189	0.2461(49)	-0.0061(98)	0.0625
CI (c)	222.1/196	0.2646(21)	-0.0443	0.0420(77)
CI (a,c)	219.7/190	0.2645(22)	-0.0444	0.0420
------	------	------	------	------
BB	**477.4/452**	**0.2571(48)**	**-0.0241(129)**	**0.0631(97)**
BB (a)	474.4/446	0.2544(51)	-0.0194(132)	0.0631
BB (c)	479.8/453	0.2633(24)	-0.0447	0.0538(77)
BB (ac)	478.1/447	0.2627(25)	-0.0449	0.0538

Rad. corr. on

← **final result**

(f_{atom}, $a_0 - a_2$, a_2 free in the fit)

Cusp Results on $a_0 - a_2$ and a_2

$a_2 m_+$

with rad. corr.

CI

BB

ChPT constraint

CI

BB

without rad. corr.

$(a_0 - a_2)m_+$
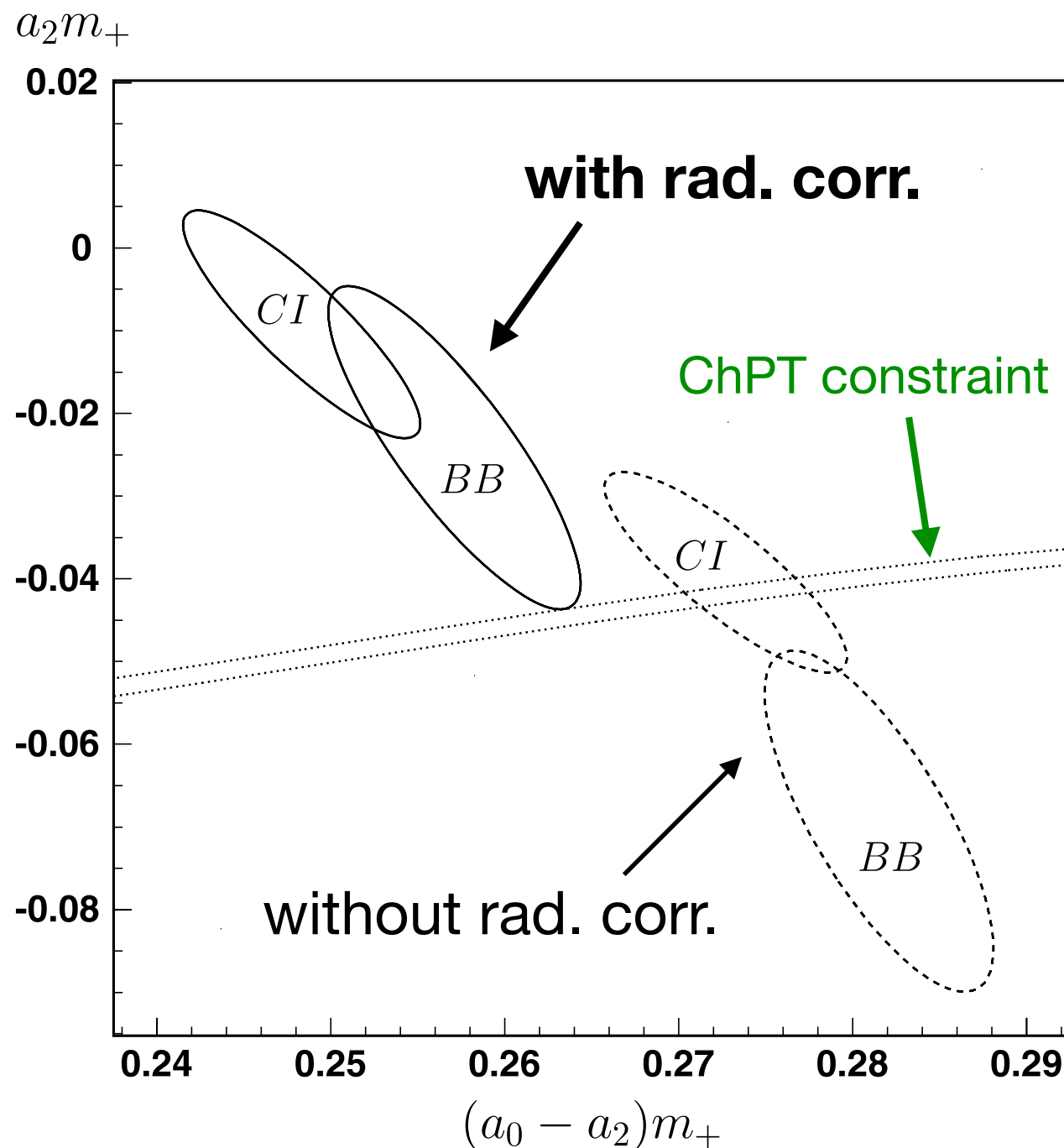
(only statistical uncertainties shown)

Final result: (EPJC 64 (2009) 589)

$a_0-a_2 = 0.257(5)_{stat}(3)_{sys}(1)_{ext}$

$a_2 = -0.024(13)_{stat}(9)_{sys}(2)_{ext}$

(statistical correlation -0.839)

With ChPT constraint:

$a_0-a_2 = 0.2633(24)_{stat}(14)_{sys}(19)_{ext}$
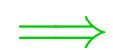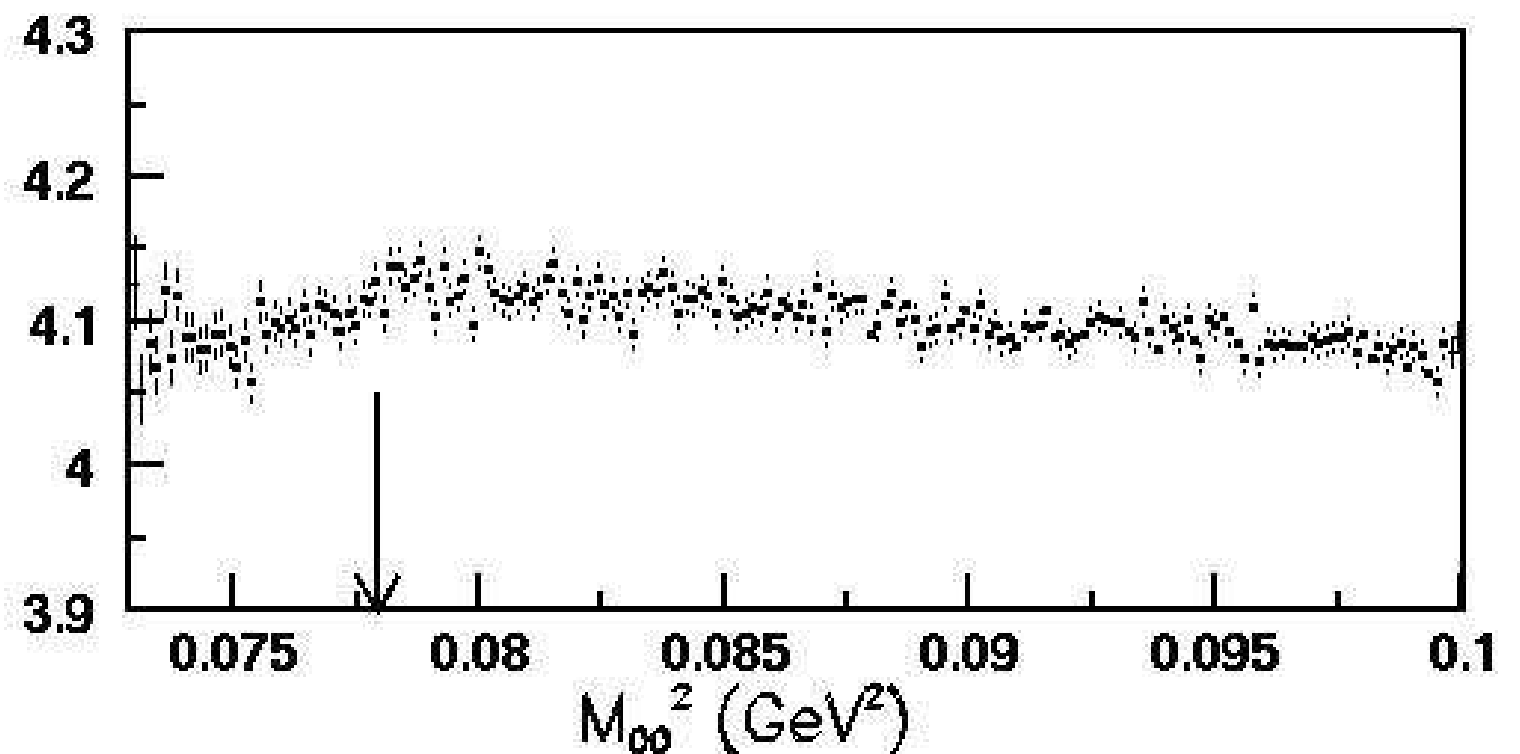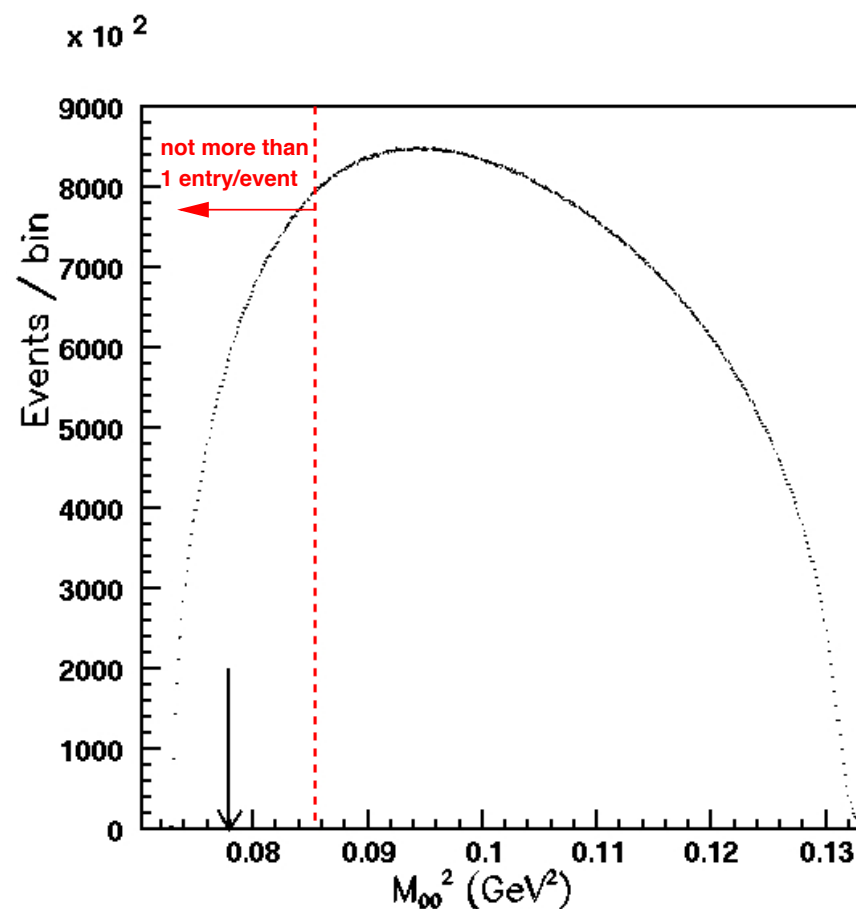
ChPT prediction:

