Anomalous dynamical back-action in interferometers

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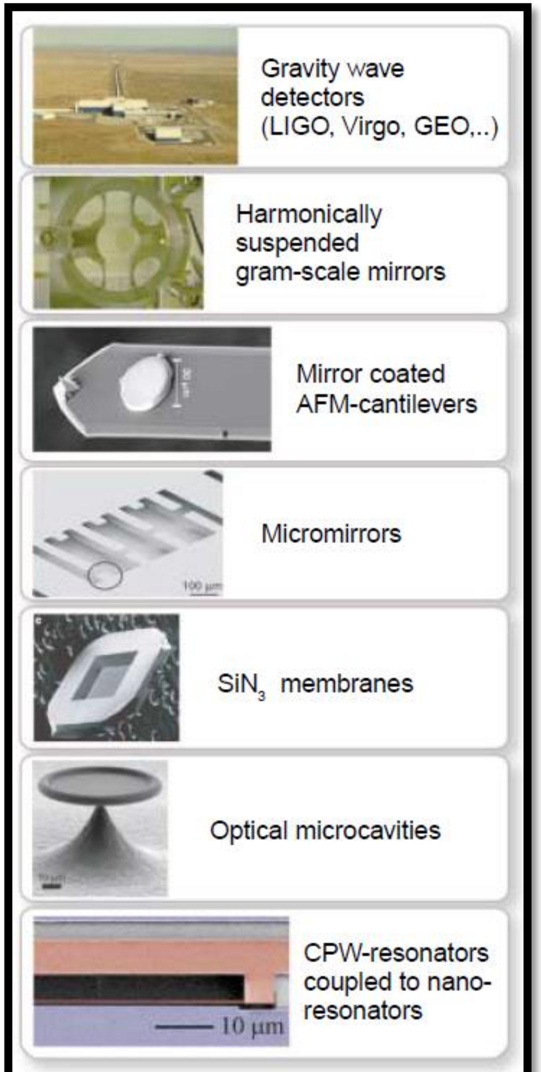
Basic theory behind dynamic back-action



T. J. Kippenberg et al., Science ***321****, 1172 (2008);*

- Light wave in an optical apparatus interacts with a mechanical probe (test mass) via radiation pressure.
- Imposes measurement back-action on the probe.
- Back-action noise due to quantum fluctuations of electromagnetic field. Causes SQL together with measurement shot noise.
- Dynamic back-action due to motion of the probe: test mass moves → redistribution of energy → ponderomotive force alters dynamics of the test mass.
  - *V.B. Braginsky, A.B. Manukin, Sov. Phys. JETP* ***25****, 653 (1967);*
  - *V.B. Braginsky et al., Sov. Phys. JETP* ***31****, 829 (1970).*

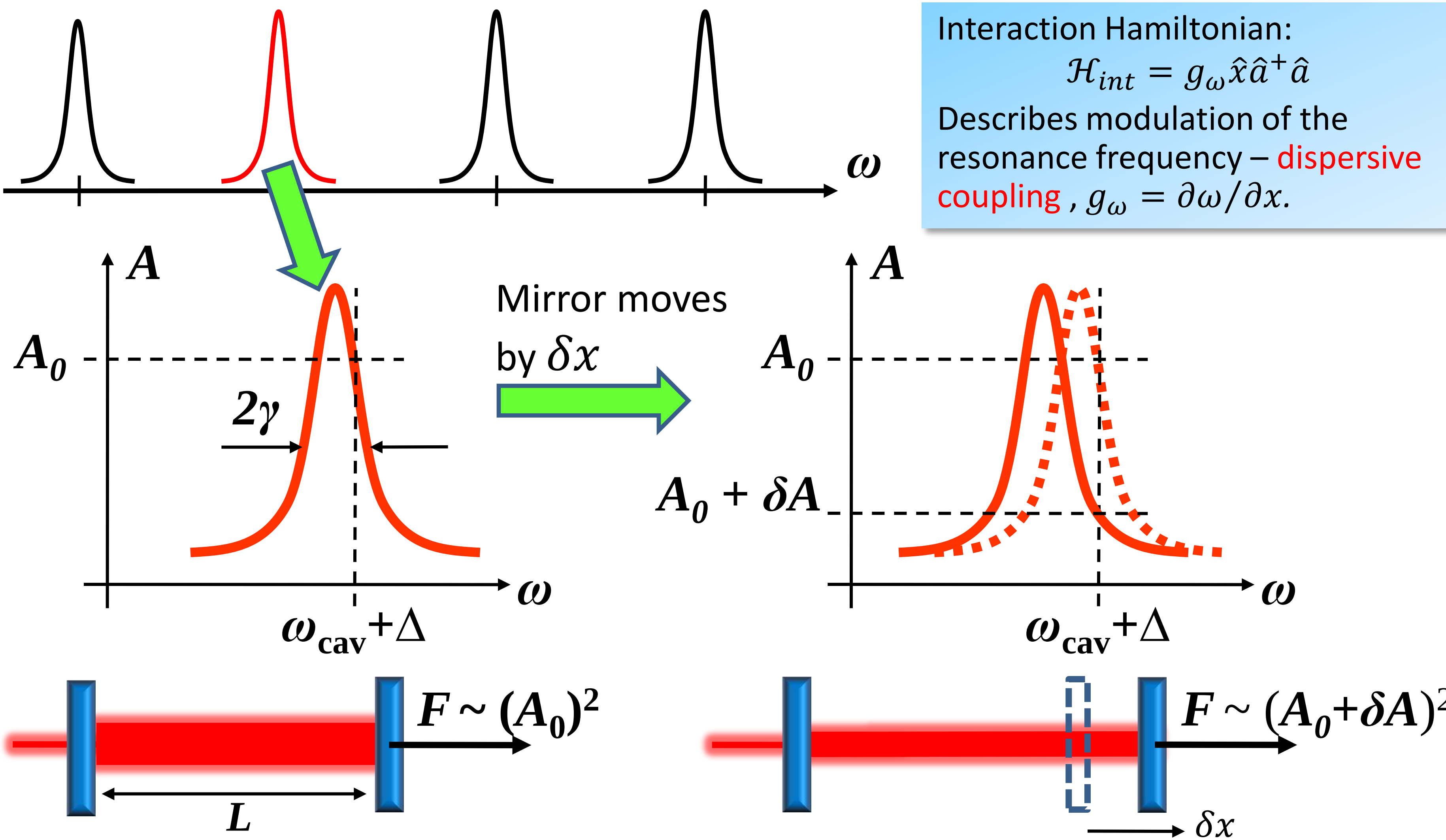
- Effective *Fabry-Perot cavity* in optomechanics with micro-and nano-oscillators (topologies of 'movable mirror', 'membrane-in-the-middle', etc.).
- Scaling law for gravitational-wave detectors: noise and dynamics of interferometers operated on dark port are equivalent to those of a *Fabry-Perot cavity*.
  - *A. Buonanno, Y. Chen, Phys. Rev. D* ***67****, 062002 (2003).*

Basic theory behind dynamic back-action

Interaction Hamiltonian:

$$\mathcal{H}_{int} = g_\omega \hat{x}\hat{a}^+\hat{a}$$

Describes modulation of the resonance frequency – dispersive coupling , $g_\omega = \partial\omega/\partial x$.



Ponderomotive force: $\delta F(\Omega) \sim A_0 \delta A = -\mathcal{K}(\Omega)\delta x = -\delta x[K(\Omega) - 2i\Omega\Gamma(\Omega)]$

Canonical dynamic back-action

- Complex optical spring:

$$\mathcal{K}(\Omega) = \frac{2\omega_0\mathcal{E}}{L^2}\frac{\Delta}{\Delta^2 + (\gamma - i\Omega)^2}$$

- Real optical spring (rigidity):

$$K(\Omega) = \Re[\mathcal{K}(\Omega)] =$$
$$= \frac{2\omega_0\mathcal{E}}{L^2}\frac{\Delta(\Delta^2 + \gamma^2 - \Omega^2)}{|\Delta^2 + (\gamma - i\Omega)^2|^2}$$

  – Crosses zero once, if $\Omega < \gamma$ (GWD)

  – Crosses zero thrice, if $\Omega > \gamma$ (OM)

- Optical damping:

$$\Gamma(\Omega) = -\Im[\mathcal{K}(\Omega)]/2\Omega$$
$$= -\frac{2\omega_0\mathcal{E}}{L^2}\frac{\Delta\gamma}{|\Delta^2 + (\gamma - i\Omega)^2|^2}$$

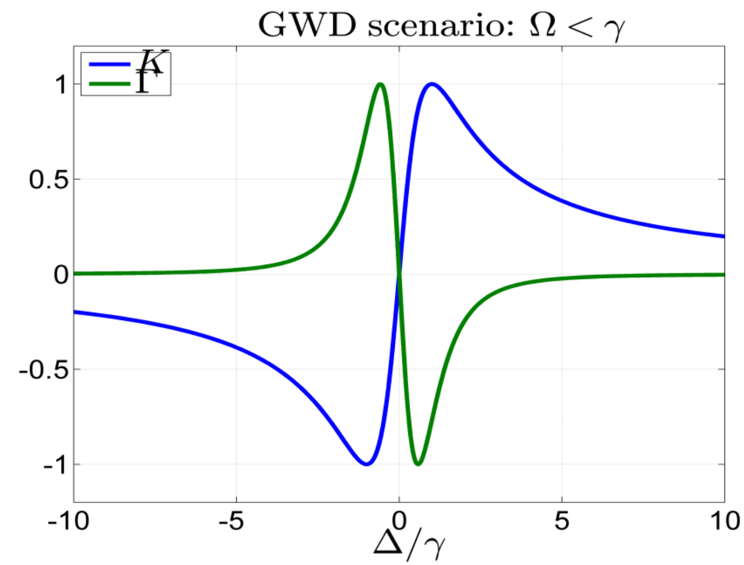
  – Crosses zero only once

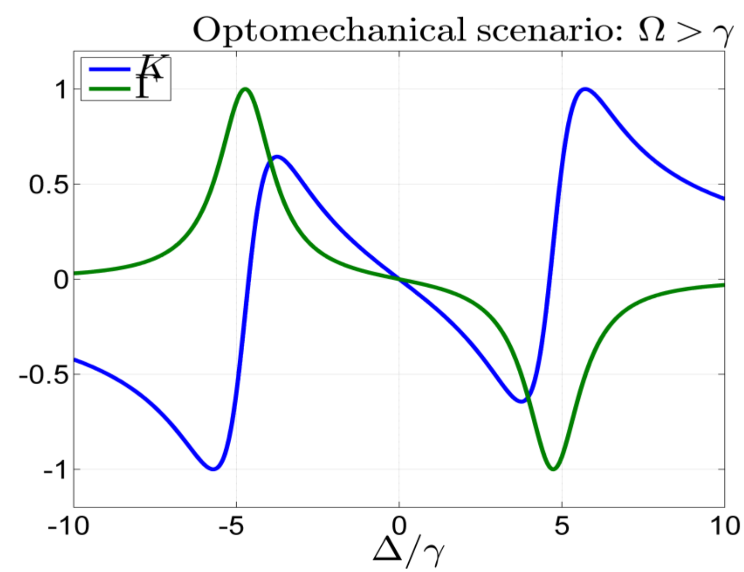
Both vanish on resonance, $\Delta = 0$.

V.B. Braginsky et al., Phys. Lett. A ***232****, 340 (1997);*

F.Ya. Khalili, Phys. Lett. A ***288****, 251 (2001);*

A. Buonanno, Y. Chen, Phys. Rev. D ***65****, 042001 (2002).*

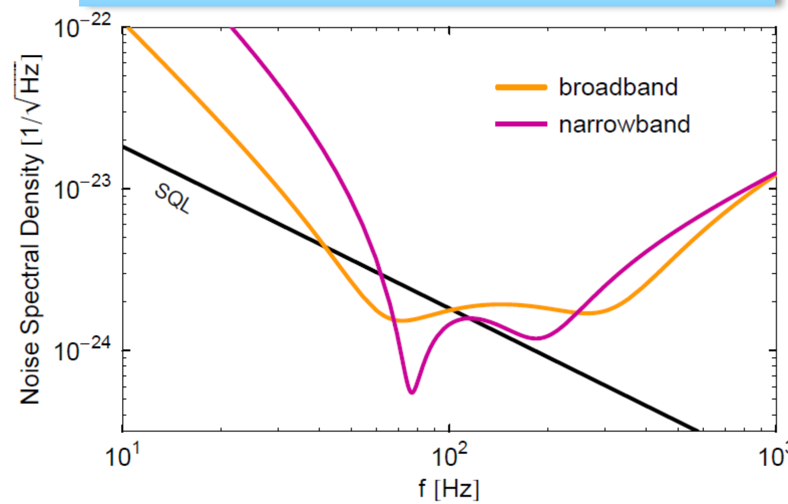


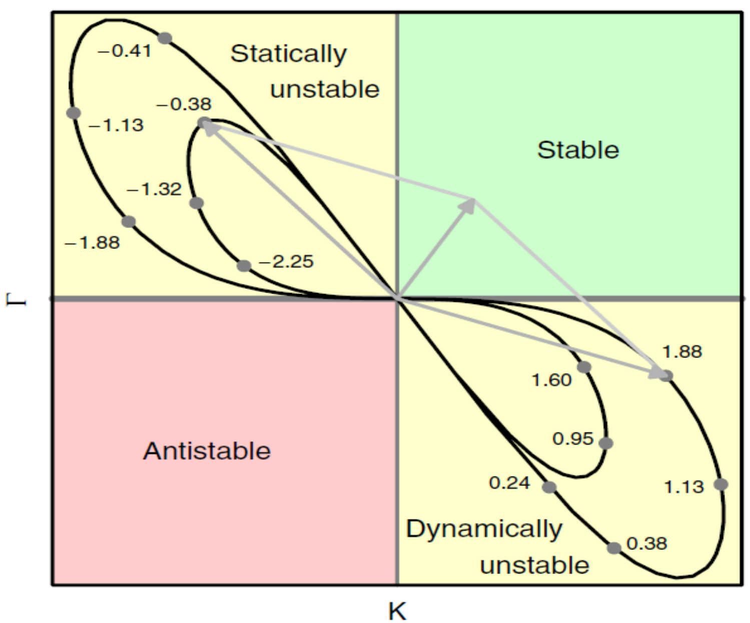


Canonical dynamic back-action in GWDs

- Optical spring K allows reduction of quantum noise in certain freq. band via modifying dynamics of the test masses.
  Free mass: $\mathcal{S}_F^{SQL} = 2\hbar m\Omega^2$,
  Oscillator: $\mathcal{S}_F^{SQL} = 2\hbar m(\Omega^2 - \omega_m^2)$.
- Optical spring makes signal amplification rather than back-action noise cancelation – much more tolerant to optical losses.

H. Mueller-Ebhardt, PhD thesis, 2009.





- A single-carrier optical spring, as a set of K and Γ, is unstable for any detuning. Feedback/control needed for stabilization.
- Two laser drives can create stable spring.
  - *H. Rehbein et al., PRD **78**, 062003 (2008);*
  - *T. Corbitt et al., PRL **98**, 150802 (2007).*
- Broadband reduction of quantum noise via dual-carrier negative optical inertia.
  - *F. Khalili et al., PRD **83**, 062003 (2011).*

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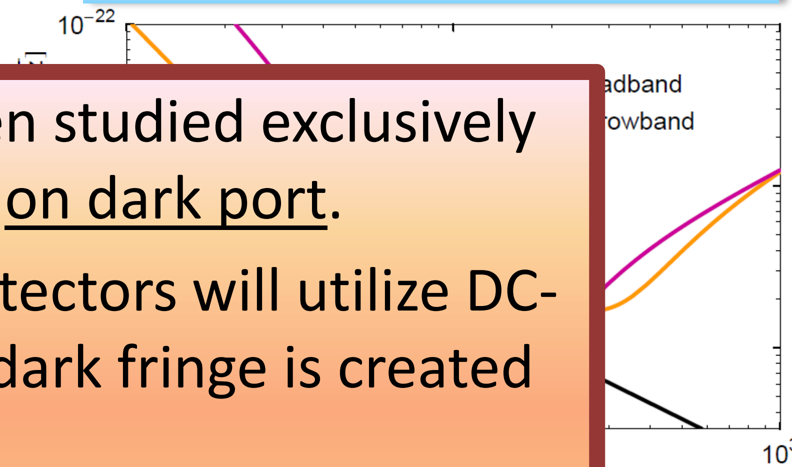
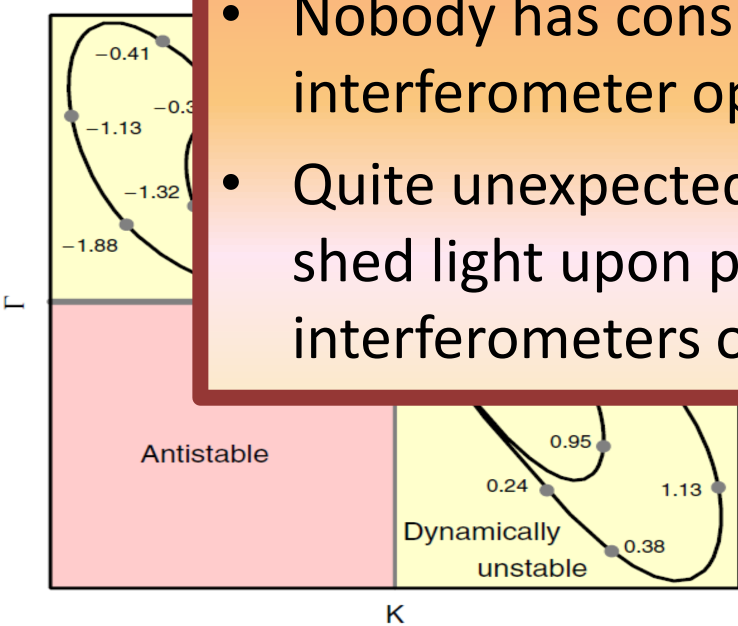


- So far, optical spring has been studied exclusively in interferometers operated <u>on dark port</u>.
- Although already 2^{nd}-gen detectors will utilize DC-readout, where offset from dark fringe is created on purpose.
- Nobody has considered if new effects rise in an interferometer operated off dark fringe.
- Quite unexpectedly, a micromechanical model shed light upon physics in large-scale interferometers operated <u>off dark port</u>.



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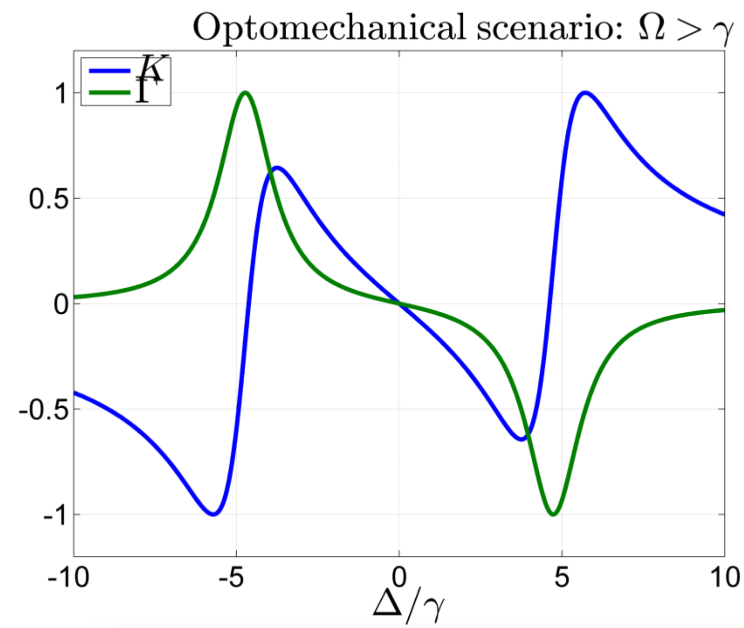
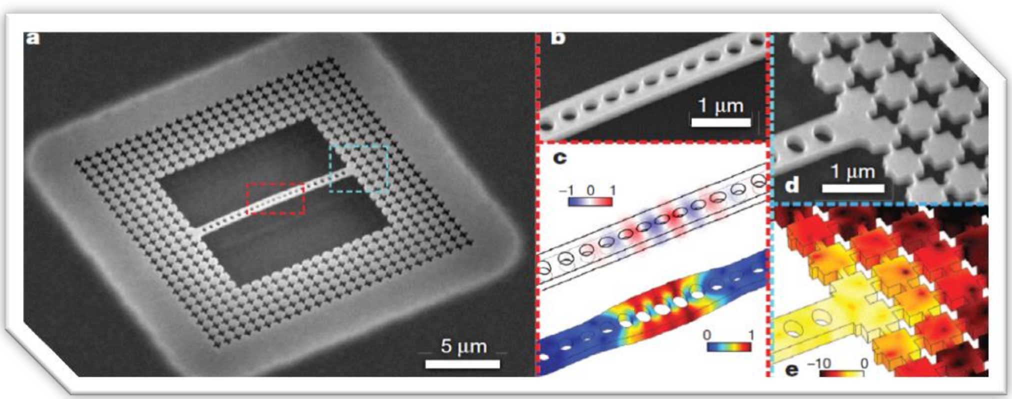
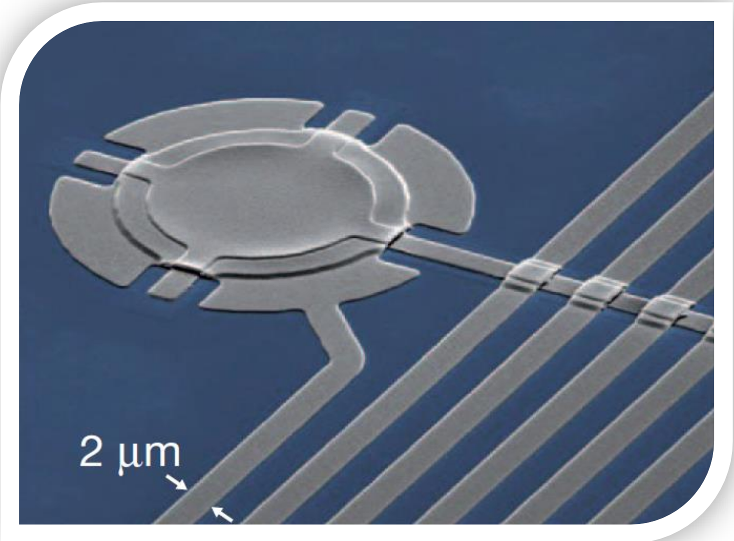
Canonical dynamic back-action in OM

Optomechanics with micro-/nano-oscillators:

- Shift of mechanical frequency caused by optical spring K is mostly negligible compared to intrinsic resonance freq. ω_m.
- Cooling by optical damping Γ, preferably at deeply resolved sideband, $\Omega = \omega_m \gg \gamma$.
  - *I. Wilson-Rae et al., PRL **99,** 093901 (2007);*
  - *F. Marquardt et al., PRL **99,** 093902 (2007).*
- Experiments on resolved-sideband ground-state cooling of micro-oscillators:
  - *J.D. Teufel et al., Nature **475**, 359 (2011);*
  - *J. Chan et al., Nature **478**, 89 (2011);*





