# Lepton Energy Moments in Semileptonic Charm Decays


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# Motivation: CKM Unitarity Analysis

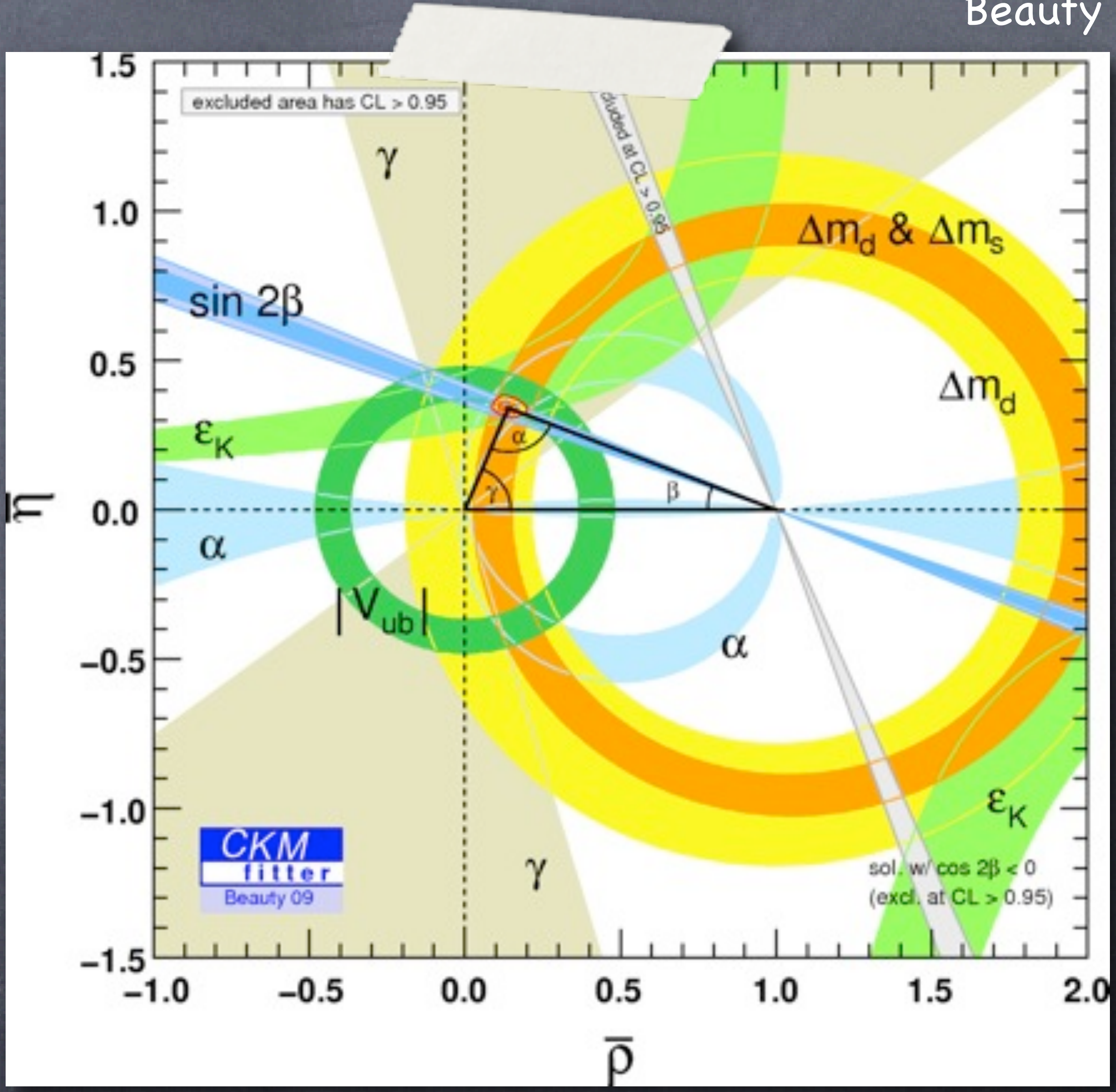
See talk by P. Paradisi

- UTA within the SM

$$\epsilon_K, \Delta m_d, \left|\frac{\Delta m_s}{\Delta m_d}\right|, \left|\frac{V_{ub}}{V_{cb}}\right|$$

