QED corrections to hadronic processes in Lattice QCD

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QED corrections to hadronic processes in Lattice QCD

N.Carrasco, VL, G.Martinelli, C.T.Sachrajda, N.Tantalo, C.Tarantino, M.Testa

PRD 91 (2015) 074506, arXiv: 1502.00257

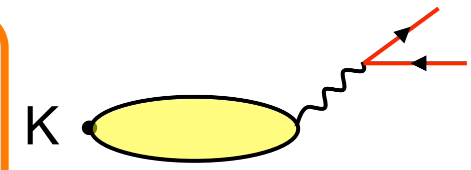
Outline of the talk

- Phenomenological motivations
- The method
- (Very) preliminary numerical results (not in the paper)

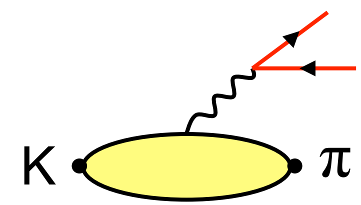
Motivations

- Consider the determination of V_{us} and V_{ud} from leptonic and semileptonic K and π decays

$$\frac{\Gamma\left(K^+ \to \ell^+ \nu_\ell(\gamma)\right)}{\Gamma\left(\pi^+ \to \ell^+ \nu_\ell(\gamma)\right)} = \left(\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi}\right)^2 \frac{m_K\left(1-m_\ell^2/m_K^2\right)^2}{m_\pi\left(1-m_\ell^2/m_\pi^2\right)^2}\left(1+\delta_{EM}+\delta_{SU(2)}\right)$$



$$\Gamma\left(K \to \pi \ell \nu(\gamma)\right) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{EW}\left(|V_{us}|\, f_+^{K^0\pi^-}(0)\right)^2 I_{K\ell}\left(1+\delta_{EM}^{K\ell}+\delta_{SU(2)}^{K\pi}\right)^2$$



- From the experimental measurements of the decay rates

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = 0.2758(5)$$

$$|V_{us}|\, f_+^{K^0\pi^-}(0) = 0.2163(5)$$

FlaviA net Kaon WG

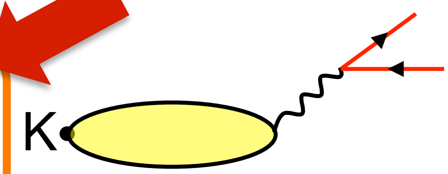
The accuracy is at the level of **0.2%** for both determinations

M.Antonelli *et al.*, EPJ C69 (2010) 399

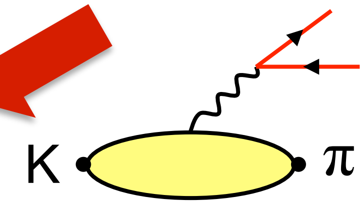
Electromagnetic and isospin breaking effects

- An important source of uncertainty are long distance electromagnetic and SU(2) breaking corrections

$$\frac{\Gamma\left(K^+\to \ell^+\nu_\ell(\gamma)\right)}{\Gamma\left(\pi^+\to\ell^+\nu_\ell(\gamma)\right)} = \left(\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi}\right)^2 \frac{m_K\left(1-m_\ell^2/m_K^2\right)^2}{m_\pi\left(1-m_\ell^2/m_\pi^2\right)^2}\boxed{\left(1+\delta_{EM}+\delta_{SU(2)}\right)}$$



$$\Gamma\left(K\to\pi\ell\nu(\gamma)\right) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{EW}\left(|V_{us}|\, f_+^{K^0\pi^-}(0)\right)^2 I_{K\ell}\boxed{\left(1+\delta_{EM}^{K\ell}+\delta_{SU(2)}^{K\pi}\right)^2}$$



For $\Gamma_{Kl2}/\Gamma_{\pi l2}$

At leading order in ChPT both δ_{EM} and $\delta_{SU(2)}$ can be expressed in terms of physical quantities (e.m. pion mass splitting, f_K/f_π, …)

- $\boxed{\delta_{EM} = -0.0069\,(17)}$ 25% of error due to higher orders ➡ 0.2% on $\Gamma_{Kl2}/\Gamma_{\pi l2}$

M.Knecht *et al.*, EPJ C12 (2000) 469; V.Cirigliano, H.Neufeld, PLB 700 (2011) 7

- $\boxed{\delta_{SU(2)} = \left(\frac{f_{K^+}/f_{\pi^+}}{f_K/f_\pi}\right)^2 - 1 = -0.0044\,(12)}$ 25% of error due to higher orders ➡ 0.1% on $\Gamma_{Kl2}/\Gamma_{\pi l2}$

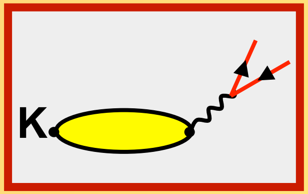
J.Gasser, H.Leutwyler, NPB 250 (1985) 465; V.Cirigliano, H.Neufeld, PLB 700 (2011) 7

ChPT is not applicable to D and B decays. Estimates are model dependent.

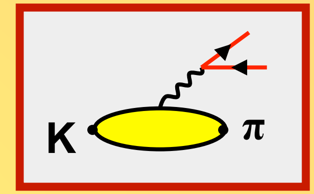
$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = 0.2758(5)$$ 0.2%

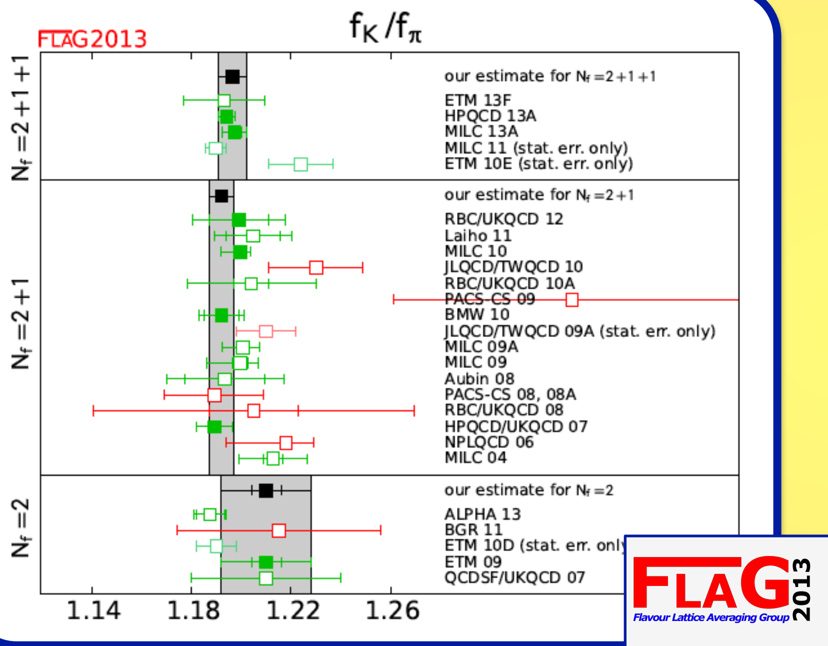
FlaviA net Kaon WG

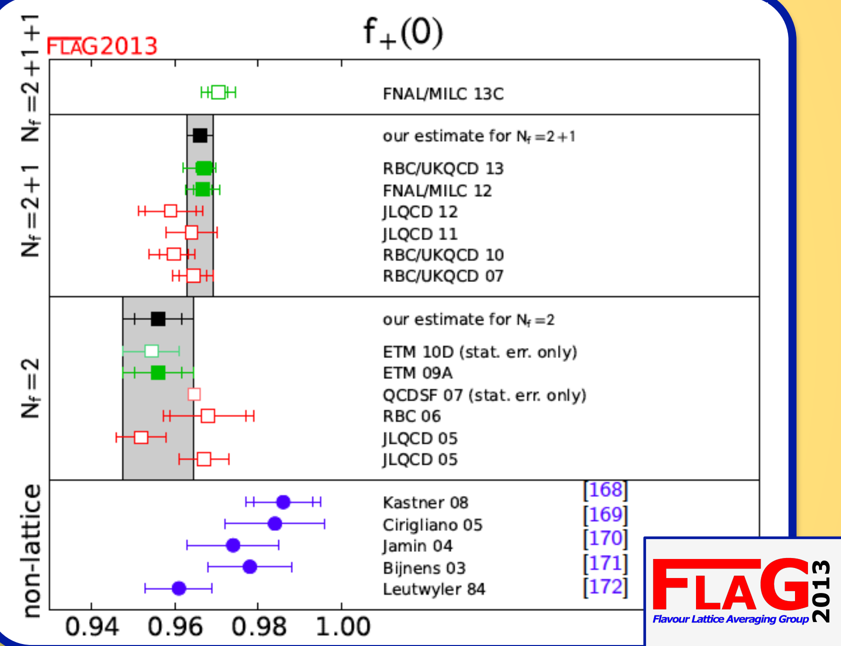
$$|V_{us}|\, f_+^{K^0\pi^-}(0) = 0.2163(5)$$ 0.2%



Lattice results for f_K/f_π and $f_+(0)$







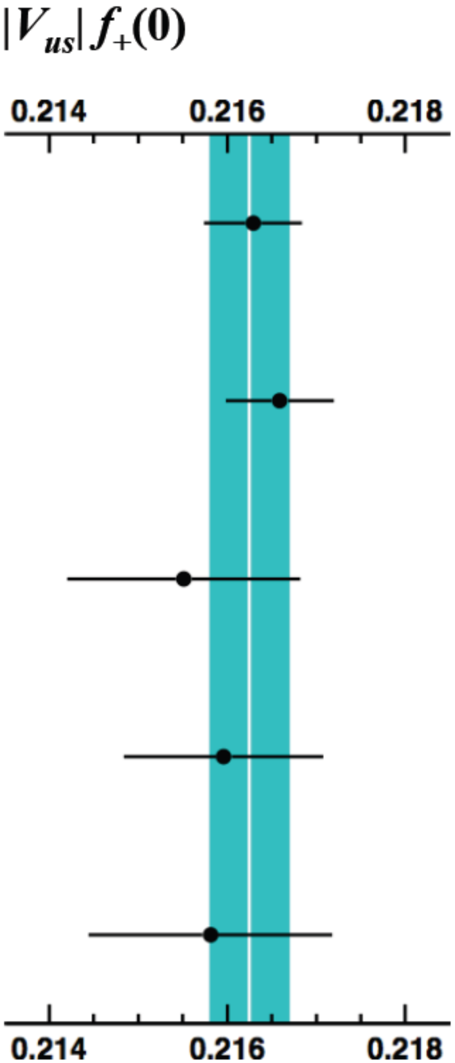
$f_{K^+}/f_{\pi^+} = 1.194(5)$ Nf=2+1+1 0.4%

$f_{K^+}/f_{\pi^+} = 1.192(5)$ Nf=2+1

$f_+(0) = 0.970(3)$ Nf=2+1+1 0.3%

$f_+(0) = 0.966(3)$ Nf=2+1

| Vus | f+(0) from world data: 2012



Approx. contrib. to % err from:

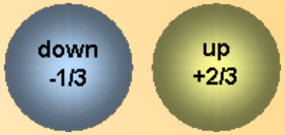
		% err	BR	τ	$\delta_{SU,EM}$	Int
K_Le3	**0.2163(5)**	**0.26**	0.09	0.20	0.11	0.05
$K_L\mu3$	**0.2166(6)**	**0.28**	0.15	0.18	0.11	0.06
K_Se3	**0.2155(13)**	**0.61**	0.60	0.02	0.11	0.05
$K^{\pm}e3$	**0.2160(11)**	**0.52**	0.31	0.09	0.41	0.04
$K^{\pm}\mu3$	**0.2158(13)**	**0.63**	0.47	0.08	0.41	0.06

Average: $|V_{us}|\,f_+(0)$ = 0.2163(5) **χ^2/ndf = 0.84/4 (93%)**

LEADING ISOSPIN BREAKING EFFECTS ON THE LATTICE

These effects are small because:

$m_u \neq m_d$: $O[(m_d - m_u)/\Lambda_{QCD}] \approx 1/100$ "Strong"



$Q_u \neq Q_d$: $O(\alpha_{e.m.}) \approx 1/100$ "Electromagnetic"

The electromagnetic and isospin breaking part of the Lagrangian can be treated as a perturbation

Expand in: $m_d - m_u$ + α_{em}

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Isospin breaking effects due to the up-down mass difference in lattice QCD

RM123 collaboration

PHYSICAL REVIEW D 87, 114505 (2013)

Leading isospin breaking effects on the lattice

M. de Divitiis,[1,2] R. Frezzotti,[1,2] V. Lubicz,[3,4] G. Martinelli,[5,6] R. Petronzio,[1,2] G. C. Rossi,[1,2] F. Sanfilippo,[7] S. Simula,[4] and N. Tantalo[1,2]

(RM123 Collaboration)   arXiv:1303.4896

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RM123 Collaboration

The (md-mu) expansion

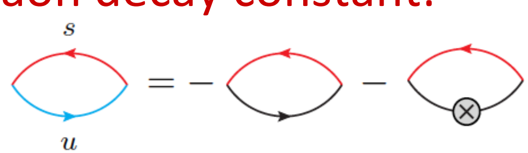
G.M.de Divitiis *et al.*, RM123 collaboration, JHEP 04 (2012) 124

- Identify the isospin breaking term in the action and expand in $\Delta m = (m_d - m_u)/2$

$$S_m = \sum_x \left[m_u \bar{u}u + m_d \bar{d}d \right] = \sum_x \left[\frac{1}{2}(m_u + m_d)(\bar{u}u + \bar{d}d) - \frac{1}{2}(m_d - m_u)(\bar{u}u - \bar{d}d) \right] = S_0 - \Delta m \hat{S}$$

$$\langle O \rangle = \frac{\int D\phi \, O \, e^{-S_0 + \Delta m \hat{S}}}{\int D\phi \, e^{-S_0 + \Delta m \hat{S}}} \overset{1st}{\simeq} \frac{\int D\phi \, O \, e^{-S_0} \left(1 + \Delta m \, \hat{S}\right)}{\int D\phi \, e^{-S_0} \left(1 + \Delta m \, \hat{S}\right)} \simeq \frac{\langle O \rangle_0 + \Delta m \langle O \, \hat{S} \rangle_0}{1 + \Delta m \cancel{\langle \hat{S} \rangle_0}} = \langle O \rangle_0 + \Delta m \langle O \, \hat{S} \rangle_0$$

- For the kaon decay constant:

$C_{K^+K^-}(t) = -$  $+ \mathcal{O}(\Delta m_{ud})^2$

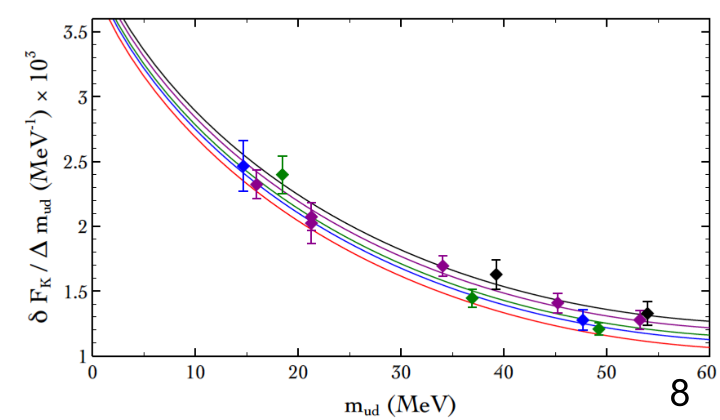
$$\boxed{\delta_{SU(2)} = -0.0080(7)}$$

Lattice - Nf=2
RM123 collab. (2012)

which is ~2.6 σ larger than

$$\boxed{\delta_{SU(2)} = -0.0044(12)}$$

ChPT
Cirigliano, Neufeld (2011)

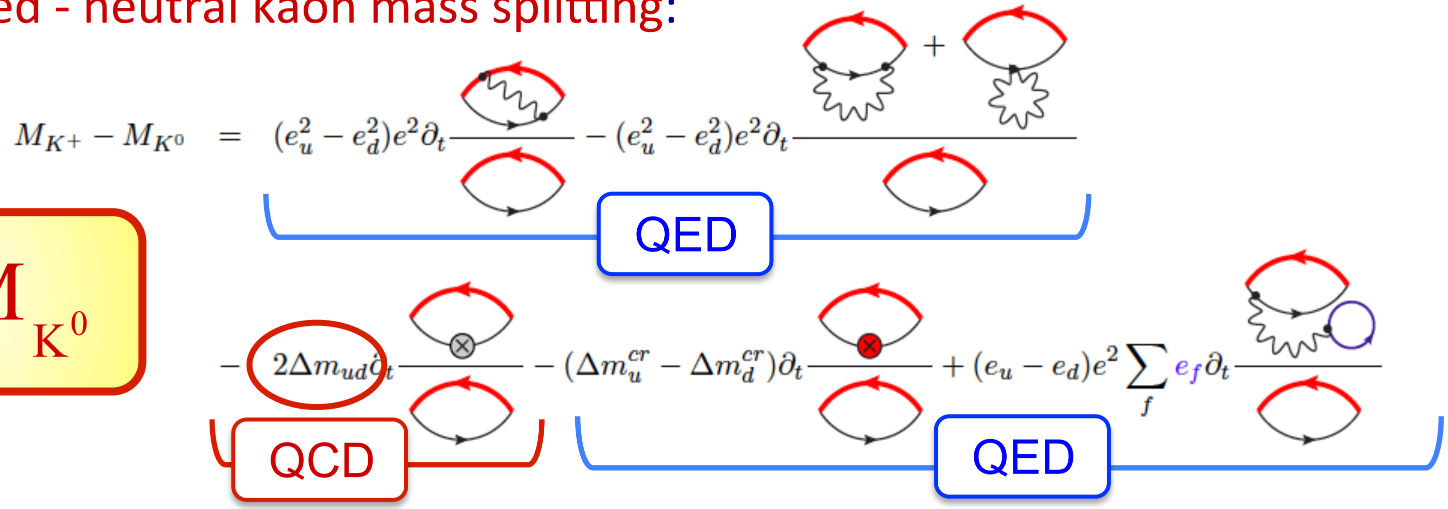


Electromagnetic corrections

G.M.de Divitiis *et al.*, RM123 collaboration, PRD 87 (2013) 114505

- The expansion can be generalized to include the electromagnetic corrections. For the charged - neutral kaon mass splitting:

$$M_{K^+} - M_{K^0}$$



$$\left[M_{K^+} - M_{K^0}\right]^{QED} = 2.3(2)(2)\ \text{MeV} \quad , \quad \left[M_{K^+} - M_{K^0}\right]^{QCD} = -6.2(2)(2)\ \text{MeV}$$

$$(\bar{m}_d - \bar{m}_u) = 2.39(8)(17)\ \text{MeV}$$

$$\bar{m}_u / \bar{m}_d = 0.50(2)(3)$$

- This is an alternative approach to full QCD+QED lattice simulations.

See talk by A. Portelli

QED corrections to hadronic decay rates

In collaboration with:

N. Carrasco, G. Martinelli, C. T. Sachrajda,

N. Tantalo, C. Tarantino, M. Testa

PRD 91 (2015) 074506, arXiv: 1502.00257

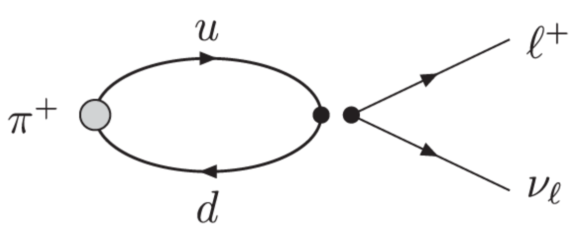
The strategy

- To be specific, we consider the leptonic decay of a charged pion, but the method is general (it can be extended for example to semileptonic decays).

- The rate at $O(\alpha^0)$ is:

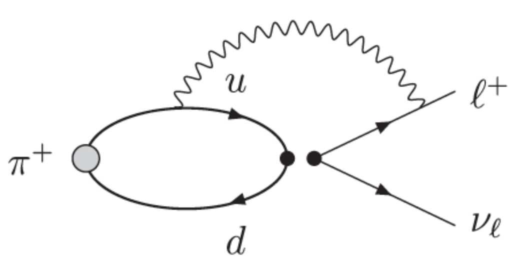
$$\Gamma_0^{\text{tree}}\left(\pi^+ \to \ell^+ \nu_\ell\right) = \frac{G_F \left|V_{ud}\right|^2 f_\pi^2}{8\pi} m_\pi m_\ell^2 \left(1 - \frac{m_\ell^2}{m_\pi^2}\right)^2$$

In the absence of electromagnetism, the nonperturbative QCD effects are contained in a single number, the decay constant:



$$\left\langle 0 \left| \bar{d}\gamma_\mu(1-\gamma_5)u \right| \pi^+(p) \right\rangle = i p_\mu f_\pi$$

- In the presence of electromagnetism, because of the contributions of diagrams like this one, it is not even possible to give a physical definition of f_π.