$a_0-a_2 = 0.265(4)$
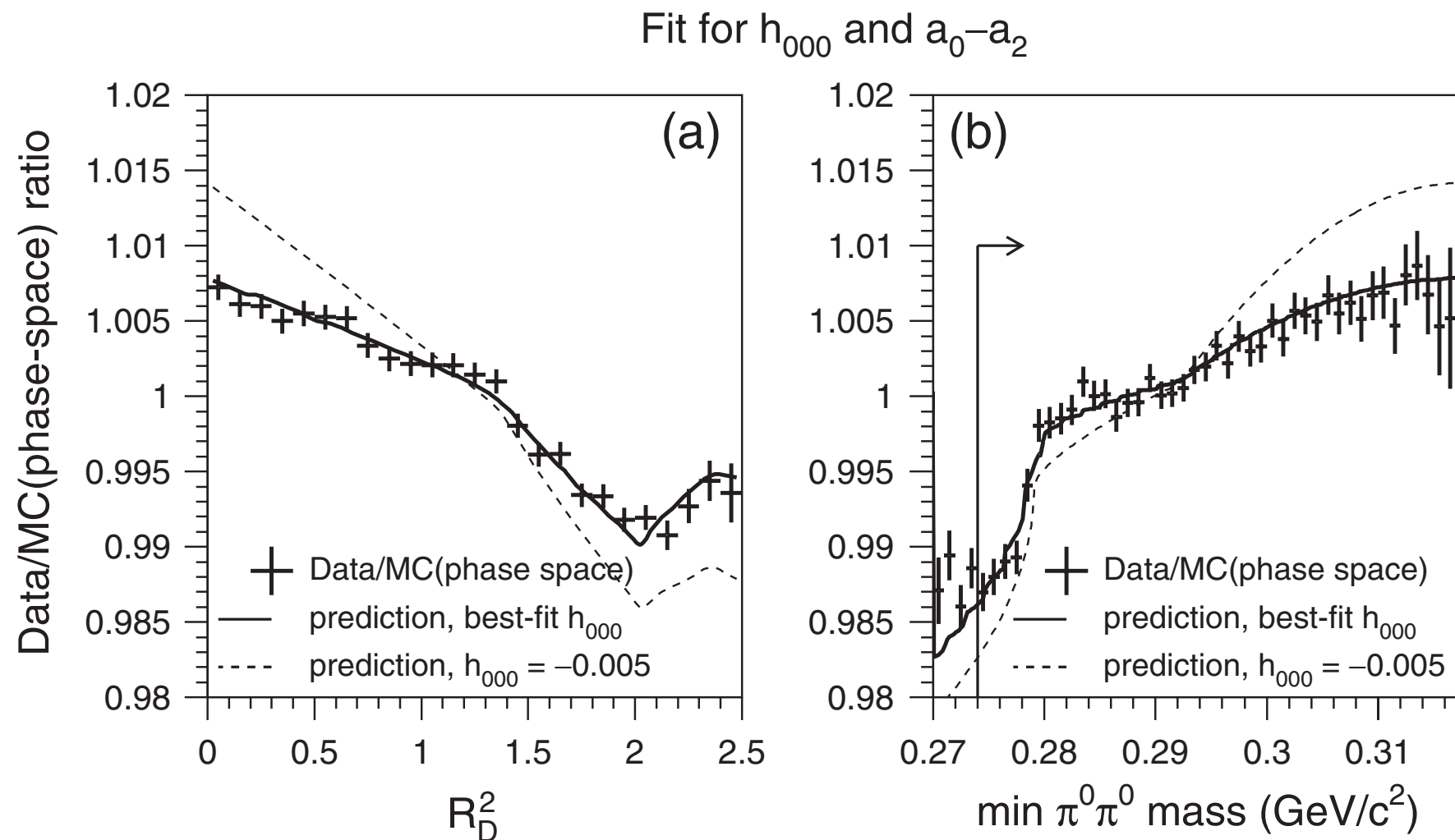
Rescattering in $K_L \to \pi^0\pi^0\pi^0$ (NA48/2)

NA48/2 data taking in 2000:

$\Longrightarrow$ **Evidence for a change in slope near the cusp point.**

Rescattering in $K_L \to \pi^0\pi^0\pi^0$ (KTeV)
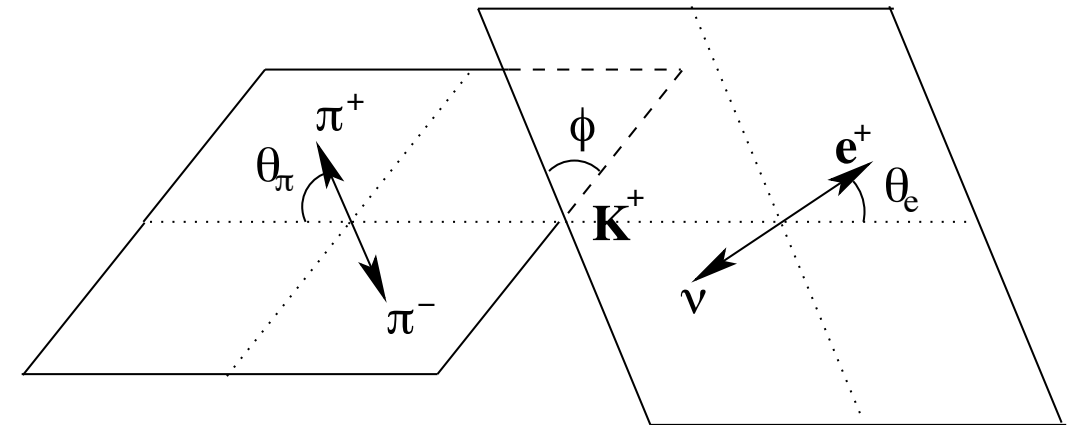
Fit for h_{000} and $a_0 - a_2$

$$m_{\pi^+}(a_0 - a_2) = 0.215 \pm 0.014_{\text{stat}} \pm 0.025_{\text{syst}} \pm 0.006_{\text{ext}}$$

$$h_{000} = (-2.09 \pm 0.62_{\text{stat}} \pm 0.72_{\text{syst}} \pm 0.28_{\text{ext}}) \times 10^{-3}$$

$\pi\pi$ Scattering Lengths from K_{e4} Decays

K_{e4} decay:

- $\mathbf{K_{e4} \; = \; K^{\pm} \rightarrow \pi^+ \, \pi^- \, e^{\pm} \, \nu}$

- Very rare: **Br(K_{e4}) ~ 4 x 10^{-5}**

Measurement of $\pi\pi$ scattering in $\mathbf{K_{e4}}$:

- K_{e4} decay amplitude depends on two complex phases:

  — $\delta_0 = \pi\pi$ scattering phase shift for $I = 0$, $l = 0$ (S-wave)

  — $\delta_1 = \pi\pi$ scattering phase shift for $I = 1$, $l = 1$ (P-wave)

  ($I = 2$ suppressed by $\Delta I = \frac{1}{2}$ rule)

- Decay rate depends on difference $\delta = \delta_0 - \delta_1$, with $\delta = \delta(m_{\pi\pi})$.