  - relying on theoretical calculations of hadronic matrix elements

K.Trabelsi (CKMFitter) Beauty 2009



# Motivation: CKM Unitarity Analysis

T. Browder et al.
0710.3799

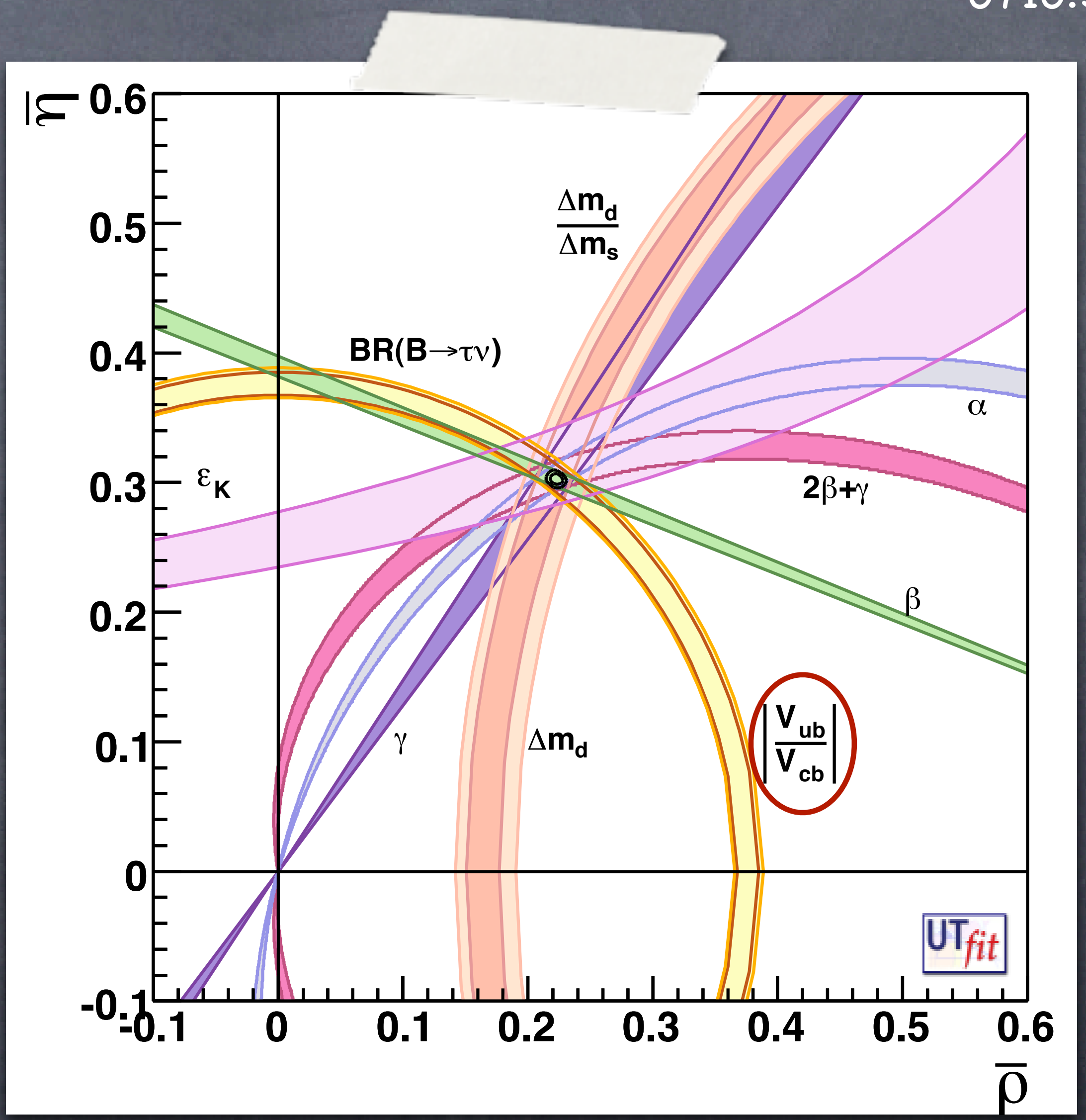
See talk by
P. Paradisi

- UTA within the SM

$$\epsilon_K, \Delta m_d, \left|\frac{\Delta m_s}{\Delta m_d}\right|, \left|\frac{V_{ub}}{V_{cb}}\right|$$

  - relying on theoretical calculations of hadronic matrix elements

- Projected Super Flavour Factory sensitivity
  - $V_{ub}$ (exclusive): 3-5%
  - $V_{ub}$ (inclusive): 2-6%



# Status of $B \to X_u \, l \, \nu$

See talk by N. Gagliardi

Lange, Neubert and Paz [hep-ph/0504071]

Andersen and Gardi [hep-ph/0509360]

Gambino, Giordano, Ossola, Uraltsev [arXiv:0707.2493]

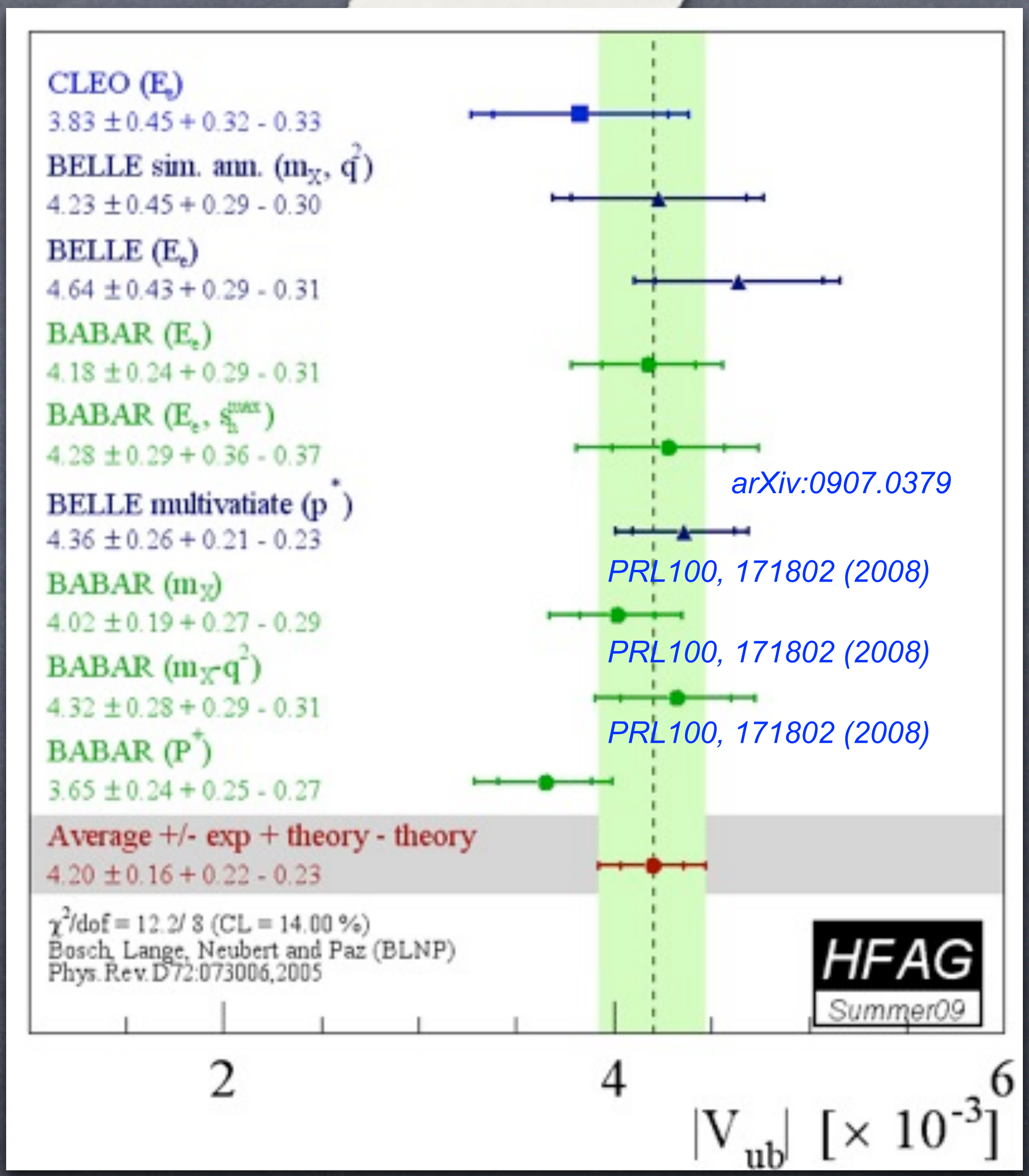
Aglietti, Di Lodovico, Ferrera, Ricciardi [arXiv:0711.0860]