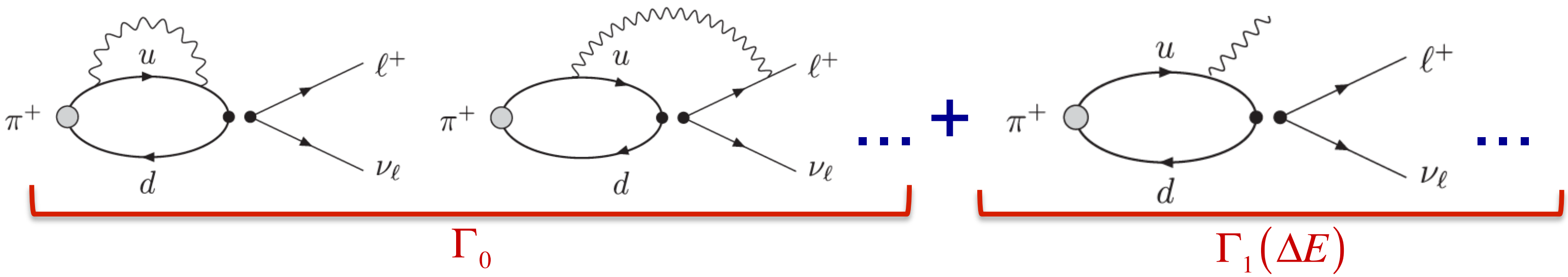


For a discussion on this point based on ChPT see

J. Gasser and G.R.S. Zarnauskas, PLB 693 (2010) 122.

The strategy

- At $O(\alpha)$, the rate Γ_0 contains infrared divergences. One has to consider:



$$\boxed{\Gamma(\Delta E) = \Gamma\left(\pi^+ \to \ell^+\nu_\ell\right) + \Gamma\left(\pi^+ \to \ell^+\nu_\ell\gamma(\Delta E)\right) \equiv \Gamma_0 + \Gamma_1(\Delta E)}$$

with $0 \leq E_\gamma \leq \Delta E$. The sum is infrared finite

F. Bloch and A. Nordsieck, PR 52 (1937) 54

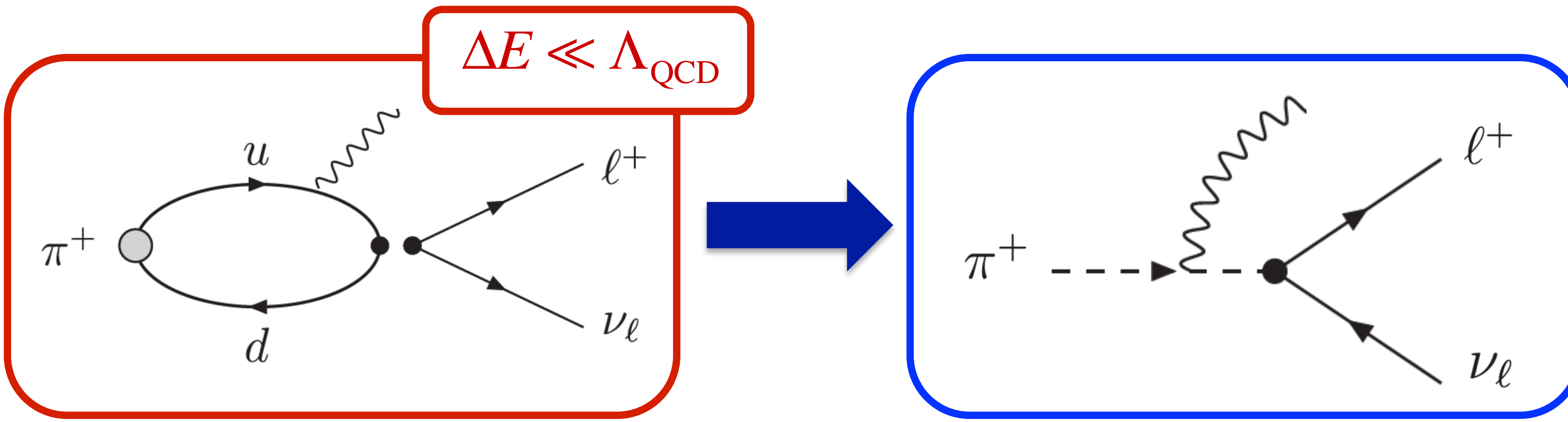
- In principle, both Γ_0 and $\Gamma_1(\Delta E)$ can be evaluated in lattice simulations.

But $\Gamma_1(\Delta E)$ is very challenging, due to discretized photon momenta: the integral up to ΔE replaced by a finite sum, many correlation functions…

We thus propose a different strategy ➡

The strategy

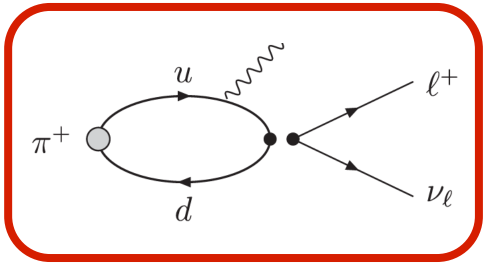
- We propose to consider sufficiently soft photons such that they do not resolve the internal structure of the pion. Then the pointlike approximation can be used to compute $\Gamma_1(\Delta E)$ in perturbation theory.

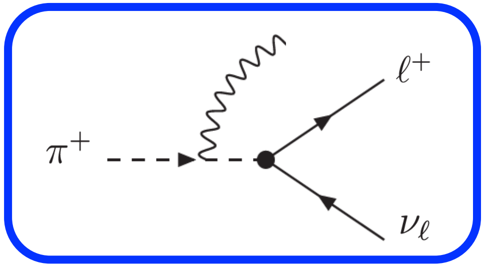


➡ A cut-off $\boxed{\Delta E \sim O(20\ \mathrm{MeV})}$ appears to be appropriate, both experimentally and theoretically

F. Ambrosino et al., KLOE Collaboration,
PLB 632 (2006) 76; EPJC 64 (2009) 627; 65 (2010) 703(E)

The strategy

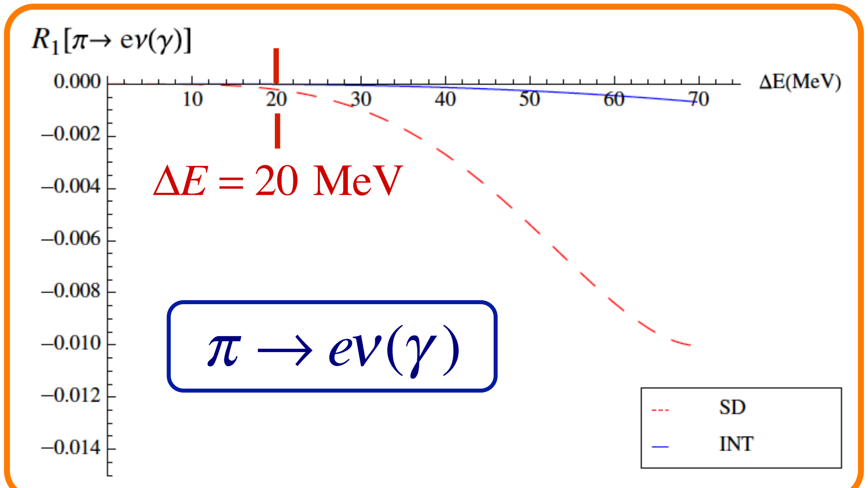


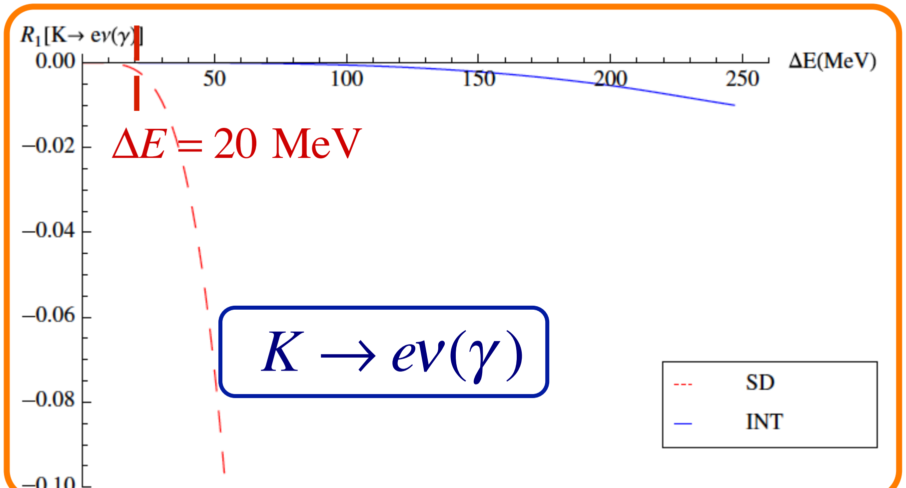


$\Delta E \sim O(20\ \text{MeV})$

- The size of the neglected structure-dependent contributions can be estimated, as a function of ΔE, using chiral perturbation theory

J. Bijnens, G. Ecker, J. Gasser, NPB 396 (1993) 81; V.Cirigliano, I.Rosell, JHEP 0710 (2007) 005





$$R_1^A(\Delta E) = \frac{\Gamma_1^A(\Delta E)}{\Gamma_0^{\alpha,pt} + \Gamma_1^{pt}(\Delta E)} \quad , \quad A = \{SD, INT\}$$

+ ChPT

$$F_V = \frac{m_P}{4\pi^2 f_\pi} \quad , \quad F_A = \frac{8 m_P}{f_\pi}\left(L_9^r + L_{10}^r\right)$$

The strategy

$$\Gamma(\Delta E) = \Gamma_0 + \Gamma_1^{\text{pt}}(\Delta E) \qquad \Delta E \sim O(20\text{ MeV})$$

$\Gamma(\pi^+ \to \ell^+ \nu_\ell)$

Montecarlo simulation

Lattice QCD

$\Gamma(\pi^+ \to \ell^+ \nu_\ell \gamma(\Delta E))$

Perturbation theory

pointlike pion

- In order to ensure the cancellation of IR divergences with good numerical precision, an intermediate step is required. We then rewrite:

$$\Gamma(\Delta E) = \underbrace{\lim_{V\to\infty}\left(\Gamma_0 - \Gamma_0^{\text{pt}}\right)} + \underbrace{\lim_{V\to\infty}\left(\Gamma_0^{\text{pt}} + \Gamma_1^{\text{pt}}(\Delta E)\right)}$$

$\Gamma_0^{\text{pt}} = \Gamma(\pi^+ \to \ell^+ \nu_\ell)^{\text{pt}}$ is an unphysical quantity

The strategy

$$\Gamma(\Delta E) = \underbrace{\lim_{V\to\infty}\left(\Gamma_0 - \Gamma_0^{\mathrm{pt}}\right)} + \underbrace{\lim_{V\to\infty}\left(\Gamma_0^{\mathrm{pt}} + \Gamma_1^{\mathrm{pt}}(\Delta E)\right)}$$

- The second term is calculated in perturbation theory directly in infinite volume. The sum $\Gamma_0^{\mathrm{pt}} + \Gamma_1^{\mathrm{pt}}(\Delta E)$ is IR finite.
- $\Gamma_0 - \Gamma_0^{\mathrm{pt}}$ in the first term is calculated, in the intermediate step, in the finite volume. The contributions from small virtual photon momenta to Γ_0 and Γ_0^{pt} are the same, and the first term is IR finite.
- ⟹ IR divergences cancel separately in each of the two terms, and so we can calculate each of these terms separately. We also use different IR regulators: the finite volume for the first term and a photon mass for the second term.
- The two terms are also separately gauge invariant.

