Meson resonances on the lattice

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Hadron Spectrum Collaboration

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MESON SPECTRUM

PRL103 262001 (2009)	$I=1$
PRD82 034508 (2010)	$I=1,\ K^\star$
PRD83 111502 (2011)	$I=0$
JHEP07 126 (2011)	$c\bar{c}$
PRD88 094505 (2013)	$I=0$
JHEP05 021 (2013)	$D,\ D_s$

BARYON SPECTRUM

PRD84 074508 (2011)	$(N,\Delta)^\star$
PRD85 054016 (2012)	$(N,\Delta)_{\mathrm{hyb}}$
PRD87 054506 (2013)	$(N\ldots\Xi)^\star$
PRD90 074504 (2014)	$\Omega^\star_{ccc}$
arXiv:1502.01845	$\Xi^\star_{cc}$

HADRON SCATTERING

PRD83 071504 (2011)	$\pi\pi\ I=2$
PRD86 034031 (2012)	$\pi\pi\ I=2$
PRD87 034505 (2013)	$\pi\pi\ I=1,\ \rho$
PRL113 182001 (2014)	$\pi K, \eta K$
PRD91 054008 (2015)	$\pi K, \eta K$

"TECHNOLOGY"

PRD79 034502 (2009)	lattices
PRD80 054506 (2009)	distillation
PRD85 014507 (2012)	$\vec{p}>0$

MATRIX ELEMENTS

arXiv:1501.07457	$M' \to \gamma M$
PRD90 014511 (2014)	$f_{\pi^\star}$

Hadron spectroscopy

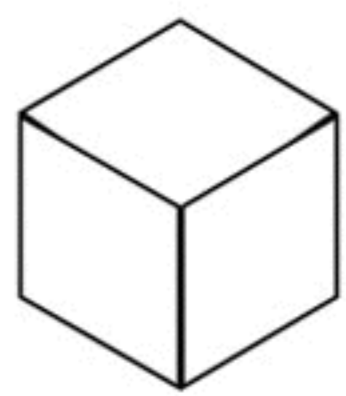
- Determination of hadron spectrum of Quantum Chromodynamics (QCD) a central goal in NP
- Several experiments worldwide

Finite volume QCD & the hadron spectrum

- Compute **correlation functions** as an average over field configurations

e.g. $$\int \mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}A_\mu \; \bar{\psi}\Gamma\psi(t)\;\bar{\psi}\Gamma\psi(0)\;\; e^{-\int d^4x\, \mathcal{L}_{\mathrm{QCD}}(\psi,\bar{\psi},A_\mu)}$$

'sum' 'field correlation' 'probability weight'



Field integration within a finite, but continuous, hypercube

Need some kind of ultraviolet regulator....

- Spectrum from two-point correlation functions

$$\begin{aligned} C(t) &= \langle 0|\mathcal{O}(t)\;\mathcal{O}^\dagger(0)|0\rangle \\ &= \sum_{\mathfrak{n}} e^{-E(\mathfrak{n})t}\,\langle 0|\mathcal{O}(0)|\mathfrak{n}\rangle\,\langle \mathfrak{n}|\mathcal{O}^\dagger(0)|0\rangle \end{aligned}$$

Lattice QCD & the hadron spectrum

- Compute **correlation functions** as a Monte Carlo average over field configurations

e.g. $$\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A_\mu \; \bar{\psi}\Gamma\psi(t)\, \bar{\psi}\Gamma\psi(0) \;\; e^{-\int d^4x\, \mathcal{L}_{\mathrm{QCD}}(\psi,\bar{\psi},A_\mu)}$$

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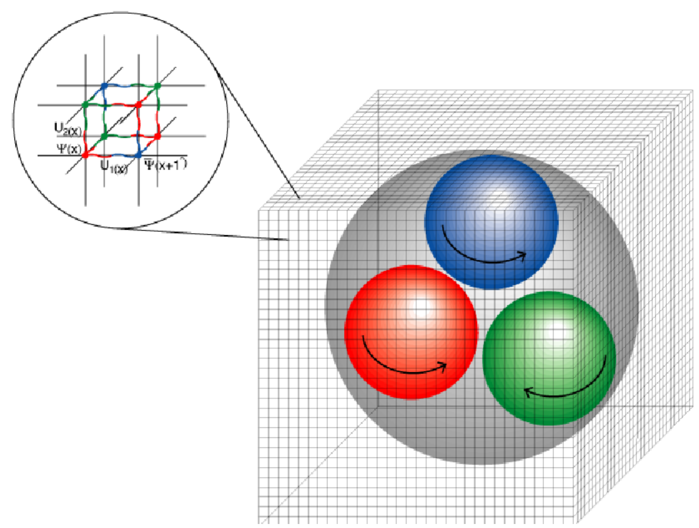


Discretize the action over sites

Serves as an ultraviolet regulator

- Spectrum from two-point correlation functions

$$\begin{aligned} C(t) &= \langle 0 | \mathcal{O}(t)\, \mathcal{O}^\dagger(0) | 0 \rangle \\ &= \sum_{\mathfrak{n}} e^{-E(\mathfrak{n})t} \langle 0 | \mathcal{O}(0) | \mathfrak{n} \rangle \langle \mathfrak{n} | \mathcal{O}^\dagger(0) | 0 \rangle \end{aligned}$$

Excited states from correlators

- how to get at excited QCD eigenstates ?
  - optimal operator for state $|\mathfrak{n}\rangle$: $\Omega_{\mathfrak{n}}^{\dagger} \sim \sum_i v_i^{(\mathfrak{n})} \mathcal{O}_i^{\dagger}$

    for a basis of meson operators $\{\mathcal{O}_i\}$
  - can be obtained (in a variational sense) from the matrix of correlators

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\, \mathcal{O}_j^{\dagger}(0) | 0 \rangle$$

  - by solving a generalized eigenvalue problem

‘diagonalize the correlation matrix’

$$C(t) v^{(\mathfrak{n})} = C(t_0) v^{(\mathfrak{n})} \lambda_{\mathfrak{n}}(t)$$

eigenvalues $\lambda_{\mathfrak{n}}(t) \sim e^{-E_{\mathfrak{n}}(t-t_0)}$

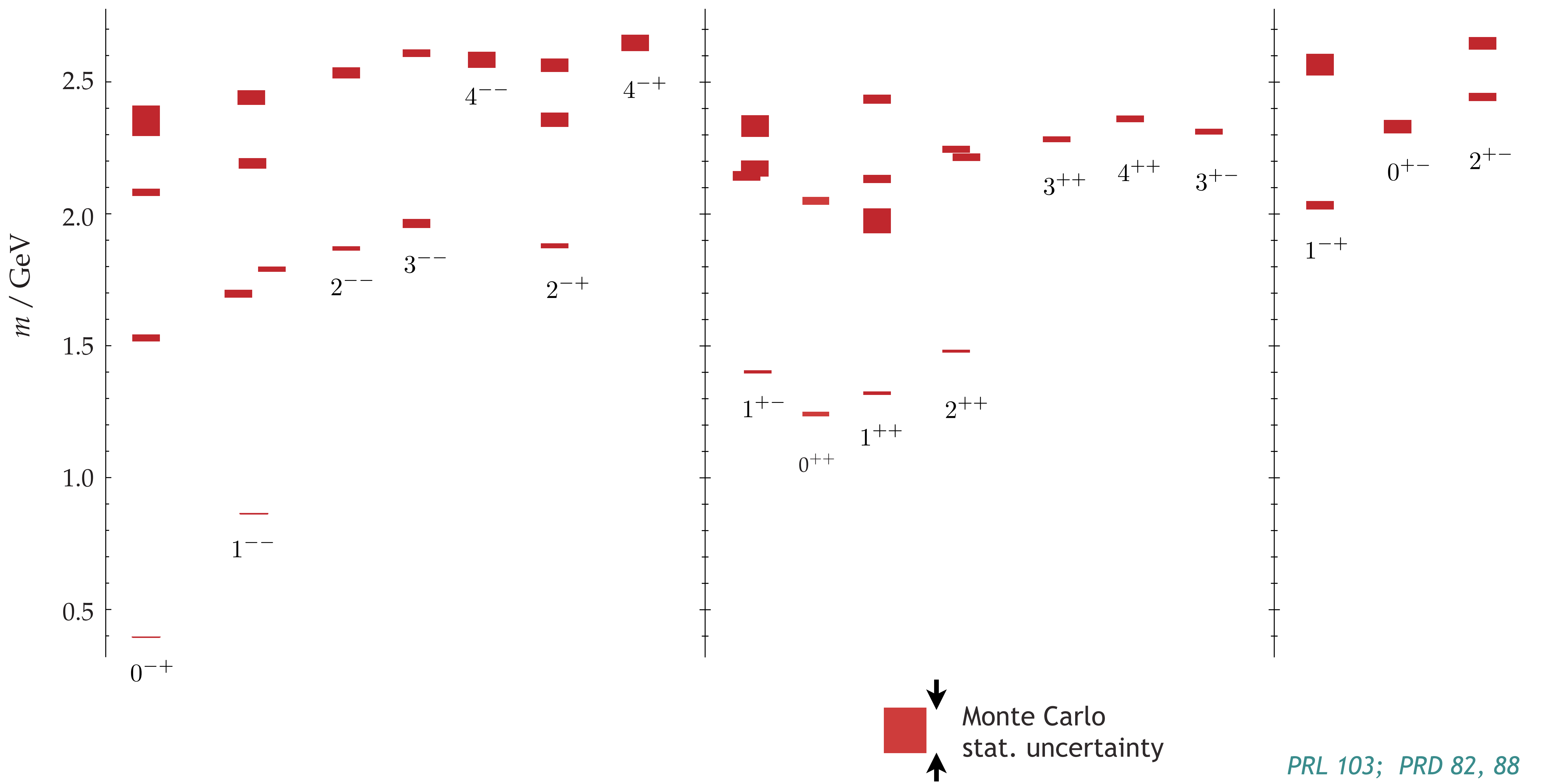
  - a large basis can be constructed using covariant derivatives :

$$\mathcal{O} \sim \bar{\psi} \Gamma \overleftrightarrow{D} \ldots \overleftrightarrow{D} \psi$$

Glimpse of meson spectrum from lattice QCD

- Appears to be some $q\bar{q}$-like near-degeneracy patterns - isovectors

$m_\pi \sim 391\,\mathrm{MeV}$



PRL 103; PRD 82, 88

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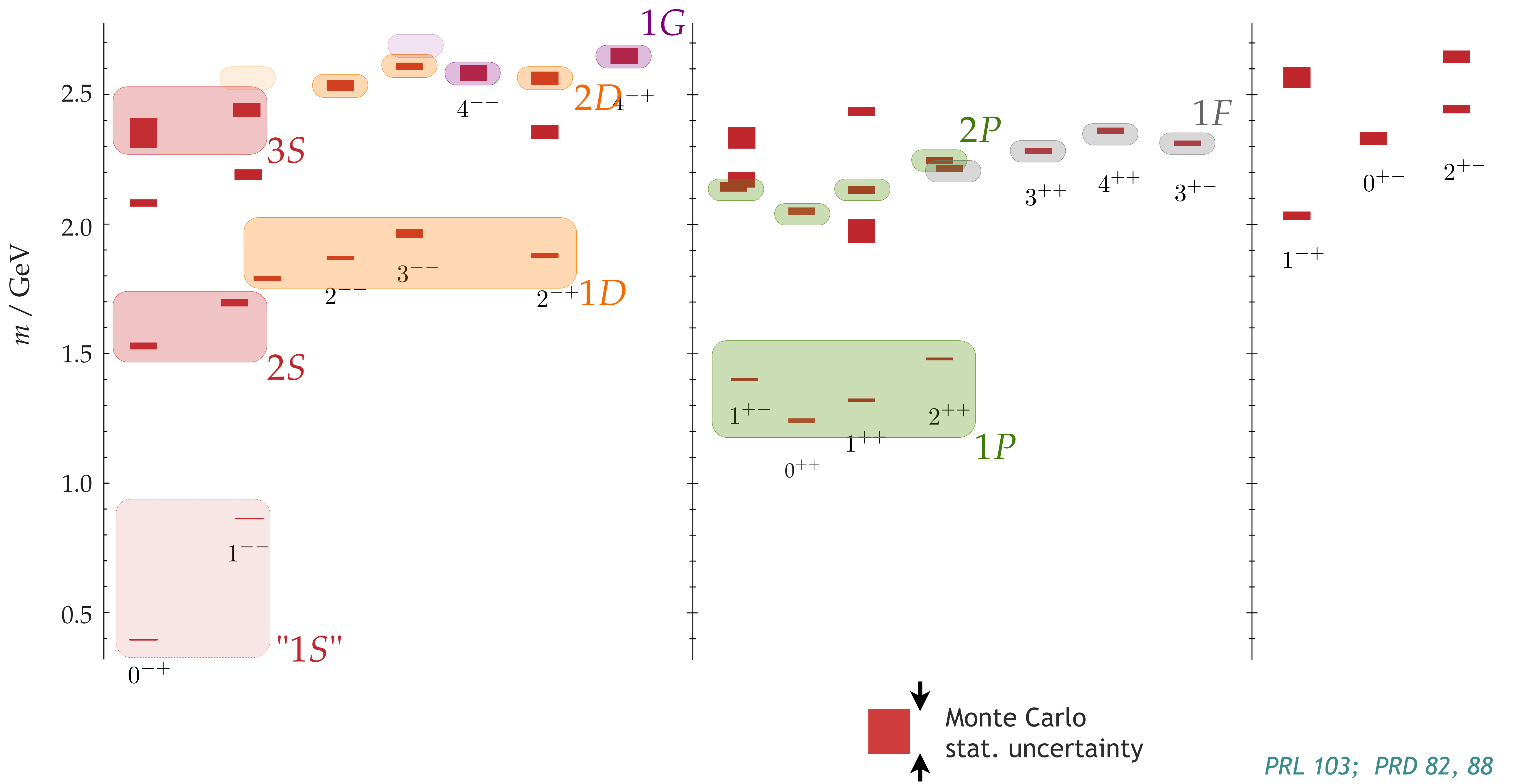
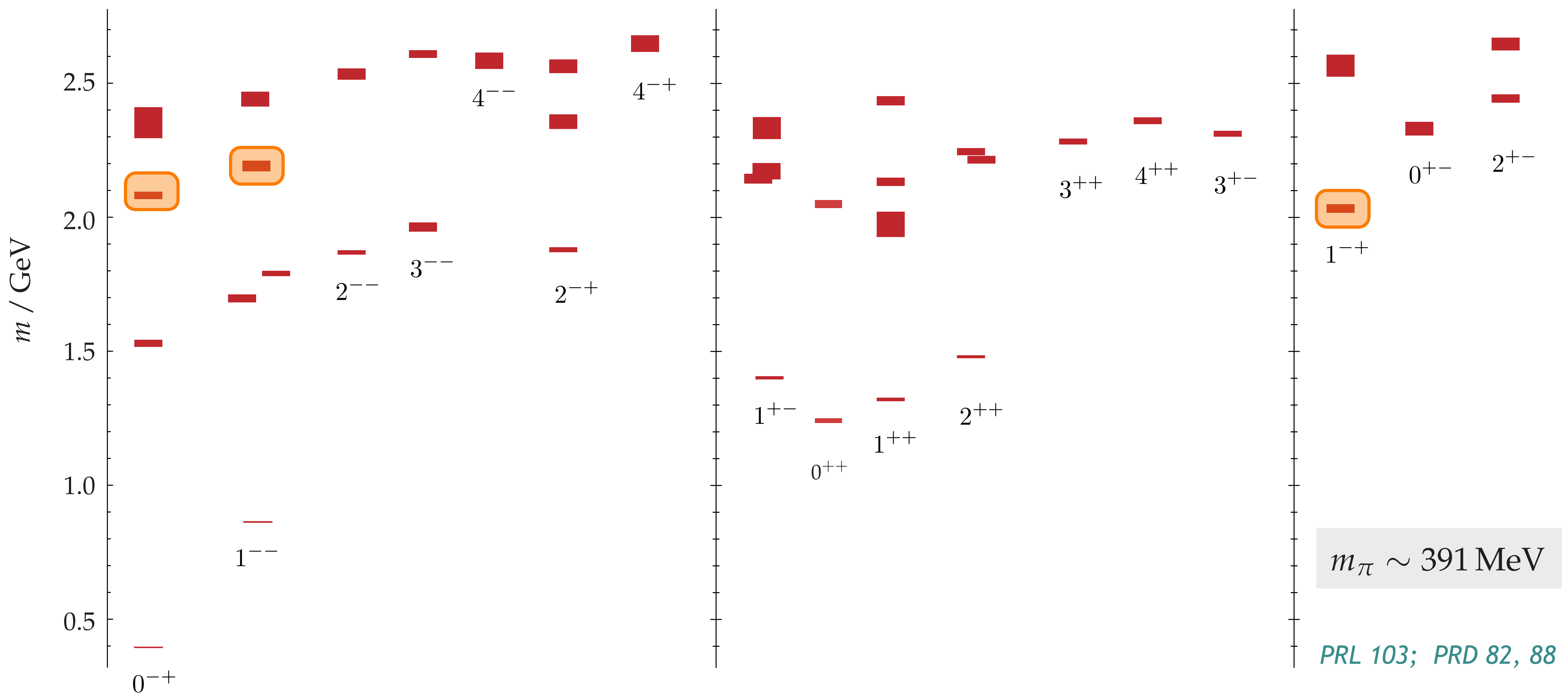


PRL 103; PRD 82, 88

Glimpse of meson spectrum from lattice QCD

- ‘super’-multiplet of **hybrid mesons** roughly 1.2 GeV above the ρ

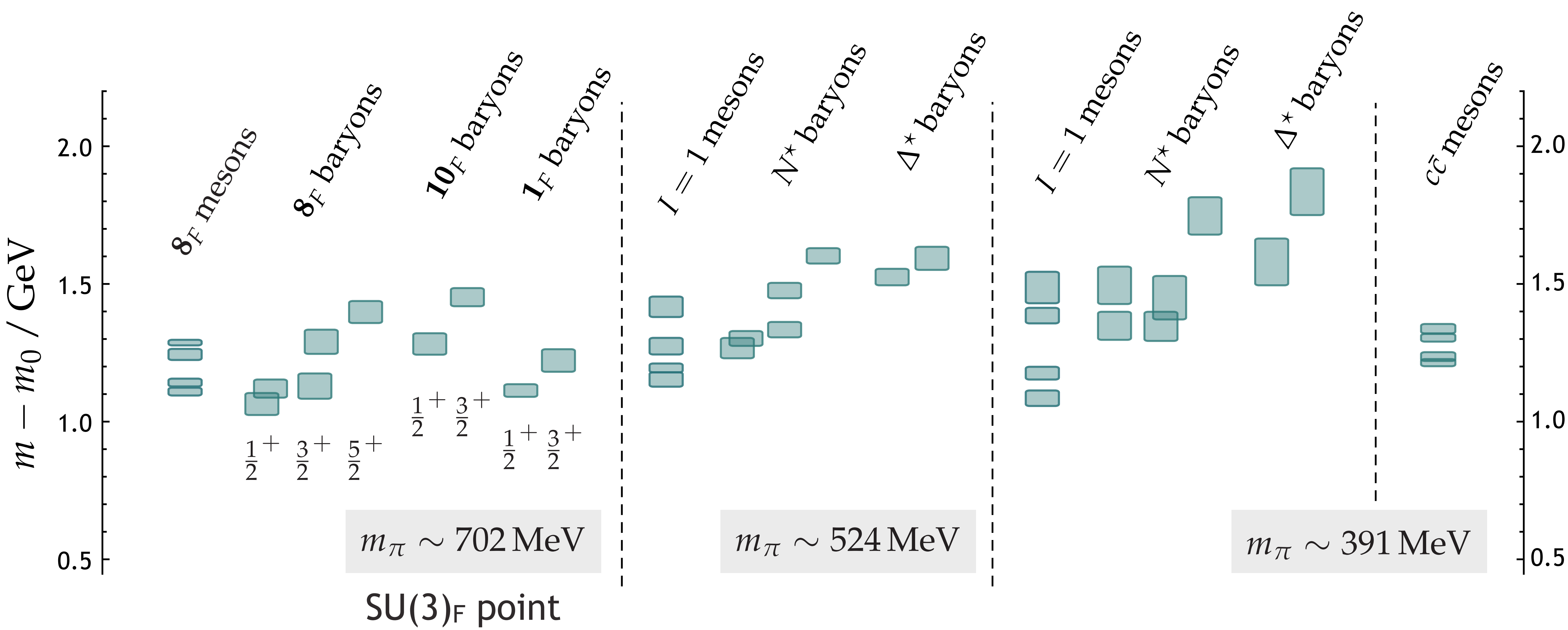
$\boxed{(0,1,2)^{-+}, 1^{--}}$



- these states have a dominant overlap onto $\bar{\psi}\Gamma[D,D]\psi \sim [q\bar{q}]_{\mathbf{8}_c} \otimes B_{\mathbf{8}_c}$

Chromo-magnetic excitation

- Subtract the 'quark mass' contribution



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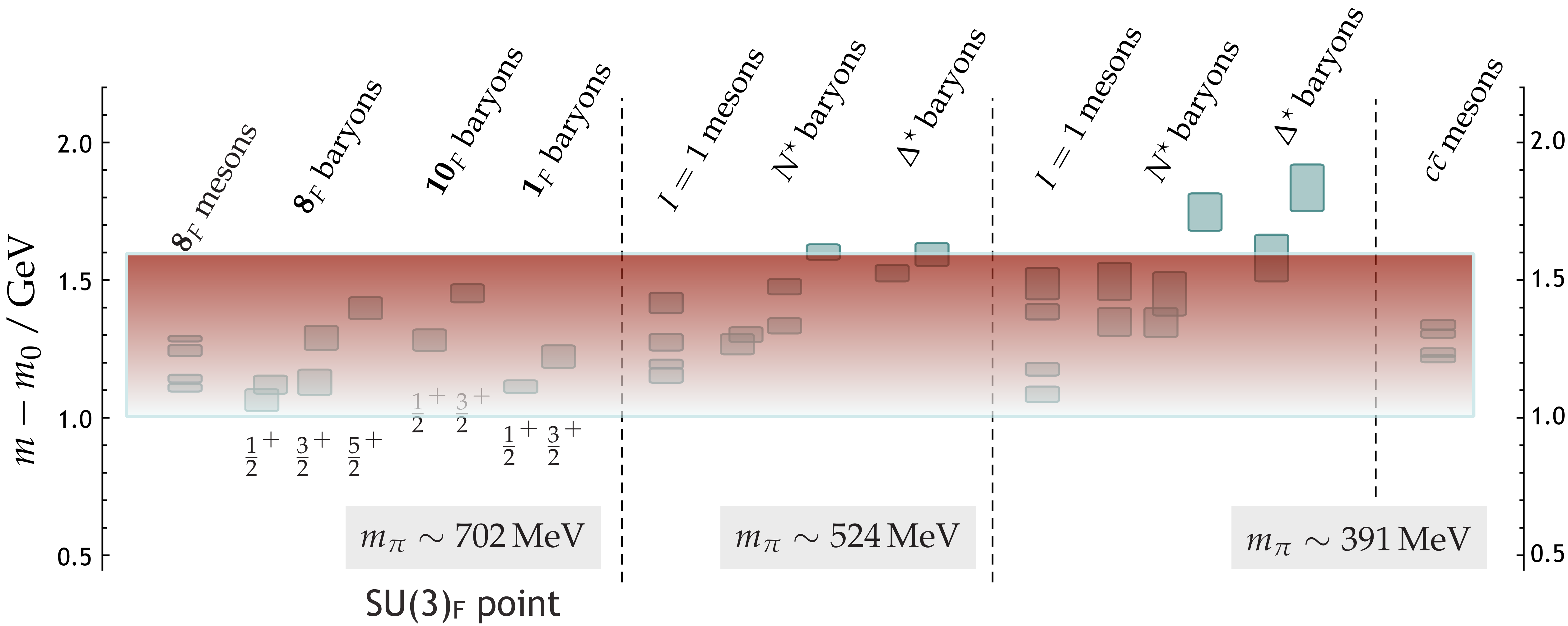
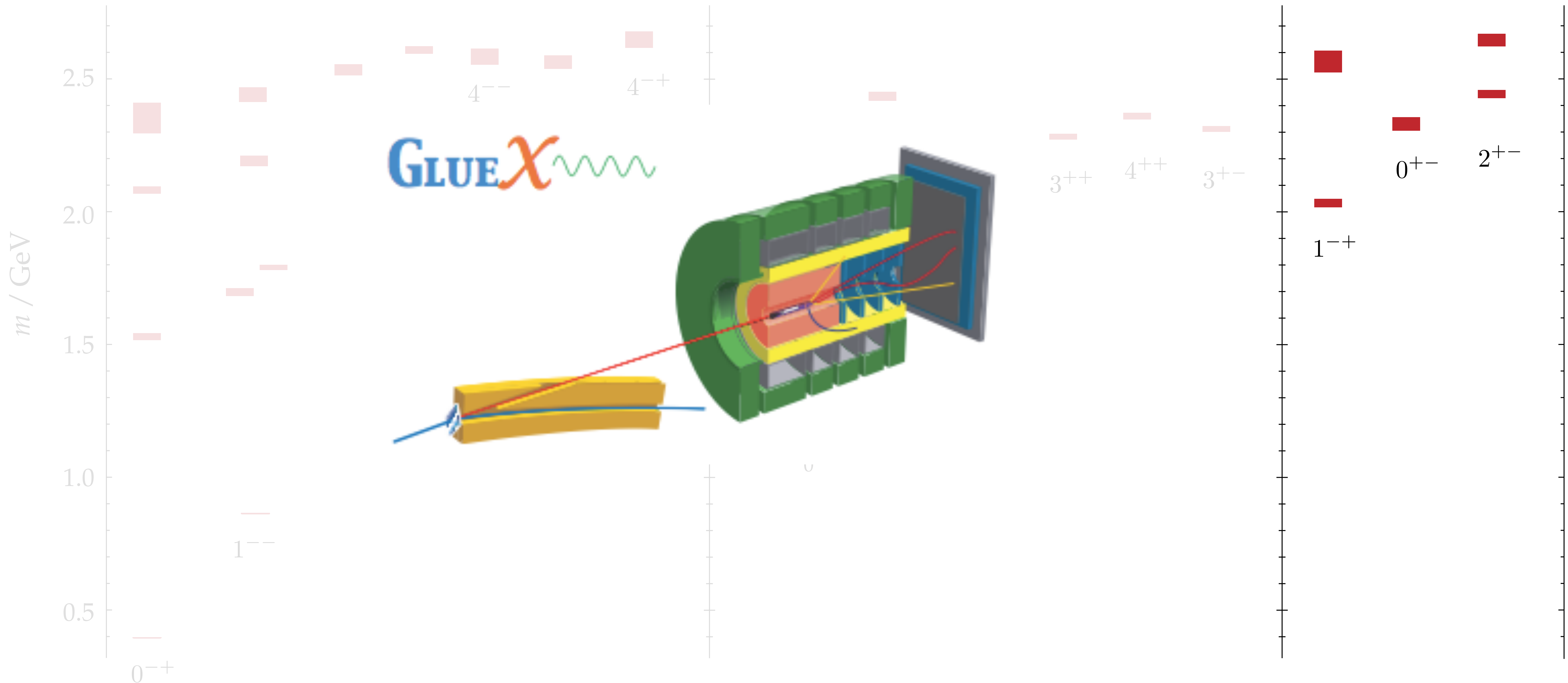


– ***Common energy scale of gluonic excitation*** $\sim 1.3\,\mathrm{GeV}$

Glimpse of meson spectrum from lattice QCD

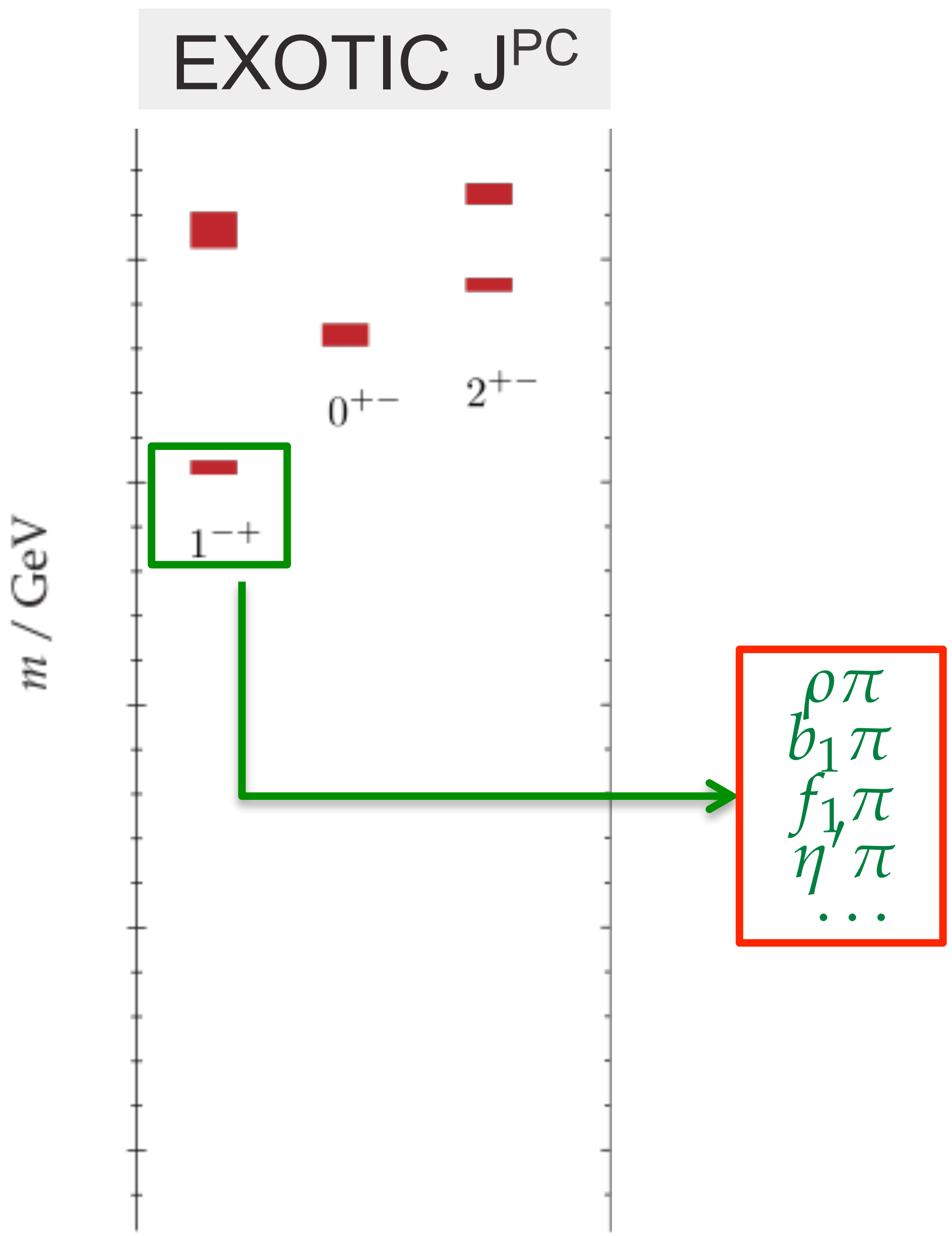
Multiple exotic mesons within range of GlueX

EXOTIC MESONS



PRL 103; PRD 82, 88

Major objective - exotic meson decays



- LQCD suggests existence of exotic mesons
- Expt. measurement in many decay channels
- Present LQCD calculations missing this info

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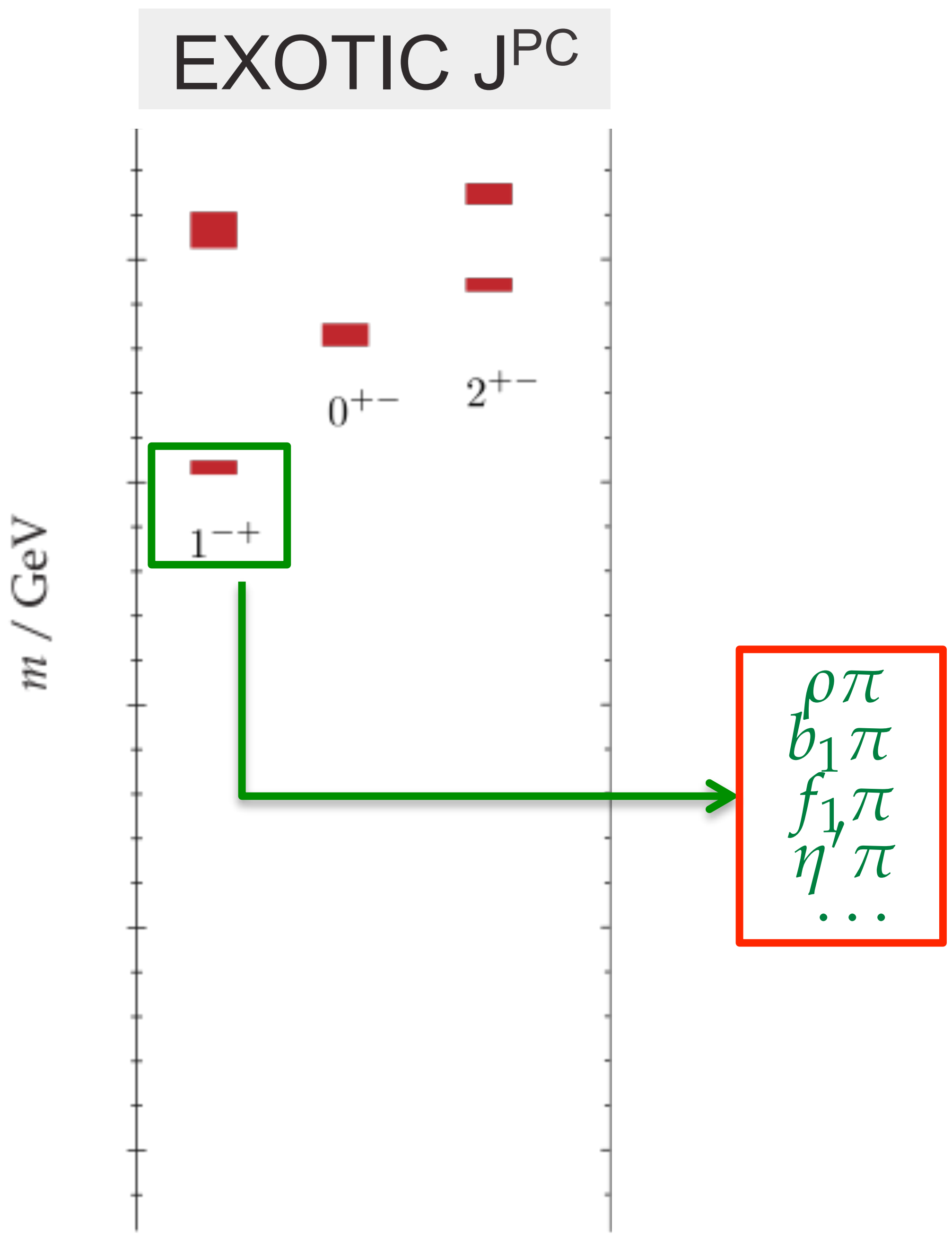


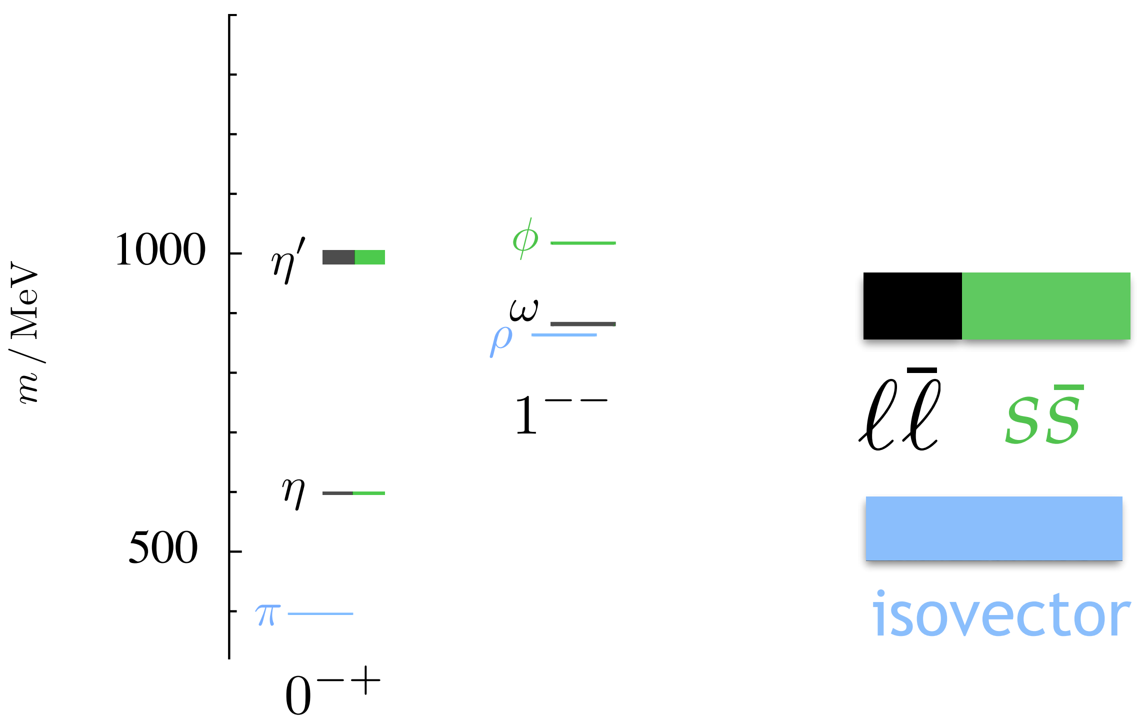
- LQCD suggests existence of exotic mesons
- Expt. measurement in many decay channels
- Present LQCD calculations missing this info

- Objective is to compute them ahead of expt.
  - » Guide expt. analysis

JLab expt. beam has started

Can study many other channels

- Many come as a pre-requisite before tackling exotics
- In particular, low-energy sector of QCD



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- Getting to the physical pion mass **not the most pressing concern here**
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e.g. Some of the simple low-lying resonances: $f_2(1270) \quad M > 8m_\pi$

$a_2(1320) \quad M > 9m_\pi$

the number of open channels
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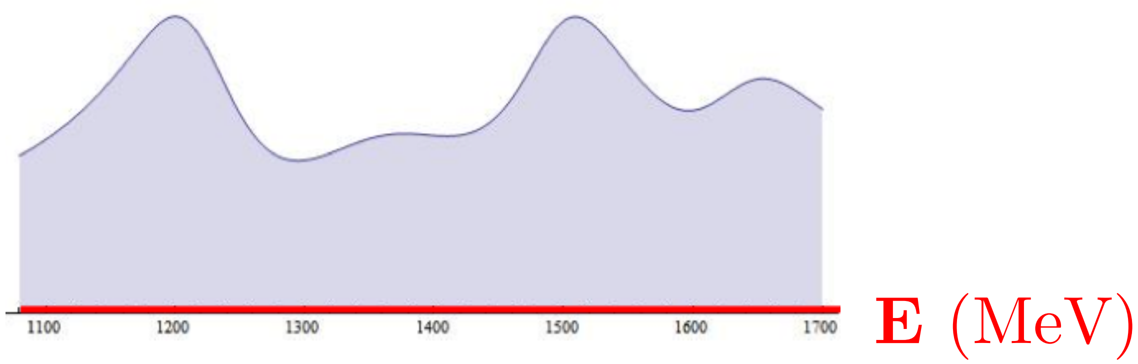
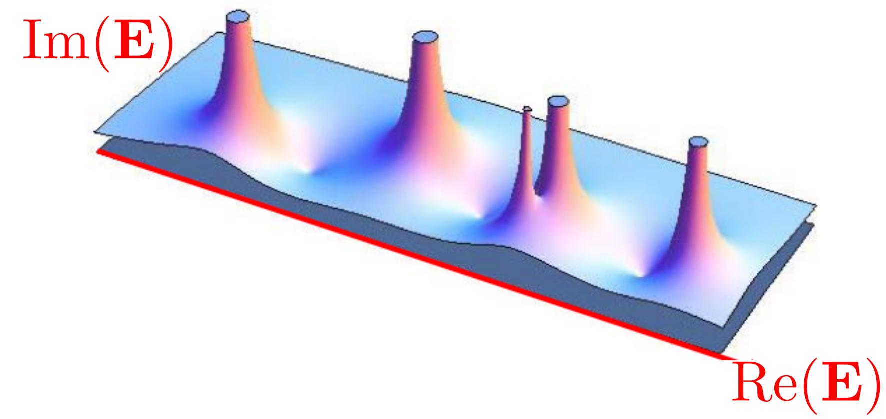
- Development of three-body formalism required

e.g., Hansen & Sharpe, Beane/Briceno/Rusetsky/Savavge,... - progress

Resonances

- Most hadrons are resonances
  - E.g., πN πN





  - Formally defined as a pole in a partial-wave scattering amplitude

$$t_l(s) \sim \frac{R}{s_0 - s} + \ldots$$

  - Different channels should have same pole location
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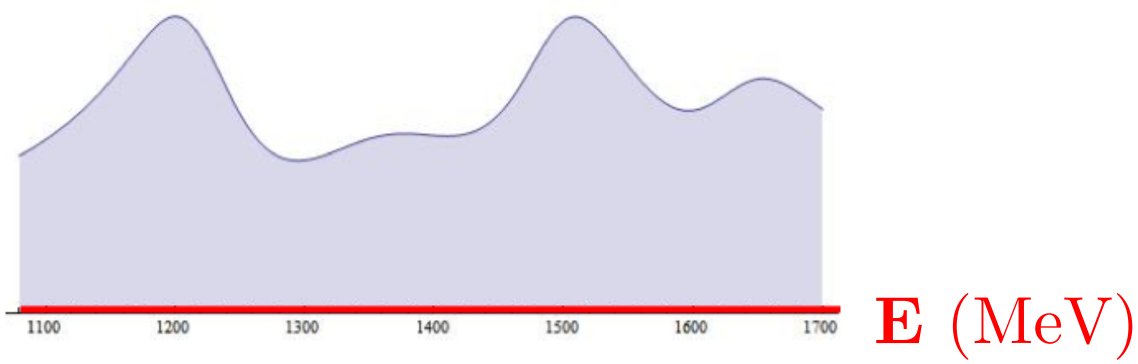

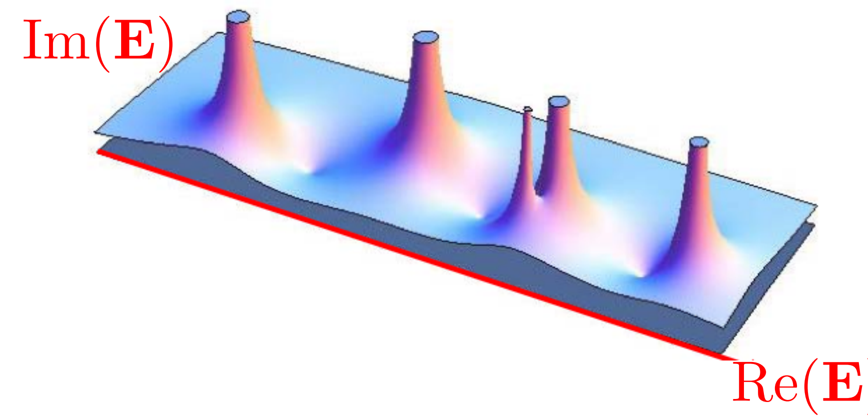

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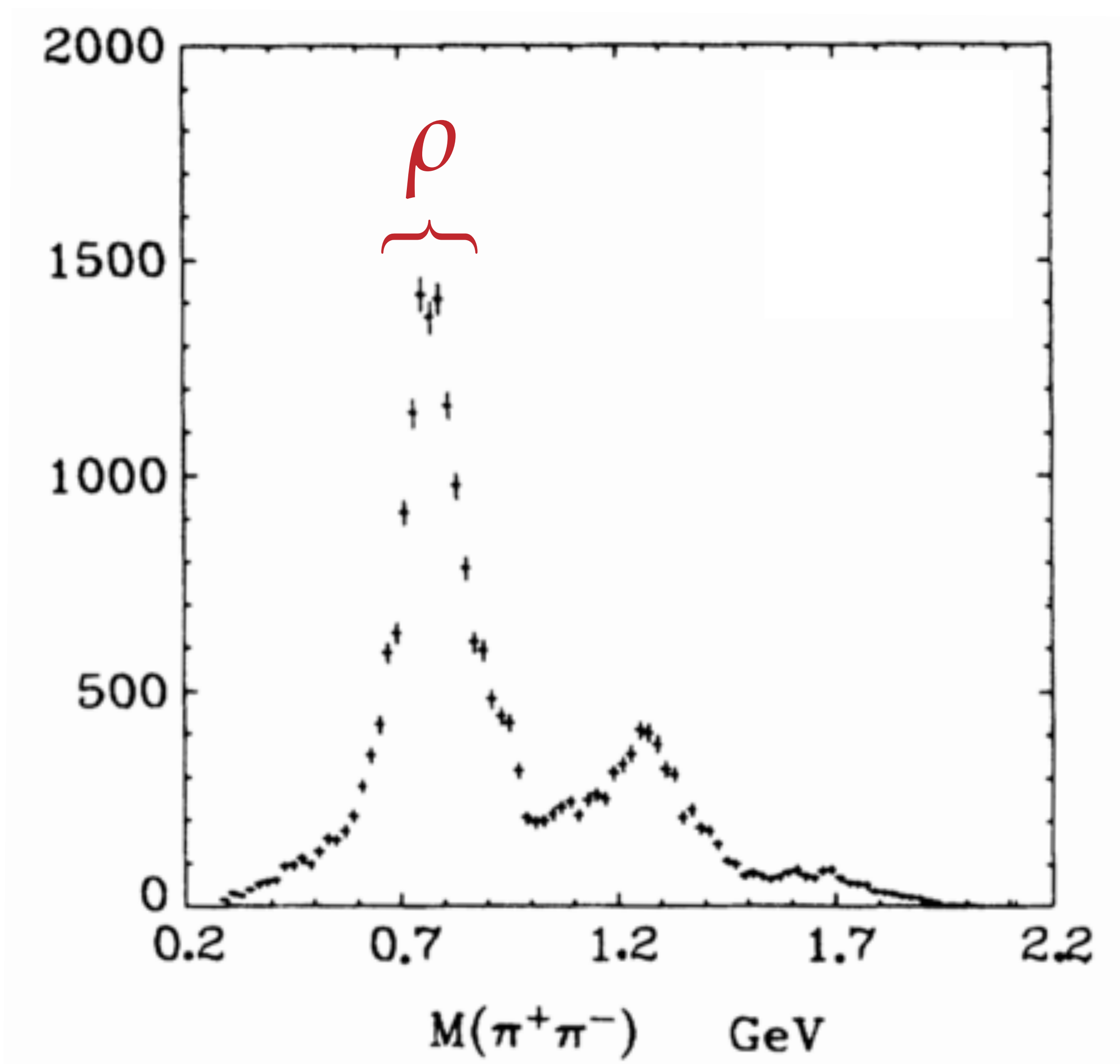
- Can we predict hadron properties from first principles?

Isospin=1 ππ P-wave



expand angular dependence
in *partial waves*

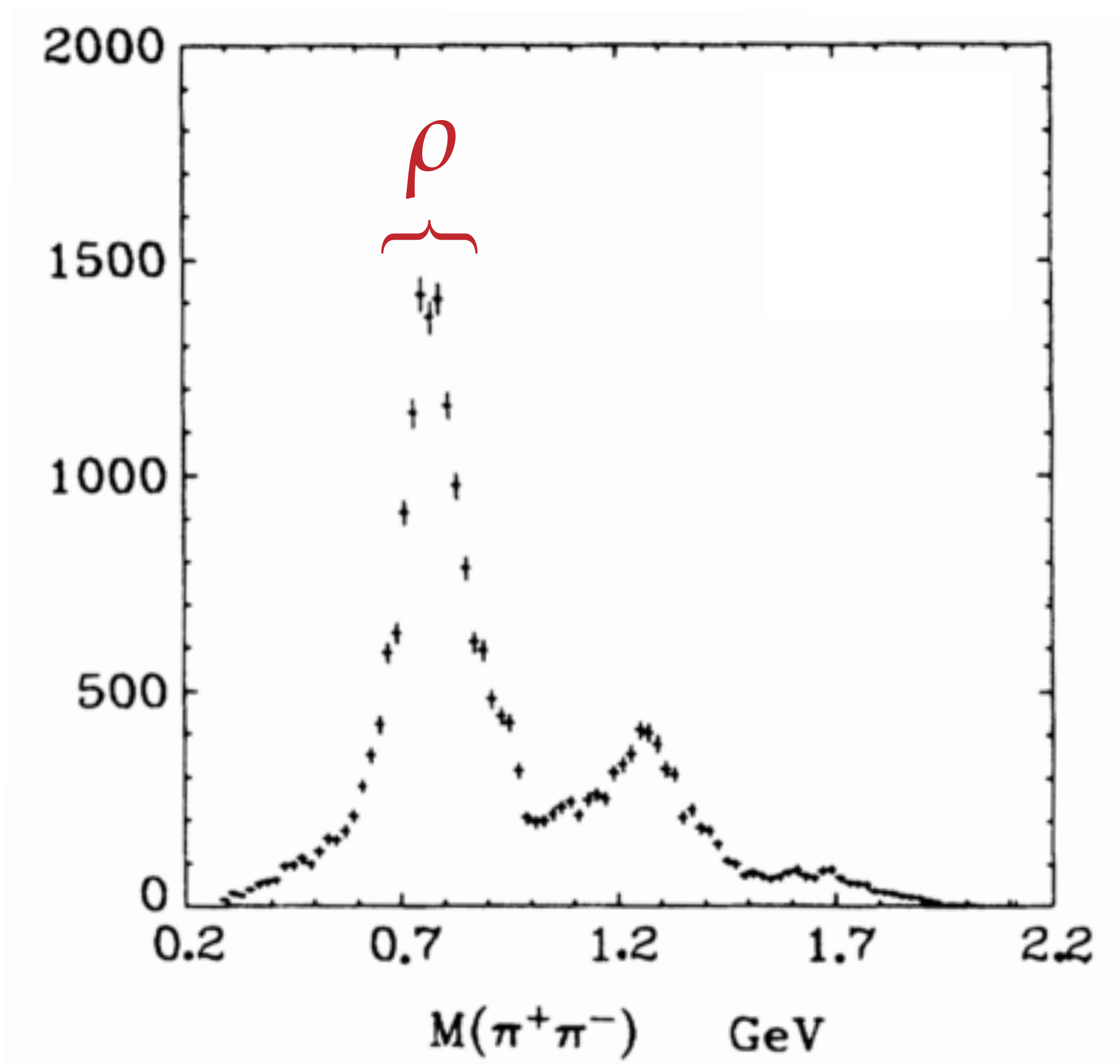
PARTIAL WAVE AMPLITUDE

$$f_\ell = \frac{1}{2i}\left(\eta_\ell\, e^{2i\delta_\ell} - 1\right)$$

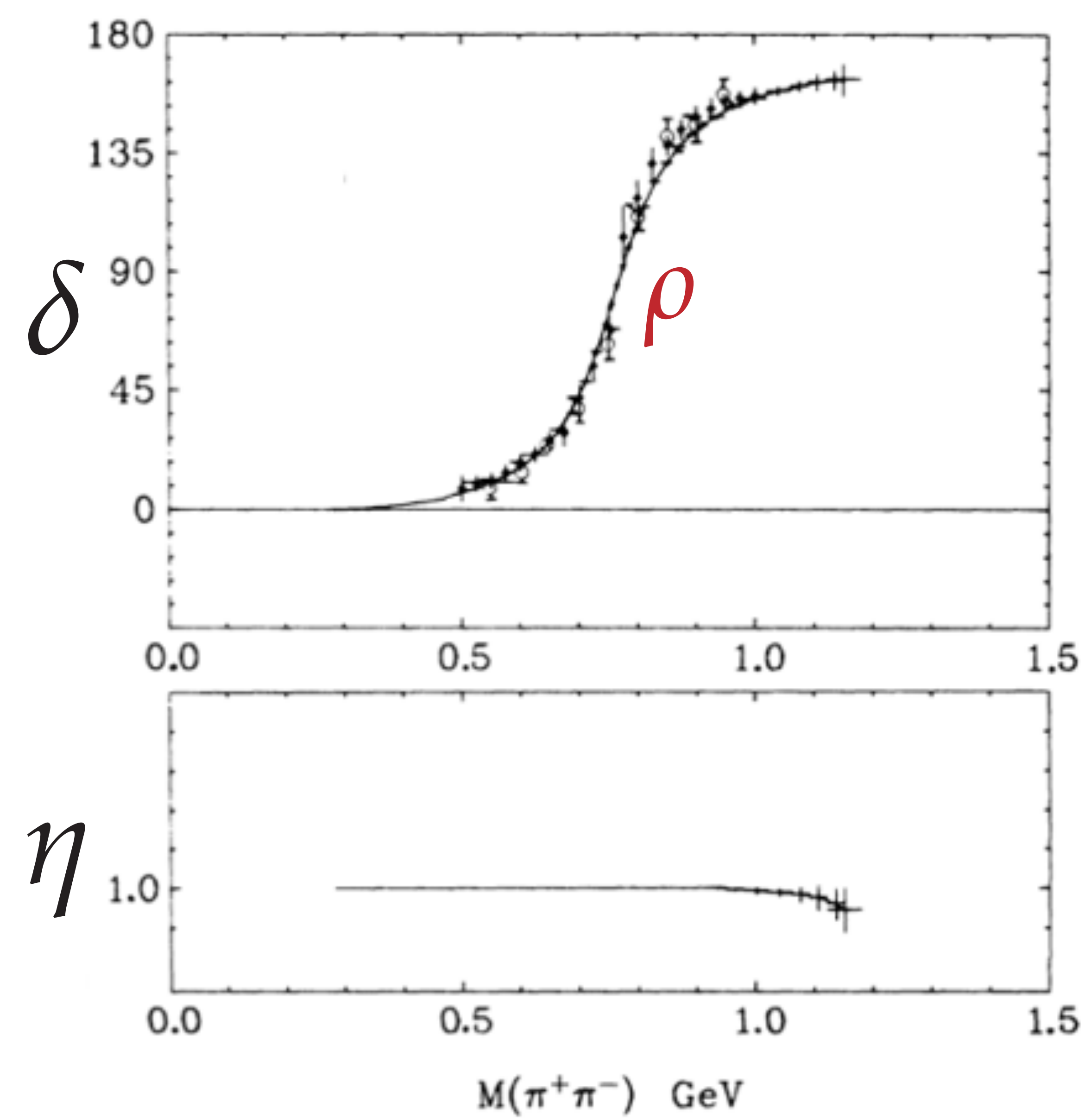
$\eta = 1$ elastic

$\eta \leq 1$ inelastic

Isospin=1 ππ P-wave



RESONANT PHASE SHIFT



Finite-volume

- Where’s the meson-meson continuum ?
  - **there isn’t one !**

Finite-volume

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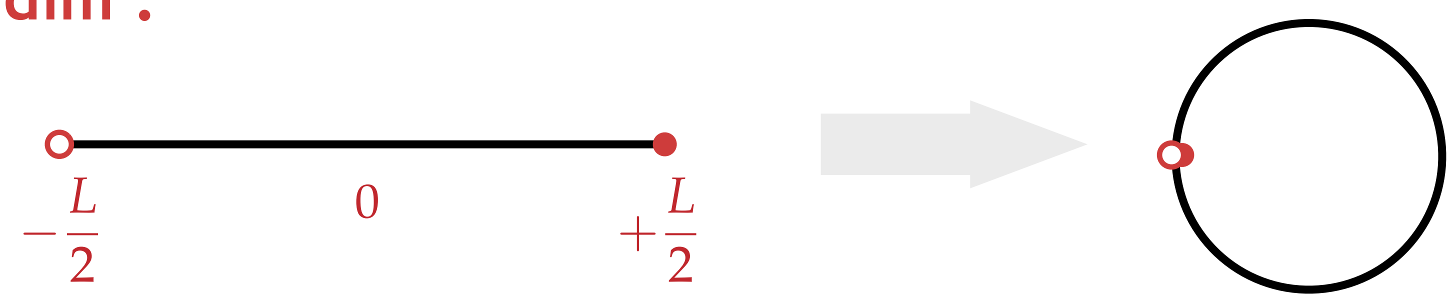
  - **there isn't one !**
  - in a finite-volume the spectrum is discrete

one-dim :



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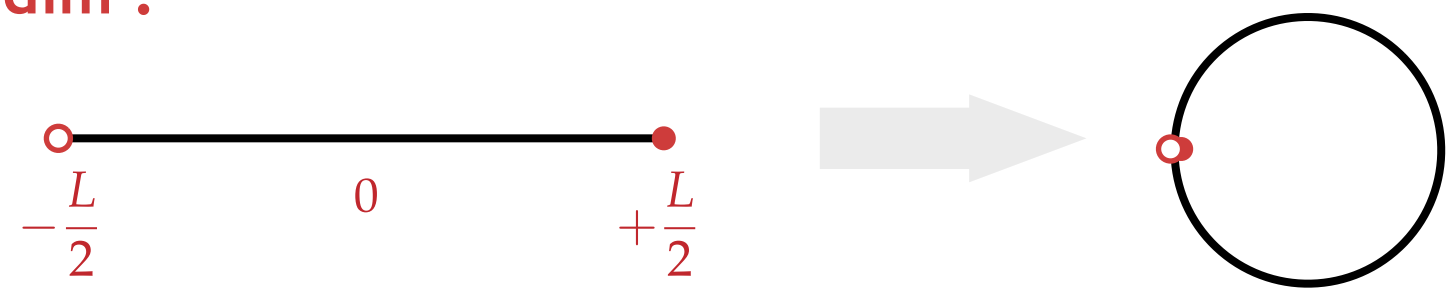

e.g. a free particle

$$\psi(x) \sim e^{ipx}$$

» periodic boundary condition

$$\psi(x) = \psi(x+L)$$
$$e^{ipx} = e^{ip(x+L)}$$
$$e^{ipL} = 1$$

$$p = \frac{2\pi}{L} n$$

discrete energy spectrum

Interacting particles in a finite-volume

- Two identical bosons **interacting** through a finite-range potential

$$\psi(z) \sim \cos\left[\, p|z| + \delta(p)\right]$$ outside the range of the potential, $|z| > R$

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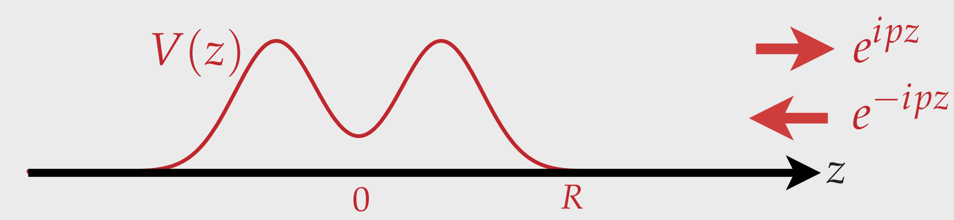

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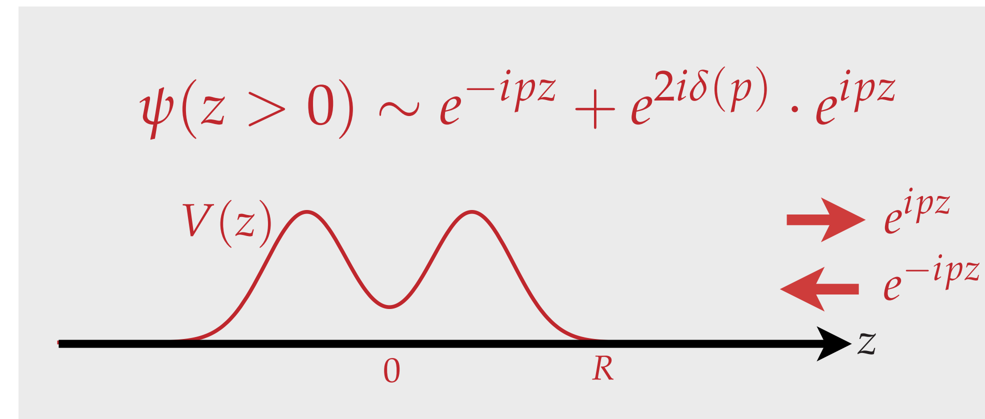

  - periodic boundary conditions

$$\left.\begin{aligned} \psi\left(-\tfrac{L}{2}\right) &= \psi\left(\tfrac{L}{2}\right) \\ \frac{d\psi}{dz}\left(-\tfrac{L}{2}\right) &= \frac{d\psi}{dz}\left(\tfrac{L}{2}\right) \end{aligned}\right\} \implies 0 = \sin\left[\frac{pL}{2} + \delta(p)\right]$$

$$\frac{pL}{2} + \delta(p) = n\pi$$

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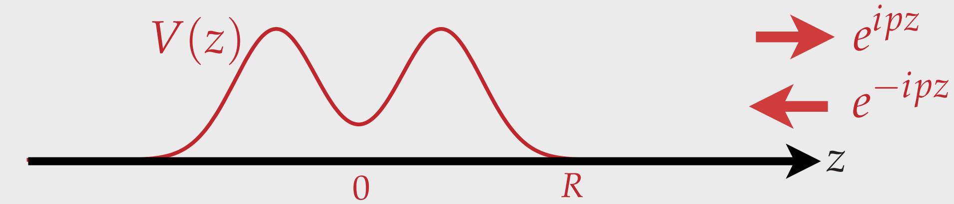

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discrete energy spectrum

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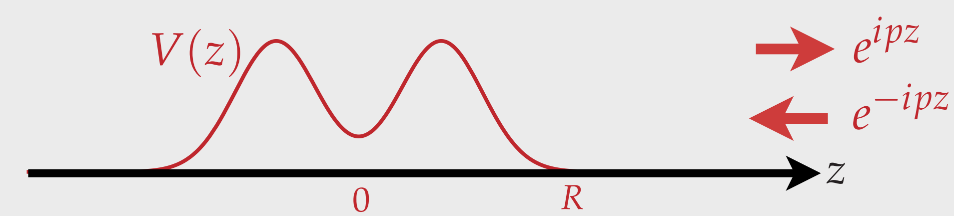

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discrete energy spectrum

discrete energy spectrum is determined by the scattering amplitude

(or vice-versa)

Scattering in a finite cubic volume

- Expect a discrete spectrum in a finite periodic volume $\psi(x+L) = \psi(x)$

e.g. free particle $e^{ip(x+L)} = e^{ipx}$

quantized momentum $p = \frac{2\pi}{L} n$

- For an interacting theory $\cot \delta_\ell(E) = \mathcal{M}_\ell(E, L)$ LÜSCHER ...

elastic scattering phase-shift

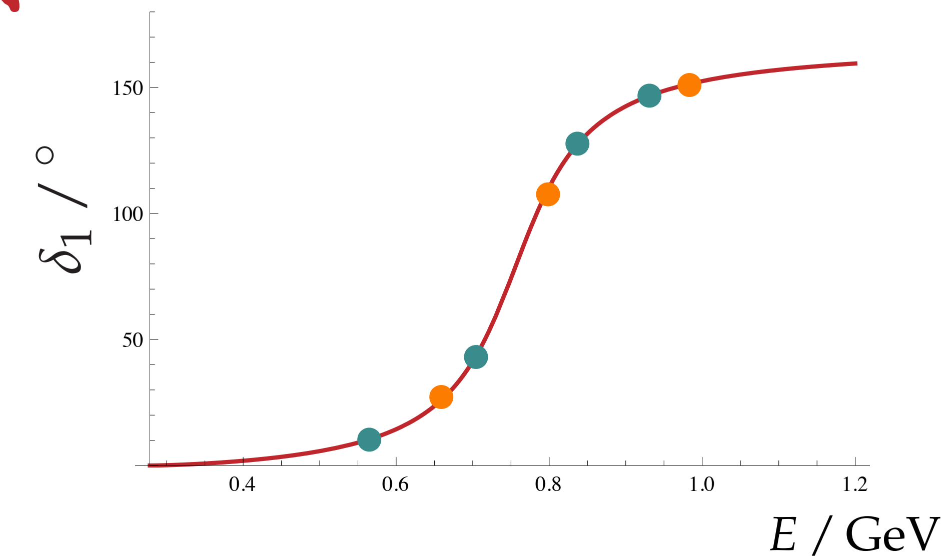
known function

Discrete energies in a finite-volume $\longleftrightarrow$ Discrete values of the phase-shift

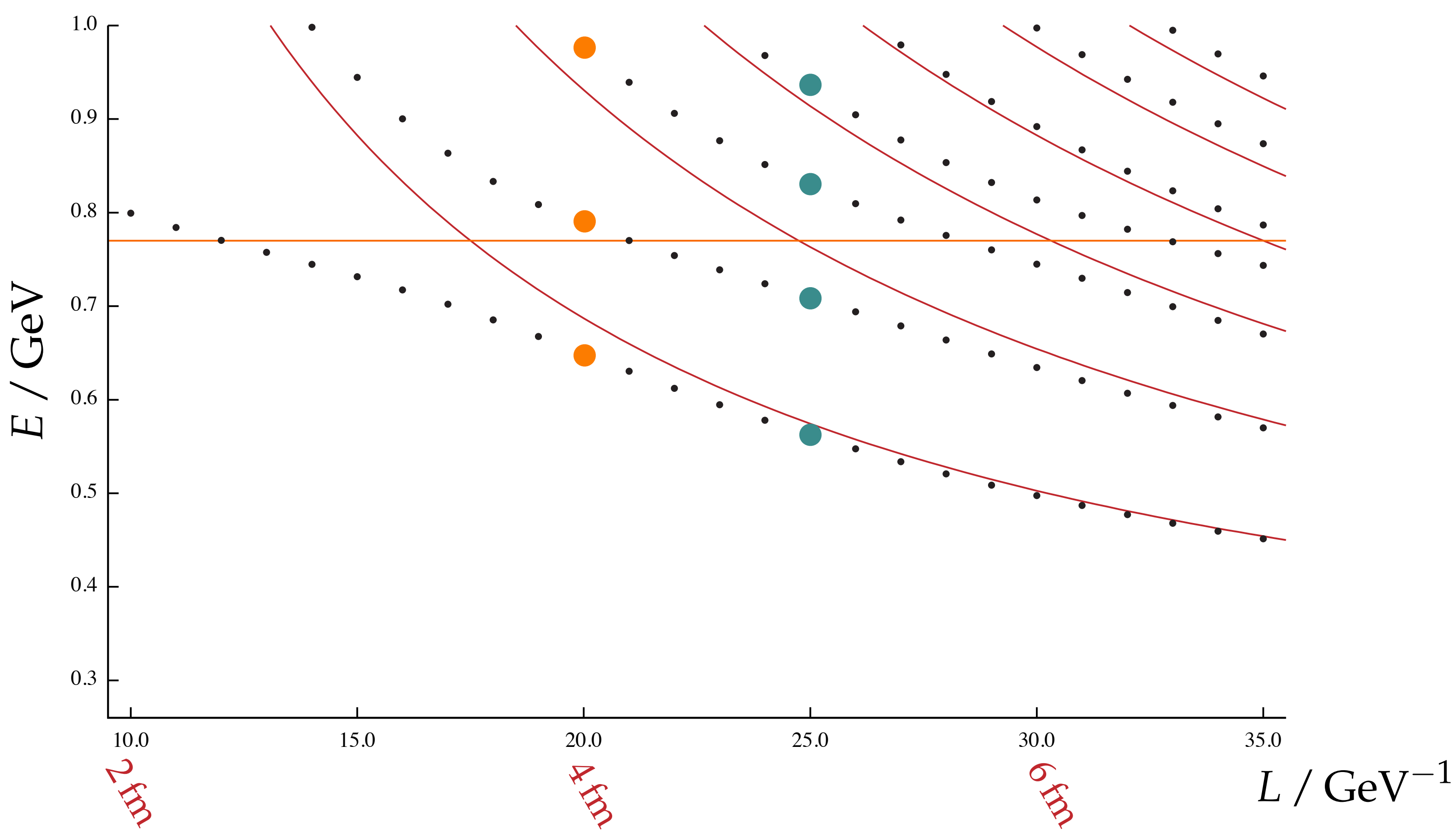
Scattering in a finite cubic volume

- Experimental $\pi\pi$ I=1 P-wave scattering amplitude

P-WAVE PHASE SHIFT



CUBIC BOX SPECTRUM



Coupled-channel scattering

HE, JHEP 0507 011
HANSEN, PRD86 016007
BRICENO, PRD88 094507
GUO, PRD88 014051

- Finite-volume formalism recently derived (multiple methods)

$$\det\left[\left(\left[t^{(\ell)}(E)\right]^{-1}_{ij} + i\rho_i(E)\,\delta_{ij}\right) - \delta_{ij}\,\mathcal{M}_\ell\big(p_i(E)L\big)\right] = 0$$

scattering matrix

phase space

known functions

matrices in partial-wave space ...

- However, this is **one equation** for **multiple unknowns** (per energy level) $\frac{1}{2}N(N+1)$ for *N* channels
  - parameterize the energy dependence of *t*
  - try to describe a spectrum globally

"Energy-dependent" analysis

Including multi-meson operators

- Form correlator matrix with both $\bar{\psi}\mathbf{\Gamma}\psi$ and $\pi\pi$-like

- Include operators which resemble a pair of pions and also kaons

$$\bar{d}\mathbf{\Gamma}u$$

$$\sum_{\hat{k}_1,\hat{k}_2} C(\Lambda,\vec{P},\vec{k}_1,\vec{k}_2)\pi^\dagger(\vec{k}_1)\pi^\dagger(\vec{k}_2)$$

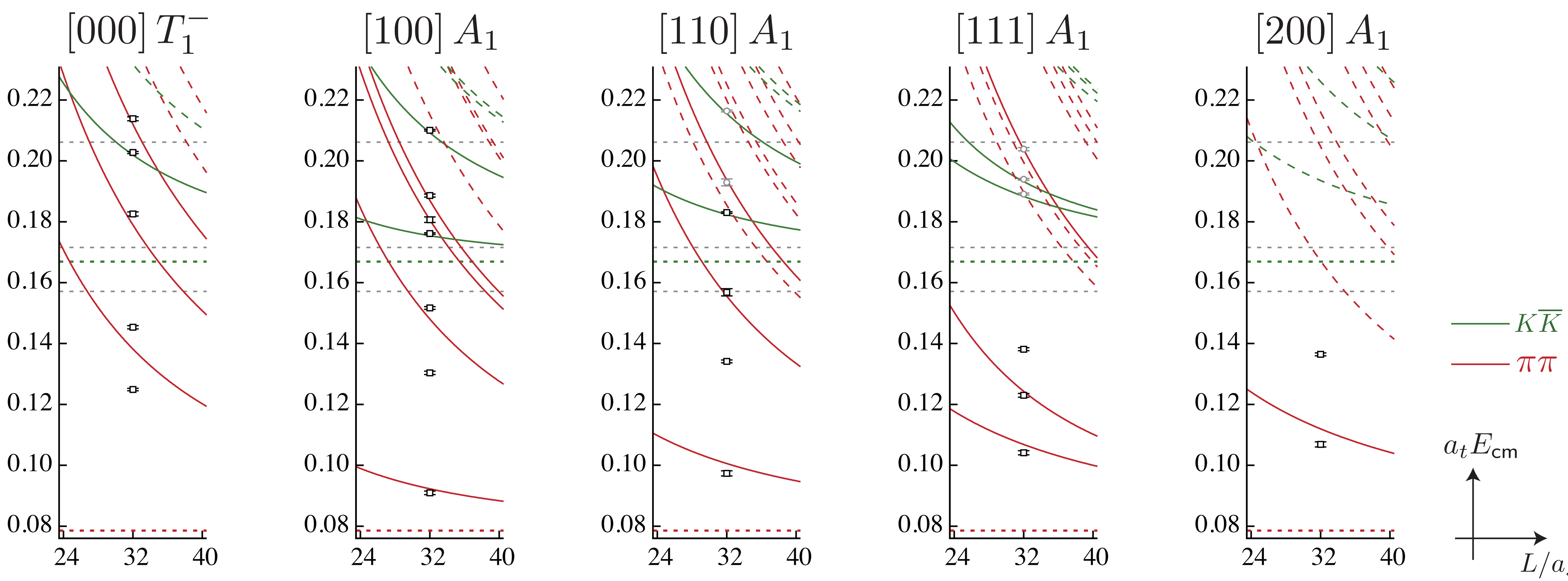
$$\sum_{\hat{k}_1,\hat{k}_2} C(\Lambda,\vec{P},\vec{k}_1,\vec{k}_2)K^\dagger(\vec{k}_1)\bar{K}^\dagger(\vec{k}_2)$$

Finite-volume spectrum - moving frames

- Non-interacting thresholds and energies as a function of $\vec{k}$

$m_\pi \sim 236$ MeV

Momentum & lattice irrep labels: $[\vec{k}]\,\Lambda$



$\pi\pi/KK$ scattering

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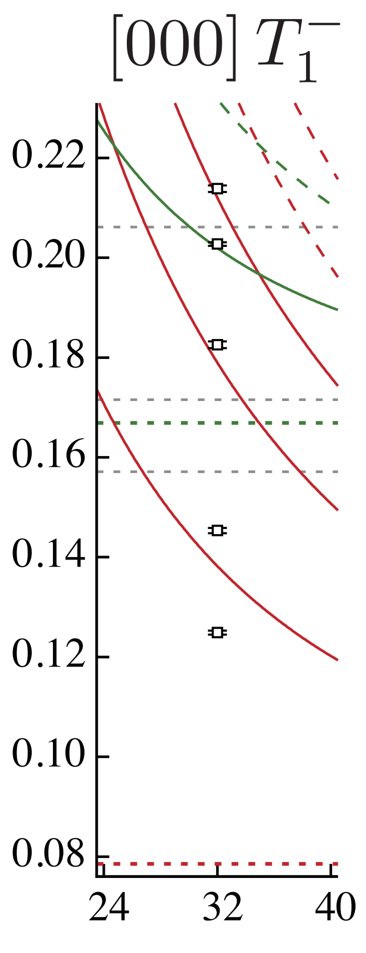

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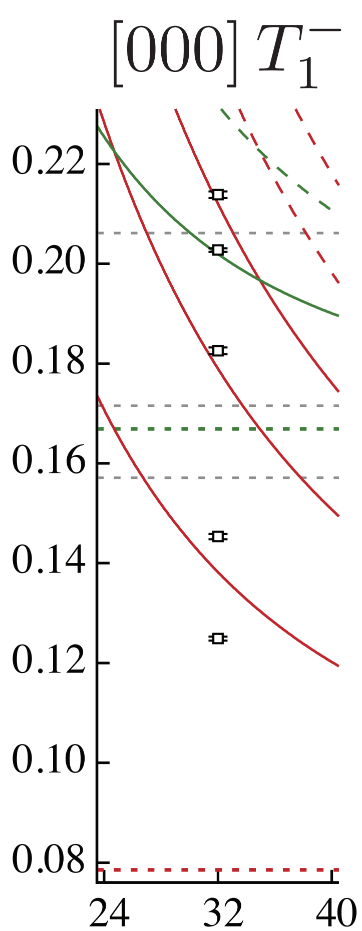

- Consider size of operator overlaps $\langle \mathfrak{n} | \mathcal{O}_i^\dagger | \varnothing \rangle$

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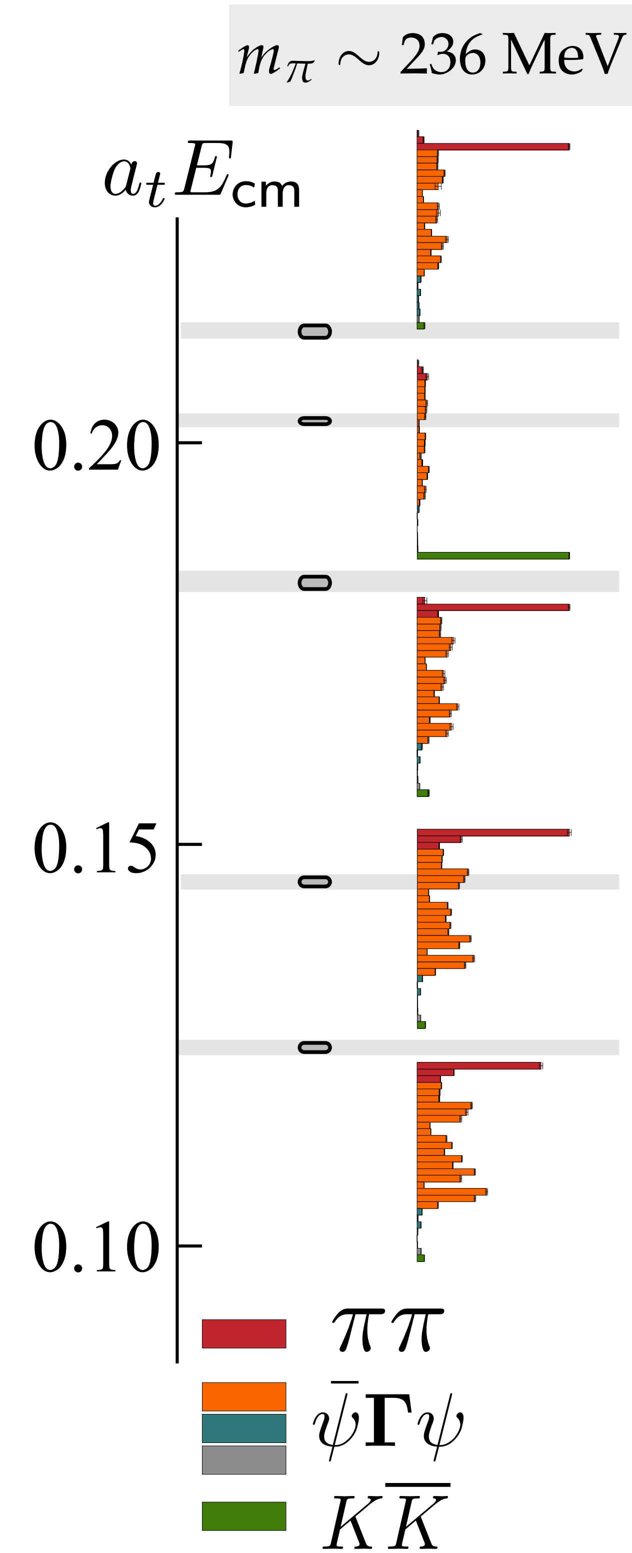

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» Finite-V: ad-mixture of single & two-particle

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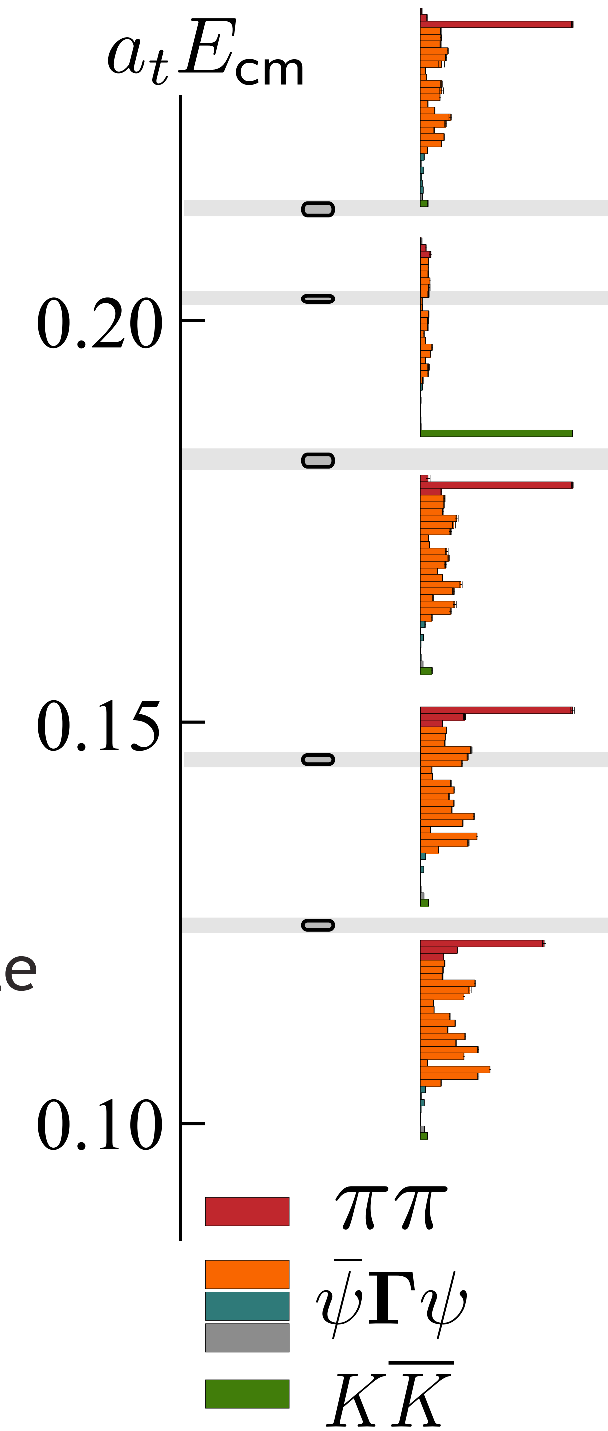
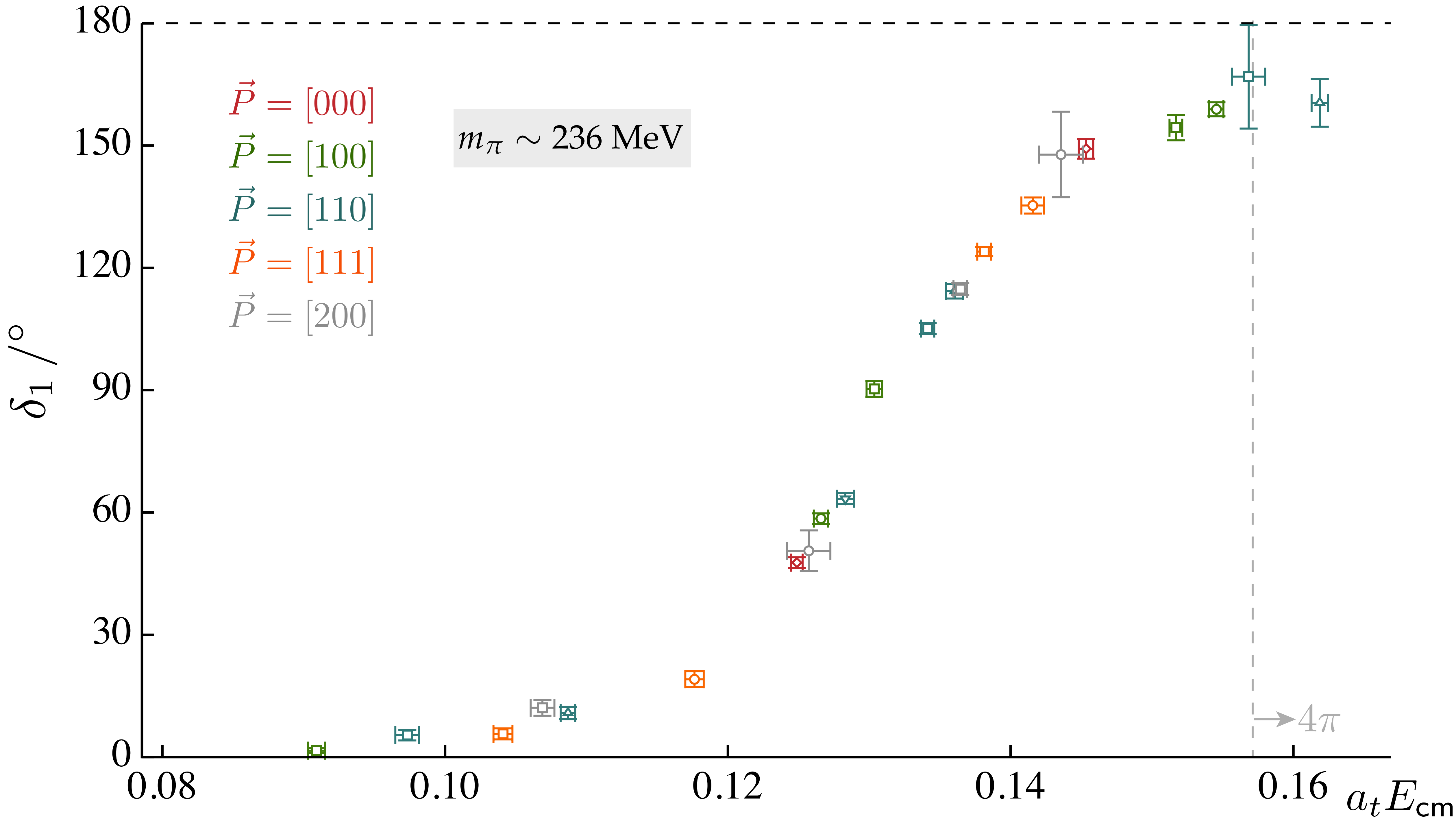


$\pi\pi$ *P*-wave phase-shift

- Restrict to elastic region below $K\bar{K}$ threshold



To appear very soon...

ππ P-wave phase-shift

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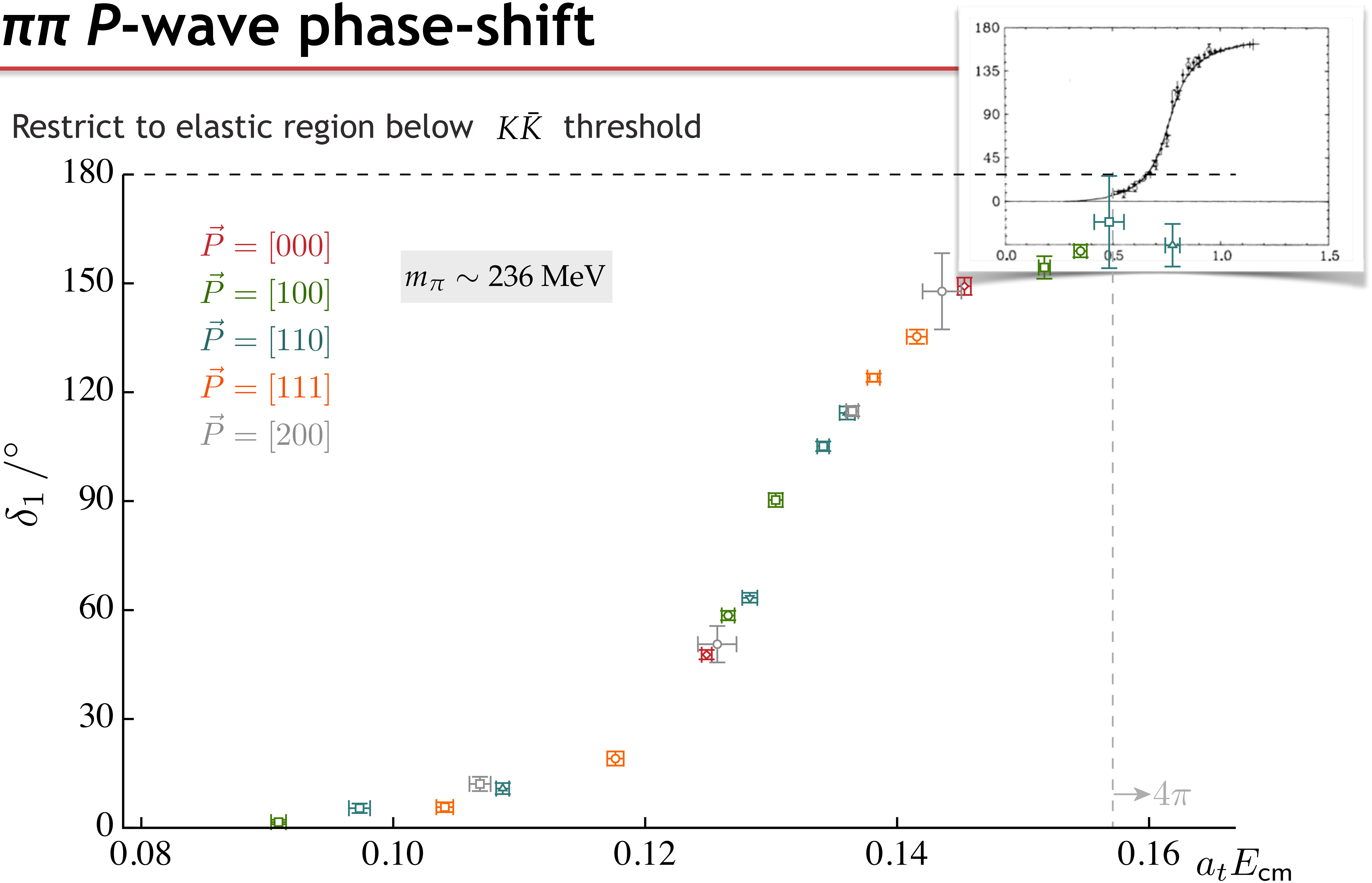

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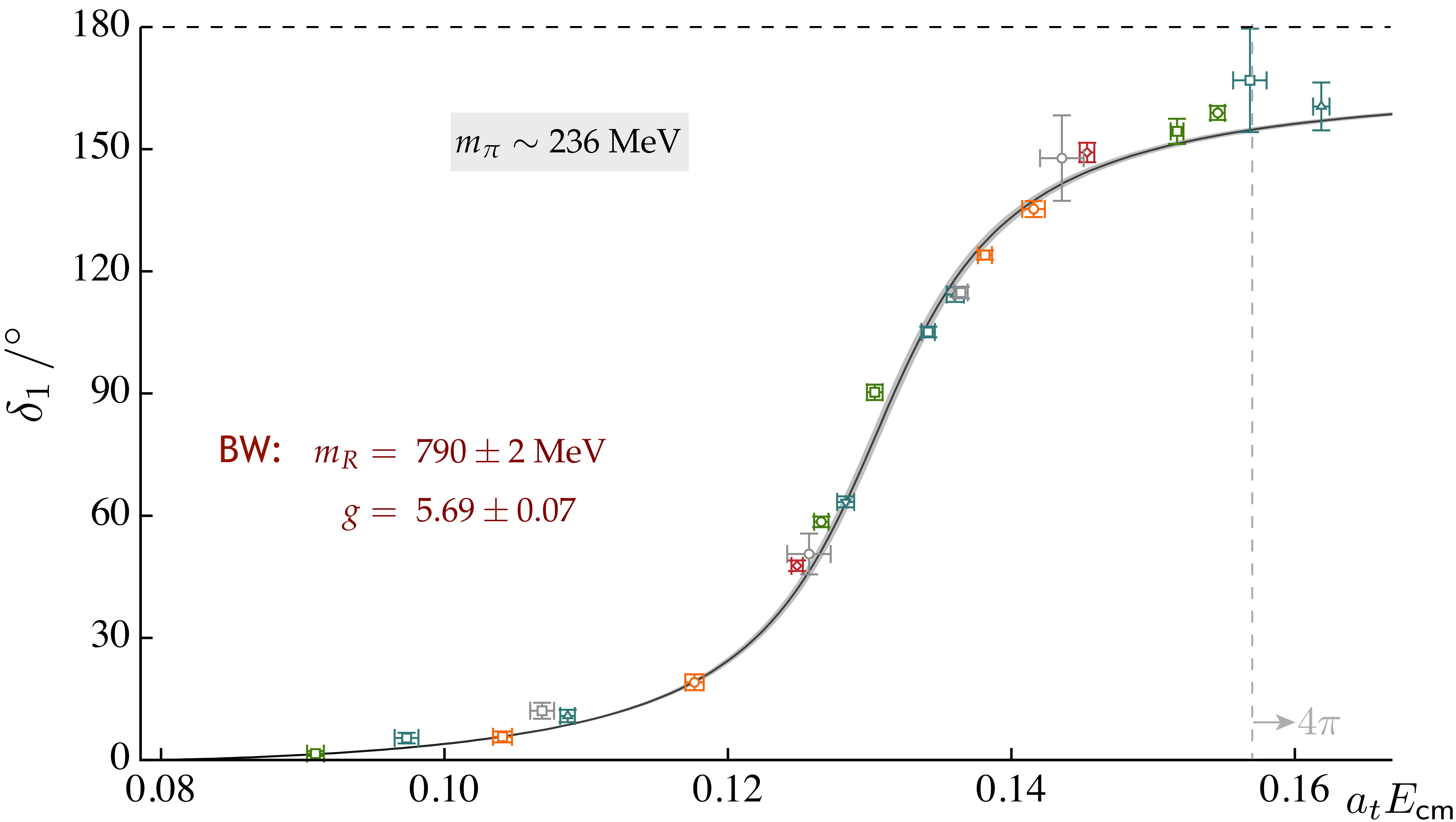
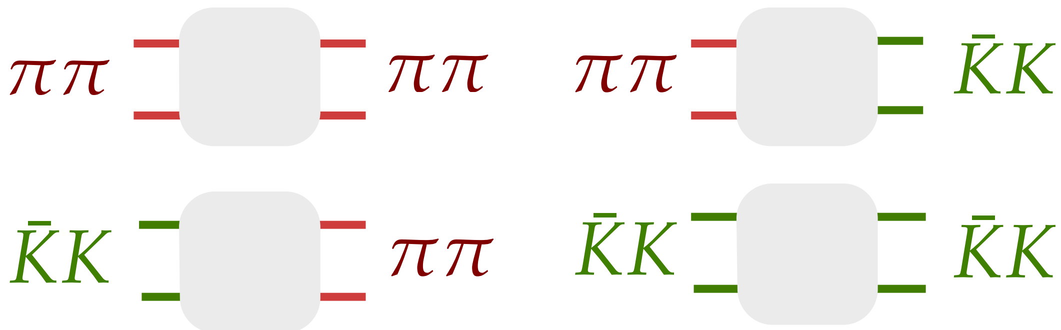


To appear very soon...

ρ resonance as a coupled channel system

- Parameterize the t-matrix in a unitarity conserving way



- Compute finite-volume spectrum

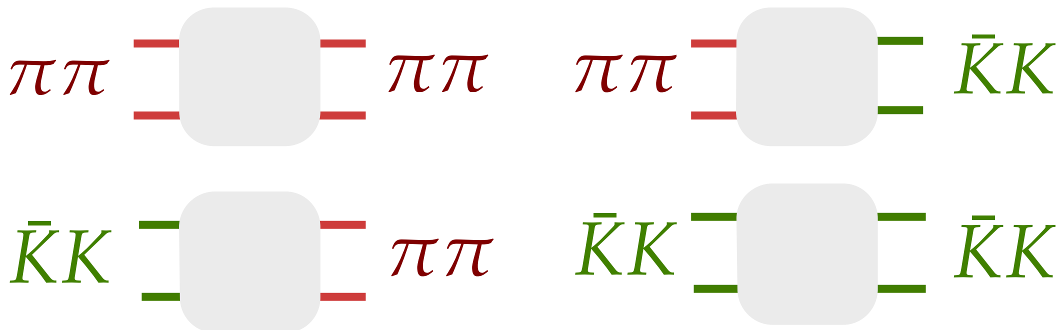
$$\bar{d}\mathbf{\Gamma}u$$

$$\sum_{\hat{k}_1,\hat{k}_2} C(\Lambda,\vec{P},\vec{k}_1,\vec{k}_2)\,\pi^\dagger(\vec{k}_1)\,\pi^\dagger(\vec{k}_2)$$

$$\sum_{\hat{k}_1,\hat{k}_2} C(\Lambda,\vec{P},\vec{k}_1,\vec{k}_2)\,K^\dagger(\vec{k}_1)\,\bar{K}^\dagger(\vec{k}_2)$$

ρ resonance as a coupled channel system

- Parameterize the *t*-matrix in a unitarity conserving way



$$t^{-1}_{ij}(E) = K^{-1}_{ij}(E) + \delta_{ij}\, I_i(E)$$

$$K_{ij}(E) = \frac{g_i\, g_j}{m^2 - E^2} + \gamma_{ij}$$

  - Vary the parameters, solving

$$\det\left[\left([t^{(\ell)}(E)]^{-1}_{ij} + i\rho_i(E)\,\delta_{ij}\right) - \delta_{ij}\,\mathcal{M}_\ell(E,L)\right] = 0$$

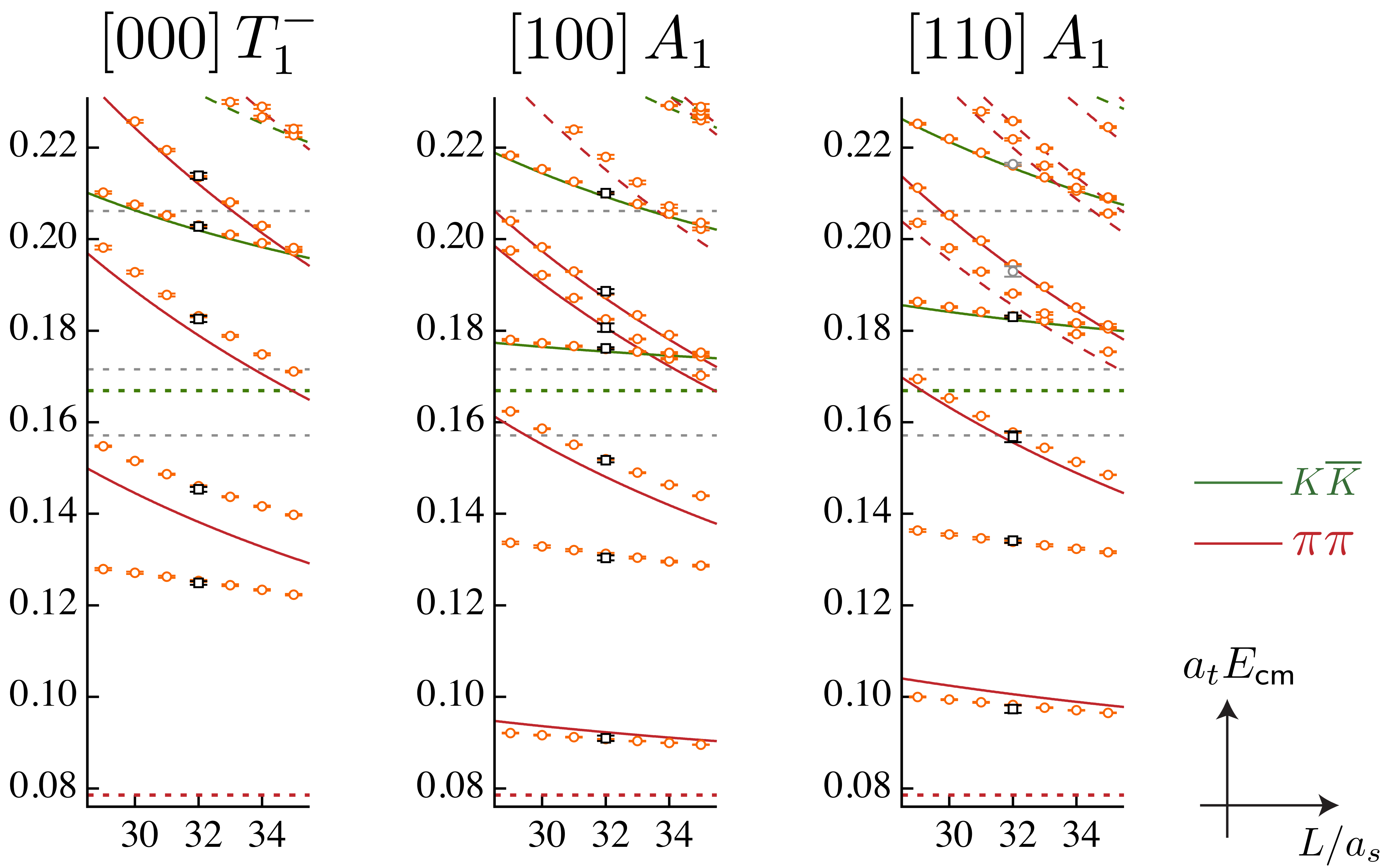
for the spectrum in each irreducible representation & momentum

Want pole mass and couplings of t-matrix

ππ/KK scattering

- Data points (black) compared to parameterization (gold)

$m_\pi \sim 236$ MeV



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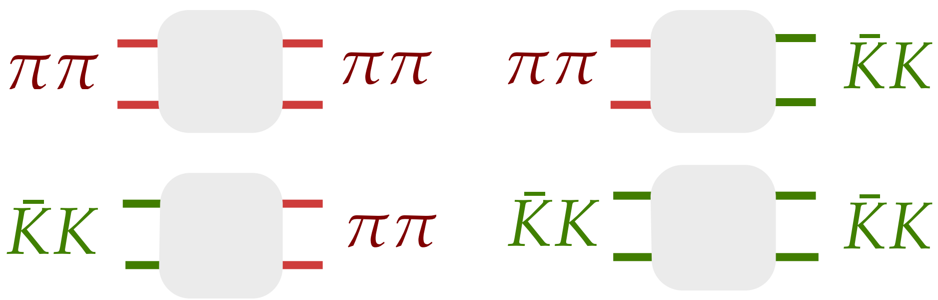
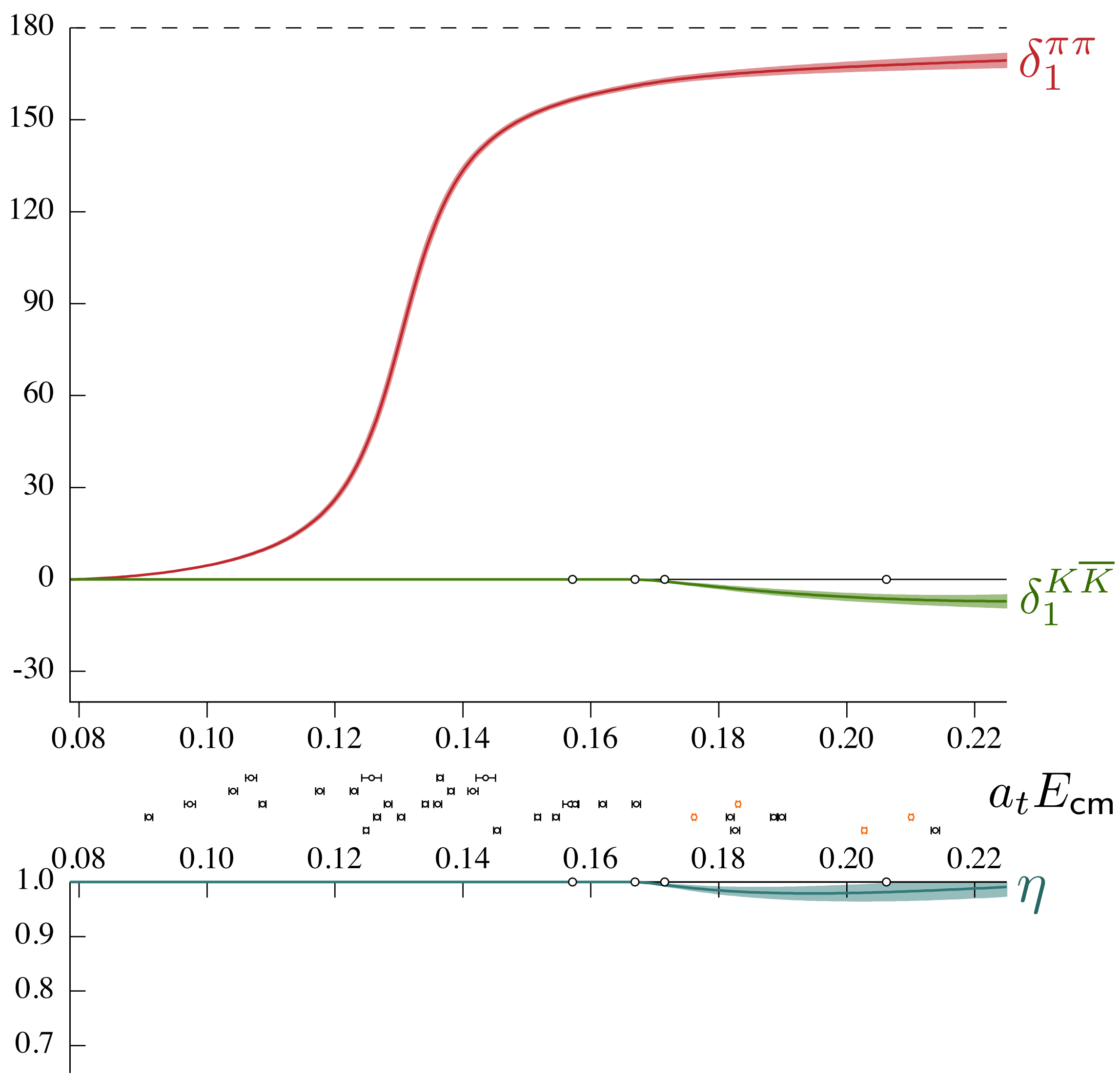
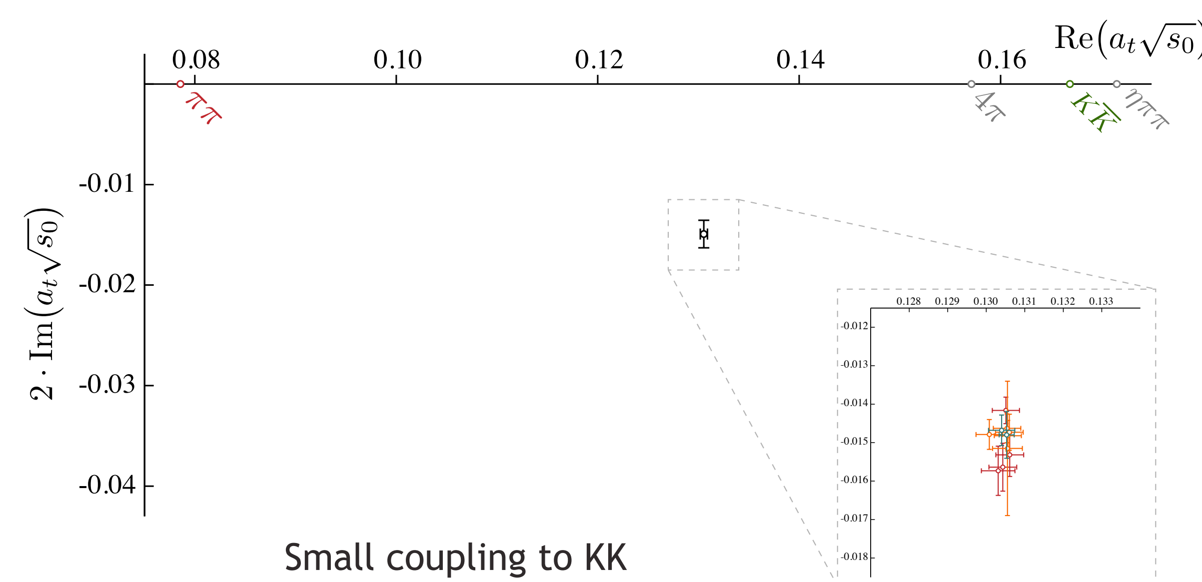


Phase shifts & inelasticity



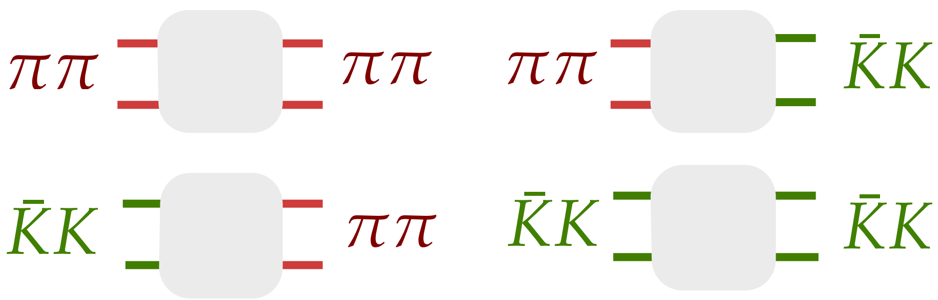
$m_\pi \sim 236$ MeV

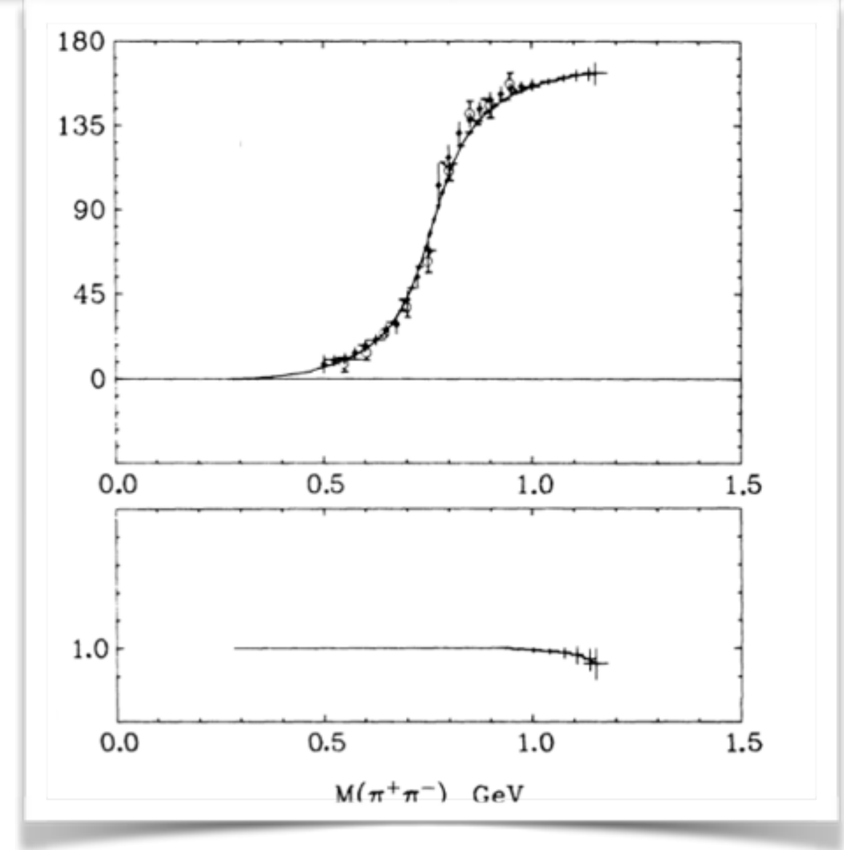
t-matrix pole location



Small coupling to KK

ρ resonance as a coupled channel system





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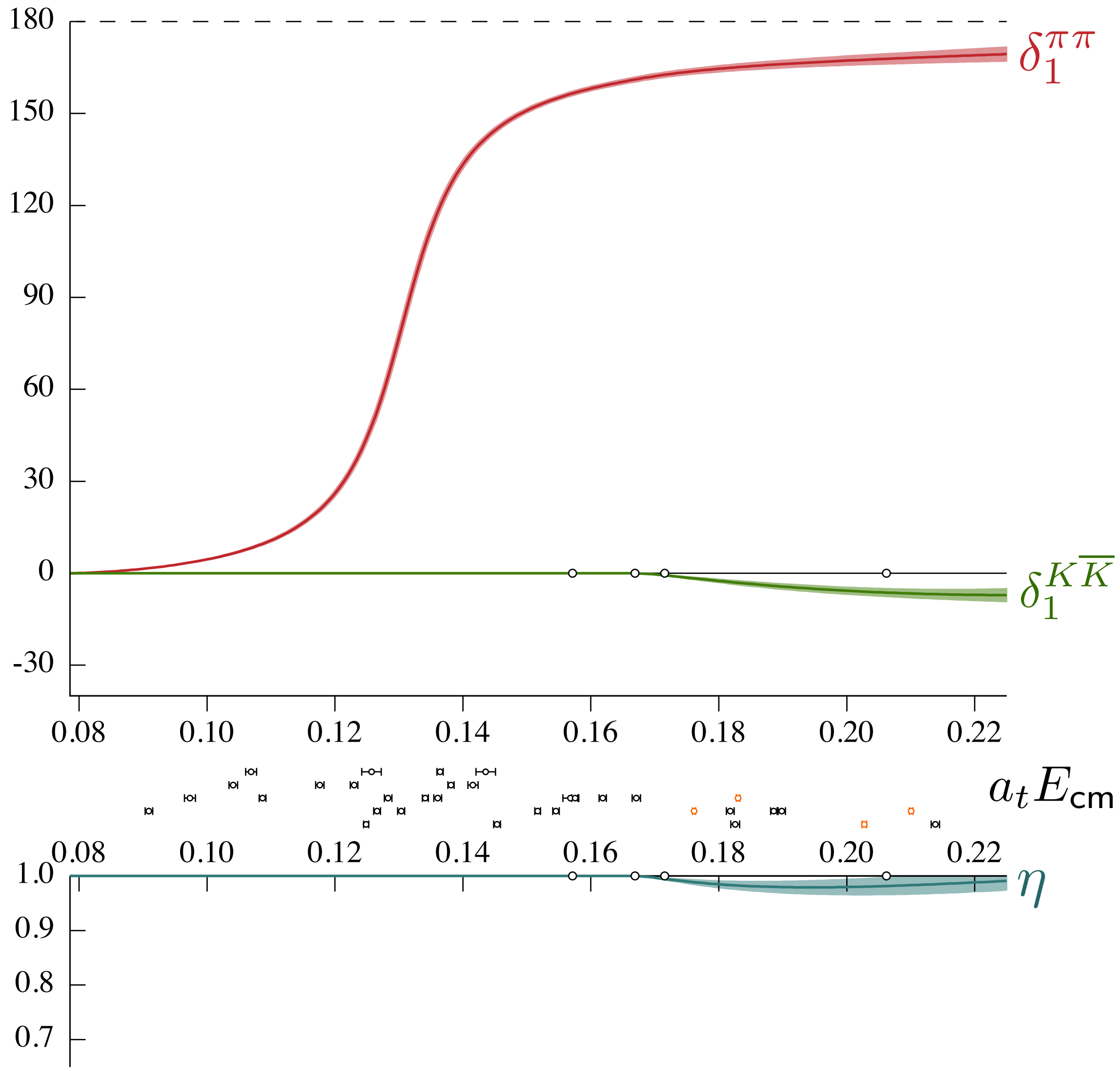

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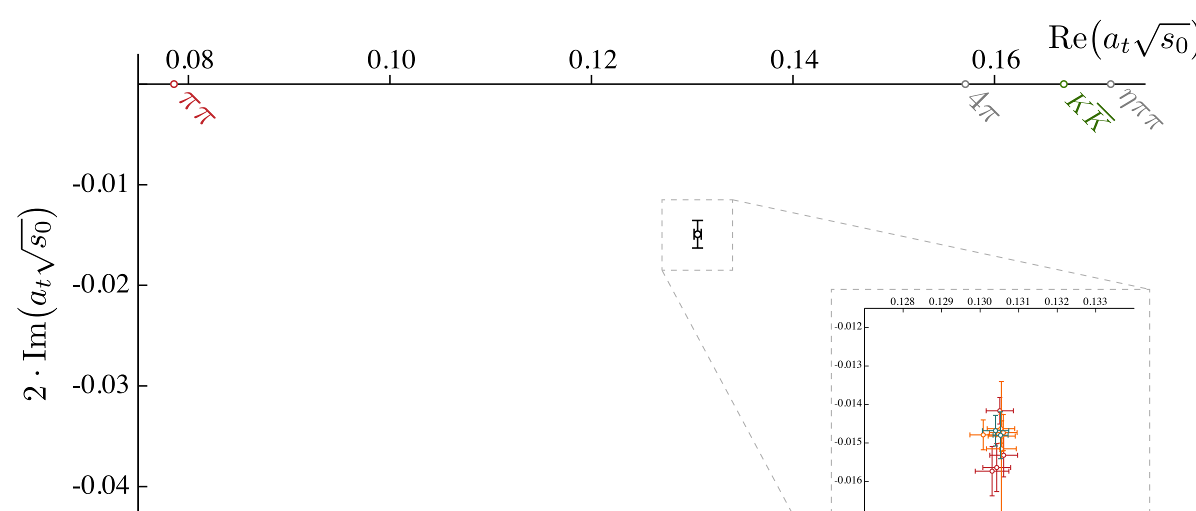
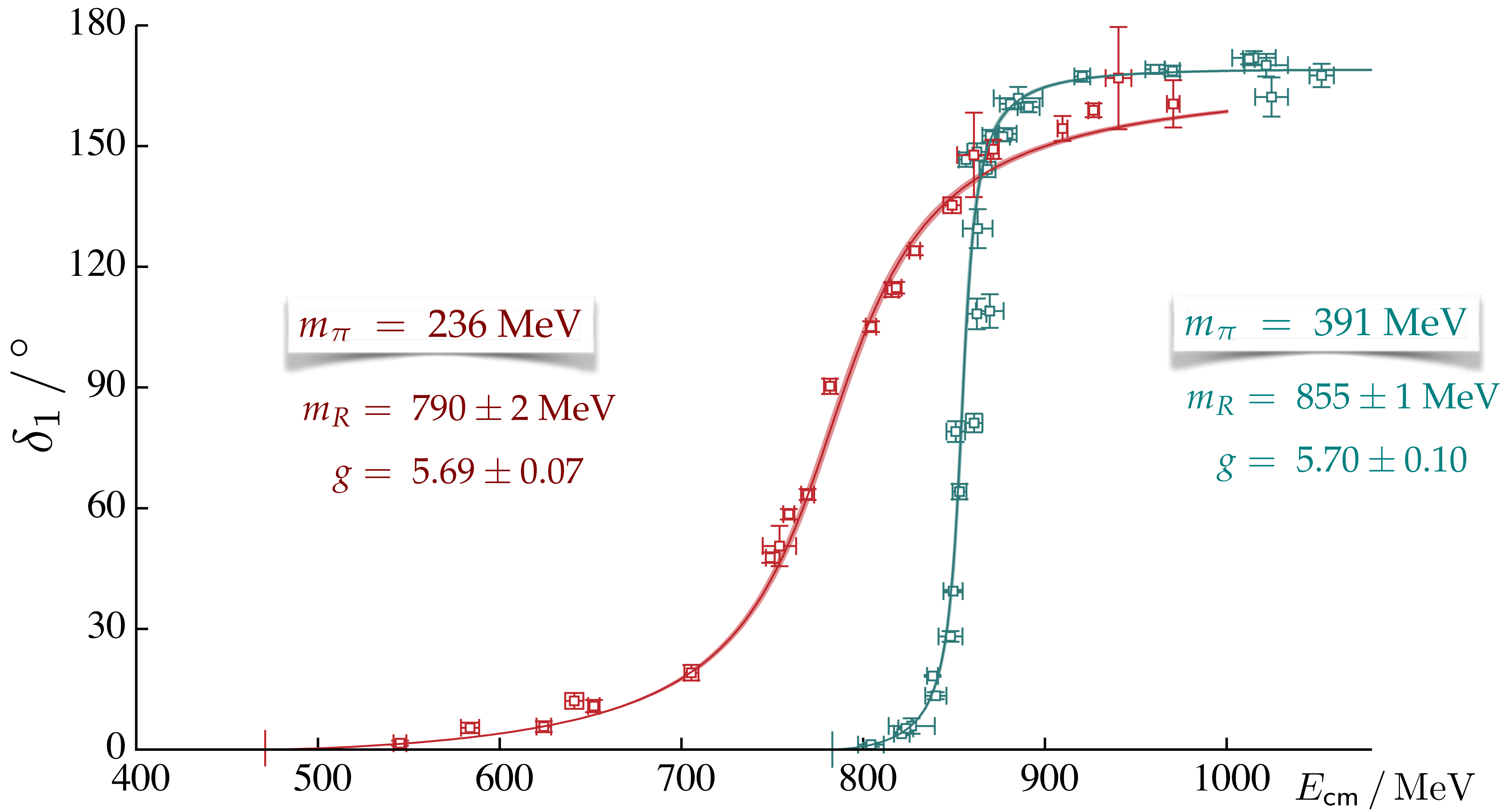


Small coupling to KK

ρ resonance at different pion masses

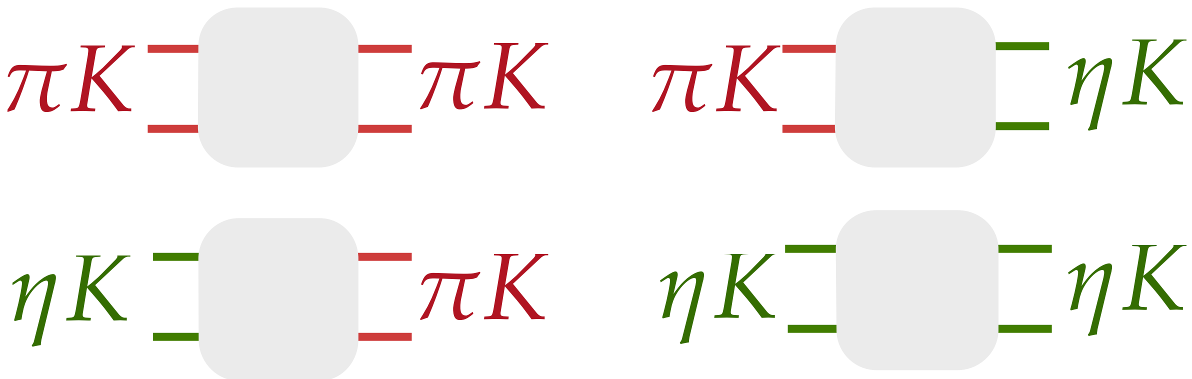
- BW couplings nearly constant in pion mass (will come back to this later...)



PRD 87 034505

$\pi K / \eta K$ scattering & kaon resonances

- Example of coupled-channel scattering



- Compute finite-volume spectrum

$$\bar{u}\mathbf{\Gamma}s$$

$$\sum_{\hat{k}_1,\hat{k}_2} C(\Lambda,\vec{P};\vec{k}_1,\vec{k}_2)\ \pi^\dagger(\vec{k}_1)\ K^\dagger(\vec{k}_2)$$

$$\sum_{\hat{k}_1,\hat{k}_2} C(\Lambda,\vec{P};\vec{k}_1,\vec{k}_2)\ \eta^\dagger(\vec{k}_1)\ K^\dagger(\vec{k}_2)$$

PRL 113 182001
PRD 91 054008

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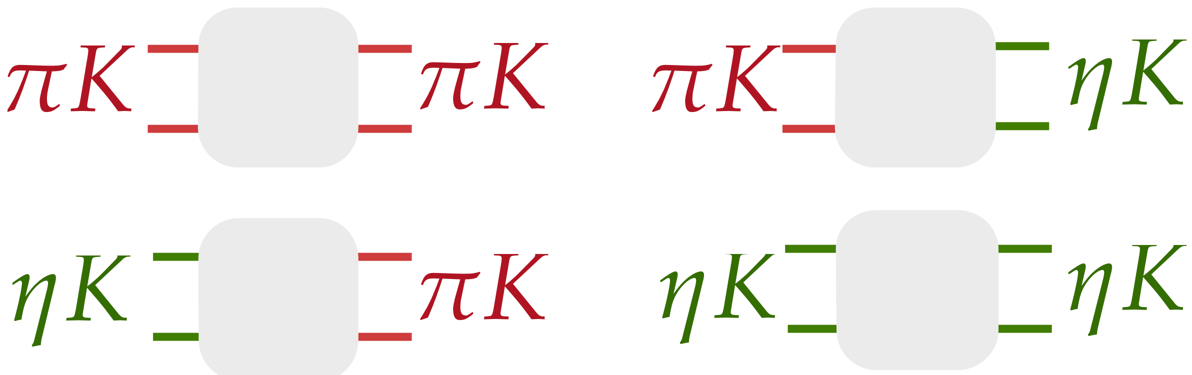

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PRL 113 182001
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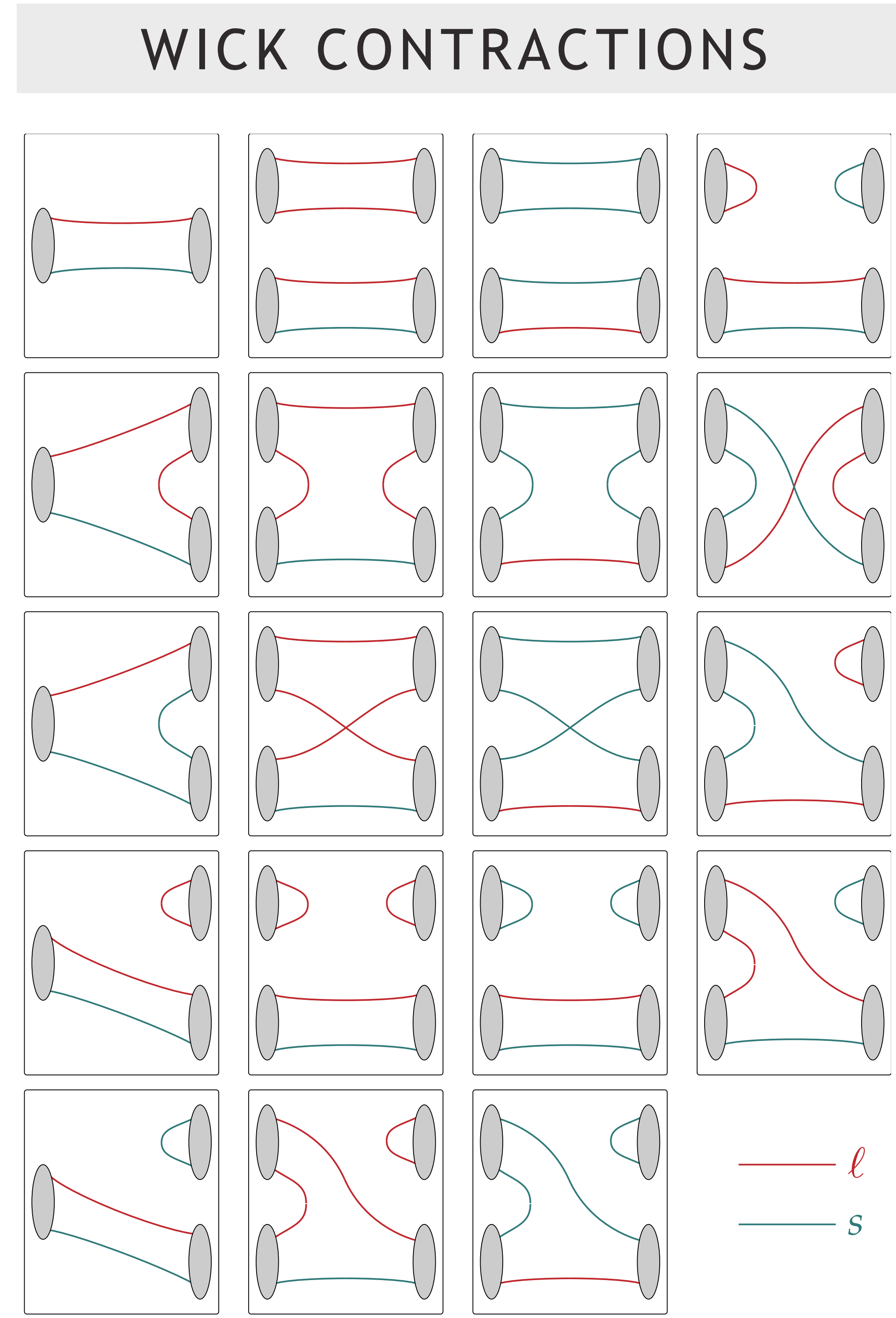

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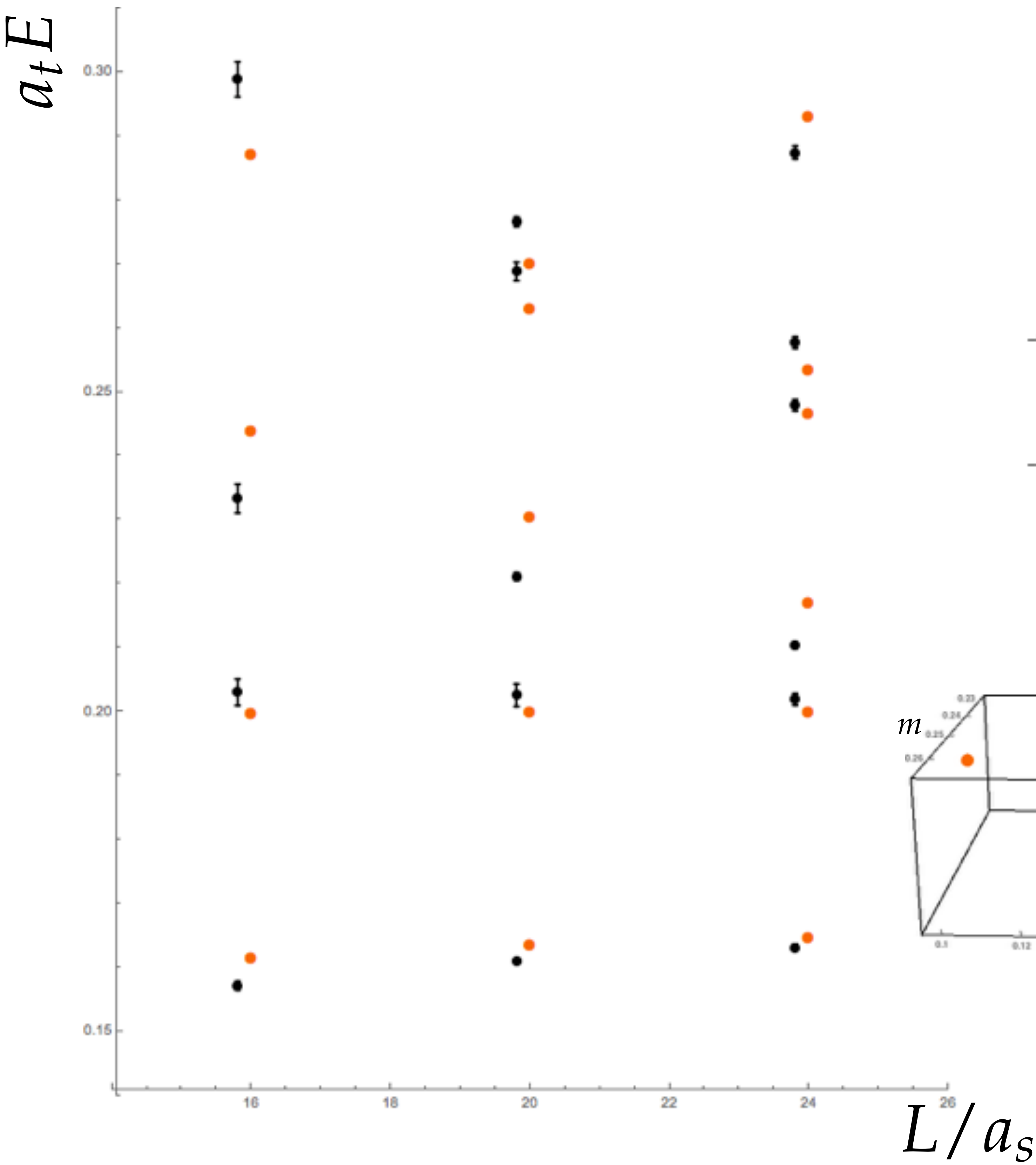

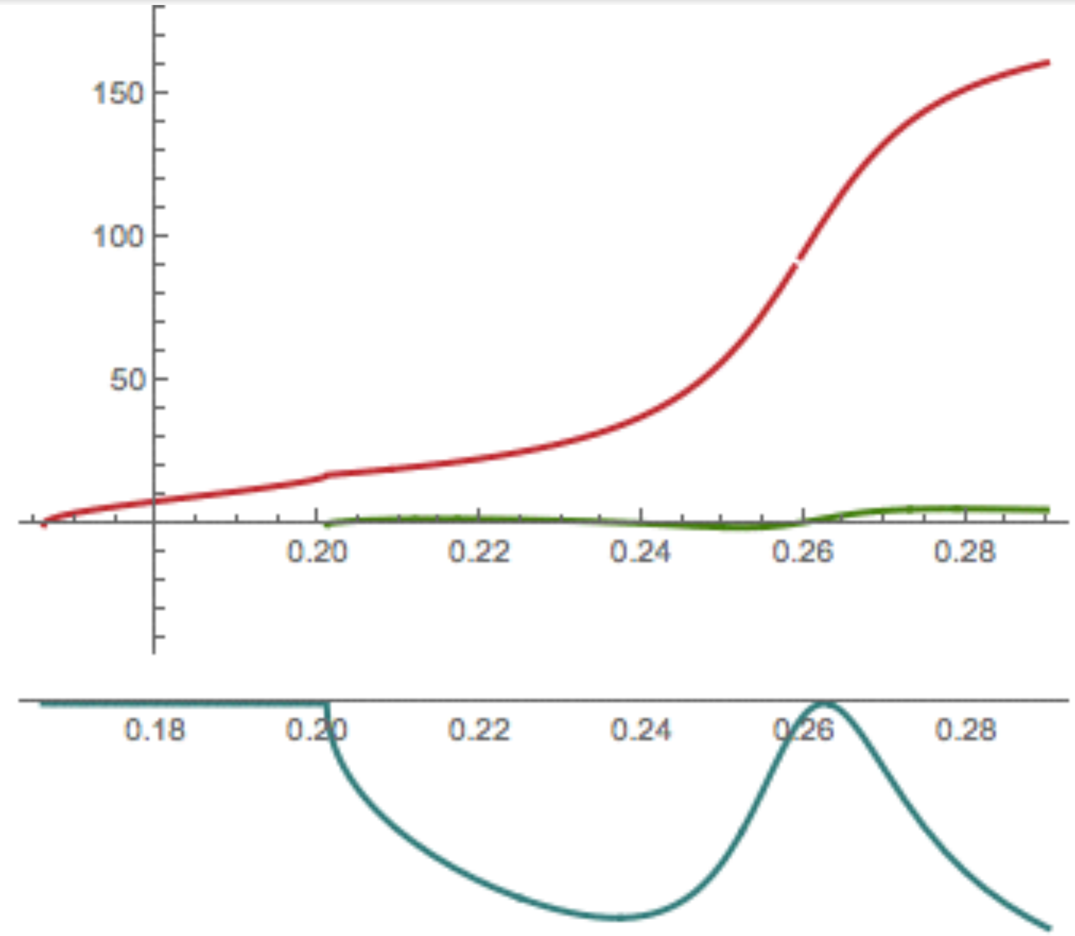

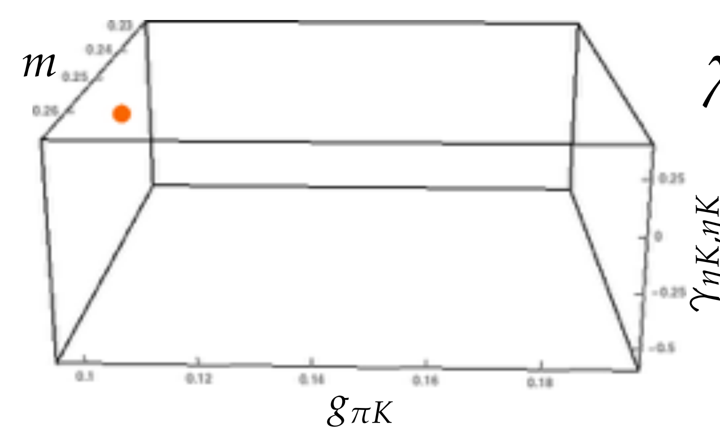

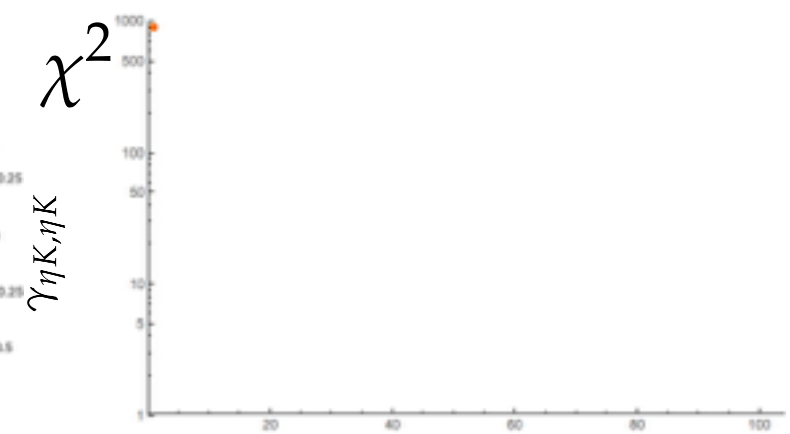

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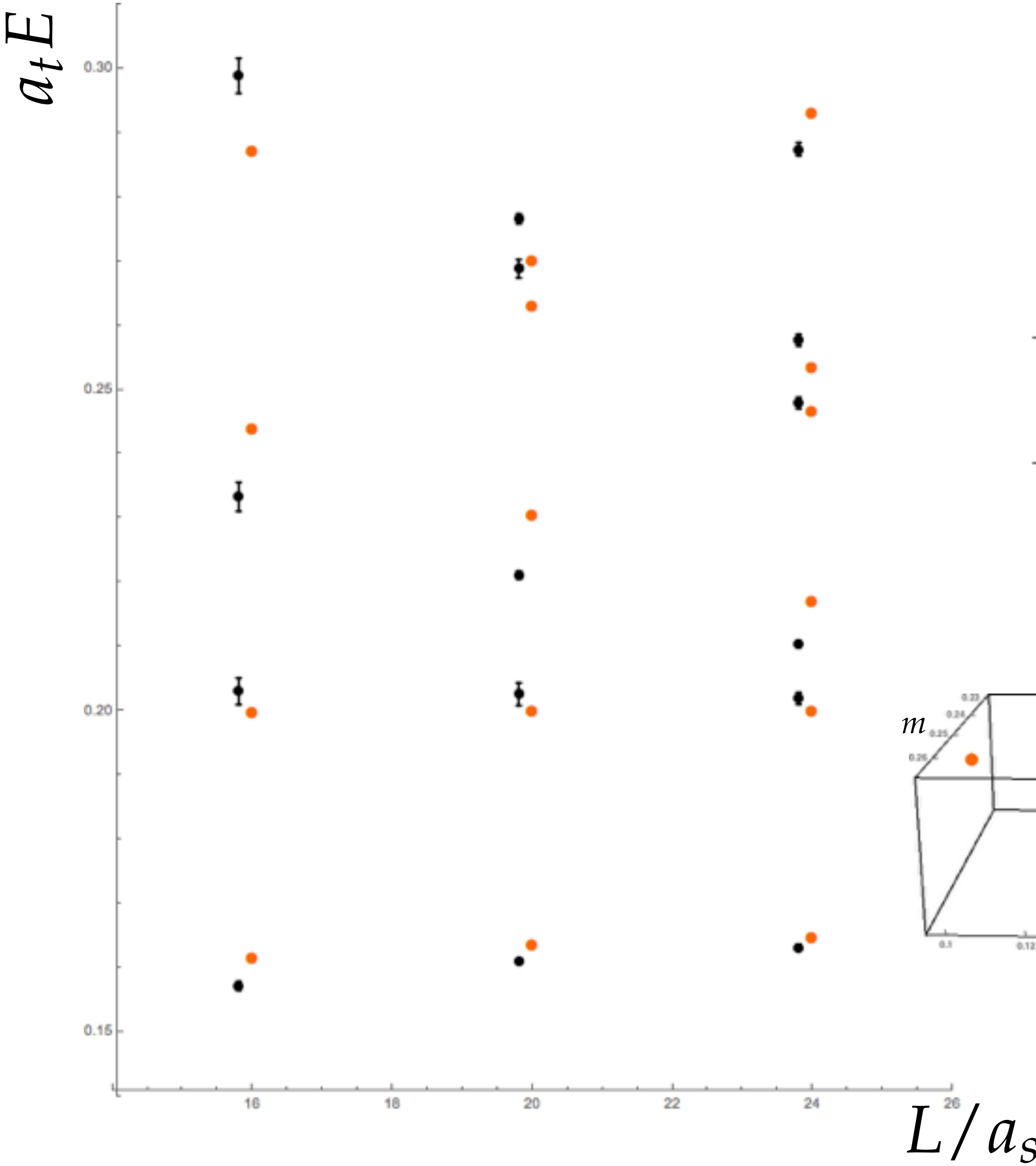
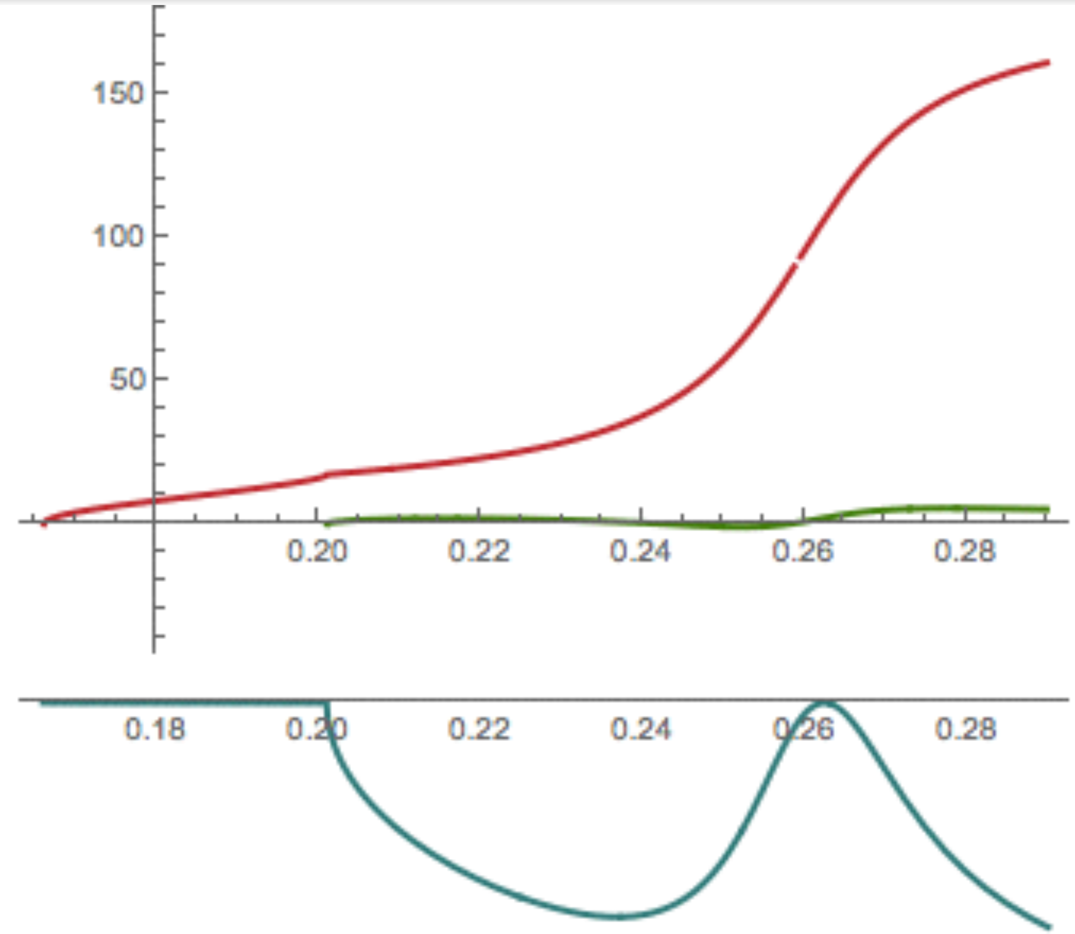
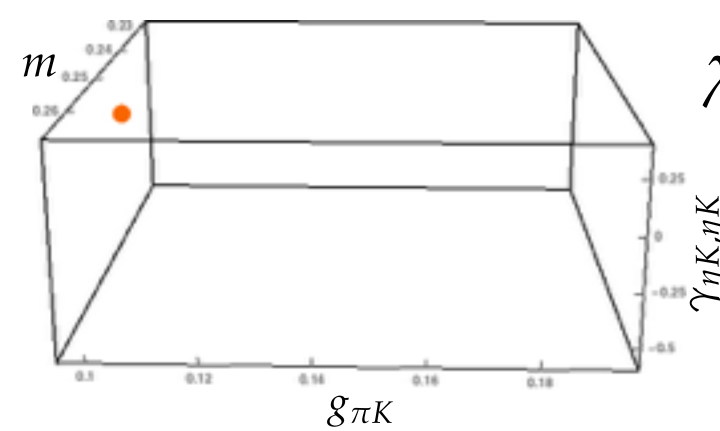
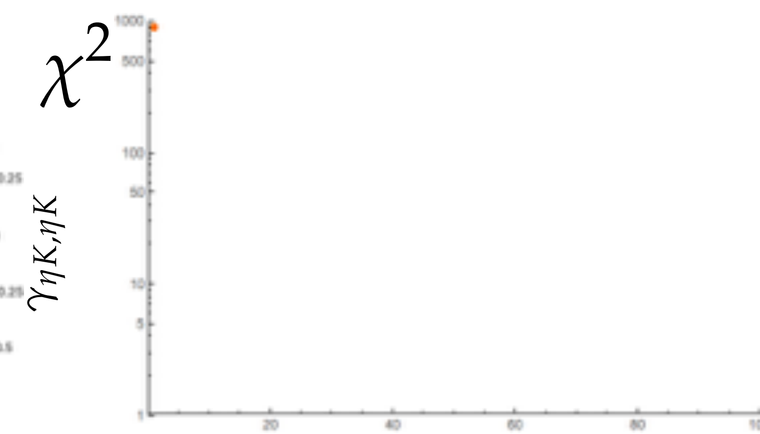




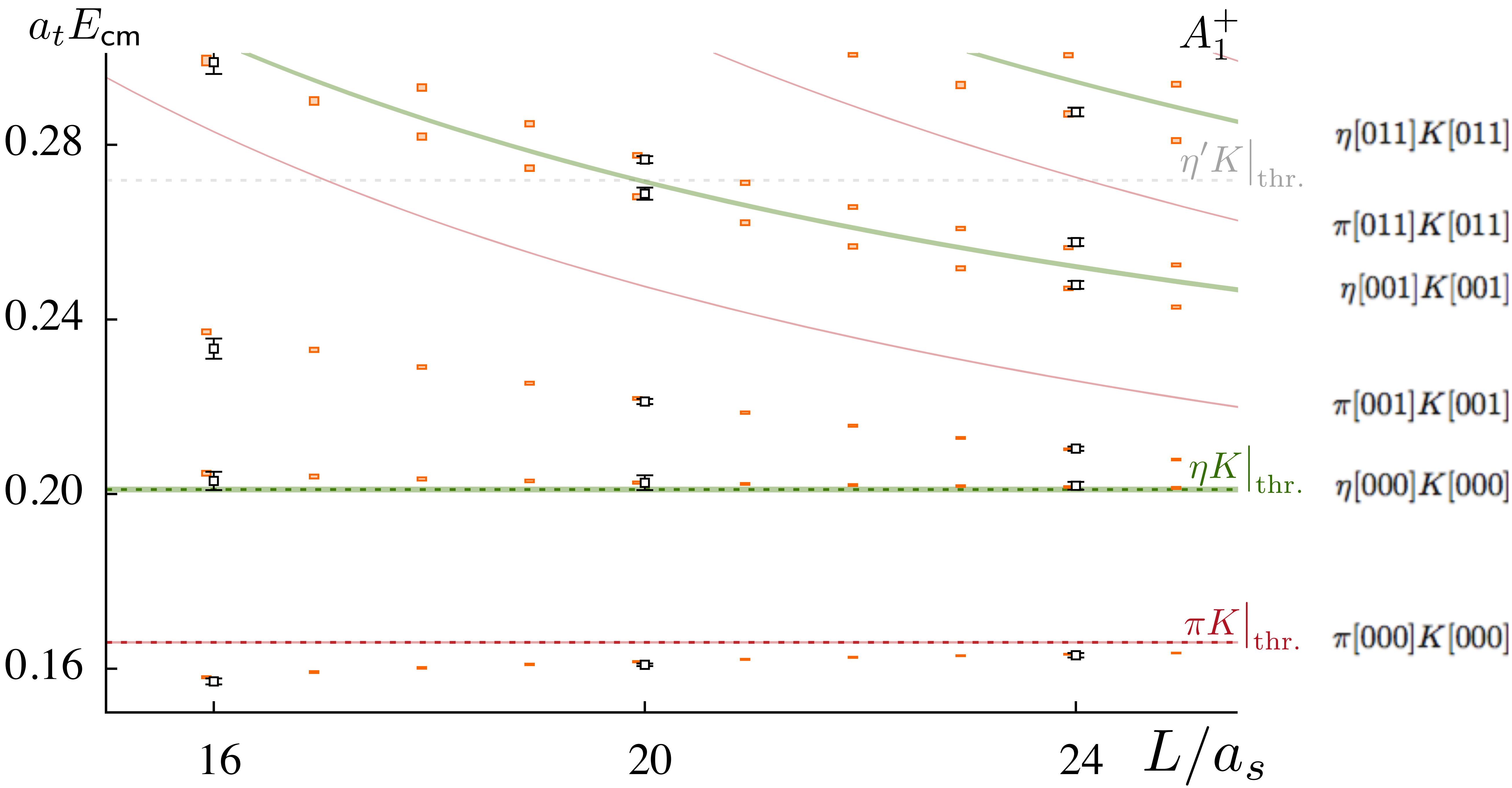




$\pi K/\eta K$ scattering

$$\chi^2/N_{\mathrm{dof}} = \frac{6.40}{15-6} = 0.71$$

$m_\pi \sim 391\,\mathrm{MeV}$



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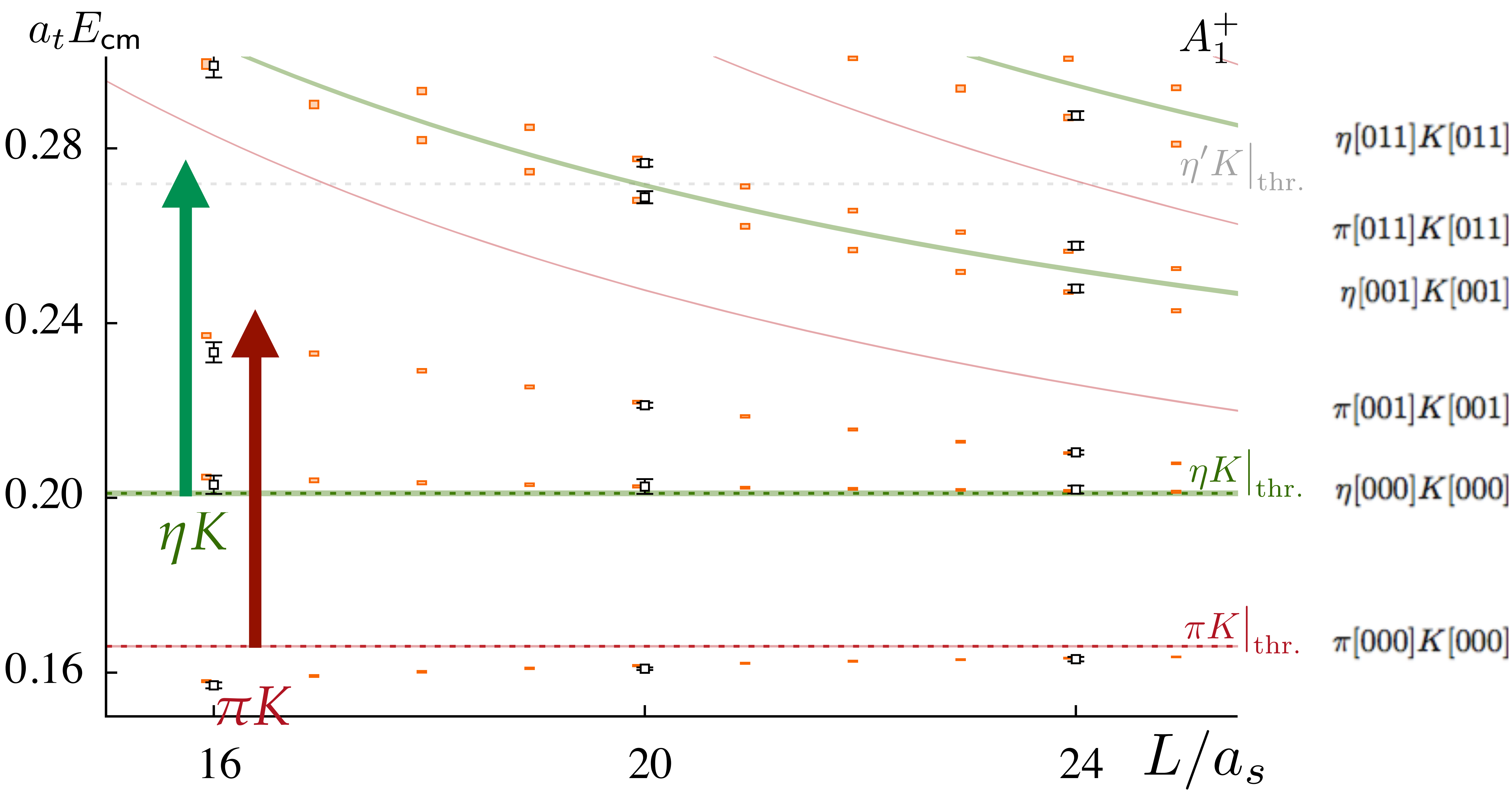

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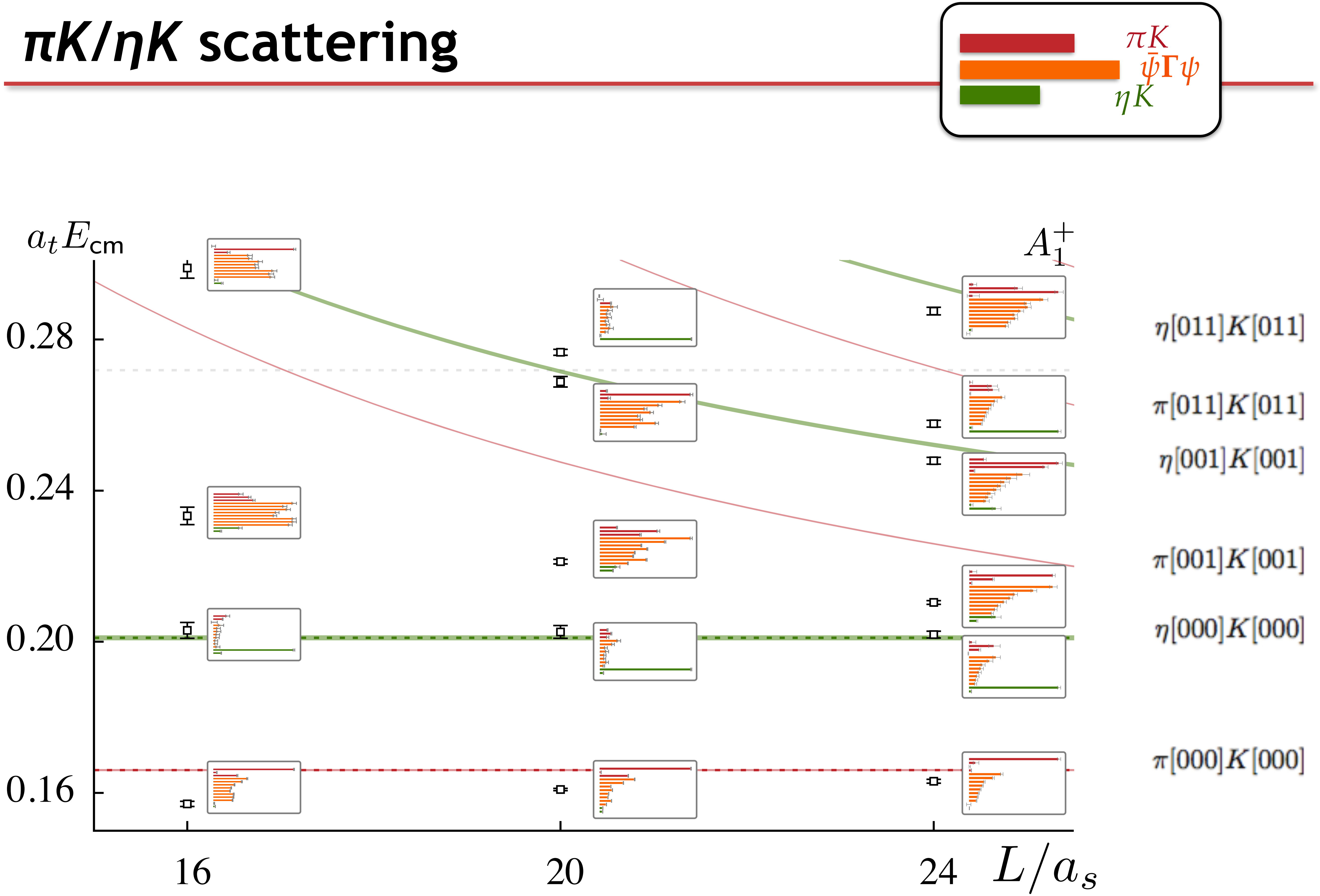


$\pi K/\eta K$ scattering

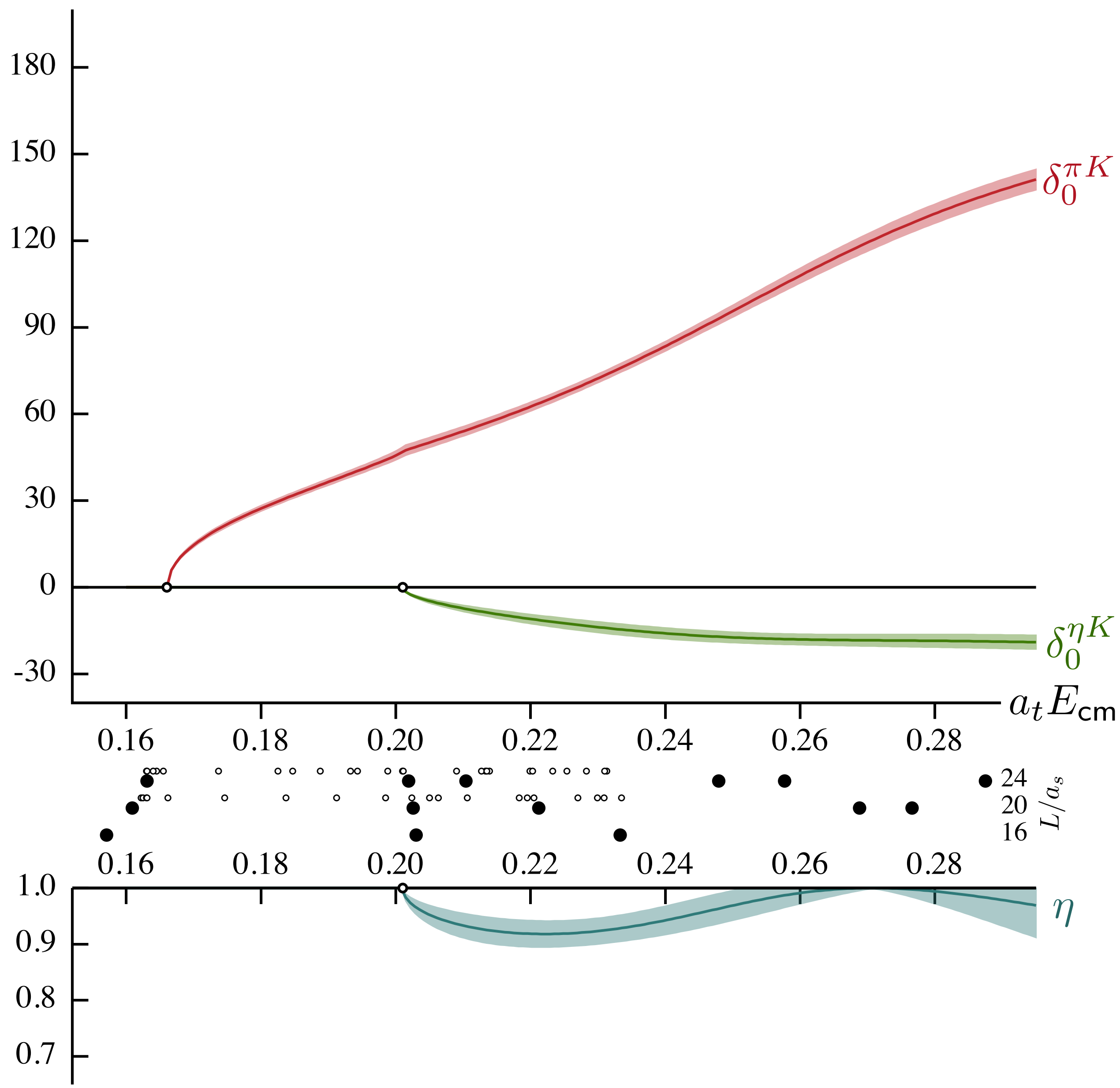
$m_\pi \sim 391\,\mathrm{MeV}$

- Describe all the finite-volume spectra

$$\chi^2/N_{\mathrm{dof}} = \frac{49.1}{61-6} = 0.89$$

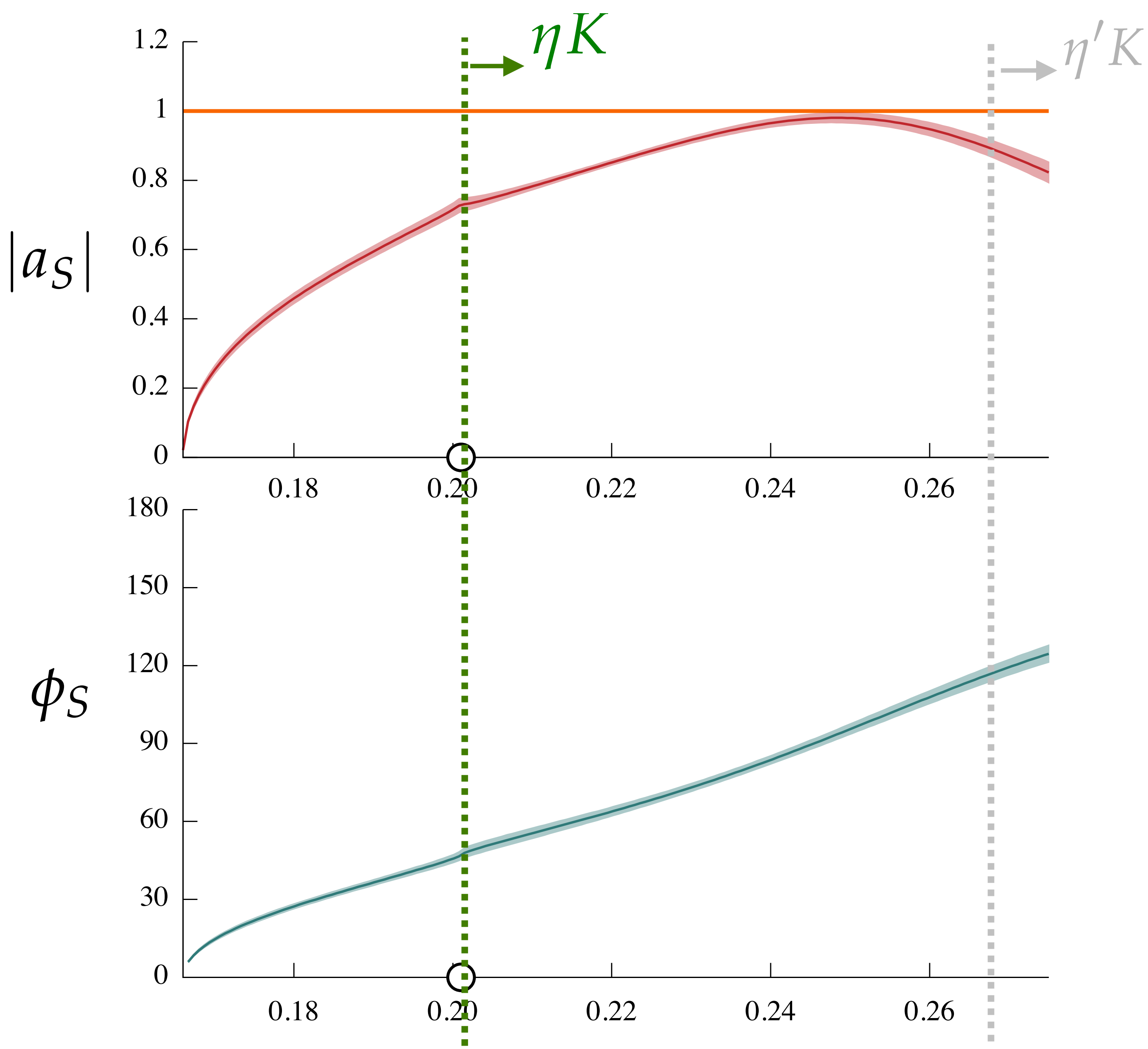
$$S_{\pi K,\pi K} = \eta\, e^{2i\delta^{\pi K}}$$
$$S_{\eta K,\eta K} = \eta\, e^{2i\delta^{\eta K}}$$

S-WAVE πK/ηK SCATTERING



Versus experimental scattering

S-WAVE πK→πK AMPLITUDE



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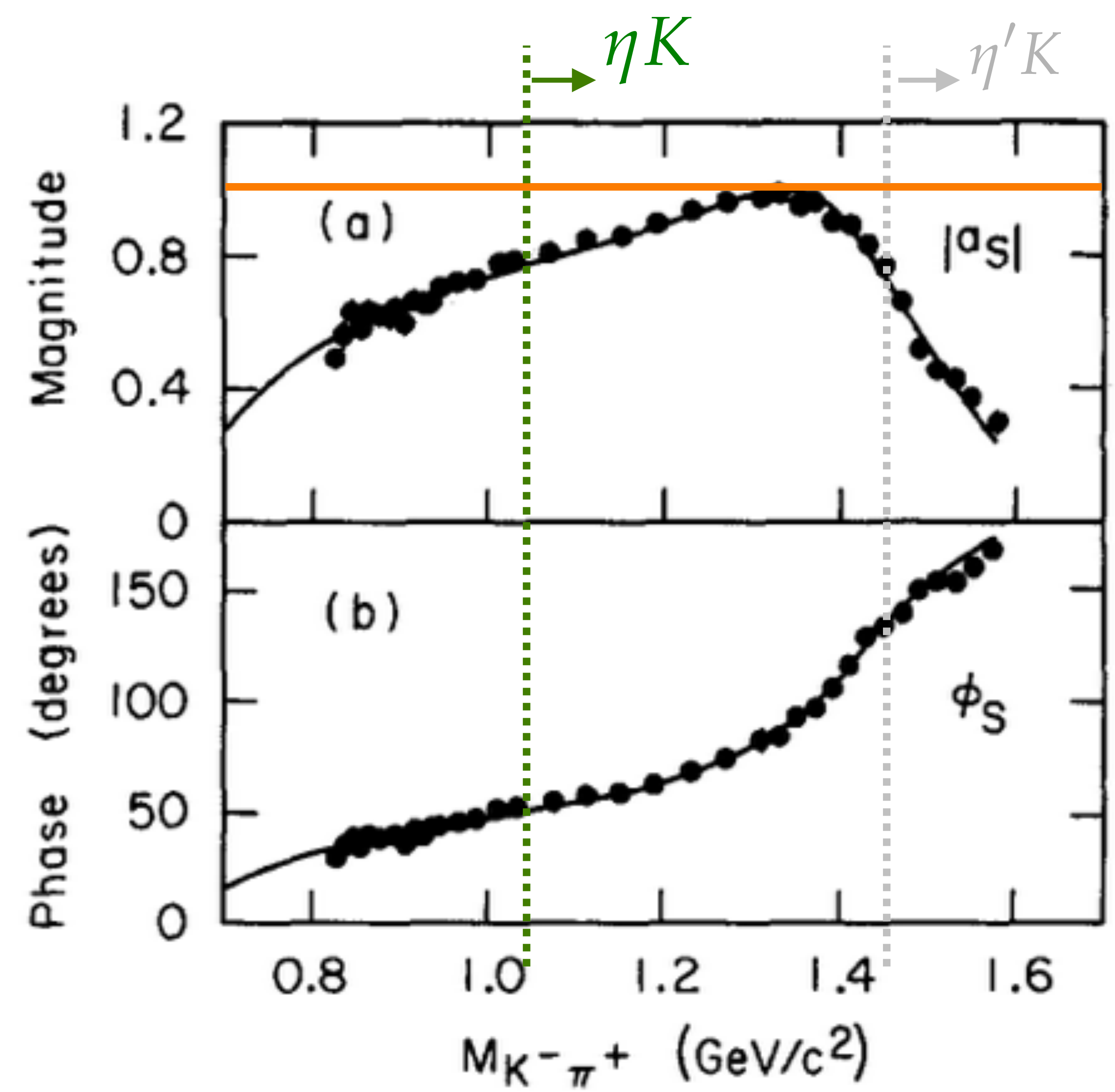
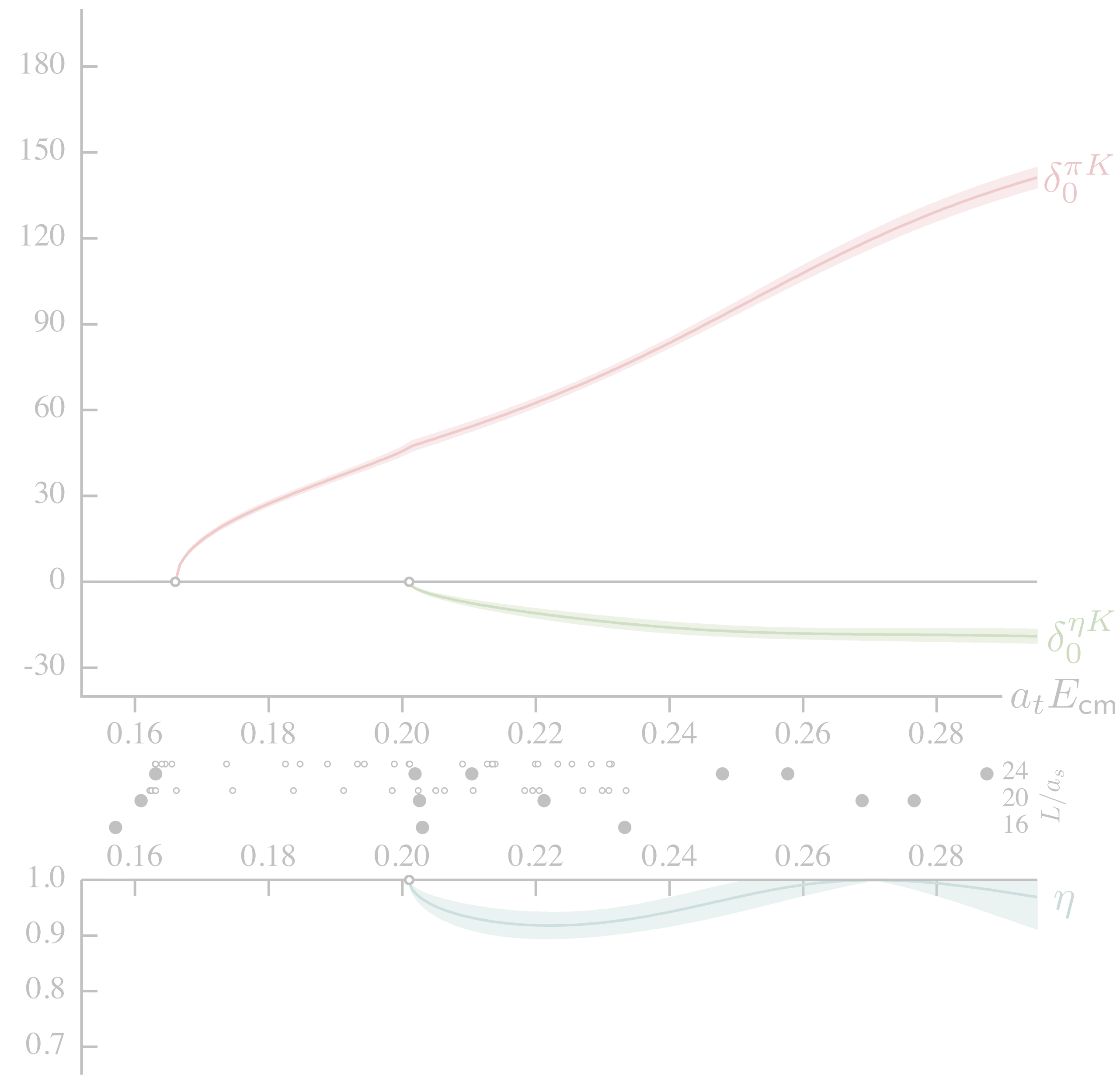


LASS, NPB296 493

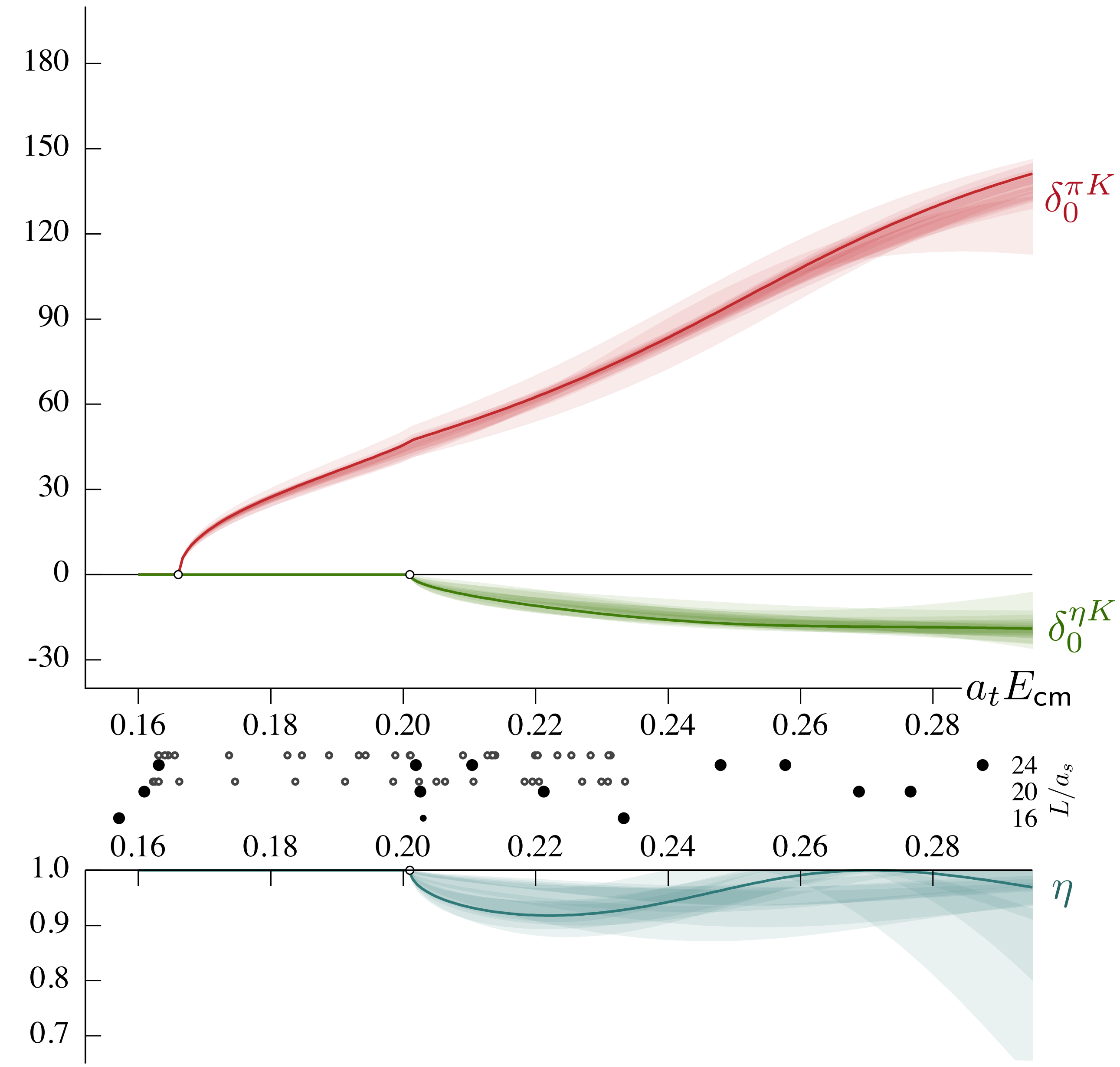
πK/ηK scattering

$m_\pi \sim 391\,\mathrm{MeV}$

- Are the result parameterization dependent ?
  - Try a range of parameterizations …



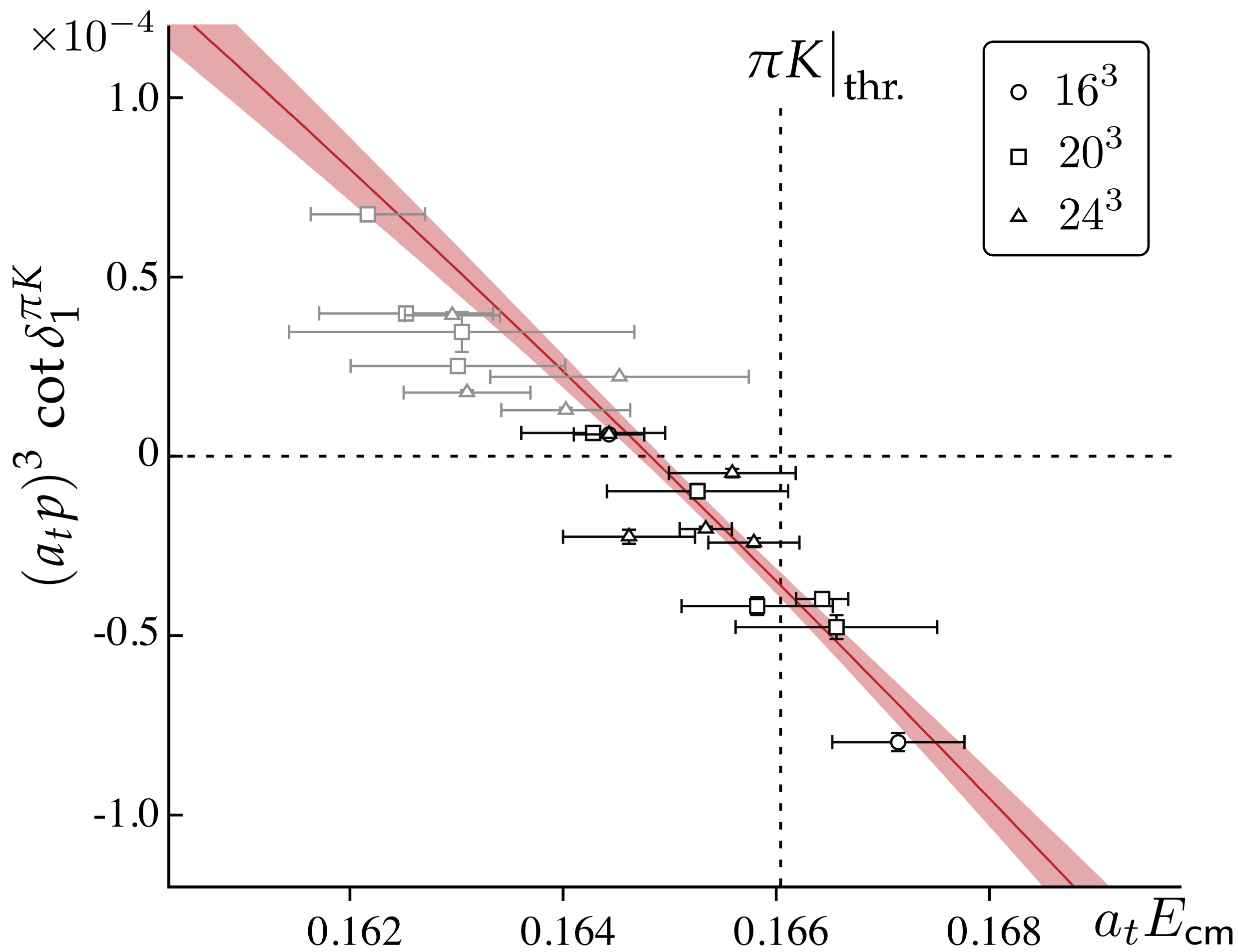
S-WAVE πK/ηK SCATTERING



- gross features are robust

$\pi K/\eta K$ scattering amplitudes

P-WAVE πK SCATTERING



Vector (shallow) bound-state

Use a Breit-Wigner with a subthreshold mass

$$k^3 \cot \delta_1 = (m_R^2 - s)\frac{6\pi\sqrt{s}}{g_R^2}$$

$$\longrightarrow \quad g = 5.93(30)$$

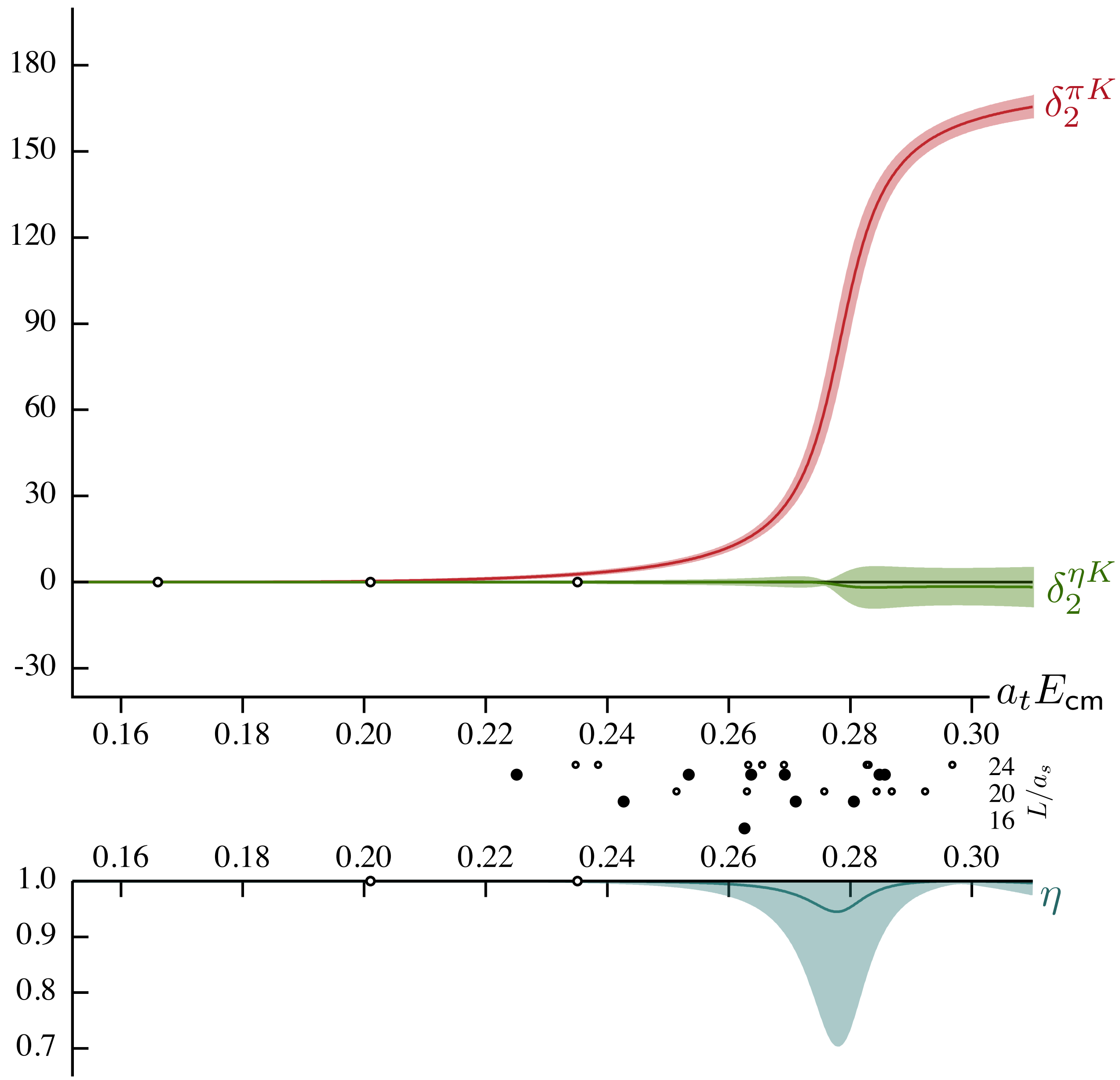
Quark mass accident that it lies so close to threshold …

$g_{\text{phys.}} = 5.5(2)$ *PDG*

$\pi K/\eta K$ scattering

- Clear narrow resonance in *D*-wave scattering

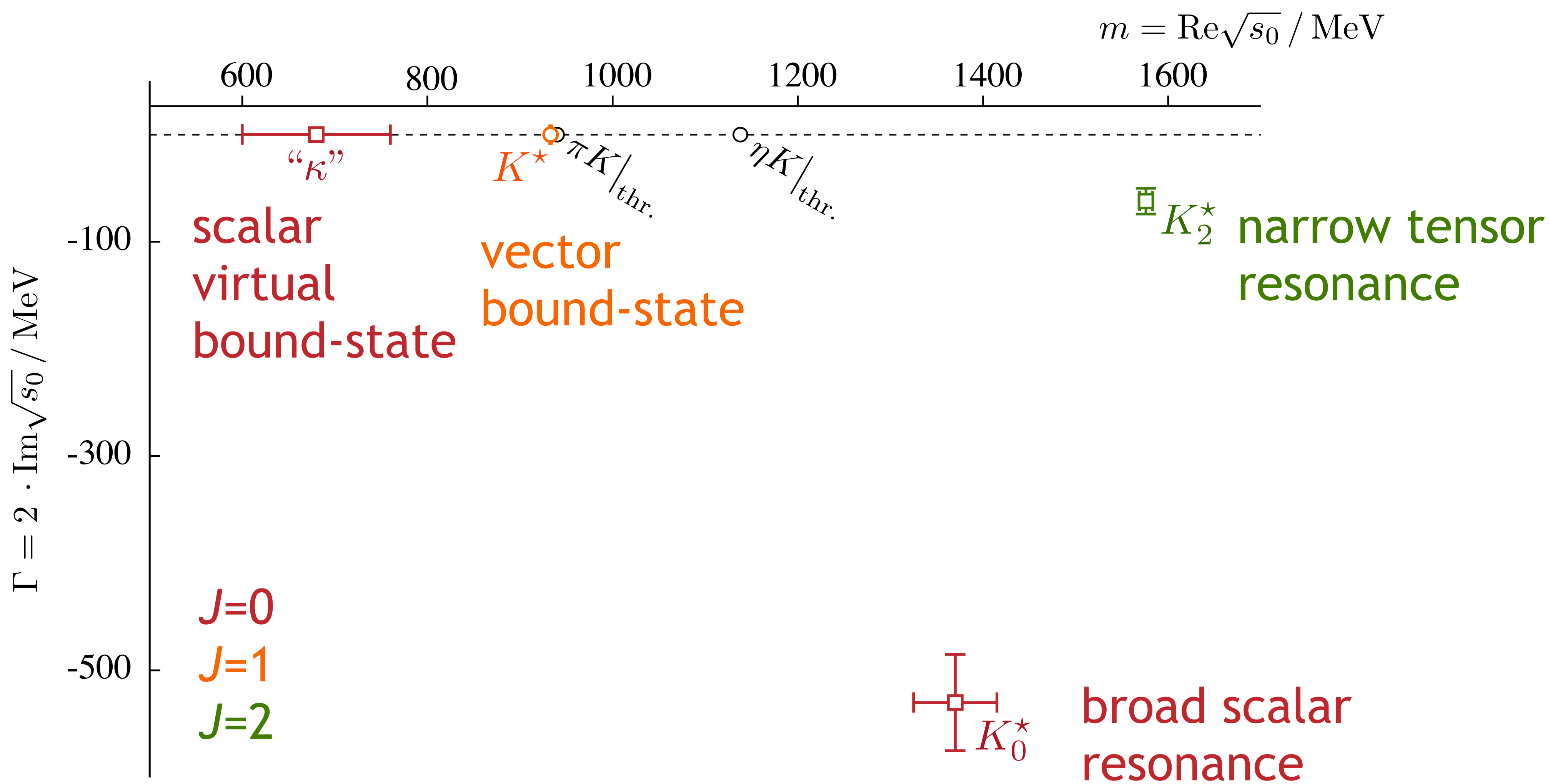
D-WAVE πK/ηK SCATTERING



$m_\pi \sim 391\,\mathrm{MeV}$

Singularity content

- extract ***t*-matrix poles** from partial waves

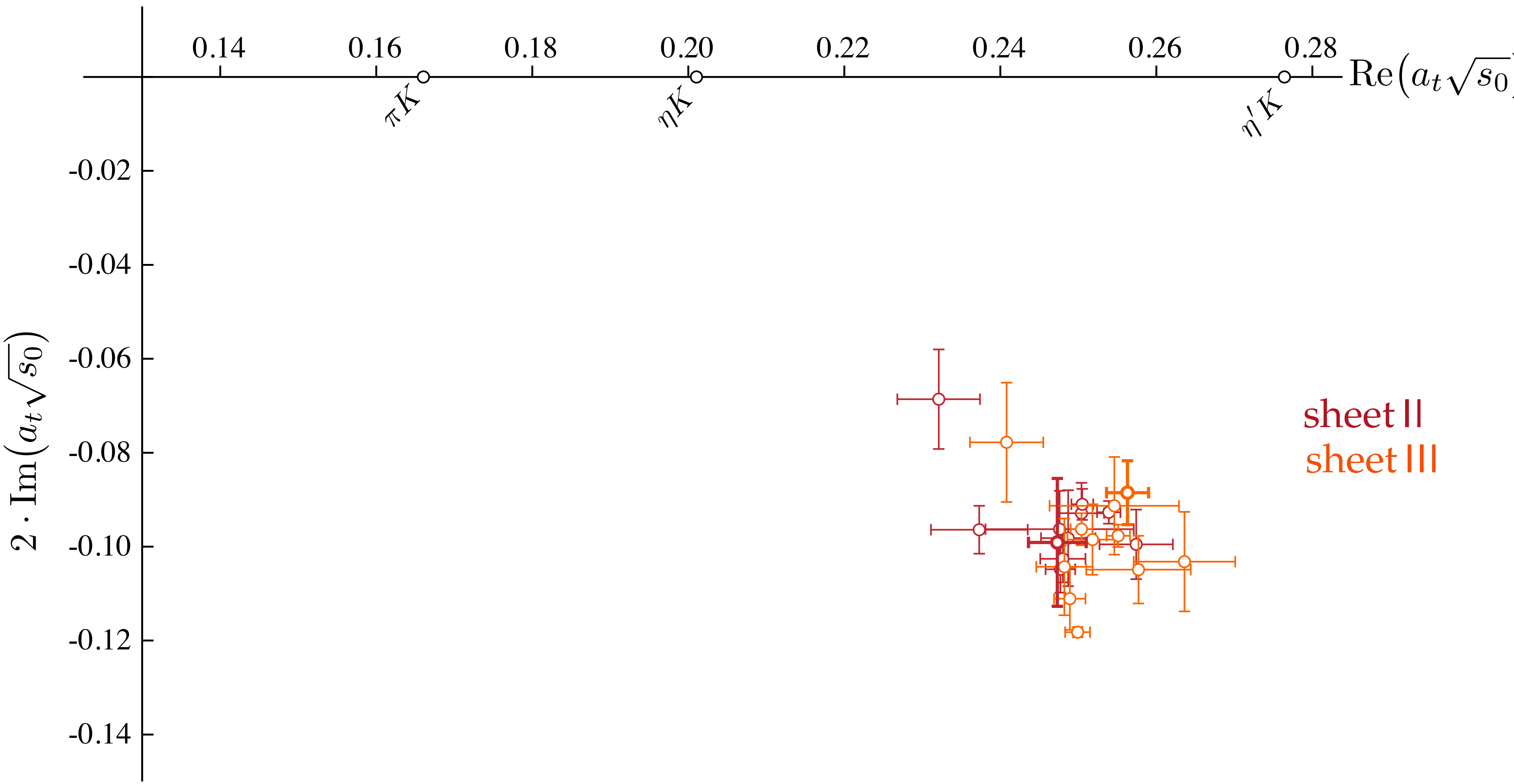


$m_\pi \sim 391\,\mathrm{MeV}$

PRL 113 182001
PRD 91 054008

$\pi K/\eta K$ S-wave resonance

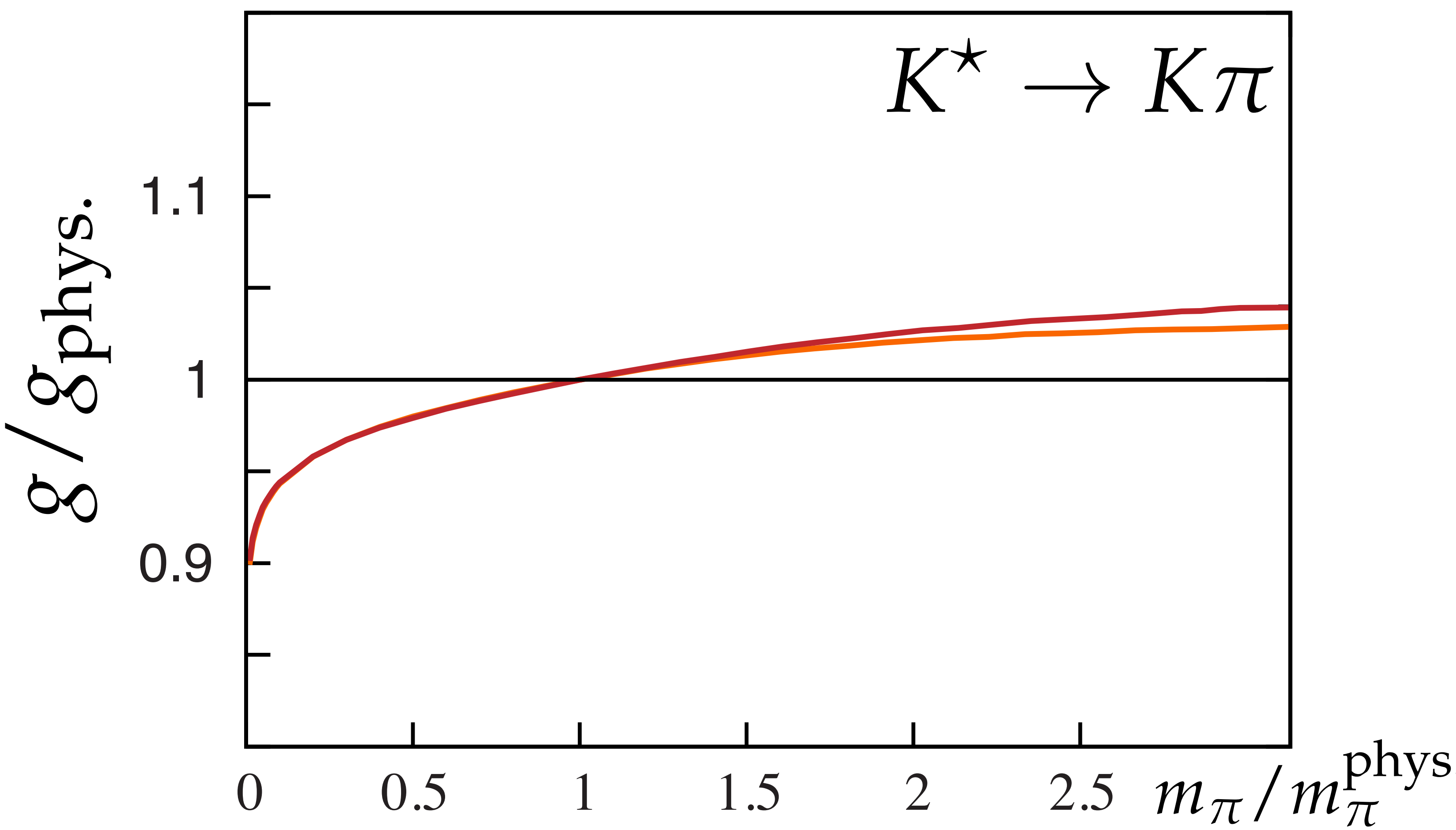
- t-matrix pole position under variation of parameterization



K*πK coupling with changing quark mass

- Unitarized SU(3)$_F$ chiral perturbation theory

NEBREDA & PELAEZ
PRD81 054035 (2010)



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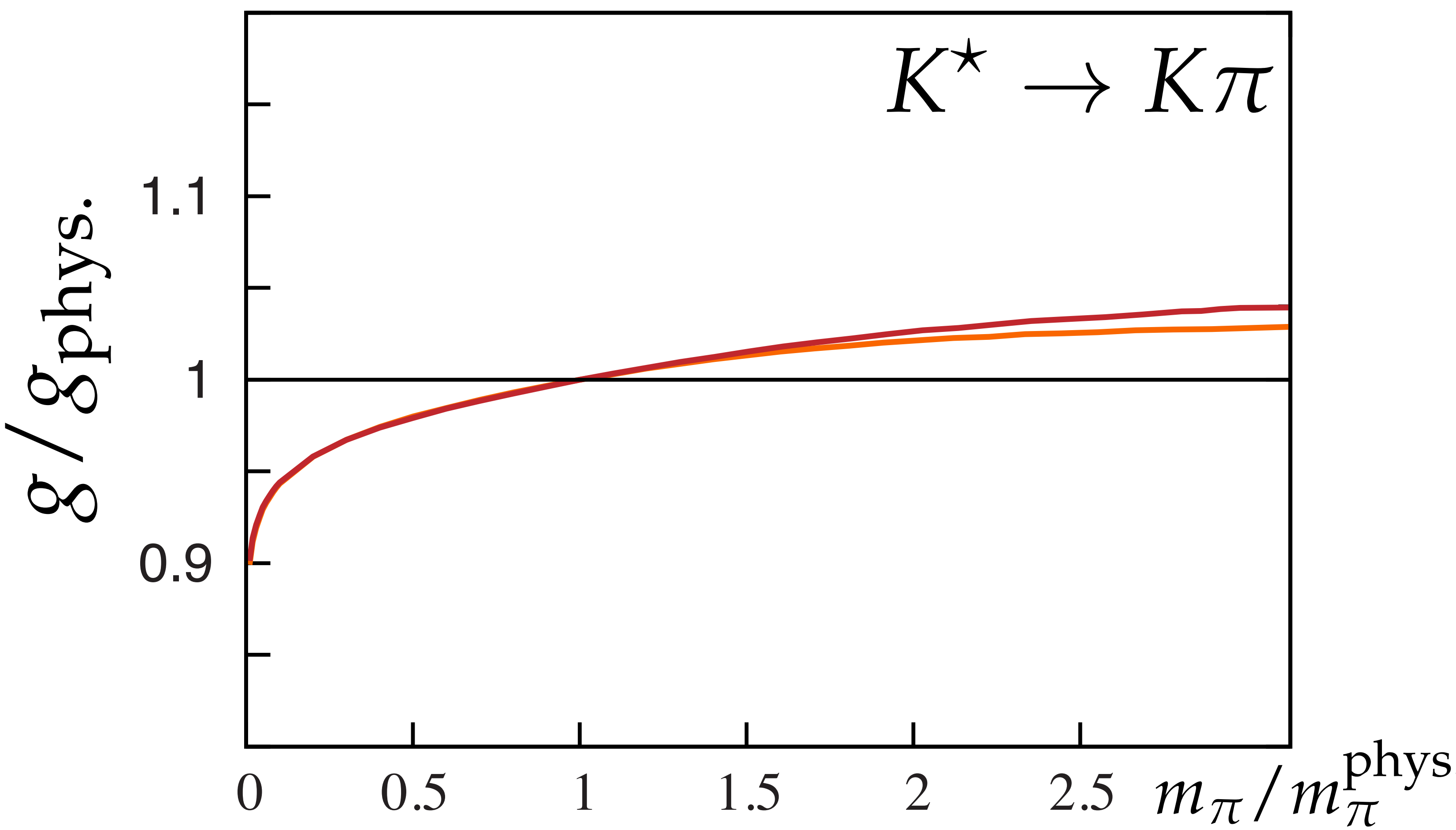


$g_{\text{phys.}} = 5.5(2)$ *PDG*

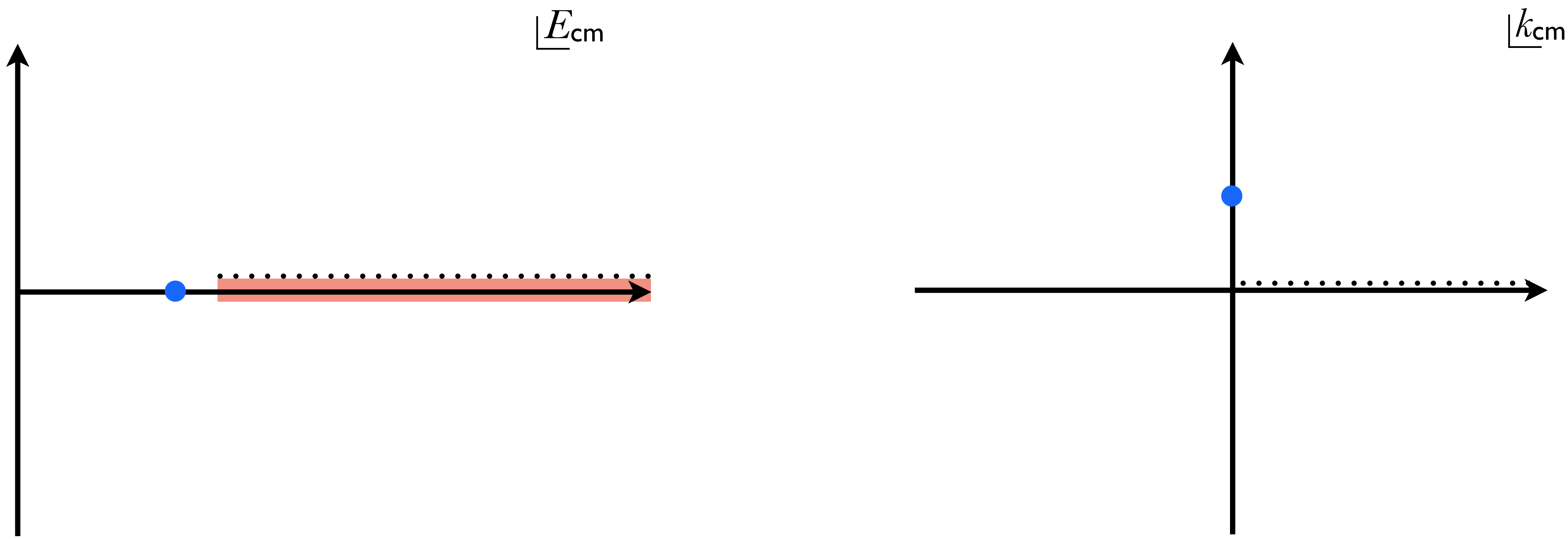
At $m_\pi \sim 391\,\text{MeV}$ we have shallow bound state: $g = 5.93 \pm 0.26$

That's the P-wave - what about S-wave?

Varieties of poles

- Multi-sheet structure around a cut: single channel case

$$k_{cm} = \pm\frac{1}{2}\sqrt{E_{cm}^2 - 4m^2}$$



Bound state

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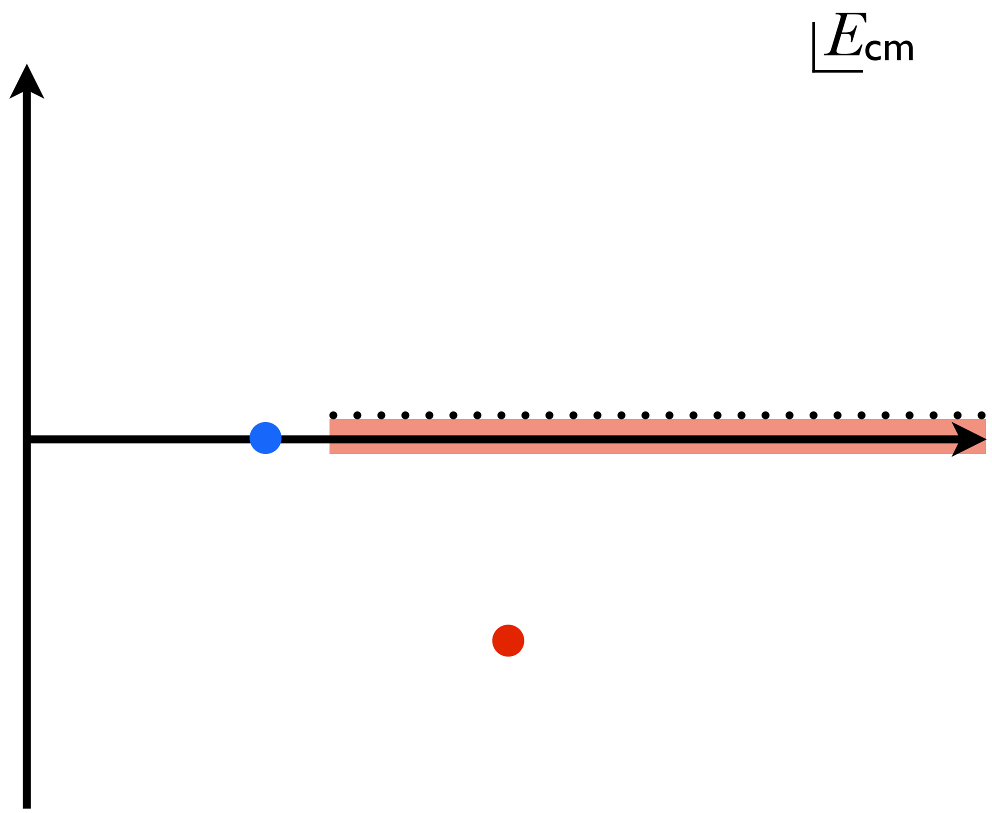

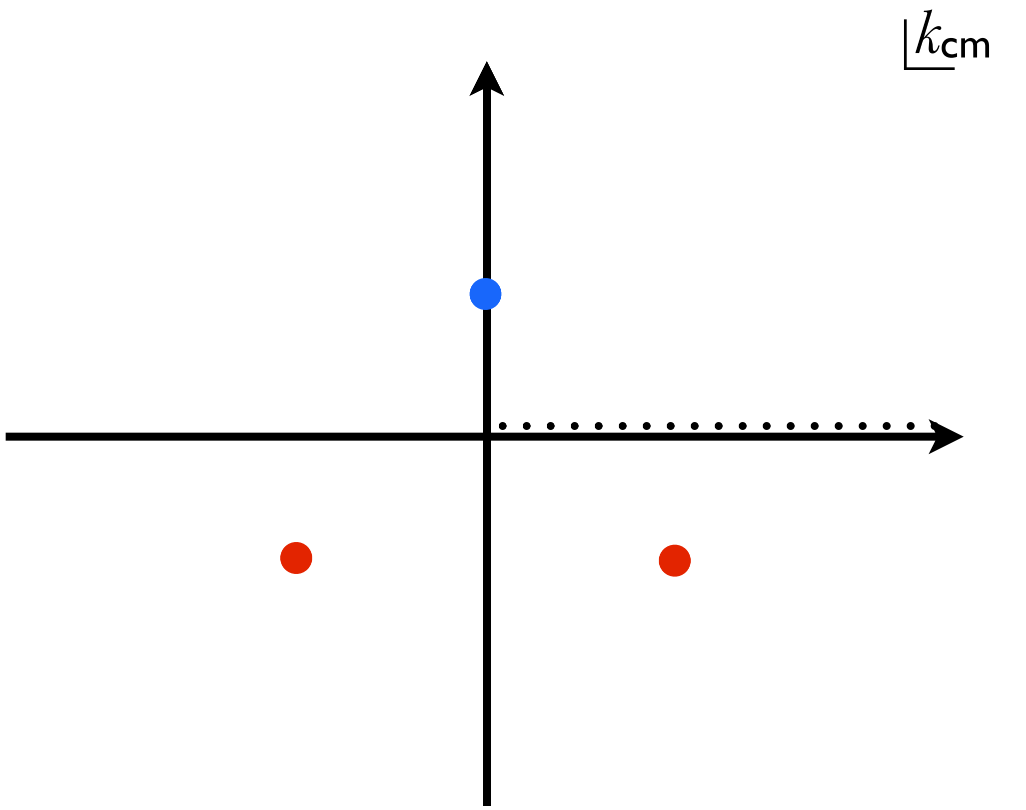

Bound state

Resonance

Varieties of poles

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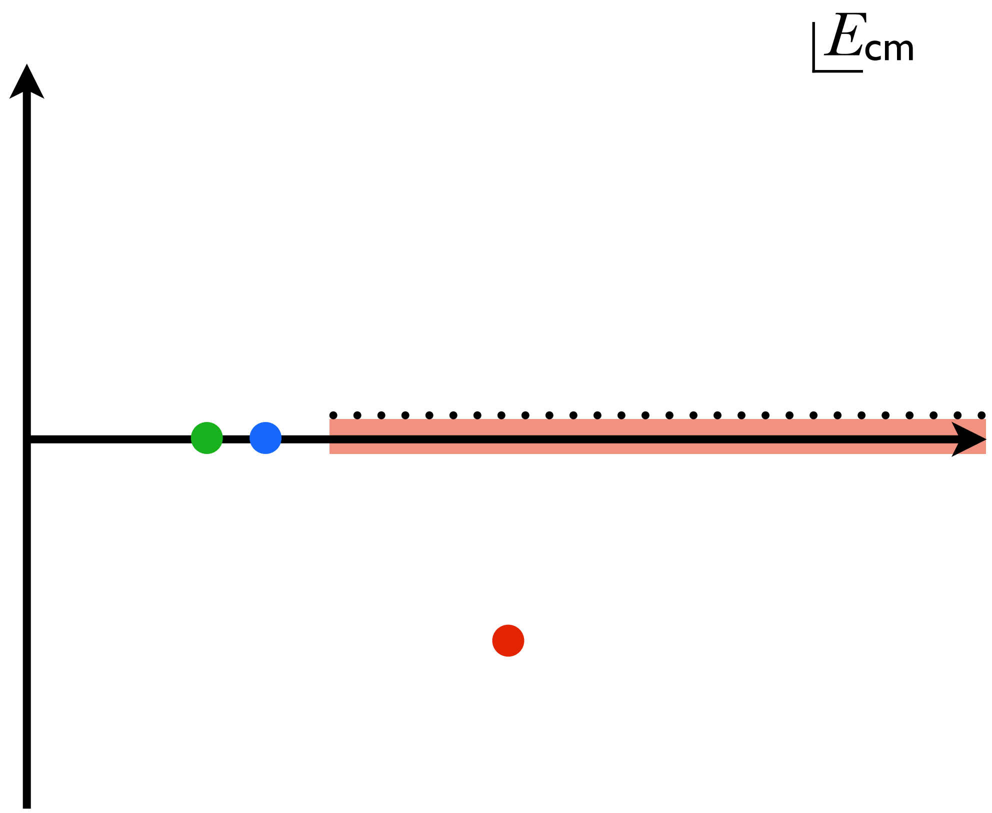
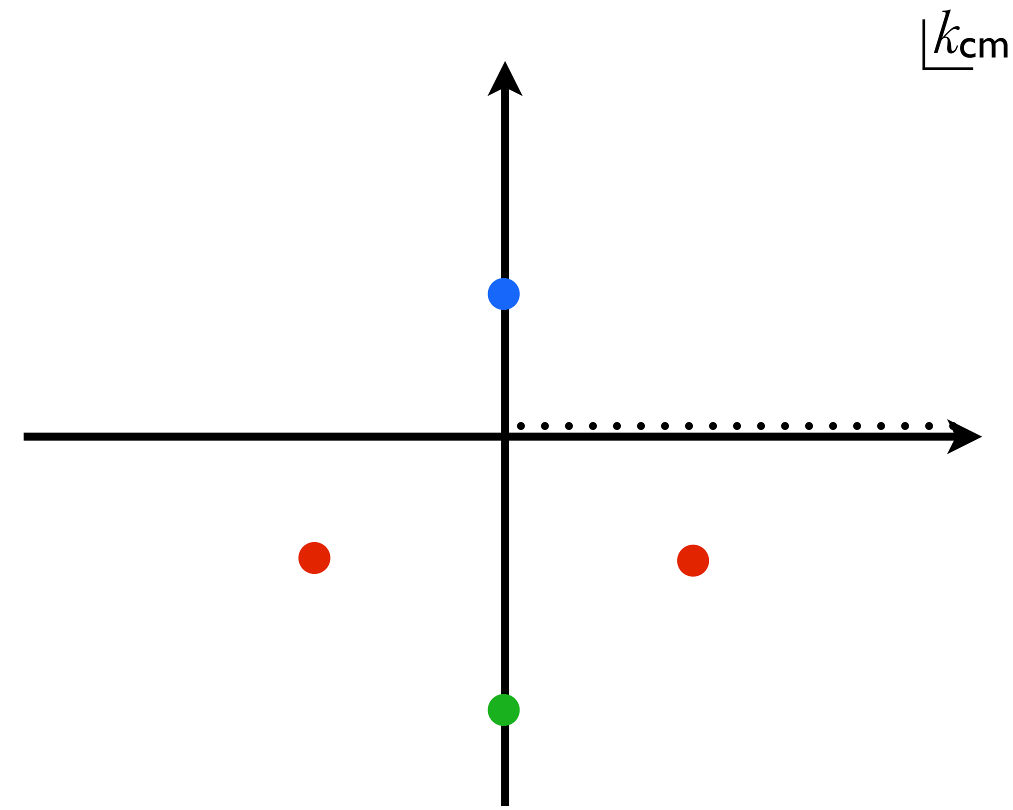




Bound state

Resonance

Virtual Bound state

Familiar examples:

N-N $^3S_1$ deuteron

ρ

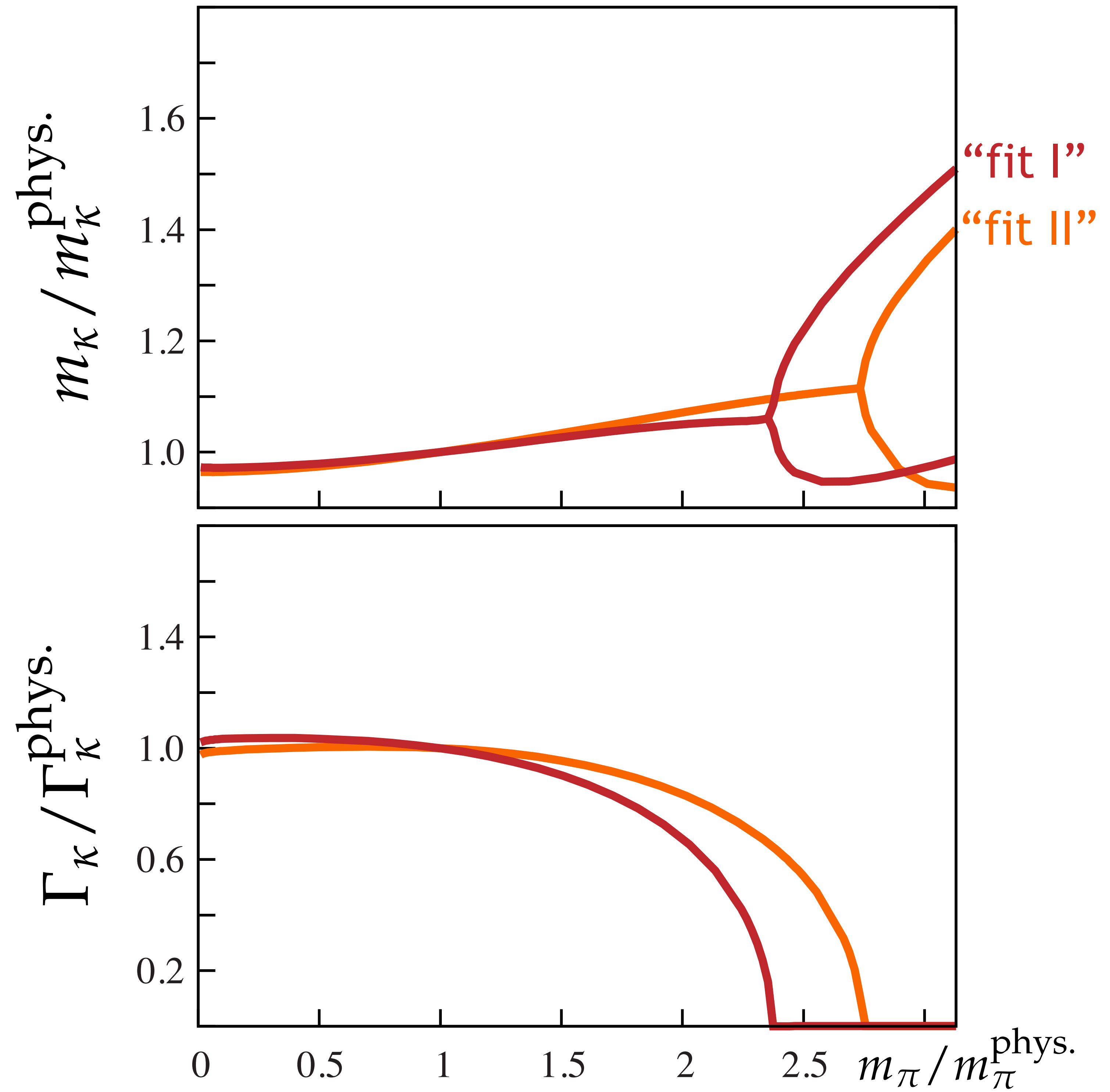
N-N $^1S_0$

κ (kappa) pole with changing quark mass

- Unitarized SU(3)$_F$ chiral perturbation theory

NEBREDA & PELAEZ
PRD81 054035 (201

$\sqrt{s_0} = m + \frac{i}{2}\Gamma$



- Resonance poles become virtual bound states somewhere near $m_\pi \sim 2.5\, m_\pi^{phys}$

- At higher pion mass virtual bound-state becomes bound

DESCOTES-GENON

$$\sqrt{s_0} = 660(20) + \frac{i}{2}\, 550(25)\ \text{MeV}$$

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NEBREDA & PELAEZ
PRD81 054035 (201

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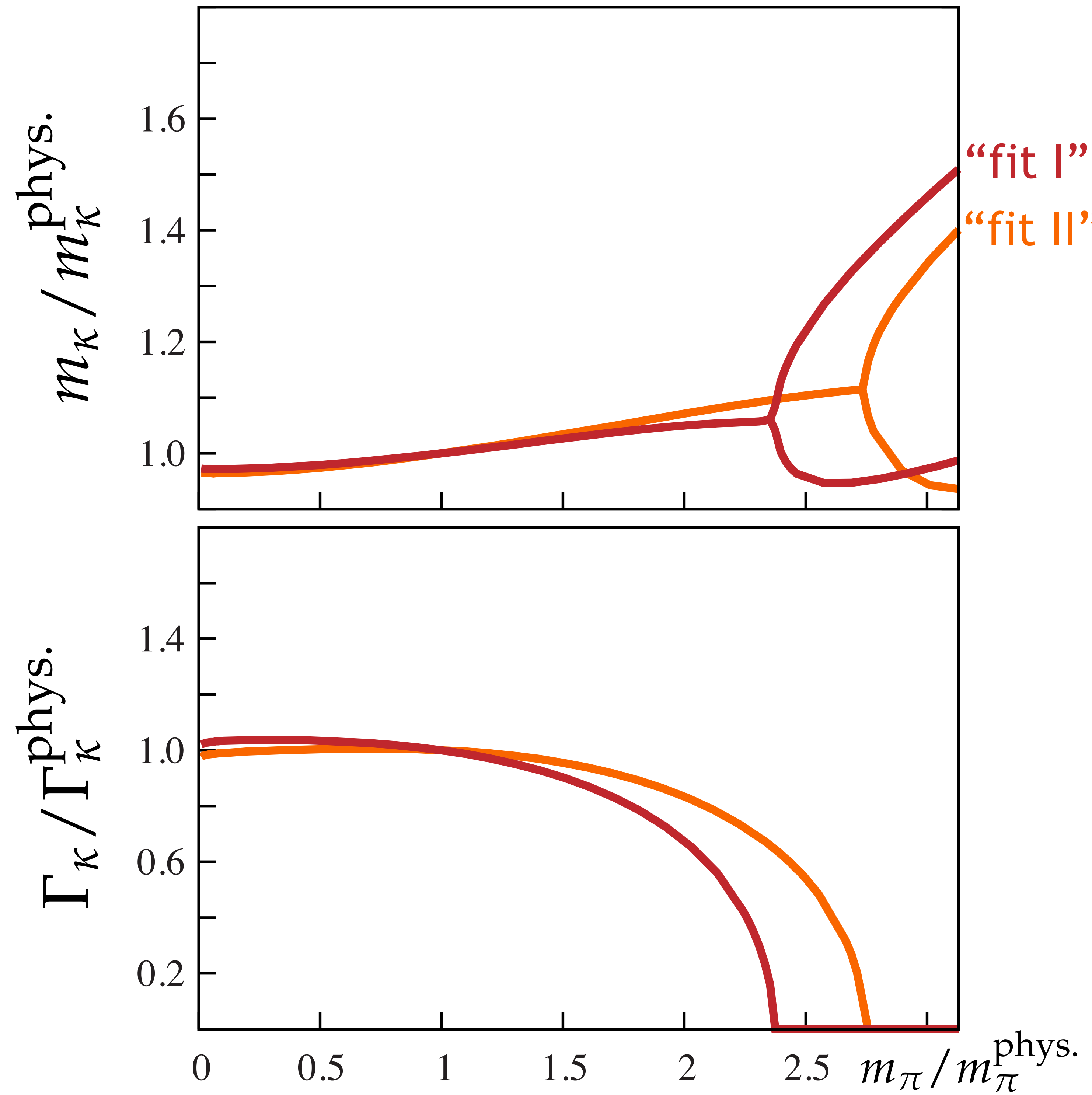

DESCOTES-GENON

$$\sqrt{s_0} = 660(20) + \tfrac{i}{2}\, 550(25)\ \mathrm{MeV}$$

At a lower pion mass, do we see a resonance?
Stay tuned !

Summary

- LQCD spectroscopy program maturing. First phase:
  - With only "single-hadron" operators obtain sketch of hadron spectrum
  - Suggests rich spectrum of mesons & baryons - exotic & non-exotic hybrids

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  - Including multi-hadron operators leads to richer spectrum
  - Demonstrated viability of finite-volume methods
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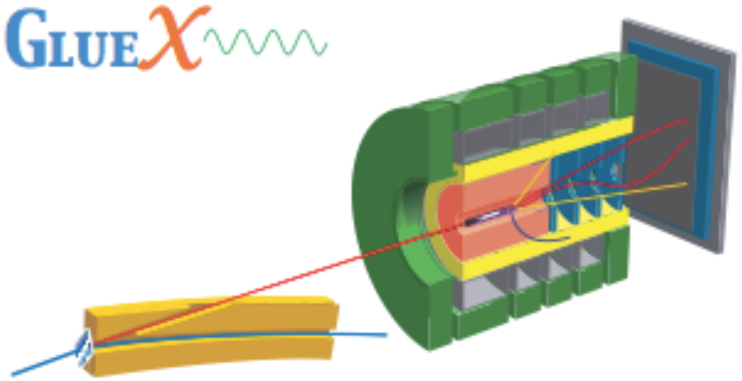
- *Goal is to compute resonance information - decays & branching fractions*
  - Including multi-hadron operators leads to richer spectrum
  - Demonstrated viability of finite-volume methods
  - S-matrix formalism increasingly important

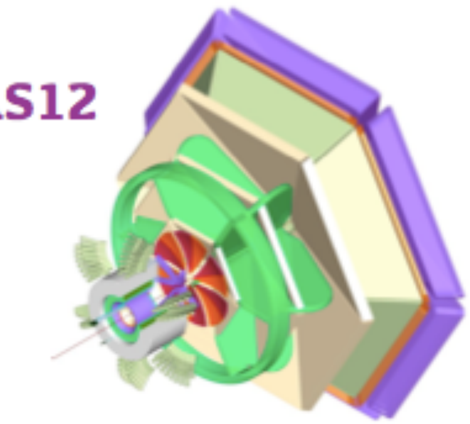
- Ultimately, determine underlying structure
  - Besides exotics - disentangle scalar spectrum (in progress)
  - Consider external currents coupled to resonances (in progress)

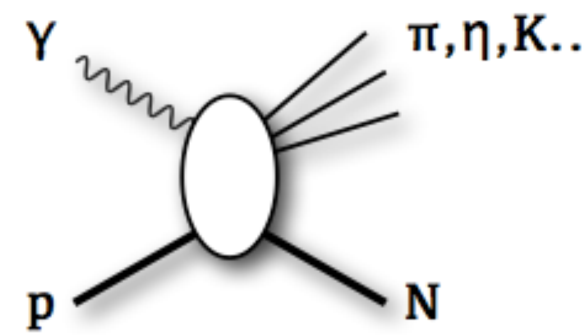
SHULTZ 2015
BRICENO & HANSEN 2015

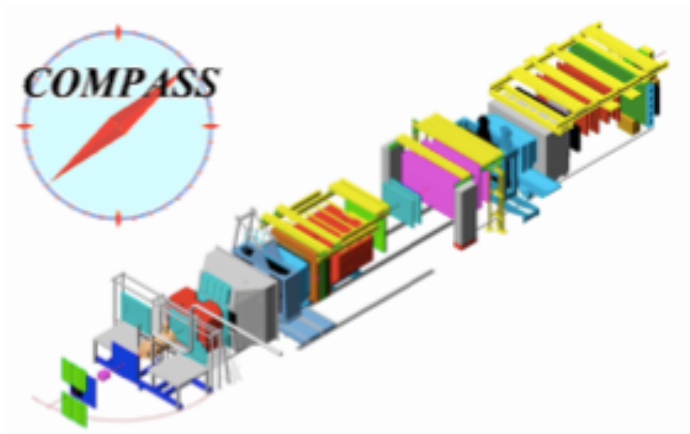
Spectrum - light meson experiments

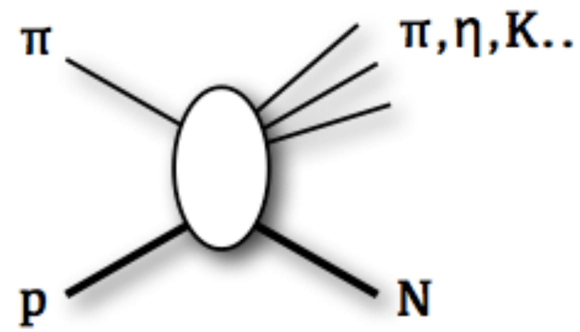


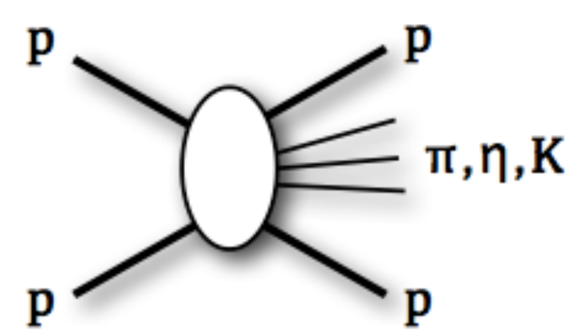
CLAS12



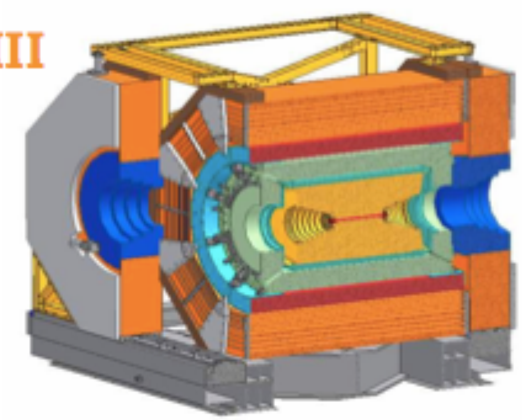


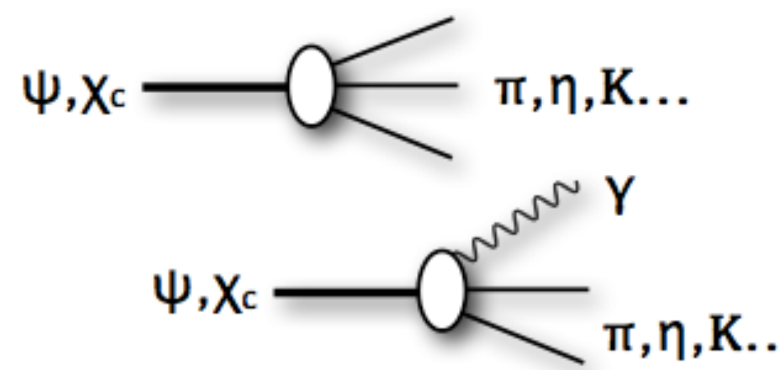






BES III





Scattering with external currents ?