- $\delta \neq 0$ implies asymmetric distribution of lepton w.r.t. $\pi\pi$ plane.
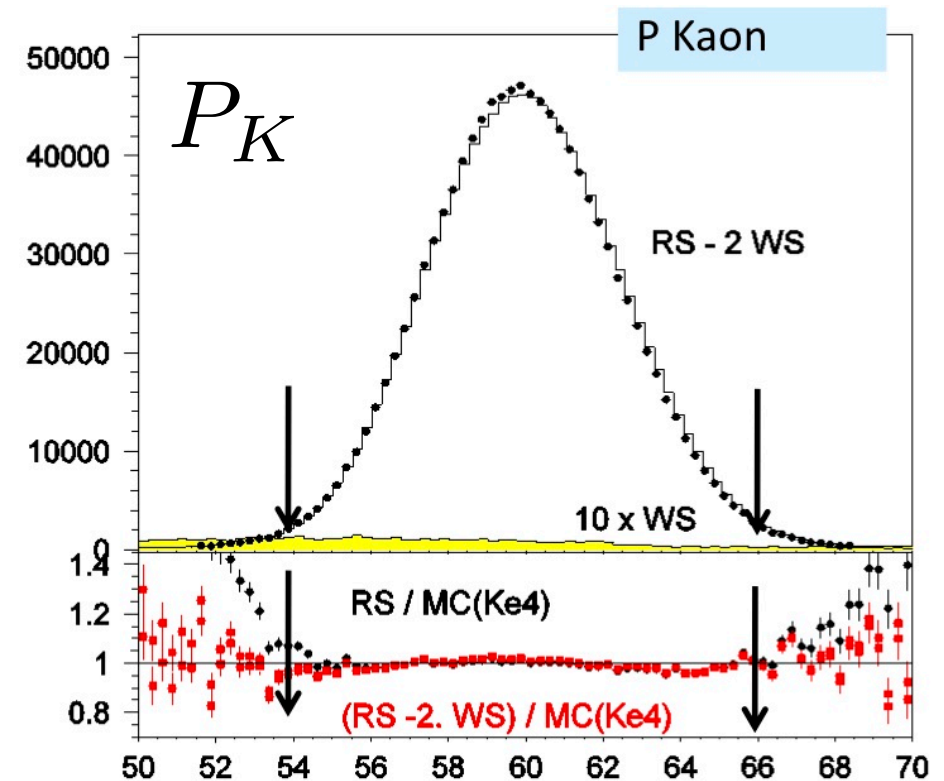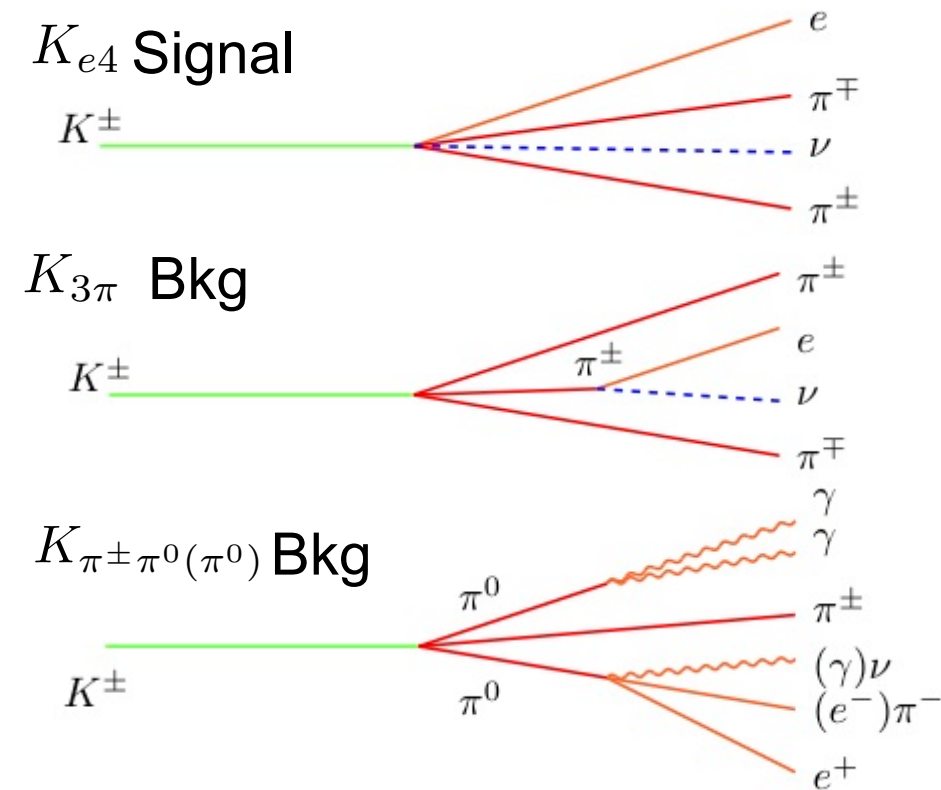
K$_{e4}$ Selection

K$_{e4}$ selection:

- 3 charged tracks and 1 good vertex
- 2 opposite-sign pions, 1 electron ($E/p \sim 1$)
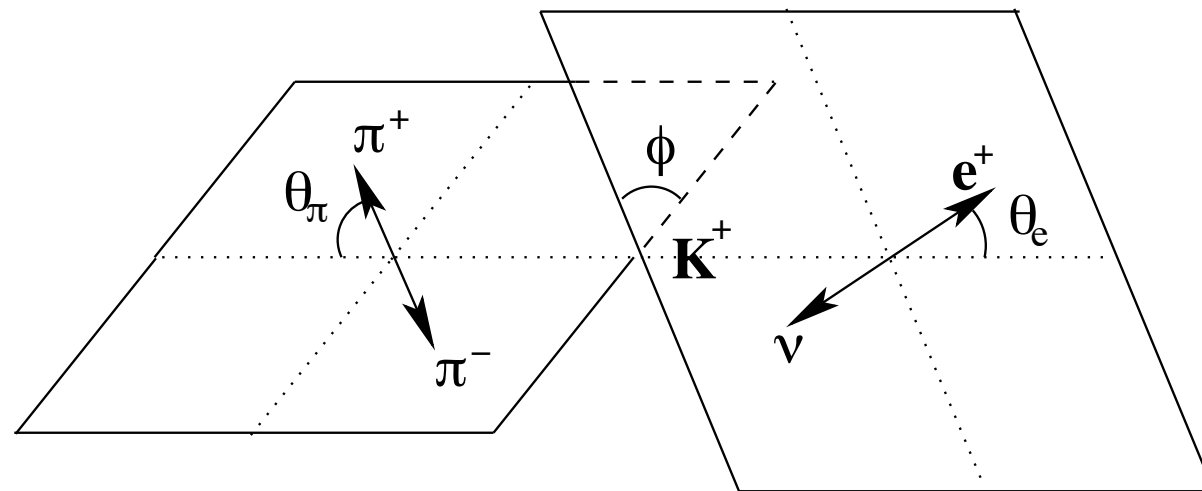- missing transverse momentum
- kaon momentum close to 60 GeV/c

Background:

- $\mathbf{K^{\pm} \rightarrow \pi^{\pm} \pi^{+} \pi^{-}}$ with $\pi \rightarrow e\nu$ or mis-identified pion
- $\mathbf{K^{\pm} \rightarrow \pi^{\pm} \pi^{0} (\pi^{0})}$ with $\pi^0 \rightarrow e^+ e^- \nu$ and mis-identified electron
- **Background estimation** from wrong-sign $\pi^+ \pi^+ e^-$ events

➜ **Background ~ 0.6 %**

K_{e4} Signal

$K_{3\pi}$ Bkg

$K_{\pi^{\pm}\pi^{0}(\pi^{0})}$ Bkg

P_K

P Kaon

RS - 2 WS

10 x WS

RS / MC(Ke4)

(RS -2. WS) / MC(Ke4)

Kinematic Variables in K_{e4}

K_{e4} is 4-body decay $\implies$ 5 independent kinematic variables.
(Cabibbo-Maksymowicz variables)

$s_\pi = M_{\pi\pi}^2$ Invariant di-pion mass squared.

$s_e = M_{e\nu}^2$ Invariant di-lepton mass squared.

θ_π Angle of π^+ w.r.t. $\pi\pi$ direction of flight in $\pi\pi$ rest frame.

θ_e Angle of e^+ w.r.t. $e\nu$ direction of flight in $e\nu$ rest frame.

ϕ Angle of $\pi\pi$ decay plane w.r.t. $e\nu$ decay plane.