Bauer, Ligeti and Luke [hep-ph/0107074]

- Inclusive determination of $V_{ub}$ using OPE and HQE
  - Expansion in $\alpha_s$ and $1/m_b$
- Present precision around 6-7%
  - however 15% tension with UTA
  - dominant source of theoretical uncertainty due to shape-function modeling (kinematical phase-space cuts)

Antonelli et al. 0907.5386

- A fully inclusive analysis would carry a tiny 2-3% theoretical error



# Status of $B \to X_u\, l\, \nu$

- At $1/m_b^3$ leading spectator effects due to dimension 6 four quark operators (WA contributions)
  - $16\pi^2$ phase space enhanced compared to LO & NLO contributions
    Not present at dim=7*
    [Dassinger et al. hep-ph/0611168]
  - Affect both the total rate and spectra (expected to populate the $q^2$ / lepton energy endpoint region)
  - Cannot be extracted from inclusive $B \to X_c\, l\nu$ analysis
    - Nor completely from comparing $B^+$ and $B^0$ decay modes
  - Difficult to study non-perturbatively

Bigi & Uraltsev
hep-ph/9310285

Dikeman & Uraltsev
hep-ph/9703437

Bigi, Dikeman & Uraltsev
hep-ph/9706520

Uraltsev
hep-ph/9905520

Voloshin
hep-ph/0106040

D. Becirevic
hep-ph/0110124

D. Becirevic et al.
0804.1750

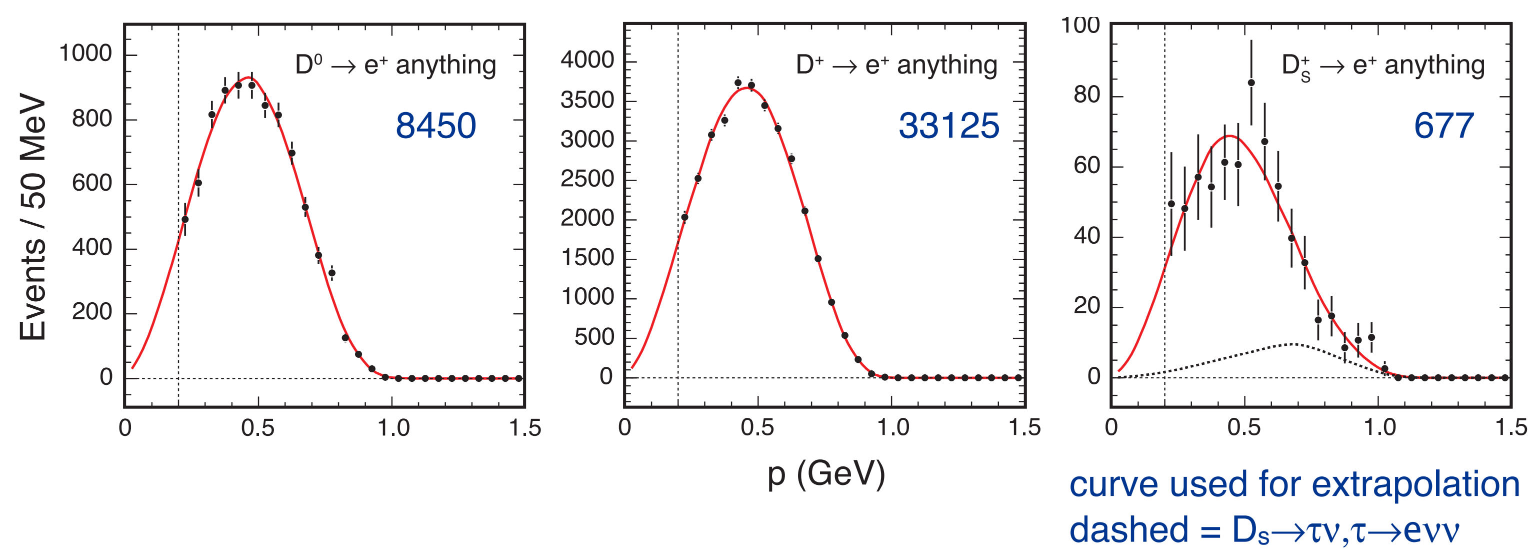
**Existing estimates spread between 3-10%**

# Inclusive Semileptonic Charm Decays

- $D_q \to X\, l\, \nu$
- Recently determined experimentally
  - $$B(D^+ \to Xe\nu) = (16.13 \pm 0.20 \pm 0.33)\%$$ $$B(D^0 \to Xe\nu) = (6.46 \pm 0.17 \pm 0.13)\%$$
  - Similar results for muons
- Very recently results also for $D_s$ decays

$$B(D_s \to Xe\nu) = (6.52\pm, 0.39 \pm 0.15)\%$$

  - Including spectra




N. E. Adam et al. [CLEO] hep-ex/0604044

M. Ablikim et al. [BES] arXiv:0804.1454

Asner et al. [CLEO] 0912.4232

# Inclusive Semileptonic Charm Decays

- Ratio of $D_s$ and $D^0$ rates shows significant [17(6)%] deviation from unity

Asner et al. [CLEO] 0912.4232

$$\Gamma(D^+ \to X e^+ \nu)/\Gamma(D^0 \to X e^+ \nu) = 0.985(28) ,$$
$$\Gamma(D_s^+ \to X e^+ \nu)/\Gamma(D^0 \to X e^+ \nu) = 0.828(57)$$

- Signs of WA in $D_s$ decays?
- How to disentangle from possible SU(3) violation?

# SU(3) violation in Charm (Two examples)

- Hyperfine mass splitting $\Delta^{hf}_{D_q} = 3(m^2_{D^*_q} - m^2_{D_q})/4$

$$\Delta^{hf}_{D^+} = 0.409(1)\mathrm{GeV}^2\,, \qquad \Delta^{hf}_{D^0} = 0.413(1)\mathrm{GeV}^2\,, \qquad \Delta^{hf}_{D_s} = 0.440(2)\mathrm{GeV}^2\,.$$

  - SU(3) violation at 10%

- Decay constants

Bazavov et al.
[Fermilab & MILC]
0912.5221

  - Lattice estimates: $f_{D_s} = 260(10)\mathrm{MeV}$ $f_D = 217(10)\mathrm{MeV}$
  - SU(3) violation at 20%

# Inclusive Semileptonic Charm Decays in OPE

- Treating charm quark mass as heavy, one can attempt an expansion in $\alpha_s(m_c)$, $\Lambda/m_c$
  - Need to estimate local operator matrix elements between hadronic states
    - First appear at $1/m_c^2$ <- sources of SU(3) violation
  - Heavy quark symmetry relates these estimates between the charm and beauty sectors
    - Quantitative translation (renormalization) not straight-forward

I. I. Bigi & N. G. Uraltsev, Phys. Lett. B 280 (1992)

- Alternative approach involves an educated sum over known exclusive modes

Gronau & Rosner 0903.2287

# OPE for the rate & leptonic moments

- Rate & leptonic energy moments in HQE & OPE
  - $x=2E/m_c$, $r=(m_s/m_c)^2$

$$\Gamma^{(n)} \equiv \int_0^{(1-r)} \frac{d\Gamma}{dx} x^n dx = \frac{G_F^2 m_c^5}{192\pi^3}|V_{cs}|^2 \left[ f_0^{(n)}(r) + \frac{\alpha_s}{\pi} f_1^{(n)}(r) + \frac{\alpha_s^2}{\pi^2} f_2^{(n)}(r) + \frac{\mu_\pi^2}{m_c^2} f_\pi^{(n)}(r) + \frac{\mu_G^2}{m_c^2} f_G^{(n)}(r) + \frac{\rho_{LS}^3}{m_c^3} f_{LS}^{(n)}(r) + \frac{\rho_D^3}{m_c^3} f_D^{(n)}(r) + \frac{32\pi^2}{m_c^3} B_{WA}^{(n)s} \right],$$

A. Pak & A. Czarnecki 0803.0960,

K. Melnikov 0803.0951

V. Aquila et al. hep-ph/0503083

Czarnecki & Jezabek hep-ph/9402326

Gremm and Kapustin hep-ph/9603448

Dassinger et al. hep-ph/0611168

- $\alpha_s$ corrections known up to $\alpha_s^2$ for the total rate ($\alpha_s^2\beta_0$ for the higher moments)
- $1/m_c$ corrections known up to $1/m_c^4$ (all present analyses use $1/m_c^3$)
- Cabibbo suppressed modes contribute to the total rate at the level of 5%, but their effect is highly suppressed in the normalized moments

# WA in OPE

- WA contributions to the rate can be related to matrix elements of dim=6 four quark operators

$$\langle H_{Q\bar{q}}|O^{q'}_{V-A}|H_{Q\bar{q}}\rangle \equiv \langle H_{Q\bar{q}}|\bar{Q}\gamma_\mu(1-\gamma_5)q'\,\bar{q}'\gamma^\mu(1-\gamma_5)Q|H_{Q\bar{q}}\rangle$$

$$\langle H_{Q\bar{q}}|O^{q'}_{S-P}|H_{Q\bar{q}}\rangle \equiv \langle H_{Q\bar{q}}|\bar{Q}(1-\gamma_5)q'\,\bar{q}'(1-\gamma_5)Q|H_{Q\bar{q}}\rangle$$

  - In the SU(3) limit one distinguishes between isosinglet/triplet contributions - only the later can be estimated from the rate differences of $B^+$ and $B^0$

- Conventionally one parametrizes deviations from VSA: bag parameters

$$\langle D|O_{V-A}|D\rangle = f_D^2 m_D^2 B_1\,,$$

$$\langle D|O_{S-P}|D\rangle = f_D^2 m_D^2 B_2$$

P. Gambino et al. hep-ph/0505091, 0707.2493

- Renormalization scale dependent, mix with the Darwin contributions at LO

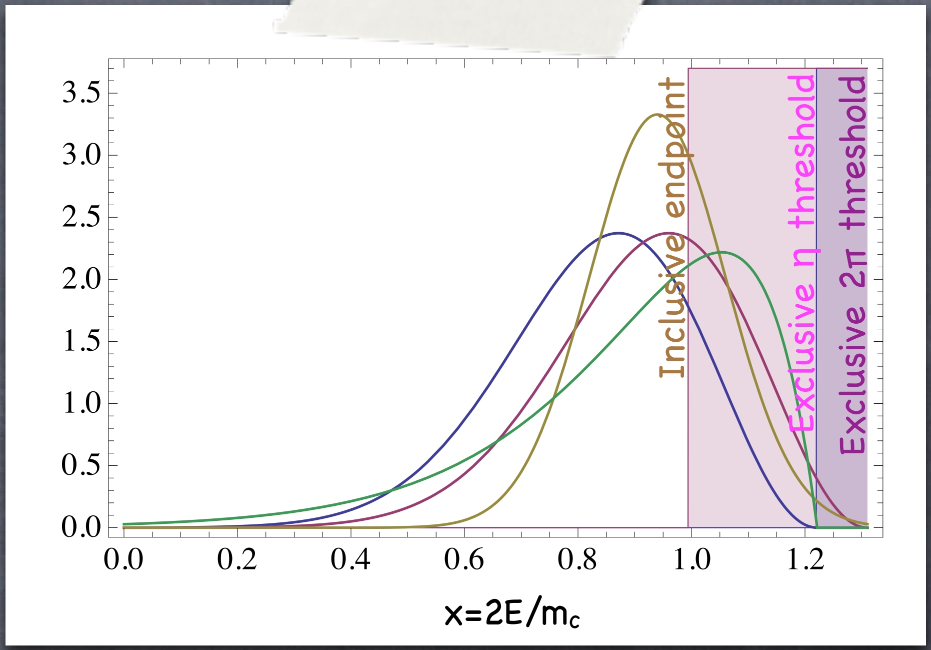
$$\delta\Gamma \sim \left[C_{WA}B_{WA}(\mu_{WA}) - \left(8\ln\frac{m_c^2}{\mu_{WA}^2} - \frac{77}{6}\right)\frac{\rho_D^3}{m_c^3} + \mathcal{O}(\alpha_s)\right]$$

  - can be used to estimate WA contributions to the rate

I. I. Bigi et al. 0911.3322

# Modeling WA in leptonic moments

- WA contributions to the weak current correlators vanish in the OPE - need to model




Bigi & Uraltsev hep-ph/9310285

- Expected to populate the spectrum endpoint

A. K. Leibovich et al. hep-ph/0205148]

- Develop a perturbative tail & non-perturbative smearing

- Possible phase-space suppression by hadronic thresholds

Gronau & Rosner 0902.1363

  - Can be studied directly using exclusive channels ($D_s \to \omega\, l\, \nu$)

