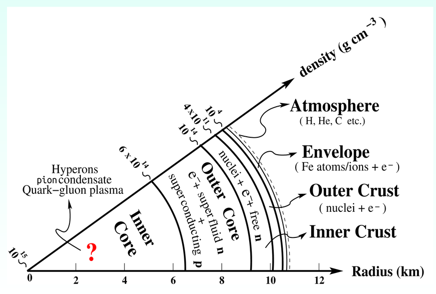
Dispersion relations of charged and uncharged pions in presence of weak magnetic field

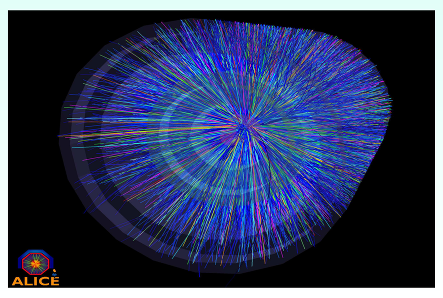
Souvik Priyam Adhya

Experimental High Energy Physics & Applications Group,
Variable Energy Cyclotron Centre, Kolkata, India.

Introduction

- Recent years have born testimony to the fact that the role of external magnetic field can have significant effect on the study of mass modification of pions. The distinct phase of dense matter in the QCD diagram invoke the concept of introduction of the magnetic field effects into the phenomenology of compact stars which are laboratories of high density matter and magnetic fields alike; with fields as high as $eB \sim 1MeV^2$ in some magnetars. On other hand, for off-central heavy ion collisions, the intensity of the magnetic field due to presence of charged species can be as high as $eB \sim m_\pi^2 \sim 0.02GeV^2$ (at RHIC) and $eB \sim 15m_\pi^2 \sim 0.3GeV^2$ (at LHC).
- In this work, we have derived the expression of the self energy of π^0 and $\pi^\pm$ in the limit of weak external magnetic field ($eB << m_\pi^2$). For our purpose, we have calculated the results up to one loop order in self energy diagrams.





Lagrangians in PS and PV couplings

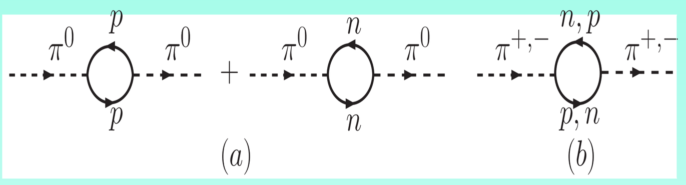


FIGURE 2: (a) represents the one-loop self-energy diagram for π^0 and (b) represents the same for $\pi^\pm$.

- The Lagrangian density for pseudo- scalar coupling ($\Gamma(q) = -i\gamma_5 g_\pi$):

$$\mathcal{L}_{int}^{PS} = -ig_\pi \bar{\Psi}\gamma_5(\vec{\tau}\cdot\vec{\Phi}_\pi)\Psi$$

$$\mathcal{L}_{int}^{PS} = -\sqrt{2}ig_{NN\pi}[\bar{\psi}_p\gamma_5\psi_n\pi^{(-)} - \bar{\psi}_n\gamma_5\psi_p\pi^{(+)}] - ig_{NN\pi}[\bar{\psi}_p\gamma_5\psi_p\pi^{(0)} - \bar{\psi}_n\gamma_5\psi_n\pi^{(0)}]$$

- The Lagrangian density for pseudo- vector coupling ($\Gamma(q) = -i(f_\pi/m_\pi)\gamma_5\not{q}$):

$$\mathcal{L}_{int}^{PV} = -\frac{f_\pi}{m_\pi}\bar{\Psi}'\gamma_5\gamma^\mu\partial_\mu(\tau.\Phi'_\pi)\Psi'$$

- Notations: $q^2 = q_\parallel^2 - q_\perp^2$ and $g^{\mu\nu} = g_\parallel^{\mu\nu} - g_\perp^{\mu\nu}$

Fermionic propagators in weak field limit

- The fermionic propagator $G(k)$ up to order $(eB)^2$ in weak magnetic field can be written as,

$$G(k) = G^{(0)}(k) + eB\, G^{(1)}(k) + (eB)^2\, G^{(2)}(k) + ...$$

where,

$$\begin{aligned} G^{(0)}(k) &= \frac{\not{k}+m}{k^2-m^2}, \\ G^{(1)}(k) &= \frac{i\gamma_1\gamma_2(\gamma.k_{||}+m)}{(k^2-m^2)^2} \quad \text{and} \\ G^{(2)}(k) &= \frac{-2k_\perp^2}{(k^2-m^2)^4}[\not{k}+m-\frac{\gamma.k_\perp}{k_\perp^2}(k^2-m^2)]. \end{aligned}$$

- Eqs.[6] and [7] are the weak field corrections to the free propagator.
- Let us consider the magnetic field along the z direction with the choice of vector potential $\vec{A} = (-By/2, Bx/2, 0)$

Self energy of pions in weak field limit for PS coupling

- The one loop contribution to the π self energy is given as,

$$\Pi_\pi(q) = -i\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\{i\Gamma(q)\}iS_a(k)\{i\Gamma(-q)\}iS_b(k+q)]$$
$$\Pi_{\pi^0} = \Pi_{\pi^0}^{(0,0)} + (eB)^2\Pi_{\pi^0}^{(1,1)} + (eB)^2\Pi_{\pi^0}^{(2,0)} + (eB)^2\Pi_{\pi^0}^{(0,2)}.$$
$$\Pi_{\pi^\pm} = \Pi_{\pi^\pm}^{(0,0)} + 0 + (eB)^2\Pi_{\pi^\pm}^{(2,0)}$$

- On evaluation of Dirac traces, all the terms proportional to (eB) have vanishing traces either due to odd no. of γ matrices or off-diagonal elements of metric tensor.

Vacuum contribution to the self energy

$$\begin{aligned} \Pi_{\pi^0}^{(0,0)}(q) &= -ig_\pi^2\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\gamma_5\, iS_p^{(0)}(k)\,\gamma_5\, iS_p^{(0)}(k+q)] + [p\to n] \\ \Pi_{\pi^0}^{(0,0)}(q) &= = \frac{g_\pi^2}{4\pi^2}\Big[\frac{q^2}{3} + \big[1+\frac{1}{\varepsilon} - \gamma_E + \log(4\pi\mu^2)\big](m_p^2 - \frac{q^2}{2}) \\ &\quad - \int_0^1 dx\big(m_p^2 - 3x(1-x)q^2\big)\log[m_p^2 - x(1-x)q^2]\Big] + [p\to n] \end{aligned}$$

Renormalisation of the pion self energy

- For $\Pi^\pm$, the corresponding term for n will be absent. Here, $\varepsilon = 2-\frac{N}{2}$ and μ is an arbitrary scaling parameter. γ_E is the Euler-Mascheroni constant. It is clearly seen that ε contains the singularity and it diverges as $N \to 4$.

$$\Pi_{\pi\,\mathbf{R}}^{(0,0)}(q) = \Pi_\pi^{(0,0)}(q) - \beta_1(q^2-m_\pi^2) - \beta_2$$
$$\beta_1 = \left(\frac{\partial\Pi_\pi^{(0,0)}(q)}{\partial q^2}\right)_{q^2=m_\pi^2} \quad \text{and} \quad \beta_2 = \left(\Pi_\pi^{(0,0)}(q)\right)_{q^2=m_\pi^2}$$

The renormalised part

$$\begin{aligned} \Pi_{\pi^0\mathbf{R}}^{(0,0)} &= -\frac{g_{pp\pi^0}^2}{4\pi^2}\int_0^1 dx\Bigg[\frac{(q^2-m_{\pi^0}^2)\,x(1-x)\big[m_p^2 - 3m_{\pi^0}^2x(1-x)\big]}{m_p^2 - m_{\pi^0}^2x(1-x)} \\ &\quad + \big[m_p^2 - 3q^2\,x(1-x)\big]\log\frac{\Delta_R}{m_p^2 - m_{\pi^0}^2\,x(1-x)}\Bigg] + [p\to n], \end{aligned}$$

here $\Delta_R = m_p^2 - q^2x(1-x)$ and m_{π^0} is mass of the neutral pion.

The magnetic field contribution in PS coupling

- The first term contribution at $(eB)^2$ is given as,

$$\begin{aligned} \Pi_{\pi^0}^{(1,1)}(q) &= -ig_\pi^2\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\gamma_5\, iS_p^{(1)}(k)\,\gamma_5\, iS_p^{(1)}(k+q)] \\ \Pi_{\pi^0}^{(1,1)}(q) &= -\frac{g_\pi^2}{4\pi^2}\int_0^1 dx\, x(1-x)\Big[\frac{1}{\Delta_R} + \frac{m_p^2 + x(1-x)q_{||}^2}{\Delta_R^2}\Big] \end{aligned}$$

- $\Pi_{\pi^\pm}^{(1,1)}(q)$ contribution is absent.
- The second term contribution at $(eB)^2$ is given as,

$$\begin{aligned} \Pi_{\pi^0}^{(2,0)}(q) &= -ig_\pi^2\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\gamma_5\, iS_p^{(2)}(k)\,\gamma_5\, iS_p^{(0)}(k+q)] \\ \Pi_{\pi^0}^{(2,0)}(q) &= -\frac{g_\pi^2}{4\pi^2}\Bigg[\int_0^1 dx(1-x)^3\Big[\frac{1}{\Delta_R} + \frac{q^2x(1-x) + q_\perp^2x(4x-1) + m_p^2}{3\Delta_R^2} \\ &\quad + \frac{2x^2q_\perp^2[q^2x(1-x)+m_p^2]}{3\Delta_R^3}\Big] + \int_0^1 dx(1-x)^2\Big[\frac{1}{\Delta_R} - \frac{q_\perp^2x(1-x)}{\Delta_R^2}\Big]\Bigg] \end{aligned}$$

Pion self energy in PV coupling

- Calculating the vacuum part and subsequent re-normalisation,

$$\begin{aligned} \Pi_{\pi^0\mathbf{R}}^{(0,0)}(q) = (\frac{f_m}{m_\pi})^2\frac{q^2}{4\pi^2}4m^2\Bigg[&\frac{\sqrt{4m^2-q^2}}{q}\tan^{-1}(\frac{q}{\sqrt{4m^2-q^2}}) \\ &-\frac{\sqrt{4m^2-m_{\pi^0}^2}}{m_{\pi^0}}\tan^{-1}\frac{m_{\pi^0}}{\sqrt{4m^2-m_{\pi^0}^2}}\Bigg] \end{aligned}$$

- Magnetic contribution to pion self energy in pseudo vector coupling,

$$\begin{aligned} \Pi_{\pi^0}^{(1,1)}(q) &= -(\frac{f_\pi}{m_\pi})^2\frac{1}{4\pi^2}\int_0^1 x(1-x)\,dx\,\Big[\frac{x(1-x)q^2(2q_{||}^2-q^2) + m^2q_{||}^2}{\Delta_R^2}\Big]. \\ \Pi_{\pi^0}^{(2,0)}(q) &= (\frac{f_\pi}{m_\pi})^2\frac{1}{4\pi^2}\Bigg[\int_0^1 dx\frac{(1-x)^3}{3}q^2\Big[\frac{3}{2\Delta_R} + x(1-x)\frac{q_\perp^2}{\Delta_R^2} - [x(1-x)q^2 \\ &\quad + m^2](\frac{1}{\Delta_R^2} + 2x^2\frac{q_\perp^2}{\Delta_R^3})\Big] + \int_0^1 dx(1-x)^2\Big[\frac{q^2+q_\perp^2}{\Delta_R} + x(1-x)\frac{q^2q_\perp^2}{\Delta_R^2}\Big]\Bigg]. \end{aligned}$$

- The value of $\Pi_{\pi^0}^{(0,2)}(q)$ is identical with $\Pi_{\pi^0}^{(2,0)}(q)$ because we consider $m_p = m_n = m$.

Pion dispersion relation

- In order to estimate the effective pion mass, i.e mass modification of the pion in constant field, we need to introduce the pion dispersion relation,

$$\omega^2 - \vec{q}^2 - m_\pi^2 + \Pi(\omega, \vec{q}) = 0 \qquad (1)$$

where m_π is the bare pion mass and $Q^\mu = (\omega, \vec{q})$ is the 4-momentum of the pions. In the limit of vanishing momenta,

$$m_\pi^{*2} = m_\pi^2 - Re\Pi \qquad (2)$$

where, in the self energy modification, we have taken into account the Landau level quantizations through the effective Fermion propagators. The vertex factor for pseudo vector coupling is $(\frac{f_\pi^2}{4\pi} = 0.08)$.

What we have found ?

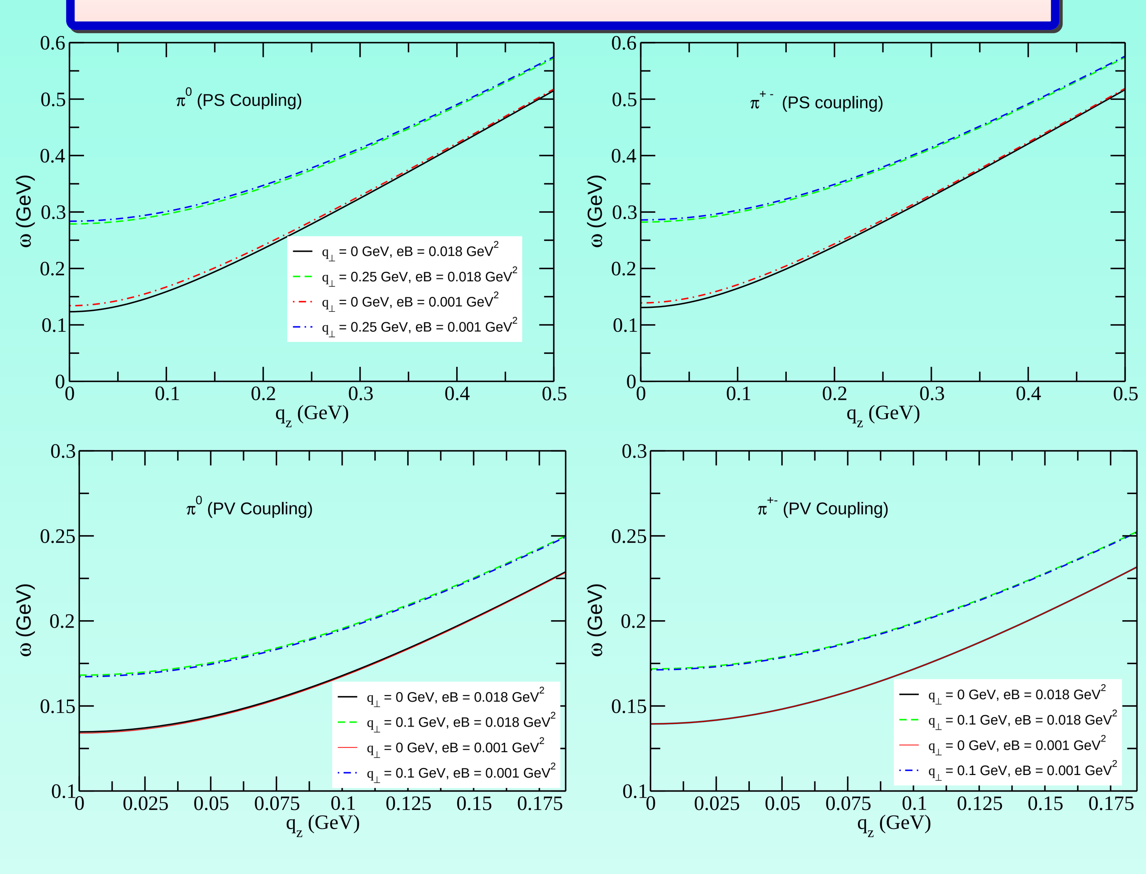


FIGURE 3: The upper left panel shows a comparison of the π_0 mass (pseudo- scalar coupling) with q_z for magnetic fields of strength $(eB) = 0.001GeV^2$ and $(eB) = 0.018GeV^2$ and for $q_\perp = 0, 0.25GeV$. The upper right panel shows a similar comparison for the $\pi_\pm$ mass. The lower left panel shows the dispersion relation for neutral pion with the z- component of momentum for pseudovector coupling. The lower right panel shows a similar plot for the case of charged pions.