K_{e4} Fitting Procedure

- **Full event sample** (2003+2004): **1.13 million K_{e4} decays**

- **Fit** in *iso-populated boxes* in the 5-dim. CM variables:

$$10(\mathbf{M_{\pi\pi}}) \times 5(\mathbf{M_{e\nu}}) \times 5(\cos\theta_{\mathbf{e}}) \times 5(\cos\theta_\pi) \times 12(\phi) = 15000 \text{ Boxes}$$

- Assuming constant form factors, K^+ and K^- samples fitted separately in **10 independent $M_{\pi\pi}$ bins** and then combined in each $M_{\pi\pi}$ bin.
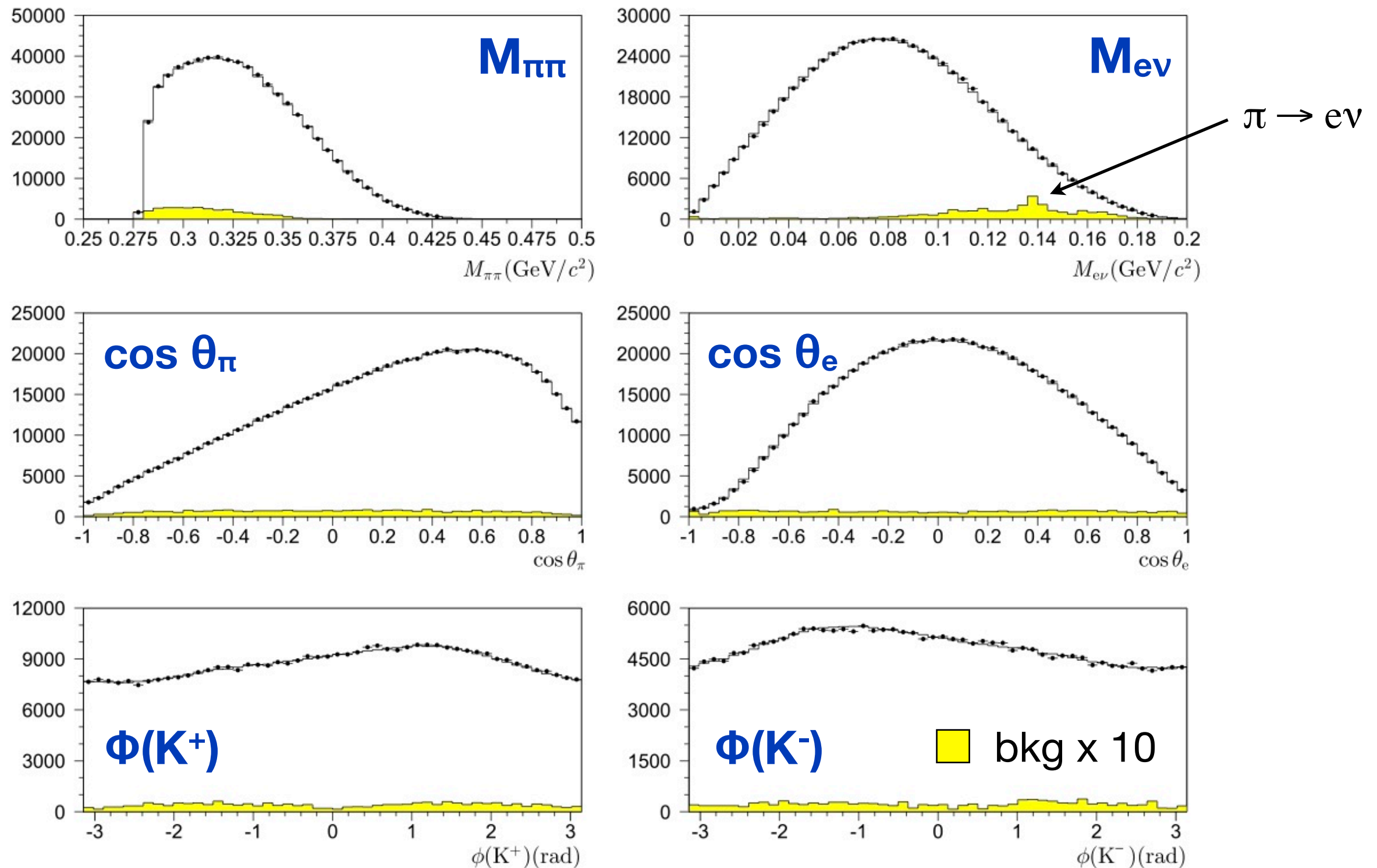
Data:

K^+ sample: 726 400 events	→	48 events/box
K^- sample: 404 400 events	→	27 events/box

MC:

K^+ sample: 17.4×10^6 events	→	1160 events/box
K^- sample: 9.7×10^6 events	→	650 events/box

K$_{e4}$ Fit Results

K$_{e4}$ Form Factor Results

Partial wave expansion of form factors:

$$F = F_s e^{i\delta_s} + F_p e^{i\delta_p} \cos\theta_\pi + d \text{ wave} \ldots$$

$$G = G_p e^{i\delta_g} + d \text{ wave} \ldots$$

$$H = H_p e^{i\delta_h} + d \text{ wave} \ldots$$

Single form factors parametrized in Taylor expansion:

$$F_s = f_s + f_s' q^2 + f_s'' q^4 + f_e' S_e/4m_\pi^2 + \ldots$$

$$F_p = f_p + f_p' q^2 + \ldots$$

$$G_p = g_p + g_p' q^2 + \ldots$$

$$H_p = h_p + h_p' q^2 + \ldots$$

$$\delta(q^2) = \delta_s - \delta_p.$$

$$q^2 = (S_\pi/4m_\pi^2) - 1$$

- All form factors measured w.r.t. f_s

- Systematics from acceptance and background control

$$f_s'/f_s = 0.152 \pm 0.007_{\text{stat}} \pm 0.005_{\text{syst}}$$

$$f_s''/f_s = -0.073 \pm 0.007_{\text{stat}} \pm 0.006_{\text{syst}}$$

$$f_e'/f_s = 0.068 \pm 0.006_{\text{stat}} \pm 0.007_{\text{syst}}$$

$$f_p/f_s = -0.048 \pm 0.003_{\text{stat}} \pm 0.004_{\text{syst}}$$

$$g_p/f_s = 0.868 \pm 0.010_{\text{stat}} \pm 0.010_{\text{syst}}$$

$$g_p'/f_s = 0.089 \pm 0.017_{\text{stat}} \pm 0.013_{\text{syst}}$$

$$h_p/f_s = -0.398 \pm 0.015_{\text{stat}} \pm 0.008_{\text{syst}}$$

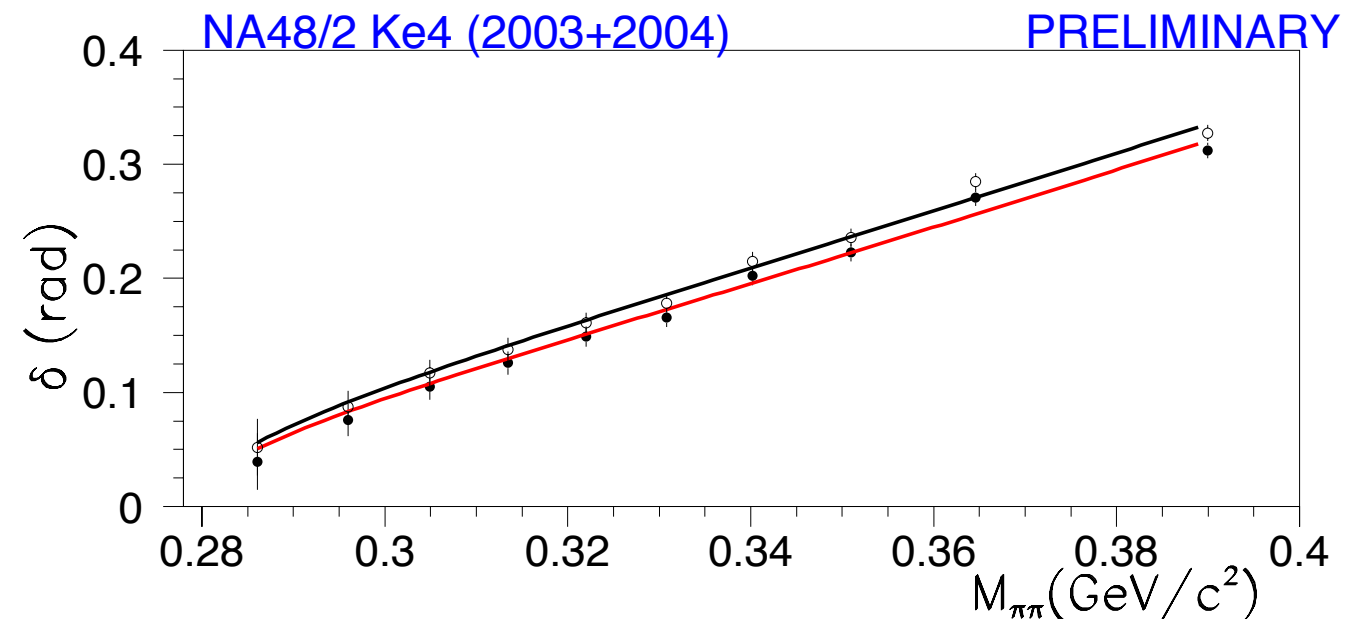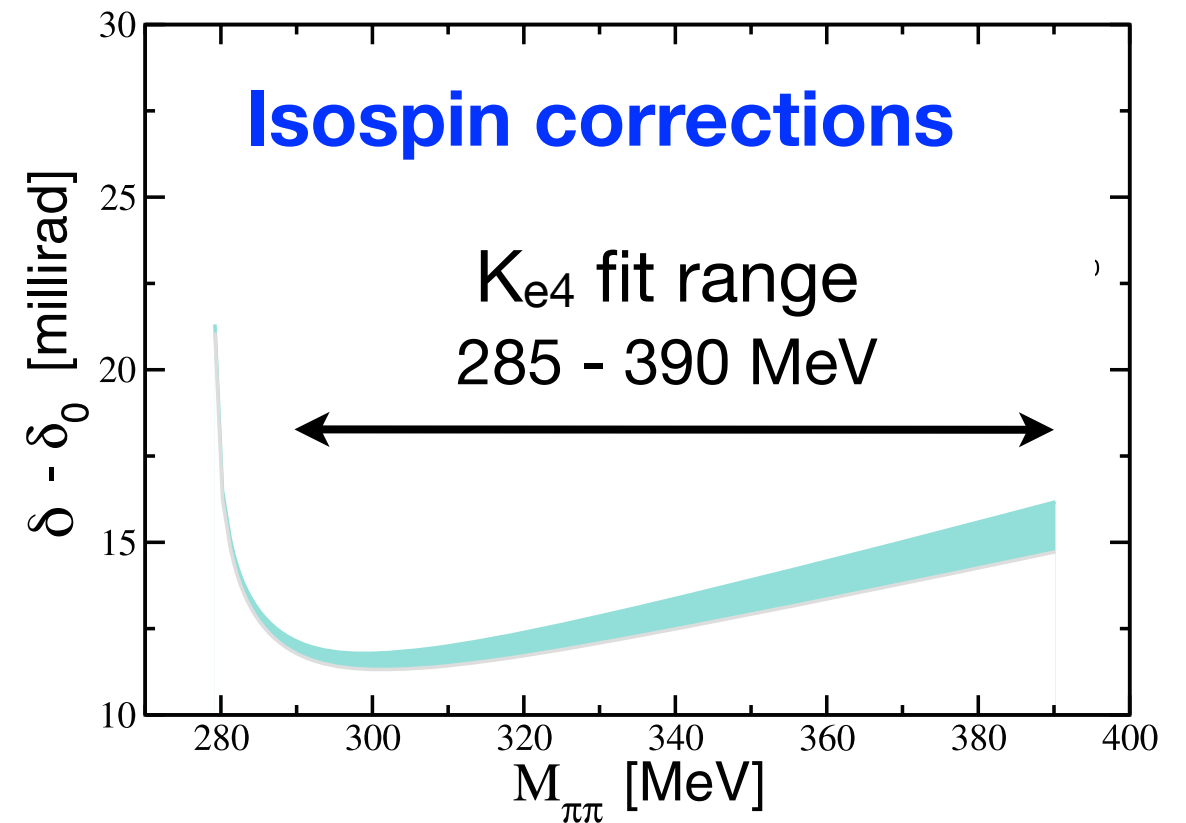➜ **First evidence of non-zero f'_e and f_p !**

From phase shifts to scattering lengths

Corrections to be applied:

- ■ **Radiative effects:**
  Included in the simulation
  (Coulomb attraction, IB).

- ■ **Mass effects:**
  Isospin corrections have
  to be applied to δ.
  Developed in close
  collaboration with NA48/2.
  (Colangelo, Gasser, Rusetsky,
  EPJC 59 (2009) 777)

  ⬇

  Effect of 10-15 mrad on δ
  (stat. precision ~7-8 mrad)

Isospin corrections

K_{e4} fit range
285 - 390 MeV

$\delta - \delta_0$ [millirad] vs $M_{\pi\pi}$ [MeV]

NA48/2 Ke4 (2003+2004)   PRELIMINARY

δ (rad) vs $M_{\pi\pi}$ (GeV/c^2)

NA48/2 Fit of Phase Shift $\delta = \delta_0 - \delta_1$
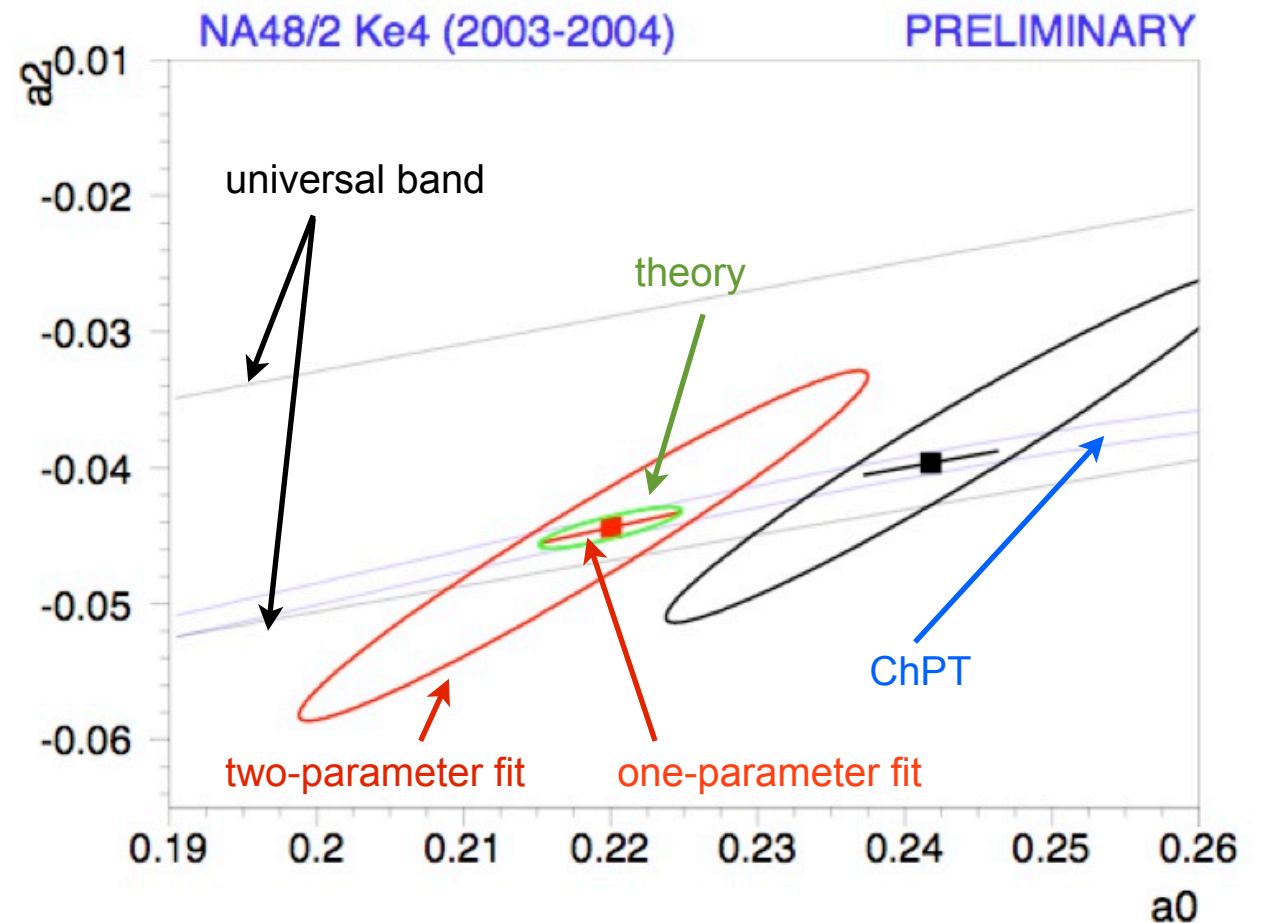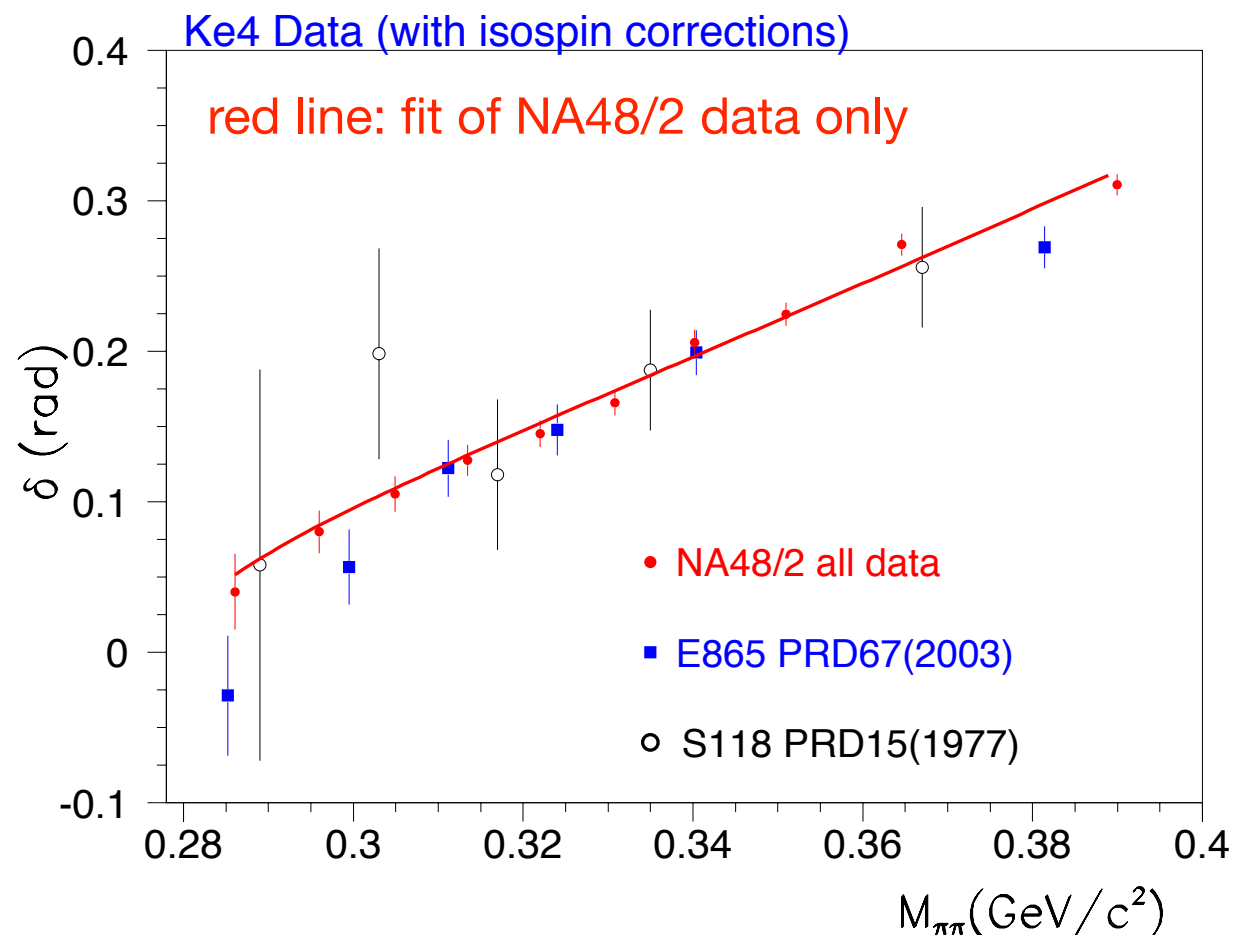
Two-parameter fit:

$$a_0 = 0.2220 \pm 0.0128_{\text{stat}} \pm 0.0050_{\text{syst}} \pm 0.0037_{\text{theo}}$$
$$a_2 = -0.0432 \pm 0.0086_{\text{stat}} \pm 0.0034_{\text{syst}} \pm 0.0028_{\text{theo}}$$

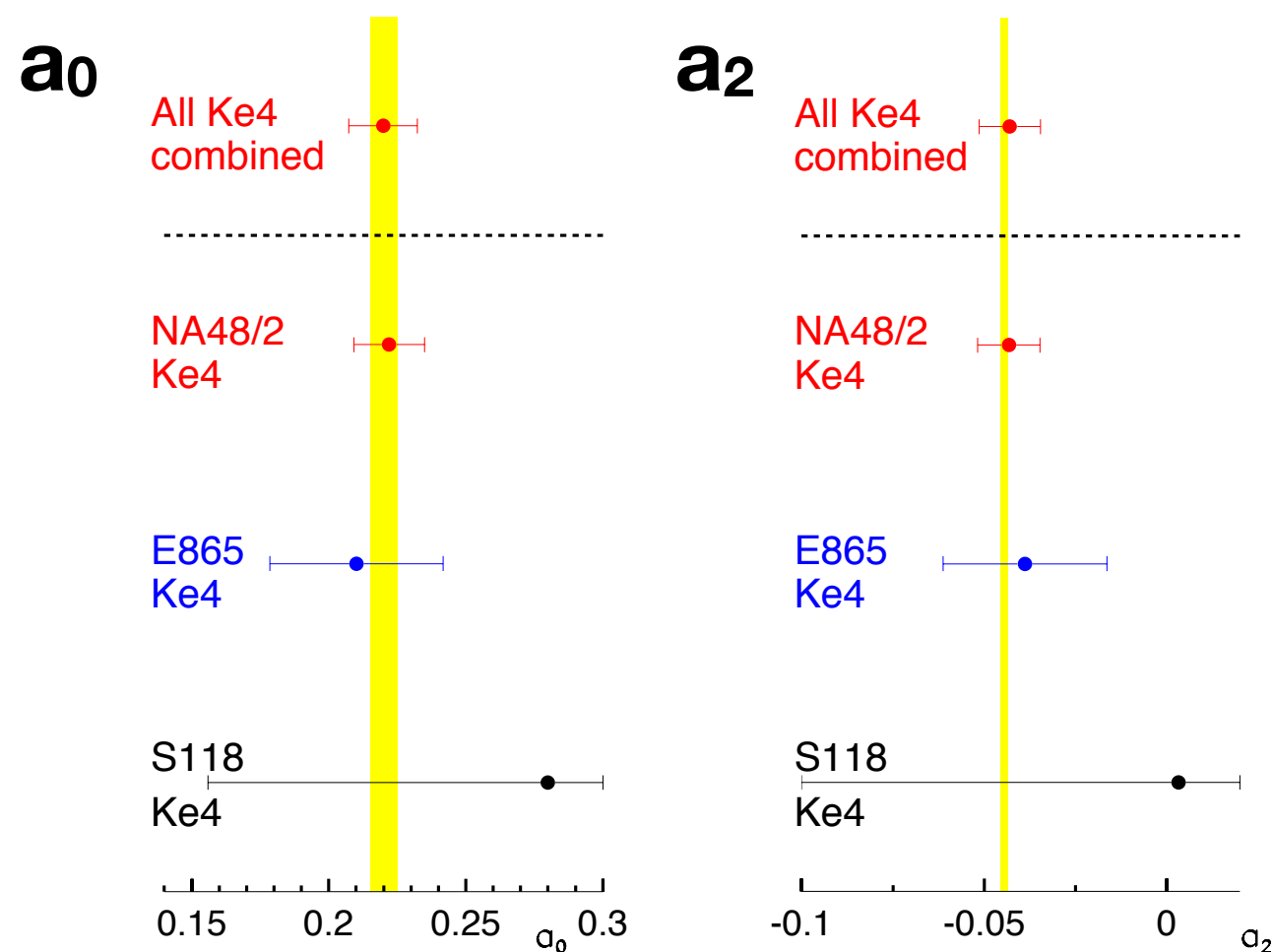**One-parameter fit:**
(with ChPT constraint)

$$a_0 = 0.2206 \pm 0.0049_{\text{stat}} \pm 0.0018_{\text{syst}} \pm 0.0064_{\text{theo}}$$

Comparison of **K_{e4} measurements**
(without ChPT constraint, old experiments isospin corrected):