Outline

$$\Gamma(\Delta E) = \lim_{V\to\infty}\left(\Gamma_0(L) - \Gamma_0^{\text{pt}}(L)\right) + \lim_{V\to\infty}\left(\Gamma_0^{\text{pt}} + \Gamma_1^{\text{pt}}(\Delta E)\right)$$

- 1. General strategy ✓
- 2. Calculation of $\Gamma_0^{\text{pt}} + \Gamma_1^{\text{pt}}(\Delta E)$ ←
- 3. Calculation of Γ_0
  - G_F and the UV matching
  - Lattice calculation
- 4. Calculation of $\Gamma_0^{\text{pt}}(L)$
- 5. Estimates of structure dependent contributions to $\Gamma_1(\Delta E)$
- 6. Conclusions

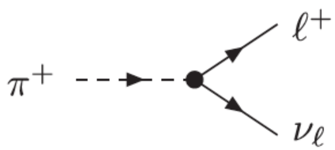
Calculation of $\Gamma^{\rm pt}(\Delta E)$

$$\Gamma(\Delta E) = \lim_{V\to\infty}\left(\Gamma_0 - \Gamma_0^{\rm pt}\right) + \boxed{\lim_{V\to\infty}\left(\Gamma_0^{\rm pt} + \Gamma_1^{\rm pt}(\Delta E)\right)}$$

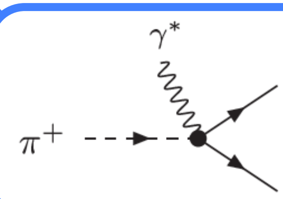
- $\Gamma^{\rm pt}(\Delta E)$ is calculated in perturbation theory with a pointlike pion

$$\mathcal{L}_{\pi-\ell-\nu_\ell} = iG_F f_\pi V_{ud}^*\{(\partial_\mu - ieA_\mu)\pi\}\left\{\bar\psi_{\nu_\ell}\frac{1+\gamma_5}{2}\gamma^\mu\psi_\ell\right\}$$

+ QED for π and l^+



$$= -iG_F f_\pi V_{ud}^*\, p_\pi^\mu\, \tfrac{1+\gamma^5}{2}\gamma_\mu$$



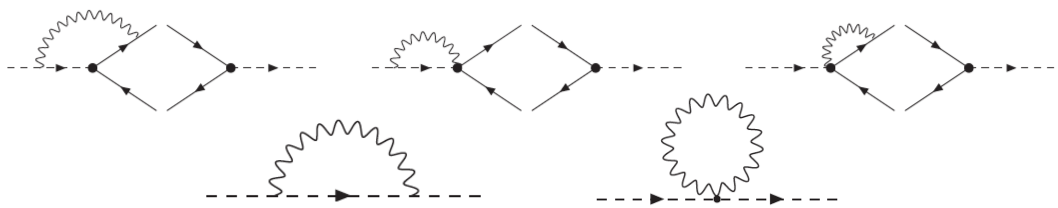
$$= ie\,G_F f_\pi V_{ud}^*\, g^{\mu\nu}\, \tfrac{1+\gamma^5}{2}\gamma_\mu$$

- UV divergences in $\Gamma_0^{\rm pt}$ are regularized with the "W-regularization" (more on this point later)

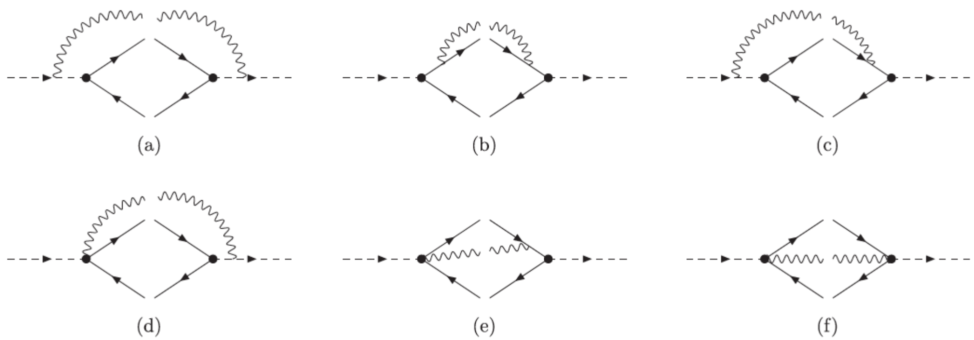
$$\frac{1}{k^2} \to \frac{M_W^2}{M_W^2 - k^2}\frac{1}{k^2}$$

- IR divergences are regularized with the a photon mass

Calculation of $\Gamma^{\rm pt}(\Delta E)$



$\Gamma_0^{\rm pt}$ +



$\Gamma_1^{\rm pt}(\Delta E)$

$= \Gamma^{\rm pt}(\Delta E)$

NEW

$$\Gamma^{\rm pt}(\Delta E) = \Gamma_0^{\rm tree} \times \left(1 + \frac{\alpha}{4\pi}\left\{\underbrace{3\log\left(\frac{m_\pi^2}{M_W^2}\right)}_{} + \log(r_\ell^2) - 4\log(r_E^2) + \frac{2-10r_\ell^2}{1-r_\ell^2}\log(r_\ell^2) - 2\frac{1+r_\ell^2}{1-r_\ell^2}\log(r_E^2)\log(r_\ell^2)\right.\right.$$

$$- 4\frac{1+r_\ell^2}{1-r_\ell^2}{\rm Li}_2(1-r_\ell^2) - 3 + \left[\frac{3+r_E^2-6r_\ell^2+4r_E(-1+r_\ell^2)}{(1-r_\ell^2)^2}\log(1-r_E) + \frac{r_E(4-r_E-4r_\ell^2)}{(1-r_\ell^2)^2}\log(r_\ell^2)\right.$$

$$\left.\left.\left.- \frac{r_E(-22+3r_E+28r_\ell^2)}{2(1-r_\ell^2)^2} - 4\frac{1+r_\ell^2}{1-r_\ell^2}{\rm Li}_2(r_E)\right]\right\}\right).$$

$r_\ell = m_\ell/m_\pi$

$r_E = 2\Delta E/m_\pi$

$0 \le r_E \le 1 - r_\ell^2$

- CHECK: For $\Delta E = \Delta E_{MAX}$ we obtain the well known result for total rate as in S. M. Berman, PRL 1 (1958) 468 and T. Kinoshita, PRL 2 (1959) 477

Outline

$$\Gamma(\Delta E) = \lim_{V\to\infty}\left(\Gamma_0(L) - \Gamma_0^{\mathrm{pt}}(L)\right) + \lim_{V\to\infty}\left(\Gamma_0^{\mathrm{pt}} + \Gamma_1^{\mathrm{pt}}(\Delta E)\right)$$

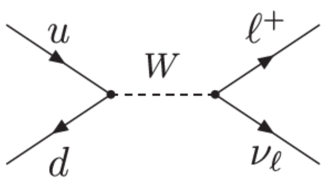
- 1. General strategy ✓
- 2. Calculation of $\Gamma_0^{\mathrm{pt}} + \Gamma_1^{\mathrm{pt}}(\Delta E)$ ✓
- 3. Calculation of Γ_0 ←
  - G_F and the UV matching
  - Lattice calculation
- 4. Calculation of $\Gamma_0^{\mathrm{pt}}(L)$
- 5. Estimates of structure dependent contributions to $\Gamma_1(\Delta E)$
- 6. Conclusions

Calculation of $\Gamma_0(L)$

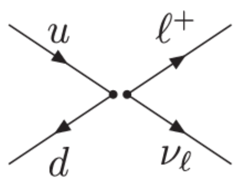
$$\Gamma(\Delta E) = \boxed{\lim_{V\to\infty}\left(\Gamma_0 - \Gamma_0^{\rm pt}\right)} + \lim_{V\to\infty}\left(\Gamma_0^{\rm pt} + \Gamma_1^{\rm pt}(\Delta E)\right)$$

- Γ_0 is calculated in the finite volume with a lattice simulation

- At lowest order in electromagnetic (and strong) perturbation theory the amplitude can be rewritten in terms of a four-fermion local interaction



$\Rightarrow$



$$H_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{ud}^* \left(\bar{d}\gamma^\mu(1-\gamma^5)u\right)\left(\bar{\nu}_\ell\gamma_\mu(1-\gamma^5)\ell\right)$$

This replacement is necessary in a lattice calculation, since $\boxed{1/a \ll M_W}$

- When including the O(α) corrections, the UV contributions to the matrix element of the local operator are different from those in the Standard Model:

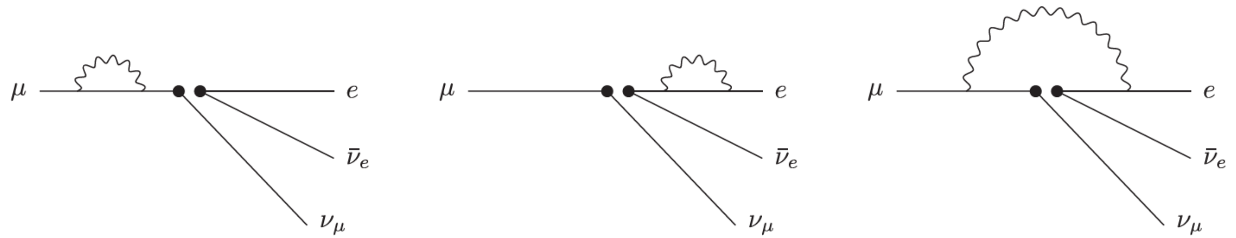
→ a matching between the two theories is required

SM electroweak corrections to muon decay

- The Fermi constant G_F is conventionally taken from the muon lifetime using

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}\left(1-\frac{8m_e^2}{m_\mu^2}\right)\underbrace{\left[1+\frac{\alpha}{2\pi}\left(\frac{25}{4}-\pi^2\right)\right]} \quad \Rightarrow \quad \boxed{G_F = 1.16634\times 10^{-5}\ \mathrm{GeV}^{-2}}$$

Electromagnetic corrections obtained in the local effective theory: [UV finite]



+ diagrams with the real photon

S.M.Berman, PR 112 (1958) 267; T.Kinoshita and A.Sirlin, PR 113 (1959) 1652

- In the Standard Model, it is convenient to write the (Feynman gauge) photon propagator as:

$$\frac{1}{k^2} = \underbrace{\frac{1}{k^2-M_W^2}} + \underbrace{\frac{M_W^2}{M_W^2-k^2}\frac{1}{k^2}}$$

A. Sirlin, RMP 50 (1978) 573, PRD 22 (1980) 971

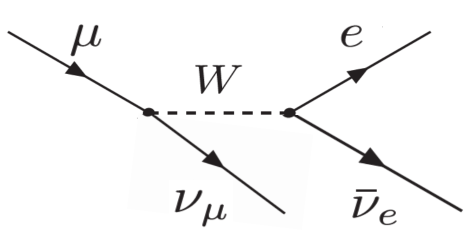
UV divergent. Absorbed in the definition of G_F together with the other EW corrections

UV convergent. Equal to the corresponding diagrams in the eff. theory with the W-regularization