# The WA interpretation of rate differences

Bigi et al. 0911.3322

- Without resorting to quantitative OPE predictions, one can estimate WA from rate differences

$$\begin{aligned}\Gamma_{WA}(D^0) &\propto \cos^2\theta_c B^s_{WA}(D^0) + \sin^2\theta_c B^d_{WA}(D^0)\,,\\ \Gamma_{WA}(D^+) &\propto \cos^2\theta_c B^s_{WA}(D^+) + \sin^2\theta_c B^d_{WA}(D^+)\,,\\ \Gamma_{WA}(D_s) &\propto \cos^2\theta_c B^s_{WA}(D_s) + \sin^2\theta_c B^d_{WA}(D_s)\,,\end{aligned}$$

- By equating the difference between $D_s$ and $D^0$ rates with the isotriplet component of WA
  - assumes SU(3) violating effects are sub-leading
- Isosinglet component unconstrained

# Confronting OPE convergence in charm

Ligeti et al. 1003.1351

J.F.K. 0909.2755

Gambino & J.F.K 1004.0114

- In order to constrain WA fully, need to explicitly compute semileptonic rates and/or distribution moments - compare with exp.
- Perturbative corrections known in the pole scheme

$$\Gamma = \Gamma_0 \left[1 - 0.72\,\alpha_s - 0.29\,\alpha_s^2\beta_0 - 0.60\,\mu_G^2 - 0.20\,\mu_\pi^2 + 0.42\,\rho_D^3 + 0.38\,\rho_{LS} + 80B_{WA}^{(0)}\right],$$

$$< E > \;=< E >_0 \left[1 - 0.03\,\alpha_s - 0.03\,\alpha_s^2\beta_0 - 0.07\,\mu_G^2 + 0.20\,\mu_\pi^2 + 1.4\,\rho_D^3 + 0.29\,\rho_{LS} + 135\bar{B}_{WA}^{(1)}\right],$$

$$< E^2 > \;=< E^2 >_0 \left[1 - 0.07\,\alpha_s - 0.05\,\alpha_s^2\beta_0 - 0.14\,\mu_G^2 + 0.52\,\mu_\pi^2 + 3.5\,\rho_D^3 + 0.66\,\rho_{LS} + 204\bar{B}_{WA}^{(2)}\right],$$

$$\sigma_E^2 = (\sigma_E^2)_0 \left[1 - 0.09\,\alpha_s - 0.05\,\alpha_s^2\beta_0 - 0.14\,\mu_G^2 + 1.7\,\mu_\pi^2 + 9.4\,\rho_D^3 + 1.4\,\rho_{LS} + 641\bar{B}_{WA}^{(\sigma)}\right],$$

c.f. Antonelli et al. 0907.5386

- Renormalon ($\Lambda/m_c$) ambiguity of pole mass
  - all moments affected (n-th scales as $m_c^n$)
- Better to use a short distance - threshold mass definition

# Convergence of perturbative corrections

Ligeti et al. 1003.1351

- Marginal in the pole scheme ($\alpha_s(m_c)\approx 0.35$)

$$\frac{\Gamma}{\Gamma_0\left[m_c^{\text{pole}}\right]} = 1 - 0.269\,\epsilon - 0.360\,\epsilon^2_{\text{BLM}} + 0.069\,\epsilon^2 + \ldots,$$ ($\epsilon$[=1] - pert. order counting parameter)

- Improves in short distance $m_c$ schemes

$$\frac{\Gamma}{\Gamma_0\left[m_c^{1S}\right]} = 1 - 0.133\,\epsilon - 0.006\,\epsilon^2_{\text{BLM}} - 0.017\,\epsilon^2.$$

- One can try to soften the strong dependence on the charm quark mass using information from inclusive B decays

$$\frac{\Gamma}{\Gamma_0\left[m_b^{1S} - \Delta\right]} = 1 - 0.075\epsilon - 0.013\,\epsilon^2_{\text{BLM}} - 0.021\,\epsilon^2,$$ $(\Delta = m_b - m_c)$

# Convergence of perturbative corrections

Gambino & J.F.K
1004.0114

- In schemes with explicit IR cut-off, one needs to choose proper (low) IR scale (0.5-0.8 GeV)

HFAG
winter '09 update

- Need to translate OPE parameters as well (from global B fits)
- Perturbative and OPE corrections translated to kinetic scheme

$$\begin{aligned}
\Gamma_{kin} &= 1.2(3)10^{-13}\text{GeV}\left\{1 + 0.23\,\alpha_s + 0.18\,\alpha_s^2\beta_0 - 0.79\,\mu_G^2 - 0.26\mu_\pi^2 + 1.45\,\rho_D^3 + 0.56\rho_{LS}^3 + 120B_{WA}^{(0)}\right.\\
< E_\ell >_{kin} &= 0.415(21)\text{GeV}\left\{1 + 0.03\,\alpha_s + 0.02\,\alpha_s^2\beta_0 - 0.09\,\mu_G^2 + 0.26\mu_\pi^2 + 2.7\rho_D^3 + 0.44\rho_{LS}^3 + 203\bar{B}_{WA}^{(1)}\right\},\\
< E_\ell^2 >_{kin} &= 0.192(20)\text{GeV}^2\left\{1 + 0.001\,\alpha_s + 0.02\,\alpha_s^2\beta_0 - 0.18\,\mu_G^2 + 0.68\mu_\pi^2 + 6.6\rho_D^3 + 0.99\rho_{LS}^3 + 307\bar{B}_{WA}^{(2)}\right\}\\
\sigma_{E,kin}^2 &= 0.019(2)\text{GeV}^2\left\{1 - 0.53\,\alpha_s - 0.17\,\alpha_s^2\beta_0 - 0.18\mu_G^2 + 2.2\mu_\pi^2 + 17\rho_D^3 + 2.1\rho_{LS}^3 + 961\bar{B}_{WA}^{(\sigma)}\right\},
\end{aligned}$$

- Rate uncertainty dominated by $m_c$ & $\mu_G$
- Higher leptonic moments by $\rho_D$