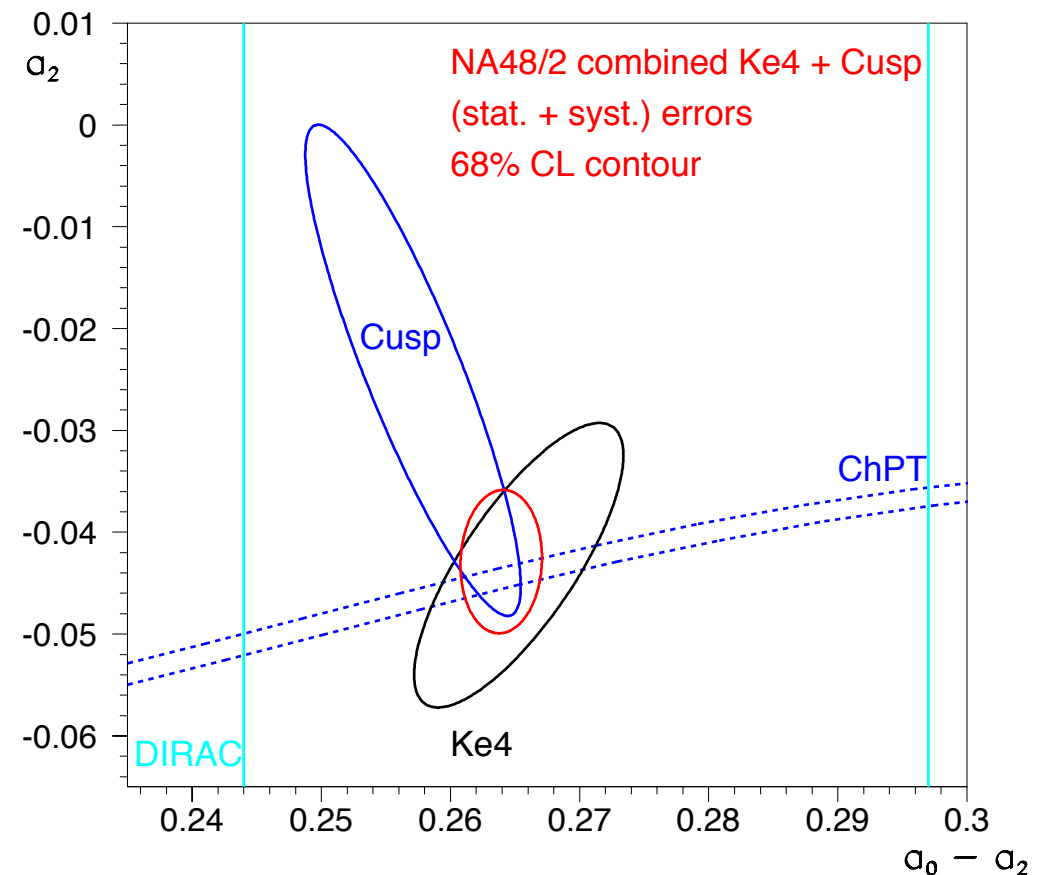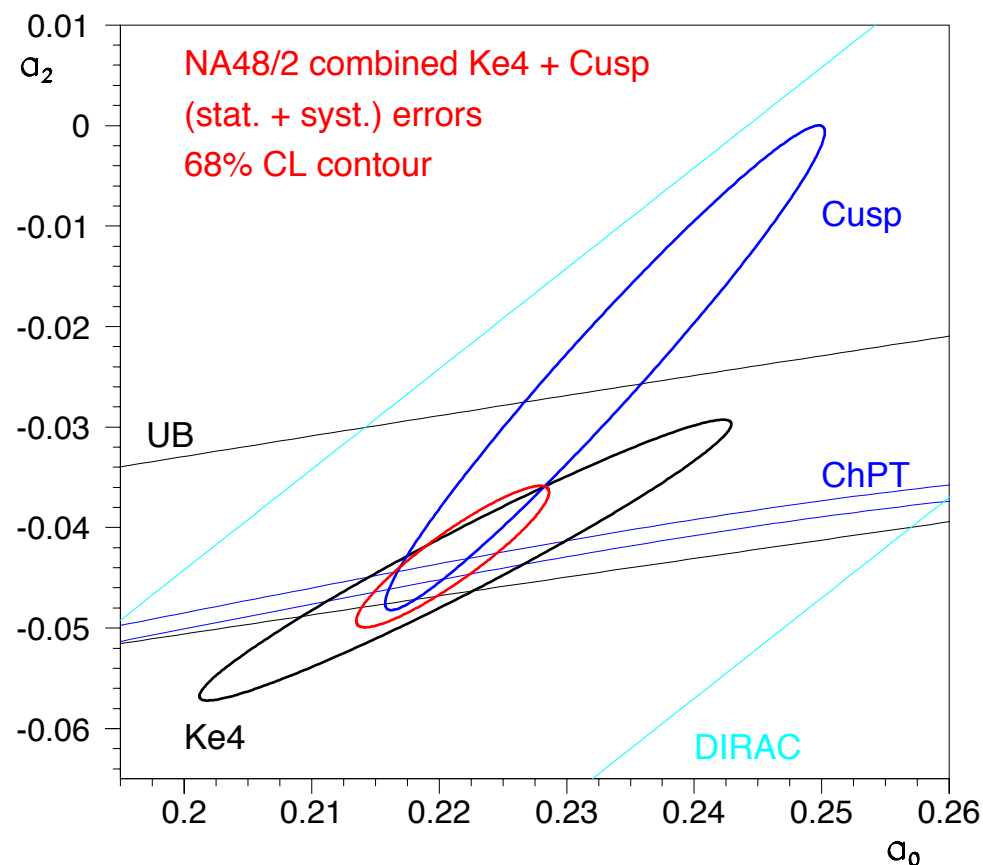■ NA48/2 dominates K_{e4} measurements

■ **Perfect agreement with ChPT prediction!**

Yellow lines: Theory prediction

(*not* combination of measurements)
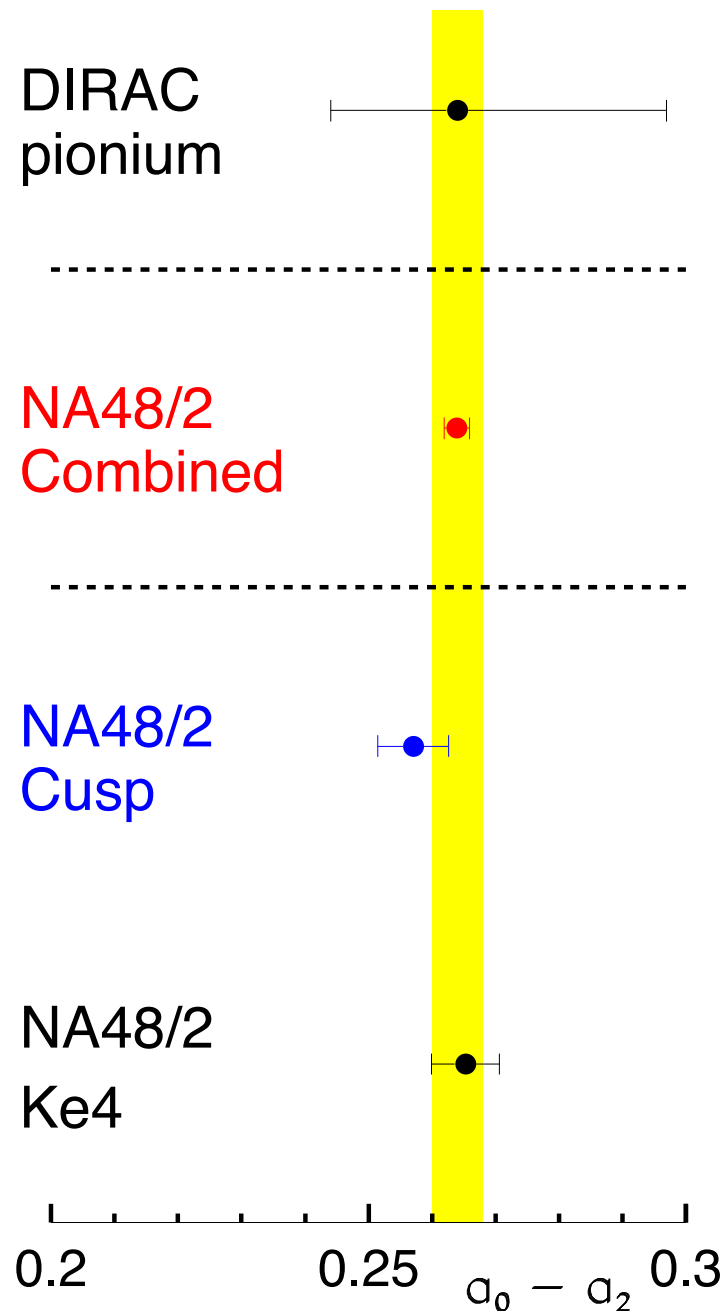
Combination of Cusp and K$_{e4}$

Two independent measurements with different samples, different systematics and different theory:

$$
\begin{aligned}
a_0 &= 0.2210 \pm 0.0047_{\text{stat}} \pm 0.0040_{\text{syst}} \\
a_2 &= -0.0429 \pm 0.0044_{\text{stat}} \pm 0.0028_{\text{syst}} \\
a_0 - a_2 &= 0.2639 \pm 0.0020_{\text{stat}} \pm 0.0015_{\text{syst}}
\end{aligned}
$$

Combination with other Measurements

All experiments:

DIRAC
pionium

NA48/2
Combined

NA48/2
Cusp

NA48/2
Ke4

0.2   0.25 $a_0 - a_2$ 0.3

**Experimental data
has reached
theoretical precision**

**Perfect agreement
with ChPT prediction**

Yellow line: Theory prediction

(*not* combination of measurements)

(no uncertainty from theory on cusp result)

Conclusions

- **Kaon decays** give unique possibility to study low-energy hadronic interactions with high precision

- Due to very high statistics, **NA48/2** can check ChPT predictions with high accuracy using both $K^\pm \to \pi^\pm \pi^0 \pi^0$ and $K^\pm \to \pi^+ \pi^- e^\pm \nu$ (K_{e4}) decays.