- E.g. $\pi\gamma \to \pi\pi$ in *P*-wave : the ρ appears as a resonance
  - The observables are the amplitudes $A_\ell(E_{\pi\pi}, Q^2)$
  - Can be obtained from correlation functions **in finite-volume**

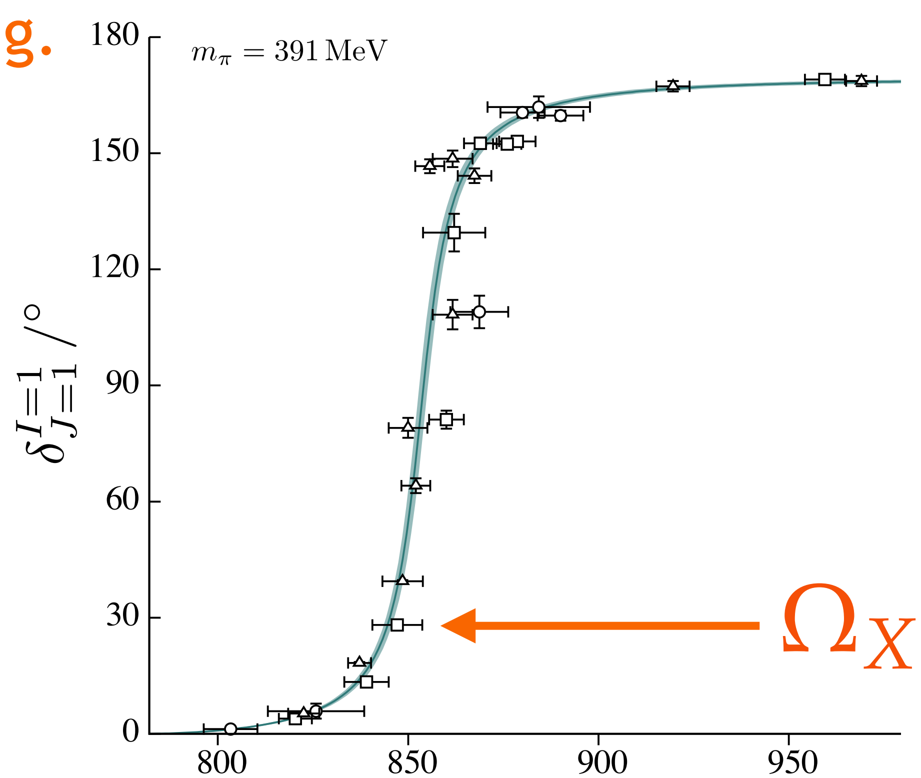
$$\langle 0 | \Omega_X(t_f)\, j^\mu(t)\, \Omega_\pi^\dagger(0) | 0 \rangle$$

PHYSICAL REVIEW D **91,** 034501 (2015)

Multichannel 1 → 2 transition amplitudes in a finite volume

Raúl A. Briceño,[1,*] Maxwell T. Hansen,[2,†] and André Walker-Loud[1,3,‡]

E.g.

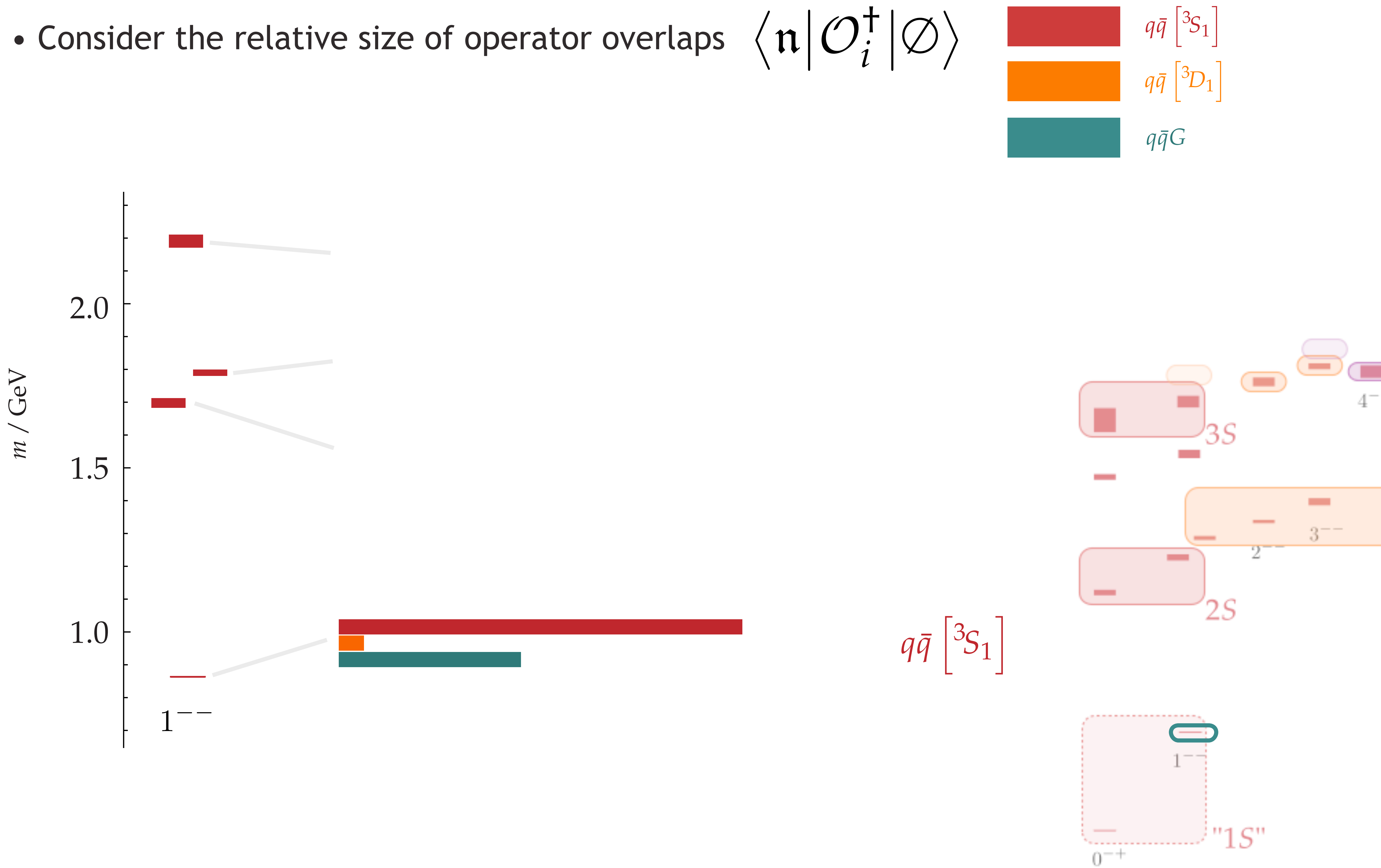


First results to appear soon…

$$\Omega_X = \sum_i a_i \left[\bar{\psi}\Gamma\psi\right]_i + \sum_j b_j \left[\pi\pi\right]_j$$

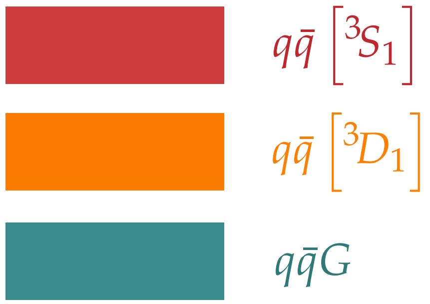
1^{--} operator overlaps

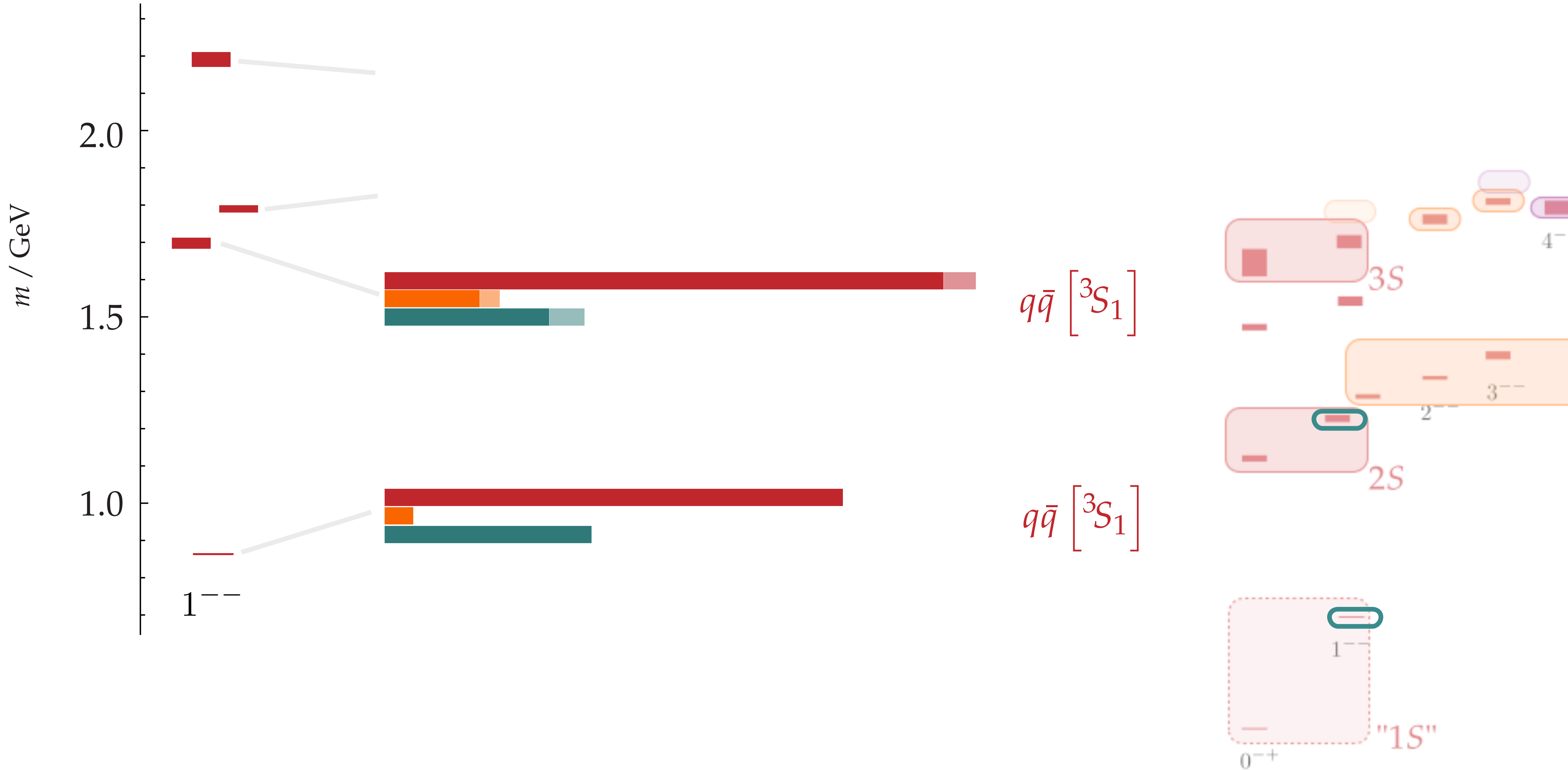
- Consider the relative size of operator overlaps $\langle \mathfrak{n} | \mathcal{O}_i^\dagger | \varnothing \rangle$



1^{--} operator overlaps

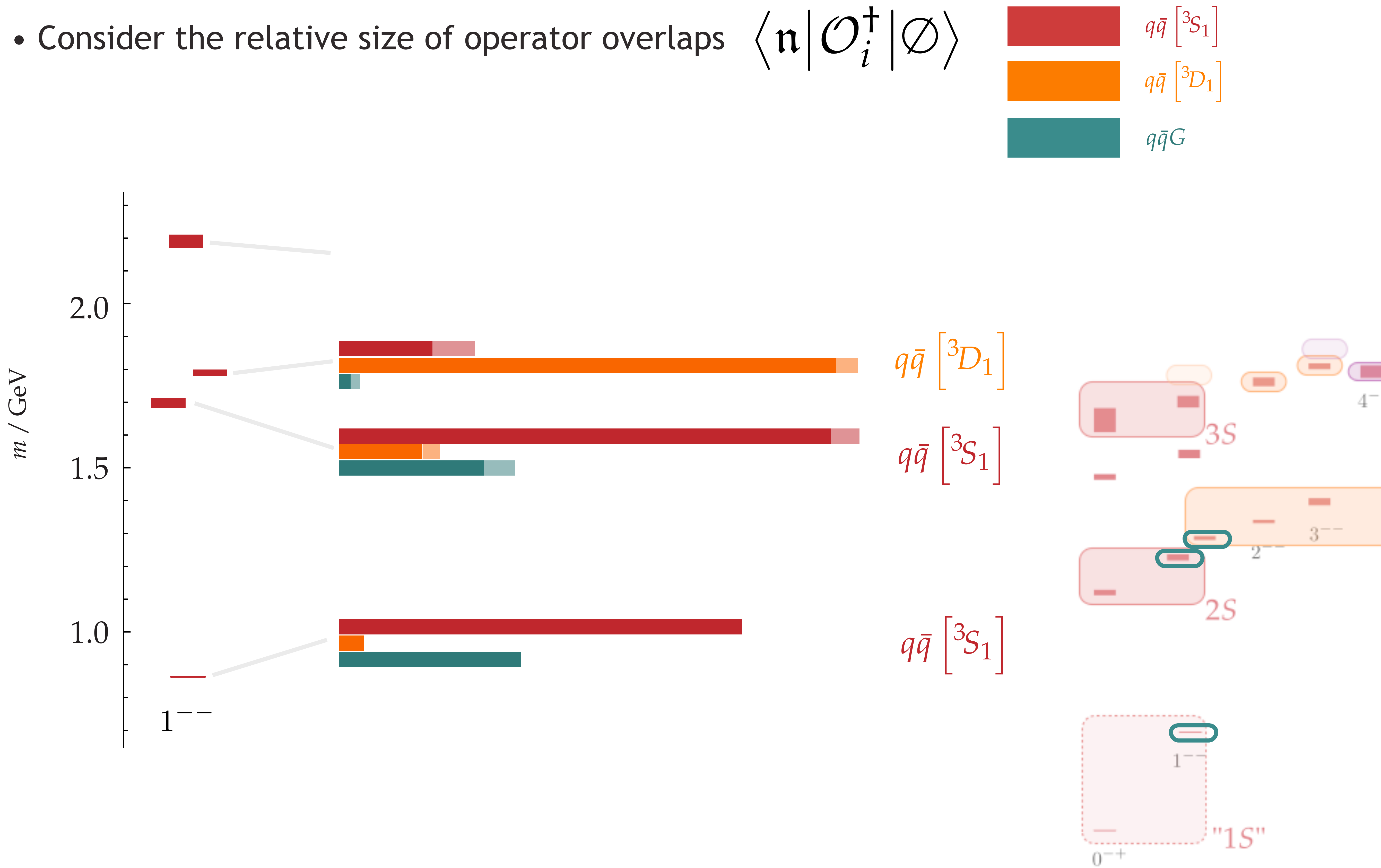
- Consider the relative size of operator overlaps $\langle \mathfrak{n} | \mathcal{O}_i^\dagger | \varnothing \rangle$





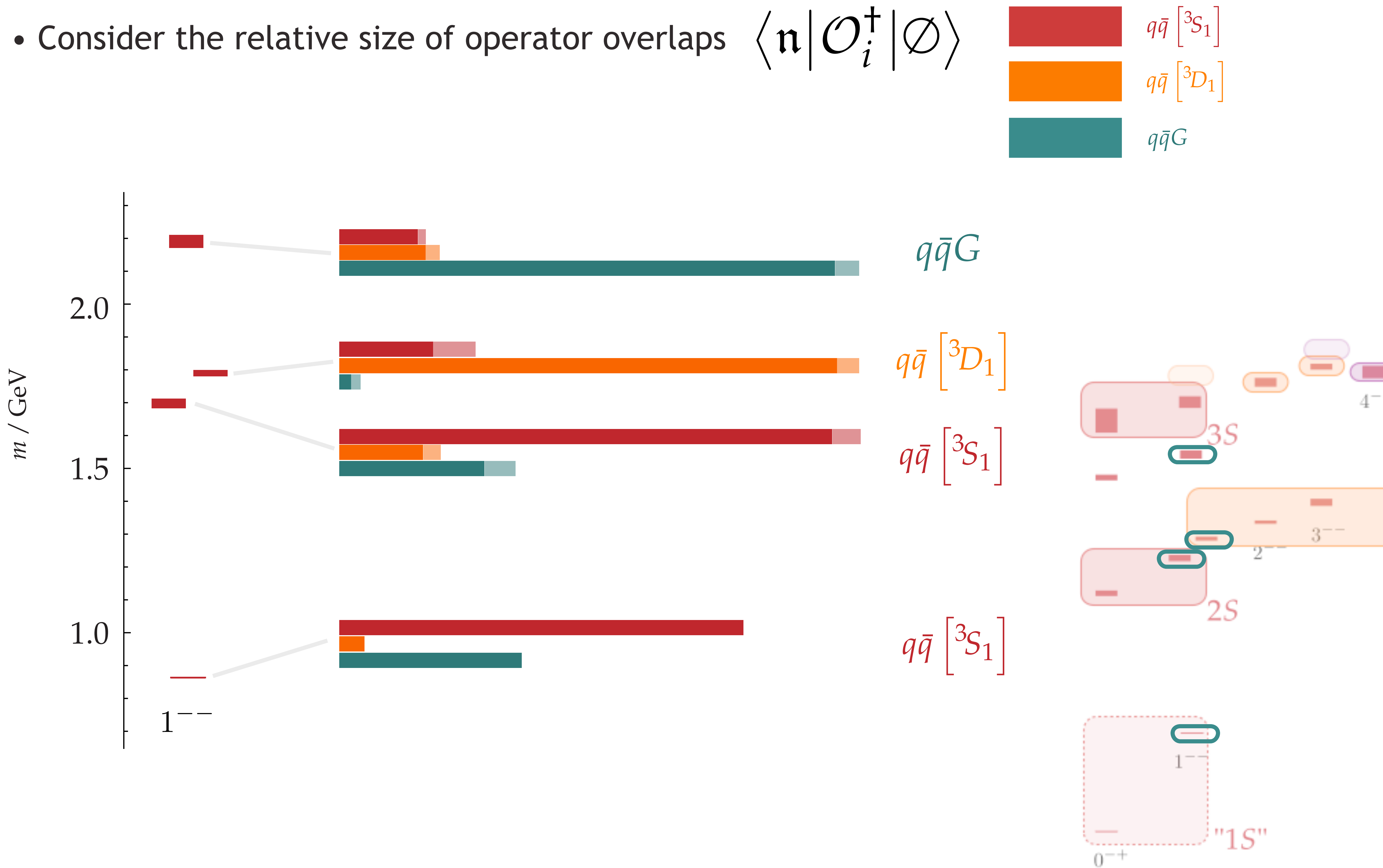
1^{--} operator overlaps

- Consider the relative size of operator overlaps $\langle \mathfrak{n} | \mathcal{O}_i^\dagger | \varnothing \rangle$



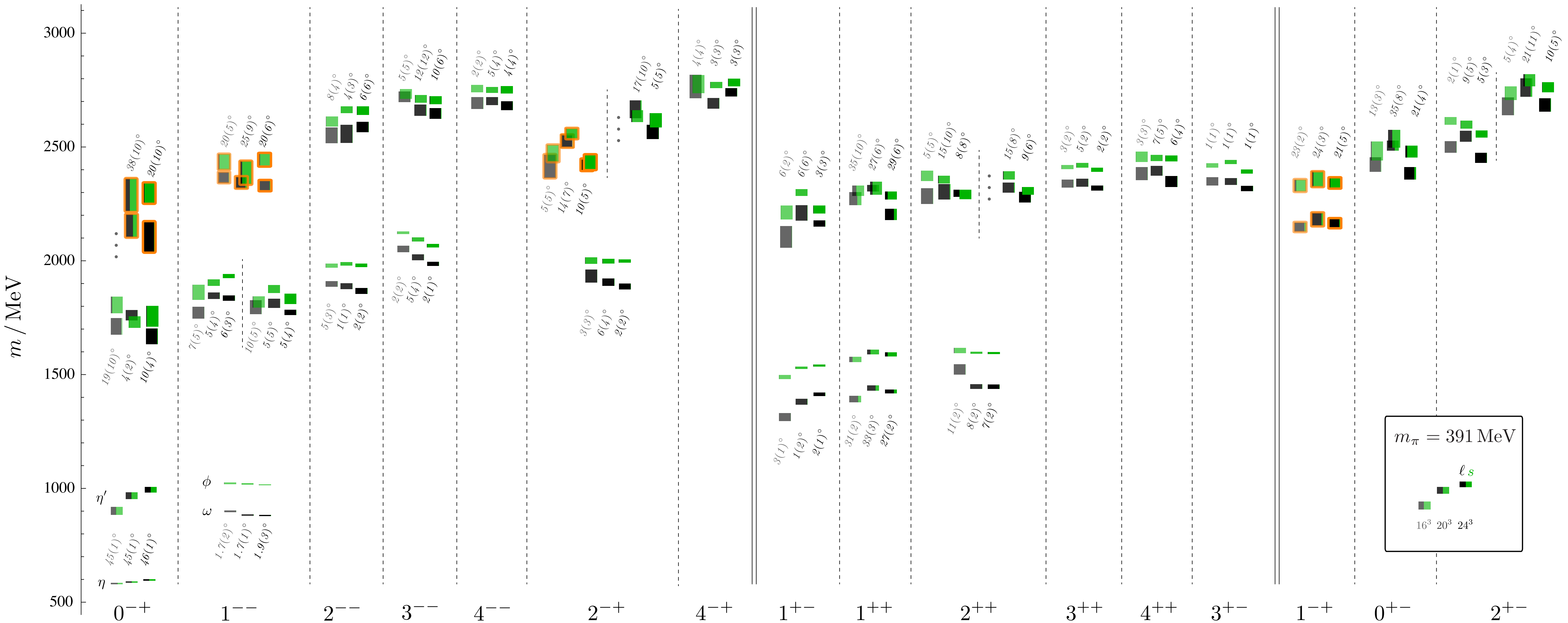
1^{--} operator overlaps

- Consider the relative size of operator overlaps $\langle \mathfrak{n} | \mathcal{O}_i^\dagger | \varnothing \rangle$



volume dependence

- meson spectrum using large $\bar{\psi}\Gamma D \ldots D\psi$ operator basis (no *meson-meson-like*)

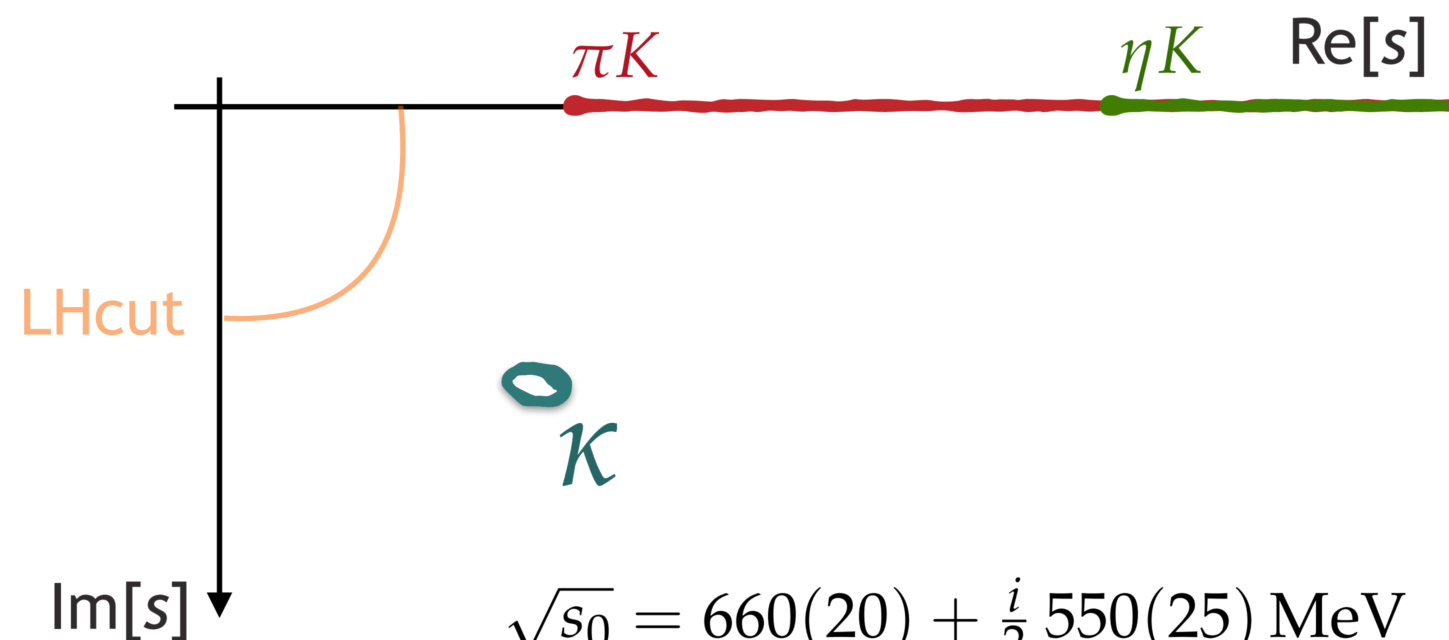


little volume dependence - probably picking out narrow resonances

Roy-Steiner & the κ (kappa)

- scattering data: $\pi K \to \pi K$, $\pi\pi \to K\bar{K}$
  + analyticity, unitarity, crossing-symmetry

- dispersive representation

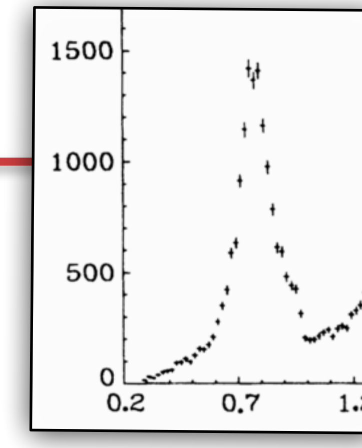


$$\sqrt{s_0} = 660(20) + \tfrac{i}{2}\,550(25)\ \mathrm{MeV}$$

DESCOTES-GENON & MOUSSALLAM, EPJC48 553 (2006)

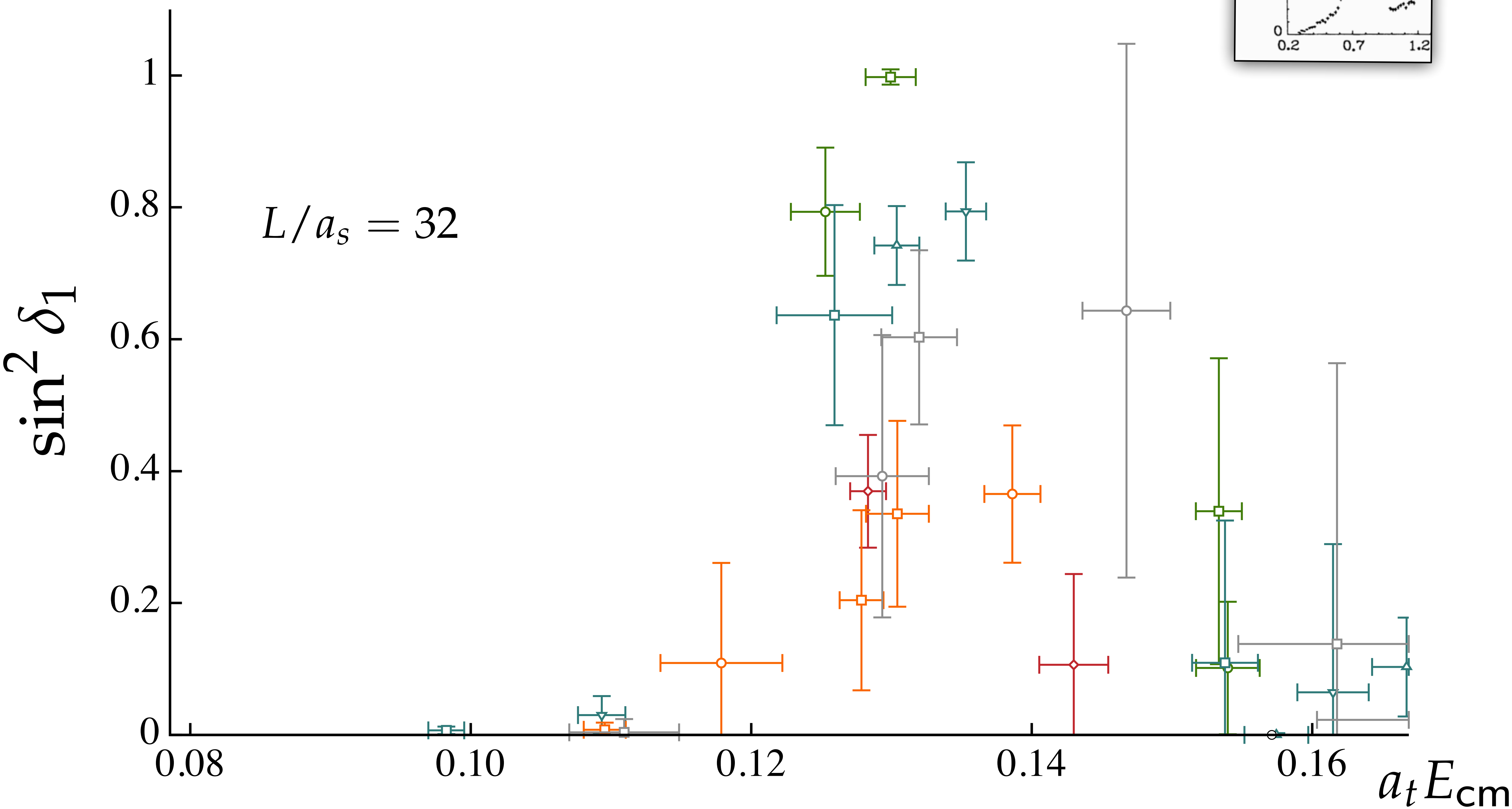
- resonance pole, but deep in the complex plane

ρ resonance - comparison



$m_\pi \sim 230\,\mathrm{MeV}$

Stochastic method

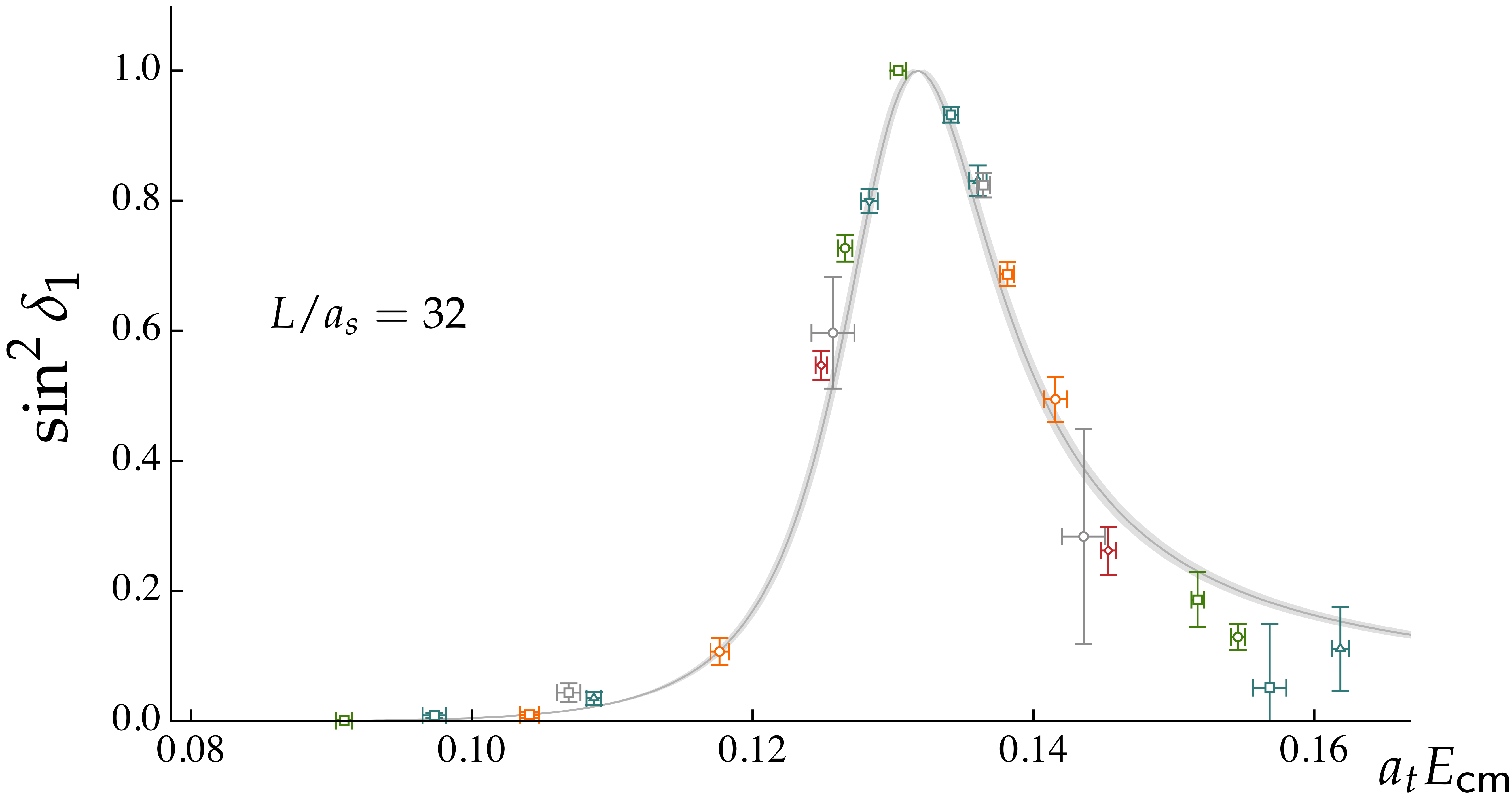


Fahy, et.al. arXiv:1410.8843

ρ resonance - comparison

$m_\pi \sim 230\,\mathrm{MeV}$

Full distillation method



PRELIMINARY … to appear soon