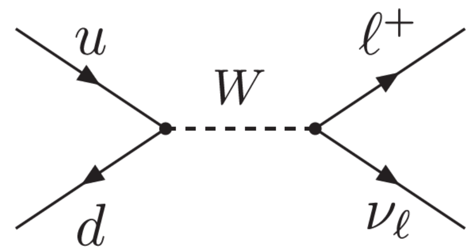
SM electroweak corrections to pion decay

- Most of the terms which are absorbed into the definition of G_F are common to other processes, including the leptonic decays of pseudoscalar mesons

muon decay



pion decay



Same weak isospin charges

- Some short-distance contributions, however, do depend on the electric charges of the fields in the four-fermion operators. These lead to a correction factor of

$$\boxed{H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ud}^* \left(1 + \frac{3\alpha}{4\pi} \underbrace{(1+2\bar{Q})} \log\frac{M_Z}{M_W} \right) \left(\bar{d}\gamma^\mu(1-\gamma^5)u\right)\underbrace{\left(\bar{\nu}_\ell\gamma_\mu(1-\gamma^5)\ell\right)}}$$

W-regularization

$$2\bar{Q} = (Q_{\nu_\mu} + Q_\mu) = -1$$

$$\boxed{1+2\bar{Q}=0}$$

muon decay

pion decay

$$2\bar{Q} = (Q_u + Q_d) = 1/3$$

$$\boxed{1+2\bar{Q}=4/3}$$

A.Sirlin, NP B196 (1982) 83; E.Braaten & C.S.Li, PRD 42 (1990) 3888

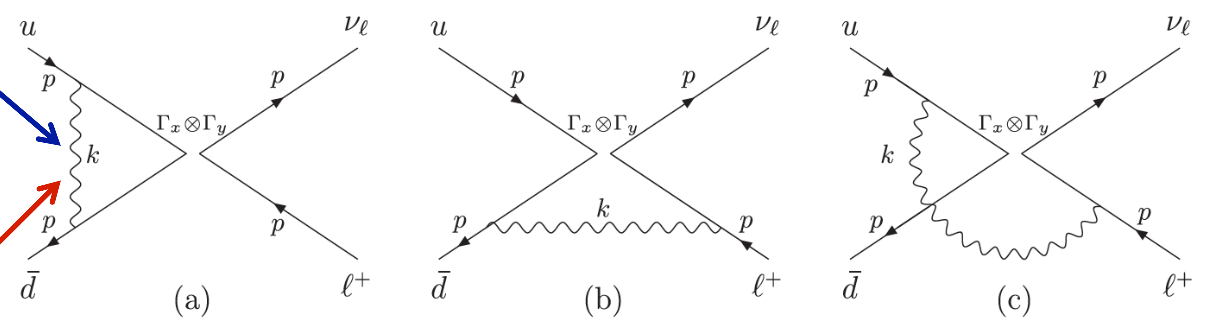
Matching the W and lattice regularizations

- The W regularization cannot be implemented directly in present day lattice simulations since $\boxed{1/a \ll M_W}$

- The relation between the Fermi effective Hamiltonian in the lattice and W regularizations can be computed in perturbation theory:

W regul. $\left(\dfrac{M_W^2}{M_W^2 - k^2}\right)\dfrac{1}{k^2}$

Lattice $1/\left[\sum_\rho \dfrac{4}{a^2}\sin^2\left(\dfrac{ak_\rho}{2}\right)\right]$



- The result, with the lattice Wilson action for both gluons and fermions, is:

$$O_1^{\text{W-reg}} = \underbrace{\left(1 + \frac{\alpha}{4\pi}\left(2\log(a^2M_W^2) - 15.539\right)\right)O_1^{\text{bare}}} + \frac{\alpha}{4\pi}\left(0.536\,O_2^{\text{bare}} + 1.607\,O_3^{\text{bare}} - 3.214\,O_4^{\text{bare}} - 0.804\,O_5^{\text{bare}}\right),$$

$$\begin{aligned}
O_1 &= (\bar{d}\gamma^\mu(1-\gamma^5)u)(\bar{\nu}_\ell\gamma_\mu(1-\gamma^5)\ell),\\
O_2 &= (\bar{d}\gamma^\mu(1+\gamma^5)u)(\bar{\nu}_\ell\gamma_\mu(1-\gamma^5)\ell),\\
O_3 &= (\bar{d}(1-\gamma^5)u)(\bar{\nu}_\ell(1+\gamma^5)\ell),\\
O_4 &= (\bar{d}(1+\gamma^5)u)(\bar{\nu}_\ell(1+\gamma^5)\ell),\\
O_5 &= (\bar{d}\sigma^{\mu\nu}(1+\gamma^5)u)(\bar{\nu}_\ell\sigma_{\mu\nu}(1+\gamma^5)\ell)
\end{aligned}$$

Outline

$$\Gamma(\Delta E) = \lim_{V\to\infty}\left(\Gamma_0(L) - \Gamma_0^{\mathrm{pt}}(L)\right) + \lim_{V\to\infty}\left(\Gamma_0^{\mathrm{pt}} + \Gamma_1^{\mathrm{pt}}(\Delta E)\right)$$

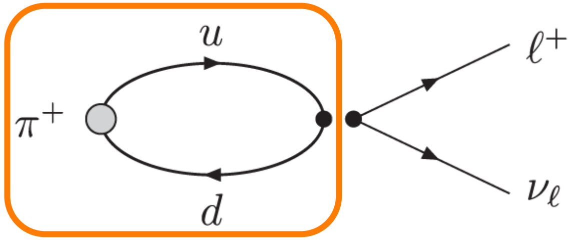
- 1. General strategy ✓
- 2. Calculation of $\Gamma_0^{\mathrm{pt}} + \Gamma_1^{\mathrm{pt}}(\Delta E)$ ✓
- 3. Calculation of Γ_0 ✓
  - G_F and the UV matching ✓
  - Lattice calculation ←
- 4. Calculation of $\Gamma_0^{\mathrm{pt}}(L)$
- 5. Estimates of structure dependent contributions to $\Gamma_1(\Delta E)$
- 6. Conclusions

Lattice calculation of $\Gamma_0(L)$

- The lattice calculation at O(α^0), i.e. without electromagnetism, is standard

$$M_0 = \frac{G_F}{\sqrt{2}} V_{ud}^* \langle 0 | \bar{d}\gamma^\mu\gamma^5 u | \pi^+(p_\pi)\rangle \left[u_{\nu_\ell}(p_{\nu_\ell})\gamma_\mu(1-\gamma^5)v_\ell(p_\ell)\right] =$$

$$= \frac{iG_F}{\sqrt{2}} V_{ud}^* f_\pi p_\pi^\mu \left[u_{\nu_\ell}(p_{\nu_\ell})\gamma_\mu(1-\gamma^5)v_\ell(p_\ell)\right]$$



The amplitude is obtained from the 2 point correlation function

$$\boxed{C_0(t) \equiv \sum_{\vec{x}} \langle 0 | \left(\bar{d}(\vec{0},0)\gamma^\mu\gamma^5 u(\vec{0},0)\right)\phi^\dagger(\vec{x},-t)|0\rangle \simeq \frac{Z_0^\phi}{2m_\pi^0} e^{-m_\pi^0 t} \mathcal{A}_0}$$ for large t

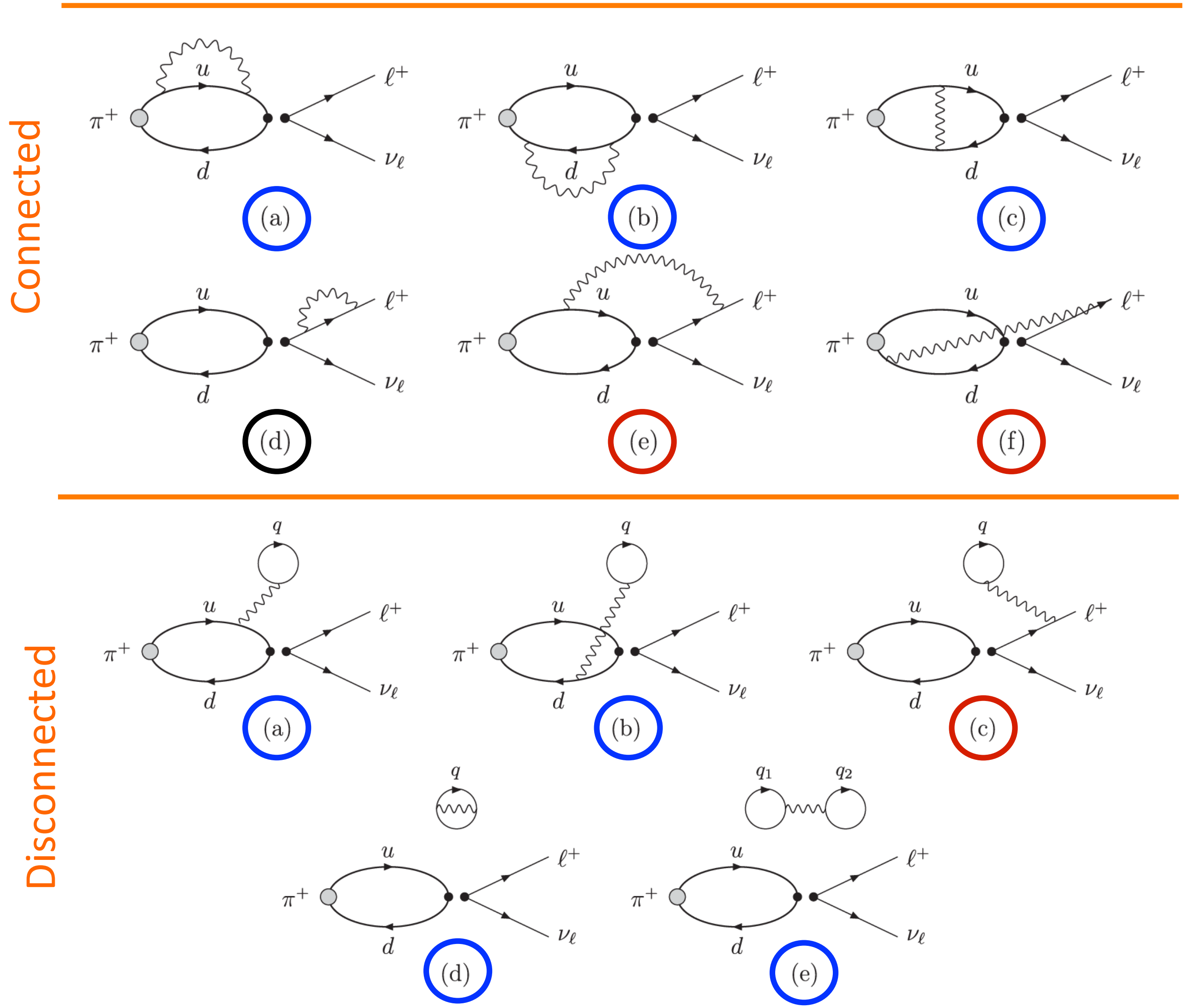
where: $Z_0^\phi \equiv \langle \pi^+(\vec{0}) | \phi^\dagger(\vec{0},0)|0\rangle$

$$\boxed{\mathcal{A}_0 \equiv \langle 0 | \bar{d}\gamma^\mu\gamma^5 u | \pi^+(\vec{0})\rangle_0}$$

$$C_0^{\phi\phi}(t) \equiv \sum_{\vec{x}} \langle 0 | \phi(\vec{0},0)\phi^\dagger(\vec{x},-t)|0\rangle \simeq \frac{\left(Z_0^\phi\right)^2}{2m_\pi^0} e^{-m_\pi^0 t}$$

Lattice calculation of $\Gamma_0(L)$ at $O(\alpha)$

- The Feynman diagrams at O(α) can be divided in 3 classes



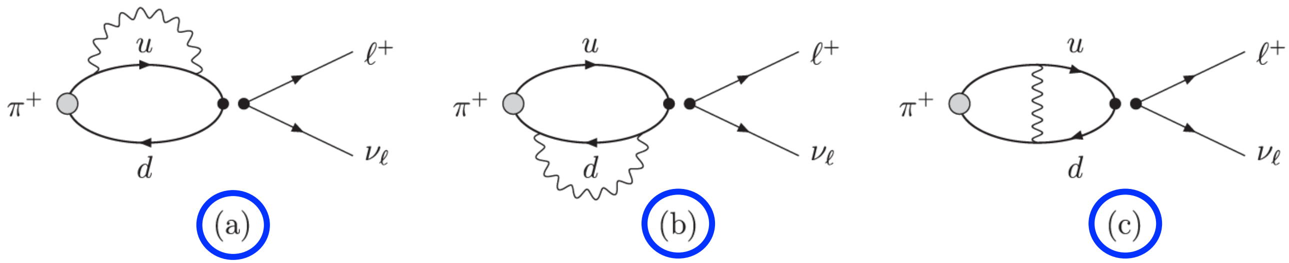
1. The photon connects two quark lines

2. The photon connects one quark and one charged lepton line

3. Leptonic wave function renormalization. It cancels in $\Gamma_0(L) - \Gamma_0^{\mathrm{pt}}(L)$

Lattice calculation of $\Gamma_0(L)$ at $O(\alpha)$ [I]

- Let us consider the Feynman diagrams of the 1st class:



+ disconnected diagrams

The leptonic contribution to the amplitude is factorized and we need to compute

weak | e.m. | pion | photon

$$C_1(t) = -\frac{1}{2}\int d^3\vec{x}\, d^4x_1\, d^4x_2 \sum_{\vec{x}} \langle 0|T\left\{J_W^\nu(0)\, j_\mu(x_1)\, j_\mu(x_2)\, \phi^\dagger(\vec{x},-t)\right\}|0\rangle\, \Delta(x_1,x_2)$$

- For sufficiently large t the correlation function is dominated by the ground state

$$C_0(t)+C_1(t) \simeq \frac{Z^\phi}{2m_\pi} e^{-m_\pi t}\mathcal{A} \simeq \frac{\left(Z_0^\phi+\delta Z^\phi\right)}{2\left(m_\pi^0+\delta m_\pi\right)} e^{-m_\pi^0 t}\left(1-\delta m_\pi t\right)\left(\mathcal{A}_0+\delta\mathcal{A}\right)$$

$$C_1(t)/C_0(t) \simeq c_1 t + c_2$$

and similarly

$$C_1^{\phi\phi}(t)/C_0^{\phi\phi}(t) \simeq c_1 t + c_2^{\phi\phi}$$

$$\delta m_\pi = -c_1$$

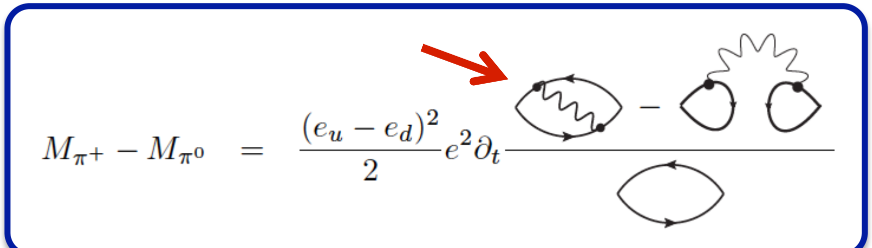
e.m. mass shift

$$\delta\mathcal{A} = \mathcal{A}_0\left(c_2 - \frac{c_2^{\phi\phi}}{2} - \frac{c_1}{2m_\pi^0}\right)$$

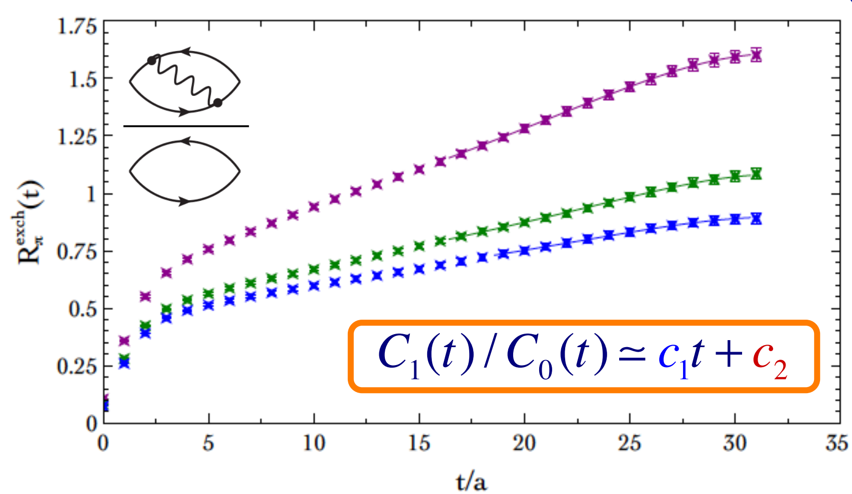
e.m. “correction to f_π” (unphys.)

The charged-neutral pion mass splitting

- Only 2 diagrams contribute to the pion mass splitting



- The 1st diagram also contributes to the π^+ decay rate
- The 2nd diagram comes from the neutral pion. It is $O(\alpha\, m_{ud})$ and it has been neglected



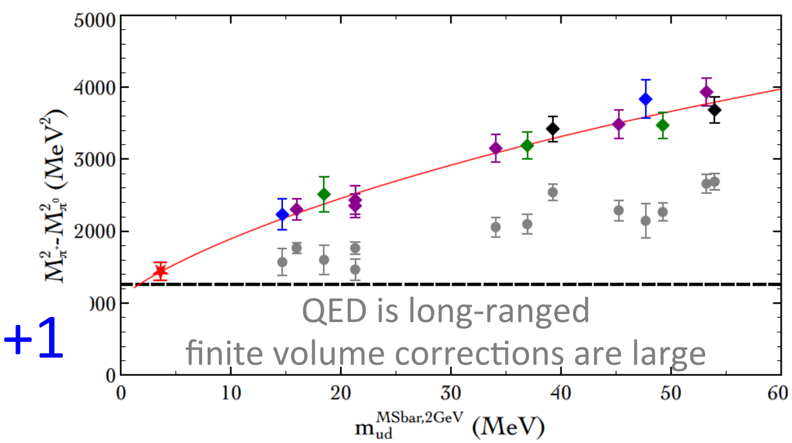
- From the linear slope in time c_1 we find

$$\boxed{M_{\pi^+} - M_{\pi^0} = 5.33(76)\ \mathrm{MeV}}$$ Nf=2

G.M.de Divitiis *et al.*, RM123 collaboration, PRD 87 (2013) 114505

$$\boxed{M_{\pi^+} - M_{\pi^0} = 4.28(39)\ \mathrm{MeV}}$$ Nf=2+1+1

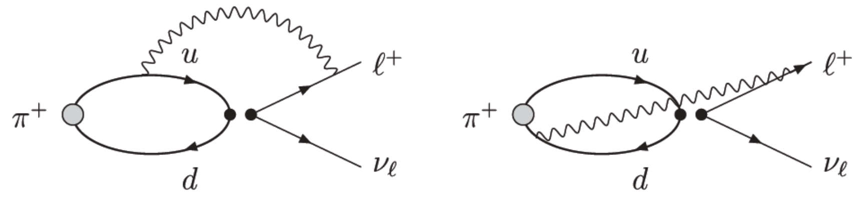
RM123 collaboration, 2015, preliminary



in good agreement with $\left(M_{\pi^+} - M_{\pi^0}\right)_{exp} = 4.59\ \mathrm{MeV}$

Lattice calculation of $\Gamma_0(L)$ at $O(\alpha)$ [II]

- For these diagrams the leptonic and hadronic contributions do not factorize



(e) (f)

+ disconnected diagrams

- The amplitude is obtained from the following Euclidean space correlation function

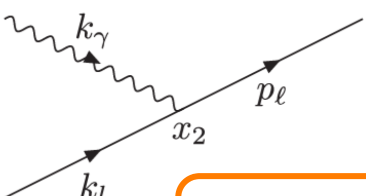
$$C_1(t)_{\alpha\beta} = -\int d^3\vec{x}\, d^4x_1\, d^4x_2 \,\langle 0|T\left\{J_W^\nu(0)\, j_\mu(x_1)\, \phi^\dagger(\vec{x},-t)\right\}|0\rangle$$

$$\times\, \Delta(x_1,x_2)\left(\gamma_\nu(1-\gamma^5)S(0,x_2)\gamma_\mu\right)_{\alpha\beta} e^{E_\ell t_2 - i\,\vec{p}_\ell\cdot\vec{x}_2}$$

weak e.m. pion

photon weak lepton e.m.

- We need to ensure that the t_2 integration converges as $t_2 \to \infty$. The large t_2 behavior is given by the factor $\exp\left[\left(E_\ell - \omega_\ell - \omega_\gamma\right)t_2\right]$



$$E_\ell = \sqrt{\vec{p}_\ell^{\,2} + m_\ell^2} \qquad \omega_\ell = \sqrt{\vec{k}_\ell^{\,2} + m_\ell^2} \qquad \omega_\gamma = \sqrt{\vec{k}_\gamma^{\,2} + m_\gamma^2} \qquad \vec{k}_\ell + \vec{k}_\gamma = \vec{p}_\ell$$

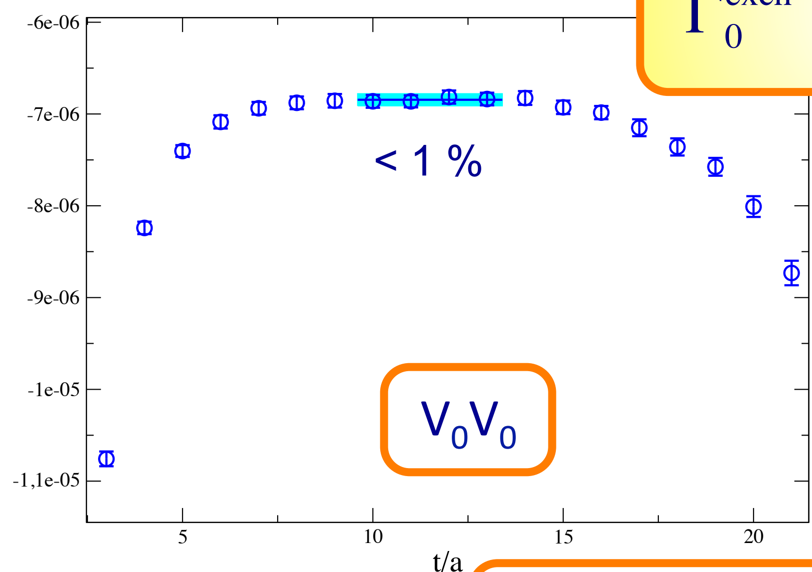
$$\left(\omega_\ell + \omega_\gamma\right)_{\min} = \sqrt{\left(m_\ell^2 + m_\gamma^2\right) + \vec{p}_\ell^{\,2}} > E_\ell$$

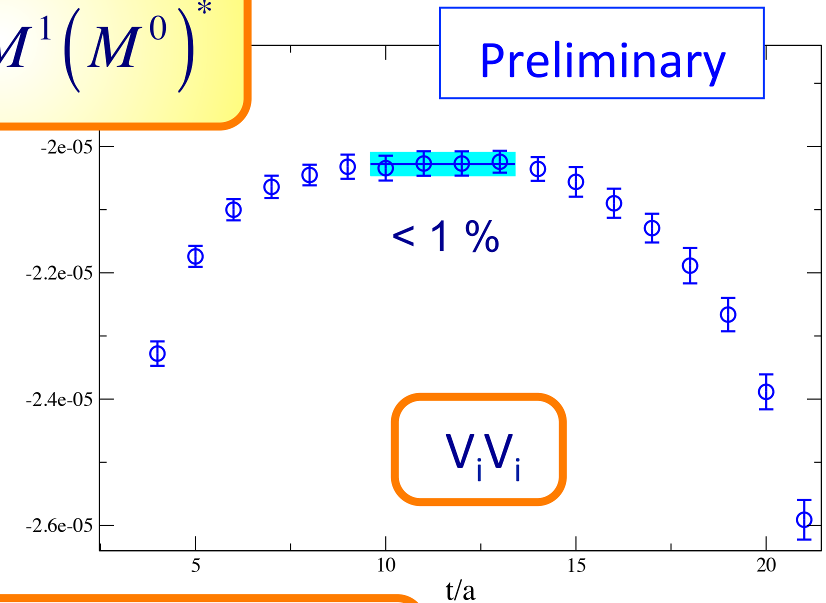
The integral is convergent and the continuation from Minkowski to Euclidean space can be performed

Lattice calculation of $\Gamma_0(L)$ at $O(\alpha)$ [II]

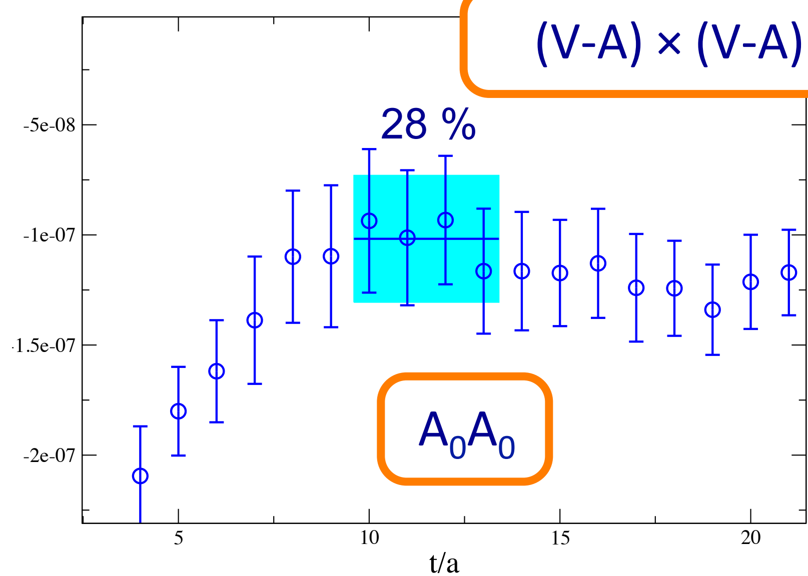
$$\Gamma_0^{\mathrm{exch}} \propto \overline{M^1 \left(M^0\right)^*}$$

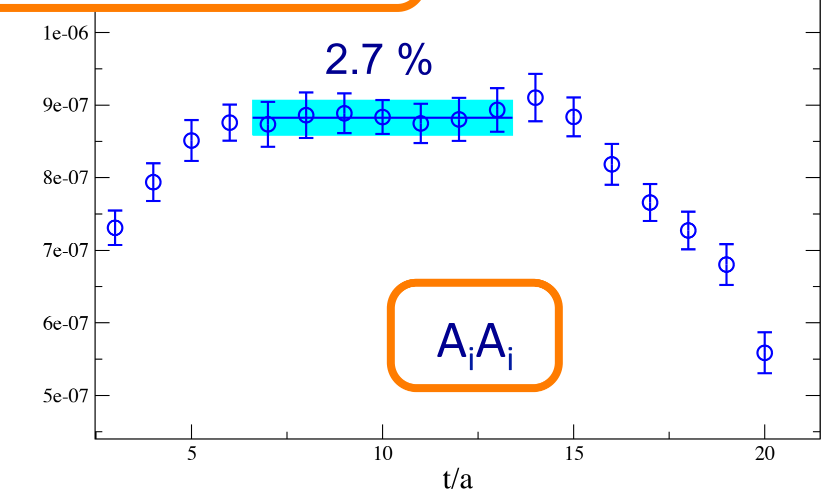
Preliminary





(V-A) × (V-A) = VV + AA - VA - AV





Outline

$$\Gamma(\Delta E) = \lim_{V\to\infty}\left(\Gamma_0(L) - \Gamma_0^{\rm pt}(L)\right) + \lim_{V\to\infty}\left(\Gamma_0^{\rm pt} + \Gamma_1^{\rm pt}(\Delta E)\right)$$

- 1. General strategy ✓
- 2. Calculation of $\Gamma_0^{\rm pt} + \Gamma_1^{\rm pt}(\Delta E)$ ✓
- 3. Calculation of Γ_0 ✓
  - G_F and the UV matching ✓
  - Lattice calculation ✓
- 4. Calculation of $\Gamma_0^{\rm pt}(L)$ ←
- 5. Estimates of structure dependent contributions to $\Gamma_1(\Delta E)$
- 6. Conclusions

Calculation of $\Gamma_0^{\mathrm{pt}}(L)$

$$\Gamma(\Delta E) = \boxed{\lim_{V\to\infty}\left(\Gamma_0 - \Gamma_0^{\mathrm{pt}}\right)} + \lim_{V\to\infty}\left(\Gamma_0^{\mathrm{pt}} + \Gamma_1^{\mathrm{pt}}(\Delta E)\right)$$

- $\Gamma_0^{\mathrm{pt}}(L)$ is calculated in perturbation theory with a pointlike pion

- UV divergences are regularized with the W-regularization
- IR divergences are regularized by the finite volume (same of $\Gamma_0(L)$)

- For the pion self energy, the result is:

Preliminary

$$\frac{1}{\pi}\left(\frac{2\pi}{L}\right)^3 \sum_{\vec q}\left\{\frac{1}{\left(M_W^4 - 4m_\pi^2 E_{W,\vec q}^2\right)^2}\left[16 m_\pi^4\left(\frac{\vec q^{\,2}}{E_{W,\vec q}} + \frac{M_W^2}{E_{W,\vec q}} + \frac{M_W^2}{E_{\pi,\vec q}}\right) + M_W^4\left(\frac{4\vec q^{\,2}}{E_{W,\vec q}} - \frac{4\vec q^{\,2}}{E_{\pi,\vec q}} + \frac{M_W^2}{E_{W,\vec q}} + \frac{M_W^2}{E_{\pi,\vec q}}\right)\right.\right.$$

$$\left.\left. -4M_W^2 m_\pi^2\left(\frac{3\vec q^{\,2}}{E_{W,\vec q}} - \frac{3\vec q^{\,2}}{E_{\pi,\vec q}} + \frac{2M_W^2}{E_{W,\vec q}} + \frac{2M_W^2}{E_{\pi,\vec q}}\right)\right] - \left(M_W \to 0\right)\right\} \qquad \boxed{\vec q = \frac{2\pi}{L}\left(n_x, n_y, n_z\right)} \qquad E_{X,\vec q} = \sqrt{M_X^2 + \vec q^{\,2}}$$

Outline

$$\Gamma(\Delta E) = \lim_{V\to\infty}\left(\Gamma_0(L) - \Gamma_0^{\mathrm{pt}}(L)\right) + \lim_{V\to\infty}\left(\Gamma_0^{\mathrm{pt}} + \Gamma_1^{\mathrm{pt}}(\Delta E)\right)$$

- 1. General strategy ✓
- 2. Calculation of $\Gamma_0^{\mathrm{pt}} + \Gamma_1^{\mathrm{pt}}(\Delta E)$ ✓
- 3. Calculation of Γ_0 ✓
  - G_F and the UV matching ✓
  - Lattice calculation ✓
- 4. Calculation of $\Gamma_0^{\mathrm{pt}}(L)$ ✓
- 5. Estimates of structure dependent contributions to $\Gamma_1(\Delta E)$ ✗
- 6. Conclusions ←

Conclusions and outlook

- We have presented, for the first time, a method to compute electromagnetic effects in hadronic processes with lattice QCD.

- The implementation of the method is challenging but within reach of present lattice technology. Preliminary numerical results have been already obtained.

- Since the effects we are calculating are of O(1%), computing the electromagnetic corrections to a precision of 20% or so would already be more than sufficient.

Phyiscal results expected soon !

Supplementary slides

Outline

$$\Gamma(\Delta E) = \lim_{V\to\infty}\left(\Gamma_0(L) - \Gamma_0^{\mathrm{pt}}(L)\right) + \lim_{V\to\infty}\left(\Gamma_0^{\mathrm{pt}} + \Gamma_1^{\mathrm{pt}}(\Delta E)\right)$$

- 1. General strategy ✓
- 2. Calculation of $\Gamma_0^{\mathrm{pt}} + \Gamma_1^{\mathrm{pt}}(\Delta E)$ ✓
- 3. Calculation of Γ_0 ✓
  - G_F and the UV matching ✓
  - Lattice calculation ✓
- 4. Calculation of $\Gamma_0^{\mathrm{pt}}(L)$ ✓
- 5. Estimates of structure dependent contributions to $\Gamma_1(\Delta E)$ ←
- 6. Conclusions

Estimates of structure dependent contributions to $\Gamma_1(\Delta E)$

- We estimate the size of the neglected structure-dependent contributions to the decay $K^+/\pi^+ \to \ell \nu_\ell \gamma$ using chiral perturbation theory at $O(p^4)$

J. Bijnens, G. Ecker, J. Gasser, NPB 396 (1993) 81; V.Cirigliano, I.Rosell, JHEP 0710 (2007) 005

- Start with the decomposition in terms of Lorenz invariant form factors of the hadronic matrix element

$$\boxed{H^{\mu\nu}(k,p_\pi) = \int d^4x\, e^{ikx} \langle 0| T\left(j^\mu(x)\, J_W^\nu(0) \right) |\pi(p_\pi)\rangle}$$

and separate the contribution corresponding to the approximation of a pointlike pion $H_{\rm pt}^{\mu\nu}$ from the structure dependent part $H_{\rm SD}^{\mu\nu}$

$$\boxed{H^{\mu\nu} = H_{\rm pt}^{\mu\nu} + H_{\rm SD}^{\mu\nu}}$$

- $H_{\rm pt}^{\mu\nu}$ is simply given by:

$$H_{\rm pt}^{\mu\nu} = f_\pi \left[g^{\mu\nu} - \frac{(2p_\pi - k)^\mu (p_\pi - k)^\nu}{(p_\pi - k)^2 - m_\pi^2} \right] \qquad \left(k_\mu H_{\rm pt}^{\mu\nu} = f_\pi p_\pi^\nu \right)$$

Estimates of structure dependent contributions to $\Gamma_1(\Delta E)$

- The structure dependent component $H_{\mathrm{SD}}^{\mu\nu}$ can be parametrized by four independent invariant form factors which we define as

$$H_{\mathrm{SD}}^{\mu\nu} = H_1\left[k^2 g^{\mu\nu} - k^\mu k^\nu\right] + H_2\left[\left(k\cdot p_\pi\right)k^\mu - k^2 p_\pi^\mu\right]\left(p_\pi - k\right)^\nu$$

$$-i\,\frac{F_V}{m_\pi}\varepsilon^{\mu\nu\alpha\beta}k_\alpha p_{\pi\beta} + \frac{F_A}{m_\pi}\left[\left(k\cdot p_\pi - k^2\right)g^{\mu\nu} - \left(p_\pi - k\right)^\mu k^\nu\right] \qquad \left(k_\mu H_{\mathrm{SD}}^{\mu\nu} = 0\right)$$

For the decay into a real photon, only F_V and F_A contribute

- At $O(p^4)$ in chiral perturbation theory F_V and F_A are constant:

$$\boxed{F_V = \frac{m_P}{4\pi^2 f_\pi}} \quad , \quad \boxed{F_A = \frac{8 m_P}{f_\pi}\left(L_9^r + L_{10}^r\right)}$$

J. Bijnens, G. Ecker, J. Gasser, NPB 396 (1993) 81

- For our estimates we use:

Direct measurement → PDG 2014

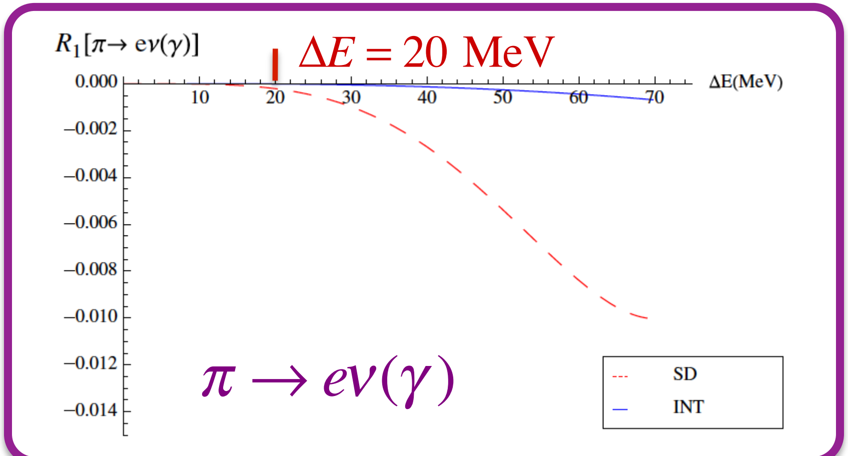
$$\boxed{\begin{aligned} F_V^{(\pi)} &= 0.0254 \\ F_A^{(\pi)} &= 0.0119 \end{aligned}} \qquad \boxed{\begin{aligned} F_V^{(K)} &= 0.096 \\ F_A^{(K)} &= 0.042 \end{aligned}}$$

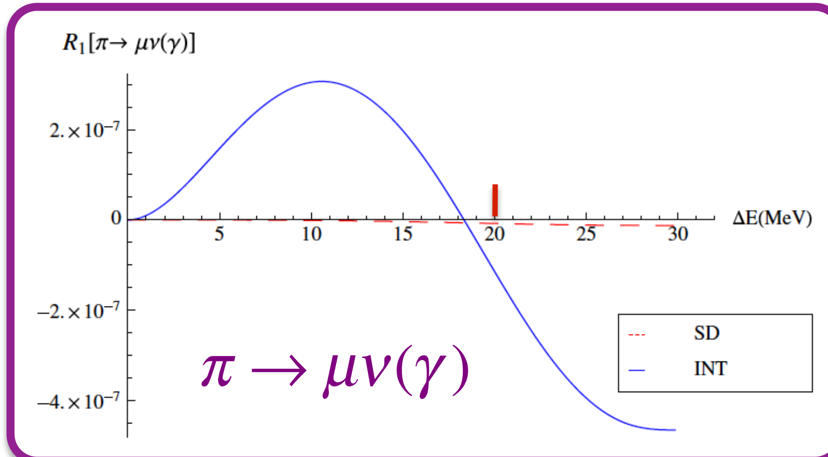
← ChPT

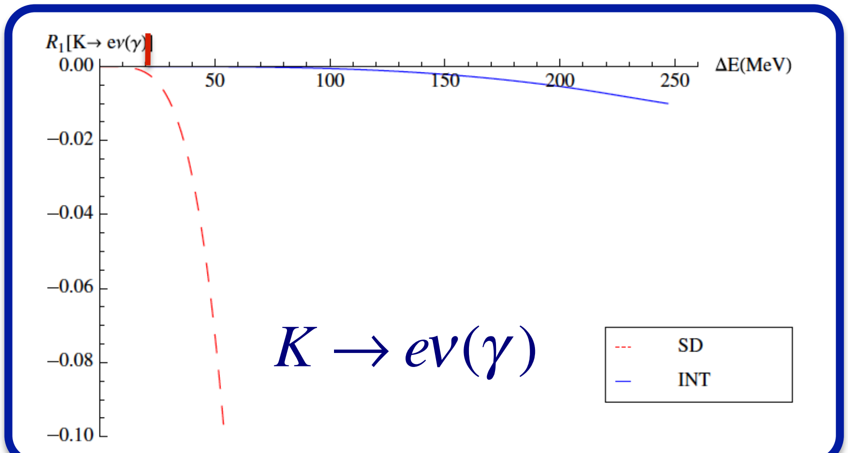
$$R_1^A(\Delta E) = \frac{\Gamma_1^A(\Delta E)}{\Gamma_0^{\alpha,pt} + \Gamma_1^{pt}(\Delta E)} \quad , \quad A = \{SD, INT\}$$

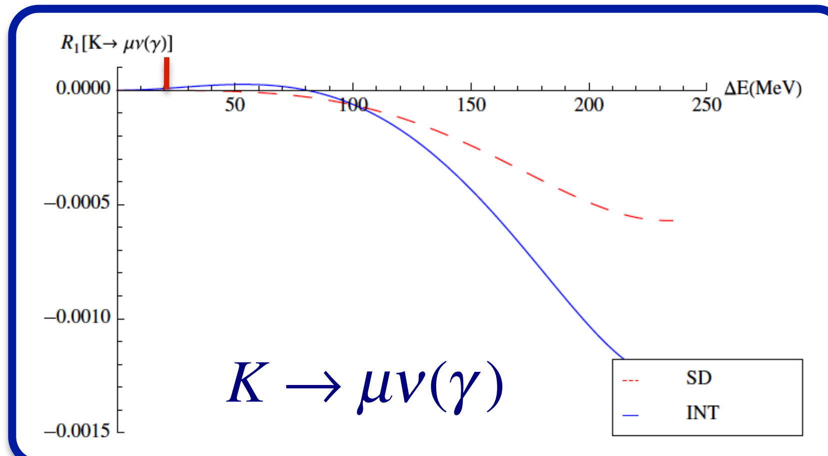
SD = structure dependent

INT = interference







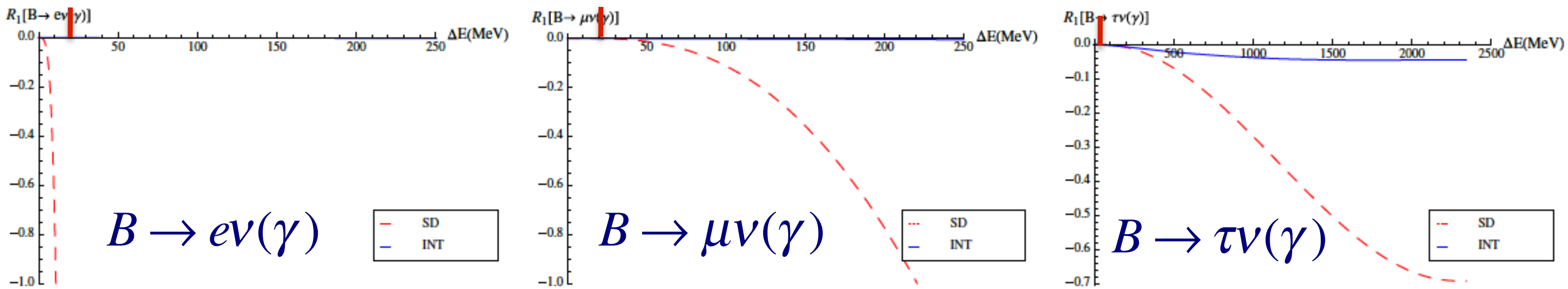


- Interference contributions are negligible in all the decays
- Structure-dependent contributions can be sizable for $K \to e\nu(\gamma)$ but they are negligible for $\Delta E < 20$ MeV (which is experimentally accessible)

Structure dependent contributions to decays of D and B mesons

- For the studies of D and B mesons decays we cannot apply ChPT
- For B mesons in particular we have another small scale, $m_{B^*} - m_B \simeq 45\ \mathrm{MeV}$
  ➡ the radiation of a soft photon may still induce sizeable SD effects
- A phenomenological analysis based on a simple pole model for F_V and F_A confirms this picture

D. Becirevic, B. Haas, E. Kou, PLB 681 (2009) 257



$$F_V \simeq \frac{\tilde{C}_V}{1-(p_B-k)^2/m_{B^*}^2}$$

$$F_A \simeq \frac{\tilde{C}_A}{1-(p_B-k)^2/m_{B_1}^2}$$

Under this assumption the SD contributions to $B \to e\nu(\gamma)$ for $E_\gamma \simeq 20$ MeV can be very large, but are small for $B \to \mu\nu(\gamma)$ and $B \to \tau\nu(\gamma)$

- A lattice calculation of F_V and F_A would be very useful