- Achieved experimental precision has reached theoretical precision.
  - ➜ **very strong test of the theory**

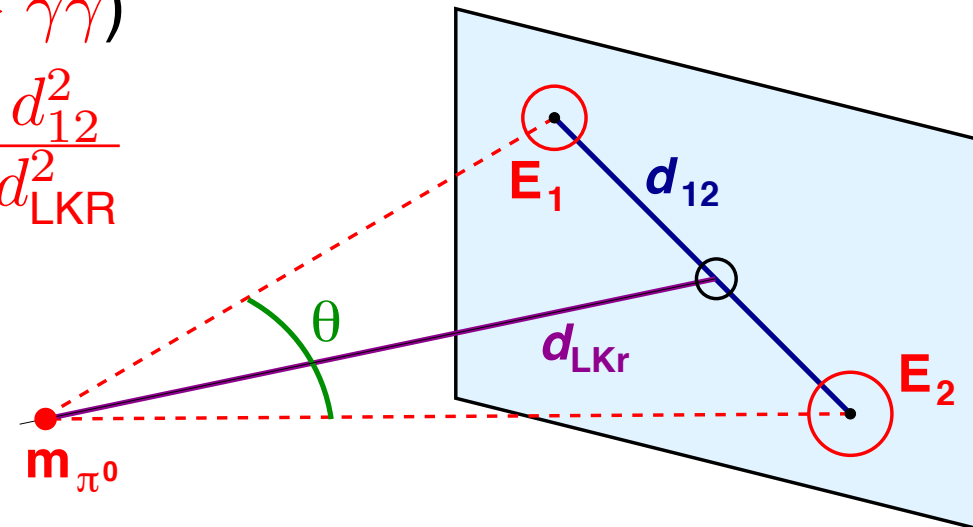- **Very good agreement with theoretical prediction from ChPT.**

Spares

K$^\pm \to \pi^\pm\pi^0\pi^0$: M($\pi^0\pi^0$) Reconstruction

Decays in two photons: (z.B. $\pi^0 \to \gamma\gamma$)

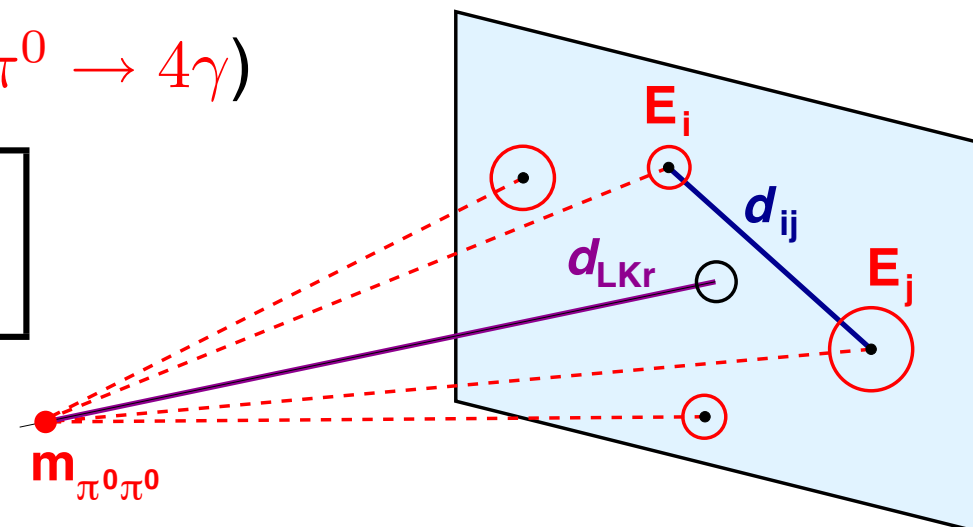$$m_{\pi^0}^2 = 2\,E_1\,E_2\,(1 - \cos\theta) = E_1\,E_2\,\frac{d_{12}^2}{d_{\text{LKR}}^2}$$

If mass m_{π^0} known:

$$d_{\text{LKR}} = \frac{1}{m_{\pi^0}}\sqrt{E_1\,E_2\,d_{12}^2}$$

Decays in many photons: (z.B. $\pi^0\pi^0 \to 4\gamma$)

$$d_{\text{LKR}} = \frac{1}{m_{\pi^0\pi^0}}\sqrt{\sum_{i,j;\,i>j} E_i\,E_j\,d_{ij}^2}$$

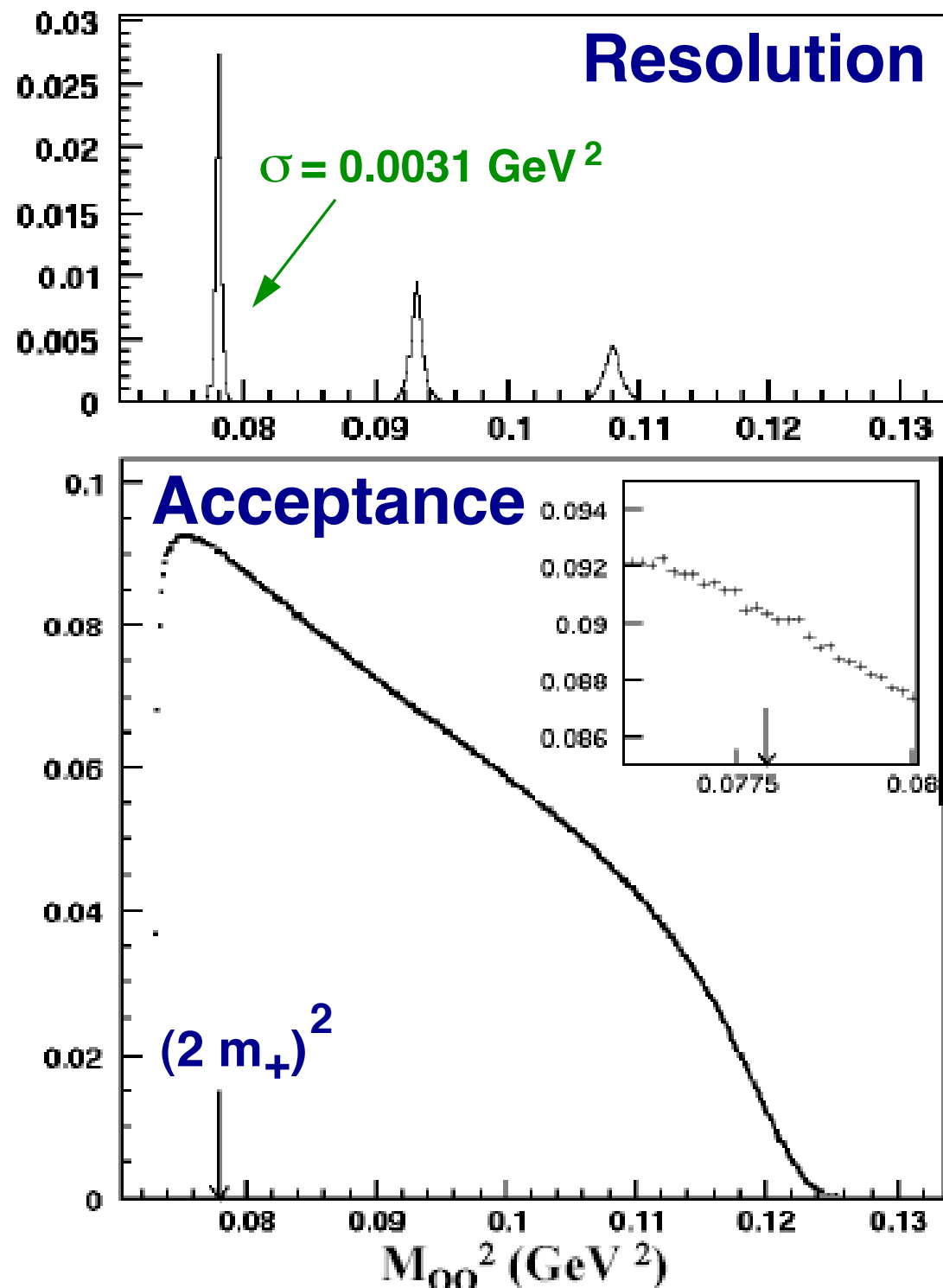**Turning it around:**

Vertex d_{LKR} always the same!

$\implies$ Invariant mass $m_{\pi^0\pi^0}$ only from LKr information:

$$m_{\pi^0\pi^0} = m_{\pi^0}\sqrt{\sum_{i,j;\,i>j} E_i\,E_j\,d_{ij}^2}\bigg/\frac{1}{2}\left(\sqrt{E_1\,E_2\,d_{12}^2} + \sqrt{E_3\,E_4\,d_{34}^2}\right)$$

$K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ Acceptance and Resolution

Resolution on $m^2_{\pi^0\pi^0}$:

- ■ Determined by **LKR resolution.**

- ■ Best at low $m^2_{\pi^0\pi^0}$ (due to kinematical constraints).

- ■ At $\mathbf{m^2_{\pi^0\pi^0} = (2\,m_{\pi^+})^2}$:

  $$\sigma = 0.0031\ \mathbf{GeV^2}$$

Acceptance:

- ■ Acceptance $\approx$ linearly varying around $(2\,m_{\pi^+})^2$.

- ■ Modelled by **Monte Carlo simulation.**

Rescattering in $K_L \to \pi^0 \pi^0 \pi^0$

NA48/2 data taking in 2000: