Radiation of Non-relativistic Particle on a Conducting Sphere and a String of Spheres

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The radiation emitting under charged particle crossing the boundary between two media with different electrodynamic properties is known as transition radiation (**TR**), whereas for the particle traveling near the boundary of the spatially localized target without crossing the emitting radiation is called as diffraction radiation (**DR**).

DR and TR of a charge on a perfectly conducting sphere (as well as on a periodic string of the spheres) are studied in the present report.

Various aspects of DR and TR on spherical targets had been considered in numerous papers, see, e.g.,

- N.F. Shul'ga, S.N. Dobrovol'sky, V.V. Syshchenko // Russian Physics Journal 44, No. 3 (2001) 317.
- F.G. Garc´ıa de Abajo // Phys. Rev. E 61 (2000) 5743.
- K.V. Lekomtsev, M.N. Strikhanov, A.A. Tishchenko // Journal of Physics: Conference Series 236 (2010) 012023.
- V.A. Astapenko, S.V. Sakhno, Yu.A. Krotov // Journal of Physics: Conference Series 732 (2016) 012025.

However, these papers either deal with some special cases or present the result in rather awkward form.

Here we propose the simple and economic method for computation of the radiation characteristics (spectral-angular density as well as polarization, if needed) based on the following idea.

One of the ways to describe these types of radiation is the application of the boundary conditions to the Maxwell equations solutions for the field of the moving particle in two media. It becomes evident that the boundary conditions could be satisfied only after addition the solution of free Maxwell equations that corresponds to the radiation field.

The conditions on the boundary between vacuum and ideal conductor could be satisfied in some cases via introduction of one or more fictive charges along with the real charged particle; this approach to electrostatic problems is known as the method of images, see, e.g.,

J.D. Jackson, *Classical Electrodynamics*, Wiley, New York, 1999.

Namely the method of images had been used in the pioneering paper

V.L. Ginzburg, I.M. Frank // J. Phys. USSR 9 (1945) 353.

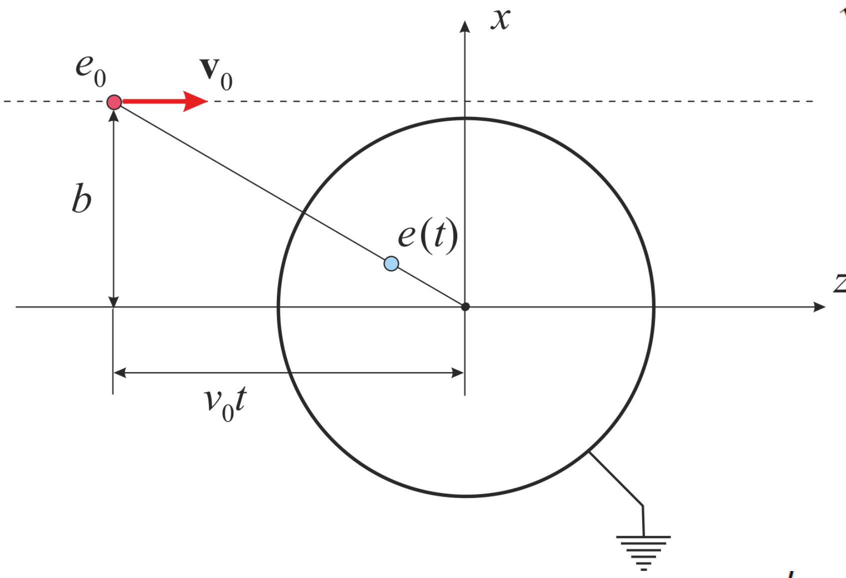
where TR on a metal plane had been predicted. The method of images had been used also in

G.A. Askaryan // JETP 29 (1955) 388 *(in Russian)*.

for consideration of TR under passage of the particle through the center of the ideally conducting sphere in dipole approximation.

Method of images in DR description

$$e(t) = -e_0 \frac{R}{\sqrt{b^2 + v_0^2 t^2}}, \qquad x(t) = \frac{R^2 b}{b^2 + v_0^2 t^2}, \qquad z(t) = \frac{R^2 v_0 t}{b^2 + v_0^2 t^2},$$



Consider the real charge e_0 passing by the grounded conducting sphere of the radius R. Its "image" $e(t)$ has to be placed in the point with coordinates $x(t)$, $z(t)$.

So, while the incident particle moves uniformly, its "image" will move accelerated,

$$v_x = \frac{dx}{dt} = -\frac{2R^2 b v_0^2 t}{(b^2 + v_0^2 t^2)^2}, \qquad v_z = \frac{dz}{dt} = \frac{R^2 v_0 (b^2 - v_0^2 t^2)}{(b^2 + v_0^2 t^2)^2}.$$

The radiation produced by non-uniform motion of the fictive charge will be described by well-known formula

$$\frac{d\mathcal{E}}{d\omega d\Omega} = \frac{1}{4\pi^2 c} \left|[\mathbf{k}, \mathbf{I}]\right|^2 = \frac{1}{4\pi^2 c} \left\{ \mathbf{k}^2 |\mathbf{I}|^2 - |\mathbf{k} \cdot \mathbf{I}|^2 \right\}, \qquad \mathbf{I} = \int_{-\infty}^{\infty} e(t)\, \mathbf{v}(t)\, e^{i(\omega t - \mathbf{k r}(t))}\, dt$$

(it could be easily seen that it is applicable to the case of time-varying charge $e(t)$ as well as to the case of the constant one). Note also that for the isolated (in contrast to grounded) sphere the method of images requires introducing another fictive charge of the value $-e(t)$ resting in the center of the sphere. However, the last equation shows that such rest charge does not produce any radiation.

Substitution of $e(t)$, $x(t)$, $z(t)$, $v_x(t)$, $v_y(t)$ gives the following integrals:

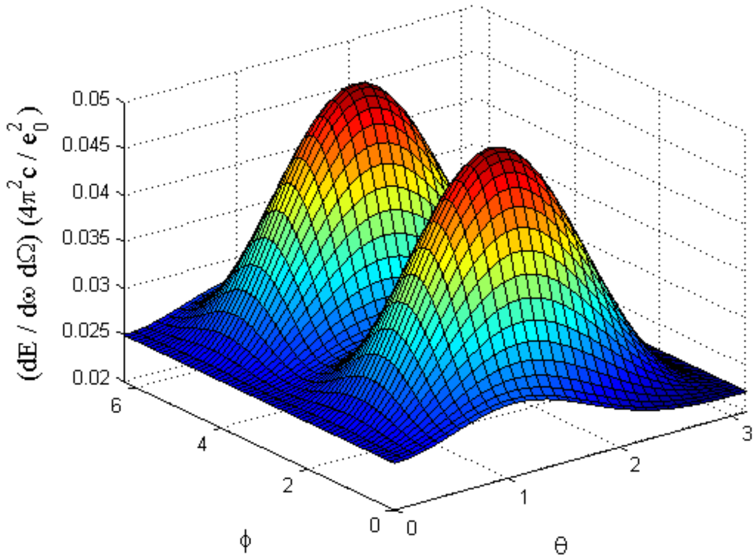
$$I_x = 2e_0 b R^3 v_0^2 \int_{-\infty}^{\infty} \frac{t}{(b^2 + v_0^2 t^2)^{5/2}} \exp\left\{ i \left[\omega t - \frac{k_x R^2 b}{b^2 + v_0^2 t^2} - \frac{k_z R^2 v_0 t}{b^2 + v_0^2 t^2} \right] \right\} dt,$$

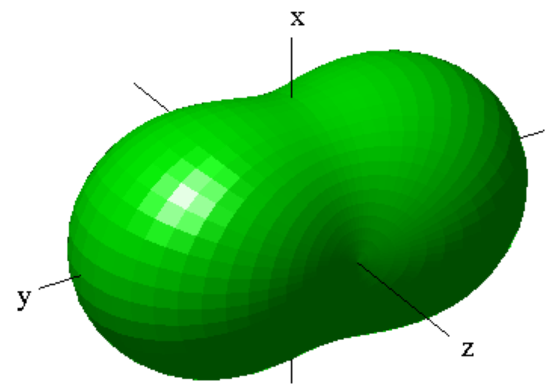
$$I_z = -e_0 R^3 v_0 \int_{-\infty}^{\infty} \frac{b^2 - v_0^2 t^2}{(b^2 + v_0^2 t^2)^{5/2}} \exp\left\{ i \left[\omega t - \frac{k_x R^2 b}{b^2 + v_0^2 t^2} - \frac{k_z R^2 v_0 t}{b^2 + v_0^2 t^2} \right] \right\} dt.$$

The integration can be easily performed numerically, that leads to the spectral-angular density of diffraction radiation in the form

$$\frac{d\mathcal{E}}{d\omega d\Omega} = \frac{e_0^2}{4\pi^2 c} \Phi_{DR}(\theta, \varphi, \omega),$$

where the angular distribution $\Phi_{DR}(\theta, \varphi, \omega)$ looks as follows:



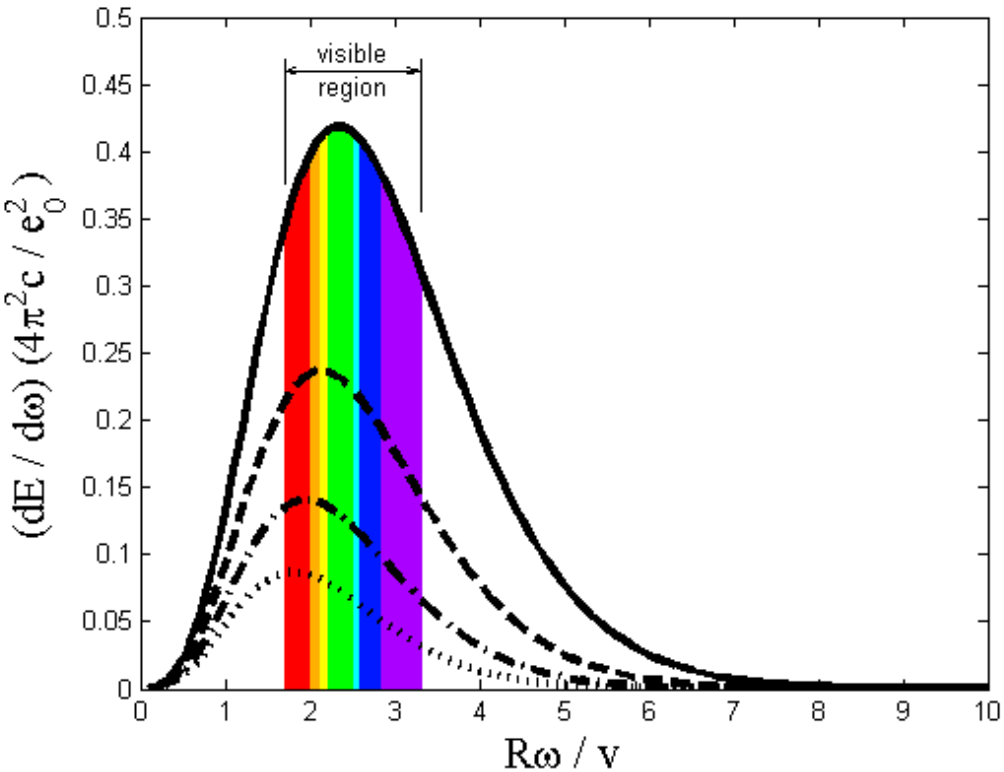


The angular dependence of DR intensity as surface plot (left) and direction diagram (right) for the passage of the real charge under $b = R$ (sliding incidence on the sphere, when DR intensity is maximal for the whole range of wavelengths) and $R\omega/v_0 = 2.34$ (this choice is due to the maximum of DR spectrum (see below) falls on $\omega b/v_0$? 2.34 and $b = R$ in the given case). This shape of the directional diagram is typical; for different values of parameters it do not vary too much.

This formula gives a good analytical approximation for DR spectral-angular density ->

$$\frac{d\mathcal{E}}{d\omega d\Omega} = \frac{4}{9\pi^2}\frac{e_0^2 R^6 \omega^6}{c^3 v_0^4}\left\{\left(1 - \sin^2\theta\cos^2\varphi\right) K_1^2\left(\frac{\omega}{v_0}b\right) + \right.$$
$$\left. + \sin^2\theta\left(\frac{v_0}{2\omega\, b}\right)^2\left[K_1\left(\frac{\omega}{v_0}b\right) + 2\frac{\omega}{v_0}\,b\,K_0\left(\frac{\omega}{v_0}b\right)\right]^2\right\}.$$

Integration over radiation angles leads to the radiation spectrum:



DR spectrum for $R = 20$ nm, $v_0 = 0.1c$ and $b = R$ (solid line), $b = 1.1R$ (dashed line), $b = 1.2R$ (dash-dotted line), $b = 1.3R$ (dotted line).

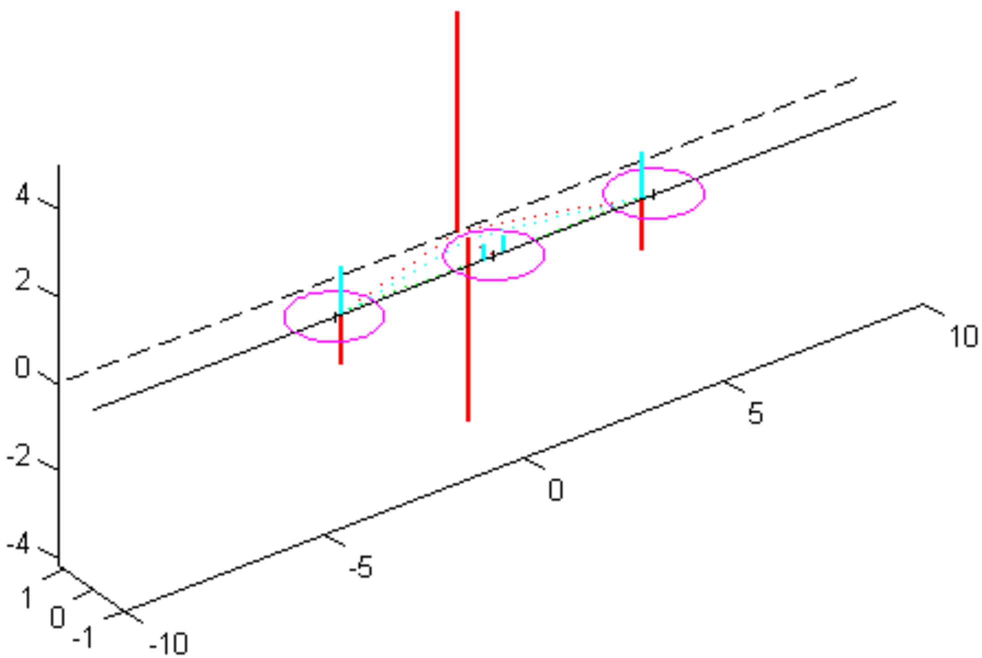
For illustrative purposes, we choose the parameters $v_0 = 0.1c$, R = 20 nm, $b = R+0$, for which the DR intensity maximum will lie in the visible spectrum. Note, however, that in this frequency domain the properties of the sphere material can be far from that of a perfect conductor. Particularly, plasma oscillations can be important.

DR on the string of spheres

Now consider the particle e_0 motion along the periodic string of the spheres. The mutual influence of the fictive charges induced in the neighboring spheres can be neglected for the string period $a > 4R$, see the Figure below; then the interference of the radiation produced on the subsequent spheres leads to the simple formula for the spectral-angular density of DR:

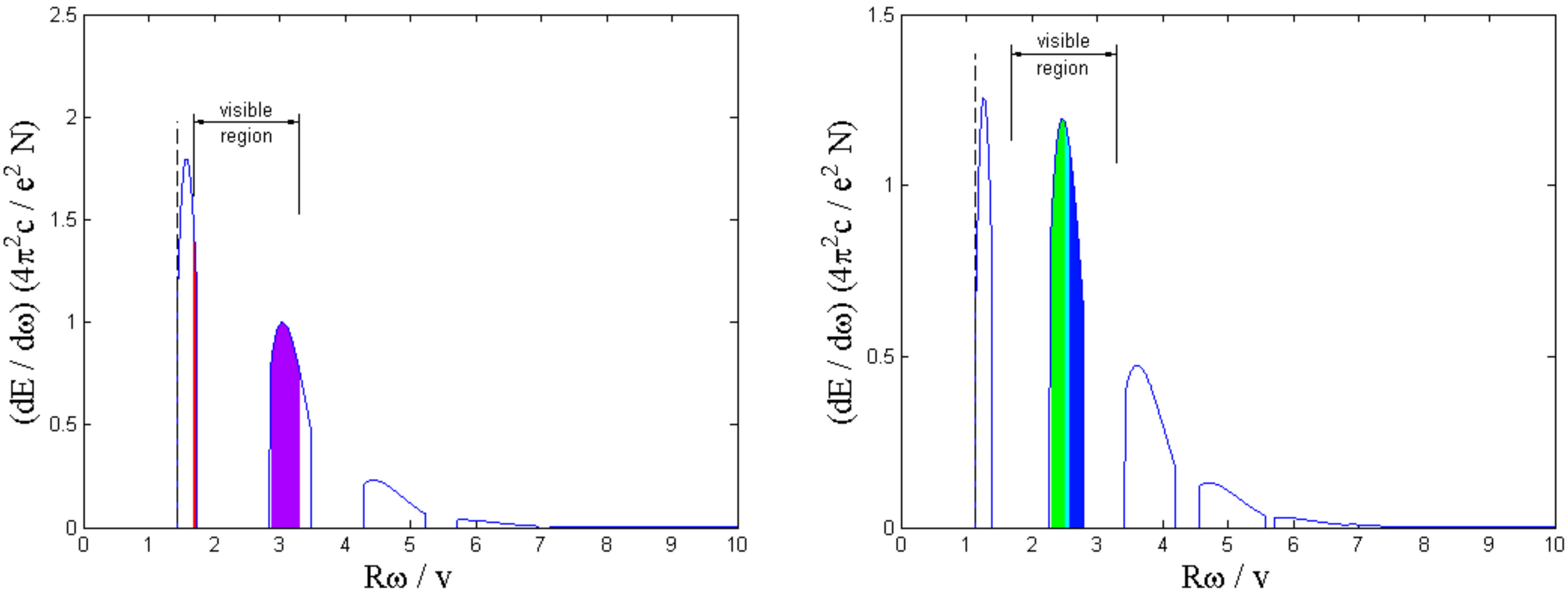
$$\frac{d\mathcal{E}}{d\omega d\Omega} = \frac{e_0^2}{4\pi^2 c}\Phi_{DR}(\theta,\varphi,\omega)\cdot 2\pi N\frac{v_0}{\omega a}\sum_{m=1}^{\infty}\delta\left(1-\frac{v_0}{c}\cos\theta - m\frac{2\pi v_0}{\omega a}\right).$$

Here the delta-function means well-known Smith-Purcell condition.



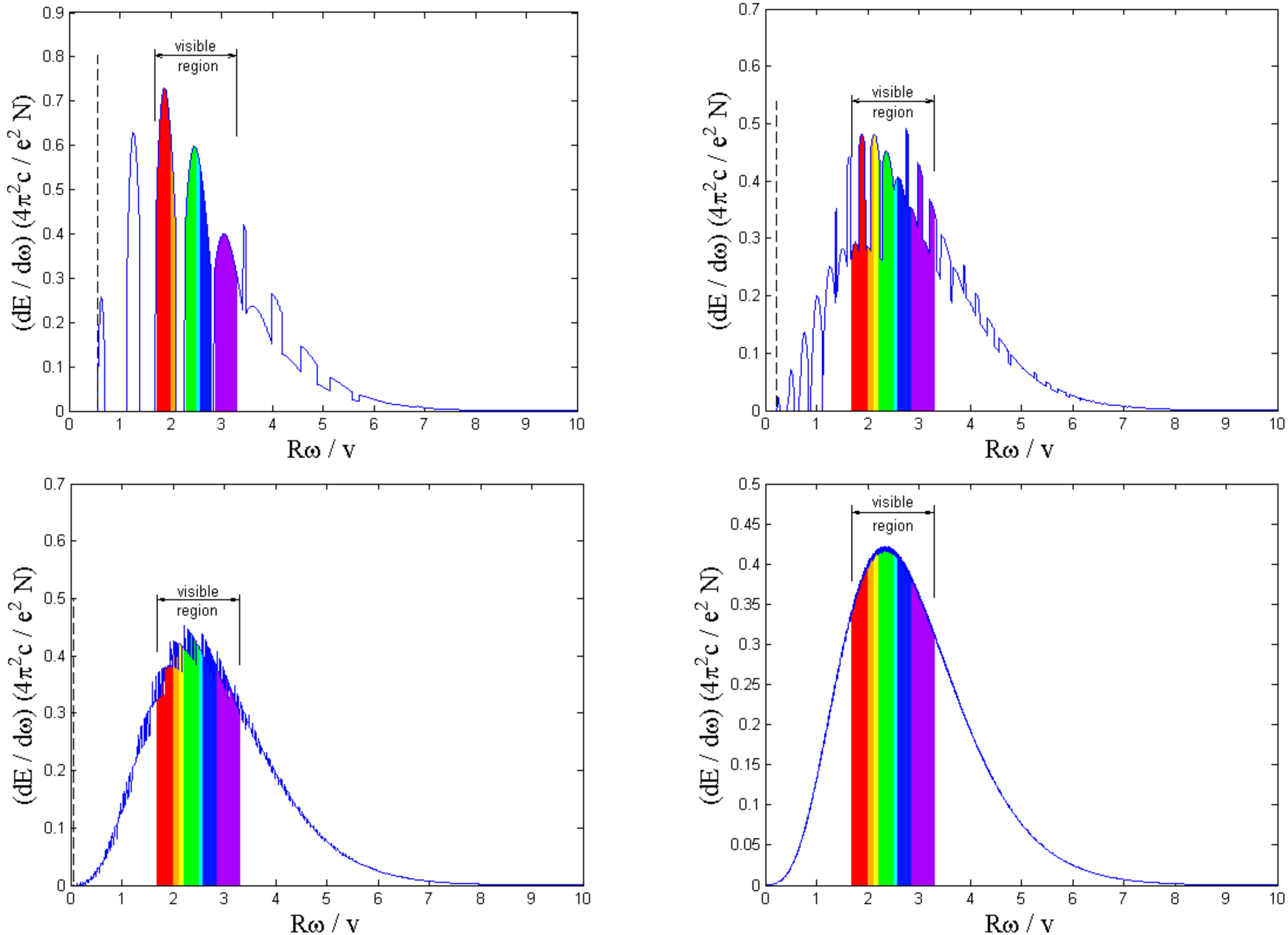
Vertical lines represent relative values of the real charge and the fictive charges induced in the neighbor spheres by the real one (red) as well as by the mutual influence of the fictive charges (blue).

For the radiation frequency ω and the string period a small enough the spectrum consists of separated bands:



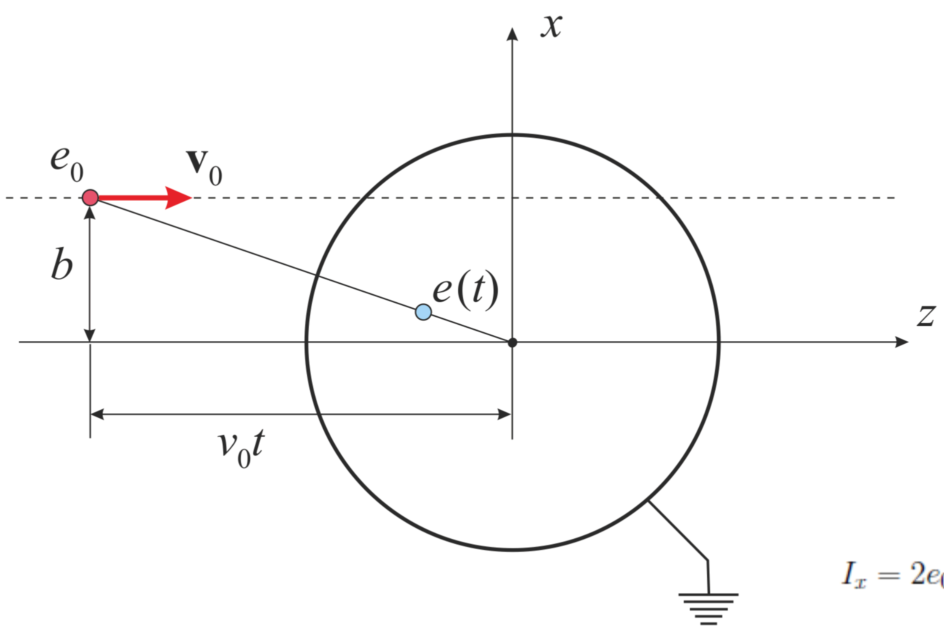
DR spectrum on the string of spheres under $b = R$, $R = 20$ nm, $v_0 = 0.1c$ $a = 80$ nm (left panel) and $a = 100$ nm (right panel). Vertical dashed line marks the long wavelength edge of the spectrum.

The bands overlap each other under increase of the string period a gradually forming the spectrum of DR on a single sphere (multiplied by the total number of the spheres N):



The same as in previous Figure, for a = 200, 500, 2000, 20000 nm

Method of images for TR description



TR arises under $b < R$, when the charge e_0 crosses the sphere.

In this case both real and fictive charges vanish while crossing the sphere (and then appear again) that complicates the formulae describing the radiation:

$$I_x = 2e_0 b R^3 v_0^2 \int_{-\infty}^{-t_0} \frac{t}{(b^2 + v_0^2 t^2)^{5/2}} \exp\left\{ i \left[\omega t - \frac{k_x R^2 b}{b^2 + v_0^2 t^2} - \frac{k_z R^2 v_0 t}{b^2 + v_0^2 t^2} \right] \right\} dt +$$

$$+ 2e_0 b R^3 v_0^2 \int_{t_0}^{\infty} \frac{t}{(b^2 + v_0^2 t^2)^{5/2}} \exp\left\{ i \left[\omega t - \frac{k_x R^2 b}{b^2 + v_0^2 t^2} - \frac{k_z R^2 v_0 t}{b^2 + v_0^2 t^2} \right] \right\} dt,$$

$$I_z = -e_0 R^3 v_0 \int_{-\infty}^{-t_0} \frac{b^2 - v_0^2 t^2}{(b^2 + v_0^2 t^2)^{5/2}} \exp\left\{ i \left[\omega t - \frac{k_x R^2 b}{b^2 + v_0^2 t^2} - \frac{k_z R^2 v_0 t}{b^2 + v_0^2 t^2} \right] \right\} dt -$$

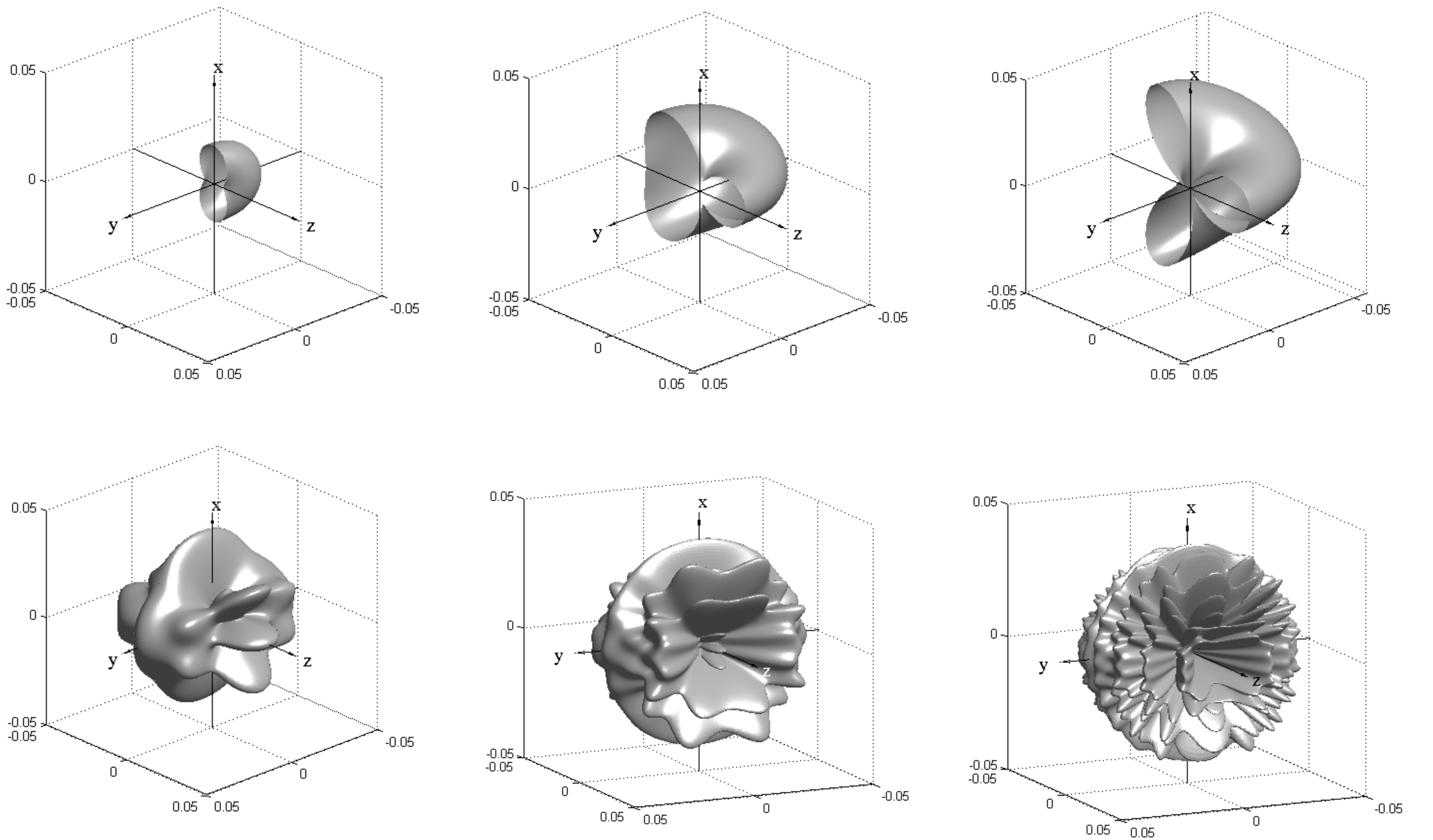
$$- e_0 R^3 v_0 \int_{t_0}^{\infty} \frac{b^2 - v_0^2 t^2}{(b^2 + v_0^2 t^2)^{5/2}} \exp\left\{ i \left[\omega t - \frac{k_x R^2 b}{b^2 + v_0^2 t^2} - \frac{k_z R^2 v_0 t}{b^2 + v_0^2 t^2} \right] \right\} dt -$$

$$- e_0 R \cdot 2 \frac{v_0}{R} (\cos k_x b - i \sin k_x b) \frac{\sin[(\omega - k_z v_0) t_0]}{\omega - k_z v_0}.$$

Numerical integration leads to the spectral-angular density of transition radiation in the form

$$\frac{d\mathcal{E}}{d\omega d\Omega} = \frac{e_0^2}{4\pi^2 c} \Phi_{TR}(\theta, \varphi, \omega),$$

where the function $\Phi_{TR}(\theta, \varphi, \omega)$ is presented for some particular cases as the directional diagram in the following Figures.

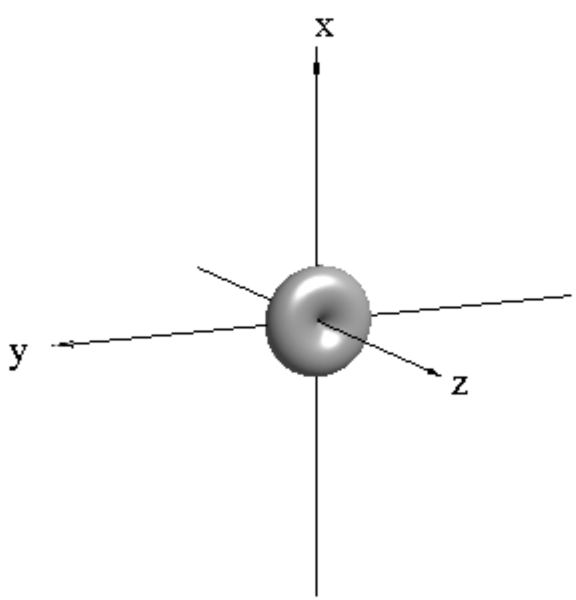


Direction diagrams of the radiation emitted under incidence of the real charge under $b = R/\boxed{?}\,2$ for $R\omega/v_0 = 1, 5, 10, 50, 100, 200$. We see very sophisticate shapes in contrast to DR case. This is due to interference of the radiation emitted by the real charge and its image while crossing two boundaries between vacuum and conductor:

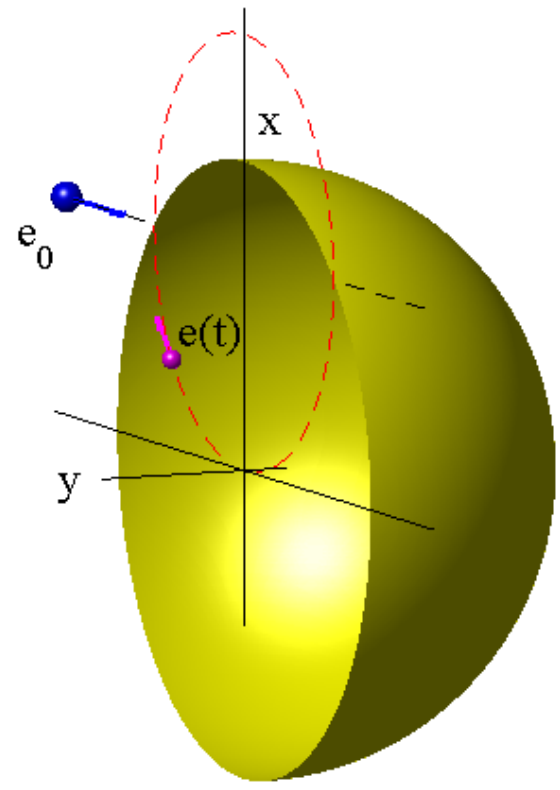
Let us trace out, how that picture forms, in the case of high frequency, when the metal surface curvature becomes negligible.

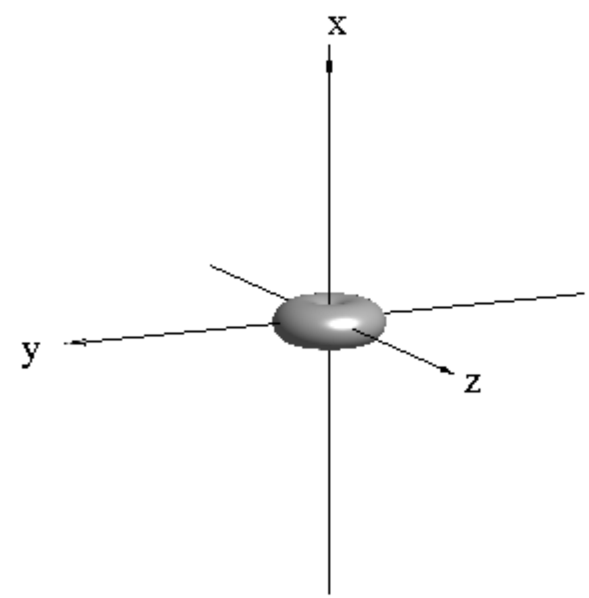
The contributions from both real and fictive charges into the radiation under each crossing of the metal surface have similar shape with axis of symmetry along their velocity.

For the incidence under 45^{o} the velocity of the fictive charge will be directed along the x axis.

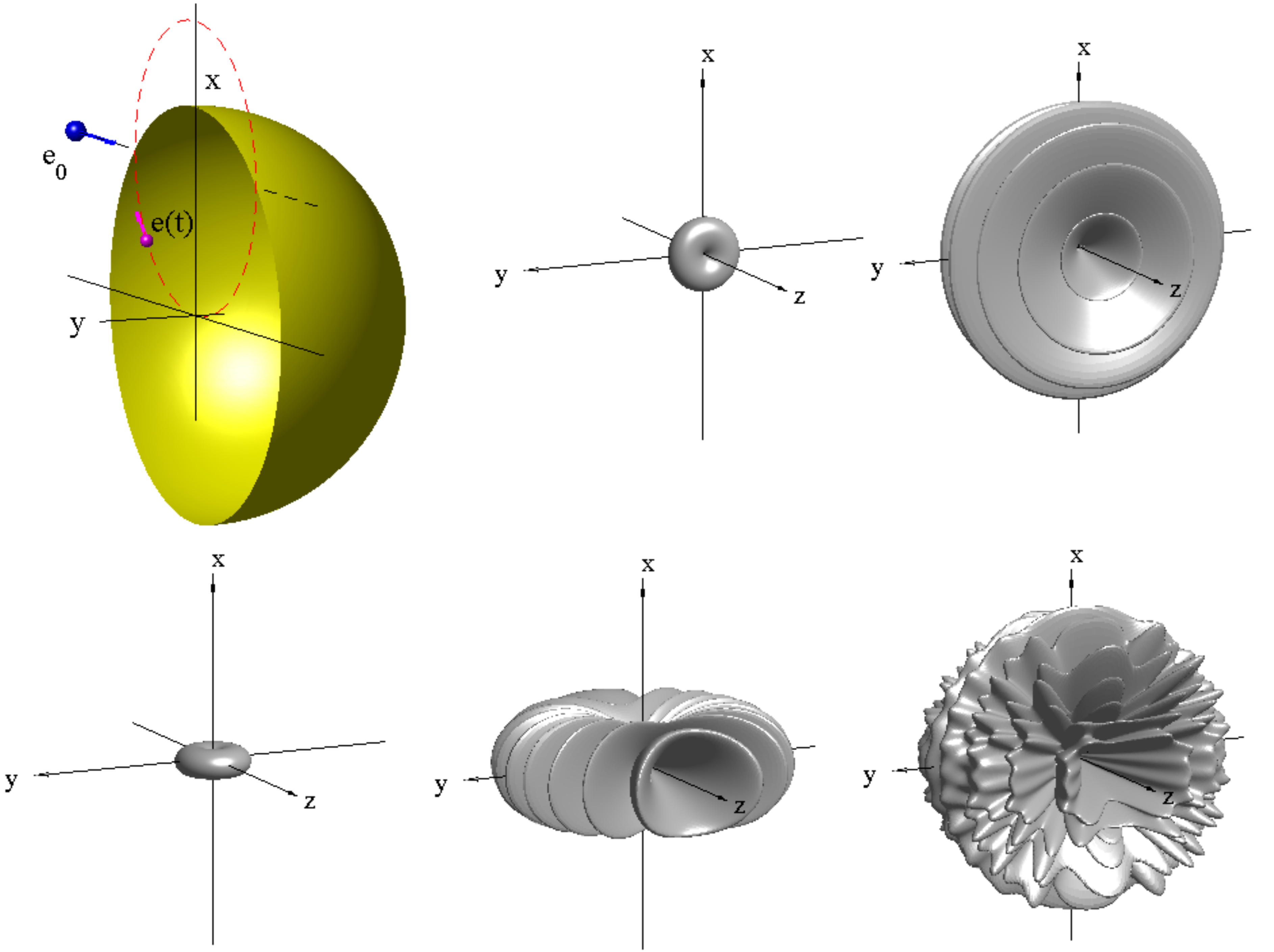


Elementary contribution to the radiation from the real charge.





Elementary contribution to the radiation from the fictive charge.


x
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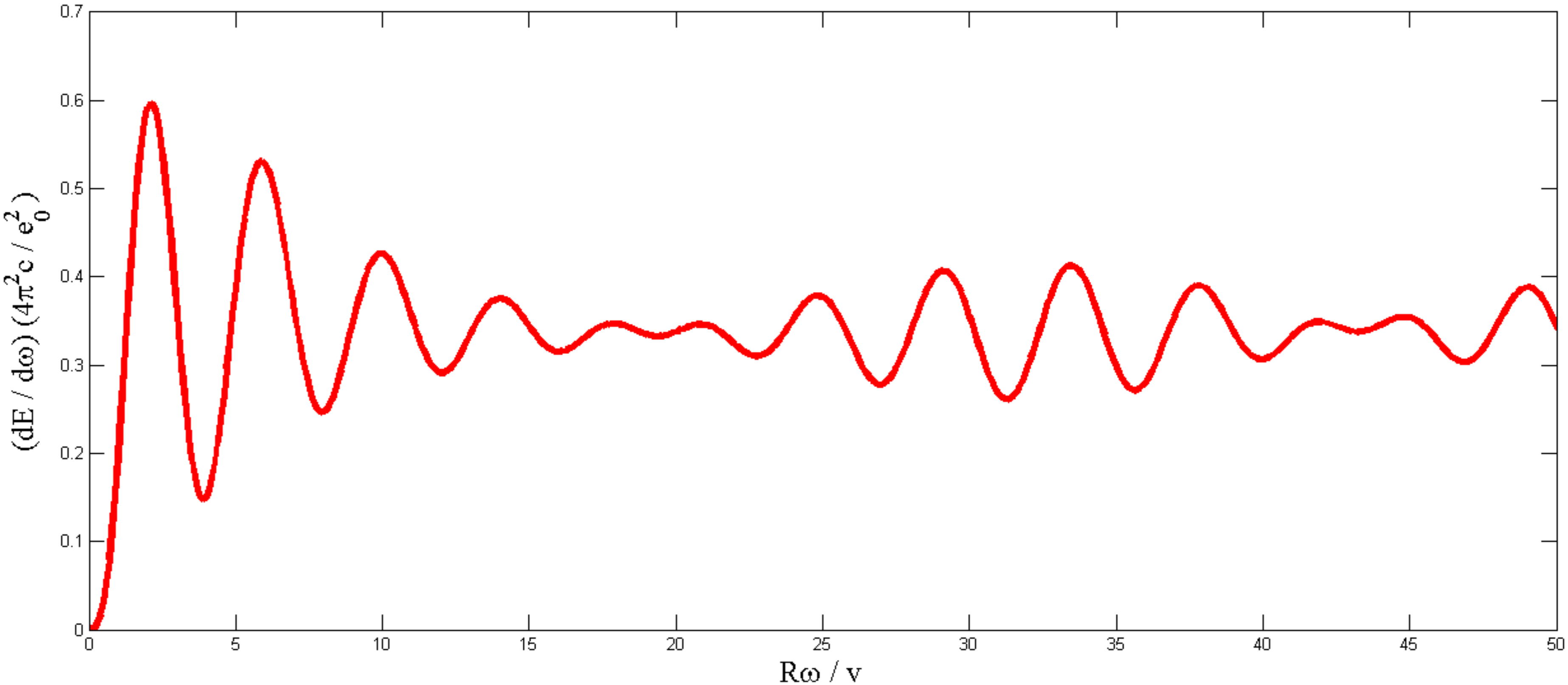
TR spectrum $\dfrac{d\mathcal{E}}{d\omega} = \dfrac{e_0^2}{4\pi^2 c} \displaystyle\int \Phi_{TR}(\theta, \varphi, \omega)\, d\Omega$ for $b = R/?\,2$ looks as follows:



Summary:

- The radiation emitting under interaction of non-relativistic particle with ideally conducting sphere is considered.
- The method of images leads to precise description of the radiation in this case. The integration in the resulting formulae can be easily performed numerically.
- When the incident particle crosses the sphere, the angular distribution of the radiation can be very complex. This is due to the interference of the waves emitted by the particle and its image under crossing the boundaries between conductor and vacuum.