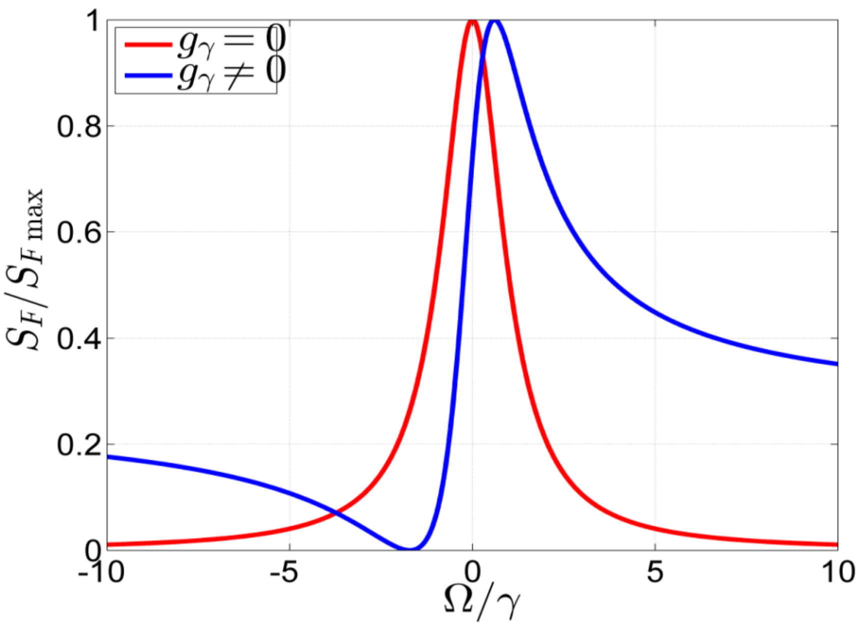


Dissipative coupling in optomechanics

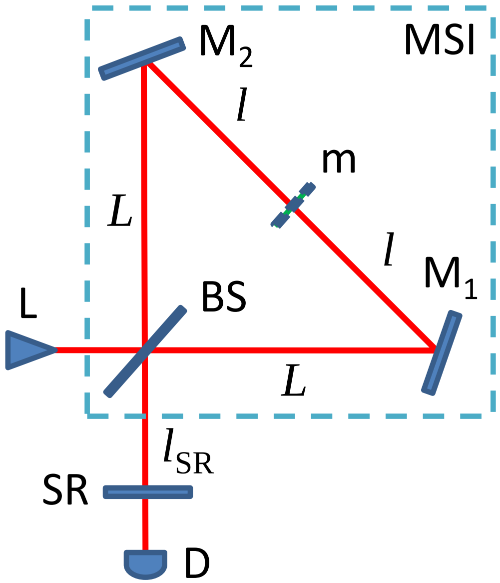
*F. Elste, S. Girvin, A. Clerk, PRL **102**, 207209, 2009*

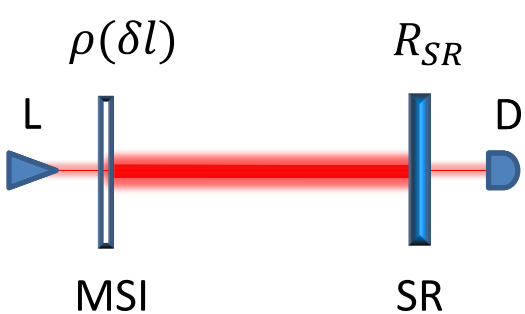
- Interaction Hamiltonian:
  $$\mathcal{H}_{int} = g_\omega \hat{x}\hat{a}^+\hat{a} + \frac{g_\gamma}{\sqrt{2\gamma}}\hat{x}\int\frac{d\omega}{2\pi}\left[\hat{a}^+(\omega)\hat{a} - \mathrm{h.c.}\right].$$
  Modulation of the resonance frequency – *dispersive coupling* ($g_\omega = \partial\omega/\partial x$), and of linewidth – *dissipative coupling* ($g_\gamma = \partial\gamma/\partial x$).
- Unsymmetrized back-action noise spectral density:
  $$S_F(\Omega) \sim g_\gamma \frac{\left[\Omega + 2\Delta - A\left(g_\omega/g_\gamma\right)\gamma\right]^2}{\gamma^2 + (\Delta+\Omega)^2}$$
  Fano resonance! Interference of white (input) noise and Lorentz-filtered (intracavity) noise.

- Optical damping:
  $\Gamma = x_{\mathrm{ZPF}}^2\left[S_F(\omega_m) - S_F(-\omega_m)\right]/\hbar^2.$
- Absorption rate $S_F(-\omega_m)$ vanishes at $\Delta = \omega_m/2 + A\left(g_\omega/g_\gamma\right)\gamma/2.$
- Ground-state cooling is possible for arbitrary ratio ω_m/γ. For pure dispersive coupling only if $\omega_m \gg \gamma$.



Michelson-Sagnac interferometer





Michelson-Sagnac interferometer as an effective mirror.

- Reflectance:

$$\rho = R_m\left(T_{BS}^2 e^{ik\delta l} + R_{BS}^2 e^{-ik\delta l}\right) + 2iT_m R_{BS} T_{BS},$$

- Transmittance:

$$\tau = -iR_m R_{BS} T_{BS}\left(e^{ik\delta l} - e^{-ik\delta l}\right) + T_m\left(T_{BS}^2 - R_{BS}^2\right).$$

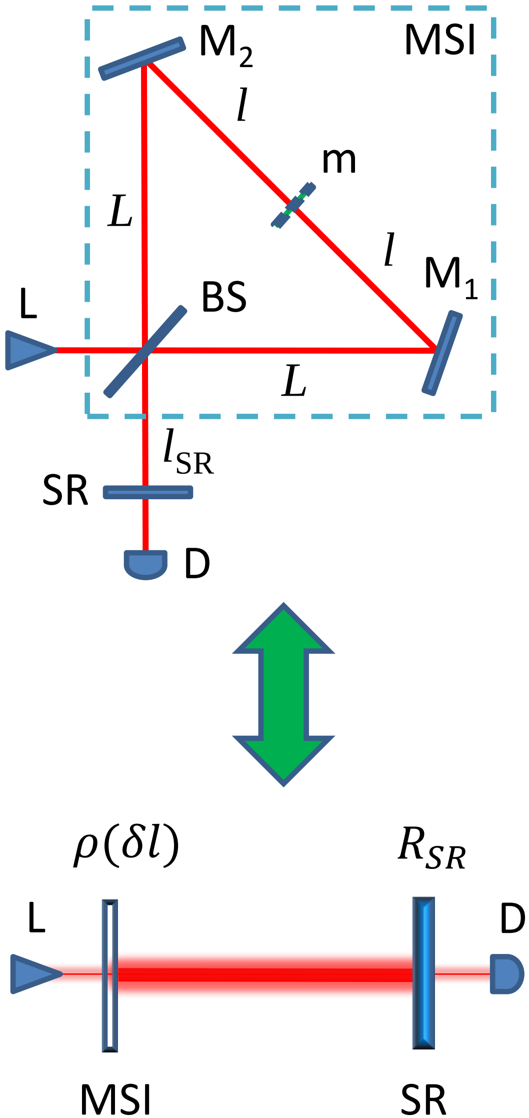
Reflectance/transmittance of the MSI depends on the position of the membrane $x = \delta l/2$.

- *K. Yamamoto et al., Phys. Rev. A **81**, 033849 (2010);*
- *D. Friedrich et al., New J. Phys. **13**, 093017 (2011).*

*A. Xuereb, R. Schnabel, K. Hammerer, PRL **107**, 213604 (2011):*

In the effective cavity approach, signal-recycled MSI features strong and tunable dispersive and dissipative couplings. Coupling strengths g_ω and g_γ can be varied independently via tuning of beamsplitter imbalance $\left|R_{BS}^2 - T_{BS}^2\right|$, and membrane position $x = \delta l/2$.

Michelson-Sagnac interferometer



Signal-recycled MSI as an effective cavity.

- Cavity resonance factor ($\mathcal{L} = L + l + l_{SR}$):
$$\frac{1}{1 - R_{SR}\rho e^{2ik\mathcal{L}}} = \frac{1}{1 - R_{SR}|\rho| e^{2i\delta\mathcal{L}/c + i\arg\rho}}.$$
- Detuning of carrier from cavity resonance:
$$\Delta = \delta + \frac{\arg[\rho]^{(\text{off DP})}}{2\mathcal{L}/c},$$
dispersive coupling via $\arg[\rho(\delta l)]$.
- Cavity half-linewidth:
$$\gamma = \frac{1 - R_{SR}|\rho|}{2\mathcal{L}/c} \approx \frac{cT_{SR}^2}{4\mathcal{L}} + \frac{c\tau^2}{4\mathcal{L}},$$
<u>dissipative coupling</u> via $\tau = \tau(\delta l)$.
- Using transfer matrix approach in freq. domain, calculate fields on the membrane as linear functions of input fields → radiation pressure force → ponderomotive force $\delta F(\Omega) = -\mathcal{K}(\Omega)x(\Omega)$.

Anomalous dynamic back-action in MSI

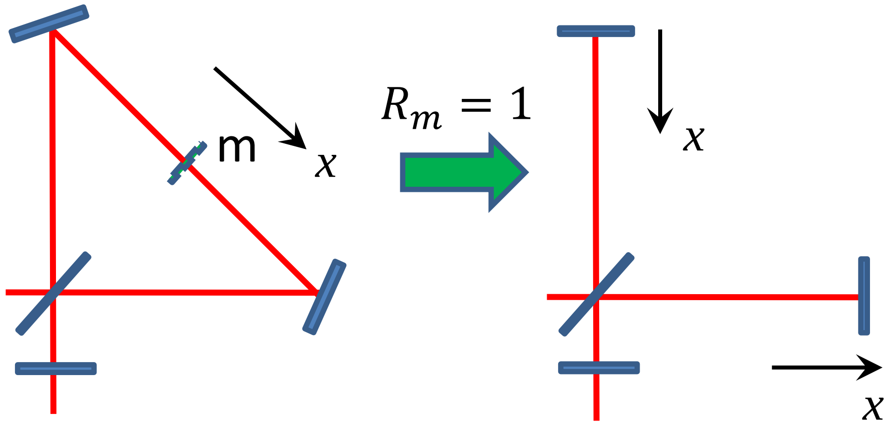
Notations:

- Offset from dark fringe: $\xi = \delta l - \delta l_{DP}, \quad \delta l_{DP} = n\lambda_0/2,$
- Linewidth due to SRM transmittance: $\gamma_{SR} = cT_{SR}^2/4\mathcal{L},$
- Linewidth due to offset from dark port: $\gamma_m = cR_m^2(k_0\xi)^2/4\mathcal{L},$
- Total half-linewidth: $\gamma = \gamma_{SR} + \gamma_m,$
- Detuning due to SRM position at dark port: $\delta_{SR} = \omega_0 - \omega_c,$
- Detuning due to offset from dark port: $\delta_m = \pm cR_mT_m(k_0\xi)^2/4\mathcal{L},$
- Total detuning: $\Delta = \delta_{SR} + \delta_m.$

- Complex optical spring (single mode, narrow band, small dark-fringe offset):

$$\mathcal{K}(\Omega) = \frac{4\omega_0 R_m^2 P_{in}}{c\mathcal{L}} \frac{1}{\gamma^2 + \Delta^2} \times \frac{\delta_{SR}[\gamma^2 + \Delta^2 - 4(\gamma\gamma_m + \Delta\delta_m)] + 2i(\gamma_{SR}\delta_m + \gamma_m\delta_{SR})\Omega + \delta_m\Omega^2}{\Delta^2 + (\gamma - i\Omega)^2}.$$

Michelson-Sagnac → Michelson



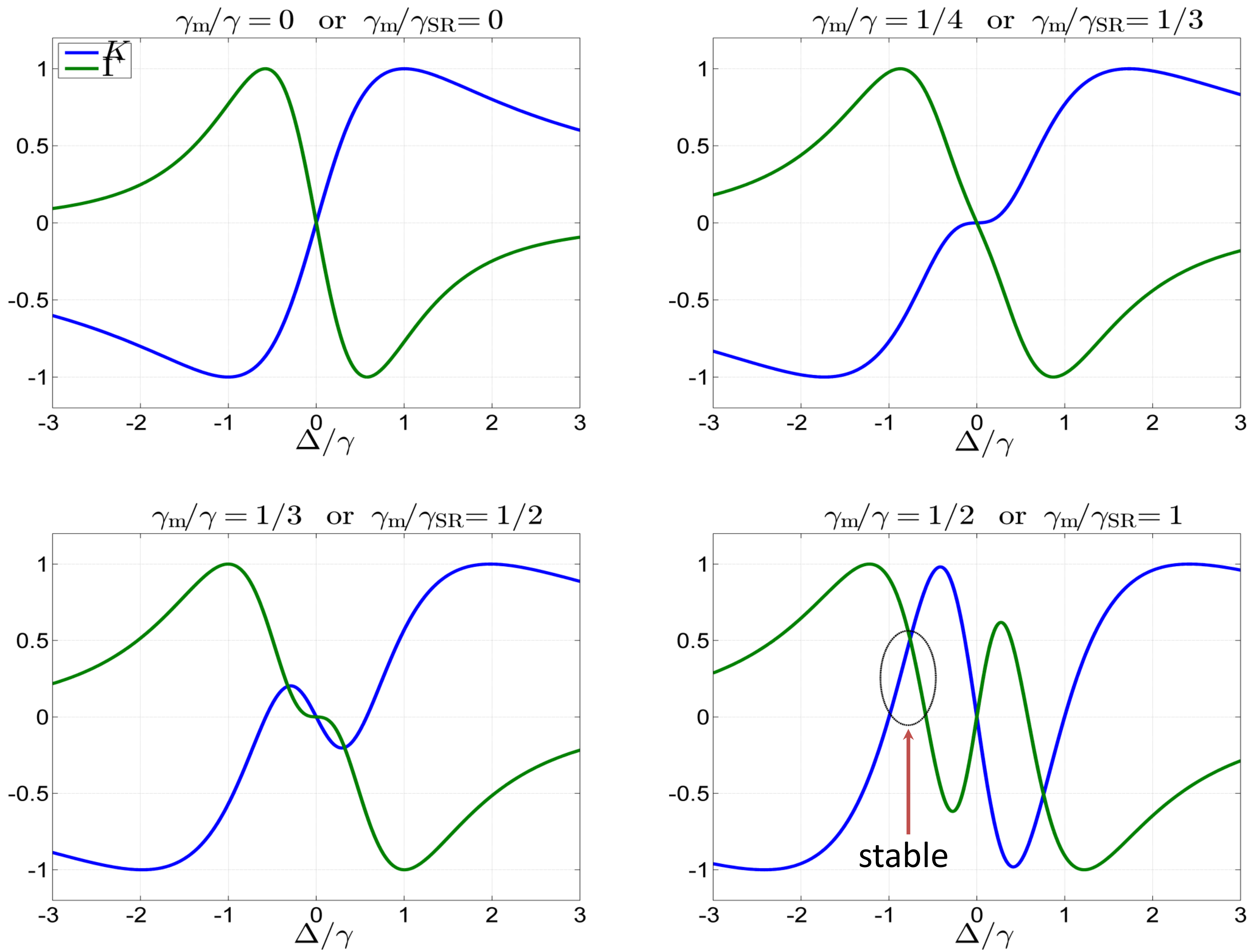
- MSI with the 100% reflecting membrane is equivalent to a Michelson interferometer.
- Motion of the membrane corresponds to the differential motion of the end-mirrors.

- Complex optical spring ($T_m = 0$, hence $\delta_m \sim T_m = 0$ and $\Delta \equiv \delta_{SR}$):
$$\mathcal{K}(\Omega) = \frac{4\omega_0 P_{in}}{c\mathcal{L}} \frac{1}{\gamma^2 + \Delta^2} \frac{\Delta(\gamma^2 + \Delta^2 - 4\gamma\gamma_m) + 2i\gamma_m \Delta\Omega}{\Delta^2 + (\gamma - i\Omega)^2}$$
- Optical spring, $\Omega \to 0$: $K = \Re[\mathcal{K}] = \frac{4\omega_0 P_{in}}{c\mathcal{L}} \frac{\Delta}{\gamma^2+\Delta^2} \left[1 - \frac{4\gamma\gamma_m}{\gamma^2+\Delta^2}\right]$

  crosses zero thrice, if $\gamma_m > \gamma/4$, or equivalently, $\gamma_m > \gamma_{SR}/3$.
- Optical damping, $\Omega \to 0$: $\Gamma = -\Im[\mathcal{K}]/2\Omega = -\frac{4\omega_0 P_{in}}{c\mathcal{L}} \frac{\gamma\Delta}{(\gamma^2+\Delta^2)^2} \left[1 - \frac{\gamma_m}{\gamma} \frac{3\gamma^2 - \Delta^2}{\gamma^2+\Delta^2}\right]$

  crosses zero thrice, if $\gamma_m > \gamma/3$, or equivalently, $\gamma_m > \gamma_{SR}/2$.

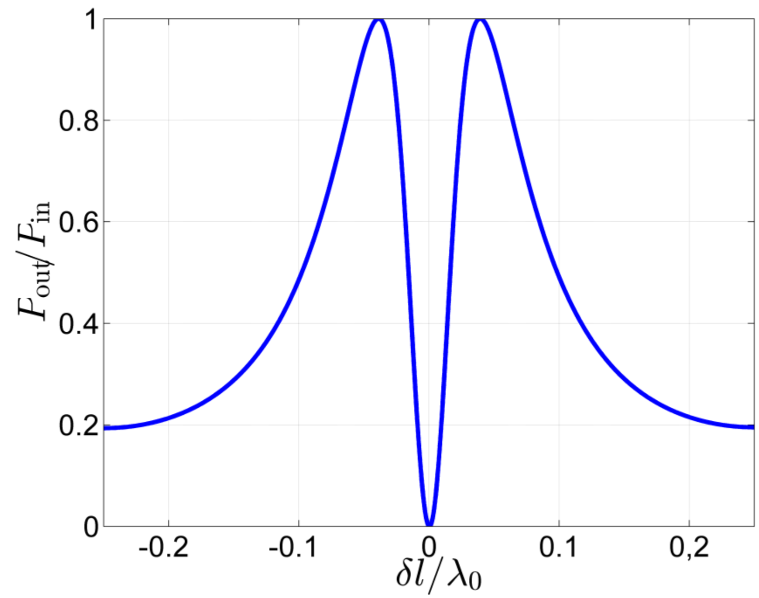
Both vanish at $\Delta = 0$. Intersections of positive/negative regions.



$\gamma_m/\gamma = 0$ or $\gamma_m/\gamma_{SR} = 0$
K
Γ
$\gamma_m/\gamma = 1/4$ or $\gamma_m/\gamma_{SR} = 1/3$
$\gamma_m/\gamma = 1/3$ or $\gamma_m/\gamma_{SR} = 1/2$
$\gamma_m/\gamma = 1/2$ or $\gamma_m/\gamma_{SR} = 1$
stable
Δ/γ

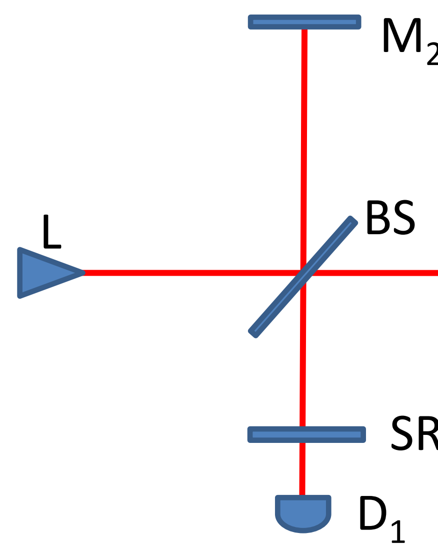
Anomalous dynamic back-action in GWDs

- Problem: regimes with $\gamma_m \sim \gamma_{SR}$, or equivalently, $\delta l \sim \lambda_0/\mathcal{F}$ (large offset from dark fringe) correspond to large values of transmitted power!
- DC readout operates at much lower offsets.



For anomalous optical spring to be manifest, some changes in the topology will be required. Possible solutions:

- Large dark fringe offset → common mode leaks into detector port, differential mode leaks into laser port. Perform detection in laser port.
- Use intracavity topologies – optical bars/levers, local readout.
  - *V.B. Braginsky et al., PLA **232**, 340 (1997);*
  - *F.Ya. Khalili, PLA **298**, 308 (2002);*
  - *H. Rehbein et al., PRD **76**, 062002 (2007).*



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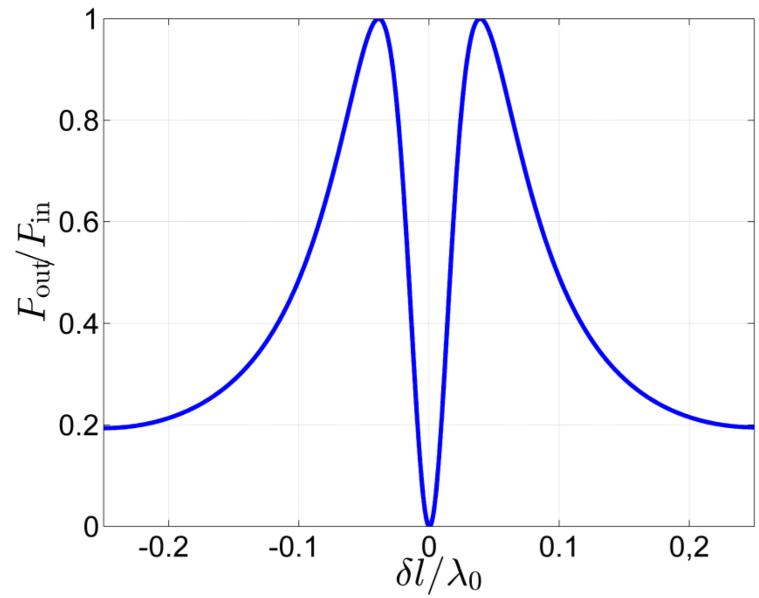

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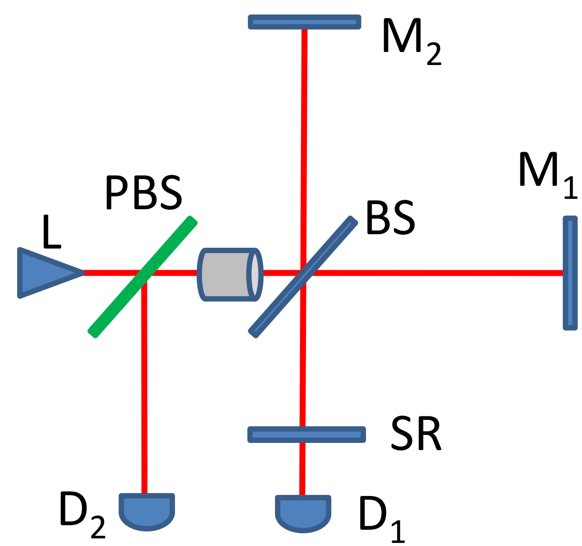

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- Detuning due to SRM position at dark port: $\delta_{SR} = \omega_0 - \omega_c,$
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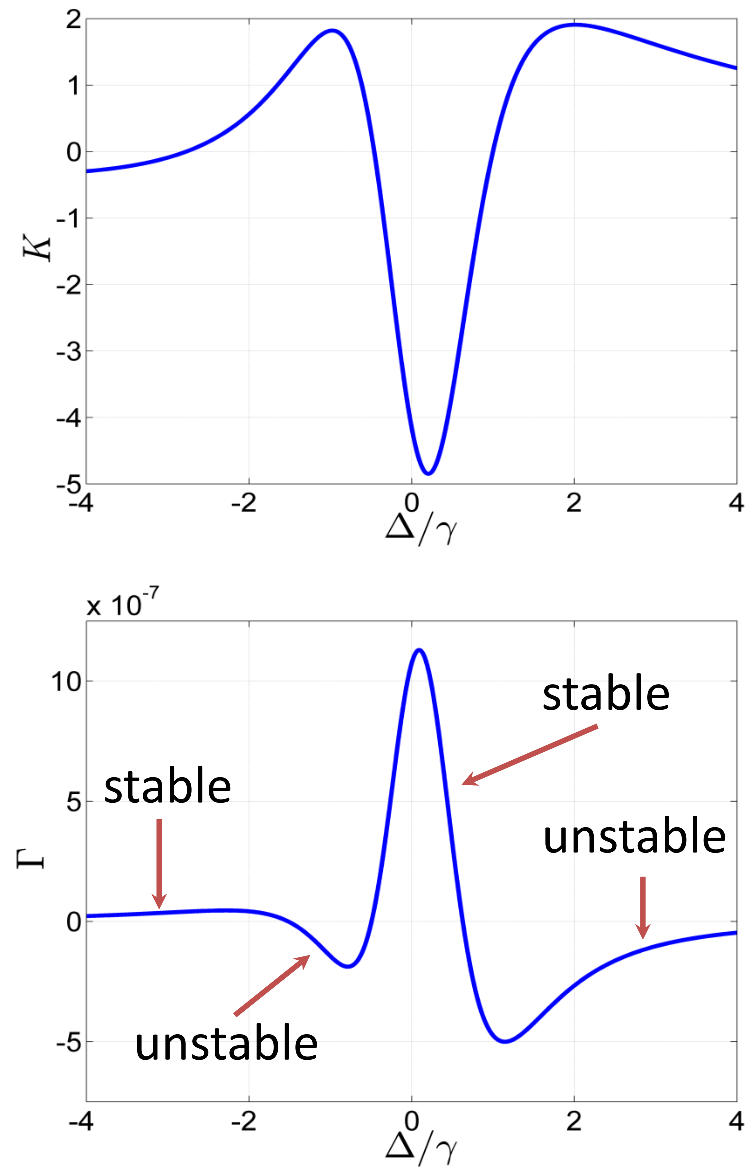
- Complex optical spring (single mode, narrow band, small dark-fringe offset):

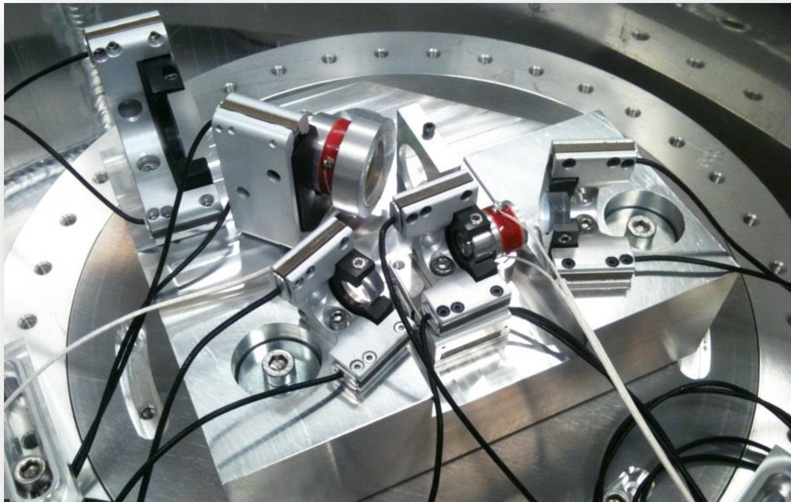
$$\mathcal{K}(\Omega) = \frac{4\omega_0 R_m^2 P_{in}}{c\mathcal{L}} \frac{1}{\gamma^2 + \Delta^2} \times \frac{\delta_{SR}[\gamma^2 + \Delta^2 - 4(\gamma\gamma_m + \Delta\delta_m)] + 2i(\gamma_{SR}\delta_m + \gamma_m\delta_{SR})\Omega + \delta_m\Omega^2}{\Delta^2 + (\gamma - i\Omega)^2}.$$