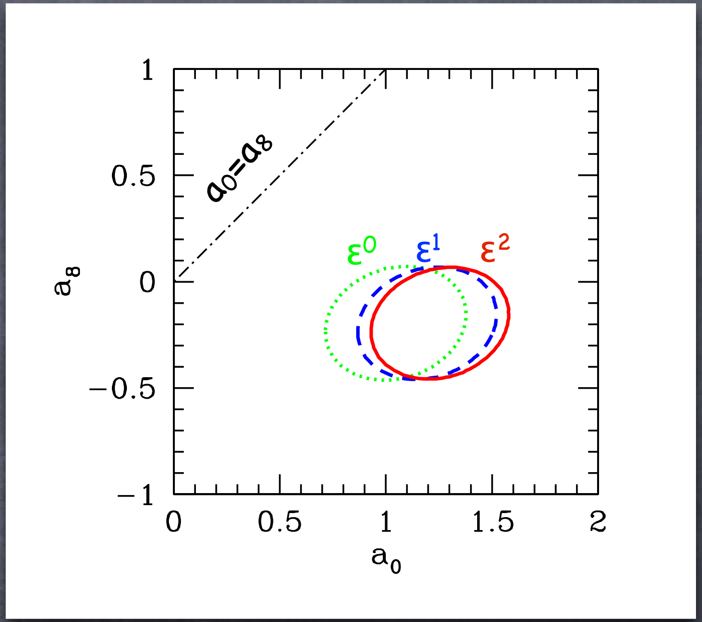
# Extraction of WA contributions

Ligeti et al. 1003.1351

- Comparing theoretical expressions with experimental rates (in 1S scheme)
  - using OPE parameters and masses as extracted from global B decay fits
  - neglecting possible SU(3) violations
- Indication of a non-zero isosinglet WA contribution

$$a_0 = 1.25 \pm 0.15\,,$$
$$a_8 = -0.20 \pm 0.12\,,$$

  - Translates into O(1-2%) effect in B->$X_u$ l ν rate

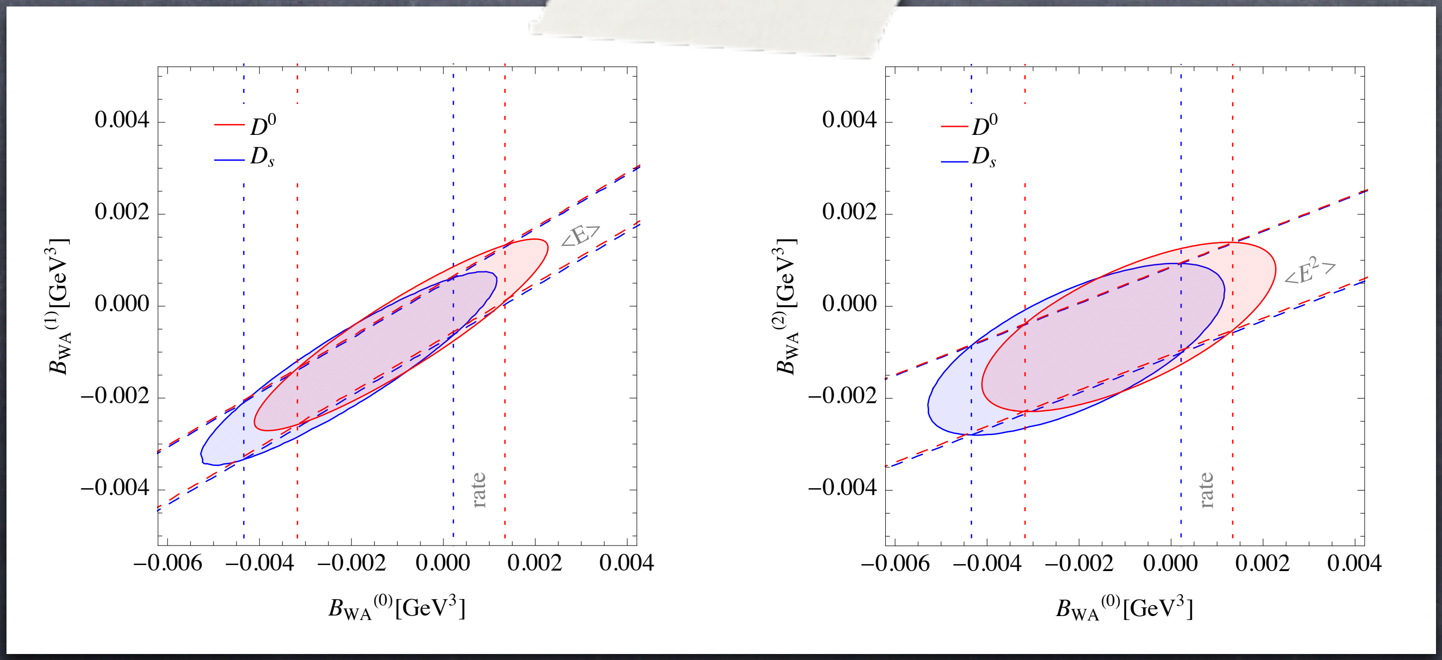



$$a_{0,8} = \frac{m_c^2\, m_D f_D^2}{m_c^5}\, 16\pi^2 \big(B_2^{s,ns} - B_1^{s,ns}\big),$$

# Extraction of WA contributions

Gambino & J.F.K
1004.0114

- Including information on the leptonic energy moments
  - Different dependence of moments on the OPE parameters allows to possibly disentangle SU(3) violating effects from WA contributions
  - Introduces dependence due to the modeling of the WA shape in the spectra
  - Correlated WA determination from the rate and the moments



# Extraction of WA contributions

Gambino & J.F.K
1004.0114

- Including information on the leptonic energy moments
  - Different dependence of moments on the OPE parameters allows to possibly disentangle SU(3) violating effects from WA contributions
  - Introduces dependence due to the modeling of the WA shape in the spectra
  - Correlated WA determination from the rate and the moments
    - Allowing for O(20%) SU(3) violation in OPE parameters
    - Largest uncertainty due to $\rho_D$ - linear (scale dependent) combination of $\rho_D$ and WA contributions determined precisely
    - For $\mu_{WA}$≈1GeV no clear indication of non-zero WA contributions

$$B^s_{WA} = -0.0003(25)\mathrm{GeV}^3$$

    - Translates into O(2%) uncertainty in $B\to X_u\, l\, \nu$ decay rate

# Conclusions

- Inclusive semileptonic charm decays can be used as a laboratory to test the OPE techniques used in the extraction of $|V_{ub}|$ and $|V_{cb}|$ from inclusive B decays
  - perturbative convergence seems to be surprisingly good
- Use several observables to over-constrain the OPE parameter uncertainties and test OPE convergence
- Indications that WA related uncertainties in inclusive $|V_{ub}|$ extraction smaller than previously expected [O(1%)]
- More tests possible in the future with additional experimental inputs (experimentally determined leptonic energy and hadronic invariant mass moments) from Cleo and BESIII

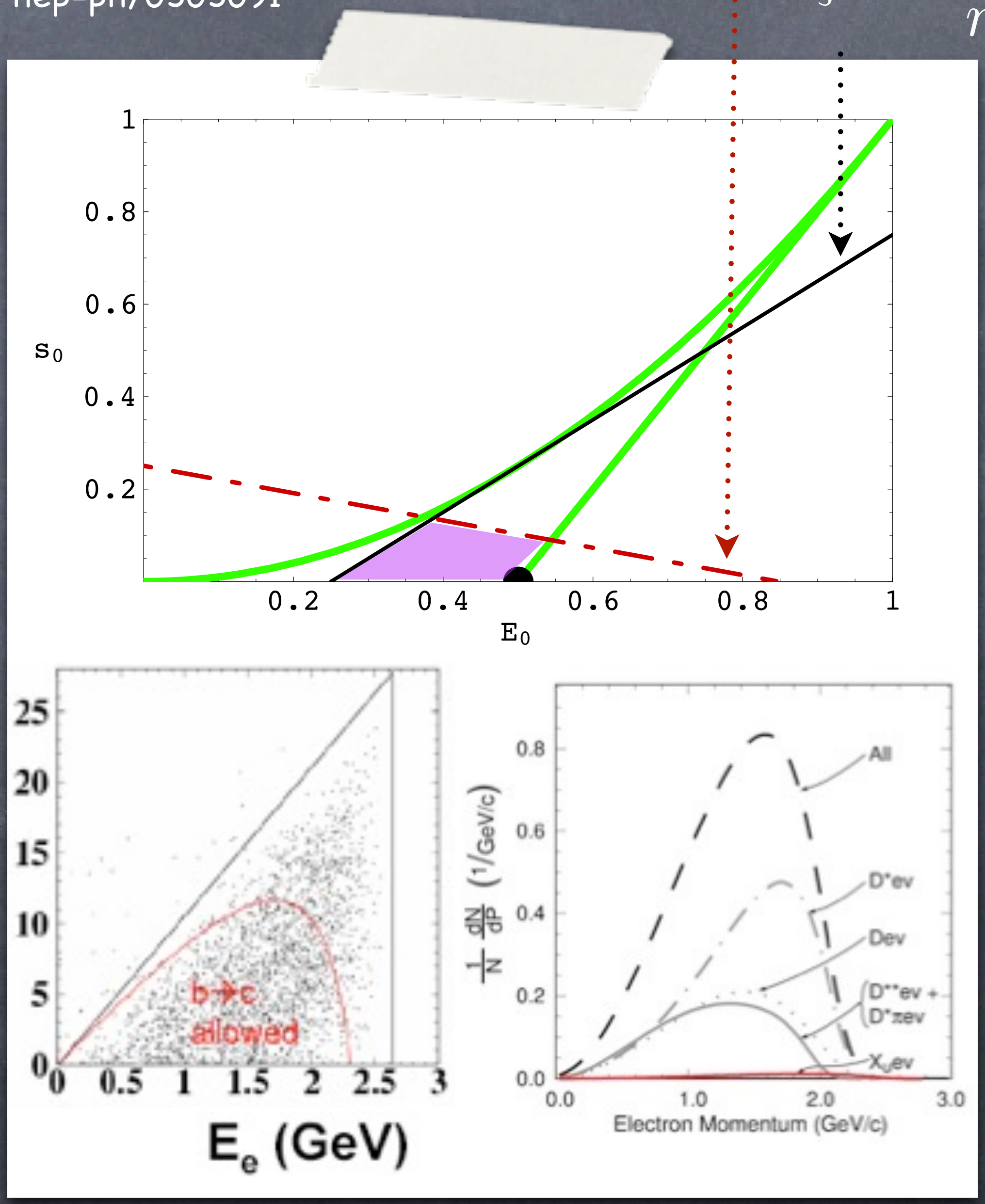
# Backup Slides

# Status of B → $X_u$ l ν

$$M_X^{cut} = 2.4\,\mathrm{GeV}$$

$$\xi = \frac{2E_\ell^{cut}}{m_b} = 0.5$$

P. Gambino, G. Ossola
hep-ph/0505091




- Experimental cuts on the leptonic energy and hadronic invariant mass to suppress dominant charm final state contributions
  - Introduce theoretical sensitivity to effects beyond the OPE
  - Modeled by s.c. shape-functions
- A fully inclusive analysis would carry a tiny 2-3% theoretical error

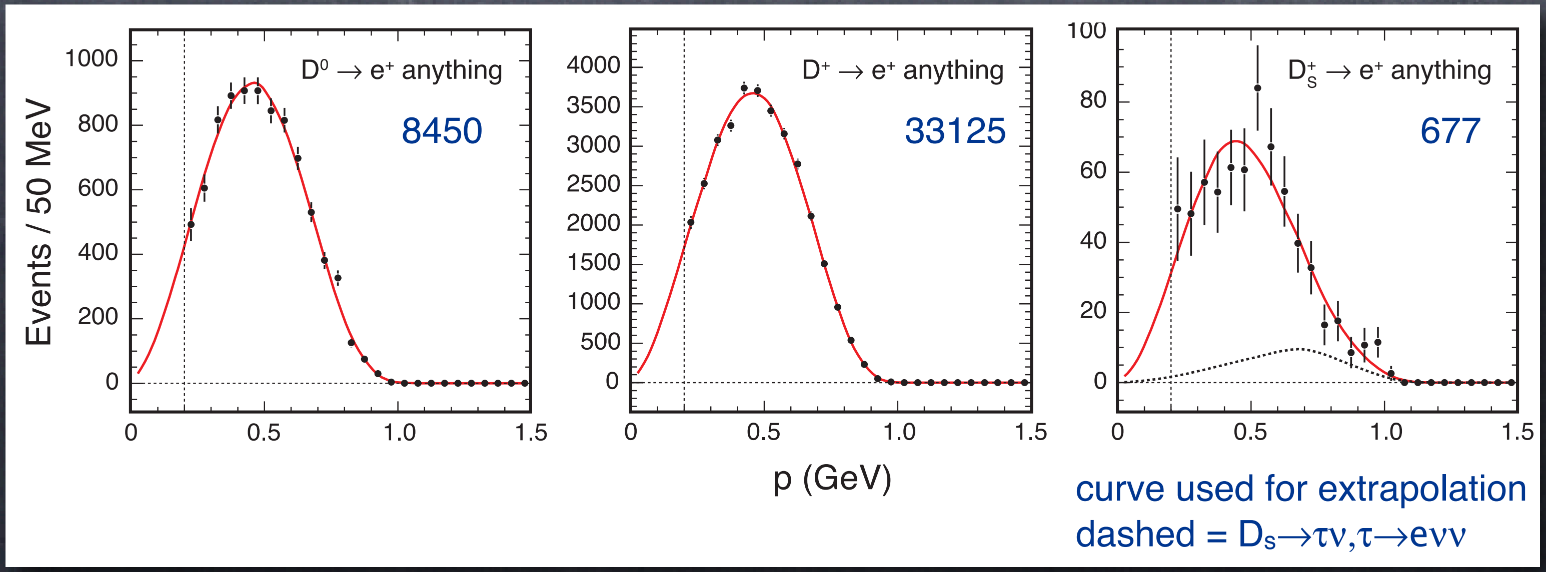
Antonelli et al.
0907.5386

# Playing the experimentalist

- One would want to compare completely inclusive leptonic energy moments in the rest-frame of the decaying hadron

Asner et al. [CLEO] 0912.4232

- This is not what Cleo presently provide:
  - do not compute the leptonic energy moments
  - spectra given in the lab frame
  - involve a lower $E_e$=0.2 GeV cut
  - do subtract the $D_s$ -> τ ν leptonic background



# Playing the experimentalist

- One would want to compare completely inclusive leptonic energy moments in the rest-frame of the decaying hadron

Gambino & J.F.K
1004.0114

- We try to compensate:
  - extrapolate the spectra down to $E_e$=0 using inclusive model shapes
  - compute the leptonic energy moments from extrapolated spectra (in the lab frame)
  - boost the moments to the D frame by directional averaging

$$< E'_e >= \gamma < E_e > \quad < {E'}_e^2 >= \gamma^2(1+\beta^2/3) < E_e^2 >$$

    - D's produced in pairs at $E_{CM}$=3774MeV
    - $D_s$'s produced associated with $D_s$*'s and through their decays

# OPE and heavy quark expansion

Bigi et al.
[hep-ph/9207214]

Manohar and Wise,
[hep-ph/9308246]
...

- Optical theorem

$$\Gamma(H_{Q\bar{q}}) = \frac{1}{2m_H}\langle H_{Q\bar{q}}|\mathcal{T}|H_{Q\bar{q}}\rangle$$

$$\mathcal{T} = \mathrm{Im}\, i \int d^4x\, T\{\mathcal{H}_{eff}(x)\mathcal{H}_{eff}(0)\}$$

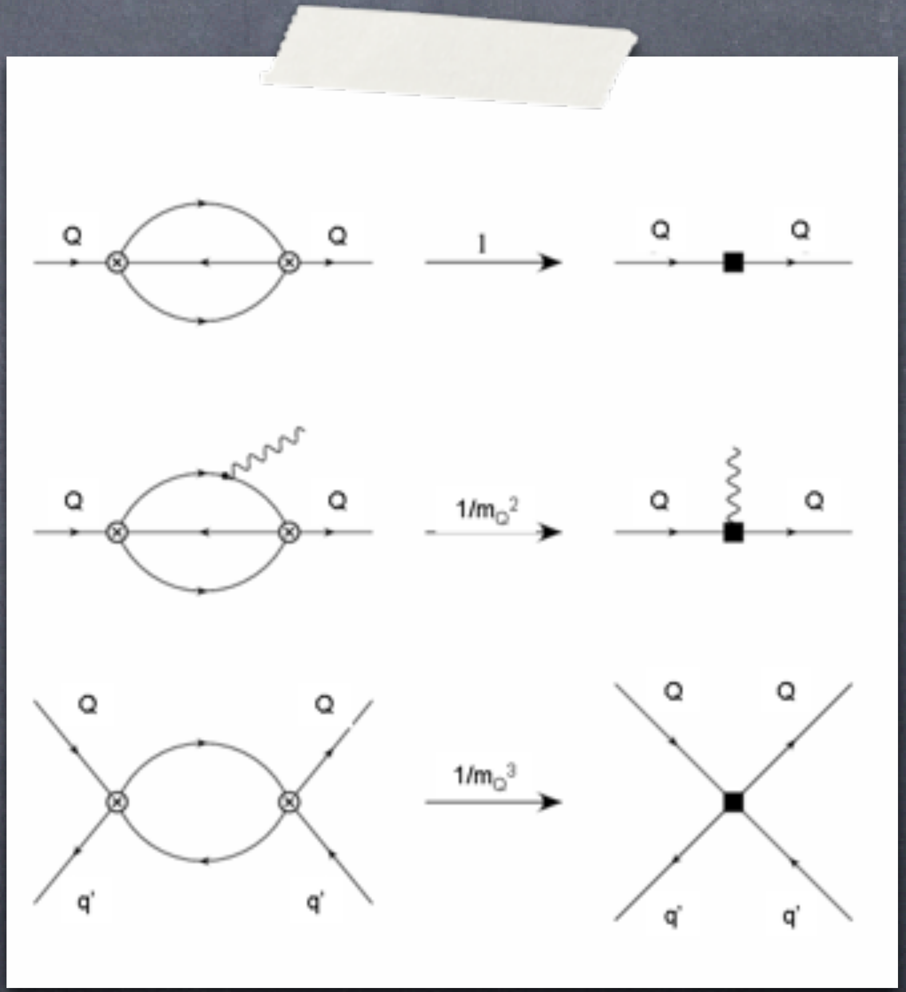
- (Global) quark-hadron duality, HQE & OPE
  - Equations of motion

$$\bar{c}c = \bar{c}\not{v}c + \frac{1}{2m_c^2}\left(\bar{c}(iD_\perp)^2 c + \bar{c}\frac{g_s}{2}\sigma.Gc\right) + \mathcal{O}(1/m_c^3)$$

  - HQE parameters

$$\mu_\pi^2 = -\frac{1}{2m_D}\langle D|\bar{c}(iD_\perp)^2 c|D\rangle$$
$$\mu_G^2 = \frac{1}{2m_D}\langle D|\bar{c}\frac{g_s}{2}\sigma.Bc|D\rangle$$

- Only applicable for the total rate



# OPE and heavy quark expansion

Bigi et al.
[hep-ph/9207214]

Manohar and Wise,
[hep-ph/9308246]
...

- Analogously define current correlator whose imaginary part gives the hadronic tensor contributing to inclusive semileptonic spectra
- Again use HQE & OPE
- Requires local quark-hadron duality to hold
  - Can be softened by instead computing spectral moments
  - Any spectral cuts will reintroduce sensitivity to contributions beyond OPE