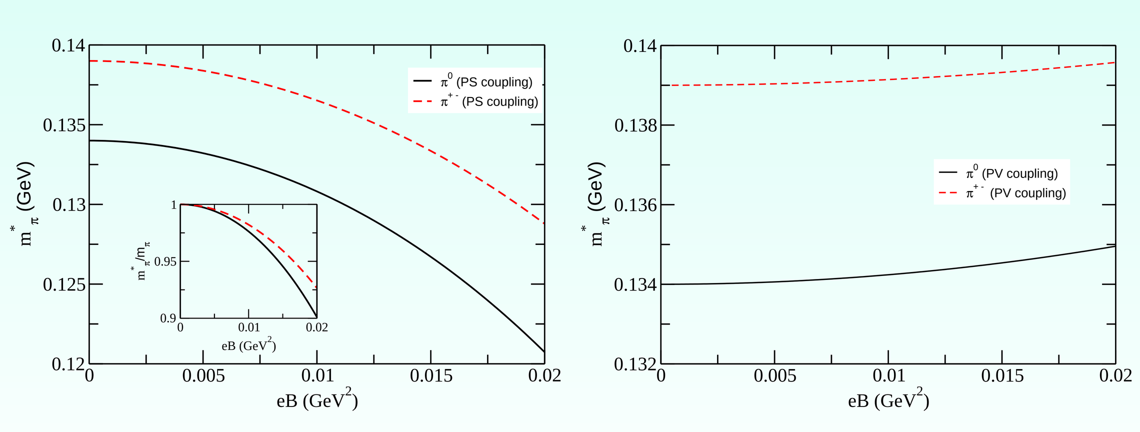


FIGURE 4: The left panel shows a comparison of the π_0 and the $\pi_\pm$ mass with magnetic field for pseudo- scalar coupling. The right panel shows a similar comparison for pseudo- vector coupling.

Summary and Conclusions

- In this work, we have re-visited the modification of the pion dispersion relations by the introduction of the external magnetic field on the charged and neutral pions. For our purpose, we have used Schwinger's proper time method of fermion propagator in presence of background magnetic field. The effect of the external magnetic field appears as corrections of order $(eB)^2$ over the vacuum contribution to the pion self energy which are relevant for the study of neutron stars and relativistic heavy ion collisions.
- The phenomenology of pions in nuclear matter is generally described by a chiral invariant pion-nucleon interaction which leads to the additional Lagrangian term $L_{\pi NN} = -(g_{\pi NN}/2g_Am_N)^2\bar{\Psi}_N\gamma^\mu\tau\Psi_N(\vec{\Pi}\times\delta_\mu\vec{\Pi})$ known as the Weinberg-Tomozawa term in the literature. We have found that the contribution for the corresponding diagram for the $\pi\pi NN$ interaction vanishes at the (eB) and $(eB)^2$ order of the external magnetic field.
- From the numerical estimates, we conclude decreasing nature of effective pion masses in case of PS coupling while an increasing nature is noticed for PV coupling.
- The results obtained here serve as a theoretical framework for study at finite density and/ or temperature in presence of arbitrary magnetic field. Finally, it should be noted that we have not incorporated the nucleon's magnetic moment in the present work. Inclusion of this will contribute in (eB) order. We do not include the medium modifications in our calculation which will be reported soon in a future work [5].

References

- T. Matsui and B. D. Serot, Ann. Phys. (N.Y.) **144**, 107(1979).
- C. S. Machado et. al. , Phys. Rev. D, **89**, 074027 (2014).
- J. Schwinger, Phys. Rev. **82**, 664 (1951).
- S. P. Adhya, M. Mandal, S. Biswas and P. K. Roy, Phys. Rev. **D 93**, 074033 (2016).
- S. P. Adhya et. al. (in preparation).

- S.P.A. would like to thank Prof. Alejandro Ayala for valuable discussions and suggestions regarding different aspects of this work.