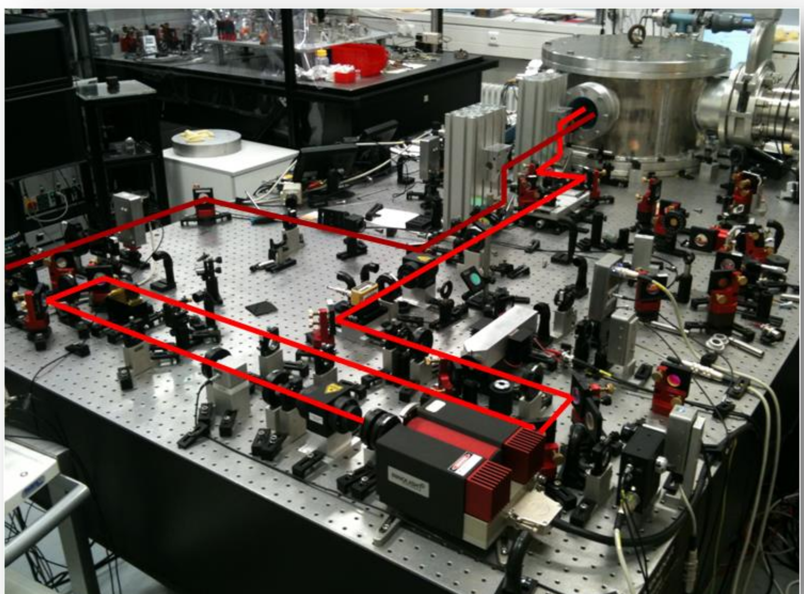
Anomalous dynamic back-action in MSI

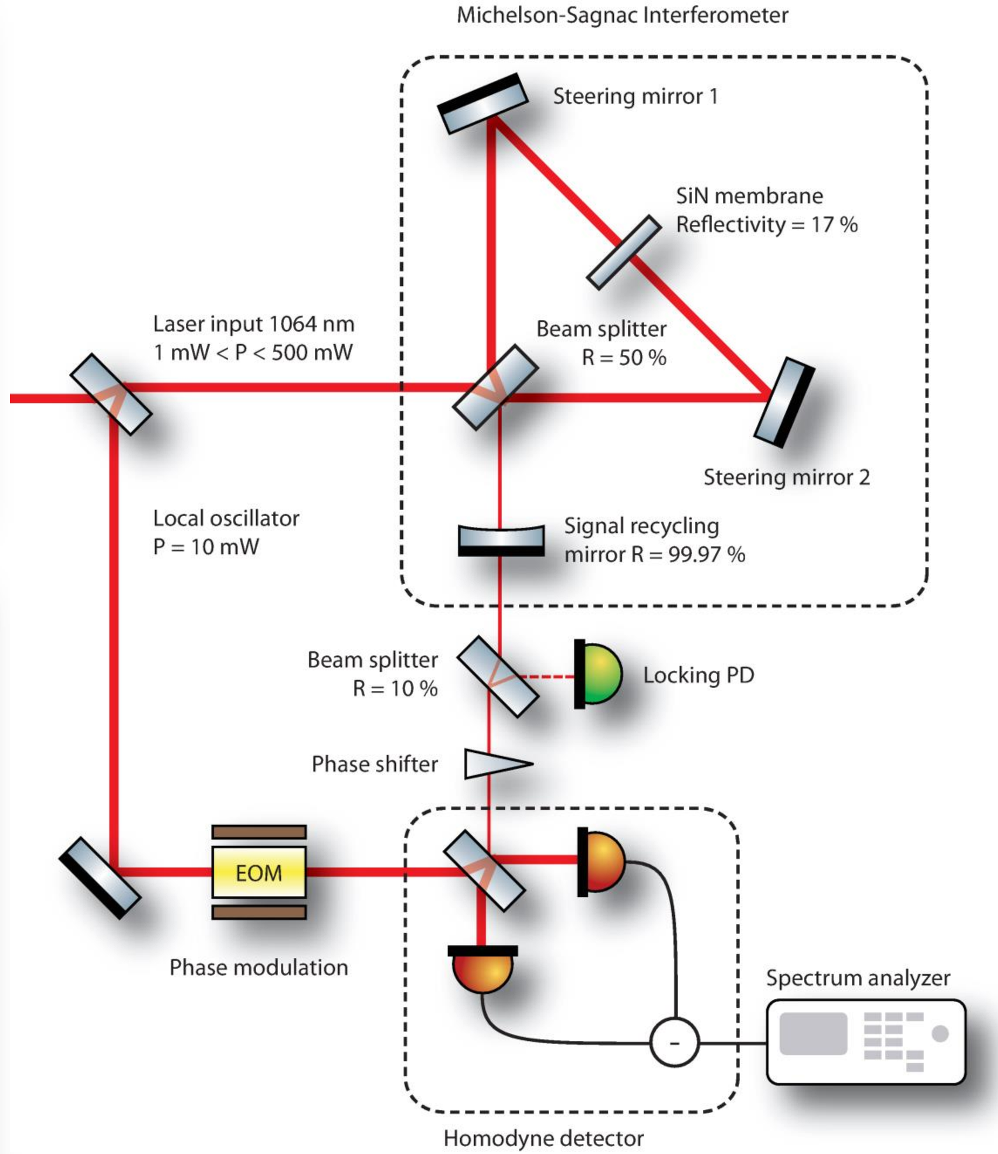
- Optical spring (usually small):
  - Non-zero on resonance – shift of mechanical freq. on resonance,
  - Two positive and negative regions.
- Optical damping:
  - Non-zero on resonance – cooling/heating of the membrane on resonance,
  - Two stable and unstable regions – new regimes of cooling/heating.
- Line shapes of spring and damping can be tuned via several parameters:
  - Offset from dark fringe,
  - SR transmittance,
  - Membrane transmittance,
  - Beamsplitter imbalance.



Membrane experiment @ AEI







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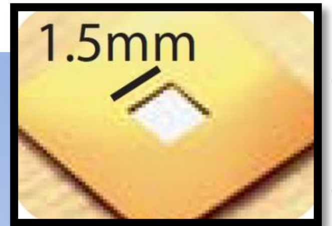
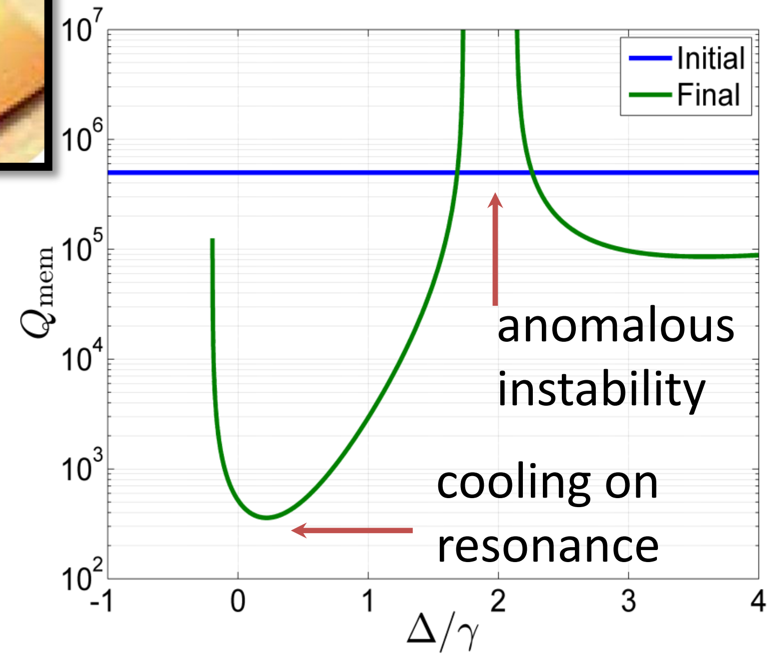
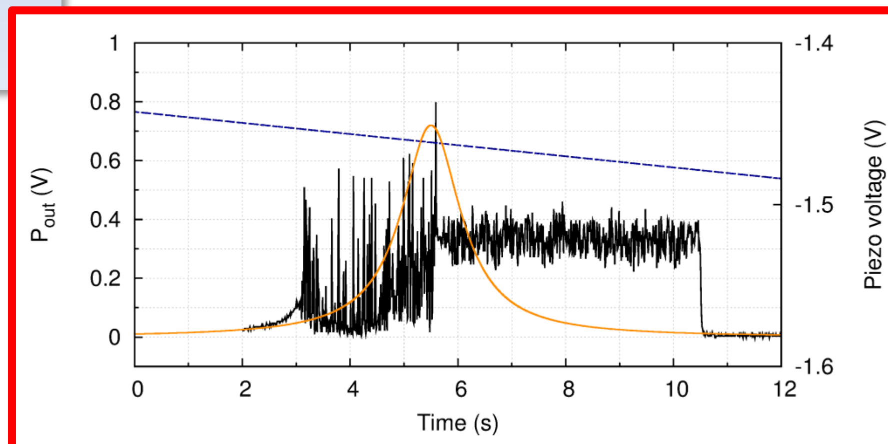


Parameters:

- Cavity length $\mathcal{L} = 8.7$ cm
- Carrier wavelength $\lambda_0 = 1064$ nm
- Input power $P_{in} = 1 \div 500$ mW
- Membrane power reflect. $R_m^2 = 0.17$
- SRM power transmit. $T_{SR}^2 = 3 \times 10^{-4}$
- Optical losses $3 \div 5 \times 10^{-3}$
- Membrane mass $m = 80$ ng
- Fundamental freq. $f_m = 130$ kHz
- Intrinsic Q-factor $Q = 5 \times 10^5$



- Anomalous instability region confirmed.
- Limit cycle observed.
- Cooling on resonance?



Summary and conclusions

- Interferometers operated off dark port feature:
  - Anomalous dynamic back-action and violation of scaling law,
  - Strong dissipative coupling in the sense of cavity optomechanics.
- For optomechanics with micro- and nano-mechanical oscillators:
  - Optical damping acquires non-zero value on resonance – cooling/heating of the oscillator on resonance,
  - Additional stability/instability region – another regime of cooling/heating,
  - Shift of the mechanical frequency on resonance.
- For gravitational-wave detectors:
  - Intersecting regions of positive/negative values of optical spring and damping. Problem of control in DC-readout schemes?
  - Stable optical spring for a single carrier (some changes in topology needed),
  - Optical inertia with a single carrier? (to be explored),
- Power recycling and arm cavities? (under investigation),
- QND games? (to be explored),
- Emergence of anomalous instability confirmed experimentally,
- Available at: arXiv:1212.6242 [quant-ph].

Thank you for your attention!